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Design Procedures for Fiber Composite Structural Components: Rods, Columns, and Beam Columns

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DESIGN PROCEDURES FOR FIBFR COMPOSITE STRUCTURAL COMPONENTS:

RODS, COLUMNS, AND BEAM COLUMNS

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ABSTRACT

Step-by-step procedures are described which can be used to design structural components (rods, columns, and beam columns) subjected to steady state mechanical loads and hygrothermal environments. Illustrative examples are presented for structural components designed for static tensile and compressive loads, and fatigue as well as for moisture and temperature effects. Each example is set up as a sample design illustrating the detailed steps that can be used to design similar components.

1.0 INTRODUCTION

The design of fiber composite structural components requires analysis methods and procedures which relate the structural response of the structural component to the specified loading and environmental conditions. Subsequently, the structural response is compared to given design criteria for strength, displacement, buckling, vibration frequencies etc. in order to ascertain that the component will perform satisfactorily.

Though there are several recent books on composite mechanics available (refs. 1-6), none of these books cover design procedures in sufficient detail to be used for designing fiber composite structural components. Herein sample designs are presented in step-by-step detail to illustrate procedures for designing structural components such as rods, columns and beam columns as well as other similar components. This is accomplished by assuming a cross-section for the component and then checking to verify that it meets all the specified design requirements. In this respect the section selected is not unique. In describing the sample designs, it is assumed that the reader has some familiarity with mechanics of materials and fiber composites. The data used in the sample designs are typical for the composites summarized in Table 1. For other designs, comparable properties need to be used. Allowable stress as used herein denotes fracture stress. The safety factor is included in the specified load or in the fatigue stress.

The specific sample designs include hanger rods, columns and beam columns. The loading conditions include static and cyclic loads and hygrothermal (moisture and temperature) environments. Limiting design requirements considered include, stresses, displacements, fatigue life, combined fatigue with static stresses, creep, buckling and frequencies. The numerical calculations are rounded to three significant figures, in general.

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The notation used is defined in each sample design and summarized under NOTATION. Some repetition is unavoidable for the sake of keeping each sample design as self-contained as possible. The sections, sample designs (SD) and steps are numbered for ease of reference. The concepts and most of the equations used are from references 7 to 13 which provide general background information, appropriate description, justification and/or correlation with experimental data.

Collectively, these sample designs provide illustrative examples which are described for the first time.

2.0 COMPOSITE HANGER RODS

Hanger rods are structural components usually with circular cross section designed to support axial tensile force (Fig. 1a). The tensile force (loading condition) can be: (1) Static; (2) Static with superimposed axial tensile fatigue; and (3) Creep (under sustained tensile load). We will present sample designs for each of these loading conditions.

SAMPLE DESIGN 2.1

structural component:	hanger rod with circular cross-section, 2 ft long
specified load component:	50 000 lb axial tension
axial displacement limit:	2-percent of length at design load
safety factor:	2 on the specified load
composite system:	Kevlar-49/epoxy unidirectional composite with 0.54 fiber volume ratio
Design procedure:	Hanger rod designed to meet stress and displacement requirements at design load.
Step 1. Design variables:	rod cross-section area
Step 2. Design load (P_d):	safety factor times specified load, or 2 x 50 000 lb = 100 000 lb
Step 3. Composite Material longitudinal properties (Table 1):	Modulus $E_{\ell 11}$ = 12.2 mpsi; Strength $S_{\ell 11T}$ = 172 000 psi

Step 4. Rod cross-section area

A_c = design load/tensile strength ($P_d/S_{\ell 11T}$)
A_c = 100 000 lb/172 000 lb/sq in.
A_c = 0.581 sq in.

Step 5. Rod diameter

$$d_c = (4A_c/\pi)^{1/2}$$
$$d_c = [4(0.581 \text{ sq in.})/\pi]^{1/2}$$
$$d_c = 0.861 \text{ in.}$$
take $d_c = 7/8$ in. diam

Step 6. Check displacement limit. The rod displacement at design load is

$$U_c = \text{design load/rod axial stiffness, } P_d/K_a = \frac{P_d l_c}{E_{\ell 11} A_c}$$

$$K_a = E_{\ell 11} A_c / l_c$$

$$K_a = 12\ 200\ 000 \text{ (lb/sq in.)} \times \frac{0.601 \text{ sq in.}}{24 \text{ in.}}$$

$$K_a \approx 305\ 508 \text{ lb/in.}$$
$$u_c = 100\ 000 \text{ lb}/305\ 508 \text{ lb/in.}$$
$$u_c = 0.327 \text{ in.}$$
$$u_c < 2 \text{ percent } l_c$$
$$0.327 \text{ in.} < 0.02 \times 24 \text{ in.}$$
$$0.327 \text{ in.} < 0.48 \text{ in.}$$

Therefore, the designed composite rod is 7/8 in. diameter Kevlar 49/epoxy unidirectional composite which and satisfies both stress and displacement requirements at design load. The rod would weigh about 0.84 lb (based on $\rho = 0.058$ lb/in^3).

SAMPLE DESIGN 2.2

structural component:	hanger rod with circular cross-section, 2 ft long (same as SD2.1)
specified load:	cyclic axial tensile load with maximum amplitude of 20 000 lb
axial displacement limit:	1 percent at maximum amplitude
safety factor:	2 on maximum fatigue stress amplitude
rod fatigue life:	survive 10 000 000 cycles
composite system:	Kevlar 49/epoxy unidirectional composite with 0.54 fiber volume ratio (same as SD2.1)
Design procedure:	Hanger rod designed to meet fatigue life and displacement limit at design maximum load amplitude
Step 1. Design variables:	rod cross-section area

Step 2. Design load: equal to specified cyclic load (safety factor applied to fatigue stress allowable)

Step 3. Composite material longitudinal properties (Table 1): modulus, $E_{\ell 11}$ = 12.2 mpsi; strength $S_{\ell 11T}$ = 172 000 lb/sq in.

Step 4. Determine Fatigue

stress allowable for Kevlar composite: (ref. 10)

$$\frac{S_N}{S_{\ell 11T}} = [1.0 - 0.03 \text{ Log } N]$$

S_N = Fatigue stress to be determined

$S_{\ell 11T}$ = Static tensile strength = 172 000 psi

N = Number of cycles = 10 000 000 or (10^7)

S_N = (1.0 - 0.03 Log 10^7) x 172 000 psi

S_N = [(1.0 - 0.03(7)] x 172 000 psi

S_N = 136 000 psi

And the fatigue stress allowable is:

S_{NA} = $S_N/2$ = 136 000 psi/2

S_{NA} = 68 000 psi

Step 5. Rod cross-section area

$A_c = P_{max}/S_{NA}$

A_c = 20 000 lb/68 000 psi

A_c = 0.294 sq in.

Step 6. Rod diameter

$d_c = (4\ A_c/\pi)^{1/2}$

d_c = (4 x 0.294 sq in./$\pi)^{1/2}$

d_c = 0.612 in.

take d_c = 5/8 in. diam.

Step 7. Check displacement limit. The rod maximum displacement will occur at maximum cyclic load amplitude (neglecting damping and inertial effects).

