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APPLICATION OF KALMAN FILTERS TO ROBOT CALIBRATION

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16. Abstract

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The Kalman filter allows the robot to read its sensors plus external sensors and "learn" from its experience. In effect, the robot is given primitive intelligence. The study, which is applicable to any type of powered kinematic linkage, focuses on the calibration of a manipulator.

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APPLICATION OF KALMAN FILTERS TO ROBOT CALIBRATION

by

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PREFACE

This report explores new uses of Kalman filter theory in manufacturing systems (robotics in particular).

The Kalman filter is a recursive algorithm to update estimates of the system's state variables using the current sensor measurements. The Kalman filter allows the robot to read its sensors plus external sensors and "learn" from its experience. In effect, the robot is given a primitive intelligence. This work will focus on one application of Kalman filtering, namely, the calibration of a manipulator. This work is applicable to any type of powered kinematic linkage.

The second section reviews the Kalman filter and its modification for nonlinear systems. Section 3 gives a framework for generating the Kalman filter equations from the robot's kinematic equations. Once the equations are in the Kalman filter format, then the report demonstrates how the analysis of a calibration routine is done. The last section gives the derivation of the Kalman filter equations for a manipulator designed by the Computer Aided Design section of the Goddard Space Flight Center. Its unique kinematics caused special problems, and the solution to those problems is given. In the Appendix, the algebraic equations for evaluating the Kalman filter parameters are given.

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LIST OF SYMBOLS

Section 1

$\underline{f}(\ )$	Vector-valued function
$\underline{h}(\ )$	Vector-valued function
k	Time index; "time step k"
$\underline{k}(\ )$	Vector valued (updating) function
$\underline{u}(k)$	Control vector; input vector at time step k
$\underline{v}(k)$	Process noise vector; the vector of sensor noise at time step k
$\underline{w}(k)$	Process noise vector at time step k
$\underline{x}$	System state vector; the set of variables that completely describes the system
$\underline{x}(k)$	State vector at time step k
$\hat{\underline{x}}(k)$	Estimated state vector at time step k
$\underline{z}(k)$	Vector of measurements at time step k

Section 2

B	Input weighting matrix. Zero matrix if parameters are time invariant (or B(k))
E[ ]	Expectation operator
$\underline{h}(k)$	Output function at time k
H	Output weighting matrix. Cannot be a function of $\underline{x}$ or $\underline{u}$

I	Identity matrix
J	Performance criterion created by summing the square of the differences $\underline{x}_i$ and $\hat{\underline{x}}_i$ and minimizing J with respect to K
K(k)	Kalman gain matrix
M(k)	Estimation error covariance matrix before measurement
P(k)	Estimation error covariance matrix after measurement
Q	Process noise covariance matrix; zero matrix if system is time invariant
R	Sensor noise covariance matrix
$\hat{\underline{x}}(k)(+)$	The estimate of $\underline{x}(k)$ after a measurement
$\hat{\underline{x}}(k)(-)$	The estimate of $\underline{x}(k)$ prior to a measurement
Φ	State-transition matrix. May be time varying - if $\underline{x}$ is time invariant, Φ is the identity matrix. May <u>not</u> be a function of $\underline{x}$ or $\underline{u}$.

<u>Superscripts</u>

T	Matrix transpose

<u>Section 3</u>

A	Observability matrix
$\overline{B}$	Transformation that describes an object or feature relative to the basic coordinate frame
$\overline{E}$	Transformation that describes the state of the end effector
f( )	General output function, matrix arguments
G	Transformation that describes the tool tip or the end effector relative to B (linking tool tip to object or target)

H_i	(observability vector)
r_i	Estimation error (± range for uniformly distributed noise process)
t	Time
T_6	Transformation that describes the state of the manipulator
$\underline{u}$	Vector of actuator inputs
$\underline{v}$	Vector of zero-mean, Gaussian white noise
$\underline{x}$	A parameter to be estimated
$\underline{Z}$	Transformation that describes the position of the manipulator base relative to the origin or basic coordinate frame. (Transformation relating manipulator base to origin of space)
σ_i^2	Variance
σ_i	Standard deviation of noise at time i

<u>Subscript</u>

i	Index of measurement numbers
m	Number of measurements
n	Number of parameters to be estimated

<u>Section 4</u>

a	Remaining 3 state variables after Eq. 4-27
A_i	Homogeneous transformation matrix
a_j	Encoder Bias (offset in encoder j)

B_i	= ?
B_j	= $\partial h_j/\partial z_j$ (scalar)
BOT	Transformation relating bottom link mounting base to bottom coordinate fra...
C_i	= $\partial \underline{f}_i/\partial \underline{t}$ (a 6 × 6 matrix)
Euler ( )	Euler angle rotation transformation (angle of top joint of a link)
D_i	= $\partial g_i/\partial \underline{f}_i$ (1 × 6 matrices)
$\underline{E}$	
E_n	= $-\partial \underline{h}/\partial x_n$
$\underline{f}_i$	Function for manipulator transformations to link parameters $\underline{j}_i$
$\underline{F}_{in}$	= $-\partial \underline{f}_i/\partial x_n$ (a 6 element vector)
$\underline{g}$	Vector functions of g_i
g_i	Equation for link i relating link parameters $\underline{j}_i$ to actuator extension u_i
$\underline{G}$	= $\begin{bmatrix} D_1 C_1 \\ \vdots \\ D_6 C_6 \end{bmatrix} \frac{\partial \underline{t}}{\partial x_n}$ = state variables appearing in $\underline{g}$ = 6 element vector evaluated from $\underline{G}_n$
$\underline{G}_n$	= (-) $\partial \underline{g}/\partial x_n$
h_j	Function relating link parameters $\underline{j}_i$ to sensor output z_j
$\underline{h}$	Vector of the h_j functions
H	Matrix

j_i	Vector that comprises a set of six independent parameters of the transformation JT (vector of link parameters for link i)
$\underline{j\ell}$	Vector of parameters describing transformation JL
JL	Relationship of the lower bases to the bottom pivot points of the links (transformation relating bottom mounting base coordinates to lower pivot point coordinates)
JT	Transformation of a single line (direct transformation from the bottom bearing coordinate frame to the top bearing coordinate frame)
JU	Relationship of the upper bases to the upper pivot points of the links (transformation relating top mounting base to top pivot point)
L	Link's unextended length (when u = 0)
L1,L2,L3	Bases relative to the bottom platform
N	Matrix operator
N_i	Transformation differentiation operator
P_i	Variable function of JU, TOP, T, BOT, and JL
$\underline{q}$	Vector in the top platform coordinate frame known as $\underline{r}$ in the base coordinate frame by $\underline{r} = T \cdot \underline{q}$
$\underline{r}$	Vector in the base platform coordinate frame
$r^{\bullet}_{y}$	Variable functions of JU, TOP, T, BOT, and JL
Rot ( )	Rotation transformation
s	Pitch of the (link's) lead screw
$\underline{t}$	Vector of parameters describing the transformation T
t(4)	Fourth element of vector $\underline{t}$

T	The 4 × 4 transformation that relates the top platform to the bottom platform (transformation relating the origin of top platform to the origin of bottom platform)
$\underline{tb}$	Vector defining the transformation of each bottom base relative to the origin of its platform (vector of parameters describing BOT)
TOP	Transformation relating the origin of the top platform to one of the top mounting bases
$\underline{tp}$	Vector defining the transformation of each top base relative to the origin of its platform (vector of parameters describing top)
Trans( )	Translational transformation
u	Actuator angle
$\underline{u}$	Actuator positions
U1,U2,U3	Bases relative to the top platform
$\underline{x}$	State vector
x_n	n^{th} element of the state vector
z	Sensor outputs
$\underline{z}$	Output vector
α	Angle of rotation about the x axis (angle of bottom joint of a link)
β	Angle of rotation about the y axis (angle of next to bottom joint of a link)
ζ	Angle of rotation about the y axis after the β rotation
η	Angle of rotation about the x axis after the α rotation (angle of next to top joint of a link
θ	Angle of rotation about the z axis. The rotation of the lead-screw nut relative to the actuator (induced rotation of lead screw)

Subscript

i	Index of link number
i	Index of transformation number as in a product of many
j	Index of the sensors
z_j	Measurement (number j)
Y_j	$= \partial h_j / \partial \underline{i}_j$

SECTION 1

INTRODUCTION

This report attempts to link the seemingly unrelated fields of optimal recursive estimation (Kalman filters) and kinematics. The need for robotic manipulator control schemes linked kinematics and control theory, so, logically, estimation theory and kinematics should be combined to provide new insights into and capabilities for robots. Robot behavior must be viewed in a new light, separate from analyzing the high-bandwidth behavior that concerns control engineers. Actually generating the robot's trajectory through space is not important; the endpoints of the trajectory are useful. This is where vital interactions between the robot and its environment usually occur. Consequently, the estimation takes place in the discrete time domain.

To implement a Kalman filter, an accurate state-space model of the system to be estimated must be formulated. This is the most challenging part of the estimation process and requires good engineering judgement. The system state vector, $\underline{x}$, is the set of variables that completely describes the system. If more variables are specified, they are redundant. If too few are specified, then some aspects of the system are ignored and this may affect both control and estimation tasks adversely. Coupled with a (nonunique) system model

$$\underline{x}(k) = \underline{f}\big(k-1, \underline{x}(k-1), \underline{u}(k-1), \underline{w}(k-1)\big) \tag{1-1}$$

where

$\underline{x}(k)$ = state vector at time step k

$\underline{f}(\ )$ = a vector-valued function

$\underline{u}(k-1)$ = a control vector; the input vector at k

$\underline{w}(k-1)$ = a process noise vector; its presence makes $\underline{x}(k)$ random

k = a time index, "time step"

the future state of the system may be predicted or extrapolated. The generic estimation task is to take the measurements Eq. (1-2) generates

$$\underline{z}(k) = \underline{h}\big(k, \underline{x}(k), \underline{u}(k-1), \underline{v}(k)\big) \tag{1-2}$$

where

$\underline{z}(k)$ = a vector of measurements at time step k

$\underline{h}(\ )$ = a vector valued function

$\underline{v}(k)$ = a process noise vector; the vector of sensor noise at time step k; its presence makes $\hat{x}(k)$ random

and generate an updated estimate

$$\hat{\underline{x}}(k) = \underline{k}\big(k, \underline{x}(k-1), \underline{u}(k-1), \underline{z}(k)\big) \tag{1-3}$$

where

$\hat{\underline{x}}(k)$ = the estimated state vector at time step k

$\underline{k}(\ )$ = a vector valued function

For a robotic system, what set of variables completely describes the robot when it is at rest? One such set could be all the link lengths, twists, etc., that describe the links in the kinematic chain.

The actuator inputs must be added to this set. If the sensor does not measure any of these state variables directly, more parameters are needed to relate the robot to the sensors, and these parameters must help describe the system. By collecting all these parameters together with all the functions that relate them to each other, the remote observer should know everything about the system at every point in time. However, not all these parameters are known exactly, and they must be either measured directly (along with some noise) or inferred from other noisy measurements. Thus, these parameters must be estimated.

The engineer must derive a model, as in Eq. (1-1) and (1-2), that includes a specification of the state variables to be estimated. The state vector should not include perfectly known quantities or control variables like actuator inputs. Then the estimator, Eq. (1-3), takes over and generates updated estimates of the state vector. The rest of this report outlines the how's and why's of creating a system model, Eq. (1-1), and a measurement model, Eq. (1-2), for robots in general and the Goddard manipulator in particular. First, Section 2 reviews the mechanics of the Kalman filter, a special form of Eq. (1-3).

SECTION 2

KALMAN FILTER THEORY

The Kalman filter is one of many estimation algorithms. It provides the best linear fit of the state estimates to the sensor outputs for a linear system.[1] A linear system must be fit to the following form

$$\underline{x}(k) = \Phi(k-1)\ \underline{x}(k-1) + B(k-1)\ \underline{u}(k-1) + \underline{w}(k-1) \tag{2-1}$$

$$\underline{z}(k) = H(k)\ \underline{x}(k) + \underline{v}(k) \tag{2-2}$$

where

Φ = the state transition matrix. It may be time varing. If $\underline{x}$ is time invariant, Φ is the identity matrix. It may not be a function of $\underline{x}$ or $\underline{u}$

B = the input weighting matrix. The zero matrix if the parameters are time invariant

H = the output weighting matrix. It cannot be a function $\underline{x}$ or $\underline{u}$

$\underline{w}$ = a zero-mean, Gaussian, white-noise process of covariance matrix Q (otherwise as defined in Section 1)

$\underline{v}$ = a zero-mean, Gaussian, white-noise process of covariance matrix R (otherwise as defined in Section 1) and

$$Q = E[(\underline{w}\underline{w}^T)] \qquad (2\text{-}3)$$

$$R = E[(\underline{v}\underline{v}^T)] \qquad (2\text{-}4)$$

where

Q = the process noise covariance matrix

R = the sensor noise covariance matrix

[E] = the expectation operator

The Kalman filter is then defined as

$$\hat{\underline{x}}(k)(+) = \hat{\underline{x}}(k)(-) + K(k)[\underline{z}(k) - H(k)\hat{\underline{x}}(k)(-)] \qquad (2\text{-}5)$$

where

$\hat{\underline{x}}(k)(+)$ = the estimate of $\underline{x}(k)$ after a measurement

$\hat{\underline{x}}(k)(-)$ = the estimate of $\underline{x}(k)$ prior to a measurement

$K(k)$ = the Kalman gain matrix

$K(k)$ is derived by assigning a performance criterion, J, which is created by summing the square of the differences between $\underline{x}_i$ and $\hat{\underline{x}}_i$ and minimizing J with respect to K. The quantities $\hat{\underline{x}}(k)(-)$ and $K(k)$ are computed as follows

$$\hat{\underline{x}}(k)(-) = \Phi(k-1)\hat{\underline{x}}(k-1)(+) + B\underline{u}(k-1) \qquad (2\text{-}6)$$

$$M(k) = \Phi(k-1)P(k-1)\Phi(k-1)^T + Q(k-1) \qquad (2\text{-}7)$$

$$K(k) = M(k)H(k)^T[H(k)\ M(k)H(k)^T + R(k)]^{-1} \qquad (2\text{-}8)$$

$$P(k) = [I - K(k)H(k)]M(k) \qquad (2\text{-}9)$$

where

$$P(k) = E\left[\left(\underline{x}(k) - \hat{\underline{x}}(k)(+)\right)\left(\left(\underline{x}(k) - \hat{\underline{x}}(k)(+)\right)^T\right]$$

= the estimation error covariance matrix after measurement

$$M(k) = E\left[\left(\underline{x}(k) - \hat{\underline{x}}(k)(-)\right)\left(\left(\underline{x}(k) - \hat{\underline{x}}(k)(-)\right)^T\right]$$

= the estimation error covariance matrix before measurement

T = a superscript indicating matrix transpose

These two variables chronicle the ignorance (or knowledge) of the filter about its state estimate over time. The greater P and M are, the less confident the filter is about its estimates. As this algorithm is recursive, the starting values of $\hat{\underline{x}}$ and P must be given

$$\hat{\underline{x}}_0 = E[\underline{x}(0)] \tag{2-10}$$

$$P_0 = E\left[\left(\underline{x}(0) - \hat{\underline{x}}_0\right)\left(\underline{x}(0) - \hat{x}_0\right)^T\right] \tag{2-11}$$

Almost paradoxically, the engineer must provide the filter with his best a priori knowledge ($\hat{\underline{x}}(0)$) about the system plus his ignorance about the system (P_0). Treating a robot as a stochastic, nondeterministic system is essentially a new concept for those who work with them.

Kalman filtering has several distinct (but not necessarily exclusive) advantages.

(1) Every measurement vector need not be stored. It is operated on once and discarded. Its information is used to update the state estimate vector.

(2) Sensor output is properly weighted according to its noisiness.

(3) The progress toward state estimate convergence to its true value can be determined directly through the P matrix. Estimation may cease when acceptably small covariance values are reached.

(4) Observability is also determined, and calibration routines may be checked to see if observability satisfied. Observability is the property of the filter that determines whether, in a fixed number of measurements, all the state estimates may be determined uniquely. Section 3 expands on this.

