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Theoretical Method for Calculating Relative Joint Geometry of Assembled Robot Arms

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INTRODUCTION

Robotics is expected to play an increasingly important role in future space missions as the complexity and the exploitive nature of the missions increase. Initially, robot systems are envisioned to perform tasks in space, such as the service and repair of satellites (ref. 1). Accomplishing these tasks either remotely (teleoperator control) or by onboard computers (machine intelligence) requires some type of control logic to maneuver the robot's arm and hand.

Considering the way people control their arms and hands, one finds that people do not consciously control individual joints in commanding hand movements. A method of mimicking this type of control in a robot arm is called resolved-rate control (ref. 2), in which commands to maneuver the robot hand are rotational and translational velocities in the hand-axis system. These velocities are then resolved analytically into individual joint rates in the robot arm to accomplish the hand command.

Positioning a robot arm with resolved-rate control requires relative joint information, which is not always known (or is not available) for commercially available robot arms. Hence, a method is needed to ascertain this information without having to disassemble these arms. The intent in this paper is to develop a method to calculate the relative joint geometry of an assembled robot arm. Specifically, the Denavit-Hartenberg parameters (ref. 3), which completely characterize this geometry, are calculated.

ANALYSIS

The objective of this analysis is to derive equations for calculating the Denavit-Hartenberg parameters, which completely characterize the relative joint geometry in robot arms. In essence, these parameters locate consecutive joint-axis systems with respect to each other, both in position and orientation.

Robot Arm

Figure 1, which is a modification of a figure in reference 4, illustrates a robot arm and joint-axis systems. To control the arm in a teleoperator mode using resolved-rate control, a distant operator commands translational velocities (V_X, V_Y, and V_Z) and rotational velocities (ω_X, ω_Y, and ω_Z) about the hand-axis system. (A list of symbols and abbreviations used in this paper appears after the references.) These hand commands are then interpreted (or resolved) in terms of the individual joint angular rates $\dot{\theta}_i$ (i = 1, 2, ..., 6) by using transformation equations based on the relative joint geometry. Angular rates $\dot{\theta}_4$ and $\dot{\theta}_6$ correspond to rotating the base of the wrist assembly and the cylindrical portion of the wrist.

Relative Joint Geometry

Consider two sequential rotational joints in a robot arm, for instance, joint i and joint i + 1. In figure 2 the geometric relationship between axis systems at these joints is completely characterized by the Denavit-Hartenberg parameters, which consist of three constant parameters a_i, α_i, and r_i and a variable joint rotational angle θ_i'. By definition, joint rotations are always about the Z-axis. The X_i-axis is directed along the common normal from Z_{i-1} to Z_i. For clarity, the Y_i- and Y_{i-1}-axis, which simply complete right-handed coordinate systems at the respective joints, are not shown in figure 2.

In this method of systematically assigning coordinate systems to successive joints, the X_0-axis direction for the first joint (or X_N for the last joint) is chosen arbitrarily. For sliding joints, the joint variable is r_i rather than θ_i'. Only rotational joints are considered in this paper.

Basic Coordinate Transformation

The relative joint geometry dictates the basic transformation equations between adjacent joints. The coordinates of a point $P(x,y,z)$ with respect to the i joint-axis system in figure 2 can be transformed to coordinates with respect to the $i - 1$ joint-axis system by using the relation

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{i-1} = A_{i-1}^{i} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_{i} \tag{1}$$

with

$$A_{i-1}^{i} = \left[\begin{array}{ccc|c} \cos\theta_i' & -\cos\alpha_i \sin\theta_i' & \sin\alpha_i \sin\theta_i' & a_i \cos\theta_i' \\ \sin\theta_i' & \cos\alpha_i \cos\theta_i' & -\sin\alpha_i \cos\theta_i' & a_i \sin\theta_i' \\ 0 & \sin\alpha_i & \cos\alpha_i & r_i \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \tag{2}$$

where A_{i-1}^{i} is the homogeneous transformation matrix from coordinate system i to $i - 1$. (See ref. 4, for example.) Equation (1) is equivalent to

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{i-1} = \begin{bmatrix} \cos\theta_i' & -\cos\alpha_i \sin\theta_i' & \sin\alpha_i \sin\theta_i' \\ \sin\theta_i' & \cos\alpha_i \cos\theta_i' & -\sin\alpha_i \cos\theta_i' \\ 0 & \sin\alpha_i & \cos\alpha_i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}_{i} + \begin{bmatrix} a_i \cos\theta_i' \\ a_i \sin\theta_i' \\ r_i \end{bmatrix} \tag{3}$$

where the second term on the right-hand side is the location vector of the origin of coordinate system i with respect to the origin of coordinate system $i - 1$.

Problem Statement

Given a set of coordinates for a point $P(x,y,z)$ with respect to the robot hand in figure 1, the corresponding coordinates of this same point with respect to the base coordinate system of the robot arm can be computed as

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_0 = A_0^1 A_1^2 A_2^3 A_3^4 A_4^5 A_5^6 \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_6 \tag{4}$$

where equation (2), with $i = 1, 2, \ldots, 6$, supplies the matrices in equation (4).

Choose point $P(x,y,z)$ as the origin of the robot hand-axis system so that

$$\begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \tag{5}$$

Now, place the hand at different locations and measure the hand's location with respect to the base coordinate system. The corresponding joint angles for each measurement are also recorded (measured or obtained from robot's computer). The problem is to use these data to calculate the parameters a_i, α_i, and r_i of the robot arm. For the robot arm in figure 1, there are 18 unknown parameters a_i, α_i, and r_i (where $i = 1, 2, \ldots, 6$).