$u_c = P_{max}/K_a$

$K_a = E_{\ell 11}A_c/\ell_c$

$$K_a = 12\ 200\ 000\ \frac{\text{lb}}{\text{sq in.}} \times \frac{0.307 \text{ sq in.}}{24 \text{ in.}}$$

K_a = 156 000 lb/in
u_c = 20 000 lb/156 000 lb/in.
u_c = 0.128 in.
u_c .128 < 1 percent l_c
0.128 in. < 0.01 x 24 in.
0.128 in. < 0.240 in.

Therefore, the designed composite rod is 5/8 in. diameter unidirectional Kevlar - 49/epoxy, and satisfies both the specified fatigue stress and displacement requirements. This composite rod would weigh about one-half lb. The static tensile load capacity of this rod is about 52 800 lb (0.307 sq in. x 172 000 psi) which is about 2.6 times the maximum allowed cyclic load of 20 000 lb for the 10 000 000 cycles. Stated differently, the cyclic load carrying capacity of this composite rod is about one-third of its corresponding static strength for a fatigue life of 10 000 000 cycles at a cyclic load stress of 65 150 psi amplitude and "zero" mean stress.

SAMPLE DESIGN 2.3

structural component:	Guy rod of circular cross-section, 40 ft long
specified load:	10 000 lb axial static tensile force for 10 yr of service and subjected to a cyclic load of 5000 lb at 1/10 cps (H_z)
displacement limit:	0.1 percent creep at 10 yr
safety factor:	2 on axial load and 2 fatigue stress allowable
composite system:	S-glass/epoxy, about 0.72 fiber volume ratio
Design procedure:	Guy rod designed to survive 10 000 lb tensile for 10 yr with a cyclic load of 5000 lb and a maximum creep displacement of 0.1 percent at 10 yr
Step 1. Design variables:	rod cross-section area A
Step 2. Design static load (P_d):	safety factor times axial static tensile load or 2 x 10 000 lb = 20 000 lb
Step 3. Composite material properties, Table 1:	fiber volume ratio = 0.72 density ρ = 0.077 lb/cu in. modulus $E_{\ell 11T}$ = 8.8 mpsi Tensile strength $S_{\ell 11T}$ = 187 000 psi creep parameters (estimates)

n = 0.16; time-dependent modulus time exponent E_t = 200 mpsi (ref. 12)

Step 4. Determine fatigue stress allowable: (ref. 10)

$$\frac{S_N}{S_{\ell 11T}} = [1.0 - 0.10 \text{ Log } N]$$

$$N = \frac{1}{10}\frac{\text{cyc}}{\text{sec}} \times \frac{3600 \text{ sec}}{\text{hr}} \times \frac{24 \text{ hrs}}{\text{day}} \times \frac{365 \text{ days}}{\text{yr}} \times 10 \text{ yr}$$

$$N = 3.154\text{x}10^7 \text{ cycles}$$

$$S_N = [1.0 - 0.10 \log (3.154\text{x}10^7)] \times 187\ 000 \text{ psi}$$

$$S_N = 46\ 700 \text{ psi}$$

And the fatigue stress allowable is:

$$S_{NA} = S_N/2 = 46\ 700 \text{ psi}/2$$
$$S_{NA} = 23\ 400 \text{ psi}$$

Step 5. Rod cross-section area:
The rod cross-section area is determined from the equation of the normalized Goodman Diagram (graphical representation of the combined cyclic stress (σ_{cyc}) and mean design stresses, fig. 2 (ref. 12; also ref. 14, pg. 690).

For this sample design:

$$\frac{\sigma_{cyc}}{S_{NA}} + \frac{\sigma_d}{S_{\ell 11T}} = \frac{P_{cyc}/A_c}{S_{NA}} + \frac{P_d/A_c}{S_{\ell 11T}}$$

and $$A_c = \frac{P_{cyc}}{S_{NA}} + \frac{P_d}{S_{\ell 11T}}$$

where: P_{cyc} = cyclic load = 5000 lb

P_d = design load = 20 000 lb
S_{NA} = the fatigue stress allowable = 23 400 psi
$S_{\ell 11T}$ = the static tensile strength = 187 000 psi

$$A_c = \frac{5000 \text{ lb}}{23\ 400 \text{ (lb/sq in.)}} + \frac{20\ 000 \text{ lb}}{187\ 000 \text{ (lb/sq in.)}}$$

$$A_c = 0.213 \text{ sq in} + 0.107 \text{ sq in}$$
$$A_c = 0.320 \text{ sq in}$$

Step 6. Guy rod diameter

$$d_c = (4A_c/\pi)^{1/2}$$
$$d_c = [4 \times (0.320 \text{ sq in.})/\pi]^{1/2}$$
$$d_c = 0.638 \text{ in.}$$

Take d_c = 3/4 in. diam.

Step 7. Check the creep displacement at the end of 10 yr assuming that only the design static load contributes to creep. The creep displacement is given by:

$u_{cr} = \varepsilon_{cr} l_c$

ε_{cr} = creep strain at the end of 10 yr under the sustained stress

l_c = the composite guy rod length in inches

The creep strain is approximately given by:

$$\varepsilon_{cr} = \frac{\sigma_d}{E_o} \left(1 + t^n \frac{E_o}{E_t}\right)$$

σ_d = Sustained axial stress

$$\sigma_c = \frac{20\ 000 \text{ lb}}{(\pi/4)(3/4 \text{ in.})^2}$$

σ_d = 45 300 psi

E_o = $E_{\ell 11}$ = 8 800 000 psi

t = time in yr

n = 0.16

E_t = time dependent modulus

E_t = 200 000 000 psi

$$\varepsilon_{cr} = \frac{45\ 300 \text{ lb}}{\text{sq in.}} \times \frac{\text{sq in.}}{8\ 800\ 000 \text{ lb}} \left[1 + 10^{0.16} \left(\frac{8\ 800\ 000}{200\ 000\ 000}\right)\right]$$

ε_{cr} = 0.00548 in./in.

And the corresponding creep displacement is:

$$u_{cr} = \varepsilon_{cr} l_c$$
$$= 0.00548 \text{ (in/in)} \times 40 \text{ ft} \times \frac{12 \text{ in}}{\text{ft}}$$

u_{cr} = 2.63 in

$u_{cr} \overset{?}{<} 1 \text{ percent } l_c$

2.63 in $\overset{?}{<}$ 0.01 x (40 ft x 12 in/ft)

2.63 in < 4.80 in.

Therefore, the designed composite guy rod is 3/4 in. diameter S-glass/epoxy unidirectional composite and satisfies the specified static axial load, the cyclic load and the creep displacement. This guy rod would weigh about 16 lb (density x volume). The static fracture load of the composite guy rod is about 82 600 lb which is about four times the design static load.

It is important to note that creep calculations described here were based on Findley's simplified equation and estimated material property time constants N and E_t (ref. 12). These calculations are to be used only as a guide. Appropriate creep tests must be conducted for evaluating actual case final designs.

Another important point to note is that the sustained stress was assumed to be the stress due to the design static load. For this sample design, the design static load is 20 000 lb which is 33 percent greater than the combined axial static load (10 000 lb) and maximum cyclic load (5000 lb), and provides a conservative estimate on the 10-year creep displacement.

