NASA Technical Memorandum 85642

DESCRIPTION OF THE COMPUTATIONS AND PILOT PROCEDURES
FOR PLANNING FUEL-CONSERVATIVE DESCENTS WITH A SMALL
PROGRAMMABLE CALCULATOR

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MAY 1983

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SUMMARY

The Federal Aviation Administration (FAA) has implemented an automated, time-based metering form of air traffic control with profile descent procedures for arrivals into the terminal area. These concepts provide fuel savings by matching the airplane-arrival flow to the airport acceptance rate through time-control computations and by allowing the pilot to descent at his discretion from cruise altitude to a designated metering fix in an idle-thrust, clean-configuration (landing gear up, flaps zero, and speed brakes retracted). Substantial fuel savings have resulted from these procedures, but air traffic control (ATC) workload is high since the radar controller maintains time management for each airplane through either speed control or path stretching with radar vectors. Pilot workload is also high since the pilot must plan for an idle-thrust descent to the metering fix using various rules of thumb.

The National Aeronautics and Space Administration (NASA) has developed an airborne descent algorithm compatible with time-based metering procedures and profile descent procedures designed to improve the accuracy of delivering an airplane to a metering fix at a time designated by the ATC system. This algorithm provides open-loop guidance for an airplane to make an idle-thrust, clean-configured descent to arrive at the metering fix at a predetermined time, altitude, and airspeed. The algorithm may also be used for planning fuel-conservative descents when time is not a consideration.

The algorithm was programmed on a small programmable calculator for use with a DC-10 airplane. This report describes the descent algorithm computations and the required vertical performance modeling for the DC-10 airplane.

INTRODUCTION

In an effort to improve the efficiency of terminal area operations, the Federal Aviation Administration (FAA) has implemented an automated time-based metering form of air traffic control with profile descent procedures. The time-based metering concept is based upon airplane arrivals crossing a metering fix (typically 30 to 40 n. mi. from the airport) at a specified altitude, airspeed and time. This results in derandomizing the arrivals at high altitudes resulting in a reduction of the low altitude, high fuel consumption flight required for sequencing airplanes to a common final approach path. In addition, time-based metering allows the traffic to absorb delays at cruise altitudes or while on the ground prior to takeoff, resulting in even greater fuel savings. The proper sequencing and spacing of enroute traffic also allows for increased airport productivity (ref. 1 and 2). The profile descent procedure allows the pilot to plan and fly a descent which is fuel conservative for his particular airplane resulting in an additional fuel savings.

With the current time-based metering/profile descent procedures, the air traffic controller is responsible for the time management of each aircraft. The controller can adjust the airplane's time of arrival at the metering fix by increasing the flight path through heading changes or by requesting the pilot to change speed. The pilot is responsible for crossing the metering fix at the proper airspeed and altitude and must plan the descent carefully if fuel is to be conserved. The time management and the descent planning are both workload intensive and interdependent. Since limited or no guidance is available to either the controller or the pilot, their tasks must be accomplished independently through various rules of thumb and past experience. With this present operational concept, airplanes will typically cross the metering fix with a time accuracy between 1 and 2 minutes (ref. 3).

During the summer of 1979, the National Aeronautics and Space Administration (NASA) developed and flight tested in its Transport Systems Research Vehicle (TSRV), previously designated the Terminal Configured Vehicle B-737 research airplane, a flight-management descent algorithm designed to provide closed-loop guidance for a fuel conservative descent and to reduce the metering fix crossing time dispersion. The flight management algorithm descent computations were based on an idle-thrust, clean-configured descent (landing gear up, flaps and spoilers retracted), to arrive at the metering fix at the proper altitude, airspeed, and ATC-designated time. The results of the flight tests showed that the closed-loop guidance provided by the guidance and display system could reduce the crossing time dispersion to approximately 12 seconds (ref. 4).

This research was continued in June of 1981 with a T-39A (Sabreliner) airplane to determine if similar results could be obtained with open-loop guidance and a conventional complement of cockpit instrumentation (ref. 5). A version of the flight management descent algorithm was implemented on a small programmable calculator. Open-loop guidance was provided by the calculator in the form of the Mach number and airspeed at which the descent should be flown and the point at which the pilot was to reduce the thrust to flight idle and begin the descent. The descent was then flown with reference to the airplane Mach

and airspeed indicators. Flight tests using this open-loop guidance resulted in a time dispersion crossing the metering fix of approximately 20 seconds.

Having determined the viability of the open-loop descent guidance, additional research has been planned to evaluate the feasibility of such a descent planning tool in an airline operational environment. The airplane selected for this study was the McDonnell Douglas DC-10. This report contains a description of the programmable calculator software and of the DC-10 descent performance model used in the algorithm computations. A later report will present the results of the feasibility study.

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4

SYMBOLS AND ABBREVIATIONS

ATC	air traffic control
a_0, a_1, a_2	coefficients for quadratic curve fits of altitude as a function of time
b_0	modeled vertical speed at sea level, ft/sec
b_1	slope of the linear model for h_{IAS}, ft/sec/ft
c_0	constant in the model for $\dot{h}_{M_d}$, $\sec^2$/ft
c_1	constant in the model for $\dot{h}_{M_d}$, ft
DME	distance measuring equipment
$D_{w,h}$	magnetic wind direction evaluated at altitude h, deg
$D_{w,s}$	magnetic wind direction computed for sea-level altitude, deg
$\dfrac{dD_w}{dh}$	wind direction gradient with respect to altitude, deg/ft
$\dfrac{dS_w}{dh}$	wind speed gradient with respect to altitude, knots/ft
GW	gross weight, lb
GSc	ground speed at cruise altitude, knots
H	pressure altitude, ft
h	geopotential altitude, ft
h_c	cruise altitude, ft
h_{MF}	metering-fix altitude, ft
h_{XO}	altitude at transition from constant-Mach descent to constant-airspeed descent, ft
$\dot{h}$	rate of change of altitude, ft/sec

$\dot{h}_{IAS_d}$	rate of change of altitude evaluated at indicated airspeed IAS_d, ft/sec
$\dot{h}_{M_d}$	rate of change of altitude evaluated at Mach number M_d, ft/sec
IAS	indicated airspeed, knots
IAS_d	indicated airspeed used during descent, knots
$IAS_{d,initial}$	initial descent indicated airspeed for speed iteration computations, knots
$IAS_{d,i}$	descent indicated airspeed computed on ith iteration, knots
$IAS_{d,max}$	maximum operational descent indicated airspeed, knots
$IAS_{d,min}$	minimum operational descent indicated airspeed, knots
IAS_{MF}	indicated airspeed to cross metering fix, knots
IDL DME	calculator display showing point where thrust should be reduced to flight idle, n. mi.
K	interpolation factor computed for speed iteration purposes
K_{gw}	gross-weight multiplication factor for altitude rate
ℓ_t	distance between entry fix and metering fix, n. mi.
M	Mach number
M/IAS	Mach number and indicated airspeed
$M/IAS_{d,initial}$	initial Mach number and indicated airspeed for speed iteration computations, knots
MSL	mean sea level
M_c	cruise Mach number
M_d	descent Mach number
$M_{d,initial}$	initial descent Mach number for speed iteration purposes
OAT	outside air temperature, $^\circ C$
$S_{w,h}$	wind speed evaluated at altitude h, knots

$S_{w,s}$	wind speed computed for seal-level altitude, knots
t	time, sec
TAS	true airspeed, knots
TRK	airplane magnetic track angle along ground, deg
TSRV	Transport Systems Research Vehicle
T_c	temperature measured at cruise altitude, ^{O}K
$T_{isa,h}$	International Standard Atmospheric temperature at altitude h, ^{O}K
T_o	standard sea-level air temperature, ^{O}K
T'_o	nonstandard sea-level air temperature, ^{O}K
$T_{st,c}$	static air temperature measured at cruise altitude, ^{O}K
$T_{st,h}$	static air temperature at altitude h, ^{O}K
t_E	time error for descent-speed convergence criteria, sec
t_{EF}	time that entry fix was crossed, hr:min:sec
VOR	very high frequency omnirange navigation radio
$W_{H,c}$	difference between actual and computed ground speeds at cruise altitude, knots
$W_{H,h}$	head-wind component along airplane ground track evaluated at altitude h, knots
$\ddot{x}$	acceleration, knots/sec
$\Delta \ell_j$	length of path segment j, n. mi.
ΔT	difference between actual temperature and standard temperature, ^{O}K
Δt_j	time required to fly on path segment j, sec
Δt_{req}	time required to fly between entry fix and metering fix, sec

DESCRIPTION OF FLIGHT-MANAGEMENT DESCENT ALGORITHM

Description of General Profile

The flight-management descent algorithm computes the parameters required to describe a seven-segment cruise and descent profile (fig. 1) between an arbitrarily located entry fix and an ATC-defined metering fix. The descent profile is computed based on empirical modeling of airplane performance for an idle-thrust, clean-configured descent. Airplane gross weight, wind, and nonstandard-temperature effects are also considered in these calculations.

Figure 1 shows the vertical-plane geometry of the path between the entry fix and the metering fix. Each path segment, starting at the metering fix, is numbered according to the order in which it is calculated by the algorithm. To be compatible with standard airline operating practices, the path is calculated based upon the descent being flown at a constant Mach number with transition to a constant indicated airspeed and all speed reductions made in level flight.

The first segment traversed on the profile is segment 7 which begins at the entry fix and is flown at constant cruise altitude and Mach number. Segment 6 is a relatively short, level-flight path segment in which the pilot reduces thrust to flight idle so the airplane will slow from the cruise Mach number to the descent Mach number. Segment 6 is eliminated if the descent and cruise Mach numbers are the same. Once the descent Mach number is attained, the constant Mach descent segment (segment 5) is started. As altitude is decreased along this path segment, the indicated airspeed will increase because of increasing air pressure. Segment 4 begins when the desired indicated airspeed is attained for descent. The descent is continued along this segment at the desired, constant indicated airspeed. When the metering fix altitude has been reached, the airplane is flown at a constant altitude along segment 3 and slowed from the descent airspeed to the designated airspeed over the metering fix. If the metering-fix altitude is below 10 000 feet MSL and the descent airspeed is greater than 250 knots, segments 1 and 2 are computed for the pilot to comply with the ATC-imposed airspeed limit of 250 knots, or less, below 10 000 feet MSL. Segment 3 then becomes a level-flight segment at 10 000 feet MSL where the airspeed is reduced to 250 knots. The descent is then continued at 250 knots along segment 2. When the metering-fix altitude has been reached, the airplane is flown at a constant altitude along segment 1 and slowed from the descent airspeed to the designated airspeed over the metering fix. This path segment is eliminated if the descent and metering-fix airspeeds are the same.

