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TURBULENCE IN TRANSIENT SOLAR PHENOMENA

FINAL   REPORT

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January 1, 1980 to July 30, 1982

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INTRODUCTION

This project applied modern theories of turbulence and fluid mechanics
to astrophysical phenomena. Of special interest were periodic events on
the surface of the sun. Research and papers written during the grant are
summarized below.

1. STELLAR STRUCTURES

Streamers in the solar corona are thought to be caused by structures
in the underlying magnetic field. However, we have found that most expanding
gases will form streamers and expand non-uniformly. There must be a retarding
force (such as gravity in the case of the solar wind) and the expansion must
be subsonic.

Our work on this problem was presented at an APS meeting. The abstract
is enclosed as Appendix A. The paper titled "Natural Inhomogeneities in an
Expanding Gas" is Appendix B. We showed that the expanding gas had a linear
instability to initiate streamer formation, but were unable to use turbulence
to find a non-linear limiting process for the instability.

2. TURBULENCE IN A MAGNETIZED PLASMA

The Orr-Sommerfeld equation governs growing instabilities in a fluid
boundary layer. Its solutions determine the Reynolds number at which
turbulence develops. We have added a magnetic field and electrical resistivity.

This problem applies to plasma physics (fusion research) and to several astrophysical situations. In particular, the interior of the sun has boundary layers between regions of magnetic plasma flowing in opposite directions. It is possible that turbulence in these boundary layers could have a characteristic frequency that leads to observable phenomena on the sun's surface.

Appendix C is titled "Onset of Turbulence in a Magnetized Boundary Layer." We found that a higher Reynolds number is needed to start turbulence in the presence of a magnetic field because energy is required to bend the field lines attached to the fluid.

3. COSMOLOGICAL TURBULENCE

Appendix D is titled "Phase Coherent Turbulence in the Early Cosmos." Turbulence in fluids has prefered frequencies and eddy sizes. If the cosmological gas was turbulent shortly after the big bang, then galaxies could have been formed by turbulent eddies. This hypothesis is still open because the Reynolds number is very high in astrophysics. It is not clear how to extrapolate from laboratory experiments at $R_e < 10^6$ to astrophysics at $Re > 10^{14}$.

Appendix A

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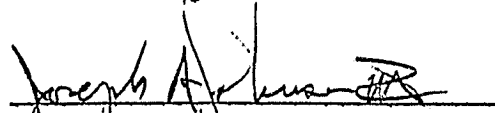
Coronal Structures without a Magnetic Field.*

M. A. CROSS, Grambling U. and J. A. JOHNSON III,
Rutgers U.
—It is believed that structures in the
atmosphere of the sun are controlled by the background
magnetic field. We show that coronal streamers can be
formed without a magnetic field. If theta dependence
is kept in the Navier-Stokes equations for the solar
wind, then a density enhancement will grow. This growth
is followed in the non-linear equations until a streamer
is formed. Viscosity stops the streamer's growth when
there is a large difference in speeds inside and outside
of the streamer. The viscosity needed to reproduce
observed streamers is compared with the classical
viscosity.

Submitted by

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Appendix B

Natural Inhomogeneities in an Expanding Gas

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ABSTRACT

Using classical fluid mechanics and a latitude dependent hydrodynamical model, we find that unmagnetized perturbed flow evolves into high and low density regions. The growth mechanisms for density enhancements are discussed along with a new nonlinear numerical solution for their large amplitude development.

Examples of expanding gases occur in many astrophysical events, e.g. the Big Bang, novae, stellar winds, and planetary atmospheres. These processes are usually modeled with a restriction to spherically symmetric hydrodynamics. It is impossible for such models to produce structures whose parameters depend on latitude and longitude. The structures actually formed in these expanding gases range from galaxies on the cosmic scale down to streamers and bursts on smaller scales. Some of these structures might be formed by hydrodynamic processes alone. In this paper we study the characteristic structures that might evolve from the unrestricted Navier-Stokes equations. Two hypotheses are investigated: "The expanding gas would still have structures even if there were no magnetic field," and "Any subsonic expanding gas might break up into structures."

Stellar streamers are now thought to arise from the regions in stellar atmosphere where collisional phenomena predominate (Kaplan, Pikelner, and Tystovich, 1974). In such regions, a hydrodynamical approach is appropriate for the analysis of the evolution of density perturbations (Lemaire 1978). This approach should be particularly relevant to those stars with weak fields or to stellar atmospheres for which the onset of amplified fluctuations is only weaky dependent on the strength of the local magnetic field. By way of illustration, the calculation will be applied specifically to the solar corona because its parameters are well known and because it has a subsonic region. We shall use solar parameters, even though the model is not applicable to the solar corona

where the magnetic field dominates. Of course, the parameters can easily be changed as new data become avilable to apply to other astrophysical situations.

The existence of streamers in an expanding gas shows that spherical symmetry is broken when random density perturbations grow and become stable configurations. Ignoring explicit time dependence (as is the convention), the latitude dependent Navier-Stokes equations are

$$\frac{1}{r^2}\frac{\partial}{\partial r}(nur^2) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(nv\sin\theta) = D\nabla^2 n \tag{1}$$

$$mnu\frac{\partial u}{\partial r} = -\frac{\partial}{\partial r}(nkT) - \frac{GM_\odot mn}{r^2} + \frac{mnv}{r}\left(v - \frac{\partial u}{\partial \theta}\right) + (VI) \tag{2}$$

$$nu\frac{\partial v}{\partial r} = -\frac{nv}{r}\left(\frac{\partial v}{\partial \theta} + u\right) - \frac{1}{mr}\frac{\partial}{\partial \theta}(nkT) + (VI) \tag{3}$$

$$\frac{3}{2}nku\frac{\partial T}{\partial r} = kTu\frac{\partial n}{\partial r} - \frac{kv}{r}\left(\frac{3}{2}n\frac{\partial T}{\partial \theta} - T\frac{\partial n}{\partial \theta}\right) + (TC) + (VI) \tag{4}$$

where: n is the total particle density; m is the average mass per particle; u is the radial speed; v is the latitude component of velocity; (TC) and (VI) symbolize thermal conductivity and viscosity terms respectively; and the transport coefficients are (Scarf and Noble, 1965) $\mu = 8.3 \times 10^{-17}T^{5/2}$ gm/cm-sec, $D = \mu/mn$, $K = 6 \times 10^{-9}T^{5/2}$ erg/cm-sec-K for our purposes. Details of the thermal conductivity and viscosity terms may be found in Yuan (1967).