3.0 COMPOSITE COLUMNS AND BEAM COLUMNS

Tubular structural components are frequently used to support axial compressive forces (Fig. 1b). The compressive forces (loading conditions) can be (1) static; (2) static with superimposed compression-compression fatigue; and (3) tension-compression (bending) fatigue combined with axial static load. We will describe sample designs for each of these loading conditions.

SAMPLE DESIGN 3.1

structural component:	thin tubular member 1.5 ft long and 3 in diam max.
specified load:	100 000 lb axial compression
axial displacement limit:	2 percent of length at design load
safety factor:	2 on specified load
composite system:	graphite-fiber/epoxy unidirectional with 0.60 fiber volume ratio
Design procedure:	Tubular compression component designed to meet (1) stress, (2) displacement, and (3) buckling requirement at design load
Step 1. Design Variables:	(1) tube diameter, (2) tube thickness, (3) specific composite system
Step 2. Design load (P_d):	safety factor times specified load, or 2 x 100 000 lb = 200 000 lb
Step 3. Composite Material longitudinal properties	select AS graphite fiber/epoxy (AS/E) as a trial composite material.

(Table 1): Modulus: $E_{\ell 11} = 16.0$ mpsi;

$E_{\ell 22} = 2.2$ mpsi;

$S_{\ell 11C} = 180\ 000$ psi;

density $\rho_\ell = 0.060$ lb/cu in.

Step 4. Tube cross section area

A_c = design load/compression strength $(P_d/S_{\ell 11C})$

$A_c = 200\ 000$ lb/180 000 lb/sq in.

$A_c = 1.111$ sq in.

Step 5. Tube diameter and thickness

$A_c = \pi t\ (d_o - t)$

take $t = 1/8$ in

then $d_o = A_c/\pi^t + t$

$$= \frac{1.111 \text{ sq in}}{\pi \times 0.125 \text{ in}} + 0.125 \text{ in.}$$

$d_o = 2.954$ in. or

take $d_o = 3.00$ in. which is the maximum allowed

Step 6. Check displacement at design load

$u_c = P_d/K_a$

$K_a = E_{\ell 11}\ A_c/\ell_c$

$E_{\ell 11} = 16\text{x}10^6$ psi

$A_c = \pi\ t\ (d_o - t)$

$= \pi\ (0.125 \text{ in})\ (3.00 - 0.125)$ in.

$A_c = 1.129$ sq in

$\ell_c = 1.5 \times 12$ in. $= 18$ in.

$$K_a = 16\text{x}10^6 \text{ (lb/sq in.) } \frac{1.129 \text{ sq in.}}{18 \text{ in.}}$$

$K_a = 1\ 004\ 000$ lb/in.

$$u_c = \frac{200\ 000 \text{ lb}}{1\ 004\ 000 \text{ (lb/in.)}}$$

$u_c = 0.199$ in.

$u_c < 2$ percent ℓ_c

0.199 in. < 2 percent 18 in.

0.199 in. < 0.36 in. O.K.

Step 7. Check tube buckling load (assume pinned ends (ref. 14, pg. 570))

$$P_b = \pi^2 E_{\ell 11} \; I_c / \ell_c^2$$

$$E_{\ell 11} = 16 \times 10^6 \text{ psi}$$

$$I_c = \pi \, (d_o^4 - d_i^4)/64$$

$$d_o = 3.00 \text{ in.}$$

$$d_i = 3.00 - 2 \times 0.125 = 2.75 \text{ in.}$$

$$I_c = \frac{\pi}{64} (3.00^4 - 2.75^4) \text{ in}^4 = 1.169 \text{ in}^4$$

$$\ell_c = 1.5 \times 12 \text{ in.} = 18 \text{ in.}$$

$$P_b = \frac{\pi^2 (16 \times 10^6 \text{psi}) \times 1.169 \text{ in}^4}{18^2 \text{ sq in.}}$$

$$P_b = 570\,000 \text{ lb}$$

$$P_d < P_b$$

200 000 lb < 570 000 lb
200 000 lb < 570 000 lb

Therefore, the designed composite thin tube is 3.00 in. outside diam. with 1/8 in. wall thickness. It satisfies the stress and displacement design requirements at design load and has about 300 percent of the buckling load design requirement. The tube would weigh about 1.22 lb.

SAMPLE DESIGN 3.2:

structural component:	tubular column, 3 ft long, 10 in. maximum outside diameter
specified load:	(1) 10 000 lb static compression (2) 5000 lb maximum amplitude cyclic compression - compression load.
axial displacement limit:	0.5 percent of column length at design load and maximum amplitude of cyclic load
safety factors:	2 for static load 2 on fatigue stress allowable
column service life:	1 000 000 cycles
composite system:	T300-graphite fiber/epoxy unidirectional composite with 0.70 fiber volume ratio

design procedure: Tubular column designed to meet combined static and cyclic stress, displacement and buckling requirements

Step 1. Design variables: (1) tube diameter, and (2) tube wall thickness

Step 2. Design static load (P_d): safety factor for static load times specified load 2 x 10 000 lb = 20 000 lb

Step 3. Composite Material longitudinal properties (Table 1):

$E_{\ell 11}$ = 26.3 mpsi
$S_{\ell 11C}$ = 247 000 psi
ρ_ℓ = 0.058 lb/cu in.

Step 4. Determine compression fatigue stress allowable: $S_N = [1.0 - 0.1 \text{ Log } N] \times S_{\ell 11C}$

$S_N = [1.0 - 0.1 \text{ Log } (1.0 \times 10^6)] \times 247\,000$ psi
$S_N = 98\,800$ psi

And the compression fatigue stress allowable is the fatigue stress divided by the safety factor or:

$S_{NA} = S_N/2 = 98\,800/2$ psi
$S_{NA} = 49\,400$ psi

Step 5. Column cross-section area:

P_{cyc} = cyclic load = 5000 lb
P_d = design load = 20 000 lb
S_{NA} = fatigue stress allowable = 49 400 psi
$S_{\ell 11C}$ = compression strength = 247 000 psi

$$A_c = \frac{P_{cyc}}{S_{NA}} + \frac{P_d}{S_{\ell 11C}}$$

$$A_c = \frac{5000 \text{ lb}}{49\,400 \text{ (lb/sq in.)}} + \frac{20\,000 \text{ lb}}{247\,000 \text{ (lb/sq in.)}}$$

A_c = 0.101 sq in. + 0.081 sq in.
A_c = 0.182 sq in.

Step 6. Tubular column diameter and thickness

$A_c = \pi t(d_o - t)$
d_o = tube outer diameter
t = tube wall thickness

Assume t = 1/8 in.

$$d_o = \frac{A_c}{\pi t} + t$$

$$d_o = \frac{0.182 \text{ sq. in.}}{\pi \times 0.125 \text{ in.}} + 0.125 \text{ in.}$$

$$d_o = 0.588 \text{ in.}$$

The tube cross-section area is adjusted to 5/8 in outer diameter with 1/8 in. wall thickness.