Procedure

A major task in extracting the relative joint parameters is finding a manageable way to look at the problem. The basic idea used in this paper is indicated in figure 1. The first problem is to determine the joint parameters a_1, α_1, and r_1, which relate the joint coordinate systems (x_0,y_0,z_0) and (x_1,y_1,z_1). To do this, fix the angle θ_1 and vary θ_2 while holding all the other joint angles constant. The resulting hand positions of the robot arm in base coordinates (x_0,y_0,z_0) are then used to extract these parameters (a_1, α_1, and r_1). A second problem is to extract the parameters a_2, α_2, and r_2, which relate the joint coordinate systems (x_1,y_1,z_1) and (x_2,y_2,z_2). Hence, fix θ_2 and vary θ_3 with all the other joint angles held constant. Notice that this second problem would be analogous to the first if the hand positions in (x_1,y_1,z_1) coordinates were known. But, since a_1, α_1, and r_1 have been computed in the first problem, transformation equations allow (x_1,y_1,z_1) to be computed from (x_0,y_0,z_0). This process is repeated up the arm. The mathematics used to extract the parameters in this process are developed in this paper.

Forward Recursive Transformation Equations

The coordinates in figure 2 are related by the following scalar transformation equations (which may be obtained from eq. (3) or derived from fig. 2):

$$x_i(k) = x_{i-1}(k) \cos \theta_i'(k) + y_{i-1}(k) \sin \theta_i'(k) - a_i \tag{6}$$

$$y_i(k) = [y_{i-1}(k) \cos \theta_i'(k) - x_{i-1}(k) \sin \theta_i'(k)]\cos \alpha_i + [z_{i-1}(k) - r_i]\sin \alpha_i \tag{7}$$

$$z_i(k) = [x_{i-1}(k) \sin \theta_i'(k) - y_{i-1}(k) \cos \theta_i'(k)]\sin \alpha_i + [z_{i-1}(k) - r_i]\cos \alpha_i \tag{8}$$

where an index argument k has been introduced to label sets of measurement data; that is, for each set of joint angles, there are corresponding coordinate positions. Recall that a_i, α_i, and r_i are constant parameters. For this analysis, equations (7) and (8) are expressed in different forms because the r_i in equations (7) and (8) is not always computable during the parameter-extraction process. The modified forms are

$$y_i(k) = [y_{i-1}(k) \cos \theta_i'(k) - x_{i-1}(k) \sin \theta_i'(k)]\cos \alpha_i + z_{i-1}^*(k) \sin \alpha_i - r_i^* \sin \alpha_i \tag{9}$$

$$z_i^*(k) = [x_{i-1}(k) \sin \theta_i'(k) - y_{i-1}(k) \cos \theta_i'(k)]\sin \alpha_i + z_{i-1}^*(k) \cos \alpha_i \tag{10}$$

where

$$z_i^*(k) = z_i(k) + r_i^* \cos \alpha_i \tag{11}$$

$$r_i^* = r_i + r_{i-1}^* \cos \alpha_{i-1} \tag{12}$$

That equations (9) and (10) are equivalent to equations (7) and (8) is easily verified by substituting equations (11) and (12) into equations (9) and (10).

In this study, the recursive application of equations (6), (9), and (10) has the following prerequisites:

1. $z_0^* = z_0$, $r_0^* = 0$, and $\alpha_0 = 0$ to start the sequential recursive process.

2. x_0, y_0, and z_0 are known values from measurements.

3. Joint angles θ_i' are known.

4. Values of $r_i^* \sin \alpha_i$, $\cos \alpha_i$, $\sin \alpha_i$, and a_i have been calculated.

Equations (6), (9), and (10) are later used recursively to transform the measurement values x_0, y_0, and z_0 to other coordinate systems in the arm. By design, r_i does not explicitly appear in these equations.

Equations for Parameter Calculations

The scalar components of equation (3) are

$$x_{i-1}(k) = [x_i(k) + a_i]\cos\theta_i'(k) - [y_i(k)\cos\alpha_i - z_i(k)\sin\alpha_i]\sin\theta_i'(k) \tag{13}$$

$$y_{i-1}(k) = [x_i(k) + a_i]\sin\theta_i'(k) + [y_i(k)\cos\alpha_i - z_i(k)\sin\alpha_i]\cos\theta_i'(k) \tag{14}$$

$$z_{i-1}^*(k) = y_i(k)\sin\alpha_i + z_i(k)\cos\alpha_i + r_i^* \tag{15}$$

where, as obtained from equation (11),

$$z_{i-1}^*(k) = z_{i-1}(k) + r_{i-1}^*\cos\alpha_{i-1} \tag{16}$$

and the r^* terms are given by equation (12). Again, k has been introduced to label data sets.

At this point, the objective is to calculate a_i, α_i, and r_i. Assume that $x_{i-1}(k)$, $y_{i-1}(k)$, and $z_{i-1}^*(k)$ have been previously calculated from equations (6), (9), and (10). Setting $i = i + 1$ in equations (13), (14), and (15) gives the transformation equations from the $i + 1$ joint-axis system to the i joint-axis system as

$$x_i(k) = [x_{i+1}(k) + a_{i+1}]\cos\theta_{i+1}'(k) - [y_{i+1}(k)\cos\alpha_{i+1} - z_{i+1}(k)\sin\alpha_{i+1}]\sin\theta_{i+1}'(k) \tag{17}$$

$$y_i(k) = [x_{i+1}(k) + a_{i+1}]\sin\theta_{i+1}' + [y_{i+1}(k)\cos\alpha_{i+1} - z_{i+1}(k)\sin\alpha_{i+1}]\cos\theta_{i+1}'(k) \tag{18}$$

$$z_i^*(k) = y_{i+1}(k)\sin\alpha_{i+1} + z_{i+1}(k)\cos\alpha_{i+1} + r_{i+1}^* \tag{19}$$