In robot calibration, the states of the system that must be estimated are the robot's geometric parameters. These parameters are not expected to vary over the duration of the calibration. Hence, $\Phi(k)$ is the identity matrix and Q and B are zero matrices. As will be shown, the greatest emphasis is on generating an output equation of the form of Eq. (1-2). To use a Kalman filter, Eq. (1-2) must be linearized to the form of Eq. (2-2) by utilizing

$$H(k) = \left. \frac{\partial \underline{h}(k)}{\partial \underline{x}(k)} \right|_{\hat{\underline{x}}(k)(-)} \qquad (2-12)$$

SECTION 3

TRANSFORMING KINEMATIC EQUATIONS INTO KALMAN FILTER FORM

3.1 Overview

Kalman filter calibration of a given manipulator or robot consists of five steps:

(1) Geometric modeling.

(2) Selection of test sequence.

(3) Design of fixtures and instrumentation.

(4) Execution of test sequence and data collection.

(5) Data reduction by Kalman filtering.

The data reduction can be accomplished either on or off line as the control vectors are generated in an open-loop fashion. If the updated estimates were required for generating control vectors, then on-line filtering would be necessary. The product of the filtering is updated estimates of geometric parameters such as link length, encoder/actuator biases, joint locations, etc.

Selecting the parameters to be estimated requires an important engineering judgement. If too many parameters are selected, then computation time increases and limits on available data memory are reached. The amount of storage needed increases as the square of the number of states (parameters). On the other hand, selecting too few states may neglect important errors and the Kalman filter will attempt to project these errors onto the wrong parameters. The parameters should be selected on the basis of the likelihood of being in error (relative to current, uncalibrated estimates) and the importance of an error if it exists.

3.2 Geometric Modeling

The first task in calibrating the manipulator is to generate an accurate kinematic model. The desired form of the model will be the matrix equation

$$Z T_6 \overline{E} = \overline{B} G \tag{3-1}$$

where

T_6 = the transformation that describes the state of the manipulator itself. For any manipulator, T_6 is a function of the actuator inputs and the kinematics of the manipulator

$\overline{E}$ = the transformation that describes the state of the end effector. For example, on the Goddard manipulator, this would be the compliant instrument. The sensor outputs are a function of this transformation

$\overline{B}$ = the transformation that describes an object or feature relative to the basic coordinate frame

G = the transformation that describes the tool tip or end effector relative to $\overline{B}$

as suggested in Reference 2.

For Kalman filtering, the highest interest is in generating equations of the form

$$\underline{z} = \underline{f}(T_6, Z, B, G, E) + \underline{v} \tag{3-2}$$

where $\underline{z}$ and $\underline{v}$ are as described in Sections 1 and 2. For example, it may be derived from

$$\overline{E}(\underline{z}) = T_6^{-1} Z^{-1} \overline{B} G \tag{3-3}$$

which is Eq. (3-1) premultiplied by Z^{-1} and T_6^{-1}, if the sensors were in the wrist. Equation (3-2) is derived by algebraic manipulation. The net result should be a set of equations of the form

$$\underline{z} = \underline{h}(t, \underline{x}, \underline{u}) + \underline{v} \tag{3-4}$$

where

$\underline{x}$ = a vector of parameters to be estimated

$\underline{u}$ = a vector of actuator inputs

This is an explicit function of time, t, as the configuration may be changed in some way, in addition to changing actuator inputs.

It may be desirable to calibrate the end effector/sensor and the manipulator separately. In this case, the instrumentation and equations will be quite different. It is, however, possible to calibrate both units simultaneously.

3.3 Selection of Test Sequences

The most important characteristic of the test sequence is that it satisfies the observability criterion. Thus, all errors in the parameter estimates are reflected in the sensor outputs and these errors are distinguishable from one another. Mathematically, this can be determined as follows: create the observability matrix, A, where A is defined as

$$A = \begin{bmatrix} H_0 \\ H_1 \\ \cdot \\ \cdot \\ \cdot \\ H_m \end{bmatrix} \tag{3-5}$$

where

$$H_i = \frac{\partial \underline{h}_i}{\partial \underline{x}_i}, \quad i = 1,2,\ldots,m$$

If the rank of A = n, where n is the number of parameters to be estimated, then the system is observable with the given set of m measurements. One cannot know, a priori, what the values of x are, so their initial estimates must be substituted when computing A. Should the matrix A have a rank less than n, then new measurements or configurations must be added until A has a rank of n. The nonlinear characteristics of $\underline{h}$, in general, will make this condition easy to satisfy as almost every parameter influences every sensor output to some degree.

After observability has been satisfied, the test sequence can be repeated until sufficient convergence has been obtained. Some measures of convergence are the diagonal elements of the matrix, P, the estimation error covariance matrix. The smaller the value, the better known the parameter or state variable that corresponds to that diagonal element. A rough rule of thumb to translate the variance, σ_i^2, into a ± range (from a normal to uniform probability distribution with identical variances) is

$$r_i = (3/4)^{1/2}\sigma_i \qquad (3\text{-}6)$$

where the estimate is in error by $\pm r$. If the variance is not getting smaller, then the observability criterion is not being met or the system is so noisy that the initial estimate is better than subsequent ones. The latter case is relevant only if the parameters change with time.

Some parameters converge very slowly because their errors make small projections into the sensor output. This can be seen in the column of the H matrix that corresponds to that state variable. If the value is small relative to that variable, then convergence will be slow. The only way to counter this is to find inputs and configurations that will boost the values in that column.

3.4 Design of Instrumentation and Fixtures

The key words for successful calibration are control, simplicity, and accuracy. Any errors not associated with the parameters to be estimated will be attributed erroneously to the estimated parameters. Hence, the estimates will be wrong. So, control of the calibration is important. Fixtures must be tight and rigid, instruments must be as accurate and bias free as possible. Simplicity is important because complex calibration apparatus that also needs to be calibrated is unwanted. It is better to take more measurements from fewer sensors than to try to control and read many sensors a fewer number of times. Use of gauge blocks and other fixed references is desirable if there is a compliance to absorb the position errors.

Whether the instruments are man read or machine read is not important as long as they are accurate, properly placed, and properly modeled. Naturally, computers often can read sensors much faster than humans.

3.5 Data Acquistion

Once the sensors are in place, the computer must be ready to accept the data. For off-line computation, the sensor readings and the commands to the actuators must be recorded. For on-line computation, the instrument readings and the actuators' control vector may be used directly in the Kalman filter. When computation is complete for one point, the next actuator control vector may be computed and sent to the manipulator.

3.6 Conclusion

Kalman filtering offers the following advantages:

(1) It does not require storage of individual data points.

(2) It allows prior knowledge to be included.

(3) It indicates convergence.

(4) It allows for noisy sensors and multiple sensors.

(5) It verifies selection of a fully observable ensemble of calibration points.

As with any calibration, knowledge of the calibrated device, careful control in data collection and fixturing, and accuracy of instrumentation is important for success.

3.6 Review

This section has discussed in general terms how the kinematic equations should be manipulated for the Kalman filter. In practice, finding Eq. (3-4) may be difficult, if not impossible. At best, the equations are very complex and nonlinear. This is the price to be paid for a detailed calibration. The next section derives the detailed equations for calibrating the Goddard manipulator.

SECTION 4

ANALYSIS OF THE GODDARD MANIPULATOR

4.1 Generating the Algebraic Equations

The fact that the Goddard manipulator, Figure 4-1, is a closed kinematic linkage makes its analysis quite difficult. An analysis done using Eq. (3-1) is more applicable to open kinematic linkages that characterize most industrial robots. 4 × 4 transformations will be used in a

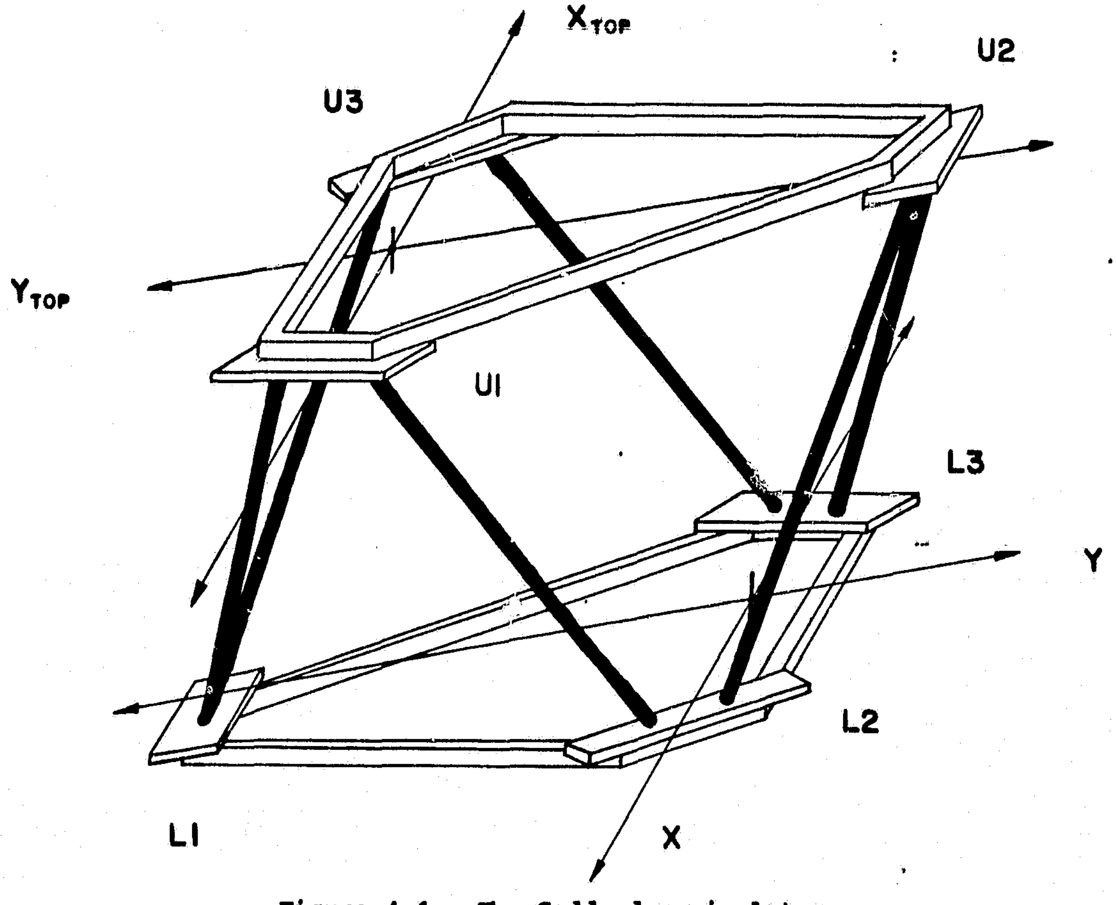

Figure 4-1. The Goddard manipulator.

slightly modified form. This section introduces a substantial amount of new nomenclature. Reference 2 defines the terms Trans [ ], Rot [ ], and Euler [ ]; the other nomenclature used herein maintains the standard that t(4) is the fourth element of vector $\underline{t}$, etc.

First, it is necessary to find all of the many unique transformations that characterize the manipulator kinematically. Little precision in manufacture is presumed; this necessitates the calibration. For example, each mounting pad of each link has a fixed offset transformation relative to one of two hypothetical origins (either for the fixed base or the moving platform). Another transformation relates the mounting-pad origin to the origin of the coordinate frame in which each joint at the link ends is found (see Figure 4-2). Each of these offset transformations is a function of no more than six variables, and in total, these 33 variables provide most of the state vector shown in Eq. (1-1). This effort results in a general relationship for finding the transformations that describe the telescoping links (see Figure 4-3).

Second, one finds the functions that relate the link transformations to the actuator positions, $\underline{u}$. This involves seven more state variables than those found in the first set of 33 equations.

The third part of the analysis derives three equations that relate the sensor outputs to the link transformations. This adds three more state variables which yields a total of 43 possible state variables. The sensor equations presume that potentiometers are mounted at the bottom pivots of three links.

Fourth, one combines the equations and derives a set of three equations for the sensor outputs, $\underline{z}$, as a function of the state variables, $\underline{x}$, and the actuator inputs, $\underline{u}$. There is no obvious method for producing this combination, and, fortunately, one need not exist. It is necessary only to compute $\underline{z}$ and the derivative of the output function with respect to the state vector. $\underline{z}$ can be computed using the previous mass of equations, and the derivative of $\underline{x}$ can be found using the chain rule. This process, however, yields a 45 × 45-element matrix which, apparently, must be inverted.

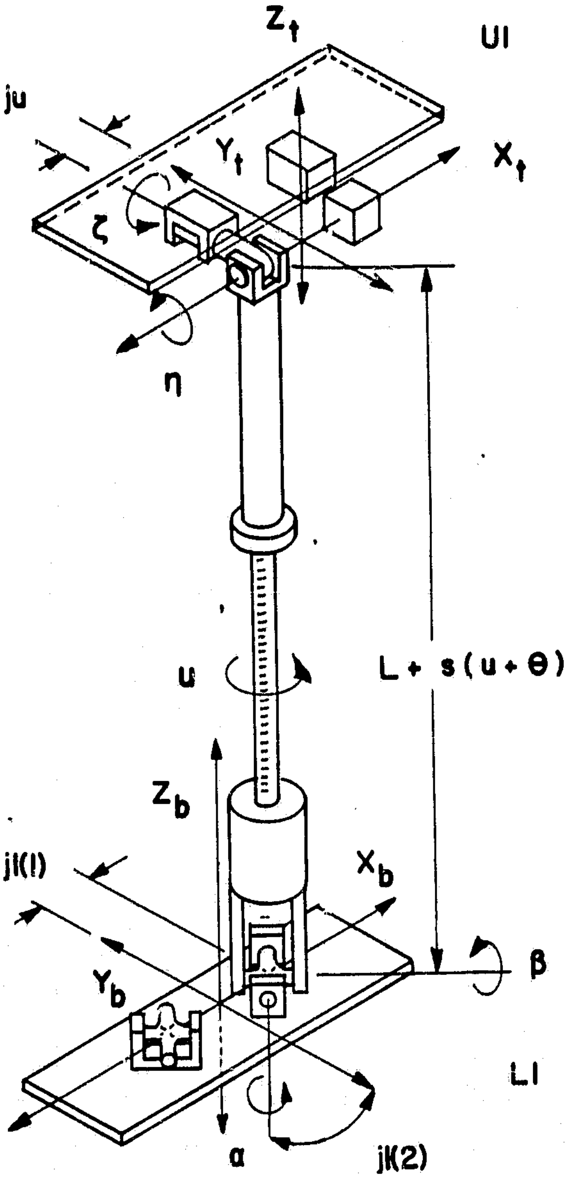

Figure 4-2. Detail of a single link.

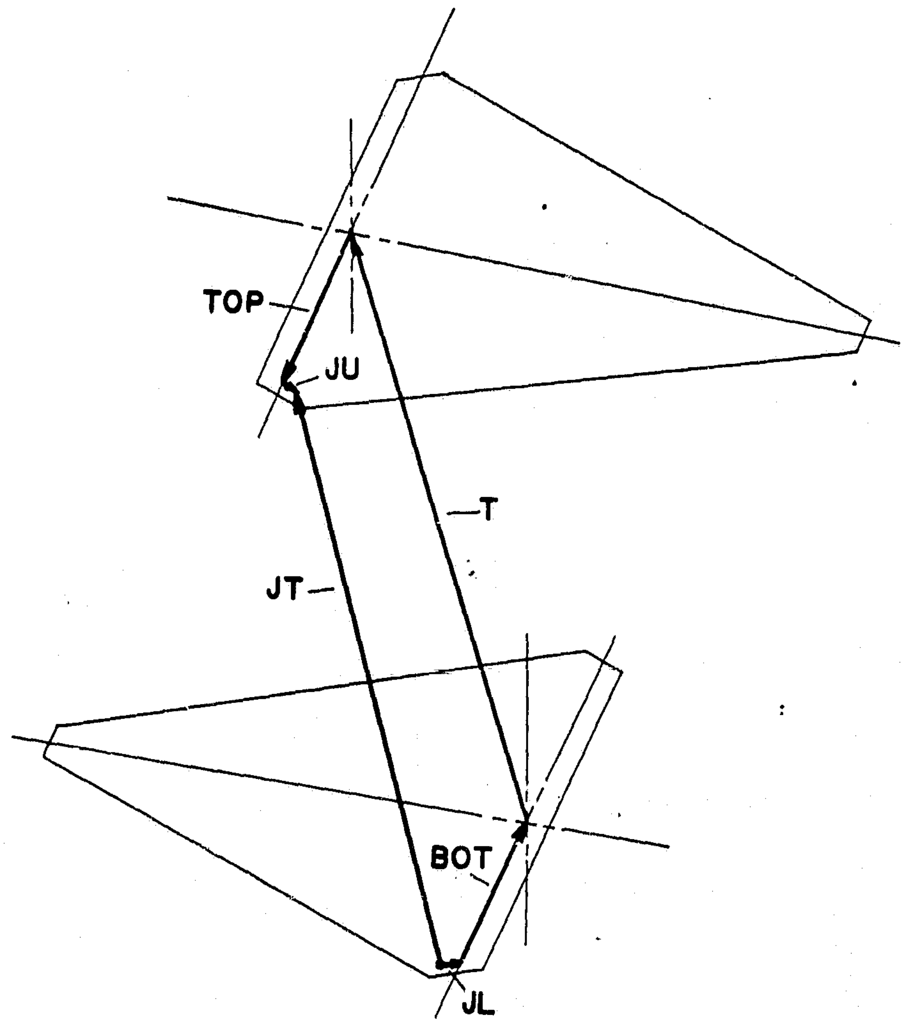

Figure 4-3. Transformation graph.