Linear Treatment. (See Cross and Blockwood 1974.) The zero order unperturbed solution ignores theta dependent terms in

eqs. (1) - (4). We add a perturbation (indicated by the subscript 1) which peaks at angle θ_0 with width τ : at θ_0, $(\partial^2 n/\partial\theta^2) = - (n_1 /\tau^2)$, $(\partial n/\partial\theta) = 0$ with similar forms for T and u perturbations. The fourth function v is zero at θ_0 but has nonzero $\partial v/\partial\theta$ since the perturbed pressure (from n_1 and T_1) causes sideways expansion away from θ_0. For a rapidly growing perturbation, we assume a small radial scale length, i.e. $(\partial n_1/\partial r) = \alpha n_1 \gg (n_1/r)$. Equations (1) - (4) in linear form without viscosity, diffusion, or thermal conductivity are:

$$\alpha u\, n_1 + \alpha n u_1 + (n/r)(\partial v/\partial\theta) \simeq 0 \tag{5}$$

$$m n u \alpha\, n_1 \simeq -k\alpha(n_1 T + n T_1) \tag{6}$$

$$\alpha n u (\partial v/\partial\theta) \simeq (k/m r\, \tau^2)(n_1 T + n T_1) \tag{7}$$

$$n_1 T \simeq (3/2)\, n T_1 \tag{8}$$

respectively. The gravitational term which would have appeared in eq. (6) has been dropped because it is much smaller than the pressure term; thermal energy is comparable to gravitational energy ($kT \approx GMm/r$) but the perturbed pressure should have a steep gradient. That is, $(dn_1/dr) = \alpha n_1 \gg (n_1/r)$. Eliminating T_1 and both components of velocity from eqs. (5) - (8), and assuming $mu^2 \ll 5kT/3$, produces $\alpha = 1/r\tau$ or $n_1(r,\theta) = n_1(\theta)\, r^{1/\tau}$. Since τ is the width of the

perturbation, narrow perturbations or streamers will grown faster as they are convected outward by a stellar wind.

Physical Mechanism . The density perturbation grows because of interactions between eqs. (5) – (7). Ignoring the temperature change in an isothermal corona reduces the radial momentum eq. (6) to

$$\frac{1}{u}\frac{du_l}{dr} \simeq -\frac{kT}{mu^2}\frac{1}{n}\frac{dn_l}{dr} \qquad \text{ORIGINAL PAGE IS OF POOR QUALITY} \qquad (9)$$

that is, fractional change in velocity is proportional to the fractional change in density. Recall that $kT \gg mu^2$ below about 5 $R_\odot$; thus velocity changes have the larger magnitude below 5 $R_\odot$. Eq. (9) also shows that changes in density make a pressure gradient which drives the velocity.

When the continuity equation is applied to a small volume at a fixed position in the corona, one sees a density enhancement convected upward into the volume by the stellar wind. This makes a locally higher pressure which is smoothed out by sideways expansion out of the volume governed by eq. (7). As the density enhancement passes through the volume, it loses flux due to sideways expansion. The particle flux nur^2 becomes successively smaller after passing through volume elements at successively larger r. The only way to both decrease the flux and also satisfy eq. (9) is for the speed to decrease and the density to increase, i.e., for the density enhancement to grown.

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<u>Nonlinear Numerical Solution.</u> Models of expanding gases usually assume initial conditions at one radius and then eqs. (1), (2), and (4) are numerically integrated to solve for n, u and T at other radii. We add eq. (3) and the θ component of velocity, V, to this process.

Recall that the solar wind has a critical point at the supersonic transition where $5kT/3 \simeq m\,u^2$. Whang, Liu and Chang (1966) have shown that the singularity can·be avoided by retaining the second order terms $\partial^2 u/\partial r^2$ in the viscosity terms of eq. (2) and $\partial^2 T/\partial r^2$ in the thermal conductivity terms in eq. (4). For this reason the numerical work reported here was done with eqs. (2) and (4) rearranged to use second order radial derivatives.

Equation (1) was left at first order in $\partial n/\partial r$ by ignoring diffusion on the right side. Finally, in eq. (3) it was assumed that the viscous term $\partial^2 V/\partial r^2$ was unimportant and could be approximated numerically from changes in $\partial V/\partial r$ at previous (lower) radii. Therefore eq. (3) was kept in the first order form for $\partial V/\partial r$ as written above.

The functions of θ were calculated on grids of 30 to 180 points equally spaced in latitude from 0° to 90°. Symmetry about the equator (at 90°) was assumed. The original pressure was preserved by using both a density perturbation n_1 and a temperature perturbation $T_1/T_0 = -n_1/n_0$. The numerical solutions used eqs. (1) – (4) to calculate the radial derivatives at all points in the theta grid at a

-6-

starting radius. Then these derivatives were used to find the functions at a higher radius. We made the usual check of changing the radial step size; starting from a large radial step, the step was decreased until the solution was independent of step size.

The perturbation was a Gaussian in latitude. Other initial conditions were typical of solar wind models. Figure 1 shows the total praticle density plotted three times as the solutions precede outward in radius. We see that a density perturbation grows as it is convected outward. Figure 2 shows the history of the amplitude of the central maximum relative to the two minima on either side. Data from the numerical solution are plotted in this figure and compared favorably with the curve (amplitude) $\sim r^{1/\zeta}$, with $\zeta = 5.7^{\circ}$. Notice that the 5.7° wide Gaussian would also be a reasonable fit to the peak in Figure 1 (c). Since the initial perturbation has a Gaussian form, the short wavelengths in a Fourier decomposition of the perturbation should predominate and give the growing structure an increasingly narrower width. (See the linear growth rate derived above.) The results shown in Figures 1 and 3 confirm the expected behavior with the peaks getting progressively narrower and the distance between the central maximum and the two minima on either side gradually decreasing.