With a substitution of equations (17) and (18), equations (13) and (14) may be expressed as

$$x_{i-1}(k) = [c_0 + c_1\cos\theta_{i+1}'(k) - c_2\sin\theta_{i+1}'(k)]\cos\theta_i'(k) - [c_3\sin\theta_{i+1}'(k) + c_4\cos\theta_{i+1}'(k) - c_5]\sin\theta_i'(k) \tag{20}$$

$$y_{i-1}(k) = [c_0 + c_1 \cos \theta'_{i+1}(k) - c_2 \sin \theta'_{i+1}(k)]\sin \theta'_i(k)$$

$$+ [c_3 \sin \theta'_{i+1}(k) + c_4 \cos \theta'_{i+1}(k) - c_5]\cos \theta'_i(k) \qquad (21)$$

where

$$c_0 = a_i \qquad (22)$$

$$c_1 = x_{i+1} + a_{i+1} \qquad (23)$$

$$c_2 = y_{i+1} \cos \alpha_{i+1} - z_{i+1} \sin \alpha_{i+1} \qquad (24)$$

$$c_3 = c_1 \cos \alpha_i \qquad (25)$$

$$c_4 = c_2 \cos \alpha_i \qquad (26)$$

$$c_5 = z_i \sin \alpha_i \qquad (27)$$

and where k has been dropped in equations (22) to (27) because of the important assumption that all joint angles, except for $\theta'_{i+1}(k)$ and possibly $\theta'_i(k)$, are held fixed at arbitrary values (perhaps convenient for making measurements). With this assumption, no joint angle above θ'_{i+1} is varied. Consequently, x_{i+1}, y_{i+1}, and z_{i+1} remain constant. In addition, since the joint parameters a_i, α_i, and r_i are constant, it follows that equations (22) to (27) represent constants. At this point, depending on what angles are physically attainable by the robot arm under consideration, equations (20) and (21) may be handled in different ways to calculate the constants c_0 to c_5.

Parameter Solution Approach

The basic procedure is explained by letting $\theta'_i(k) = 180°$ to simplify equations (20) and (21) to

$$x_{i-1}(k) = -c_0 - c_1 \cos \theta'_{i+1}(k) + c_2 \sin \theta'_{i+1}(k) \qquad (28)$$

$$y_{i-1}(k) = c_5 - c_3 \sin \theta'_{i+1}(k) - c_4 \cos \theta'_{i+1}(k) \qquad (29)$$

First, consider equation (28). Again, for the sake of discussion, let $\theta'_{i+1}(1) = 0°$. Then, with $k = 1$, equation (28) becomes

$$x_{i-1}(1) = -c_0 - c_1 \qquad (30)$$

Let $\theta'_{i+1}(2) = 180°$ in equation (28) to get

$$x_{i-1}(2) = -c_0 + c_1 \tag{31}$$

Adding equations (30) and (31) yields

$$c_0 = -\frac{1}{2}[x_{i-1}(1) + x_{i-1}(2)] \tag{32}$$

which is the value of a_i in equation (22). Subtracting equation (30) from equation (31) produces

$$c_1 = -\frac{1}{2}[x_{i-1}(1) - x_{i-1}(2)] \tag{33}$$

Then, from equation (28),

$$c_2 = [x_{i-1}(k) + c_0 + c_1 \cos \theta'_{i+1}(k)]/\sin \theta'_{i+1}(k) \tag{34}$$

where c_0 and c_1 are now known and $\theta'_{i+1}(k)$ is any attainable angle as long as $\sin \theta'_{i+1}(k) \neq 0$. For example, $\theta'_{i+1}(k)$ in equation (34) may be selected as 120°. Analogously, from equation (29),

$$c_5 = \frac{1}{2}[y_{i-1}(1) + y_{i-1}(2)] \tag{35}$$

$$c_4 = -\frac{1}{2}[y_{i-1}(1) - y_{i-1}(2)] \tag{36}$$

$$c_3 = -[y_{i-1}(k) - c_5 + c_4 \cos \theta'_{i+1}(k)]/\sin \theta'_{i+1}(k) \qquad (\sin \theta'_{i+1}(k) \neq 0) \tag{37}$$

Determination of $\cos \alpha_i$.- If $c_1 \neq 0$ in equation (25),

$$\cos \alpha_i = \frac{c_3}{c_1} \tag{38}$$

or, if $c_2 \neq 0$ in equation (26),

$$\cos \alpha_i = \frac{c_4}{c_2} \tag{39}$$

In actual computations, if $|c_2| < |c_1|$, then equation (38) is used; otherwise, unless $c_1 = c_2 = 0$, equation (39) is applied.

Deflected extension attached to hand of robot arm.- The situation wherein both $c_1 = 0$ and $c_2 = 0$ is avoidable. For example, an extension can be attached to the hand and deflected to vary the constant value of x_{i+1} in equation (23) so that $c_1 \neq 0$. If an extension is used, measurements are made relative to a point on the extension rather than on the hand. The extension length or orientation need not be known in this process.

Equations (32) and (33) and equations (35) and (36) change if θ'_{i+1} takes on values other than 180° and 0°. With choices of θ'_{i+1} which allow solutions, a generalized matrix-inverse computer routine will furnish the solutions and, at the same time, provide a single common solution routine for all the legitimate cases.

Determination of $\sin \alpha_i$.- For convenience, equation (15) is expressed as

$$z^*_{i-1}(k) = d_0 \; y_i(k) + d_1 \tag{40}$$

where

$$d_0 = \sin \alpha_i \tag{41}$$

$$d_1 = z_i(k) \cos \alpha_i + r^*_i \tag{42}$$

Equation (40) is a straight-line equation with ordinate $z^*_{i-1}(k)$ and abscissa $y_i(k)$. The constant slope of this line is d_0 and the ordinate intercept is d_1. The coordinate $z_i(k)$ in equation (42) is constant in the parameter-calculation procedure in this paper, as can be shown with equations (19) and (11). The two constant parameters d_0 and d_1 in equation (40) can be determined by using two known points on the line, that is, a combination of values of $z^*_{i-1}(k)$ and $y_i(k)$ corresponding to two different values of $\theta'_{i+1}(k)$. The $y_i(k)$ value will vary according to equation (18), which may be rewritten as

$$y_i(k) = c_1 \sin \theta'_{i+1}(k) + c_2 \cos \theta'_{i+1}(k) \tag{43}$$

The value of $z^*_{i-1}(k)$ results from a recursive process using equation (10).

Determination of α_i.- This constant joint parameter is computed as

$$\alpha_i = \tan^{-1}(\sin \alpha_i / \cos \alpha_i) \tag{44}$$

where $\sin \alpha_i$ is given by equation (41) and $\cos \alpha_i$ is given by equation (38) or (39). The correct quadrant for α_i is readily ascertained from the signs of $\sin \alpha_i$ and $\cos \alpha_i$. Actually, $\sin \alpha_i$ and $\cos \alpha_i$ are needed in the transformation equations rather than α_i itself.