Fifth, however, this formidable problem can, in this case, be reduced to one of inverting a 6 × 6 matrix. The form of the transformation equations and selecting proper intermediate variables makes the reduction possible.

Sixth, an easier method of computing the necessary differentials is described. The Appendix contains the set of analytic equations used to compute the differentials. The need for a simpler computation scheme is self evident.

The first task is to establish a set of coordinate frames. Referring to Figure 4-1, the origin of the manipulator base will be defined as a point in the plane defined by the three points midway between the center of rotation of the pivots at the base of adjacent links. This midpoint for base L1 is on the y axis and on the x axis for the L2 and L3 bases. The top of the manipulator has a similarly defined origin, base U2 is on the x axis and U1 and U3 are on the y axis of the upper coordinate frame. The 4 × 4 transformation that relates the top platform to the bottom platform is T such that

$$T = \text{Trans}\,[t(1), t(2), t(3)] \cdot \text{Euler}\,[t(4), t(5), t(6)] \tag{4-1}$$

where

Trans [ ] = the translational transformation

Euler [ ] = the Euler angle transformation

In matrix form

$$\text{Trans}\,[(t(1), t(2), t(3)] = \begin{bmatrix} 1 & 0 & 0 & t(1) \\ 0 & 1 & 0 & t(2) \\ 0 & 0 & 1 & t(3) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4-2}$$

and

$$\text{Euler}\,[t(4), t(5), t(6)] = \text{Rot}\,[z,t(4)]\ \text{Rot}\,[y,t(5)]\ \text{Rot}\,[z,t(6)] \tag{4-3}$$

The Rot tranformations are

$$\mathrm{Rot}\,[z,t(4)] = \begin{bmatrix} \cos t(4) & -\sin t(4) & 0 & 0 \\ \sin t(4) & \cos t(4) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (4\text{-}4)$$

and

$$\mathrm{Rot}\,[y,t(5)] = \begin{bmatrix} \cos t(5) & 0 & \sin t(5) & 0 \\ 0 & 1 & 0 & 0 \\ -\sin t(5) & 0 & \cos t(t) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (4\text{-}5)$$

and

$$\mathrm{Rot}\,[z,t(6)] = \begin{bmatrix} \cos t(6) & -\sin t(6) & 0 & 0 \\ \sin t(6) & \cos t(6) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (4\text{-}6)$$

As a result, the vector $\underline{t}$, from Eq. (4-1) or the transformation T can describe the origin of the top platform relative to the origin of the bottom platform completely. Also, using the transformation in Eq. (4-7), the vector $\underline{q}$ in the top platform can be known as the vector $\underline{r}$ in the bottom platform.

$$\underline{r} = T \cdot \underline{q} \tag{4-7}$$

The Euler rotation transformation is one of a number of possible rotation transformations. It is used herein because it is so common.

Similarly, a platform's origin can be defined relative to each of its three bases with a transformation

$$\begin{aligned} \text{TOP} = \ & \text{Trans } [tp(1), tp(2), 0] \\ & \cdot \text{ Euler } [tp(4), tp(5), tp(6)] \end{aligned} \tag{4-8}$$

for the basis U1, U2, U3 relative to top platform and

$$\begin{aligned} \text{BOT} = \ & \text{Euler } [tb(3), tb(4), tb(5)] \\ & \cdot \text{ Trans } [tb(1), tb(2), 0] \end{aligned} \tag{4-9}$$

for the origin of the bottom platform relative to the bases L1, L2, L3. Putting the transformations in this order make tp(1), tp(2), tb(1), and tb(2) into x and y coordinates in their respective platform planes.

For any base, either tp(1) or tp(2) [tb(1) or tb(2)] may be selected uniquely. For each base, its vector $\underline{tp}$ (or $\underline{tb}$) will uniquely define its transformation relative to the origin of its respective platform. For a given base, the z axis is normal to the surface of the base, the y axis points to the center of the platform, and the x axis intersects adjacent pivots. Note that the base's origin lies above its surface.

The lower bases are, in turn, related to the bottom pivot points of the links by a simple rotation about the z axis and a translation

$$\text{JL} = \text{Trans } [j\ell(1), 0, 0] \cdot \text{Rot } [z, j\ell(2)] \tag{4-10}$$

The upper pivot points of the links are related to their bases by a translation

$$\text{JU} = \text{Trans } (ju, 0, 0) \tag{4-11}$$

Each pivot bearing lies in a different coordinate frame than the base to which it is attached. The z axes are parallel and the y axes still point inward, but the x axes of the bottom bases are the axes of rotation of the lowest bearings of the spiders. These are the axes for the angle α of Figure 4-2. The y axes of the top pivot bearings are the center of rotation of the topmost bearing, about which the angle ζ is measured. Here, the coordinate frame is just shifted along the x axis.

The variables $j\ell(1)$, $j\ell(2)$, and ju do not vary from base to base if a template is used to locate the position of the pivots relative to each other on the base. For all of the other vectors described, such uniformity may not be guaranteed, hence there are three TOPs and three BOTs, one for each base. Correspondingly, there are three <u>tb</u> vectors and three <u>tp</u> vectors.

Enough transformations have been defined to derive the transformation that represents the transformation for a single link:

$$JT = JL \cdot BOT \cdot T \cdot TOP \cdot JU \qquad (4\text{-}12)$$

The truth of this equation becomes evident if one traces the two equivalent transformation paths from the bottom bearing coordinate frame to the top bearing coordinate frame. One path is direct and is called JT. The other path goes from the bottom bearing to its base (JL), from this base to the bottom platform origin (BOT), from the bottom platform origin to the top platform origin (T), from the top platform origin to the top base (TOP), and, finally, from the top base to the top bearing coordinate frame (JU). These two paths are equivalent, hence the product of the set of relative transformations is equivalent to the direct transformation as in Eq. (4-12). This equation is the objective of the first step in the analysis.

This identity allows us to determine the extension and rotation of a link based on T and four other transformations. The transformation JT can be expressed explicitly as

$$JT = \begin{bmatrix} r_{11} & r_{12} & r_{13} & P_1 \\ r_{21} & r_{22} & r_{23} & P_2 \\ r_{31} & r_{32} & r_{33} & P_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4-13}$$

The variables r_{ij} and p_i are functions of JU, TOP, T, BOT, and JL. In addition to Eq. (4-12), JT also is a product of a third set of transformations that identify each translation and rotation of the link's parts

$$\begin{aligned} JT = & \text{Rot}\,[x,\alpha] \cdot \text{Rot}\,[y,\beta] \cdot \text{Rot}\,[z,\theta] \\ & \cdot \text{Trans}\,[z,L + s(\theta+u)] \cdot \text{Rot}\,[x,\eta] \\ & \cdot \text{Rot}\,[y,\zeta] \end{aligned} \tag{4-14}$$

The variable u is the actuator angle, s is the pitch of the lead screw and L is the link's unextended length.

The origin of this equation is an analysis similar to the one that generated Eq. (4-12). Starting at the coordinate frame of the bottom bearing, rotate the assembly α radians about the bearing's x axis. The z and y axes are then rotated in a vertical plane. Relative to the new coordinate frame, the assembly is rotated β radians about the new y axis. This is the other bearing of the spider. Consequently, the z and x axes are rotated in a plane that intersects the base at an angle of α radians. The z axis now points to the top pivot.

Now, rotate the remaining portion of the assembly θ radians about the z axis. This is the rotation of the lead screw's nut relative to the actuator. This is not the same as the actuator angle, u. The next step is a translation that pushes the top base $L + s(\theta + u)$ units along the current z axis. Two more rotations reorient the top bearing (a rotation of η radians about the newest x axis and a rotation of ζ radians about the y axis of the top bearing). As for Eq. (4-12), this exercise yields a product of relative transformatins that is equivalent to the direct transformation, JT.

The angles α, β, η, ζ, and θ are not really important; the quantity that must be known is u as a function of JT, or more explicitly, u as a function p_1, p_2, p_3, r_{12}, r_{22}, and r_{32}. Therefore, Eq. (4-14) should be solved for all actuator angles, u. Multiplying through the right side of Eq. (4-14) and equating with the components of JT, one finds six equations for the six dependent variables α, β, ζ, η, θ, and u. The three designating the translational part are

$$p_1 = \sin\beta\,\left(s(u + \theta) + L\right) \qquad (4\text{-}15)$$

$$p_2 = \sin\alpha\cos\beta\,\left(s(u + \theta) + L\right) \qquad (4\text{-}16)$$

$$p_3 = \cos\alpha\cos\beta\,\left(s(u + \theta) + L\right) \qquad (4\text{-}17)$$

Solving for sin α, sin β, and the cosines

$$\cos\alpha = \frac{p_3}{\sqrt{p_2^2 + p_3^2}} \qquad (4\text{-}18)$$

$$\cos\beta = \frac{p_2^2 + p_3^2}{\sqrt{p_1^2 + p_2^2 + p_3^2}} \qquad (4\text{-}19)$$

$$\sin\alpha = \frac{-p_2}{\sqrt{p_2^2 + p_3^2}} \tag{4-20}$$

$$\sin\beta = \frac{p_1}{\sqrt{p_1^2 + p_2^2 + p_3^2}} \tag{4-21}$$

plus

$$\sqrt{p_1^2 + p_2^2 + p_3^2} = s(u + \theta) + L \ . \tag{4-22}$$

If Eq. (4-18) through (4-21) are substituted into the transforms of Eq. (4-14), then the identity can be rewritten as

$$\begin{aligned} \mathrm{Rot}^{-1}[y,\beta] \cdot \mathrm{Rot}^{-1}[x,\alpha] \cdot JT = \ & \mathrm{Rot}[z,\theta] \\ & \cdot \mathrm{Trans}[z, L + s(\theta + u)] \\ & \cdot \mathrm{Rot}[x,\eta) \cdot \mathrm{Rot}[y,\zeta] \end{aligned} \tag{4-23}$$

and the left side is completely determined. Multiplying the right side of Eq. (4-23) through again and equating the elements of Rows 1 and 2, Column 2

$$\cos\beta\, r_{12} - \sin\beta\left(\cos\alpha\, r_{32} - \sin\alpha\, r_{22}\right) = -\cos\eta\sin\theta \tag{4-24}$$

$$r_{32}\sin\alpha + r_{22}\cos\alpha = \cos\eta\cos\theta \tag{4-25}$$

Dividing Eq. (4-24) by (4-25)

$$\tan\theta = \frac{\sin\beta\, r_{32}\cos\alpha - r_{22}\sin\alpha - r_{12}\cos\beta}{r_{32}\sin\alpha + r_{22}\cos\alpha} \tag{4-26}$$

Substituting Eq. (4-18) through (4-21) into Eq. (4-26) and inverting the function

$$\theta = \tan^{-1}\left[\frac{p_1 p_3 r_{32} + p_1 p_2 r_{22} - \left(p_2^2 + p_3^2\right) r_{12}}{\left(p_3 r_{22} - p_2 r_{32}\right)\sqrt{p_1^2 + p_2^2 + p_3^2}}\right] \tag{4-27}$$

Finally, Eq. (4-22) can be combined with Eq. (4-27) to derive

$$u = \frac{\sqrt{p_1^2 + p_2^2 + p_3^2} - L}{s} - \tan^{-1}\left[\frac{p_1 p_3 r_{32} + p_1 p_2 r_{22} - \left(p_2^2 + p_3^2\right) r_{12}}{\left(p_3 r_{22} - p_2 r_{32}\right)\sqrt{p_2^1 + p_2^2 + p_3^2}}\right] \tag{4-28}$$

This allows the actuator angle, u, to be determined for any r_{ij}, p_i which are based on T.

To review, there are six sets of six independent equations each that describe the transformations of the individual links as a function of T and a number of fixed transformations that come from applying Eq. (4-12) to each link. It is possible to define a vector $\underline{j_i}$ which comprises a set of six independent parameters of the transformation JT such that, for each link i

$$\underline{j_i} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ r_{12} \\ r_{22} \\ r_{32} \end{bmatrix}_i \tag{4-29}$$

Consequently, $\underline{j}_i$ is a function of T and the link's BOT, TOP, JL, and JU transformation are

$$\underline{j}_i = \underline{f}_i[T, TOP_i, BOT_i, JL_i, JU_i] \tag{4-30}$$

where

$$i = 1,2,3,\ldots,6$$

The subscript i is used to designate that there are six such vector valued functions, one for each link. The TOP, BOT, JL, and JU transformations are combined uniquely for each link.

Equation (4-28) gives the second set of equations that relates the transformation of an individual link to its actuator angle

$$u_i = g_i(\underline{j}_i, L_i, s_i) \tag{4-31}$$

These are six scalar equations. If these equations were combined with Eq. (4-30) and the $\underline{j}_i$'s eliminated from them, then the u_i's would be a function of T alone (assuming TOP, BOT, JL, and JU are known constant transformations). Conceivably, one could attempt to solve for T as a function of $\underline{u}$, but this may be impossible, and it is not necessary for the estimation task. The calibration needs the differential of the output vector, $\underline{z}$, with respect to the state vector, $\underline{x}$, to find the H matrix.

Thus far, what the sensors should be or where they should be placed has not been mentioned. The sensors should measure state variables directly if possible, without adding extra functions and, therefore, potential state variables. In this case, the state variables cannot be measured directly. The easiest variables to measure are the angles α and β at the base of each link. Subsequent analysis supposes that three of the α angles (of links 1, 2, and 3) were instrumented with rotary encoders or potentiometers. Then, from Eq. (4-20)

$$\sin(z_j + a_j) = \left[\frac{-p_2}{\sqrt{p_2^2 + p_3^2}}\right]_j \tag{4-32}$$

where

z_j = the measurement

a_j = a bias

j = an index of sensors

The lower case a's are the remaining three state variables, and this is the only set of equations needed to complete the third step of the analysis.