We have not determined the ultimate nonlinear development of these radial streamers. Neither have we provided a mechanism which prevents their angular widths from decreasing

indefinitely until they are unobservable. However, it is easy to speculate that a magnetic field and turbulence would complete the model which has just been introduced. The main effect of an imbedded magnetic field on the instability presented here would be that of making it difficult for particles to cross field lines. This would limit the transverse velocity and slow the growth of the instability. Furthermore, this convective instability produces neighboring streamers with very different radial speeds. The high rate of shear should lead to a transition to turbulence. The effective transport coefficients would then increase above the laminar flow values and thereby limit the ultimate evolution of the amplitude and width of a streamer.

Nonetheless, we have shown that the perturbed expanding gas might evolve into alternating streams of high density/low speed and low density/high speed regions from purely hydrodynamical considerations. This onset mechanism would apply to any appropriate astrophysical environment to the extent that hydrodynamic boundary conditions are applicable in an underlying collisional atmosphere. Wherever the expanding gas is subsonic, perturbations will grow into radially oriented structures.

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Figure Captions

Figure 1. The evolution of the latitude profile of a streamer-like density perturbation. (a) Starting conditions: At $r = 1.3 \times 10^{11}$ cm, $T = 1.45 \times 10^6$ K, $u = 1.4 \times 10^6$ cm/sec, $n = 5.1 \times 10^6$ cm^{-3}, $dT/dr = -2 \times 10^{-6}$ K/cm, and $\Delta n = (0.1)n \exp\{(\theta-\theta_0)^2/(10°)^2\}$ (b) Density profile at $r = 1.4 \times 10^{11}$ cm. (c) Density profile at $r = 1.66 \times 10^{11}$ cm. The particle density is n, the solar latitude is θ and r is radial distance drom the center of the sun. The smooth curves are the results from the numerical solution.

Figure 2. The evolution of the amplitude of a solar streamer. $\Delta n/n \equiv (n_{max}-n_{min})/(0.5)(n_{max}+n_{min})$. The results from the numerical solution are given by the symbol x; the dashed curve is a plot of $\Delta n/n = (0.0022)^{\kappa}(r/10^{11})^{1/\tau}$ where $\tau = 0.1$ rad ($=5.7°$).