Determination of $r_i^* \sin \alpha_i$.- A value for $r_i^* \sin \alpha_i$ is needed in using the recursive equation (9). Toward this end, multiply equation (15) by $\sin \alpha_i$ and rearrange as follows:

$$r_i^* \sin \alpha_i = z_{i-1}^*(k)[\sin \alpha_i] - y_i(k)[\sin \alpha_i]^2 - [z_i \sin \alpha_i \cos \alpha_i] \tag{45}$$

The terms in brackets are known from equations (41), (27), and (38) or (39). Hence, for values of $z_{i-1}^*(k)$ and $y_i(k)$, corresponding to a value of $\theta_{i+1}'(k)$, equation (45) may be evaluated. For different sets of values of $z_{i-1}^*(k)$ and $y_i(k)$, equation (45) may be solved in a least-squares sense.

At this point, a_i, α_i, $\sin \alpha_i$, $\cos \alpha_i$, and $r_i^* \sin \alpha_i$ are computable. This allows the process to be repeated since the right-hand sides of equations (6), (9), and (10) are computable. Recall that z_{i-1}^* in equation (10) is always computed in a previous iteration.

Determination of r_i^*.- The reason for introducing the z^* and r^* notations is to allow continuation of the recursive process even though r_i may not be explicitly known. If $\sin \alpha_i \neq 0$, then equations (27) and (41) reveal that

$$z_i = \frac{c_5}{d_0} \tag{46}$$

Thus, from equation (42),

$$r_i^* = d_1 - z_i(k) \cos \alpha_i \tag{47}$$

where the right-hand side of equation (47) is now computable. On the other hand, if $\sin \alpha_i = 0$, equation (15) becomes

$$z_{i-1}^*(k) = z_i(k) \cos \alpha_i + r_i^* \tag{48}$$

and there is no information in equations (13) and (14) about $z_i(k)$ to help in eliminating $z_i(k)$ from equation (47).

Determination of r_i.- The calculation of r_i is best explained by an example. Suppose $\sin \alpha_1 \neq 0$, $\sin \alpha_2 = 0$, $\sin \alpha_3 \neq 0$, and $\sin \alpha_4 \neq 0$. This means r_1^*, r_3^*, and r_4^* are computable by using equations (46) and (47), but r_2^* is not computable. As justified by equation (12) and prerequisite (1), write

$$r_1 = r_1^* \tag{49}$$

$$r_3 + r_2 \cos \alpha_2 = r_3^* - r_1^* \cos \alpha_1 \cos \alpha_2 \tag{50}$$

$$r_4 = r_4^* - r_3^* \cos \alpha_3 \tag{51}$$

In this situation, r_1, $r_3 + r_2 \cos \alpha_2$, and r_4 are computable. It appears that any r_2 and r_3 such that equation (50) holds will give the same results with respect to the transformation equations. Indeed, this is meaningful because, in locating a point with respect to the robot arm base, two parallel Z-axes will always displace the point along the common parallel Z-axis by the sum of the individual displacements. This same type of analysis holds for other situations.

Angular Measurements

The joint parameters a_i, α_i, and r_i specify the relative locations and orientations of the successive joint-axis systems in the robot arm. Until these parameters are identified, the locations and orientations of the axis systems are not known. Thus, how can the joint angle θ_i' between the X_{i-1}-axis and the X_i-axis be measured? Essentially all that is known about the robot arm is that the joints rotate and that the orientation of the X_0-axis is arbitrary.

If the rotational axis Z_i lies in or is parallel to a plane that contains the Z_{i-1}- and X_{i-1}-axis, then $\theta_i' = 90°$ by definition of how the joint angle is measured in figure 2. This is easily seen in figure 1 by aligning Z_1 with X_0. Then θ_i', which is the angle between the X_0- and X_1-axis, is 90°.

Displaced Reference Axes

The location of the robot hand (or an extension) is measured with respect to the base of the robot arm, for example, coordinates (x_0, y_0, z_0) in figure 1. However, if this is inconvenient, a displaced reference can be used where the base coordinate system is treated as just another joint-axis system, the location and orientation of which is to be determined with respect to the new displaced reference axes. Alternatively, knowing the base coordinate system, one can determine its location and orientation with respect to a more conveniently specified reference axis system.

EXAMPLE

The procedure in this paper is applied to compute the relative joint parameters of the robot arm in figure 1. This example is strictly analytical in that no physical measurements are actually made. All data are assumed to be without error.

Generating Measurement Data

For the position of the robot arm in figure 1 it is not possible to fix θ_3 and vary θ_4 to obtain changes in the location of point H in coordinates (x_2, y_2, z_2). This does not provide enough information for the desired calculations. However, if the segment HW is deflected (e.g., θ_5 is fixed at 90°), sufficient variation does result. A similar situation occurs in trying to vary θ_6 and fix θ_5 to vary the location of point H in coordinates (x_4, y_4, z_4). But, in this instance, there is no segment to deflect. To avoid this circumstance a deflected extension HF is assumed to be attached to the hand. Although no difficulty is incurred for the first three joints, measurements for these joints are also referenced to point F.

In figure 1 measurements are assumed to be made to point F, which represents a point on an extension attached to the hand of the robot arm. The location and orientation of F need not be known in a real application where measurements are taken. But, for the purpose of calculating what these measurement data should be, the segment HF in figure 1 is assumed to lie along the X_6-axis and to have a length of 6 in. Base coordinates of F are calculated for three sets (k = 1, 2, and 3) of specified joint angles by using the relative joint parameters in table I. These data are given in table II and are used as measurement data to extract the relative joint parameters.

Parameter Calculations

Parameters a_1, α_1, and r_1.- These parameters are calculated by using the three sets of data (k = 1, 2, and 3) for joint 1 in table II. Both θ_i' and θ_i are listed for convenience. Notice that θ_1' is fixed at 180° while θ_2' takes on values of 180°, 0°, and 120°. It does not matter what the other joint angles are as long as they are constant, but they are chosen as shown. Since $i = 1$ and $\theta_1' = 180°$, equation (20) becomes

$$x_0(k) = -c_0 - c_1 \cos \theta_2'(k) + c_2 \sin \theta_2'(k) \tag{52}$$

Hence, with the three sets of data in table II for joint 1, equation (52) yields three equations to be solved simultaneously for c_0, c_1, and c_2 (given in the first row of table III). Likewise, equation (21) becomes

$$y_0(k) = -c_3 \sin \theta_2'(k) - c_4 \cos \theta_2'(k) + c_5 \tag{53}$$

which, with $k = 1$, 2, and 3, is solved for c_3, c_4, and c_5. (See table III.)