The flight-management descent algorithm can be used in either of two modes. In the first mode the pilot can enter the desired M/IAS to be flown during descent. The descent profile is then computed based on this descent speed schedule without consideration of a constraint on metering-fix arrival time. This mode would be used when time-based metering is not being used.

The second mode was designed for time-metered operations. In this mode, instead of specifying the M/IAS descent schedule, the pilot enters the time that the entry fix was crossed and the metering-fix arrival time assigned by

ATC. The descent profile is then calculated based on a M/IAS descent schedule, computed through an interative process, that will closely satisfy the crossing time specified for the metering fix. The magnitude of the Mach number programmed for the descent is the same as that used for the cruise segment.

During the descent profile computations in the time-metered mode, a check is made to insure that the descent airspeed IAS_d is within the minimum and maximum speed limits for the particular airplane modeled. For the DC-10 airplane, these limits were

$$220 \leq IAS_d \leq 350 \qquad \text{[knots]}$$

There was an additional constraint that IAS_d would not be less than the airspeed at which the airplane was to cross the metering fix, so that extra fuel would not be required as thrust is added to subsequently increase airspeed. If the ATC-assigned metering-fix crossing time requires a descent speed less than the airplane's minimum descent speed limit, the profile is computed based on the minimum allowable descent speed and a message is displayed to the pilot to "hold" (delay) for an appropriate amount of time. A similar "late" message is displayed with the time error if a descent speed schedule greater than the maximum allowed is required.

Logic Flow of Profile Descent Algorithm

Figure 2 shows the general logic flow of the profile descent computations. Pilot inputs used to compute the profile may be entered prior to flight and modified, as required, prior to the descent. These parameters include cruise altitude and Mach number, airplane gross weight, outside air temperature, entry-fix and metering-fix descriptions, and the course direction to the metering fix. In addition to these parameters, the pilot may enter either a particular Mach number and indicated airspeed to be used during the descent or the entry-fix crossing time and the ATC-assigned metering-fix crossing time.

If the M/IAS descent speed schedule has been entered in the calculator, the computations will be based on a nonmetered traffic environment. The pilot initiates the computations by pushing the "compute" key. The descent profile is then computed in a single iteration, and the display gives the point where thrust should be reduced to flight idle to start the descent.

If the entry-fix crossing time and the ATC-assigned metering-fix crossing time have been entered in the calculator, the time required to fly between the fixes Δt_{req} will be computed and subsequent calculations will be based on a time-metered traffic environment. Once the pilot has initiated the computations by pushing the "compute" key, an interative process is started to determine an appropriate IAS descent speed that will satisfy the time constraints. The Mach number used for the descent is set equal to the cruise Mach number in an effort to reduce computational requirements and operational complexity.

The iterative process starts with the computation of the time to complete a descent $\left(\sum_{j=1}^{7} \Delta t_j\right)_{initial}$ at the following descent speed schedule.

$$M_{d,initial} = M_c$$

$$IAS_{d,initial} = \begin{cases} 280 \text{ knots if } IAS_{MF} \leq 280 \\ \\ IAS_{MF} \text{ otherwise} \end{cases}$$

The $IAS_{d,initial}$ value of 280 knots is used because it is the approximate mid-point between the maximum and minimum allowable descent airspeeds and because it is a descent speed typically used by the airlines.

A check is made with the following transition-time inequality to determine if the time-convergence criteria t_E has been satisfied. (For the purposes of these tests, t_E = 5 sec.)

where
$$\begin{cases} \left| \Delta t_{req} - \left(\sum_{j=1}^{7} \Delta t_j\right) \right| \leq t_E & \text{[sec]} \\ \\ \Delta t_{req} = t_{MF} - t_{EF} & \text{[sec]} \end{cases}$$

If this inequality is satisfied, the computations are complete and the idle-thrust descent point is displayed to the pilot. If the inequality is not satisfied, the descent computations will be repeated using the operational airspeed limits, as follows:

$$M_d = M_c$$

$$IAS_d = \begin{cases} 350 \text{ knots if } \Delta t_{req} < \left(\sum_{j=1}^{7} \Delta t_j\right)_{initial} \\ \\ IAS_{MF} \text{ otherwise} \end{cases}$$

A check is then made to determine if the time criteria has been satisfied or if a speed greater than 350 knots or less than IAS_{MF} would be required to satisfy the time constraints. If the time criteria is satisfied, the idle-thrust descent point is displayed to the pilot. If either the upper or lower airspeed limit must be violated to satisfy the time constraints, the

10

appropriate speed limit will be used in the descent computations and the resulting time error for crossing the metering fix will be displayed to the pilot.

If the time criteria has not been satisfied and neither airspeed limitation will be violated, a revised descent airspeed IAS_d and associated descent time will be computed and compared to Δt_{req}. The iterative process will continue until the time convergence criteria has been satisfied.

The computation of the revised IAS_d is graphically depicted in figure 3, which is a plot of the time required to fly between a specified entry fix and metering fix at a specified cruise Mach number over the complete IAS_d range of the airplane. The descent airspeed is revised through a modified linear interpolation of the desired Δt_{req} within a range of time bounded by an initial value and a computed variable value. The initial value $\left(\sum\limits_{j=1}^{7} \Delta t_j \right)_{initial}$ was the resulting time computed with the $M/IAS_{d,initial}$ descent speed schedule on the first iteration. The variable time value $\left(\sum\limits_{j=1}^{7} \Delta t_j \right)_{i-1}$ is the time computed for the $M/IAS_{d,i-1}$ descent speed schedule on the last (i-1) iteration. The revised descent speed schedule $M/IAS_{d,i}$ is computed as follows:

$$M_d = M_c$$

$$IAS_{d,i} = IAS_{d,initial} + (IAS_{d,i-1} - IAS_{d,initial})K$$

$$- \frac{(IAS_{d,i-1} - IAS_{d,initial})}{\left(\sum\limits_{j=1}^{7} \Delta t_j \right)_{initial} - \left(\sum\limits_{j=1}^{7} \Delta t_j \right)_{i-1}} \sin(180\ K) \qquad [knots]$$

where $K = \dfrac{\left(\sum\limits_{j=1}^{7} \Delta t_j \right)_{initial} - \Delta t_{req}}{\left(\sum\limits_{j=1}^{7} \Delta t_j \right)_{initial} - \left(\sum\limits_{j=1}^{7} \Delta t_j \right)_{i-1}}$

and i is the ith iteration.

11

The last term of the computations for $IAS_{d,i}$ is a compensation factor for the difference in curvature between the plot of time required to fly between the entry fix and the metering fix and the straight line used in the linear interpolation.

Empirical Representation of Airplane Performance Characteristics

Computer memory limitations with the programmable calculator preclude the use of detailed aerodynamic and performance tables to represent the airplane for profile decent calculations. Instead, an empirical model of the performance of the DC-10 airplane was developed from flight data collected during idle-thrust clean-configured descents and during level-flight speed reductions.

A portable voice recorder, a stop watch, and conventional flight instruments were used to collect data for descent-performance modeling. Altitude, speed, temperature, and time were recorded at altitude increments of approximately 500 feet during the descents. Speed, temperature, and time were recorded at 10-second intervals during constant-altitude speed changes. Gross weights were recorded at the beginning and end of each test run.

Constant IAS Descent Model.- The performance model for the constant indicated airspeed descents consisted of linear approximations of vertical speed as a function of altitude for a range of airspeeds between 220 knots and 350 knots. The vertical speed was adjusted to compensate for variations in gross weight.

The first step in the development of the vertical performance model for constant IAS_d was to approximate, for each descent, altitude as a function of time as a quadratic equation through a least squares curve fit analysis. The general form assumed for this equation was

$$h = a_2 t^2 + a_1 t + a_0 \qquad [ft]$$

Figure 4 shows a typical plot of the data and resulting curve fit for a descent flown at a constant IAS_d of 280 knots.

Vertical speed was determined by differentiating this equation with respect to time. This resulted in an equation of the following form:

$$\dot{h} = 2a_2 t + a_1 \qquad [ft/sec]$$

A plot of $\dot{h}$ as a function of h was then developed for each descent. Figure 5 shows typical data for various descent speeds. These plots indicated that vertical speed was approximately linear as a function of altitude and were modeled with an equation of the form

$$\dot{h} = b_1 h + b_0 \qquad [ft/sec]$$

12

The slope b_1 was approximately the same for all descent speeds and equal to -3.5×10^{-4} ft/sec/ft. The modeled vertical speed at sea level b_0 was corrected for gross weight variations by dividing by the gross weight correction factor K_{gw} (to be subsequently discussed). This data varied exponentially as a function of airspeed. A plot of this data and the exponential model of b_0 is shown in figure 6. The resulting model of vertical speed, corrected for gross weight variations of the DC-10 airplane was

$$\dot{h}_{IAS_d} = -3.5 \times 10^{-4} h - 3.07783 \, K_{gw} \, e^{8.158681 \times 10^{-3} \, IAS_d} \quad [ft/sec]$$

Constant Mach Number Descent Model.- The performance model for constant Mach number descents consisted of parabolic approximations of vertical speed as a function of altitude for a range of Mach numbers betwen 0.73 and 0.85. The vertical speed was adjusted to compensate for variations in gross weight.

The procedures used to develop the descent model for constant Mach number were similar to those used for development of the model for constant IAS descents. A quadratic equation of altitude as a function of time was derived for each descent through a least squares curve fit analysis. These equations were differentiated with respect to time to determine vertical speed as a function of time. The vertical speed was adjusted for gross weight variations by dividing by the gross weight correction factor K_{gw} (to be subsequently discussed).

The vertical speed $\dot{h}$ adjusted for gross weight variation, was then plotted as a function of altitude. Typical plots of this data are shown in figure 7. The resulting curves were parabolic in nature and were modeled with an equation of the following form:

$$\dot{h}_{M_d} = \sqrt{\frac{h - c_1}{c_0}} \quad [ft/sec]$$

The magnitude of the coefficient c_0 was subjectively selected based on the shape of a generic parabola that would overlay the descent data. A value of $c_0 = -1.85$ was selected and resulted in the parabola shown by the dashed line in figure 7.