Figure 3. The evolution of the width of a streamer. The width, θ_W, is defined as the angular width at half of the central maximum in the perturbation.
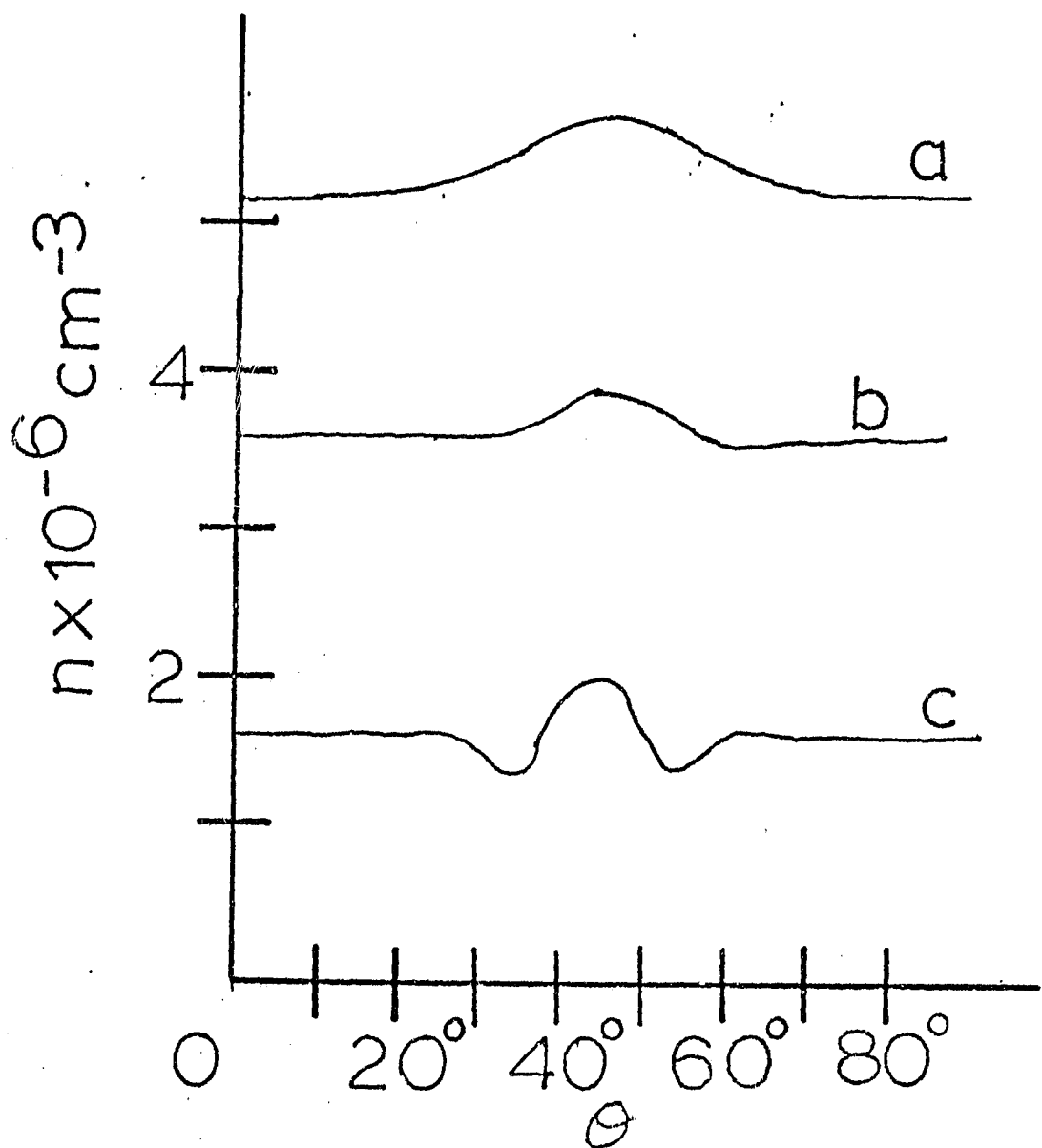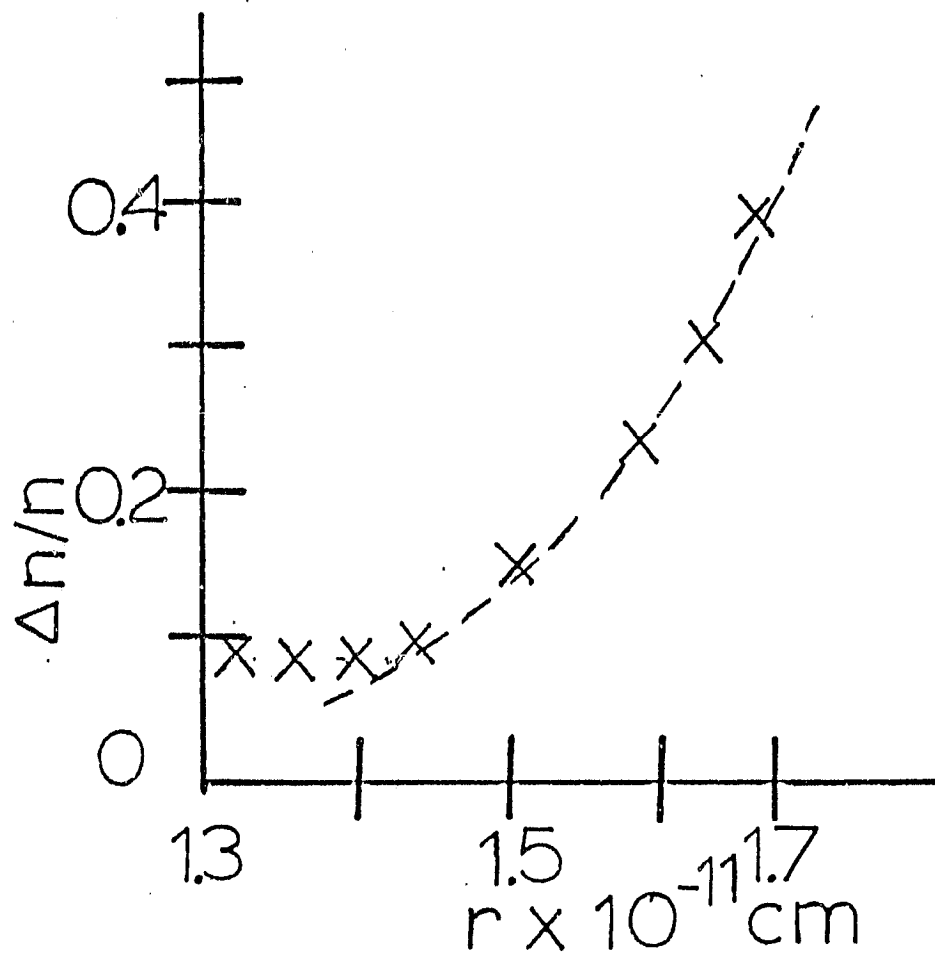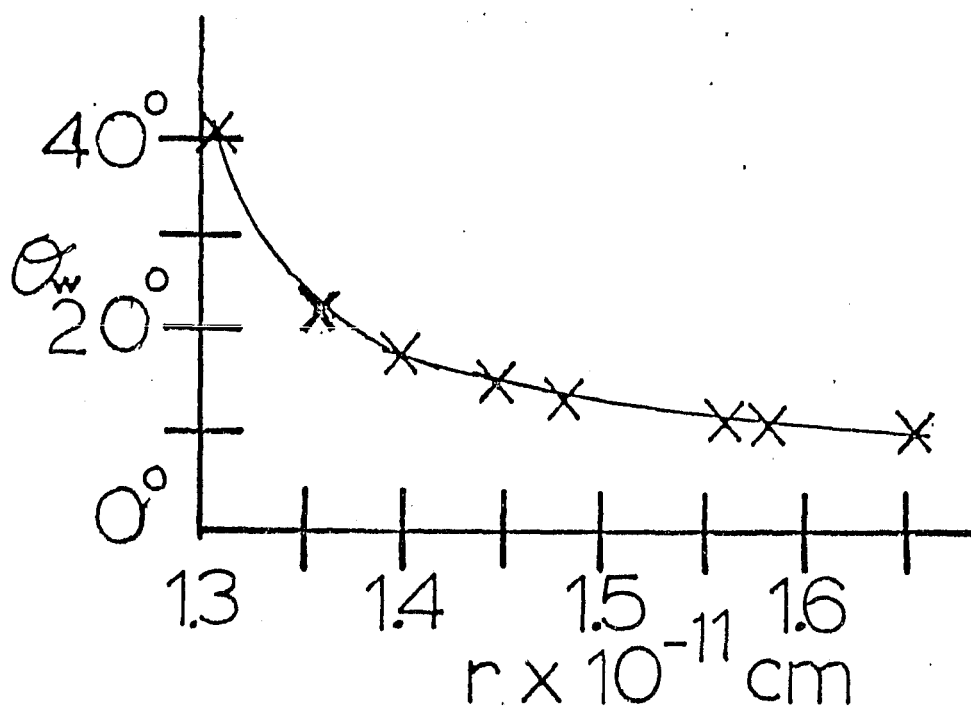
ONSET OF TURBULENCE IN A
MAGNETIZED BOUNDARY LAYER*

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ABSTRACT

When the Orr-Sommerfeld equation is derived for a conducting fluid with an imbedded magnetic.field, the first appearance of amplified perturbations occurs at a higher Reynolds number than is expected without the field.

Solutions for the Orr-Sommerfeld equation which determines the stability of small disturbances in neutral laminar flow past a wall are well known[1]. Above a critical Reynolds number the disturbances grow and eventually develop into turbulence. However, solutions for the analogous problem of boundary layer flow in a magnetic field have not been achieved. In this case there should be important implications for both astrophysical[2] and fusion[3] research since, at the present time, the impact of turbulence on the transport properties of magnetized media is not well understood. In this paper we have extended the Orr-Sommerfeld equation to the case of a weakly perturbed magnetic field in a resistive Blasius boundary layer.