Letting $i = 1$ and substituting equation (43) into equation (40) gives

$$z_0^*(k) = d_0[c_1 \sin \theta_2'(k) + c_2 \cos \theta_2'(k)] + d_1 \tag{54}$$

where c_1 and c_2 have been calculated. Letting $k = 2$ and 3 in equation (54) results in two equations which are solved simultaneously for d_0 and d_1. (See table III.)

Values for a_1, $\cos\alpha_1$, $\sin\alpha_1$, and α_1 are calculated with equations (22), (38), (41), and (44) and are listed in table IV.

Since $d_0 = \sin\alpha_1 \neq 0$, z_1 is computed from equation (46) for $i = 1$. Likewise, r_1^* results from equation (47). But, since $r_0^* = \alpha_0 = 0$ by assumption, $r_1 = r_1^*$. The values of r_1^* and r_1 are shown in table IV.

The value of $r_1^* \sin\alpha_1$ shown in table IV is computed with equation (45), where $z_1 \sin\alpha_1$ is just c_5 (eq. (27)).

Parameters a_2, α_2, and r_2.- The calculation proceeds as before with measurement data for joint 2 in table II, except that point F is needed in coordinates (x_1,y_1,z_1) rather than coordinates (x_0,y_0,z_0). The appropriate transformation equations are equations (6), (9), and (10) for $i = 1$. They are

$$x_1(k) = x_0(k)\cos\theta_1'(k) + y_0(k)\sin\theta_1'(k) - a_1 \tag{55}$$

$$\begin{aligned} y_1(k) = {} & [y_0(k)\cos\theta_1'(k) - x_0(k)\sin\theta_1'(k)]\cos\alpha_1 \\ & + z_0^*(k)\sin\alpha_1 - r_1^*\sin\alpha_1 \end{aligned} \tag{56}$$

$$\begin{aligned} z_1^*(k) = {} & [x_0(k)\sin\theta_1'(k) - y_0(k)\cos\theta_1'(k)]\sin\alpha_1 \\ & + z_0^*(k)\cos\alpha_1 \end{aligned} \tag{57}$$

At this point, a_1, $\cos\alpha_1$, $\sin\alpha_1$, and $r_1^*\sin\alpha_1$ are known (table IV) and $z_0^*(k) = z_0$ by definition. Thus, $x_1(k)$, $y_1(k)$, and $z_1^*(k)$ are computable with the $\theta_1'(k)$ values for $i = 2$ in table II.

Let $i = 2$ in equations (20), (21), and (40) to get the following equations:

$$x_1(k) = -c_0 - c_1\cos\theta_3'(k) + c_2\sin\theta_3'(k) \tag{58}$$

$$y_1(k) = -c_3\sin\theta_3'(k) - c_4\cos\theta_3'(k) + c_5 \tag{59}$$

$$z_1^*(k) = d_0[c_1\sin\theta_3'(k) + c_2\cos\theta_3'(k)] + d_1 \tag{60}$$

The constant c values are now determined as before with the data in table II for $k = 1$, 2, and 3. These values are shown in table III. Likewise, the subsequently calculated relative joint parameters are shown in table IV. Since $\sin\alpha_2 = 0$, z_2 cannot be computed with equation (46); therefore, z_2 cannot be used to compute r_2^* in equation (47).

Parameters a_3, α_3, and r_3.- The locations of point F in coordinates (x_2,y_2,z_2) are needed for these calculations. To obtain these locations, first apply equations (55), (56), and (57). Then, apply the following set of transformation equations (obtained from eqs. (6), (9), and (10) for $i = 2$):

$$x_2(k) = x_1(k) \cos \theta_2'(k) + y_1(k) \sin \theta_2'(k) - a_2 \tag{61}$$

$$y_2(k) = [y_1(k) \cos \theta_2'(k) - x_1(k) \sin \theta_2'(k)]\cos \alpha_2 + z_1^*(k) \sin \alpha_2 - r_2^* \sin \alpha_2 \tag{62}$$

$$z_2^*(k) = [x_1(k) \sin \theta_2'(k) - y_1(k) \cos \theta_2'(k)]\sin \alpha_2 + z_1^*(k) \cos \alpha_2 \tag{63}$$

where a_2, $\cos \alpha_2$, $\sin \alpha_2$, and $r_2^* \sin \alpha_2$ have been previously computed and where $z_1^*(k)$ is computed with equation (60).

Let $i = 3$ in equations (20), (21), and (40) to get the following equations:

$$x_2(k) = -c_0 - c_1 \cos \theta_4'(k) + c_2 \sin \theta_4'(k) \tag{64}$$

$$y_2(k) = -c_3 \sin \theta_4'(k) - c_4 \cos \theta_4'(k) + c_5 \tag{65}$$

$$z_2^*(k) = d_0[c_1 \sin \theta_4'(k) + c_2 \cos \theta_4'(k)] + d_1 \tag{66}$$

The constants calculated with these equations and the data in table II for $i = 3$ are shown in table III. The results of other calculations are shown in table IV. Notice that a combination value of r_3 and r_2 is given. This value is computed from equation (12) for $i = 3$ as follows (eq. (50)):

$$r_3 + r_2 \cos \alpha_2 = r_3^* - r_1^* \cos \alpha_1 \cos \alpha_2$$

This means that as far as the equations are concerned any r_3 and r_2 will give the same results as long as they satisfy equation (50). Notice that the values of r_3 and r_2 in table I satisfy this equation.

Parameters a_4, α_4, and r_4 and a_5, α_5, and r_5.- The same procedure as used for $i = 1$, 2, and 3 is used to calculate the values shown in tables III and IV.