The coefficient c_1 was calculated for each descent at an altitude approximately 2,000 feet below cruise altitude for $c_0 = -1.85$. The resulting values for c_1 are plotted as a function of the descent Mach number in figure 8. A linear regression analysis resulted in an equation of c_1 as a function of the Mach number. However, the slope of this model was increased slightly

13

to insure that c_1 would be greater than the maximum cruise altitude of the airplane (defined by the altitude limits shown on figure 8) thus insuring that an imaginary root would not be obtained from the equation for $\dot{h}$. The resulting model for c_1 is

$$c_1 = 25750 \, M_d + 22167$$

Acceleration performance model.- Acceleration performance data were obtained for idle-thrust clean-configured speed reductions on level flight paths for typical cruise and metering fix altitudes. Indicated airspeed and time data were recorded during the speed reductions. The indicated airspeeds were converted to true airspeeds to reduce computational requirements within the descent algorithm. Figure 9 shows a plot of true airspeed as a function of time that resulted during speed reductions at various altitudes and gross weights. The average slope of each of these test runs was the same. Hence, acceleration for the DC-10 airplane was modeled as constant and equal to

$$\ddot{x} = -1.3 \qquad [\text{knots/sec}]$$

Gross weight variation.- The effects of gross weight variations on the airplane's descent peformance were accounted for with a single multiplication factor applied to the vertical performance models developed for both constant airspeed and constant Mach number descents. The multiplication factor K_{gw} is a linear, nondimensional expression.

The multiplication factor was derived for the DC-10 airplane by plotting the vertical speeds obtained during descents (conducted at the same indicated airspeed) with the gross weight of the airplane. Figure 10 shows the vertical speed at sea level as a function of gross weight for descents conducted at constant airspeeds of 250 knots, 280 knots, and 340 knots. A linear curve fit was applied to the data points for the 280 knot descents since more descents were flown at this speed and since a wider range of gross weight existed in the data. This plot also shows that the model derived for the 280 knot descents may be shifted vertically (maintaining the same slope) to overlay the descent data obtained at the other speeds. Since the same slope could be maintained for all airspeeds, changes to the vertical speed due to gross weight variations were independent of airspeed. The plot in figure 10 was then nondimensionalized by dividing the abscissa by 304,000 pounds and the ordinate by -30.2 feet/second ($b_0 = -30.2$ ft/sec at an airspeed of 280 knots and a gross weight of 304,000 lbs.) This nondimensionalization process allowed the gross weight variation model derived with the 280 knot descent data to be expressed in a form useful for descents at any airspeed. The resulting multiplication factor K_{gw} for gross weight variations was:

$$K_{gw} = -3.863133 \times 10^{-6} \, GW + 2.174392369$$

Approximation of True Airspeed

It is necessary to determine true airspeed from both Mach number and calibrated airspeed, as required by the path segment, so that a headwind component can be added to obtain ground speed for time calculations. True airspeed, as a function of Mach number and static air temperature $T_{st,h}$, was represented by the following equation:

$$TAS = 29.04 \ (T_{st,h})^{1/2} M \qquad [\text{knots}]$$

True airspeed, as a function of calibrated airspeed and altitude h, was approximated as

$$TAS = \frac{IAS}{1 - (0.12 \times 10^{-4})h} \quad [\text{knots}] \qquad \left(\begin{array}{l} h \leq 42{,}000 \text{ ft} \\ 220 \leq IAS \leq 360 \text{ knots} \end{array} \right)$$

Wind Modeling Technique

A two-component linear wind model was used to represent the wind speed and the wind direction as functions of altitude. The coefficients of the wind model were computed via a linear regression analysis of winds aloft reports and forecasts in the descent area. Winds aloft data for the linear regression analysis was inserted through the calculator keyboard in a format similar to that used in standard aviation winds aloft forecasts. Though wind speed and direction from only two altitudes were required to define a wind model, the pilot could choose to insert additional wind data based on both forecasts and pilot reports.

The magnitude of the wind speed and the direction of the wind defined by the linear wind model were computed for each segment of the profile based on the middle altitude of each segment. The following equations were used for these computations.

$$S_{w,h} = \left(\frac{dS_w}{dh} \right) h + S_{w,s} \qquad [\text{knots}]$$

$$D_{w,h} = \left(\frac{dD_w}{dh} \right) h + D_{w,s} \qquad [\text{deg}]$$

A head-wind component for each segment would be computed automatically during the profile computations by multiplying the wind speed with the cosine of the angle between the airplane ground track and the wind direction. A head-wind component correction factor, based on the actual winds encountered during cruise flight could also be added to the wind model if the pilot determined it was necessary. The correction factor was obtained by first computing the difference between the actual ground speed along the cruise segment and the

15

predicted speed based on the modeled winds and cruise Mach number. This component was proportional to altitude and decreased linearly to zero at sea level (h=0). The corrected head wind component $W_{H,h}$ used in the profile computations was defined by the following equation:

$$W_{H,h} = S_{w,h} \cos(D_{w,h} - TRK) + (h/h_c) W_{H,c} \qquad \text{[knots]}$$

Compensation for Effects of Nonstandard Atmospheric Temperature

Various flight instruments, including the Mach meter, the airspeed indicator, and the altimeter, are designed to display correct indications in a standard atmosphere. However, standard atmospheric conditions are rarely encountered. This results in slight errors in indicated altitude and speed. The profile descent algorithm compensates for nonstandard temperatures as they affect the Mach number calculations and altimeter indications.

Nonstandard temperatures are computed by the algorithm based on a standard atmospheric temperature model with a bias correction based on the difference between the actual and the standard temperatures. The standard temperature model is a two-segment linear profile defined as a function of altitude. This temperature model uses a slope equal to a temperature lapse rate of -1.978×10^{-3} °C/ft for altitudes below the tropopause (h $\leq$ 36 152 ft). At higher altitudes (h > 36 152 ft), it was assumed that flight was being conducted within the tropopause where temperature remains constant with changes in altitude. The standard temperature model is represented mathematically as:

$$T_{isa,h} = 216.65 \qquad h > 36\ 152\ ft \qquad \text{[°K]}$$

$$T_{isa,h} = 216.65 + 1.978 \times 10^{-3} (36\ 152 - h) \qquad h \leq 36\ 152\ ft \qquad \text{[°K]}$$

The following bias correction ΔT, representing the difference between the actual temperature measured at cruise altitude T_c and the standard temperature for cruise altitude $T_{isa,c}$ is added to the standard atmospheric temperature profile to completely define static temperature $T_{st,h}$ at any altitude h as follows:

$$\Delta T = T_c - T_{isa,c} \qquad \text{[°K]}$$

$$T_{st,h} = T_{isa,h} + \Delta T \qquad \text{[°K]}$$

The static temperature is then used for conversion of Mach number to true airspeed.

Pressure altitudes H used to define the end points of each segment are corrected to geopotential altitudes by multiplying the pressure altitude by a temperature ratio of nonstandard and standard sea-level temperatures (ref. 6) as follows:

$$h = H \ (T_o'/T_o) \qquad [ft]$$

The standard sea-level temperature T_o is 288.15 °K; the nonstandard sea-level temperature T_o' is computed from the static-temperature model for h = 0.

Computations of Descent Path

The point where the pilot is to reduce power to idle thrust to start the descent was defined by summing the distances required to fly segments 1 to 6. Each segment length was determined by first computing the required time to traverse the segment and then multiplying by the average ground speed computed for the segment. Times for the level-flight segments requiring airspeed or Mach reductions were determined by dividing the required speed change by the deceleration capability of the airplane. Times for the path segments requiring descents were determined by integrating the reciprocal of the vertical speed model for that segment over the altitude change required. The average ground speed at which the airplane was to fly each segment was determined by summing the computed true airspeed and the head-wind component evaluated for each segment.

The cruise segment (segment 7) at level flight and constant Mach number had no influence on the location of the point where idle thrust was to begin. This segment was significant only during the time-metered mode and was used for the calculations to satisfy the time constraints. Segments 1 and 2 were computed only if the ATC-imposed limit of 250 knots indicated airspeed for flight below 10 000 ft MSL was applicable. The details of these calculations are presented in the following paragraphs.

M/IAS transition altitude.- As the airplane descends at a constant Mach number, the indicated airspeed increases because of an increase in the air pressure. The altitude at which the desired descent airspeed is obtained is called the M/IAS transition altitude and defines the point at which the constant-Mach segment ends and the constant-IAS segment begins. The general equation for this transition altitude was determined by equating true airspeed as a function of indicated airspeed and altitude with true airspeed as a function of Mach number and altitude. Solving for altitude results in the following equation to define the altitude for transition of Mach to indicated airspeed

$$h_{XO} = 1.77675 \times 10^5 - \left[(8.90046 \times 10^9) + (3.42936 \times 10^7) \ \frac{IAS_d}{M_d} \right]^{1/2} \qquad [ft]$$

Segment 1.- Path segment 1 is a level-flight segment on which the airplane is slowed from an indicated airspeed of 250 knots to the metering-fix crossing speed. If the metering fix crossing speed is 250 knots, or if the ATC-imposed

17

250 knot maximum airspeed limit for flight below 10 000 ft MSL is not applicable, this segment is not computed.
The equations for time and length in segment 1 are

$$\Delta t_1 = \frac{(IAS_{MF} - 250)}{\ddot{x} \, [1 - (0.12 \times 10^{-4}) \, h_{MF}]} \quad [\text{sec}]$$

where $\ddot{x} = -1.3$ knots/sec, and

$$\Delta \ell_1 = \left[\frac{(IAS_{MF} + 250)/2}{1 - (0.12 \times 10^{-4}) h_{MF}} - W_{H,h_{MF}} \right] \frac{\Delta t_1}{3600} \quad [\text{n. mi.}]$$

Segment 2.- Segment 2 is an idle-thrust descent flown at a constant 250 knots from 10 000 ft MSL to the metering-fix altitude. Segment 2 is not computed if the metering-fix altitude is equal to or greater than 10 000 ft MSL or if the descent speed IAS_d flown on segment 4 is 250 knots or less.

The equations for time and length in segment 2 are

$$\Delta t_2 = \frac{1}{b_1} \ln \left[\frac{h_{MF} \, b_1 + b_0}{10 \, 000 \, b_1 \, T_0'/T_0 + b_0} \right] \quad [\text{sec}]$$

where

$$b_1 = -3.5 \times 10^{-4}$$

$$b_0 = -3.07783 \, K_{gw} \, e^{250(8.158681 \times 10^{-3})}$$

$$\Delta \ell_2 = \left[\frac{250}{\left(1 - 0.12 \times 10^{-4}\right) \dfrac{10,000 + h_{MF}}{2}} - W_{H,(10 \, 000 + h_{MF})/2} \right] \frac{\Delta t_2}{3600} \quad [\text{n. mi.}]$$

Segment 3.- Segment 3 is a level-flight segment on which the airplane is slowed from the descent speed IAS_d to the metering-fix crossing speed (or 250 knots if segments 1 and 2 are computed).