Step 7. Check displacement at combined design load with maximum cyclic load

$(P_d + P_{cyc})$

$u_c = (P_d + P_{cyc})/K_a$

$K_a = E_{\ell 11} A_c / \ell_c$

$P_d + Pcyc = 20\,000 + 5000 = 25\,000$ lb

$E_{\ell 11} = 26.3$ mpsi

$A_c = \pi\ (d_o^2 - d_i^2)/4$

$= \pi(0.625^2 - 0.375^2)/4$

$A_c = 0.196$ sq. in.

$\ell_c = 3$ ft x 12 in./ft = 36 in.

$K_a = 26\,300\,000$(lb/sq in.) x 0.196 (sq in.)/36 in.

$K_a = 143\,200$ lb/in.

$$u_c = \frac{25\,000 \text{ lb}}{143\,200 \text{ (lb/in.)}} = 0.175 \text{ in.}$$

$U_c \overset{?}{<} 0.5$ percent ℓ_c

0.175 in. $\overset{?}{<}$ 0.005 x 36 in.

0.175 in. < 0.18 in. O.K.

Step 8. Check tube buckling load (assume pinned ends)

$P_b = \pi^2\ E_{\pi 11}\ I_c / \ell_c^2$

$E_{\pi 11} = 26.3$ mpsi

$$I_c = \frac{\pi}{64}\,(d_o^4 - d_i^4)$$

$$I_c = \frac{\pi}{64}\,(0.625^4 - 0.375^4) \text{ in}^4$$

$I_c = 0.00652 \text{ in}^4$

$\ell_c = 36$ in.

$$P_b = \frac{\pi^2 \times 26\ 300\ 00\ (\text{lb/sq in}) \times 0.00652\ \text{in}^4}{36 \times 36\ \text{sq in.}}$$

$P_b = 1310$ lb << 25 000 lb $\quad (P_d + P_{cyc})$

To increase the buckling load to 25 000 lb ($P_{cyc} + P_d$) and greater, I_c must be increased by about 20 times. The easiest way to achieve this increase is to increase the tube outer diameter at least 2.5 times.

Assume d_o = 1.5 in. and t = 0.20 in.

$I_c = \pi/64\ (1.5^4 - 1.1^4)\text{in}^4$
$I_c = 0.177\ \text{in}^4$

$$P_b = \frac{\pi^2 \times 26\ 300\ 00\ (\text{lb/sq in.}) \times 0.177\ \text{in}^4}{36 \times 36\ \text{sq in.}}$$

$P_b = 35\ 500$ lb

$P_b \geq (P_d + P_{cyc})$

35 500 lb > (20 000 + 5000) lb O.K.

Step. 9. Check design load and fatigue stresses. Since the tubular column dimensions were changed, the stresses need be checked again in order to determine the new stress margin.
The design load stress (σ_d) is:

$\sigma_d = P_d/A_c$
$P_d = 20\ 000$ lb
$A_c = \pi t(d_o - t)$
$A_c = \pi \times 0.20\ (1.5 - 0.20)\ \text{in}^2$
$A_c = 0.817$ sq in.
$\sigma_d = 20\ 000$ lb/0.817 sq in.
$\sigma_d = 24\ 500$ lb/sq in.

The fatigue stress (σ_{cyc}) is:

$\sigma_{cyc} = P_{cyc}/A_c$
$\sigma_{cyc} = 5000$ lb/0.817 sq in.
$\sigma_{cyc} = 6100$ lb/sq in.

Check combined stresses (Goodman Diagram)

$$\frac{\sigma_{cyc}}{S_{NA}} + \frac{\sigma_d}{S_{\ell 11C}} \leq 1$$

S_{NA} = 49 400 psi (Step.4)
$S_{\ell 11C}$ = 247 000 psi (material property, Step 3)

$$\frac{6100}{49\ 400} + \frac{24\ 500}{247\ 000} \overset{?}{<} 1.00$$

$$0.123 + 0.0992 \overset{?}{<} 1.00$$

$$0.222 < 1.00 \quad \text{O.K.}$$

Therefore, the designed tubular column is 1.5 in. outer diameter with 0.20 in. wall thickness. The tubular column satisfies all the specified design requirements: (1) less than 10 in. diameter, (2) combined stress, (3) maximum displacement, and (4) buckling load. The static compression fracture load is about 202 000 lb which is more than 8 times the combined design load and maximum cyclic load. Also the fatigue life of the tube is about 4 times greater (1.000 versus 0.222) than the fatigue stress for one million cycles. The column as designed would weigh about 1.7 lb. The tube dimensions of 1.5 in. outer diameter and 0.20 in. wall thickness are relatively small and amenable to the pultrusion fabrication process. The critical design requirement is the buckling load which is controlled by the tube bending stiffness $E_{\ell 11}I_c$. It is generally the case that structural members designed to meet buckling (elastic stability) design requirements satisfy other design requirements with wide margins. It is possible to obtain a more economical design by selecting a tubular column with a larger diameter and a smaller wall thickness. In this approach the tube would need to be sized to meet local buckling and column (Euler) buckling simultaneously.

SAMPLE DESIGN 3.3:

structural component:	thin tubular beam column 5 ft. long and 5 in. maximum outer diameter (fig. 1c).
specified load:	1) 20 000 lb axial compression 2) 10 000 lb cyclic bending load applied at the center at 3 cycles per second (Hz)
design specified limit requirements:	1) static design load stresses (less than allowable) 2) fatigue stresses (less than allowables) 3) buckling load (greater than design load) 4) midspan maximum displacement (7 percent of length) 5) fundamental frequency (5 times cyclic load frequency)
composite system:	T300-graphite fiber/epoxy unidirectional composite with 0.70 fiber volume ratio
safety factors:	1) 2 on axial compression load 2) 2 on fatigue stress allowable

Design procedure: Tubular beam column designed to meet design specified load and limit requirements

Step 1. Design variables:
1) tube diameter
2) tube wall thickness

Step 2. Design static load(P_d): safety factor for static load times specified load or $P_d = 2 \times 20\ 000$ lb
$P_d = 40\ 000$ lb

Step 3. Composite material longitudinal properties (Table 1):

$E_{\ell 11} = 26.3$ mpsi
$S_{\ell 11T} = 218\ 000$ psi
$S_{\ell 11C} = 247\ 000$ psi
$S_{\ell 12s} = 9800$ psi
$S_{SB} = 14\ 000$ psi ($S_{SB} = 1.5\ S_{\ell 12s}$)
$\rho_\ell = 0.058$ lb/cu in.