4.2 Solution of the Implicit Functions

The next question is, what next? The output Eq. (4-32) has been found, but this expression can not be differentiated with respect to state variables that do not appear in the equation. The problem is that Eq. (4-32), (4-31), and (4-30) comprise a set of implicit functions--a total of 45 scalar equations in 49 independent variables (the elements of the vector $\underline{u}$, and all the state variables) and 45 dependent variables (the elements of the vectors $\underline{j}_1$, $\underline{j}_2$, $\underline{j}_3$, $\underline{j}_4$, $\underline{j}_5$, $\underline{j}_6$, $\underline{t}$, and $\underline{z}$). The full set of state variables is

$$\underline{x}^T = \Big[\underline{tp}_1^T,\ \underline{tp}_2^T,\ \underline{tp}_3^T,\ \underline{tb}_1^T,\ \underline{tb}_2^T,\ \underline{tb}_3^T,\ a_1,\ a_2,\ a_3,\ L_1,\ldots,L_6,$$

$$s,\ ju_1,\ ju_2,\ ju_3,\ j\underline{\ell}_1^T,\ j\underline{\ell}_2^T,\ j\underline{\ell}_3^T\Big] \qquad (4\text{-}33)$$

There is a mathematical relation called the chain rule that allows one to find the partial derivative of a dependent variable with respect to an independent variable without solving the functions involved. This is important because the derivative of $\underline{z}$ (a dependent variable) with respect to the state variables (independent variables) is needed to create the H matrix used in the Kalman filter. The following is the fourth part of this analysis. First, three sets of implicit functions are created from Eq. (4-12), (4-28), and (4-32).

$$\underline{0} = \underline{f}_i(\underline{t}, \underline{tp}_i, \underline{tb}_i, \underline{ju}_i, \underline{j\ell}_i) - \underline{j}_i \tag{4-34}$$

$$0 = g_i(\underline{j}_i, L_i, s_i) - u_i \tag{4-35}$$

where

$$i = 1, 2, \ldots, 6$$

and

$$0 = h_j = \sin(z_j + a_j) + \left[\frac{p_2}{\sqrt{p_2^2 + p_3^2}}\right]_j \tag{4-36}$$

where

$$j = 1, 2, 3$$

The chain rule allows us to create

$$(-)\begin{bmatrix} \frac{\partial \underline{f}_1}{\partial x_n} \\ \vdots \\ \frac{\partial \underline{f}_6}{\partial x_n} \\ \cdots \\ \frac{\partial \underline{g}}{\partial x_n} \\ \cdots \\ \frac{\partial \underline{\underline{h}}}{\partial x_n} \end{bmatrix} = \begin{bmatrix} \frac{\partial \underline{f}_1}{\partial \underline{j}_1} & \cdots & \frac{\partial \underline{f}_1}{\partial \underline{j}_6} & \frac{\partial \underline{f}_1}{\partial \underline{t}} & \frac{\partial \underline{f}_1}{\partial \underline{z}} \\ \vdots & & \vdots & \vdots & \\ \frac{\partial \underline{f}_6}{\partial \underline{j}_1} & \cdots & \frac{\partial \underline{f}_6}{\partial \underline{j}_6} & \frac{\partial \underline{f}_6}{\partial \underline{t}} & \frac{\partial \underline{f}_6}{\partial \underline{z}} \\ \frac{\partial \underline{g}}{\partial \underline{j}_1} & \cdots & \frac{\partial \underline{g}}{\partial \underline{j}_6} & \frac{\partial \underline{g}}{\partial \underline{t}} & \frac{\partial \underline{g}}{\partial \underline{z}} \\ \frac{\partial \underline{\underline{h}}}{\partial \underline{j}_1} & \cdots & \frac{\partial \underline{\underline{h}}}{\partial j_6} & \frac{\partial \underline{\underline{h}}}{\partial \underline{t}} & \frac{\partial \underline{\underline{h}}}{\partial \underline{z}} \end{bmatrix} \cdot \begin{bmatrix} \frac{\partial \underline{j}_1}{\partial x_n} \\ \vdots \\ \frac{\partial \underline{j}_6}{\partial x_n} \\ \frac{\partial \underline{t}}{\partial x_n} \\ \frac{\partial \underline{z}}{\partial x_n} \end{bmatrix} \tag{4-37}$$

where

x_n = the n^{th} element of the state vector

See Reference 3 for details. Note that the last element of the vector on the right side of Eq. (4-37) is the variable sought. The large matrix may be computed numerically as may the vector on the left. This computation requires a 45 × 45 matrix to be inverted to find $\partial \underline{z}/\partial x_n$ which is not practical. Fortunately, this matrix may be simplified so that the largest matrix inversion is on the order of 6 × 6. The following paragraphs explain how this is done.

Refer to Figure 4-4. It is equivalent to Eq. (4-37) after the differentials have been evaluated and assigned to the given submatrices. Note that most of the matrix is empty. Some useful simplifications may then be made. This is the fifth step in the analysis. First, presume that x_n is not found in $\underline{q}$ or $\underline{h}$, which is the case for 33 of the variables. Each of the differentials

$$\frac{\partial \underline{j}_i}{\partial x_n}$$

may be found by rearranging the first six rows of submatrices to yield

$$C_i \frac{\partial \underline{t}}{\partial x_n} - \underline{F}_{in} = \frac{\partial \underline{j}_i}{\partial x_n} \tag{4-38}$$

where

$C_i = \partial \underline{f}_i/\partial \underline{t}$ = a 6 × 6 matrix

$i = 1, 2, \ldots, 6$

$\underline{F}_{in} = (-)\partial \underline{f}_i/\partial x_n$ = a 6-element vector

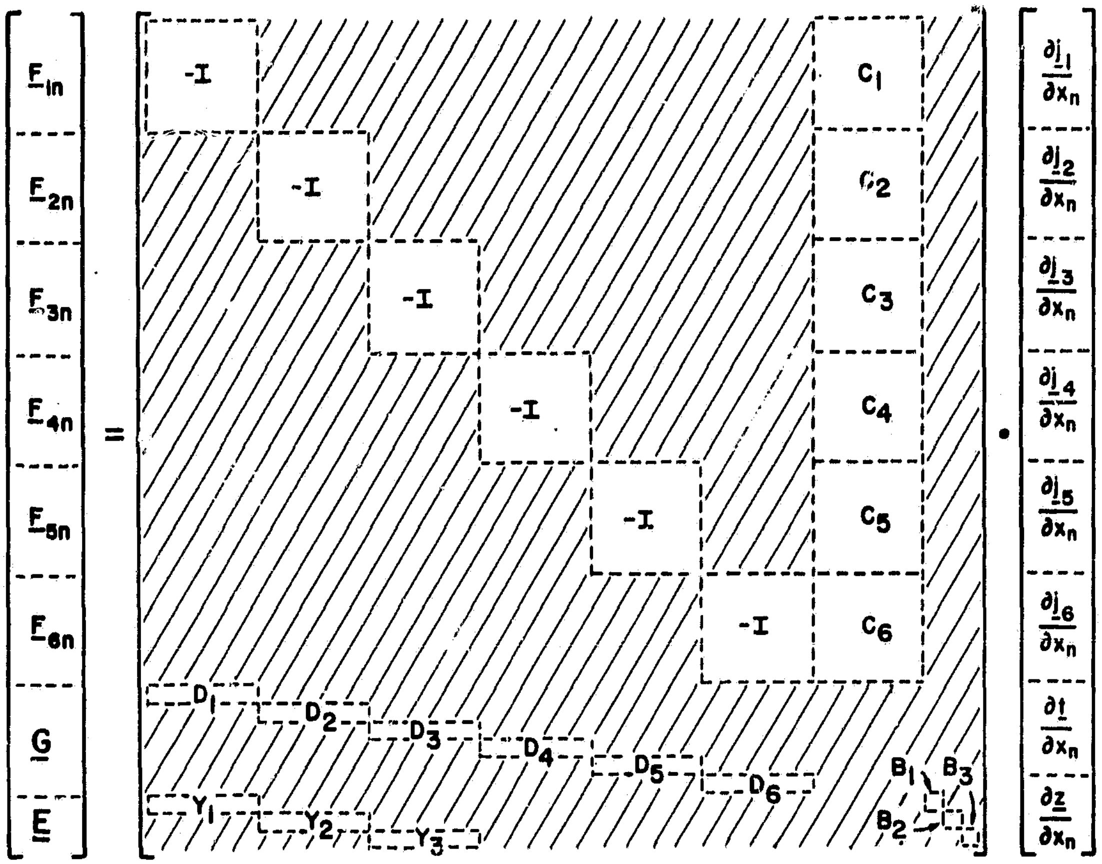

Figure 4-4. Jacobian matrix.

In turn, this relation can be inserted into the g-type equations ($\partial \underline{j}_i / \partial x_n$ is eliminated) and this vector results

$$\underline{0} = \begin{bmatrix} D_1 \left(C_1 \dfrac{\partial \underline{t}}{\partial x_n} - \underline{F}_{1n} \right) \\ \cdot \\ \cdot \\ \cdot \\ D_6 \left(C_6 \dfrac{\partial \underline{t}}{\partial x_n} - \underline{F}_{6n} \right) \end{bmatrix} \qquad (4\text{-}39)$$

where

$$D_i = \partial g_i / \partial j_i = \text{a } 1 \times 6 \text{ matrix}$$

Distributing D and factoring out $\partial \underline{t}/\partial x_n$

$$\begin{bmatrix} D_1 \underline{F}_{1n} \\ \vdots \\ D_6 \underline{F}_{6n} \end{bmatrix} = \begin{bmatrix} D_1 C_1 \\ \vdots \\ D_6 C_6 \end{bmatrix} \frac{\partial \underline{t}}{\partial x_n} \tag{4-40}$$

Then, the matrix on the right side may be inverted. It is only a 6 × 6 matrix. Inverting this matrix and premultiplying each side with it results in a numerical value for $\partial \underline{t}/\partial x_n$. If $\partial \underline{t}/\partial x_n$ is known, then the left side of Eq. (4-38) is determined completely so it may be inserted into the $\underline{h}$ equations

$$Y_1 \left(C_1 \frac{\partial \underline{t}}{\partial x_n} - \underline{F}_{1n}\right) = - B_1 \frac{\partial z_1}{\partial x_n} \tag{4-41}$$

$$Y_2 \left(C_2 \frac{\partial \underline{t}}{\partial x_n} - \underline{F}_{2n}\right) = - B_3 \frac{\partial z_2}{\partial x_n} \tag{4-42}$$

$$Y_3 \left(C_3 \frac{\partial \underline{t}}{\partial x_n} - F_{3n}\right) = - B_3 \frac{\partial z_3}{\partial x_n} \tag{4-43}$$

where

$Y_j = \partial k_j/\partial \underline{j}_j$ = a 1 × 6 matrix

$B_j = \partial k_j/\partial z_j$ = a scalar quantity

Consequently

$$\frac{\partial z_1}{\partial x_n} = - \frac{Y_1}{B_1} \left[C_1 \frac{\partial \underline{t}}{\partial x_n} - F_{1n}\right] \tag{4-44}$$

$$\frac{\partial z_2}{\partial x_n} = - \frac{Y_2}{B_2} \left[C_2 \frac{\partial \underline{t}}{\partial x_n} - F_{2n}\right] \tag{4-45}$$

$$\frac{\partial z_3}{\partial x_n} = -\frac{Y_3}{B_3}\left[C_3 \frac{\partial \underline{t}}{\partial x_n} - \underline{F}_{3n}\right] \qquad (4\text{-}45)$$

The left side of these three equations gives the elements of H that are necessary for this analysis. The matrix

$$\begin{bmatrix} D_1 \; C_1 \\ \cdot \\ \cdot \\ \cdot \\ D_6 \; C_6 \end{bmatrix}^{-1} \qquad (4\text{-}46)$$

is evaluated once each time step, the value of $\partial \underline{t}/\partial x_n$ must be evaluated for each state variable, as is $\underline{F}_{in}$, and both must be evaluated every time step. Thus

$$H(k) = \frac{\partial \underline{z}(k)}{\partial \underline{x}(k)} = \begin{bmatrix} \frac{\partial z_1}{\partial x_1} & \cdot \; \cdot \; \cdot \; \cdot \; \cdot & \frac{\partial z_1}{\partial x_{43}} \\ \frac{\partial z_2}{\partial x_1} & \cdot \; \cdot \; \cdot \; \cdot \; \cdot & \frac{\partial z_2}{\partial x_{43}} \\ \frac{\partial z_3}{\partial x_1} & \cdot \; \cdot \; \cdot \; \cdot \; \cdot & \frac{\partial z_3}{\partial x_{43}} \end{bmatrix}_{\hat{\underline{x}}(k)(-)} \qquad (4\text{-}48)$$

which is the desired form for the Kalman filter. This allows estimation of the values of the 43 state variables. If it is judged that fewer states are required, then these states may be omitted without Eq. (4-44) through (4-47) losing their generality.

There are a few state variables that only appear in functions $\underline{g}$ and $\underline{h}$. By a similar analysis as above, this relation results for state variables that appear in $\underline{g}$

$$\underline{G} = \begin{bmatrix} D_1 \ C_1 \\ \vdots \\ D_6 \ C_6 \end{bmatrix} \frac{\partial \underline{t}}{\partial x_n} \tag{4-49}$$

where

$$\underline{G} = (-)\ \partial \underline{g}/\partial x_n = \text{a 6-element vector}$$

Consequently

$$\frac{\partial \underline{t}}{\partial x_n} = \begin{bmatrix} D_1 \ C_1 \\ \vdots \\ D_6 \ C_6 \end{bmatrix}^{-1} \underline{G} \tag{4-50}$$

and

$$\frac{\partial z_1}{\partial x_n} = -\frac{Y_1}{B_1}\left[C_1 \frac{\partial \underline{t}}{\partial x_n}\right] \tag{4-51}$$

$$\frac{\partial z_2}{\partial x_n} = -\frac{Y_2}{B_2}\left[C_2 \frac{\partial \underline{t}}{\partial x_n}\right] \tag{4-52}$$

$$\frac{\partial z_3}{\partial x_n} = -\frac{Y_3}{B_3}\left[C_3 \frac{\partial \underline{t}}{\partial x_n}\right] \tag{4-53}$$

If the state variable appears in $\underline{h}$ only, like the a_j's, then

$$\underline{0} = \begin{bmatrix} D_1 \ C_1 \\ \cdot \\ \cdot \\ \cdot \\ D_6 \ C_6 \end{bmatrix} \frac{\partial \underline{t}}{\partial x_n} \qquad (4\text{-}54)$$

Since the matrix on the right side is nonsingular

$$\frac{\partial \underline{t}}{\partial x_n} = \underline{0} \qquad (4\text{-}55)$$

so

$$\frac{\partial z_1}{\partial x_n} = \frac{E_1}{B_1} \qquad (4\text{-}56)$$

$$\frac{\partial z_2}{\partial x_n} = \frac{E_2}{B_2} \qquad (4\text{-}57)$$

$$\frac{\partial z_3}{\partial x_n} = \frac{E_3}{B_3} \qquad (4\text{-}58)$$

It should be noted that additional sensors may be added with little increase in the complexity of this system. Putting sensors on all six joints would speed up the convergence of the state estimates. Another interesting feature is that the values of

$$\frac{\partial \underline{t}}{\partial \underline{u}}$$

are generated if the $\underline{u}$'s are treated like x_n's. This is the manipulator's Jacobian matrix, which relates the differential change in its Cartesian coordinates with changes in its joint angles.

The remaining challenge is to evaluate the differentials of the functions $\underline{f}_i$, $\underline{g}$, and $\underline{h}$. The MACSYMA symbolic algebraic manipulation program was used for this evaluation. The functions for evaluating the elements of the matrices $\underline{F}_{in}$, E_n, G_n, C_i, D_i, Y_i, B_i are given in the Appendix. For the differentials of the $\underline{F}_{in}$ and C_i matrices, the functions are quite lengthy. Clearly, these equations should not be programmed into a computer in order to compute the $\underline{F}_{in}$'s and C_i's. The following is the sixth and final part of the analysis.

A simpler method of deriving the differentials comes from the Teleoperator Arm Design Theory (TOAD).[4] To set up the equations for this, one must remember that determining the differential of a transformation with respect to a variable is equivalent to finding the differential with respect to each of its matrix elements. So, if the differential of the elements of JT with respect to a state variable or dependent variable is found, then the differential with respect to JT is found and vice versa. Recall Eq. (4-12). The differential of the right side with respect to the state variables would also give the differential of JT with respect to the state variables. If the right side is factored into all its single-axis rotations and single-direction translations, as for Eq. (4-1), then each state or dependent variable will appear in only one of these factors. So, if the differential of the matrix with respect to its variable is put in its place, then the new product is the differential of the old product with respect to the state variable.

The aforementioned differential matrix needs not be evaluated; a single matrix operator may be used to premultiply the transformation that will result in the differential, i.e.

$$\frac{\partial A_i}{\partial x_m} = N_i A_i \qquad (4\text{-}59)$$

where

N_i = the matrix operator

If A_i is an x translation, then

$$N = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4-60}$$

If A_i is a y translation, then

$$N = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4-61}$$

If A_i is a z translation, then

$$N = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4-62}$$

If A_i is a rotation about the x axis, then

$$N = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4-63}$$

If A_i is a rotation about the y axis, then

$$N = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4-64}$$

And, if A_i is a rotation about the z axis, then

$$N = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4-65}$$

The benefits of this technique are obvious; the equations are more compact, and they result from transformations that would have to be computed for any case.