The equations for time and length in segment 3 are

$$\Delta t_3 = \frac{IAS_{MF} - IAS_d}{\ddot{x}[1 - (0.12 \times 10^{-4})h]} \quad [\text{sec}]$$

where $\ddot{x} = -1.3$ knots/sec, and

18

$$\Delta \ell_3 = \left[\frac{IAS_d + IAS_{MF}/2}{1 - (0.12 \times 10^{-4})h} - W_{H,h} \right] \frac{\Delta t_3}{3600} \qquad [\text{n. mi.}]$$

$$h = \begin{cases} h_{MF}, & \text{if segments 1 and 2 are not computed} \\ 10\ 000\ \text{ft}, & \text{otherwise} \end{cases}$$

Segment 4.- Segment 4 is an idle-thrust descent flown at a constant indicated airspeed IAS_d. The descent begins at the transition altitude h_{XO} and ends at the metering-fix altitude (or at 10 000 ft MSL, if segments 1 and 2 are computed).

The equations for the time and length of segment 4 are

$$\Delta t_4 = \frac{1}{b_1} \ln \left[\frac{hb_1 + b_0}{h_{XO}b_1 + b_0} \right]$$

where

$$b_1 = -3.5 \times 10^{-4}$$

$$b_0 = -3.07783\ K_{gw}\ e^{IAS_d(8.15868 \times 10^{-3})} \qquad [\text{sec}]$$

$$h = \begin{cases} 10\ 000\ T'_o/T_o, & \text{if segments 1 and 2 are computed} \\ h_{MF}, & \text{otherwise} \end{cases}$$

and

$$\Delta \ell_4 = \left[\frac{IAS_d}{1 - (0.12 \times 10^{-4}) \left(\frac{h_{XO} + h}{2} \right)} - W_{H,(H_{XO} + h)/2} \right] \frac{\Delta t_4}{3600} \qquad [\text{n. mi.}]$$

Segment 5.- Segment 5 is a constant-Mach descent flown at idle-thrust power settings. This segment begins at cruise altitude h_c and ends when IAS_d is attained at the transition altitude h_{XO}.

The equations for the time and length of segment 5 are

$$\Delta t_5 = \frac{2\ c_0}{K_{gw}} \left\{ \left[(h_c - c_1)/c_0 \right]^{1/2} - \left[(h_{XO} - c_1)/c_0 \right]^{1/2} \right\} \qquad [\text{sec}]$$

where

$$c_0 = -1.85$$

$$c_1 = 25750\ M_d + 22167$$

and

$$\Delta \ell_5 = \left[29.04(T_{st,5})^{1/2} M_d - W_{H,(H_c + h_{X0})/2} \right] \frac{\Delta t_5}{3600} \quad \text{[n. mi.]}$$

The static temperature $T_{st,5}$ and the head-wind component are evaluated at the average altitude between the cruise and transition altitudes.

Segment 6.- Segment 6 is a level-flight speed change from the cruise Mach number to the descent Mach number. If the cruise and descent Mach numbers are the same, this segment is not computed. The equations for time and length of segment 6 are

$$\Delta t_6 = 29.04 \ (T_{st,c})^{1/2} \left| \frac{M_d - M_c}{\ddot{x}} \right| \quad \text{[sec]}$$

where $\ddot{x} = -1.3$ knots/sec, and

$$\Delta \ell_6 = \left[29.04 \ (T_{st,c})^{1/2} \left(\frac{M_c + M_d}{2} \right) - W_{H,h_c} \right] \frac{\Delta t_6}{3600} \quad \text{[n. mi.]}$$

Segment 7.- Segment 7 is the remaining path between the entry fix and the beginning of segment 6. The length of segment 7 is the difference between the total distance between the entry fix and metering fix ℓ_t and the sum of the distances of the remaining six segments. The length is given as follows

$$\Delta \ell_7 = \ell_t - \sum_{j=1}^{6} \Delta \ell_j \quad \text{[n. mi.]}$$

Segment 7 time Δt_7 is found by dividing the distance to be flown by the ground speed, as follows

$$\Delta t_7 = \frac{3600 \ \Delta \ell_7}{29.04(T_{st,c})^{1/2} M_c - W_{H,h_c}} \quad \text{[sec]}$$

Input/Output Requirements

The data required for the profile descent equations are obtained from the preprogrammed calculator memory and from pilot entries through the keyboard shown in figure 11. Though all the data necessary to compute the descent are entered prior to takeoff, these parameters may be updated during cruise to obtain more accurate results.
The wind data is entered through the keyboard and the wind model coefficients are automatically computed and stored in the proper memory locations. The wind data, correlated to altitude, is inserted in a data format similar to that found on an aviation weather forecast. To insert the wind data, the

20

pilot must first push the key labeled "*" followed by the key labeled "WIND."
The display will request the altitude for the wind speed and direction data
with the message "H=?FT" The altitude should be keyed into the display and
entered into memory by pushing the "New Entry" key. The calculator will then
request the wind direction and speed with the message "DIR.SPD?" Wind
direction and speed should be keyed into the display and entered into memory
by pushing the "New Entry" key. This process will be repeated until all of
the wind data had been inserted in the calculator. The linear regression
analysis will be completed after the pilot inserts a negative altitude to
indicate that no more wind data will be inserted. The calculator will then
display a "WIND IN" message.

The wind data used by the pilot to compute the wind model could contain some
errors since that information is usually based on aviation forecasts or pilot
reports. A procedure was developed that allows the pilot to modify the wind
model with a correction factor based on the difference between the computed
and actual ground speeds along the cruise segment (segment 7). The computed
ground speed used in the profile descent computations may be displayed by
pushing the "*" key followed by the "*GSc" key. The difference between the
displayed ground speed and the actual ground speed represents the wind
modeling error along the magnetic course of the airplane to the metering
fix. If the ground speeds are different, the actual ground speed may be keyed
into the display. Then, by pushing the "New Entry" key, the difference
between the ground speeds is computed and stored in memory for use in subse-
quent descent and ground speed computations.

The operational parameters affected by ATC constraints or pilot desires, and
not accurately known until prior to the start of descent, were designed to be
single key inputs. To enter these data, the pilot presses the particular key
dedicated to the parameter to be changed. After the key has been pressed, the
display will show the name of the parameter and its current value stored in
the calculator. Another numerical value may be keyed on the display, then
stored in the proper memory location by simply pressing the "New Entry" key.
If the current value shown is satisfactory, no more keyboard actions will be
required for that parameter.

The operational parameters may be inserted in any order, or may be changed at
any time prior to initiating the descent calculations. When the magnitudes of
the parameters are satisfactory to the pilot, the profile descent computations
are initiated by pressing the "Profile" key. Computations typically require
less than 2 minutes for completion in the time-metered mode of operation and
approximately 25 seconds in the nonmetered mode.

The operational parameters to be entered by the flight crew through the
keyboard as well as their symbology as presented on the keyboard and the
display, are as follows:

Keyboard symbology	Display symbology	Operational parameter
Mc	Mc	Cruise Mach number
Hc	Hc	Cruise altitude, ft
*GSc	GSc	Ground speed at cruise altitude, knots
Md	Md	Descent Mach number
IASd	IASd	Descent indicated airspeed, knots
Time MF	MF TM	Time assigned by ATC to cross metering fix, hr:min:sec
Time EF	EF TM	Time to cross entry fix, hr:min:sec
GW	GW	Airplane gross weight at top of descent, lbs
OAT	OAT	Static outside air temperature, °C
Metering Fix	H MF	Crossing altitude at metering fix, ft
Metering Fix	IAS MF	Indicated airspeed to cross metering fix, knots
Metering Fix	MF DME	DME indication defining metering-fix location, n. mi.
Entry Fix	EF DME	DME indication defining entry-fix location - this mileage must be relative to same DME station used to define metering fix, n. mi.
Entry Fix	MAGCRS	Magnetic course from entry fix to metering fix, deg
Entry Fix	MAGVAR	Magnetic variation in the descent area, deg

If the descent speed schedule has been specified, the entry-fix and metering-fix crossing times must remain unassigned. If these times are specified through the keyboard, the proper descent speed schedule will be computed and stored in the correct memory location for recall by the pilot.

When the computations are completed, the display will normally show the DME indication where thrust should be reduced to flight idle for the descent to the metering fix. If the assigned metering-fix crossing time cannot be attained in the time-metered mode because of airplane operational speed limitations, a message will be displayed indicating the amount of time required to delay (hold) before starting the descent or the amount of time that the airplane will arrive late at the metering fix.

After the profile descent computations have been completed, the value of any operational parameters, including those required for input, may be displayed by pressing the particular designated key on the keyboard. Parameters that may be displayed after the descent computations, and their designated names, are:

Keyboard symbology	Display symbology	Operational parameter
Md	Md	Descent Mach number required to satisfy entry-fix and metering-fix time constraints
IASd	IASd	Descent indicated airspeed required to satisfy entry-fix and metering-fix time constraints, knots
Idle DME	IDL DME	DME indication showing point where thrust should be reduced to flight idle, n. mi.
Late	LATE	Amount of time that airplane will be late crossing metering fix, min:sec
Early	HOLD	Amount of time that airplane must delay before starting descent if crossing-time constraint at metering fix is to be satisfied, min:sec

Open-loop guidance in the form of desired altitude as a function of distance along the profile may also be computed by the pilot. This is accomplished by keying a DME mileage indication into the display and pushing the "DME↘H" key. The desired altitude corresponding to that distance will then be computed and displayed to the pilot. This information may then be used as guidance throughout the descent.