Step 4. Fatigue stress allowables:
1) tension fatigue $S_{NT} = [1.0-0.02 \text{ Log } N] \times S_{\ell 11T}$
2) compression fatigue $S_{NC} = [1.0 - 0.10 \text{ Log } N] \times S_{\ell 11C}$

Step 4a. Number of cycles (N) for 10 000 hr service life

$$N = \frac{3 \text{ cyc}}{\text{sec}} \times \frac{3600 \text{ sec}}{\text{hr}} \times 10\ 000 \text{ hrs}$$

$N = 108$ million cycles (1.08×10^8)

Step 4b.
$S_{NT} = [1.0 - 0.02 \log N] S_{\ell 11T}$
$S_{NT} = [1.0 - 0.02 \log (1.08 \times 10^8)] \times 218\ 000$ psi
$S_{NT} = 183\ 000$ psi
$S_{NTA} = S_{NT}/2$ (2 is the safety factor on fatigue stress)
$S_{NTA} = 183\ 000$ psi $/2$
$S_{NTA} = 91\ 500$ psi

Step 4c.
$S_{NC} = [1.0 - 0.10 \log N]\ S_{\ell 11C}$
$S_{NC} = [1.0 - 0.10 \log (1.08 \times 10^8)] \times 247\ 000$ psi
$S_{NC} = 48\ 600$ psi
$S_{NCA} = S_{NC}/2$ (2 is the safety factor on fatigue stress)
$S_{NCA} = 48\ 600$ psi$/2$
$S_{NCA} = 24\ 300$ psi

Step 5. Beam column cross-section area. Since the compression fatigue stress allowables are relatively low, assume that this controls the design. The cross-section area for static compression with cyclic compression can be expressed as:

$$A_c = \frac{P_d}{S_{\ell 11C}} + \frac{M_{cyc}C}{S_{NCA}(I_c/A_c)}$$

P_d = 40 000 lb

M_{cyc} = $P_{cyc}\, l_c/4$ (maximum moment at center of beam column, fig. C)

M_{cyc} = 10 000 lb x 60 in./4

M_{cyc} = 150 000 lb in.

$S_{\ell 11C}$ = 247 000 psi

S_{NCA} = 24 300 psi

$C = d_o/2$

$$I_c = \pi\,(d_o^4 - d_i^4)/64$$

$$A_c = \pi\,(d_o^2 - d_i^2)/4$$

$$A_c = \pi\,(d_o - t)$$

First trial - Assume that d_o = 5 in. and t = 0.50 in., and solve for A_c by trial and success

$A_c = \pi$ x 0.50 in. (5.0-0.5) in.
A_c = 7.07 sq in.
$I_c = \pi\,(5.00^4 - 4.00^4)\ \text{in}^4/64$
$I_c = 18.1\ \text{in}^4$
C = 5.00 in./2
C = 2.5 in.

$$7.07\ \text{in}^2 \overset{?}{=} \frac{40\ 000\ \text{lb}}{247\ 000\ \text{psi}} + \frac{150\ 000\ \text{lb in. x } 2.5\ \text{in.}}{24\ 300\ \text{psi x } (18.1\ \text{in}^4/7.07\ \text{in}^2)}$$

$7.07\ \text{in}^2 \overset{?}{=} 0.162\ \text{in}^2 + 6.03\ \text{in}^2$
$7.07\ \text{in}^2 > 6.19\ \text{in}^2$
second trial-assume d_o = 5.00 in.

$$A_c = 6.19 = \pi\,(d_o^2 - d_i^2)/4$$

$$d_i^2 = d_o^2 - \frac{4 \times 6.19}{\pi} = 5.00^2\ \text{sq in.} - \frac{(4 \times 6.19)}{\pi}\ \text{sq in.}$$

d_i = 4.14 in.

$$I_c = \pi\ (d_o^4 - d_i^4/64$$

$$I_c = \frac{\pi}{64}\ (5.00^4 - 4.14^4)\ in^4$$

$I_c = 16.3\ in^4$
$I_c/A_c = 16.3\ in^4/6.19\ in^2 = 2.63\ in^2$

$$6.19\ in^2 = \frac{40\ 000\ lb}{247\ 000\ psi} + \frac{150\ 000\ lb\ in\ x\ 2.5\ in.}{24\ 300\ psi\ x\ (16.3\ in^4/6.19\ in^2)}$$

$6.19\ in^2 = (0.162 + 5.86)\ in^2$
$6.19\ in^2 = 6.02\ in^2$

Third trial - decide on 5.00 in outside diameter with 0.45 in wall thickness. For these dimensions:

$A_c = \pi t\ (d_o - t)$
$= \pi\ (0.45\ in)\ (5.00\ in - 0.45\ in)$
$A_c = 6.43\ in^2$

$$I_c = \pi\ (d_o^4 - d_i^4)/64$$

$$I_c = \pi\ (5.00^4 - 4.10^4)\ in^4/64$$

$I_c = 16.8\ in^4$
$I_c/A_c = 16.8\ in^4/6.43\ in^2 = 2.61\ in^2$

Step 6. Check stresses:

Step 6a. The maximum compression stress is:

$$\sigma_c = -\frac{P_d}{A_c} - \frac{M_{cyc}C}{I_c}$$

$P_d = 40\ 000\ lb$
$M_{cyc} = 150\ 000\ lb\ in.$
$A_c = 6.43\ in^2$
$I_c = 16.8\ in^4$
$C = 2.50\ in.$

$$\sigma_c = -\frac{40\ 000\ lb}{6.43\ in^2} - \frac{150\ 000\ lb\ in.\ x\ 2.5\ in.}{16.8\ in^4}$$

σ_c = -28 500 psi or 28 500 psi compression and the margin of safety (MOS) IS:

$$MOS = S_{\ell 11C}/\sigma_c - 1.000$$

$$MOS = \frac{247\ 000\ psi}{28\ 500\ psi} - 1.00 = 7.67 \quad O.K.$$

Step 6b. The maximum tensile stress is:

$$\sigma_t = -\frac{P_d}{A_c} + \frac{M_{cyc}C}{I_c}$$

$$\sigma_c = -\frac{40\ 000\ lb}{6.43\ in^2} + \frac{150\ 000\ lb\ in \times 2.50\ in.}{16.8\ in^4}$$

$$\sigma_t = 16\ 100\ psi$$

$$MOS = \frac{218\ 000\ psi}{16\ 100\ psi} - 1.00 = 12.5 \quad O.K.$$

Step 6c. The maximum interlaminar shear stress is at the beam column center (ref. 14, pg. 349).

$$\sigma_s = 2.0\ (Pcyc/2)/A_c$$

$$\sigma_s = \frac{2.0 \times 10\ 000\ lb}{2 \times 6.43\ in^2}$$

$$\sigma_s = 1560\ psi$$

and $MOS = \frac{14\ 000\ psi}{1560\ psi} - 1.000 = 7.97 \quad O.K.$

Step 7. Check fatigue stresses:

Step 7a. Static compression with cyclic compression

$$1.000 \geq \frac{\sigma_c}{S_{\ell 11C}} + \frac{\sigma_{cyc}}{S_{NCA}}$$

$$1.000 \geq \frac{P_d/A_c}{S_{\ell 11C}} + \frac{M_{cyc}C/I_c}{S_{NCA}}$$

$1.000 \overset{?}{>} 6200\ psi/247\ 000\ psi + 22\ 300\ psi/24\ 300\ psi$
$1.000 > 0.025 + 0.919 = 0.944$ O.K.

Step 7b. Cyclic tension only (tensile part of the bending cycle)

$$1.000 \overset{?}{>} \frac{M_{cyc}C/I_c}{S_{NTA}} - \frac{P_d/A_c}{S_{\ell 11T}}$$

$$1.000 \overset{?}{>} \frac{22\ 300\ psi}{91\ 500\ psi} - \frac{6200\ psi}{218\ 000\ psi}$$

$1.000 > 0.244 - 0.028 = 0.216$ O.K.