The coordinate system is shown in Figure 1. The field is parallel to the flow so it has no effect on the unperturbed flow. We derive the new Orr-Sommerfeld equation using standard notation[4] where the zeroth and first order functions and the stream function are defined by

$$\vec{U}_0 = (U_0(y), 0, 0); \quad \vec{B}_0 = (B_0, 0, 0); \quad \vec{b}_1(x,y) = (b_x, b_y, 0)$$
$$\vec{u}_1(x,y) = (u_1, v_1, 0); \quad \Psi = \varphi(y) e^{i(\alpha x - \beta t)}; \quad u_1 = \frac{\partial \psi}{\partial y}; \quad v_1 = -\frac{\partial \psi}{\partial x} \quad (1)$$

in which α and β are the wavenumber and frequency of the disturbance. If both b_1 and u_1 are of the form $f(y) \exp i(\alpha x - \beta t)$ then the magnetic field equation leads to

$$\partial \vec{B}/\partial t = \nu_m \nabla^2 B + (\vec{B} \cdot \nabla) \vec{V} - (\vec{V} \cdot \nabla) \vec{B}$$
$$-i\beta b_x = \nu_m(-\alpha^2 + \partial_y^2) b_x + B_{0x} i(\alpha u_1) - u_0 i\alpha b_x + b_y \partial_y u_0$$
$$-i\beta b_y = \nu_m(-\alpha^2 + \partial_y^2) b_y + B_{0x} i\alpha v_1 - u_0 i\alpha b_y \quad (2)$$

in which $\nu_m = 1/\sigma\mu$; σ and μ are respectively the conductivity and magnetic permeability. Introducing the stream function, eq. (3) becomes

$$-i\beta b_y = \nu_m(-\alpha^2 b_y + b_y'') - u_0 i\alpha b_y + B_{0x}\alpha^2\varphi \qquad (4)$$

From conservation of momentum one finds

$$(\partial/\partial t + \vec{V}\cdot\nabla)\vec{V} = \frac{1}{\rho}\nabla P + \frac{1}{\rho\mu}(\nabla\times\vec{B})\times\vec{B} + \nu\nabla^2\vec{V} \qquad (5)$$

Taking the first order x and y components of (5).

$$-i\beta u_1 + u_0 i\alpha u_1 + v_1 \partial_y u_0 = -\frac{1}{\rho}\partial_x P_1 + \nu(-\alpha^2 u_1 + u_1'') \qquad (6)$$

$$-i\beta v_1 + u_0 i\alpha v_1 = -\frac{1}{\rho}\partial_y P_1 + \frac{B_{0x}}{\rho\mu}(\partial_x b_{1y} - \partial_y b_{1x}) + \nu(\partial_x^2 + \partial_y^2)v_1 \quad (7)$$

By eliminating the pressure between (6) and (7) and using the stream function, one achieves

$$i\beta(\varphi'' - \alpha^2\varphi) + i\alpha^3 u_0\varphi - i\alpha\varphi'' u_0 + i\alpha\varphi u_0''$$

$$= -\frac{B_0}{\rho\mu}(\alpha^2 b_y + i\alpha b_x') + \nu(-\alpha^4\varphi + 2\alpha^2\varphi'' - \varphi^{IV}) \qquad (8)$$

The dimensionless variables are:

$$\bar{x} = x/d \quad ; \quad \bar{y} = y/d \; ; \quad \bar{\alpha} = \alpha/d \; ; \quad \overline{b_i} = b_i/B_o \; ;$$

$$R = U_m d/\nu \; ; \quad R_m = U_m d/\nu_m \; ; \quad \overline{\varphi} = \varphi/U_m d \; ; \quad \overline{U_o} = U_o/U_m$$

$$\overline{\beta} = \beta d/U_m \quad N_A = (\rho\mu U_m^2 / B_o^2)^{1/2} \tag{9}$$

where d is the width of the boundary layer, R_m is the magnetic Reynolds number, N_A is the Alfven number, and U_m is the fluid velocity at $y = \infty$. We drop the use of bars and assume that all symbols after this refer to dimensionless variables. Equations (4) and (8) become

$$-i\beta b_y = (-\alpha^2 b_y + b_y'')/R_m - U_o i\alpha b_y + \alpha^2 \varphi \tag{10}$$

$$-i\alpha(\varphi'' - \alpha^2\varphi)(U_o - \tfrac{\beta}{\alpha}) - i\alpha\varphi U_o'' - (\varphi^{IV} - 2\alpha^2\varphi'' + \alpha^4\varphi)/R \tag{11a}$$

$$= -(\alpha^2 b_y - b_y'')/N_A^2$$

$$= -(R_m/N_A^2) * (\alpha^2\varphi - U_o i\alpha b_y + i b_y \beta) \tag{11b}$$

where the prime notation means $\partial/\partial y$.

The left side of eq. (11) is the usual Orr-Sommerfeld equation. The right side is our new result. It shows that the first order magnetic perturbations in the boundary layer approximation simply add a new term that depends on the Alfven number and the magnetic Reynolds number.

As $R_m \to 0$ there is no new effect because the low conductivity permits particles to freely flow across magnetic field lines.

In order to solve these equations, we have altered the method of Mack[1] to include our equations (10) and (11). At $y = \infty$:

$$U_o(\infty) = 1 \; ; \; \varphi \propto e^{-cy} \; ; \; b_y \propto e^{-cy} \tag{12}$$

Two analytical solutions are known in the free stream far from the wall. The first, called the inviscid solution, has $c = \alpha$ in eq. (12). This requires $b_y = \alpha^2 \varphi / i(\alpha - \beta)$ in eq. (10). The second "viscous" solution has $\psi \propto \exp(-py)$ with $p^2 = \alpha^2 + i R(\alpha - \beta)$ when there is no field. We find p when $B_0 \neq 0$ by first eliminating b between equations (10). and (11a). The result is linear in φ and can be solved to yield a cubic equation for p as a function of α, R, R_m, and N_A from which p is obtained numerically.