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24
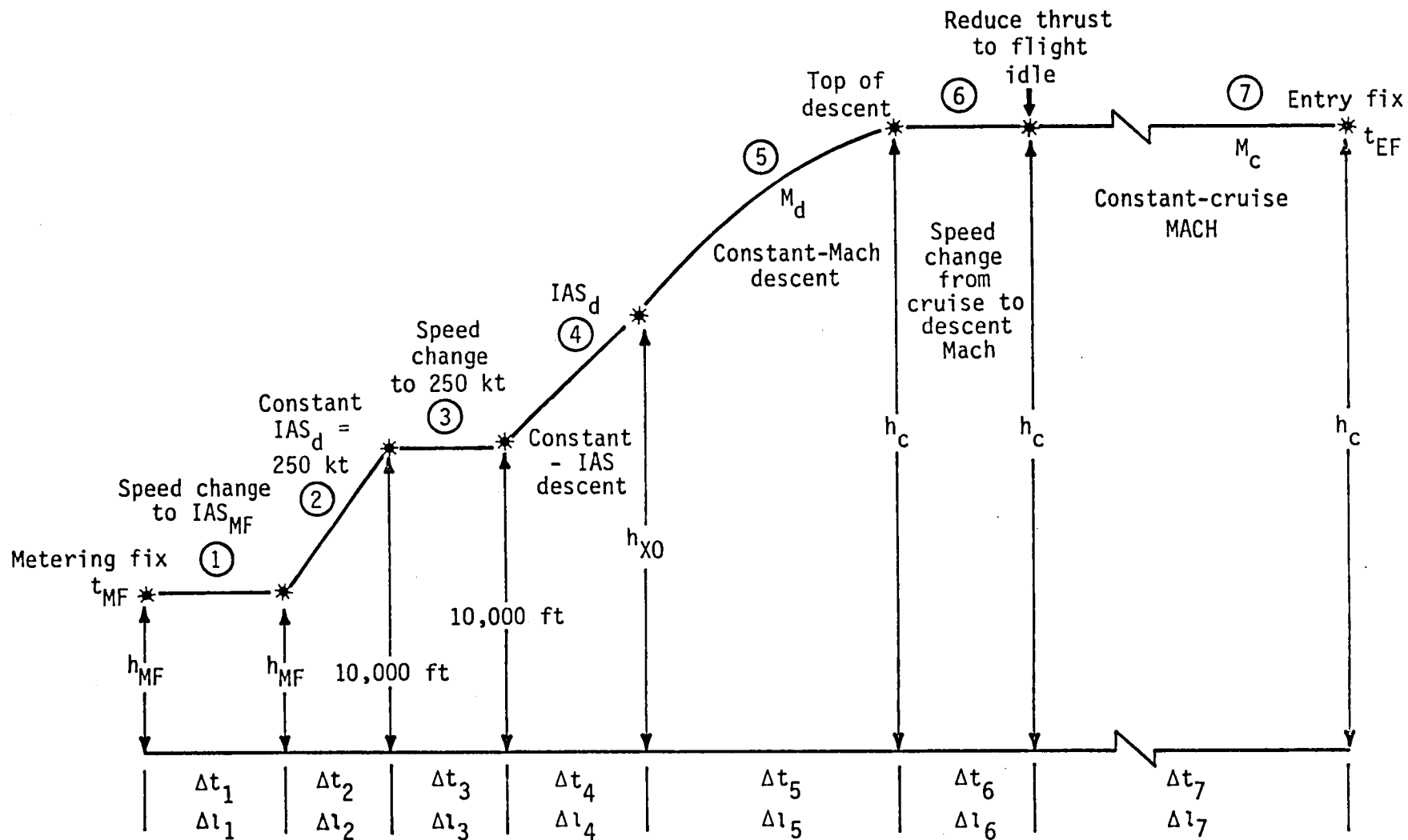
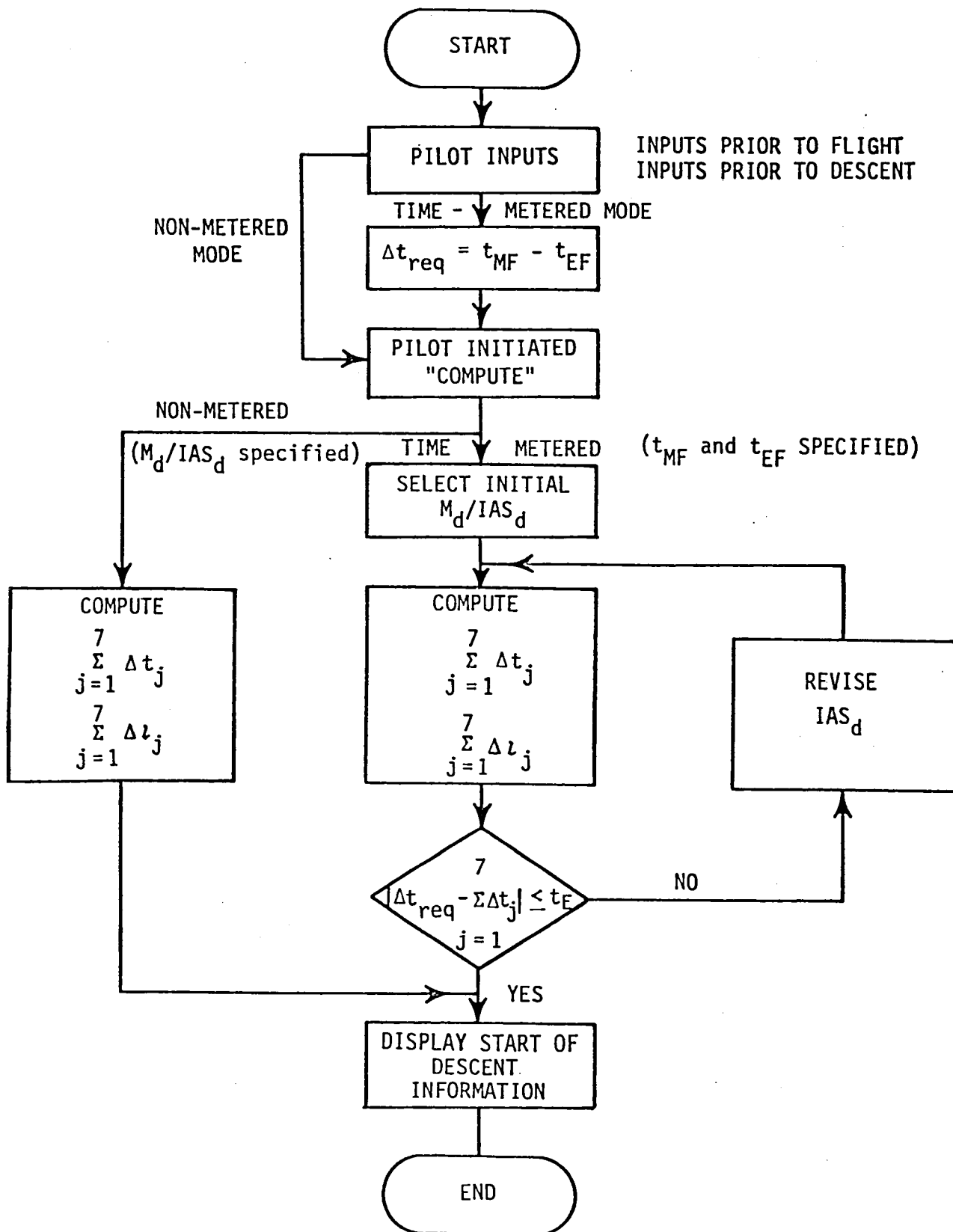Figure 1. - Vertical plane geometry of computed descent path.

START

PILOT INPUTS INPUTS PRIOR TO FLIGHT
INPUTS PRIOR TO DESCENT

NON-METERED
MODE

TIME - METERED MODE

$\Delta t_{req} = t_{MF} - t_{EF}$

PILOT INITIATED
"COMPUTE"

NON-METERED

$(M_d/IAS_d$ specified) TIME METERED (t_{MF} and t_{EF} SPECIFIED)