Step 8. Check buckling load:

The buckling load of this beam column assuming pinned ends is:

$$P_b = \pi^2 E_{\ell 11}\ Ic/\ell_c^2$$

$E_{\ell 11} = 26\ 300\ 000$ psi
$I_c = 16.8\ in^4$
$L_c = 60$ in.

$$P_b = \frac{\pi^2 \times 26\ 300\ 000\ lb/in^2 \times 16.8\ in^4}{60 \times 60\ in^2}$$

$P_b = 1\ 210\ 000$ lb

$P_b > P_d + Pcyc$

$1\ 210\ 000\ lb > (40\ 000 + 10\ 000)\ lb = 50\ 000\ lb$

and $MOS = \frac{1\ 210\ 000\ lb}{50\ 000\ lb} - 1 = 23.2$ O.K.

Step 9. Check maximum displacement:
The maximum displacement at the beam column (fig. 1C) midspan is given by (ref. 15, pg. 5):

$$w_{max} = \frac{P_{cyc}}{2P_d{}^2}\left[\tan \frac{\lambda \ell_c}{2} - \frac{\lambda \ell_c}{2}\right]$$

Pcyc = 10 000 lb maximum cyclic load
P_d = 40 000 lb axial compressive design load

$\lambda = [P_d/E_{\ell 11}\ I_c]^{1/2}$

$E_{\ell 11} = 26.3$ mpsi
$I_c = 16.8\ in^4$

$$\lambda = \left[\frac{40\ 000\ lb}{26.3x10^6\ lb/in^2 \times 16.8\ in^4}\right]^{1/2}$$

$\lambda = 0.00951/in.$

$$w_{max} = \frac{10\ 000\ lb}{2 \times 40\ 000\ lb \times 0.00951/in.} \times [\tan \frac{0.00951}{in.} \times \frac{60\ in.}{2} - \frac{0.00951}{in.} \times \frac{60\ in.}{2}$$

$w_{max} = -3.68$ in

$|-3.68|$ in < 7 percent 60 in. ($|\ |$ absolute value sign)

3.68 in. < 4.2 in. O.K.

Note this relatively large quasi-static mid-span displacement is acceptable since (1) it reaches this magnitude at short times, and (2) the inertial properties were not accounted for in the calculations. In the absence of the axial load, w_{max} 0.101 in which is relatively insignificant.

Step 10. Check fundamental frequency:
The fundamental frequency (f) for a beam column with axial compression load is given by (ref. 16, pg. 455):

$$\left[f = \frac{\pi^2}{l_c^2}\ \frac{E_{\ell 11}\quad I_c}{\rho_c A_c}\right]^{1/2} \times \left[1 - \frac{{}_dP_c{}^2}{\pi^2 E_{\ell 11}\ I_c}\right]^{1/2}$$

$l_c = 60$ in
$E_{\ell 11} = 26.3$ mpsi
$I_c = 16.8\ in^4$

$$\rho_c = \rho_e/g = \frac{0.058\ lb}{in^3} \times \frac{1.0}{32.2\ (ft/sec^2) \times 12(in/ft)} = \frac{1.50 \times 10\quad lb^{-4}sec^2}{in^4}$$

$A_c = 6.43\ in^2$
$P_d = 40\ 000$ lb

$$\left[\frac{E_{\ell 11} I_c}{\rho_c A_c}\right]^{1/2} = \left[\frac{26.3 \times 10^6 \text{lb/in}^2 \times 16.8 \text{ in}^4}{1.5 \times 10^{-4} \text{lb-sec}^2/\text{in}^4 \times 6.43 \text{ in}^2}\right]^{1/2} = 6.77 \times 10^5 \text{in}^2/\text{sec}$$

$$\left[\frac{P_d \ell_c^2}{\pi^2 E_{\ell 11} I_c}\right] = \left[\frac{40\,000 \text{ lb} \times 3600 \text{ in}^2}{\pi^2 \times 26.3 \times 10^6 \text{lb/in}^2 \times 16.8 \text{ in}^4}\right] = 0.033$$

$$f = \left(\frac{\pi^2}{3600 \text{ in}^2}\right) (6.77 \times 10^5 \text{in}^2/\text{sec}) [1 - 0.033]^{1/2}$$

f = 1830 cyc/sec.
1830 cyc/sec >> 3 cyc/sec O.K.

Therefore, the designed tubular beam column is 5.0 in. outer diameter with 0.45 in. wall thickness. This beam column satisfies all the specified design limit requirements: (1) quasi-static stresses, (2) fatigue stresses, (3) buckling load (elastic stability), (4) midspan maximum displacement, and (5) fundamental frequency. This beam column would weigh about 33.4 lb. The diameter of the tube (5.0 in.) and the wall thickness (0.45 in.) are relatively small and amenable to the pultrusion fabrication process. It is interesting to note that compression fatigue stress controlled the design of this beam column. Once this was satisfied, the other specified limit design requirements were satisfied with wide margins of safety. For example, the estimated static fracture loads (ultimate loads) of this beam column are: (1) 1 590 000 lb compression, (2) 1 401 740 lb tension, (3) 1 465 000 lb in bending moment, and (4) 1 210 000 buckling load. The tubular beam column design would be tested for all these conditions as well as fatigue, midspan displacement and frequency in order to verify the design in actual design practice.

4.0 HYGROTHERMAL EFFECTS

The hygrothermal environment (moisture and temperature) affects the composite material properties which are controlled by the resin (ref.7). These properties are: (1) longitudinal compression strength, (2) transverse tension and compression - moduli and strengths, and (3) intralaminar and interlaminar shear-moduli and strengths. Thermal expansion coefficients and moisture expansion coefficients are also affected by the hygrothermal environments (refs. 11 and 13). In the sample design, we only consider the hygrothermal effects on compression strength and compression fatigue.

<u>SAMPLE DESIGN 4.1</u>:

structural component: (that designed in SD3.3)	thin tubular member (beam column) 5 ft long, outer diameter and 0.45 in wall thickness
specified load:	1) 20 000 lb axial compression 2) 10 000 lb cyclic bending load at 3) cycles/sec applied at midspan

service environment: 1) 0.8 percent moisture in the composite
2) 120° F temperature

design specified limit requirements: compression - compression fatigue

composite system safety factors: T300/epoxy unidirectional composites with 0.70 fiber volume ratio/
1) 2 on axial compression
2) 2 on fatigue stress allowable

design procedure: check beam column area sufficiency for the environmental effects and modify as needed.