The numerical computation of the submatrices $\underline{F}_{in}$ and C_i should be performed using TOAD. Factoring Eq. (4-12) into its individual single variable transformations will achieve this, and it is possible for all homogeneous transformations. Equation (4-12) starts with the five basic subtransformtions, each of which can be decomposed into up to six single variable transformations as in Eq. (4-1). Thus the derivative of any JT with respect to a single variable is the product of the 17 elemental transformations, with the approximate N-operator premultiplying the elemental transformation that contains the variable of interest. The values of the derivative of the $\underline{f}_i$'s with respect to the state variables and $\underline{t}$ are selected from the derivative of JT_i with respect to the state variables and $\underline{t}$. If the variable is a state variable, it goes in $\underline{F}_i$. If it is an element of the $\underline{t}$ vector, it goes in a C_i.

This derivative must be computed for each state variable and $\underline{t}$ element of the vector $\underline{j}_i$ for each joint transformation JT_i. This is a substantial amount of computation, but the alternative is to implement the analytic equations found in the Appendix.

The TOAD technique works for homogeneous transformations only. The submatrices D, B, Y, and the vectors $\underline{G}$, and $\underline{E}$ must be computed using the analytic equations found in the Appendix.

The following is a possible sequence of the computations needed for calibration.

(1) Select a T transformation to which the manipulator will be moved. Using the current estimates of transformations JL, JU, TOP, and BOT compute all $\underline{j}_i$ and then $\underline{u}_i$. Move the manipulator to the joint angles $\underline{u}_i$.

(2) Compute the numerical values of all D_i, Y_j, and B_j using the analytic equations, again use the current estimates of the state variables and joint variables.

(3) Use TOAD to compute the values of the submatrices C_i. This involves setting up the decomposed product of the elemental transformations for each joint. Then, insert the matrix operators into the product for each of the $\underline{t}$ elements, and the new product is evaluated. The derivatives are selected from the matrix-valued differentials yielding one column of a C_i matrix for each $\underline{t}$ element/JT_i combination.

(4) Evaluate the matrix inverse of Eq. (4-47).

Repeat the following five steps for each state variable.

(5) Compute the values of the vectors $\underline{G}$ and $\underline{E}$ using the analytic equations in the Appendix. If the state variable is not found in the equations, its derivative is zero. There are new $\underline{G}$, $\underline{E}$, and $\underline{F}_i$ vectors for each state variable.

(6) Compute the values of the vectors $\underline{F}_i$ using TOAD. These are best computed and stored when Step 3 is executed and should be retrieved now.

(7) Compute the vector

$$\frac{\partial \underline{t}}{\partial x_n}$$

using Eq. (4-40), (4-50), or (4-55).

(8) Compute the values of

$$\frac{\partial \underline{z}}{\partial x_n}$$

using Eq. (4-44), (4-51), or (4-57).

(9) Insert these values into the H(k) matrix.

When the H(k) matrix is complete, follow the next four steps.

(10) Compute the Kalman-filter gain using Eq. (2-7) to (2-9).

(11) Compute the expected value of the sensor outputs based on the value of T (or equivalently, $\underline{u}$).

(12) Read the sensors, and update the state estimates using Eq. (2-13).

(13) Return to Step 1.

This paradigm may be optimized to ease computation of course. At Step 1, the value of T was assumed to be given. Its selection for each time step is based on the ability to satisfy the observability criterion. One can estimate whether any sequence of T transformations can satisfy observability a priori, but selecting a set of T's to satisfy observability is much trickier. The only sensible way to pick T is at random and then monitor the resulting H(k) matrices for observability after the fact.

SECTION 5

CONCLUSIONS AND RECOMMENDATIONS

The sum of the work presented is the equations contained in Section 4 and the Appendix. The manner in which these equations are derived should have an analog in the analysis of all closed-link kinematic linkages. The particular equations are peculiar to this manipulator configuration. The manipulator may be scaled up or down in size, and the generality is not lost.

The quantity of computation needed is substantial. This will obviously slow the speed at which a calibration can be done. A more insidious problem will be how to guarantee the stability of the filter, expecially one with up to 43 state variables. If the initial estimates of state variables are far from their true values, the the filter could converge to unreasonable state estimates or drift about state space without converging. It is important to provide as accurate an initial estimate as possible to help justify the linearization of the equations.

How useful the Kalman filter is for solving problems of this sort and magnitude has not been simulated or experimentally verified. Should the filter fail to give reasonable results, two techniques for improving the estimation are offered:

(1) Degenerate the model and limit the number of state variables. Pick a set of critical state variables and presume the others are fixed and accurate. The estimated state variables in the new model will reflect any errors in the

unestimated state variables in the full model. In the paradigm described at the end of Section 4, steps 5 through 9 will be executed a fewer number of times. The encoder biases, a_j, and the unextended joint lengths, L_i, are likely candidates for this type of state-variable subset. They are, respectively, offsets in sensors and inputs (actuators). These yield quick fixes for whatever working kinematic model is used in practice.

(2) Compute the H(k) matrix based on the initial estimates of the state variables. Should the state estimate diverge, the Kalman gains will still be based on reasonable values of the state.

All experimental data should be saved, so the examples can be rerun with a simpler model or different initial conditions.

The numerical values of $\underline{F_i}$ and C_i should be computed using TOAD. The numerical values of $\underline{G}$, $\underline{E}$, D_i, Y_j, and B_j should be found by using analytic equations found in the Appendix. This avoids programming a very large set of equations into a computer. The programmer must find the best way to implement the equations given.

APPENDIX

This appendix contains formulas for the differentials of Eqs. (4-33), (4-34), (4-35). The differentials for $\frac{\partial f}{\partial j_u}$, $\frac{\partial f}{\partial j_{1(1)}}$, and $\frac{\partial f}{\partial j_{1(2)}}$ were not computed.

Key

$\frac{\partial g}{\partial *}$ ≡ partial derivative of the expression g, with respect to *

$\frac{\partial h}{\partial *}$ ≡ partial derivative of the expression h, with respect to *

$\frac{\partial f(k)}{\partial *}$ ≡ partial derivative of the kth element of the vector-valued function, f, with respect to *.

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g-equations (Eq. 4.30)

$$
-\frac{\partial g}{\partial p1} = \left(\frac{p3\ r32 + p2\ r22}{\mathrm{sqrt}(p3^2 + p2^2 + p1^2)\ (p3\ r22 - p2\ r32)} - \frac{p1\ (p1\ p3\ r32 + p1\ p2\ r22 - (p3^2 + p2^2)\ r12)}{(p3^2 + p2^2 + p1^2)^{3/2}\ (p3\ r22 - p2\ r32)}\right) \Big/ \left(\frac{(p1\ p3\ r32 + p1\ p2\ r22 - (p3^2 + p2^2)\ r12)^2}{(p3^2 + p2^2 + p1^2)\ (p3\ r22 - p2\ r32)^2} + 1\right) - \frac{p1}{\mathrm{sqrt}(p3^2 + p2^2 + p1^2)\ s}
$$

$$
-\frac{\partial g}{\partial p2} = \left(\frac{p1\ r22 - 2\ p2\ r12}{\sqrt{p3^2 + p2^2 + p1^2}\ (p3\ r22 - p2\ r32)} - \frac{p2\ (p1\ p3\ r32 + p1\ p2\ r22 - (p3^2 + p2^2)\ r12)}{(p3^2 + p2^2 + p1^2)^{3/2}\ (p3\ r22 - p2\ r32)} + \frac{r32\ (p1\ p3\ r32 + p1\ p2\ r22 - (p3^2 + p2^2)\ r12)}{\sqrt{p3^2 + p2^2 + p1^2}\ (p3\ r22 - p2\ r32)^2}\right) \Big/ \left(\frac{(p1\ p3\ r32 + p1\ p2\ r22 - (p3^2 + p2^2)\ r12)^2}{(p3^2 + p2^2 + p1^2)\ (p3\ r22 - p2\ r32)^2} + 1\right)
$$

$$
- \frac{p2}{\sqrt{p3^2 + p2^2 + p1^2}\ s}
$$

$$
\frac{\partial g}{\partial p3} = \frac{\left(\dfrac{p1\ r32 - 2\ p3\ r12}{\sqrt{p3^2 + p2^2 + p1^2}\ (p3\ r22 - p2\ r32)} - \dfrac{p3\ (p1\ p3\ r32 + p1\ p2\ r22 - (p3^2 + p2^2)\ r12)}{(p3^2 + p2^2 + p1^2)^{3/2}\ (p3\ r22 - p2\ r32)} - \dfrac{r22\ (p1\ p3\ r32 + p1\ p2\ r22 - (p3^2 + p2^2)\ r12)}{\sqrt{p3^2 + p2^2 + p1^2}\ (p3\ r22 - p2\ r32)^2} \right)}{\left(\dfrac{(p1\ p3\ r32 + p1\ p2\ r22 - (p3^2 + p2^2)\ r12)^2}{(p3^2 + p2^2 + p1^2)\ (p3\ r22 - p2\ r32)^2} + 1 \right)} - \frac{p3}{\sqrt{p3^2 + p2^2 + p1^2}\ s}
$$

$$-\frac{\partial g}{\partial r_{12}} = (-p3^2 - p2^2)/\left(\mathrm{sqrt}(p3^2 + p2^2 + p1^2)\ (p3\ r22 - p2\ r32)\left(\frac{(p1\ p3\ r32 + p1\ p2\ r22 - (p3^2 + p2^2)\ r12)^2}{(p3^2 + p2^2 + p1^2)\ (p3\ r22 - p2\ r32)^2} + 1\right)\right)$$

$$-\frac{\partial g}{\partial r_{22}} = \left(\frac{p1\ p2}{\mathrm{sqrt}(p3^2 + p2^2 + p1^2)\ (p3\ r22 - p2\ r32)} - \frac{p3\ (p1\ p3\ r32 + p1\ p2\ r22 - (p3^2 + p2^2)\ r12)}{\mathrm{sqrt}(p3^2 + p2^2 + p1^2)\ (p3\ r22 - p2\ r32)^2}\right) / \left(\frac{(p1\ p3\ r32 + p1\ p2\ r22 - (p3^2 + p2^2)\ r12)^2}{(p3^2 + p2^2 + p1^2)\ (p3\ r22 - p2\ r32)^2} + 1\right)$$

$$\frac{\partial g}{\partial r_{32}} = \left(\frac{p1\,p3}{\mathrm{sqrt}(p3^2 + p2^2 + p1^2)\,(p3\,r22 - p2\,r32)} + \frac{p2\,(p1\,p3\,r32 + p1\,p2\,r22 - (p3^2 + p2^2)\,r12)}{\mathrm{sqrt}(p3^2 + p2^2 + p1^2)\,(p3\,r22 - p2\,r32)^2}\right) \Big/ \left(\frac{(p1\,p3\,r32 + p1\,p2\,r22 - (p3^2 + p2^2)\,r12)^2}{(p3^2 + p2^2 + p1^2)\,(p3\,r22 - p2\,r32)^2} + 1\right)$$

$$\frac{\partial g}{\partial L} = \frac{1}{s}$$

$$\frac{\partial g}{\partial s} = \frac{\mathrm{sqrt}(p3^2 + p2^2 + p1^2) - l}{s^2}$$

h-equations (Eq. 4.31)

$$\frac{\partial h}{\partial p1} = \frac{p1^2}{(p3^2 + p1^2)^{3/2}} - \frac{1}{\mathrm{sqrt}(p3^2 + p1^2)}$$

$$\frac{\partial h}{\partial p3} = \frac{p1\ p3}{(p3^2 + p1^2)^{3/2}}$$

$$\frac{\partial h}{\partial z} = \cos(z + a)$$

$$\frac{\partial h}{\partial a} = \cos(z + a)$$

f - equations (Eq. 4.29)

$$\frac{\partial f1}{\partial t1} =$$

```
cos(jl(2)) (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
 - sin(jl(2)) (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
```

$$\frac{\partial f1}{\partial t2} =$$

```
cos(jl(2)) (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
 - sin(jl(2)) (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
```

$$\frac{\partial f1}{\partial t3} =$$

```
cos(jl(2)) cos(tb(3)) sin(tb(4)) - sin(jl(2)) sin(tb(3)) sin(tb(4))
```

$$\frac{\partial f1}{\partial t4} =$$

```
cos(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
```

```
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju)
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (sin(t(4)) cos(t(5)) sin(t(6)) - cos(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
+ sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju))
- sin(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju)
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (sin(t(4)) cos(t(5)) sin(t(6)) - cos(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
+ sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju))
```

$\frac{\partial f1}{\partial t5}=$

```
    cos(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
(- sin(t(4)) sin(t(5)) cos(t(6)) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + sin(t(4)) sin(t(5)) sin(t(6))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) cos(t(5)) sin(tp(5)) cos(tp(6)) ju)
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
(- cos(t(4)) sin(t(5)) cos(t(6)) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + cos(t(4)) sin(t(5)) sin(t(6))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) cos(t(5)) sin(tp(5)) cos(tp(6)) ju)
+ cos(tb(3)) sin(tb(4)) (- cos(t(5)) cos(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ cos(t(5)) sin(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
+ sin(t(5)) sin(tp(5)) cos(tp(6)) ju))
- sin(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
(- sin(t(4)) sin(t(5)) cos(t(6)) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + sin(t(4)) sin(t(5)) sin(t(6))
```

```
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) cos(t(5)) sin(tp(5)) cos(tp(6)) ju)
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
(- cos(t(4)) sin(t(5)) cos(t(6)) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + cos(t(4)) sin(t(5)) sin(t(6))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) cos(t(5)) sin(tp(5)) cos(tp(6)) ju)
+ sin(tb(3)) sin(tb(4)) (- cos(t(5)) cos(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ cos(t(5)) sin(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
+ sin(t(5)) sin(tp(5)) cos(tp(6)) ju))
```

$\frac{\partial fl}{\partial t6}=$

```
     cos(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2)))
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (sin(t(4)) sin(t(6)) - cos(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2)))
+ cos(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ sin(t(5)) cos(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))))
- sin(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
```

```
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2)))
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (sin(t(4)) sin(t(6)) - cos(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2)))
+ sin(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ sin(t(5)) cos(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))))
```

$\frac{\partial f2}{\partial tl}=$

```
sin(jl(2)) (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
  + cos(jl(2)) (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
```

$\frac{\partial f2}{\partial t2}=$

```
cos(jl(2)) (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
+ sin(jl(2)) (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
```

$\frac{\partial f2}{\partial t3}=$

```
cos(jl(2)) sin(tb(3)) sin(tb(4)) + sin(jl(2)) cos(tb(3)) sin(tb(4))
```

$\frac{\partial f2}{\partial t4}=$

```
cos(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju)
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (sin(t(4)) cos(t(5)) sin(t(6)) - cos(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
+ sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju))
+ sin(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
```

```
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju)
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (sin(t(4)) cos(t(5)) sin(t(6)) - cos(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
+ sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju))
```

$\frac{\partial f2}{\partial t5}=$

```
    cos(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
(- sin(t(4)) sin(t(5)) cos(t(6)) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + sin(t(4)) sin(t(5)) sin(t(6))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) cos(t(5)) sin(tp(5)) cos(tp(6)) ju)
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
(- cos(t(4)) sin(t(5)) cos(t(6)) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
```

```
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + cos(t(4)) sin(t(5)) sin(t(6))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) cos(t(5)) sin(tp(5)) cos(tp(6)) ju)
+ sin(tb(3)) sin(tb(4)) (- cos(t(5)) cos(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ cos(t(5)) sin(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
+ sin(t(5)) sin(tp(5)) cos(tp(6)) ju))
+ sin(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
(- sin(t(4)) sin(t(5)) cos(t(6)) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + sin(t(4)) sin(t(5)) sin(t(6))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) cos(t(5)) sin(tp(5)) cos(tp(6)) ju)
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
(- cos(t(4)) sin(t(5)) cos(t(6)) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + cos(t(4)) sin(t(5)) sin(t(6))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) cos(t(5)) sin(tp(5)) cos(tp(6)) ju)
```

```
+ cos(tb(3)) sin(tb(4)) (- cos(t(5)) cos(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ cos(t(5)) sin(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
+ sin(t(5)) sin(tp(5)) cos(tp(6)) ju))
```