SELECT INITIAL
M_d/IAS_d

COMPUTE
$\sum_{j=1}^{7} \Delta t_j$
$\sum_{j=1}^{7} \Delta l_j$

COMPUTE
$\sum_{j=1}^{7} \Delta t_j$
$\sum_{j=1}^{7} \Delta l_j$

REVISE
IAS_d

$|\Delta t_{req} - \sum_{j=1}^{7} \Delta t_j| \leq t_E$

NO

YES

DISPLAY START OF
DESCENT
INFORMATION

END

Figure 2. - Flight management descent algorithm logic flow.
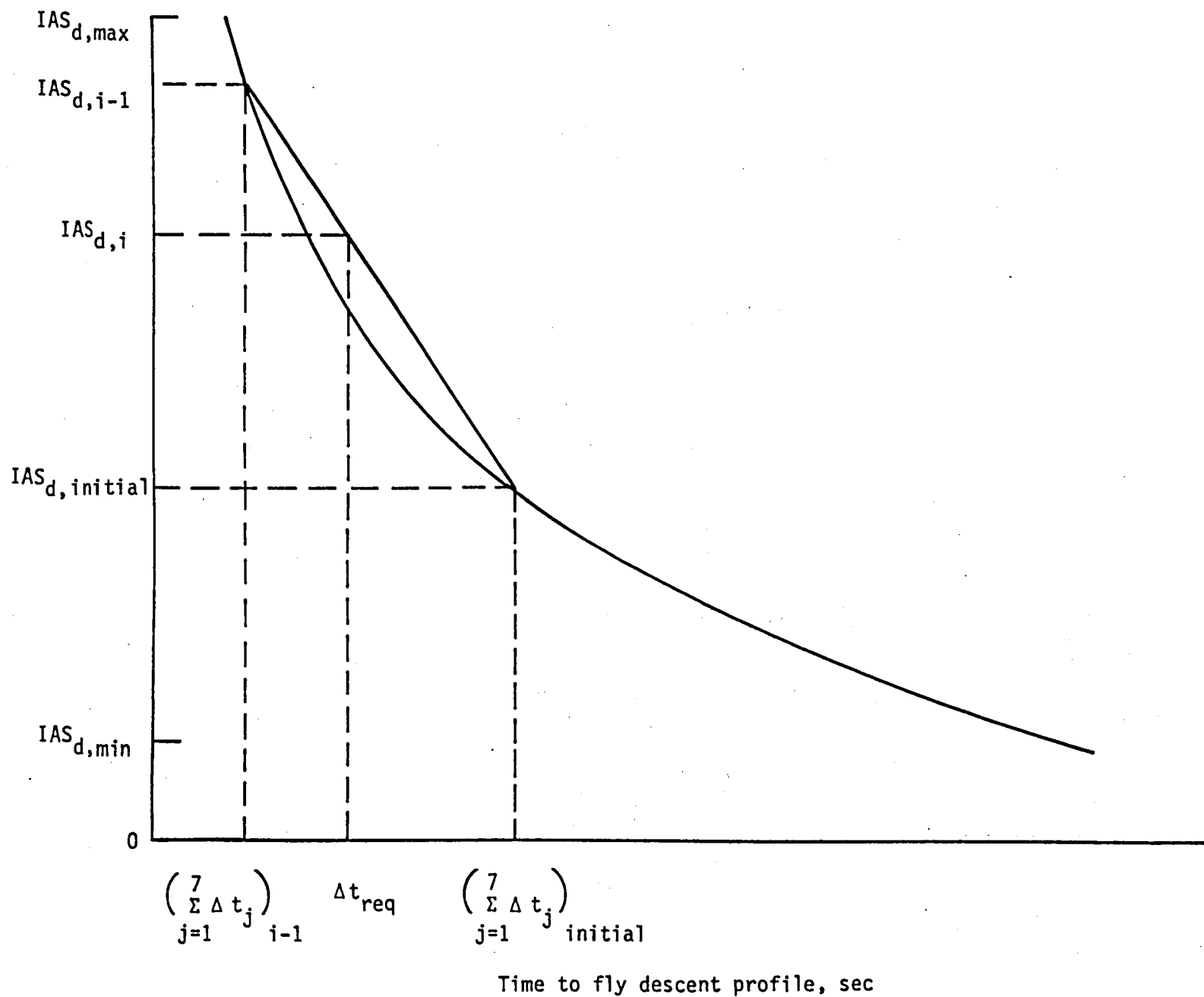
Figure 3. - Descent indicated airspeed selection via interpolation.
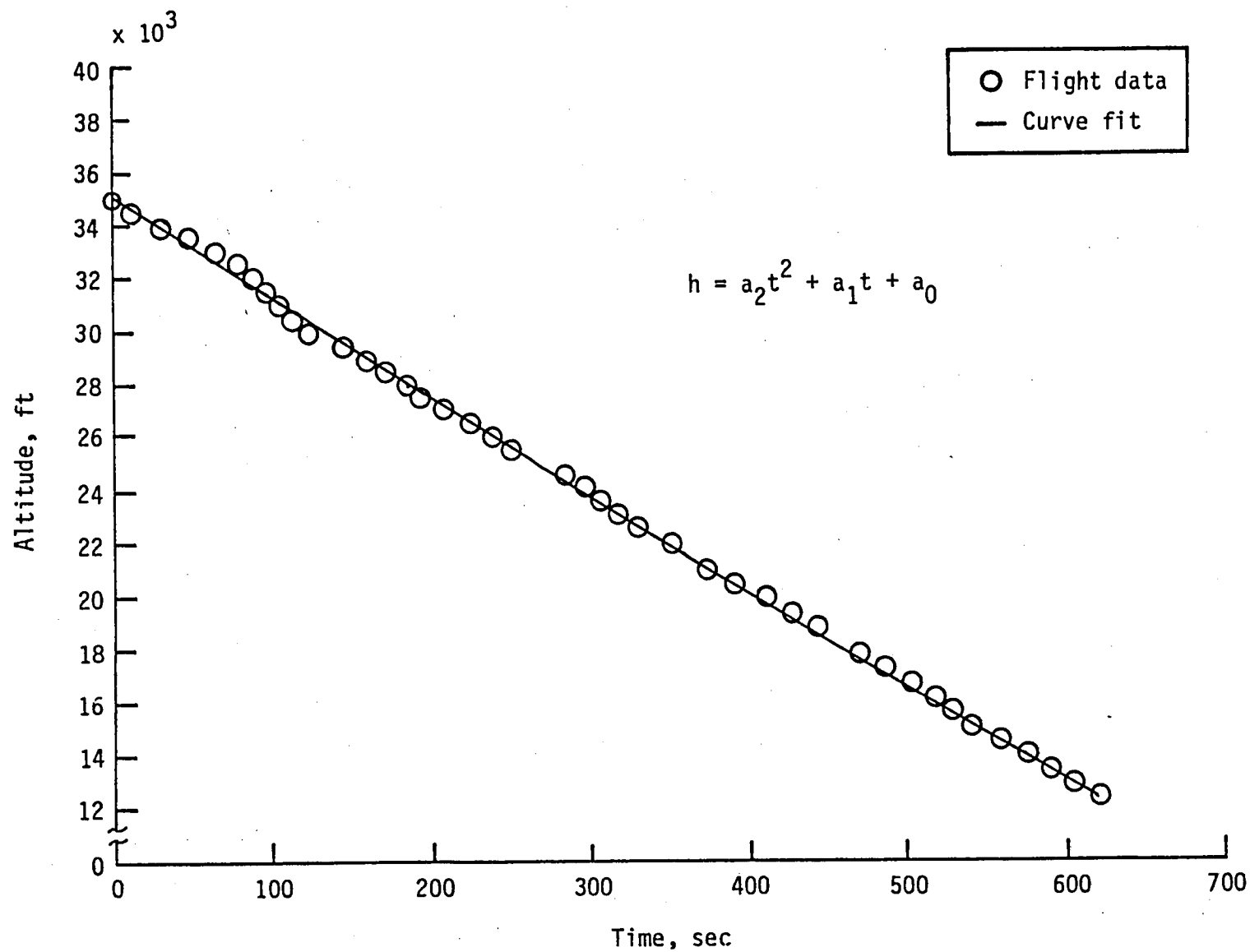
Figure 4. - Quadratic curve fit of altitude vs time for a DC-10 airplane executing a constant 280 knot indicated airspeed descent.
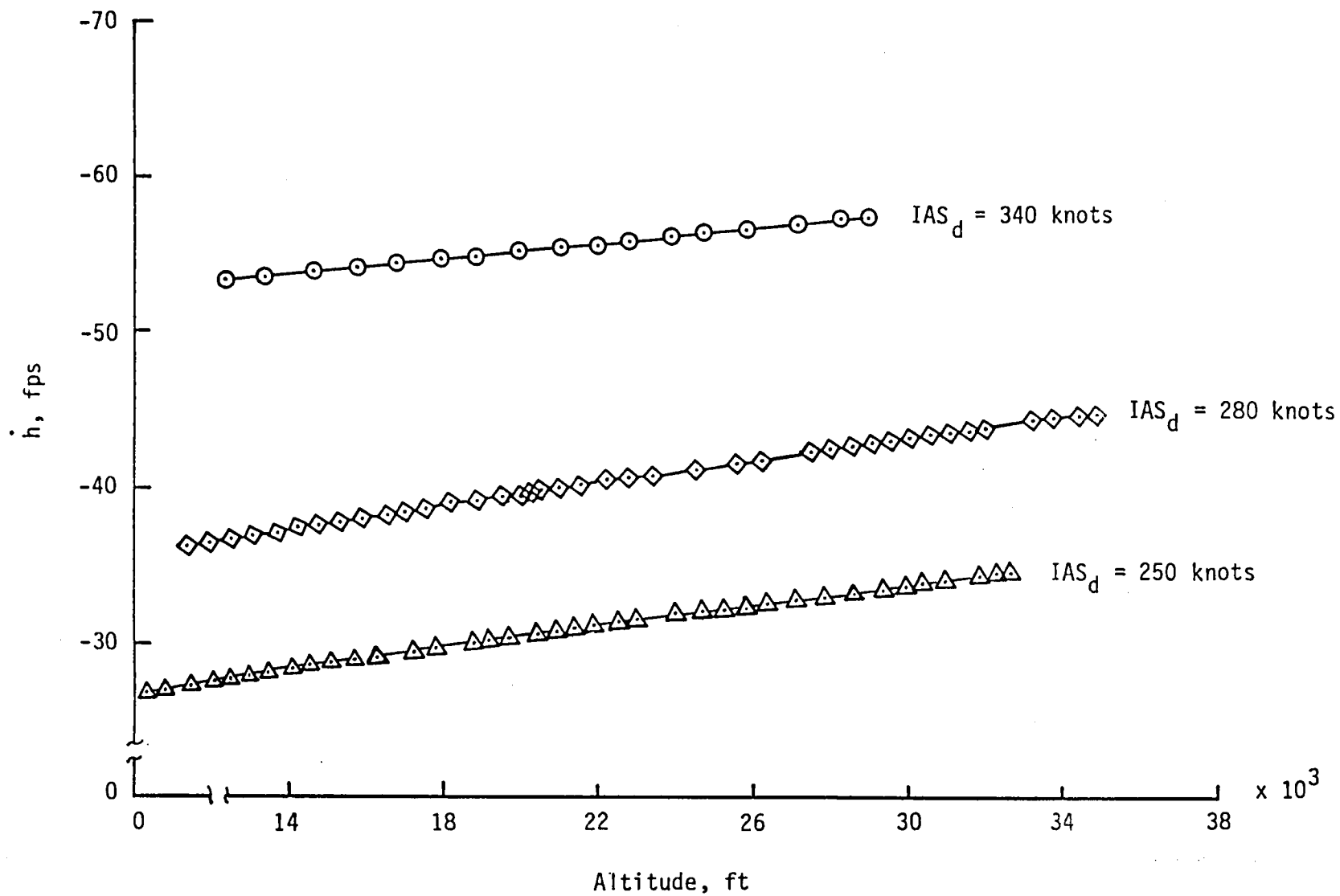
Figure 5. - Vertical speed model of the DC-10 airplane for constant indicated airspeed descents (idle thrust).
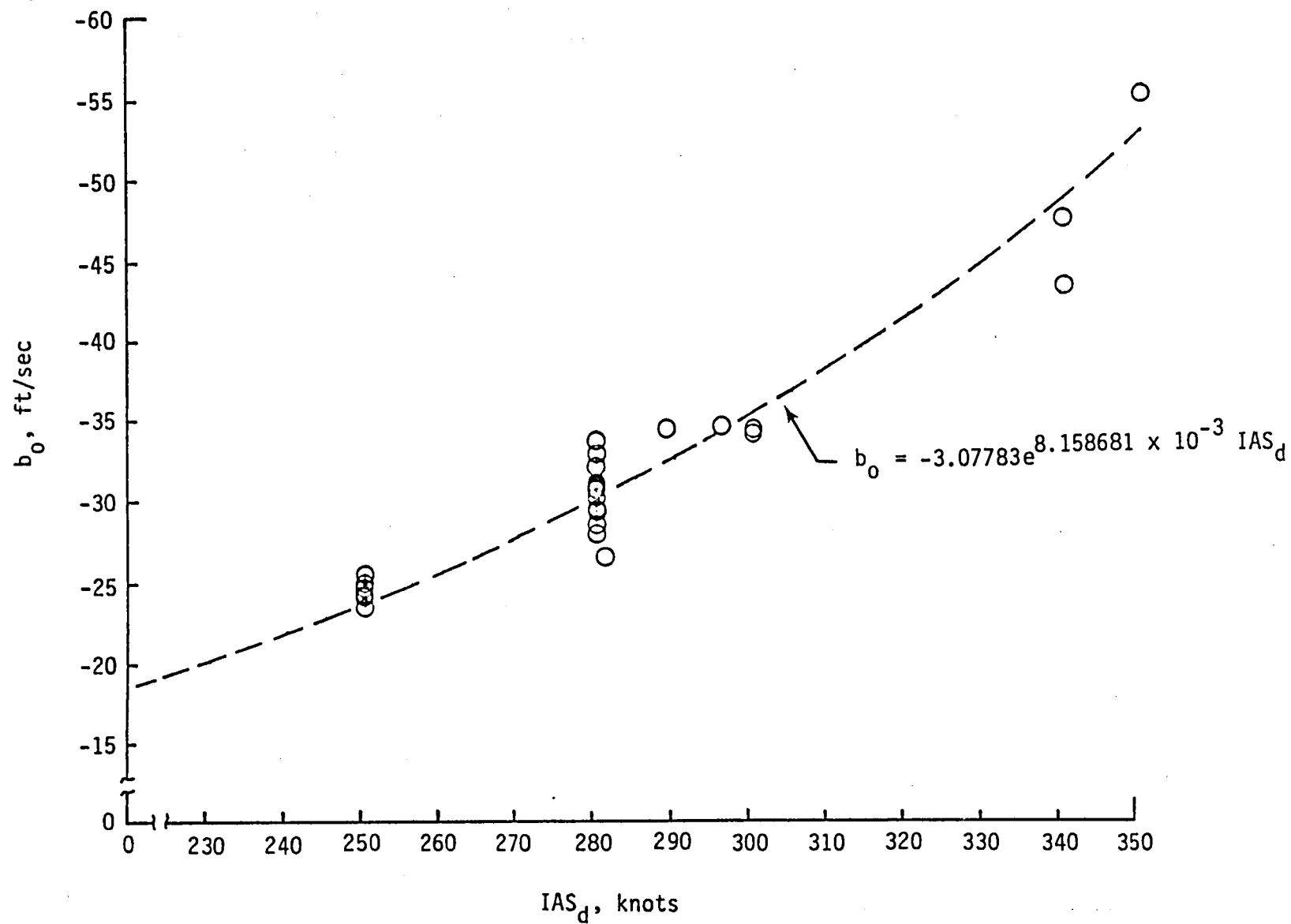
$$b_0 = -3.07783e^{8.158681 \times 10^{-3} \, IAS_d}$$