Step 1. Design variable: wall thickness

Step 2. Design static load: safety factor times specified load:
$2 \times 20\,000$ lb $= 40\,000$ lb $= P_d$

Step 3. Composite material properties at room temperature dry conditions (Table 1):

$E_{\ell 11} = 26.3$ mpsi
$S_{\ell 11C} = 247\,000$ psi
$\rho_\ell = 0.058$ lb/in^3
$T_{GD} = 420^\circ$ F

Step 4. Compression fatigue stress allowable

Step 4a. The compression fatigue stress with hygrothermal effects is given by (Ref. 10)

$$\frac{S_{NC}}{S_{\ell 11C}} = \left[\frac{T_{GW} - T}{T_{GD} - T_o}\right]^{1/2} - 0.1 \log N$$

T_{GW} = the glass transition temperature of the wet composite. If not known, it can be estimated by (ref. 13):

$T_{GW} = (0.005M_\ell - 0.10\ M_\ell + 1.0)\ T_{GD}$

M_ℓ = 0.8 percent moisture (service environment)

$T_{GD} = 420^\circ$ F (glass transition temperature of dry composite)

$T_{GW} = [0.005(0.8) - 0.10(0.8) + 1.0] \times 420^\circ$ F

$T_{GW} = 390^\circ$ F

$T = 120^\circ$ F (service environment)

$T_o = 70^\circ$ F room temperature

$N = 1.08 \times 10^8$ cycles (SD3 step 4a)

Step 4b. Substituting these numerical values in

$$\frac{S_{NC}}{S_{\ell 11C}} = \left[\frac{390^\circ\ F - 120^\circ\ F}{420^\circ\ F - 70^\circ\ F}\right]^{1/2} - 0.1 \log (1.08 \times 10^8)$$

$$\frac{S_{NC}}{S_{\ell 11C}} = 0.075$$

$S_{NC} = 0.075 \times S_{\ell 11C} = 0.075 \times 247{,}000$ psi
$S_{NC} = 18\ 500$ psi
$S_{NCA} = S_{NC}/2 = 18\ 500$ psi/2
$S_{NCA} = 9\ 250$ psi

This compression fatigue stress allowable is about 40 percent of that determined in SD3 Step 4c. This means that the moment of inertia I_c must be changed by about 2.5 times.

Step 4c. The new moment of inertia is changed by changing the outside diameter keeping the d_i at 4.1 in.

$$2.5\ I_c = \frac{\pi}{64} (d_o^4 - d_i^4)$$

$$d_o = [d_i^4 + \frac{64 \times 2.5}{\pi} I_c]^{1/4}$$

$I_c = 16.8\ in^4$ (SD3, Step 5)

$$d_o = [4.1^4 + \frac{64 \times 2.5}{\pi} (16.8)^{1/4} \text{ in.}$$

$d_o = 5.81$ in.
$t = (5.81 - 4.10)$in./2
$t \approx 0.855$ in.

Step 4d. Select 6.0 in. for the outside diameter and 1 in. wall thickness

$d_i = d_o - 2t$
$d_i = 6.0 - 2\ (1.00)$ in.
$d_i = 4.0$ in.

$$I_c = \frac{\pi}{64} (d_o^4 - d_i^4)$$

$$I_c = \frac{\pi}{64} (6.0^4 - 4.0^4) \text{ in}^4$$

$I_c = 51.1\ in^4$

$$A_c = \frac{\pi}{4} (d_o^2 - d_i^2)$$

$$A_c = \frac{\pi}{4}\,(6.0^2 - 4.0^2)\ \text{in}^2$$

$$A_c = 15.7\ \text{in}^2$$

Step 5. Check fatigue stresses at the hygrothermal environmental conditions

$$1.000 \overset{?}{\geq} \frac{\sigma_c}{S_{CHT}} + \frac{\sigma_{cyc}}{S_{NCA}}$$

Step 5a. $\sigma_c = \dfrac{P_d}{A_c} = \dfrac{40\ 000\ \text{lb}}{15.7\ \text{in}^2}$

$$\sigma_c = 2550\ \text{psi}$$

Step 5b. $S_{CHT} = \left[\dfrac{T_{GW} - T}{T_{GD} - T_o}\right]^{1/2} \times S_{\ell 11C}$ (longitudinal compression with hygrothermal effects)

$$S_{CHT} = \left[\frac{390°\ F - 120°\ F}{420°\ F - 70°\ F}\right]^{1/2} \times 247\ 000\ \text{psi}$$

$$S_{CHT} = 217\ 000\ \text{psi}$$

Step 5c.
$\sigma_{cyc} = M_{cyc}/I_c$
$M_{cyc} = 150\ 000$ lb in. (SD3, Step 6a)
$C = 3.00$ in.
$I_c = 51.1\ \text{in}^4$

$$\sigma_{cyc} = \frac{150\ 000\ \text{lb in.} \times 3.00\ \text{in.}}{51.1\ \text{in}^4}$$

$$\sigma_{cyc} = 8810\ \text{psi}$$

Step 5d. Combined condition

$$1.000 \overset{?}{\geq} \frac{\sigma_{cyc}}{S_{CHT}} + \frac{\sigma_{cyc}}{S_{NCA}}$$

$$1.000 \overset{?}{\geq} \frac{2550\ \text{psi}}{217\ 000\ \text{psi}} + \frac{8810\ \text{psi}}{9250\ \text{psi}}$$

$$1.000 \overset{?}{>} 0.0118 + 0.952$$

$$1.000 > 0.964 \quad \text{O.K.}$$

Therefore, the designed tubular beam column is 6.0 in. outside diameter with 1 in. wall thickness. The tubular beam column to satisfy the fatigue stress requirements in the specified hygrothermal environment is considerably larger than that in SD3.3. It would weigh about 55 lb. The result of this sample design is significant in that it indicates that fatigue stresses at very high cycles (10^8) combined with hygrothermal environments are severe design conditions. Actual designs for such conditions need to be based and verified on relevant experimental data. These data may well show that the fatigue degradation coefficient of 0.1 (0.1 log N) may be too severe. For example, a fatigue degradation coefficient of 0.07 will increase the fatigue stress allowable to about 39 000 psi which is about 1.5 times the fatigue stress allowable of 24 300 psi in SD3.3. Step 4c and would result in a considerably lighter tubular beam column. It is important to keep in mind that this sample design was selected to illustrate the steps to account for combined hygrothermal environmental effects with static compression and fatigue. It was also selected to demonstrate that data for composites are needed for very high cycle fatigue (10^8 cycles).

Though the sample designs described were for unidirectional composites, the design steps remain the same for similar components made from angleplied laminates. Laminate properties must be used. These may be obtained by the procedures described in references 8, 9, and 11.

5.0 CONCLUDING REMARKS

Sample designs were worked out in detail for three structural components: (1) hanger rod, (2) tubular column, and (3) tubular beam column. The loading conditions considered in these sample designs include static and cyclic. The environmental conditions included room temperature and hygrothermal 0.8 percent moisture at 120° F. Design limiting requirements considered include: (1) static strength, (2) fatigue, (3) combined static and fatigue in both room temperature and hygrothermal environments, (4) displacements, (5) creep, (6) buckling, and (7) frequencies. The composite materials considered were: (1) Kevlar/epoxy, S-glass/epoxy, AS/epoxy and T300/epoxy. All composites were made from unidirectional materials. The step-by-step design procedures used were selected to illustrate the significant aspects of the design process and to provide samples to be followed for designing more complex components. The composite data used in the various sample designs are typical for the respective composite systems and should be used only for preliminary designs.