$\frac{\partial f2}{\partial t6} =$

```
     cos(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2)))
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (sin(t(4)) sin(t(6)) - cos(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2)))
+ sin(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
```

```
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ sin(t(5)) cos(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))))
+ sin(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2)))
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (sin(t(4)) sin(t(6)) - cos(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2)))
+ cos(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ sin(t(5)) cos(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))))
```

$\frac{\partial f3}{\partial t1}=$

```
- sin(tb(4)) cos(tb(5))
```

$\frac{\partial f3}{\partial t2}=$

```
sin(tb(4)) sin(tb(5))
```

$\frac{\partial f3}{\partial t3}=$

```
cos(tb(4))
```

$\frac{\partial f3}{\partial t4}=$

```
   sin(tb(4)) sin(tb(5)) ((cos(t(4)) cos(t(5)) cos(t(6))
- sin(t(4)) sin(t(6))) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + (- cos(t(4)) cos(t(5)) sin(t(6))
- sin(t(4)) cos(t(6))) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju)
- sin(tb(4)) cos(tb(5)) ((- cos(t(4)) sin(t(6))
- sin(t(4)) cos(t(5)) cos(t(6))) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + (sin(t(4)) cos(t(5)) sin(t(6))
```

```
- cos(t(4)) cos(t(6))) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
+ sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju)
```

$$\frac{\partial f3}{\partial t5} =$$

```
    sin(tb(4)) sin(tb(5)) (- sin(t(4)) sin(t(5)) cos(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ sin(t(4)) sin(t(5)) sin(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) cos(t(5)) sin(tp(5)) cos(tp(6)) ju)
- sin(tb(4)) cos(tb(5)) (- cos(t(4)) sin(t(5)) cos(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ cos(t(4)) sin(t(5)) sin(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) cos(t(5)) sin(tp(5)) cos(tp(6)) ju)
+ cos(tb(4)) (- cos(t(5)) cos(t(6)) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + cos(t(5)) sin(t(6))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
+ sin(t(5)) sin(tp(5)) cos(tp(6)) ju)
```

$\frac{\partial f3}{\partial t6} =$

```
            sin(tb(4)) sin(tb(5)) ((cos(t(4)) cos(t(6))
        - sin(t(4)) cos(t(5)) sin(t(6))) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
        - sin(tp(4)) sin(tp(6))) ju + tp(1)) + (- cos(t(4)) sin(t(6))
        - sin(t(4)) cos(t(5)) cos(t(6))) ((cos(tp(4)) sin(tp(6))
        + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2)))
    - sin(tb(4)) cos(tb(5)) ((- cos(t(4)) cos(t(5)) sin(t(6))
    - sin(t(4)) cos(t(6))) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
    - sin(tp(4)) sin(tp(6))) ju + tp(1)) + (sin(t(4)) sin(t(6))
    - cos(t(4)) cos(t(5)) cos(t(6))) ((cos(tp(4)) sin(tp(6))
    + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2)))
    + cos(tb(4)) (sin(t(5)) sin(t(6)) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
    - sin(tp(4)) sin(tp(6))) ju + tp(1)) + sin(t(5)) cos(t(6))
    ((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2)))
```

$\frac{\partial f4}{\partial t1} = \quad \frac{\partial f4}{\partial t2} = \quad \frac{\partial f4}{\partial t3} = \quad 0$

$\frac{\partial f4}{\partial t4} =$

```
    cos(jl(2)) ((cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((sin(t(4)) cos(t(5)) sin(t(6)) - cos(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
- sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6))))
- sin(jl(2)) ((cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((sin(t(4)) cos(t(5)) sin(t(6)) - cos(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
- sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
```

```
+ (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6))))
```

$\frac{\partial f4}{\partial t5} =$

```
    cos(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
(sin(t(4)) sin(t(5)) sin(t(6)) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) - sin(t(4)) sin(t(5)) cos(t(6))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) cos(t(5)) sin(tp(5)) sin(tp(6)))
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
(cos(t(4)) sin(t(5)) sin(t(6)) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) - cos(t(4)) sin(t(5)) cos(t(6))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) cos(t(5)) sin(tp(5)) sin(tp(6)))
```

```
+ cos(tb(3)) sin(tb(4)) (cos(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
- cos(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) - sin(t(5)) sin(tp(5)) sin(tp(6))))
- sin(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
(sin(t(4)) sin(t(5)) sin(t(6)) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) - sin(t(4)) sin(t(5)) cos(t(6))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) cos(t(5)) sin(tp(5)) sin(tp(6)))
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
(cos(t(4)) sin(t(5)) sin(t(6)) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) - cos(t(4)) sin(t(5)) cos(t(6))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) cos(t(5)) sin(tp(5)) sin(tp(6)))
+ sin(tb(3)) sin(tb(4)) (cos(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
- cos(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) - sin(t(5)) sin(tp(5)) sin(tp(6))))
```

$\frac{\partial f4}{\partial t6}=$

```
    cos(jl(2)) ((cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((sin(t(4)) sin(t(6)) - cos(t(4)) cos(t(5)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ cos(tb(3)) sin(tb(4)) (sin(t(5)) cos(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ sin(t(5)) sin(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))))) - sin(jl(2))
((cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((sin(t(4)) sin(t(6)) - cos(t(4)) cos(t(5)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
```

```
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ sin(tb(3)) sin(tb(4)) (sin(t(5)) cos(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ sin(t(5)) sin(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6)))))
```

$$\frac{\partial f5}{\partial t1} = \frac{\partial f5}{\partial t2} = \frac{\partial f5}{\partial t3} = 0$$

$$\frac{\partial f5}{\partial t4} =$$

```
sin(jl(2)) ((cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((sin(t(4)) cos(t(5)) sin(t(6)) - cos(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
- sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6))))
+ cos(jl(2)) ((cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((sin(t(4)) cos(t(5)) sin(t(6)) - cos(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
- sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
```

```
+ (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6))))
```

$$\frac{\partial f5}{\partial t5} =$$

```
     cos(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
(sin(t(4)) sin(t(5)) sin(t(6)) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) - sin(t(4)) sin(t(5)) cos(t(6))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) cos(t(5)) sin(tp(5)) sin(tp(6)))
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
(cos(t(4)) sin(t(5)) sin(t(6)) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) - cos(t(4)) sin(t(5)) cos(t(6))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) cos(t(5)) sin(tp(5)) sin(tp(6)))
```

```
+ sin(tb(3)) sin(tb(4)) (cos(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
- cos(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) - sin(t(5)) sin(tp(5)) sin(tp(6))))
+ sin(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
(sin(t(4)) sin(t(5)) sin(t(6)) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) - sin(t(4)) sin(t(5)) cos(t(6))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) cos(t(5)) sin(tp(5)) sin(tp(6)))
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
(cos(t(4)) sin(t(5)) sin(t(6)) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) - cos(t(4)) sin(t(5)) cos(t(6))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) cos(t(5)) sin(tp(5)) sin(tp(6)))
+ cos(tb(3)) sin(tb(4)) (cos(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
- cos(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) - sin(t(5)) sin(tp(5)) sin(tp(6))))
.....
```

$\frac{\partial f5}{\partial t6}=$

```
    sin(jl(2)) ((cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((sin(t(4)) sin(t(6)) - cos(t(4)) cos(t(5)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ cos(tb(3)) sin(tb(4)) (sin(t(5)) cos(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ sin(t(5)) sin(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))))) + cos(jl(2))
((cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((sin(t(4)) sin(t(6)) - cos(t(4)) cos(t(5)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
```

```
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ sin(tb(3)) sin(tb(4)) (sin(t(5)) cos(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ sin(t(5)) sin(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6)))))
```

$$\frac{\partial f6}{\partial t1} = \frac{\partial f6}{\partial t2} = \frac{\partial f6}{\partial t3} = 0$$

$\frac{\partial f6}{\partial t4}=$

```
    sin(tb(4)) sin(tb(5)) ((- cos(t(4)) cos(t(5)) sin(t(6))
- sin(t(4)) cos(t(6))) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) + (cos(t(4)) cos(t(5)) cos(t(6))
- sin(t(4)) sin(t(6))) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) + cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
- sin(tb(4)) cos(tb(5)) ((sin(t(4)) cos(t(5)) sin(t(6)) - cos(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
- sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
```

$\frac{\partial f6}{\partial t5}=$

```
    sin(tb(4)) sin(tb(5)) (sin(t(4)) sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
- sin(t(4)) sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) + sin(t(4)) cos(t(5)) sin(tp(5)) sin(tp(6)))
- sin(tb(4)) cos(tb(5)) (cos(t(4)) sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
```

```
- cos(t(4)) sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) + cos(t(4)) cos(t(5)) sin(tp(5)) sin(tp(6)))
+ cos(tb(4)) (cos(t(5)) sin(t(6)) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) - cos(t(5)) cos(t(6))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
- sin(t(5)) sin(tp(5)) sin(tp(6)))
```

$\frac{\partial f6}{\partial t6}=$

```
    - sin(tb(4)) cos(tb(5)) ((sin(t(4)) sin(t(6))
- cos(t(4)) cos(t(5)) cos(t(6))) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) + (- cos(t(4)) cos(t(5)) sin(t(6))
- sin(t(4)) cos(t(6))) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6)))) + sin(tb(4)) sin(tb(5))
((- cos(t(4)) sin(t(6)) - sin(t(4)) cos(t(5)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ cos(tb(4)) (sin(t(5)) cos(t(6)) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) + sin(t(5)) sin(t(6))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
```

$\frac{\partial fl}{\partial tpl}=$

```
     cos(jl(2)) ((cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
(cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
+ (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
(cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
- cos(tb(3)) sin(tb(4)) sin(t(5)) cos(t(6)))
- sin(jl(2)) ((cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
(cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
+ (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
(cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
- sin(tb(3)) sin(tb(4)) sin(t(5)) cos(t(6)))
```

$\frac{\partial fl}{\partial tp2}=$

```
     cos(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
(cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
(- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
+ cos(tb(3)) sin(tb(4)) sin(t(5)) sin(t(6)))
- sin(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
```

```
(cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
(- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
+ sin(tb(3)) sin(tb(4)) sin(t(5)) sin(t(6)))
```

$$\frac{\partial fl}{\partial tp4} =$$

```
cos(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6))) ju)
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
```

```
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6))) ju)
+ cos(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju
- sin(t(5)) cos(t(6)) (- cos(tp(4)) sin(tp(6))
- sin(tp(4)) cos(tp(5)) cos(tp(6))) ju))
- sin(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6))) ju)
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6))) ju)
+ sin(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju
- sin(t(5)) cos(t(6)) (- cos(tp(4)) sin(tp(6))
- sin(tp(4)) cos(tp(5)) cos(tp(6))) ju))
```

$\frac{\partial fl}{\partial tp5} =$

```
    cos(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
(- sin(tp(4)) sin(tp(5)) (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
cos(tp(6)) ju - cos(tp(4)) sin(tp(5)) (cos(t(4)) sin(t(6))
+ sin(t(4)) cos(t(5)) cos(t(6))) cos(tp(6)) ju
- sin(t(4)) sin(t(5)) cos(tp(5)) cos(tp(6)) ju)
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
(- sin(tp(4)) sin(tp(5)) (- cos(t(4)) cos(t(5)) sin(t(6))
- sin(t(4)) cos(t(6))) cos(tp(6)) ju - cos(tp(4)) sin(tp(5))
(cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6))) cos(tp(6)) ju
- cos(t(4)) sin(t(5)) cos(tp(5)) cos(tp(6)) ju)
+ cos(tb(3)) sin(tb(4)) (- sin(tp(4)) sin(t(5)) sin(tp(5)) sin(t(6))
cos(tp(6)) ju + cos(tp(4)) sin(t(5)) sin(tp(5)) cos(t(6)) cos(tp(6)) ju
- cos(t(5)) cos(tp(5)) cos(tp(6)) ju))
- sin(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
(- sin(tp(4)) sin(tp(5)) (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
cos(tp(6)) ju - cos(tp(4)) sin(tp(5)) (cos(t(4)) sin(t(6))
+ sin(t(4)) cos(t(5)) cos(t(6))) cos(tp(6)) ju
- sin(t(4)) sin(t(5)) cos(tp(5)) cos(tp(6)) ju)
```

```
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
(- sin(tp(4)) sin(tp(5)) (- cos(t(4)) cos(t(5)) sin(t(6))
- sin(t(4)) cos(t(6))) cos(tp(6)) ju - cos(tp(4)) sin(tp(5))
(cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6))) cos(tp(6)) ju
- cos(t(4)) sin(t(5)) cos(tp(5)) cos(tp(6)) ju)
+ sin(tb(3)) sin(tb(4)) (- sin(tp(4)) sin(t(5)) sin(tp(5)) sin(t(6))
cos(tp(6)) ju + cos(tp(4)) sin(t(5)) sin(tp(5)) cos(t(6)) cos(tp(6)) ju
- cos(t(5)) cos(tp(5)) cos(tp(6)) ju))
```

$\frac{\partial fl}{\partial tp6}=$

```
    cos(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6))) ju
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))) ju
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)) ju)
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
```

```
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6))) ju
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))) ju
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)) ju)
+ cos(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6))) ju
- sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) ju + cos(t(5)) sin(tp(5)) sin(tp(6)) ju))
- sin(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6))) ju
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))) ju
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)) ju)
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6))) ju
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
```

```
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))) ju
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)) ju)
+ sin(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6))) ju
- sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) ju + cos(t(5)) sin(tp(5)) sin(tp(6)) ju))
```

$$\frac{\partial f2}{\partial tp1} =$$

```
     sin(jl(2)) ((cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
(cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
+ (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
(cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
- cos(tb(3)) sin(tb(4)) sin(t(5)) cos(t(6)))
+ cos(jl(2)) ((cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
(cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
+ (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
(cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
- sin(tb(3)) sin(tb(4)) sin(t(5)) cos(t(6)))
```

$\frac{\partial f2}{\partial tp2}=$

```
     cos(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
(cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
(- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
+ sin(tb(3)) sin(tb(4)) sin(t(5)) sin(t(6)))
+ sin(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
(cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
(- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
+ cos(tb(3)) sin(tb(4)) sin(t(5)) sin(t(6)))
```

$\frac{\partial f2}{\partial tp4}=$

```
     cos(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju
```

```
        + (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
        (- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6))) ju)
        + (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
        ((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
        (cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju
        + (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
        (- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6))) ju)
        + sin(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
        (cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju
        - sin(t(5)) cos(t(6)) (- cos(tp(4)) sin(tp(6))
- sin(tp(4)) cos(tp(5)) cos(tp(6))) ju))
+ sin(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6))) ju)
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
```

```
(cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6))) ju)
+ cos(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju
- sin(t(5)) cos(t(6)) (- cos(tp(4)) sin(tp(6))
- sin(tp(4)) cos(tp(5)) cos(tp(6))) ju))
```

$\frac{\partial f2}{\partial tp5}=$

```
      cos(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
(- sin(tp(4)) sin(tp(5)) (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
cos(tp(6)) ju - cos(tp(4)) sin(tp(5)) (cos(t(4)) sin(t(6))
+ sin(t(4)) cos(t(5)) cos(t(6))) cos(tp(6)) ju
- sin(t(4)) sin(t(5)) cos(tp(5)) cos(tp(6)) ju)
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
(- sin(tp(4)) sin(tp(5)) (- cos(t(4)) cos(t(5)) sin(t(6))
- sin(t(4)) cos(t(6))) cos(tp(6)) ju - cos(tp(4)) sin(tp(5))
(cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6))) cos(tp(6)) ju
```

```
    - cos(t(4)) sin(t(5)) cos(tp(5)) cos(tp(6)) ju)
    + sin(tb(3)) sin(tb(4)) (- sin(tp(4)) sin(t(5)) sin(tp(5)) sin(t(6))
    cos(tp(6)) ju + cos(tp(4)) sin(t(5)) sin(tp(5)) cos(t(6)) cos(tp(6)) ju
    - cos(t(5)) cos(tp(5)) cos(tp(6)) ju))
    + sin(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
(- sin(tp(4)) sin(tp(5)) (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
cos(tp(6)) ju - cos(tp(4)) sin(tp(5)) (cos(t(4)) sin(t(6))
+ sin(t(4)) cos(t(5)) cos(t(6))) cos(tp(6)) ju
- sin(t(4)) sin(t(5)) cos(tp(5)) cos(tp(6)) ju)
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
(- sin(tp(4)) sin(tp(5)) (- cos(t(4)) cos(t(5)) sin(t(6))
- sin(t(4)) cos(t(6))) cos(tp(6)) ju - cos(tp(4)) sin(tp(5))
(cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6))) cos(tp(6)) ju
- cos(t(4)) sin(t(5)) cos(tp(5)) cos(tp(6)) ju)
+ cos(tb(3)) sin(tb(4)) (- sin(tp(4)) sin(t(5)) sin(tp(5)) sin(t(6))
cos(tp(6)) ju + cos(tp(4)) sin(t(5)) sin(tp(5)) cos(t(6)) cos(tp(6)) ju
- cos(t(5)) cos(tp(5)) cos(tp(6)) ju))
```