The boundary conditions at $y = 0$ are:

$$\varphi(o) = \varphi' = b_y = b_y' = b_y'' = 0 \tag{13}$$

Mack's method of solution consists of beginning with an initial estimate of α and β and then numerically integrating the two known solutions from $y = \infty$ toward $y = 0$. The eigenvalues α and β are altered until a linear combination of the two solutions fits the $y = 0$ conditions. Our solutions reduce to Mack's in the limits $R_m \to 0$ and $B_0 \to 0$.

-5-

We have solved for the wave number α as a function of β, R, R_m, and N_A. (This is called the space-amplified case, i.e. a perturbation at frequency β grows spatially at the rate exp $(i\alpha x)$ as it is convected along the boundary.) The dimensionless frequency parameter $F = \beta/R$ remains constant as a perturbation is convected along the boundary layer. (Schlicting[4]). Figure 2 presents the region of instability in the F-R plane for comparison purposes. Inside the curves, α_i is negative and perturbations will grow. The curve for $R_m = 0$ reduces to the non-magnetic case investigated by others.

Next we consider changes in the magnetic energy density measured by N_A. When $b_y \ll \varphi$, eq. (11b) is linear in φ and the quantity (R_m/N_A^2) reduces to a single parameter. In Table I we keep this parameter constant. The eigenvalue α_r is nearly constant for the entire range. The growth rate α_i is constant for $N_A \ll 1$. In Figure 3 we compare a set of critical Reynolds numbers (that is, the minimum Reynolds number at which perturbations will be amplified) with a corresponding set of magnetic Reynolds numbers. Notice that the curves of Figure 2 shift toward higher R with increasing conductivity and the curves of Figure 3 show that the flow will require a new and larger R_c before perturbations are amplified. For example: given the fluid parameters $R_m = 1$ and R = 1300, turbulence could not develop if the magnetic energy density were greater than $B^2/\mu\rho v^2 = .1$.

Thus, as the energy in the magnetic field approaches the kinetic energy of the flow, its influence becomes increasingly important

through the inhibiting effect of the magnetic field on the amplication of stochastic fluctuations. Specifically, in the presence of a magnetic field, a conducting fluid experiences an increasing delay (i.e., the critical Reynolds number increases) in the onset of turbulence.

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FIGURE CAPTIONS

Figure 1.   Coordinate system for the Orr-Sommerfeld Equation. A
             viscous fluid flows past a semi-infinite wall. The
             fluid has an imbedded magnetic field.

Figure 2.   Neutral stability curves for disturbances in magnetic
             laminar flows. $F = \beta/R$ is the frequency of the
             disturbance. The imaginary part of the wavenumber is
             negative inside the curves indicating that
             disturbances will grow in space. The changed values of
             the magnetic Reynolds number, R_m are: a,$=0$;b,
             $R_m=0.05$; C, $R_m = 0.1$.

Figure 3.   Critical Reynolds number (R_c) for turbulent onset as
             a function of the magnetic Reynolds number (R_m).
             The leftmost curve, a, shows the case for magnetic
             energy density, ($B^2/\mu\rho v^2$), 10 times the kinetic
             energy density; the center curve b is for magnetic
             energy equal to kinetic energy; and for the right
             curve, c, the magnetic energy is one tenth of kinetic
   energy.

Table Caption

Table I. Sensitivity of Solution Eigenvalues to Changes in Magnetic Energy Density. Here: $R = 1280$; $\bar{\beta} = .0768$; $F = \bar{\gamma}/R = 6 \times 10^{-5}$; and $(N_A)^{-2} = B^2/\mu \rho v^2$.
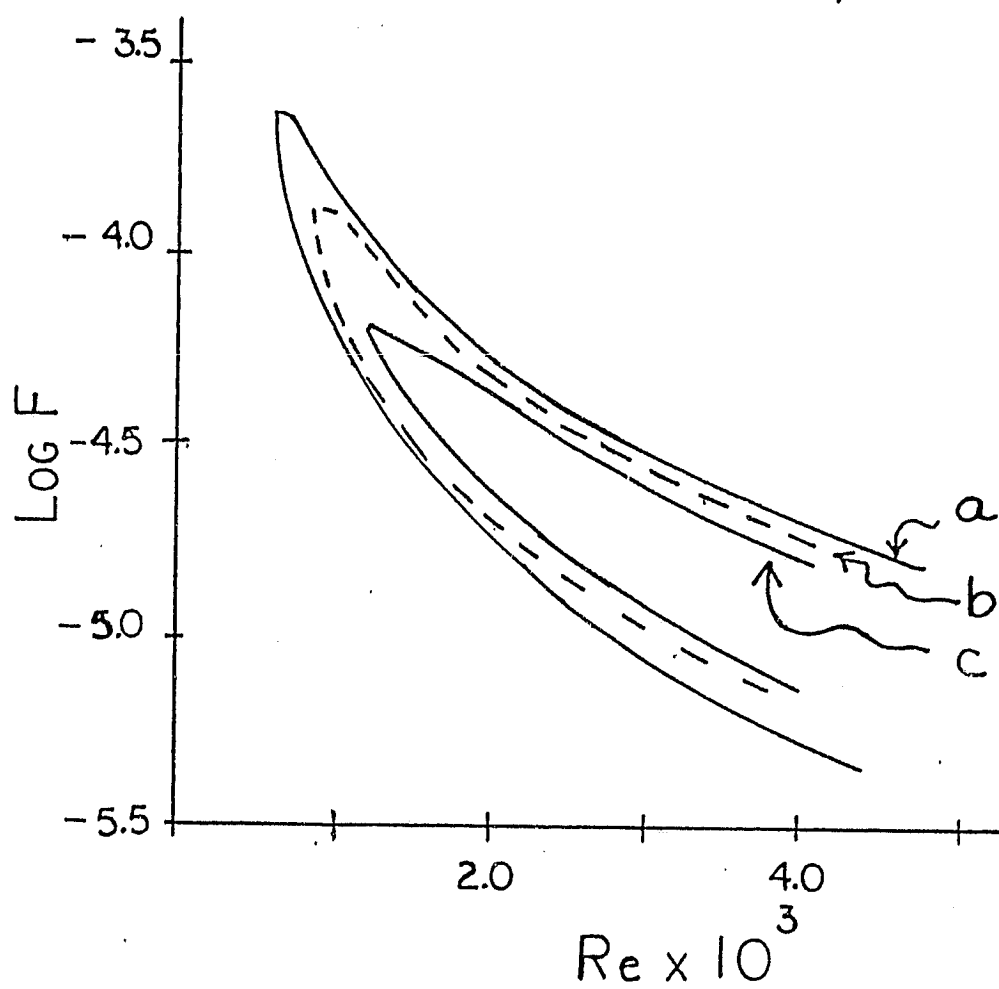
TABLE I