Parameters a_6, α_6, and r_6.- If these parameters were calculated in the same manner as the other parameters, then some means of introducing a rotational angle θ_7 about Z_6 in figure 1 would be required. This is not necessary, however, to compute a_6 and r_6. To compute these parameters find the location of point H (rather than point F) in figure 1 in coordinates (x_0,y_0,z_0). With previously computed relative joint parameters, point H in coordinates (x_5,y_5,z_5) can be computed by recursively solving equations (6), (7), and (8) with $i = 1$ to 5. The location of point H with respect to point H is zero, so $(x_6,y_6,z_6) = (0,0,0)$.

With $i = 6$, equations (6), (7), and (8) are

$$x_6(k) = x_5(k) \cos \theta_6'(k) + y_5(k) \sin \theta_6'(k) - a_6 \tag{67}$$

$$y_6(k) = [y_5(k) \cos \theta_6'(k) - x_5(k) \sin \theta_6'(k)]\cos \alpha_6 + [z_5(k) - r_6]\sin \alpha_6 \tag{68}$$

$$z_6(k) = [x_5(k) \sin \theta_6'(k) - y_5(k) \cos \theta_6'(k)]\sin \alpha_6 + [z_5(k) - r_6]\cos \alpha_6 \tag{69}$$

From equation (67),

$$a_6 = -x_6(k) + x_5(k) \cos \theta_6'(k) + y_5(k) \sin \theta_6'(k) \tag{70}$$

Add equation (68), multiplied by $\sin \alpha_6$, to equation (69), multiplied by $\cos \alpha_6$, to obtain

$$r_6 = z_5(k) - y_6(k) \sin \alpha_6 - z_6(k) \cos \alpha_6 \tag{71}$$

With $x_6 = y_6 = z_6 = 0$, equations (70) and (71) are simply

$$a_6 = -x_5 \cos \theta_6'(k) - y_5 \sin \theta_6'(k) \tag{72}$$

$$r_6 = z_5(k) \tag{73}$$

where x_5, y_5, and z_5 are coordinates of point H in figure 1. In figure 1, with $\theta_6 = 0$, point H has coordinates $(x_5,y_5,z_5) = (0,0,6)$, where the coordinates are in inches. Hence, from equations (72) and (73), $a_6 = 0$ and $r_6 = 6$ in.

Another way to compute α_6 other than by the method used to compute α_1 to α_5 is to specify the axis at point H as desired and then physically measure point F in coordinates (x_6,y_6,z_6). The coordinates (x_5,y_5,z_5) of point F are computed by using the recursive transformation equations. Then, equations (68) and (69) provide two simultaneous equations in two unknowns, $\sin \alpha_6$ and $\cos \alpha_6$. The value of r_6 is given by equation (73) as the z_5 coordinate of point H. Hence, $\tan \alpha_6$ and then α_6 can be computed. For example, let point F be moved to lie along the Z_6-axis in figure 1. Thus, the coordinates of this new point F location are $(x_6,y_6,z_6) = (0,0,6)$ and $(x_5,y_5,z_5) = (0,0,12)$ in inches. With these coordinates and $r_6 = 6$ in., equation (68) becomes $\sin \alpha_6 = 0$ and equation (69) becomes $\cos \alpha_6 = 1$. Therefore, $\tan \alpha_6 = 0$ and $\alpha_6 = 0$.

As expected, a comparison of tables I and IV shows agreement between the calculated and exact values of the relative joint parameters a_i, α_i, and r_i $(i = 1, 2, \ldots, 6)$ when simulated perfect measurement data are used.

CONCLUDING REMARKS

If an operator remotely controls the hand of a robot arm by commanding translational and rotational rates about the hand axes, then these rates must be resolved mathematically into joint rates along the arm to effect these commands (resolved-rate control). This resolution depends on the location of the joints relative to each other. This information is usually not available or is difficult to measure for assembled commercially available robot arms. But, in teleoperation studies involving the control of these arms by resolved rate, this information is required.

This paper presents a theoretical method to compute the relative joint parameters of assembled robot arms. The idea is to measure locations of the robot's hand for different joint angles and to then ascertain the parameters mathematically using these measurements. The method is illustrated for a six-degree-of-freedom robot arm. Calculated data agreed perfectly with measurement data.

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SYMBOLS

A_{i-1}^{i} homogeneous transformation matrix from coordinate system i to $i - 1$

a_i length of common normal between Z_{i-1} and Z_i

$c_0, c_1, \ldots, c_5$ constants in parameter-extraction process (see eqs. (22) to (27))

d_0, d_1 constants in parameter-extraction process (see eqs. (41) and (42))

i integer indicating the ith joint axis or parameters associated with this axis

k integer argument for labeling corresponding measurement data

N number of joints or joint-axis systems in robot arm

$P(x,y,z)$ point in Cartesian coordinates

r_i relative distance between coordinate system $i - 1$ and i along Z_{i-1}

r_i^* constant defined to eliminate explicit dependence on r_i in parameter-extraction process

V_X, V_Y, V_Z translational velocities of robot's hand

X, Y, Z coordinate axes

X_i axis directed along common normal between Z_{i-1} and Z_i (see fig. 2)

Y_i axis directed to complete right-handed-axis system with X_i and Z_i

Z_i axis of rotation of joint $i - 1$

x, y, z coordinates along X, Y, and Z

x_i, y_i, z_i coordinates along X_i, Y_i, and Z_i

$x_i(k), y_i(k), z_i(k)$ coordinates associated with data set k

$z_i^*(k)$ new variable which results when r_i^* is introduced into the parameter-extraction process

α_i angle between Z_{i-1} and Z_i, measured positive counterclockwise about X_i

θ_i joint angle with initial value corresponding to position of robot arm in figure 1

θ_i' joint angle between X_{i-1} and X_i, measured positive counterclockwise about Z_{i-1} (see fig. 2)

$\theta_i'(k)$ joint angle θ_i' associated with data set k

$\omega_X, \omega_Y, \omega_Z$ rotational velocities of the robot's hand

Abbreviations:

NO neck-to-base length

SN shoulder-to-neck length

ES elbow-to-shoulder length

WE wrist-to-elbow length

HW hand-to-wrist length

HF hand-to-finger (or extension) length

Use of a dot over a symbol indicates first derivative with respect to time.