C-2

$\frac{\partial f2}{\partial tp6} =$

```
    cos(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6))) ju
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))) ju
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)) ju)
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6))) ju
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))) ju
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)) ju)
+ sin(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6))) ju
- sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) ju + cos(t(5)) sin(tp(5)) sin(tp(6)) ju))
+ sin(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
```

```
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6))) ju
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))) ju
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)) ju)
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6))) ju
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))) ju
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)) ju)
+ cos(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6))) ju
- sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) ju + cos(t(5)) sin(tp(5)) sin(tp(6)) ju))
```

$\frac{\partial f3}{\partial tp1}=$

```
     - sin(tb(4)) cos(tb(5)) (cos(t(4)) cos(t(5)) cos(t(6))
 - sin(t(4)) sin(t(6))) + sin(tb(4)) sin(tb(5))
 (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
 - cos(tb(4)) sin(t(5)) cos(t(6))
```

$\frac{\partial f3}{\partial tp2}=$

```
     sin(tb(4)) sin(tb(5)) (cos(t(4)) cos(t(6))
 - sin(t(4)) cos(t(5)) sin(t(6))) - sin(tb(4)) cos(tb(5))
 (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
 + cos(tb(4)) sin(t(5)) sin(t(6))
```

$\frac{\partial f3}{\partial tp4}=$

```
     sin(tb(4)) sin(tb(5)) ((cos(t(4)) cos(t(6))
 - sin(t(4)) cos(t(5)) sin(t(6))) (cos(tp(4)) cos(tp(5)) cos(tp(6))
 - sin(tp(4)) sin(tp(6))) ju + (cos(t(4)) sin(t(6))
 + sin(t(4)) cos(t(5)) cos(t(6))) (- cos(tp(4)) sin(tp(6))
```

```
- sin(tp(4)) cos(tp(5)) cos(tp(6))) ju)
- sin(tb(4)) cos(tb(5)) ((- cos(t(4)) cos(t(5)) sin(t(6))
- sin(t(4)) cos(t(6))) (cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + (cos(t(4)) cos(t(5)) cos(t(6))
- sin(t(4)) sin(t(6))) (- cos(tp(4)) sin(tp(6))
- sin(tp(4)) cos(tp(5)) cos(tp(6))) ju)
+ cos(tb(4)) (sin(t(5)) sin(t(6)) (cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju - sin(t(5)) cos(t(6))
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6))) ju)
```

$\frac{\partial f3}{\partial tp5} =$

```
    sin(tb(4)) sin(tb(5)) (- sin(tp(4)) sin(tp(5))
(cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6))) cos(tp(6)) ju
- cos(tp(4)) sin(tp(5)) (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
cos(tp(6)) ju - sin(t(4)) sin(t(5)) cos(tp(5)) cos(tp(6)) ju)
- sin(tb(4)) cos(tb(5)) (- sin(tp(4)) sin(tp(5))
(- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6))) cos(tp(6)) ju
- cos(tp(4)) sin(tp(5)) (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
```

```
cos(tp(6)) ju - cos(t(4)) sin(t(5)) cos(tp(5)) cos(tp(6)) ju)

+ cos(tb(4)) (- sin(tp(4)) sin(t(5)) sin(tp(5)) sin(t(6)) cos(tp(6)) ju

+ cos(tp(4)) sin(t(5)) sin(tp(5)) cos(t(6)) cos(tp(6)) ju

- cos(t(5)) cos(tp(5)) cos(tp(6)) ju)
```

$\frac{\partial f3}{\partial tp6}=$

```
    sin(tb(4)) sin(tb(5)) ((cos(t(4)) cos(t(6))

 - sin(t(4)) cos(t(5)) sin(t(6))) (cos(tp(4)) cos(tp(6))

 - sin(tp(4)) cos(tp(5)) sin(tp(6))) ju

 + (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))

 (- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))) ju

 + sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)) ju)

 - sin(tb(4)) cos(tb(5)) ((- cos(t(4)) cos(t(5)) sin(t(6))

 - sin(t(4)) cos(t(6))) (cos(tp(4)) cos(tp(6))

 - sin(tp(4)) cos(tp(5)) sin(tp(6))) ju

 + (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))

 (- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))) ju

 + cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)) ju)
```

```
+ cos(tb(4)) (sin(t(5)) sin(t(6)) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) ju
- sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) ju + cos(t(5)) sin(tp(5)) sin(tp(6)) ju)
```

$$\frac{\partial f4}{\partial tp1} = \frac{\partial f4}{\partial tp2} = 0$$

$$\frac{\partial f4}{\partial tp4} =$$

```
    cos(jl(2)) ((cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(sin(tp(4)) cos(tp(5)) sin(tp(6)) - cos(tp(4)) cos(tp(6)))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
```

```
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(sin(tp(4)) cos(tp(5)) sin(tp(6)) - cos(tp(4)) cos(tp(6)))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ cos(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
- sin(t(5)) cos(t(6)) (sin(tp(4)) cos(tp(5)) sin(tp(6))
- cos(tp(4)) cos(tp(6))))) - sin(jl(2))
((cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(sin(tp(4)) cos(tp(5)) sin(tp(6)) - cos(tp(4)) cos(tp(6)))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(sin(tp(4)) cos(tp(5)) sin(tp(6)) - cos(tp(4)) cos(tp(6)))
```

```
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ sin(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
- sin(t(5)) cos(t(6)) (sin(tp(4)) cos(tp(5)) sin(tp(6))
- cos(tp(4)) cos(tp(6)))))
```

$\frac{\partial f4}{\partial tp5} =$

```
    cos(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
(sin(tp(4)) sin(tp(5)) (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
sin(tp(6)) + cos(tp(4)) sin(tp(5)) (cos(t(4)) sin(t(6))
+ sin(t(4)) cos(t(5)) cos(t(6))) sin(tp(6))
+ sin(t(4)) sin(t(5)) cos(tp(5)) sin(tp(6)))
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
(sin(tp(4)) sin(tp(5)) (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
sin(tp(6)) + cos(tp(4)) sin(tp(5)) (cos(t(4)) cos(t(5)) cos(t(6))
- sin(t(4)) sin(t(6))) sin(tp(6)) + cos(t(4)) sin(t(5)) cos(tp(5)) sin(tp(6)))
+ cos(tb(3)) sin(tb(4)) (sin(tp(4)) sin(t(5)) sin(tp(5)) sin(t(6)) sin(tp(6))
- cos(tp(4)) sin(t(5)) sin(tp(5)) cos(t(6)) sin(tp(6))
```

```
+ cos(t(5)) cos(tp(5)) sin(tp(6)))) - sin(jl(2))
((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
(sin(tp(4)) sin(tp(5)) (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
sin(tp(6)) + cos(tp(4)) sin(tp(5)) (cos(t(4)) sin(t(6))
+ sin(t(4)) cos(t(5)) cos(t(6))) sin(tp(6))
+ sin(t(4)) sin(t(5)) cos(tp(5)) sin(tp(6)))
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
(sin(tp(4)) sin(tp(5)) (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
sin(tp(6)) + cos(tp(4)) sin(tp(5)) (cos(t(4)) cos(t(5)) cos(t(6))
- sin(t(4)) sin(t(6))) sin(tp(6)) + cos(t(4)) sin(t(5)) cos(tp(5)) sin(tp(6)))
+ sin(tb(3)) sin(tb(4)) (sin(tp(4)) sin(t(5)) sin(tp(5)) sin(t(6)) sin(tp(6))
- cos(tp(4)) sin(t(5)) sin(tp(5)) cos(t(6)) sin(tp(6))
+ cos(t(5)) cos(tp(5)) sin(tp(6))))
```

$\frac{\partial f4}{\partial tp6}=$

```
    cos(jl(2)) ((cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(sin(tp(4)) sin(tp(6)) - cos(tp(4)) cos(tp(5)) cos(tp(6)))
```

```
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6)))
+ cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)))
+ (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(sin(tp(4)) sin(tp(6)) - cos(tp(4)) cos(tp(5)) cos(tp(6)))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)))
+ cos(tb(3)) sin(tb(4)) (- sin(t(5)) cos(t(6))
(sin(tp(4)) sin(tp(6)) - cos(tp(4)) cos(tp(5)) cos(tp(6)))
+ sin(t(5)) sin(t(6)) (- cos(tp(4)) sin(tp(6))
- sin(tp(4)) cos(tp(5)) cos(tp(6))) + cos(t(5)) sin(tp(5)) cos(tp(6))))
- sin(jl(2)) ((cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(sin(tp(4)) sin(tp(6)) - cos(tp(4)) cos(tp(5)) cos(tp(6)))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6)))
```

```
+ cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)))
+ (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(sin(tp(4)) sin(tp(6)) - cos(tp(4)) cos(tp(5)) cos(tp(6)))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)))
+ sin(tb(3)) sin(tb(4)) (- sin(t(5)) cos(t(6))
(sin(tp(4)) sin(tp(6)) - cos(tp(4)) cos(tp(5)) cos(tp(6)))
+ sin(t(5)) sin(t(6)) (- cos(tp(4)) sin(tp(6))
- sin(tp(4)) cos(tp(5)) cos(tp(6))) + cos(t(5)) sin(tp(5)) cos(tp(6))))
```

$$\frac{\partial f5}{\partial tp1} = \frac{\partial f5}{\partial tp2} = 0$$

$\frac{\partial f5}{\partial tp4} =$

```
     sin(jl(2)) ((cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(sin(tp(4)) cos(tp(5)) sin(tp(6)) - cos(tp(4)) cos(tp(6)))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(sin(tp(4)) cos(tp(5)) sin(tp(6)) - cos(tp(4)) cos(tp(6)))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ cos(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
- sin(t(5)) cos(t(6)) (sin(tp(4)) cos(tp(5)) sin(tp(6))
- cos(tp(4)) cos(tp(6))))) + cos(jl(2))
((cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(sin(tp(4)) cos(tp(5)) sin(tp(6)) - cos(tp(4)) cos(tp(6)))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
```

```
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(sin(tp(4)) cos(tp(5)) sin(tp(6)) - cos(tp(4)) cos(tp(6)))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
+ sin(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
- sin(t(5)) cos(t(6)) (sin(tp(4)) cos(tp(5)) sin(tp(6))
- cos(tp(4)) cos(tp(6)))))
```

$\frac{\partial f5}{\partial tp5} =$

```
     cos(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
(sin(tp(4)) sin(tp(5)) (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
sin(tp(6)) + cos(tp(4)) sin(tp(5)) (cos(t(4)) sin(t(6))
+ sin(t(4)) cos(t(5)) cos(t(6))) sin(tp(6))
+ sin(t(4)) sin(t(5)) cos(tp(5)) sin(tp(6)))
+ (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
```

```
(sin(tp(4)) sin(tp(5)) (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
sin(tp(6)) + cos(tp(4)) sin(tp(5)) (cos(t(4)) cos(t(5)) cos(t(6))
- sin(t(4)) sin(t(6))) sin(tp(6)) + cos(t(4)) sin(t(5)) cos(tp(5)) sin(tp(6)))
+ sin(tb(3)) sin(tb(4)) (sin(tp(4)) sin(t(5)) sin(tp(5)) sin(t(6)) sin(tp(6))
- cos(tp(4)) sin(t(5)) sin(tp(5)) cos(t(6)) sin(tp(6))
+ cos(t(5)) cos(tp(5)) sin(tp(6)))) + sin(jl(2))
((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
(sin(tp(4)) sin(tp(5)) (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
sin(tp(6)) + cos(tp(4)) sin(tp(5)) (cos(t(4)) sin(t(6))
+ sin(t(4)) cos(t(5)) cos(t(6))) sin(tp(6))
+ sin(t(4)) sin(t(5)) cos(tp(5)) sin(tp(6)))
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
(sin(tp(4)) sin(tp(5)) (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
sin(tp(6)) + cos(tp(4)) sin(tp(5)) (cos(t(4)) cos(t(5)) cos(t(6))
- sin(t(4)) sin(t(6))) sin(tp(6)) + cos(t(4)) sin(t(5)) cos(tp(5)) sin(tp(6)))
+ cos(tb(3)) sin(tb(4)) (sin(tp(4)) sin(t(5)) sin(tp(5)) sin(t(6)) sin(tp(6))
- cos(tp(4)) sin(t(5)) sin(tp(5)) cos(t(6)) sin(tp(6))
+ cos(t(5)) cos(tp(5)) sin(tp(6))))
```

$\frac{\partial f5}{\partial tp6} =$

```
     sin(jl(2)) ((cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(sin(tp(4)) sin(tp(6)) - cos(tp(4)) cos(tp(5)) cos(tp(6)))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6)))
+ cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)))
+ (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(sin(tp(4)) sin(tp(6)) - cos(tp(4)) cos(tp(5)) cos(tp(6)))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)))
+ cos(tb(3)) sin(tb(4)) (- sin(t(5)) cos(t(6))
(sin(tp(4)) sin(tp(6)) - cos(tp(4)) cos(tp(5)) cos(tp(6)))
+ sin(t(5)) sin(t(6)) (- cos(tp(4)) sin(tp(6))
- sin(tp(4)) cos(tp(5)) cos(tp(6))) + cos(t(5)) sin(tp(5)) cos(tp(6))))
+ cos(jl(2)) ((cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
```

```
(sin(tp(4)) sin(tp(6)) - cos(tp(4)) cos(tp(5)) cos(tp(6)))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6)))
+ cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)))
+ (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(sin(tp(4)) sin(tp(6)) - cos(tp(4)) cos(tp(5)) cos(tp(6)))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(- cos(tp(4)) sin(tp(6)) - sin(tp(4)) cos(tp(5)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)))
+ sin(tb(3)) sin(tb(4)) (- sin(t(5)) cos(t(6))
(sin(tp(4)) sin(tp(6)) - cos(tp(4)) cos(tp(5)) cos(tp(6)))
+ sin(t(5)) sin(t(6)) (- cos(tp(4)) sin(tp(6))
- sin(tp(4)) cos(tp(5)) cos(tp(6))) + cos(t(5)) sin(tp(5)) cos(tp(6))))
```

$$\frac{\partial f6}{\partial tp1} = \frac{\partial f6}{\partial tp2} = 0$$

$$\frac{\partial f6}{\partial tp4} =$$

```
        - sin(tb(4)) cos(tb(5)) ((cos(t(4)) cos(t(5)) cos(t(6))
     - sin(t(4)) sin(t(6))) (sin(tp(4)) cos(tp(5)) sin(tp(6))
     - cos(tp(4)) cos(tp(6))) + (- cos(t(4)) cos(t(5)) sin(t(6))
     - sin(t(4)) cos(t(6))) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
  - sin(tp(4)) cos(tp(6)))) + sin(tb(4)) sin(tb(5))
  ((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
  (sin(tp(4)) cos(tp(5)) sin(tp(6)) - cos(tp(4)) cos(tp(6)))
  + (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
  (- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6))))
  + cos(tb(4)) (sin(t(5)) sin(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
  - sin(tp(4)) cos(tp(6))) - sin(t(5)) cos(t(6))
  (sin(tp(4)) cos(tp(5)) sin(tp(6)) - cos(tp(4)) cos(tp(6))))
```

$$\frac{\partial f6}{\partial tp5} =$$

```
     sin(tb(4)) sin(tb(5)) (sin(tp(4)) sin(tp(5))
(cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6))) sin(tp(6))
+ cos(tp(4)) sin(tp(5)) (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
sin(tp(6)) + sin(t(4)) sin(t(5)) cos(tp(5)) sin(tp(6)))
```

```
- sin(tb(4)) cos(tb(5)) (sin(tp(4)) sin(tp(5))
(- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6))) sin(tp(6))
+ cos(tp(4)) sin(tp(5)) (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
sin(tp(6)) + cos(t(4)) sin(t(5)) cos(tp(5)) sin(tp(6)))
+ cos(tb(4)) (sin(tp(4)) sin(t(5)) sin(tp(5)) sin(t(6)) sin(tp(6))
- cos(tp(4)) sin(t(5)) sin(tp(5)) cos(t(6)) sin(tp(6))
+ cos(t(5)) cos(tp(5)) sin(tp(6)))
```