Figure 6. - Modeled vertical speed at sea level for the DC-10 airplane.
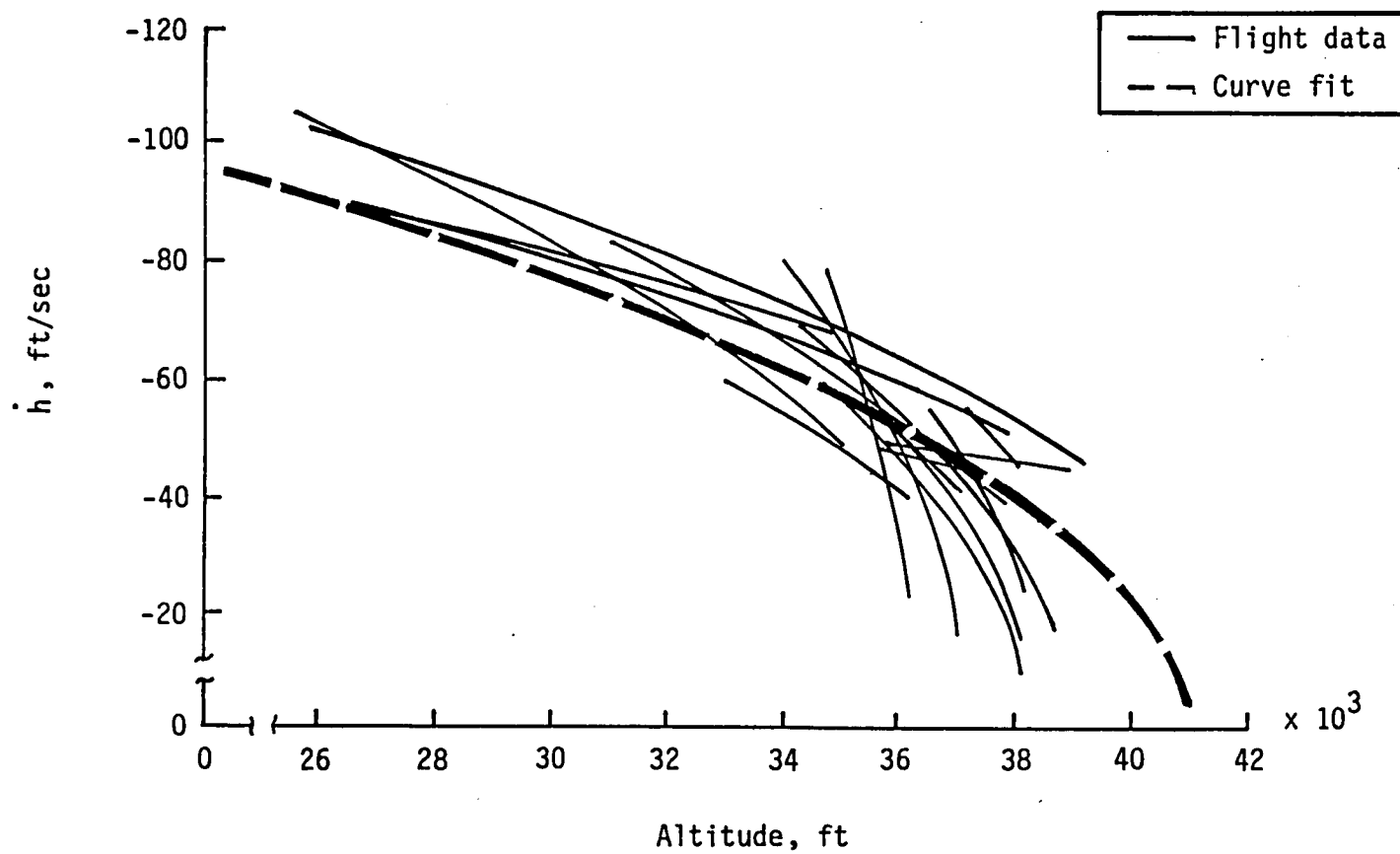
Figure 7. – Flight data and generic parabolic model with c_o = -1.85 for constant Mach number descents (idle thrust) for the DC-10 airplane.
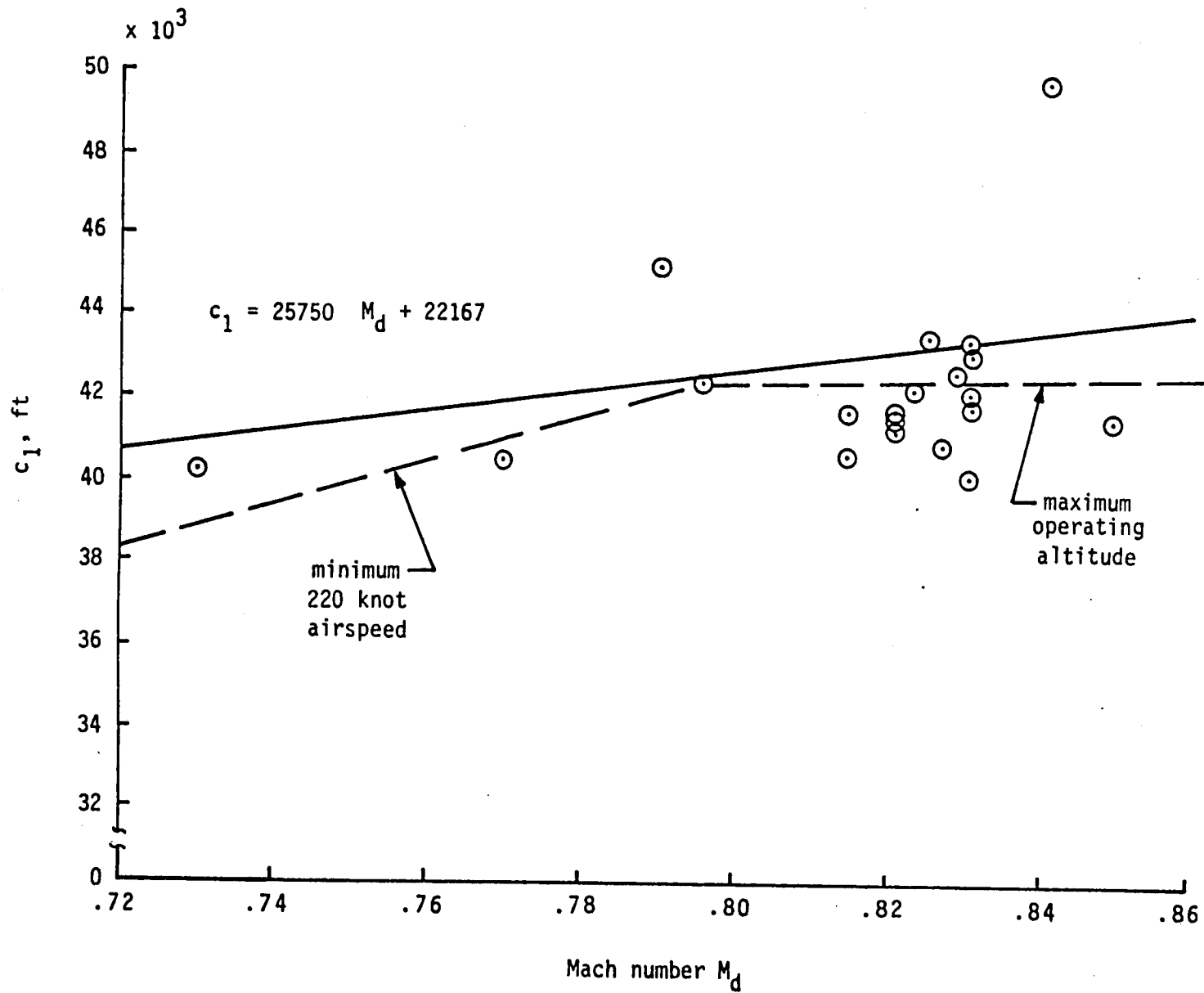
$$c_1 = 25750 \ M_d + 22167$$