TABLE I.- ASSUMED RELATIVE JOINT PARAMETERS

[From ref. 4]

Joint, i	α_i, deg	a_i, in.	r_i, in.	θ_i', deg
1	90	0	[a]26	$\theta_1 + 180$
2	0	[b]17	[c]6	$\theta_2 + 90$
3	90	0	0	$\theta_3 + 90$
4	90	0	[d]17	$\theta_4 + 180$
5	90	0	0	$\theta_5 + 180$
6	0	0	[e]6	θ_6

[a]Neck-to-base length (NO).
[b]Elbow-to-shoulder length (ES).
[c]Shoulder-to-neck length (SN).
[d]Wrist-to-elbow length (WE).
[e]Hand-to-wrist length (HW).

TABLE II.- ASSUMED MEASUREMENT DATA USED TO CALCULATE RELATIVE JOINT PARAMETERS

Joint, i	Data index, k	θ_i, deg						θ_i', deg						Coordinates of point F in fig. 1, in.		
		θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	θ_1'	θ_2'	θ_3'	θ_4'	θ_5'	θ_6'	x_0	y_0	z_0
1	1	0	90	0	0	0	0	180	180	90	180	180	0	40.00	6.00	20.00
	2	0	-90	0	0	0	0	180	0	90	180	180	0	-40.00	6.00	32.00
	3	0	30	0	0	0	0	180	120	90	180	180	0	25.19	6.00	57.64
2	1	0	90	90	0	0	0	180	180	180	180	180	0	11.00	6.00	3.00
	2	0	90	-90	0	0	0	180	180	0	180	180	0	23.00	6.00	49.00
	3	0	90	30	0	0	0	180	180	120	180	180	0	33.91	6.00	9.30
3	1	0	0	90	0	0	0	180	90	180	180	180	0	23.00	6.00	37.00
	2	0	0	90	-180	0	0	180	90	180	0	180	0	23.00	6.00	49.00
	3	0	0	90	-60	0	0	180	90	180	120	180	0	23.00	.80	40.00
4	1	0	0	0	0	0	0	180	90	90	180	180	0	6.00	6.00	66.00
	2	0	0	0	0	-180	0	180	90	90	180	0	0	-6.00	6.00	54.00
	3	0	0	0	0	-60	0	180	90	90	180	120	0	-2.19	6.00	68.19
5	1	0	0	0	0	0	180	180	90	90	180	180	180	-6.00	6.00	66.00
	2	0	0	0	0	0	0	180	90	90	180	180	0	6.00	6.00	66.00
	3	0	0	0	0	0	120	180	90	90	180	180	120	-3.00	11.19	66.00
6	1	0	0	0	0	0	0	180	90	90	180	180	0	[a]0	[a]6.00	[a]66.00

[a]Coordinates of point H in figure 1.

TABLE III.- CALCULATED CONSTANTS

Joint, i	Constants in parameter-extraction process from simulated measurements for -							
	c_0, in.	c_1, in.	c_2, in.	c_3, in.	c_4, in.	c_5, in.	d_0, in.	d_1, in.
1	0	40	60	6	0	0	1	26
2	17	6	-23	0	6	-23	0	6
3	0	-6	0	23	0	0	1	6
4	0	6	-6	0	0	0	1	17
5	0	6	0	6	0	0	1	0

TABLE IV.- CALCULATED CONSTANT PARAMETERS ASSOCIATED WITH ROBOT ARM

Joint, i	a_i, in.	cos α_i	sin α_i	α_i, deg	r_i^*, in.	r_i, in.	r_i^* sin α_i, in.
1	0	0	1	90	26	26	26
2	17	1	0	0		[a]6	0
3	0	0	1	90	6	[a]6	6
4	0	0	1	90	17	17	17
5	0	0	1	90	0	0	0
6	0	1	0	0	6	6	0

[a]$r_3 + r_2 = 6$.