$(N_A)^{-2}$	R_m	α_r	α_i
1000	.0001	.228487	$.177975 \times 10^{-4}$
100	.001	.228487	$.1776 \times 10^{-4}$
10	.01	.228487	$.1739 \times 10^{-4}$
5	.02	.228485	$.16976 \times 10^{-4}$
1	.1	.22848	$.1368 \times 10^{-4}$
.5	.2	.228474	$.97 \times 10^{-5}$
.2	.5	.228474	$-.3086 \times 10^{-5}$
.1	.1	.22842	$-.2460 \times 10^{-4}$
.01	1	.227665	$-.0005188$
.005	20	.22662	$-.001248$
0	0	.23056	$-.0087454$
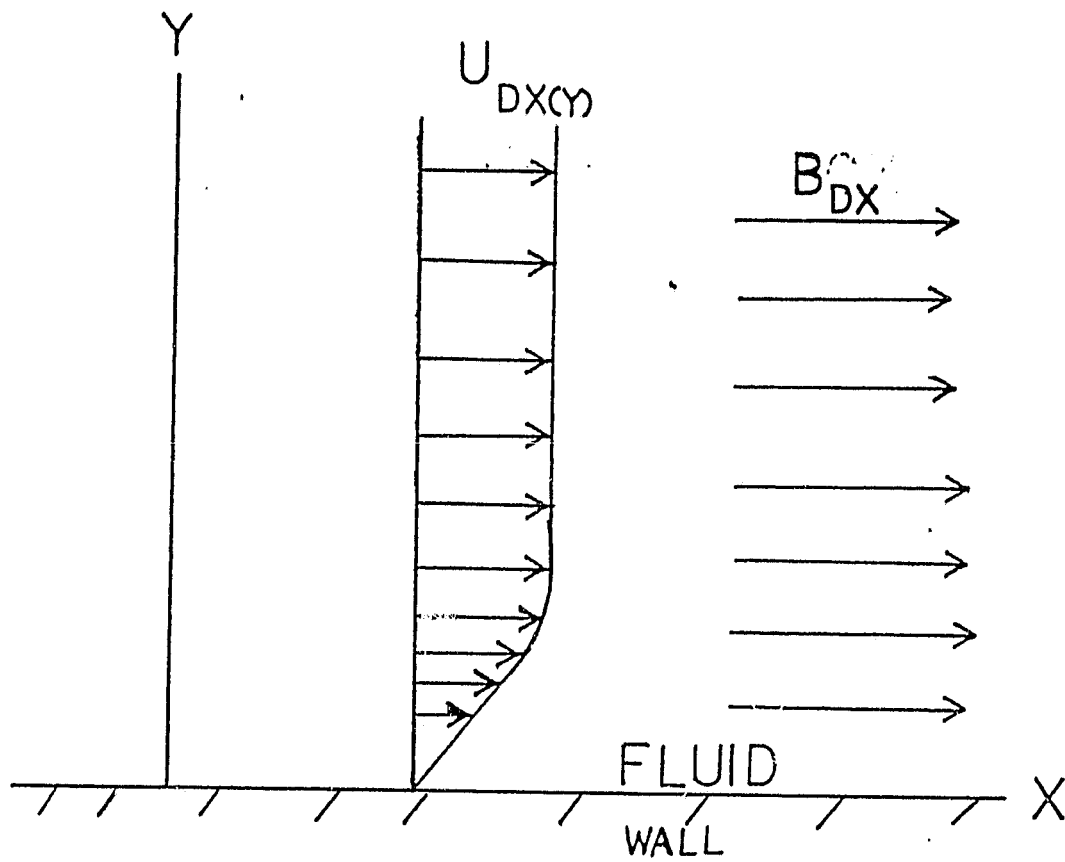
Y