minimum
220 knot
airspeed

maximum
operating
altitude

c_1, ft

Mach number M_d

Figure 8. - c_1 coefficient model for constant Mach number descents
for the DC-10 airplane.
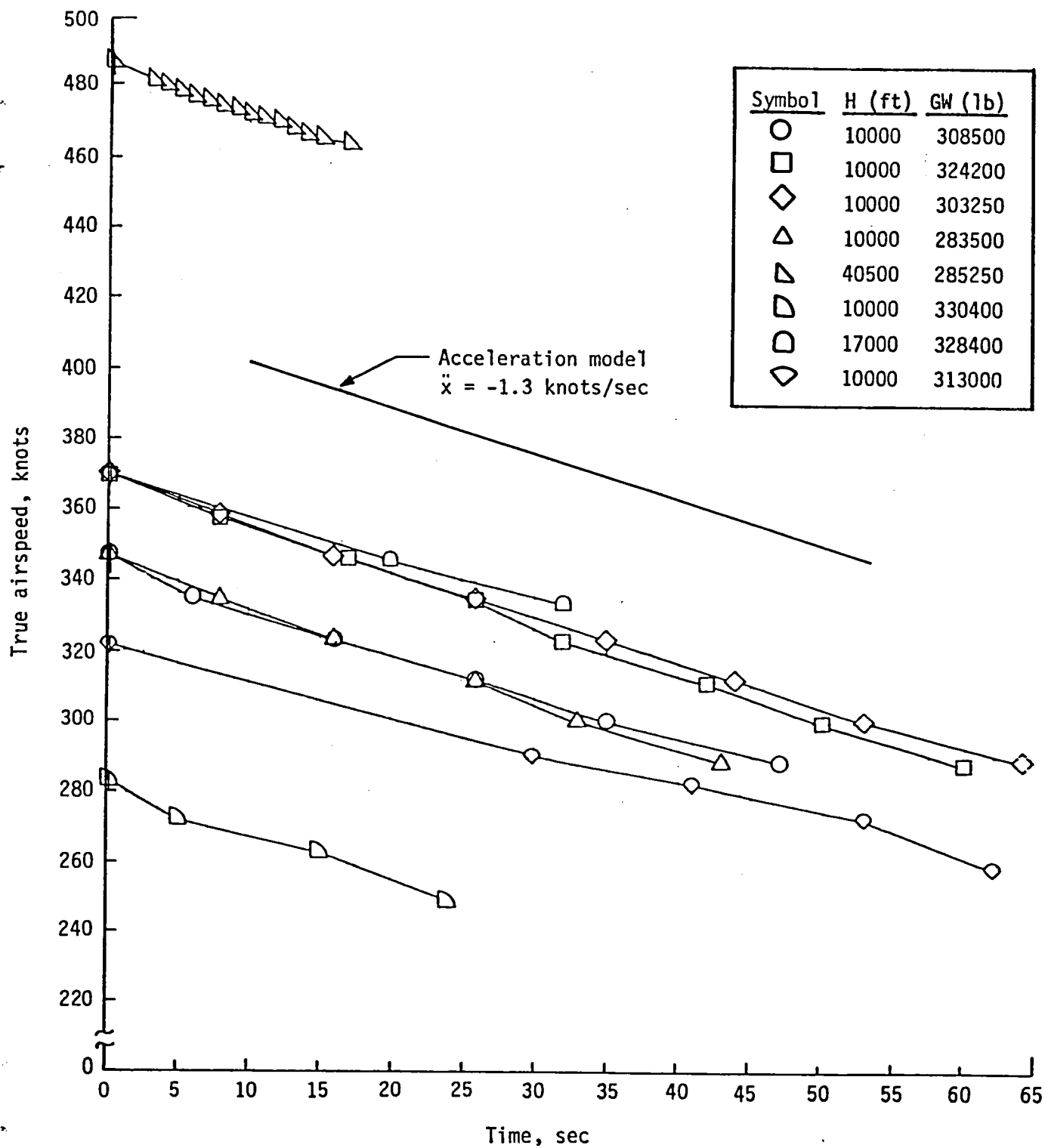
Figure 9. - Modeled and actual true airspeed reduction in level flight
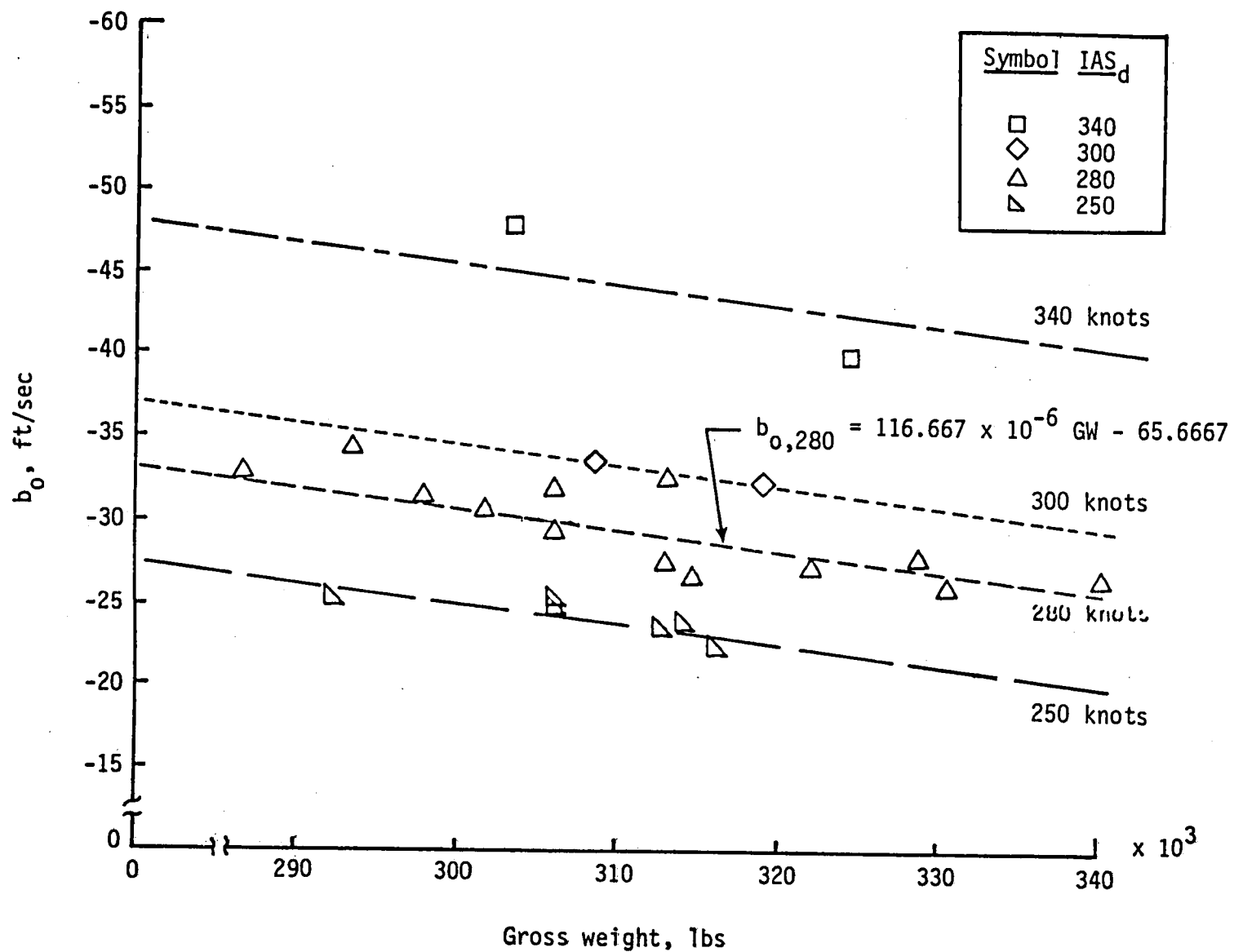(idle thrust) for the DC-10 airplane.

Figure 10. - Vertical speed at sea-level as a function of gross weight for the DC-10 airplane.
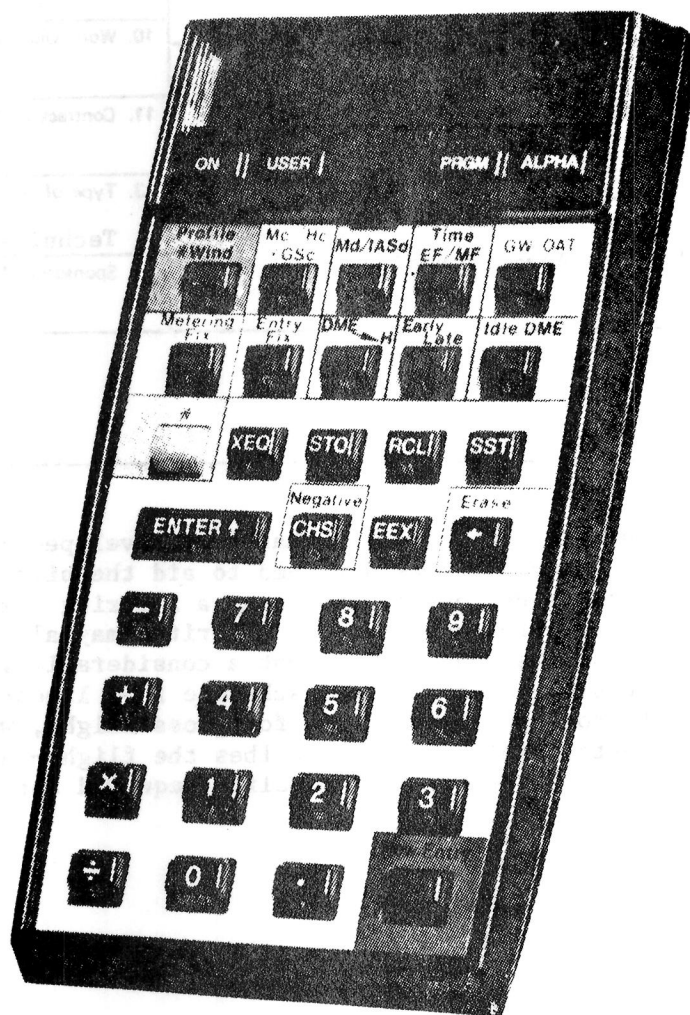
Figure 11. – Programmable Descent Calculator.

1. Report No. NASA TM-85642	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle Description of the Computations and Pilot Procedures for Planning Fuel-Conservative Descents with a Small Programmable Calculator		5. Report Date May 1983
		6. Performing Organization Code 534-04-13-52
7. Author(s) Dan D. Vicroy Charles E. Knox		8. Performing Organization Report No.
9. Performing Organization Name and Address NASA Langley Research Center Hampton, VA 23665		10. Work Unit No.
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546		13. Type of Report and Period Covered Technical Memorandum
		14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

A simplified flight-management descent algorithm has been developed and programmed on a small programmable calculator. It was designed to aid the pilot in planning and executing a fuel-conservative descent to arrive at a metering fix at a time designated by the air traffic control system. The algorithm may also be used for planning fuel-conservative descents when time is not a consideration. The descent path was calculated for a constant Mach/airspeed schedule from linear approximations of airplane performance with considerations given for gross weight, wind, and nonstandard temperature effects. This report describes the flight-management descent algorithm and the vertical performance modeling required for the DC-10 airplane.

17. Key Words (Suggested by Author(s)) fuel conservation flight management systems airplanes air traffic control airborne computer	18. Distribution Statement Unclassified - Unlimited Subject Category 06

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 36	22. Price A03