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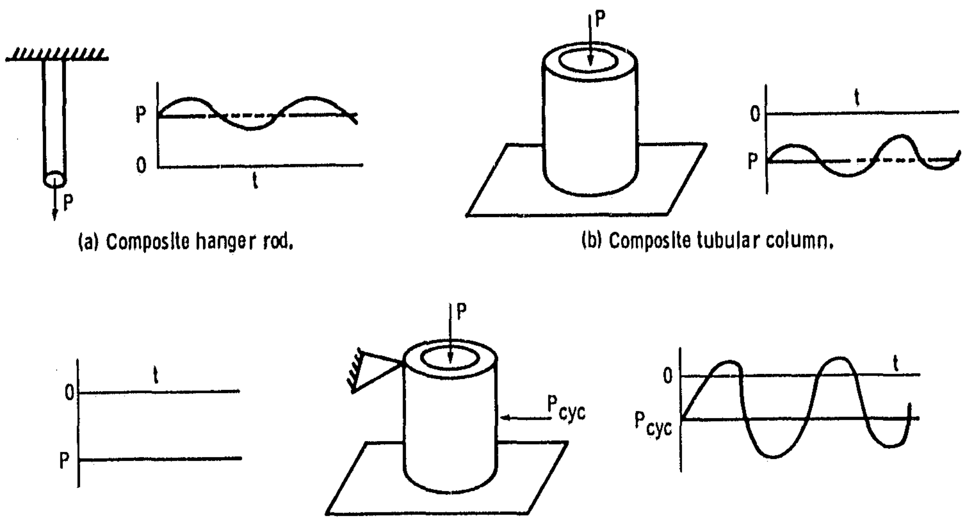
7.0 NOTATION

A_c	structural component cross-section area
AS	AS-graphite fibers
C	distance from reference to outer surface used to calculate bending stresses
d_c	component diameter
d_i	inner diameter
d_o	outer diameter
E_ℓ	unidirectional modulus-numerical subscripts denote direction
E_o	time-independent modulus for creep calculations
E_t	time-dependent modulus for creep calculations
f	frequency
I_c	component bending moment of inertia
K_a	axial stiffness
ℓ_c	component length
M_{cyc}	cyclic moment
M_ℓ	moisture, percent by weight
MOS	margin of safety
N	number of fatigue cycles
n	time exponent for creep calculations
P_b	buckling load
P_{cyc}	cyclic axial load
P_d	design load
P_{max}	maximum load
SD	sample design
$S_{\ell 11T}$	longitudinal tensile fracture stress
$S_{\ell 11C}$	longitudinal compression fracture stress
S_{SB}	interlaminar (short-beam) shear fracture stress
S_N	fatigue stress
S_{NA}	fatigue stress allowable
S_{NC}	compression fatigue stress
S_{NCA}	compression fatigue stress allowable
S_{NT}	tensile fatigue stress
S_{NTA}	tensile fatigue stress allowable

T	use temperature
T_{GD}	glass transition temperature, dry conditions
T_{GW}	glass transition temperature, wet conditions
T_o	reference temperature
T300	Thornel 300 graphite fiber
t	thickness or time
u	axial displacement
u_{cr}	axial creep displacement
w_{max}	maximum lateral displacement
ε_{cr}	creep strain
λ	defined by $\lambda = [P_d/E_{\ell 11} I_c]^{1/2}$
ρ_c	density
σ_c	compressive stress
σ_{cyc}	cyclic stress
σ_d	stress due to design load
σ_t	tensile stress
σ_s	shear stress

TABLE I. – TYPICAL PROPERTIES OF UNIDIRECTIONAL FIBER COMPOSITES AT ROOM TEMPERATURE

Properties	Symbol	Units	Boron/ Epoxy	Boron/ polyimide	Scotch ply/Epoxy	Modmor I/ epoxy	Modmor I/ polyimide	Thornel 300/ epoxy	Kevlar 49/ epoxy	Graphite AS/epoxy
1. Fiber volume ratio	k_f	———	0.50	0.49	0.72	0.45	0.45	0.70	0.54	0.60
2. Density	ρ_ℓ	lb/in^3	0.073	0.072	0.077	0.056	0.056	0.058	0.049	0.057
3. Longitudinal thermal coefficient	$\alpha_{\ell 11}$	10^{-6}in/in/°F	3.4	2.7	2.1	———	0.0	0.01	-1.60	0.40
4. Transverse thermal coefficient	$\alpha_{\ell 22}$	10^{-6}in/in/°F	16.9	15.8	9.3	18.5	14.1	12.5	31.3	16.4
5. Longitudinal modulus	$E_{\ell 11}$	10^6 psi	29.2	32.1	8.8	27.5	31.3	26.3	12.2	16.0
6. Transverse modulus	$E_{\ell 22}$	10^6 psi	3.15	2.1	3.6	1.03	0.72	1.5	0.70	2.2
7. Shear modulus	$G_{\ell 12}$	10^6 psi	0.78	1.11	1.74	0.9	0.65	1.0	0.41	0.72
8. Major Poisson's ratio	$\nu_{\ell 12}$	———	0.17	0.16	0.23	0.10	0.25	0.28	0.32	0.25
9. Minor Poisson's ratio	$\nu_{\ell 21}$	———	0.02	0.02	0.09	———	0.02	0.01	0.02	0.34
10. Longitudinal tensile strength	$S_{\ell 11T}$	psi	199 000	151 000	187 000	122 000	117 000	218 000	172 000	220 000
11. Longitudinal compressive strength	$S_{\ell 11c}$	psi	232 000	158 000	119 000	128 000	94 500	247 000	42 000	180000
12. Transverse tensile strength	$S_{\ell 22T}$	psi	8100	1600	6670	6070	2150	5850	1600	8000
13. Transverse compressive strength	$S_{\ell 22c}$	psi	17 900	9100	23 500	28 500	10 200	35 700	9400	36000
14. Intralaminar shear strength	$S_{\ell 12S}$	psi	9100	3750	6500	8900	3150	9 800	4000	10 000
15. Longitudinal moisture coefficient	$\beta_{\ell 11}$	10^{-2}in/	0.003	0.003	0.014	0.003	0.003	0.006	0.008	0.006
16. Transverse moisture coefficient	$\beta_{\ell 22}$	10^{-2}in/	0.168	0.168	0.128	0.129	0.129	0.129	0.151	0.129



(a) Composite hanger rod.

(b) Composite tubular column.

(c) Composite beam column.

Figure 1. - Schematic of composite structural components.

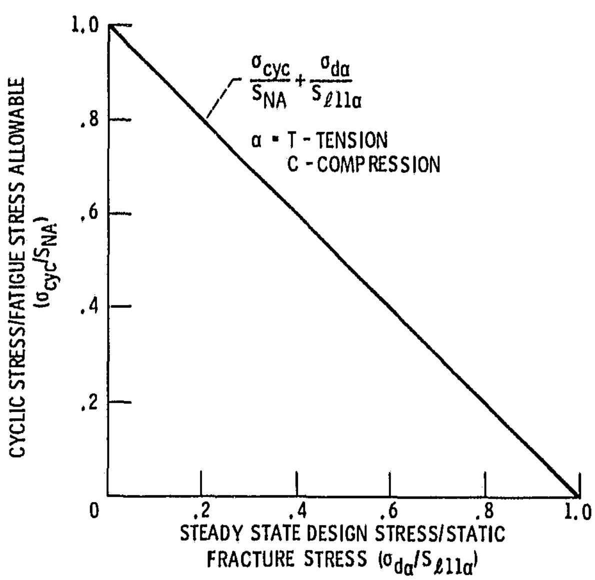


Figure 2. - Normalized Goodman diagram.