$U_{DX(Y)}$

B_{DX}

FLUID

WALL

X
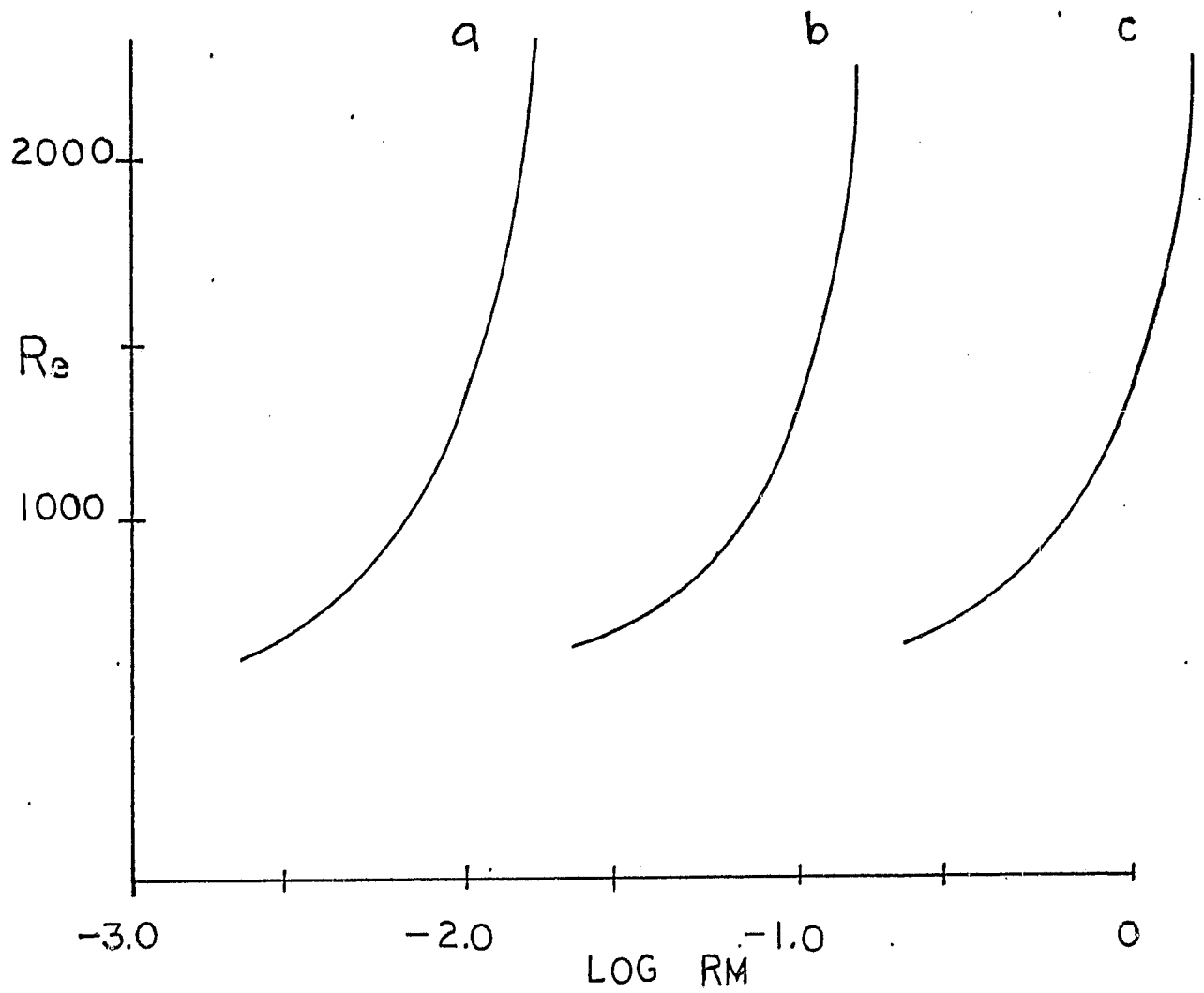
Phase Coherent Turbulence in the Early Cosmos

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In collisional fluids and plasmas, reaction-diffusion instabilities arise because of an inhibition of dispersion by a temporally competitive non-equilibrium process[1]. A use of the Klimontovich formulation has predicted the recently observed reaction distortions in unperturbed bursting nonequilibrium flow at transition to turbulence[2]. The use of a reaction·diffusion mechanism provides a good qualitative explanation of this effect[3] as well as the soliton-like turbulent bursts found in collisional shock fronts[4]. In this paper, we shall explore the implications of this mechanism for the early condensation of galactic matter.

First, lets reconsider the role which thermal fluctuations might have played in the formation of galaxies. Classically, one begins with a balance between the gravitational instability and pressure oscillations for an inhomogeneity of mass M in a universe of total initial chaos. The limits on the strengths of the initial inhomogeneities are set by the observed degree of isotropy in the microwave gackground. A critial mass is derived[5], M_J, whose evolution in time is summarized in Figure 1, from which is derived a minimum undamped mass not in disagreement with present observations. These same arguments can show that the preferred galactic shape should be pancake rather than spherical and can give a good qualitative relationship between the final radius of a galaxy, its total original mass, (in-the-beginning) and its total angular momentum.

When total initial chaos is removed, the source; of instabilities in the early cosmos become the same as that of any fluid of comparable

thermodynamic parameters. Thus, the onset mechanisms are determined by the natural evolution of appropriate local scales and the development of natural local instabilities in the case of the collision dominated epoch. In this context, therefore, natural thermal instabilities (rather than the artificially imposed hierarchy derived post hoc) can generate turbulence. Furthermore, if the front of the evolving cosmos can be treated as a tangential discontinuity, then the recent discoveries concerning the application of Orr-Sommerfeld techniques to transition at discontinuities with natural instabilities can be applied[6].

Consider turbulent eddies which are locally subsonic in the radiation era. Their importance to the growth of turbulence is determined by the local Reynolds number. If we ignore magnetic effects, the relationship between characteristic turbulent time and critical Reynolds number is shown in Figure 2 for typical cosmological values. By comparing these results with Figure 1, we see that the onset of turbulence is delayed substantially beyond the classically derived times; the local Reynolds numbers are too low. Thus, this new model predicts that galactic condensation was much later than is usually thought, well into the matter-dominated era.

Next consider turbulent eddies which are locally supersonic in the matter-dominated era. Since the turbulent intensity is still quite small, the onset mechanisms are still determined by the techniques from incompressible turbulent theory[7]. At transition, the energy of the turbulence is entirely contained in

the phase-coherent turbulent bursts. However, since the universe
is continuing to expand, nonturbulent regions are experiencing
a continuous decrease in local Reynolds number and are therefore
increasingly unlikely to produce turbulent fluctuations which
are amplified. This forces a reinterpretation of the meaning
of a new M_J, viz., it should represent a maximum as well as a
minimum on the typical galactic size, in accordance with observation.
However, the condensing turbulent bursts interact with the surrounding
sea of gases; these interactions produce, due to the naturally
probabilistic characteristics, a variety of inhomogenieties.
This variety, in turn, leads to the taxonomy of galaxies ranging
from the elliptical to the various spiral types to the irregular
shapes. Nonetheless, the frozen in feature of the spiral arms
is a resonable consequence of evolution with minimum surrounding
cea interaction and fixed initial turbulent parameters.

Two new consequences can be determined. (1) The class
or spectrum of galaxies which evolved during the early epoch must
be complete. New galaxies will, if they are produced ,result
from entirely different physical phenomena. (2) If the pre-
sently observed galaxies all developed during the early epoch
at or near the critical Reynolds number, then an upper limit
on the "missing mass" of the universe can be calculated. That
is, in this model given the size of a typical galaxy, the total
mass of the universe is no longer a free parameter. (3) Since
these onset mechanisms imply a distortion of reaction processes,
substantial differences in the distribution of nuclear matter
must exist between galatic and intergalatic space. Specific-

quantitative predictions are being calculated and will be reported
in a paper to follow.