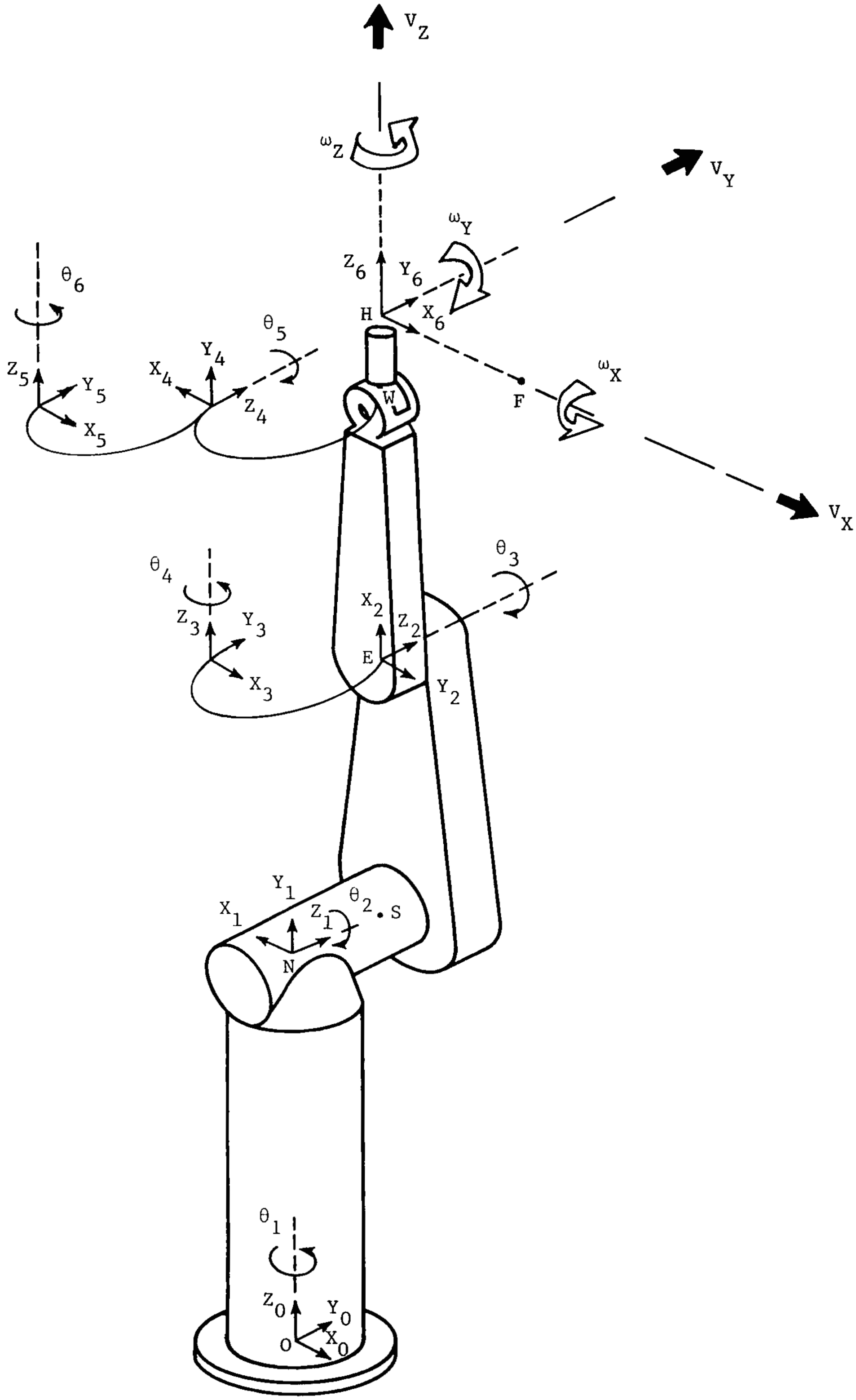

Figure 1.- Robot arm and joint-axis systems.

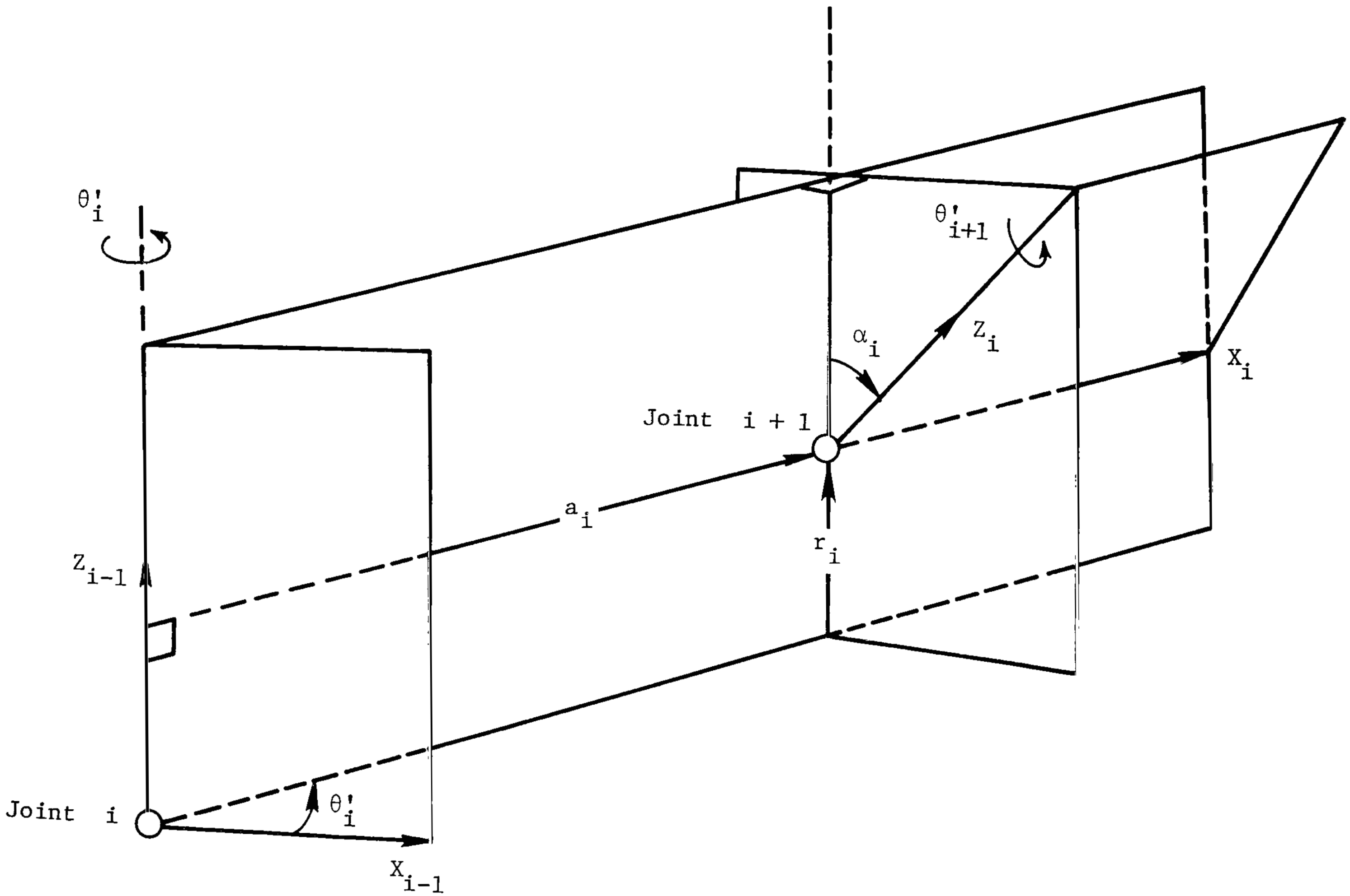

Figure 2.- Relative joint parameters a_i, α_i, and r_i.

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16. Abstract

Equations are developed to extract the relative joint parameters of an assembled robot arm. Specifically, the Denavit-Hartenberg parameters, which completely characterize the relative joint geometry, are calculated. These parameters are needed to control the hand of the robot arm by resolved rate. As an example, the parameter-extraction equations are used with perfect simulated data (no measurement noise) obtained from a mathematical model of a six-degree-of-freedom robot arm. For an actual application, measurement data needed to estimate the relative joint parameters can be generated by moving a robot arm to different positions, measuring the location of the hand (or other extension) in base coordinates, and recording the corresponding joint angles.

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