$\frac{\partial f6}{\partial tp6} =$

```
    - sin(tb(4)) cos(tb(5)) ((cos(t(4)) cos(t(5)) cos(t(6))
- sin(t(4)) sin(t(6))) (sin(tp(4)) sin(tp(6))
- cos(tp(4)) cos(tp(5)) cos(tp(6))) + (- cos(t(4)) cos(t(5)) sin(t(6))
- sin(t(4)) cos(t(6))) (- cos(tp(4)) sin(tp(6))
- sin(tp(4)) cos(tp(5)) cos(tp(6))) + cos(t(4)) sin(t(5)) sin(tp(5))
cos(tp(6))) + sin(tb(4)) sin(tb(5)) ((cos(t(4)) sin(t(6))
+ sin(t(4)) cos(t(5)) cos(t(6))) (sin(tp(4)) sin(tp(6))
- cos(tp(4)) cos(tp(5)) cos(tp(6))) + (cos(t(4)) cos(t(6))
- sin(t(4)) cos(t(5)) sin(t(6))) (- cos(tp(4)) sin(tp(6))
```

```
- sin(tp(4)) cos(tp(5)) cos(tp(6))) + sin(t(4)) sin(t(5)) sin(tp(5))
cos(tp(6))) + cos(tb(4)) (- sin(t(5)) cos(t(6))
(sin(tp(4)) sin(tp(6)) - cos(tp(4)) cos(tp(5)) cos(tp(6)))
+ sin(t(5)) sin(t(6)) (- cos(tp(4)) sin(tp(6))
- sin(tp(4)) cos(tp(5)) cos(tp(6))) + cos(t(5)) sin(tp(5)) cos(tp(6)))
```

$\frac{\partial fl}{\partial tb1} =$

```
cos(jl(2)) (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
 - sin(jl(2)) (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
```

$\frac{\partial fl}{\partial tb2} =$

```
cos(jl(2)) (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
 - sin(jl(2)) (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
```

$\frac{\partial fl}{\partial tb3} =$

```
    cos(jl(2)) ((- cos(tb(3)) sin(tb(5)) - sin(tb(3)) cos(tb(4)) cos(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(1))
+ (sin(tb(3)) cos(tb(4)) sin(tb(5)) - cos(tb(3)) cos(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
```

```
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(2))
- sin(tb(3)) sin(tb(4)) (- sin(t(5)) cos(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ sin(t(5)) sin(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(5)) sin(tp(5)) cos(tp(6)) ju + t(3))
+ tb(2) (sin(tb(3)) cos(tb(4)) sin(tb(5)) - cos(tb(3)) cos(tb(5)))
+ tb(1) (- cos(tb(3)) sin(tb(5)) - sin(tb(3)) cos(tb(4)) cos(tb(5))))
- sin(jl(2)) ((cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(1))
+ (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
```

```
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(2))
+ cos(tb(3)) sin(tb(4)) (- sin(t(5)) cos(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ sin(t(5)) sin(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(5)) sin(tp(5)) cos(tp(6)) ju + t(3))
+ tb(2) (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
+ tb(1) (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5))))
```

$\frac{\partial fl}{\partial tb4} =$

```
     cos(jl(2)) (- cos(tb(3)) sin(tb(4)) cos(tb(5))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(1))
```

```
+ cos(tb(3)) sin(tb(4)) sin(tb(5)) ((cos(t(4)) sin(t(6))
+ sin(t(4)) cos(t(5)) cos(t(6))) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + (cos(t(4)) cos(t(6))
- sin(t(4)) cos(t(5)) sin(t(6))) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(2))
+ cos(tb(3)) cos(tb(4)) (- sin(t(5)) cos(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ sin(t(5)) sin(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(5)) sin(tp(5)) cos(tp(6)) ju + t(3))
+ tb(2) cos(tb(3)) sin(tb(4)) sin(tb(5))
- tb(1) cos(tb(3)) sin(tb(4)) cos(tb(5)))
- sin(jl(2)) (- sin(tb(3)) sin(tb(4)) cos(tb(5))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
```

```
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(1))
+ sin(tb(3)) sin(tb(4)) sin(tb(5)) ((cos(t(4)) sin(t(6))
+ sin(t(4)) cos(t(5)) cos(t(6))) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + (cos(t(4)) cos(t(6))
- sin(t(4)) cos(t(5)) sin(t(6))) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(2))
+ sin(tb(3)) cos(tb(4)) (- sin(t(5)) cos(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ sin(t(5)) sin(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(5)) sin(tp(5)) cos(tp(6)) ju + t(3))
+ tb(2) sin(tb(3)) sin(tb(4)) sin(tb(5))
- tb(1) sin(tb(3)) sin(tb(4)) cos(tb(5)))
```

$\frac{\partial fl}{\partial tb5} =$

```
        cos(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(1))
+ (sin(tb(3)) sin(tb(5)) - cos(tb(3)) cos(tb(4)) cos(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(2))
+ tb(1) (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
+ tb(2) (sin(tb(3)) sin(tb(5)) - cos(tb(3)) cos(tb(4)) cos(tb(5))))
- sin(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
```

```
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(1))
+ (- cos(tb(3)) sin(tb(5)) - sin(tb(3)) cos(tb(4)) cos(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(2))
+ tb(1) (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
+ tb(2) (- cos(tb(3)) sin(tb(5)) - sin(tb(3)) cos(tb(4)) cos(tb(5))))
```

$\frac{\partial f2}{\partial tb1} =$

```
  sin(jl(2)) (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
+ cos(jl(2)) (cos(tb(3)) sin(tb(5)) + sin(tb(3)) cos(tb(4)) cos(tb(5)))
```

$\frac{\partial f2}{\partial tb2} =$

```
  cos(jl(2)) (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
+ sin(jl(2)) (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
```

$$\frac{\partial f2}{\partial tb3} =$$

```
     cos(jl(2)) ((cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(1))
+ (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(2))
+ cos(tb(3)) sin(tb(4)) (- sin(t(5)) cos(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ sin(t(5)) sin(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(5)) sin(tp(5)) cos(tp(6)) ju + t(3))
+ tb(2) (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
```

```
+ tb(1) (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5))))
+ sin(jl(2)) ((- cos(tb(3)) sin(tb(5)) - sin(tb(3)) cos(tb(4)) cos(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(1))
+ (sin(tb(3)) cos(tb(4)) sin(tb(5)) - cos(tb(3)) cos(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(2))
- sin(tb(3)) sin(tb(4)) (- sin(t(5)) cos(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ sin(t(5)) sin(t(6)) ((cos(tp(4)) sin(tp(6))
```

```
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(5)) sin(tp(5)) cos(tp(6)) ju + t(3))
+ tb(2) (sin(tb(3)) cos(tb(4)) sin(tb(5)) - cos(tb(3)) cos(tb(5)))
+ tb(1) (- cos(tb(3)) sin(tb(5)) - sin(tb(3)) cos(tb(4)) cos(tb(5))))
```

$$\frac{\partial f2}{\partial tb4} =$$

```
    cos(jl(2)) (- sin(tb(3)) sin(tb(4)) cos(tb(5))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(1))
+ sin(tb(3)) sin(tb(4)) sin(tb(5)) ((cos(t(4)) sin(t(6))
+ sin(t(4)) cos(t(5)) cos(t(6))) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + (cos(t(4)) cos(t(6))
- sin(t(4)) cos(t(5)) sin(t(6))) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(2))
```

```
+ sin(tb(3)) cos(tb(4)) (- sin(t(5)) cos(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ sin(t(5)) sin(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(5)) sin(tp(5)) cos(tp(6)) ju + t(3))
+ tb(2) sin(tb(3)) sin(tb(4)) sin(tb(5))
- tb(1) sin(tb(3)) sin(tb(4)) cos(tb(5)))
+ sin(jl(2)) (- cos(tb(3)) sin(tb(4)) cos(tb(5))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(1))
+ cos(tb(3)) sin(tb(4)) sin(tb(5)) ((cos(t(4)) sin(t(6))
+ sin(t(4)) cos(t(5)) cos(t(6))) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + (cos(t(4)) cos(t(6))
- sin(t(4)) cos(t(5)) sin(t(6))) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
```

```
- sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(2))
+ cos(tb(3)) cos(tb(4)) (- sin(t(5)) cos(t(6))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ sin(t(5)) sin(t(6)) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(5)) sin(tp(5)) cos(tp(6)) ju + t(3))
+ tb(2) cos(tb(3)) sin(tb(4)) sin(tb(5))
- tb(1) cos(tb(3)) sin(tb(4)) cos(tb(5)))
```

$\frac{\partial f2}{\partial tb5} =$

```
    cos(jl(2)) ((cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(1))
+ (- cos(tb(3)) sin(tb(5)) - sin(tb(3)) cos(tb(4)) cos(tb(5)))
```

```
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(2))
+ tb(1) (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
+ tb(2) (- cos(tb(3)) sin(tb(5)) - sin(tb(3)) cos(tb(4)) cos(tb(5))))
+ sin(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(1))
+ (sin(tb(3)) sin(tb(5)) - cos(tb(3)) cos(tb(4)) cos(tb(5)))
((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
```

```
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(2))
+ tb(1) (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
+ tb(2) (sin(tb(3)) sin(tb(5)) - cos(tb(3)) cos(tb(4)) cos(tb(5))))
```

$$\frac{\partial f3}{\partial tb1} = \quad - \sin(tb(4)) \cos(tb(5))$$

$$\frac{\partial f3}{\partial tb2} = \quad \sin(tb(4)) \sin(tb(5))$$

$$\frac{\partial f3}{\partial tb3} = \quad 0$$

$\frac{\partial f3}{\partial tb4} =$

```
    - cos(tb(4)) cos(tb(5)) ((cos(t(4)) cos(t(5)) cos(t(6))
- sin(t(4)) sin(t(6))) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + (- cos(t(4)) cos(t(5)) sin(t(6))
- sin(t(4)) cos(t(6))) ((cos(tp(4)) sin(tp(6))
+ sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(1))
+ cos(tb(4)) sin(tb(5)) ((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
+ (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(2))
- sin(tb(4)) (- sin(t(5)) cos(t(6)) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
- sin(tp(4)) sin(tp(6))) ju + tp(1)) + sin(t(5)) sin(t(6))
((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
- cos(t(5)) sin(tp(5)) cos(tp(6)) ju + t(3)) + tb(2) cos(tb(4)) sin(tb(5))
- tb(1) cos(tb(4)) cos(tb(5))
```

$\frac{\partial f3}{\partial tb5} =$

```
     sin(tb(4)) sin(tb(5)) ((cos(t(4)) cos(t(5)) cos(t(6))
 - sin(t(4)) sin(t(6))) ((cos(tp(4)) cos(tp(5)) cos(tp(6))
 - sin(tp(4)) sin(tp(6))) ju + tp(1)) + (- cos(t(4)) cos(t(5)) sin(t(6))
 - sin(t(4)) cos(t(6))) ((cos(tp(4)) sin(tp(6))
 + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
 - cos(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(1))
 + sin(tb(4)) cos(tb(5)) ((cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
 ((cos(tp(4)) cos(tp(5)) cos(tp(6)) - sin(tp(4)) sin(tp(6))) ju + tp(1))
 + (cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
 ((cos(tp(4)) sin(tp(6)) + sin(tp(4)) cos(tp(5)) cos(tp(6))) ju + tp(2))
 - sin(t(4)) sin(t(5)) sin(tp(5)) cos(tp(6)) ju + t(2))
 + tb(1) sin(tb(4)) sin(tb(5)) + tb(2) sin(tb(4)) cos(tb(5))
```

$\frac{\partial f4}{\partial tb1} = \frac{\partial f4}{\partial tb2} = 0$

$\frac{\partial f4}{\partial tb3} =$

```
     cos(jl(2)) ((sin(tb(3)) cos(tb(4)) sin(tb(5)) - cos(tb(3)) cos(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ (- cos(tb(3)) sin(tb(5)) - sin(tb(3)) cos(tb(4)) cos(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
- sin(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
- sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) + cos(t(5)) sin(tp(5)) sin(tp(6))))
- sin(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
```

```
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ cos(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
- sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) + cos(t(5)) sin(tp(5)) sin(tp(6))))
```

$$\frac{\partial f4}{\partial tb4} =$$

```
     cos(jl(2)) (cos(tb(3)) sin(tb(4)) sin(tb(5))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
- cos(tb(3)) sin(tb(4)) cos(tb(5)) ((- cos(t(4)) cos(t(5)) sin(t(6))
- sin(t(4)) cos(t(6))) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) + (cos(t(4)) cos(t(5)) cos(t(6))
- sin(t(4)) sin(t(6))) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) + cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ cos(tb(3)) cos(tb(4)) (sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
- sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) + cos(t(5)) sin(tp(5)) sin(tp(6))))
- sin(jl(2)) (sin(tb(3)) sin(tb(4)) sin(tb(5))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
```

```
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
- sin(tb(3)) sin(tb(4)) cos(tb(5)) ((- cos(t(4)) cos(t(5)) sin(t(6))
- sin(t(4)) cos(t(6))) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) + (cos(t(4)) cos(t(5)) cos(t(6))
- sin(t(4)) sin(t(6))) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) + cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ sin(tb(3)) cos(tb(4)) (sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
- sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) + cos(t(5)) sin(tp(5)) sin(tp(6))))
```

$\frac{\partial f4}{\partial tb5}=$

```
        cos(jl(2)) ((sin(tb(3)) sin(tb(5)) - cos(tb(3)) cos(tb(4)) cos(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
```

```
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6))))
- sin(jl(2)) ((- cos(tb(3)) sin(tb(5)) - sin(tb(3)) cos(tb(4)) cos(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
```

(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))

+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))

(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))

+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6))))

$$\frac{\partial f5}{\partial tb1} = \frac{\partial f5}{\partial tb2} = 0$$

$$\frac{\partial f5}{\partial tb3} =$$

sin(jl(2)) ((sin(tb(3)) cos(tb(4)) sin(tb(5)) - cos(tb(3)) cos(tb(5)))

((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))

(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))

+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))

(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))

+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))

+ (- cos(tb(3)) sin(tb(5)) - sin(tb(3)) cos(tb(4)) cos(tb(5)))

((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))

```
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
- sin(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
- sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) + cos(t(5)) sin(tp(5)) sin(tp(6))))
+ cos(jl(2)) ((- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ (cos(tb(3)) cos(tb(4)) cos(tb(5)) - sin(tb(3)) sin(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
```

```
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ cos(tb(3)) sin(tb(4)) (sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
- sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) + cos(t(5)) sin(tp(5)) sin(tp(6))))
```

$\frac{\partial f5}{\partial tb4} =$

```
    cos(jl(2)) (sin(tb(3)) sin(tb(4)) sin(tb(5))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
- sin(tb(3)) sin(tb(4)) cos(tb(5)) ((- cos(t(4)) cos(t(5)) sin(t(6))
- sin(t(4)) cos(t(6))) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) + (cos(t(4)) cos(t(5)) cos(t(6))
- sin(t(4)) sin(t(6))) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
```

```
- sin(tp(4)) cos(tp(6))) + cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ sin(tb(3)) cos(tb(4)) (sin(t(5)) sin(t(6))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
- sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) + cos(t(5)) sin(tp(5)) sin(tp(6))))
+ sin(jl(2)) (cos(tb(3)) sin(tb(4)) sin(tb(5))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
- cos(tb(3)) sin(tb(4)) cos(tb(5)) ((- cos(t(4)) cos(t(5)) sin(t(6))
- sin(t(4)) cos(t(6))) (cos(tp(4)) cos(tp(6))
- sin(tp(4)) cos(tp(5)) sin(tp(6))) + (cos(t(4)) cos(t(5)) cos(t(6))
- sin(t(4)) sin(t(6))) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) + cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ cos(tb(3)) cos(tb(4)) (sin(t(5)) sin(t(6))
```

```
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
- sin(t(5)) cos(t(6)) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
- sin(tp(4)) cos(tp(6))) + cos(t(5)) sin(tp(5)) sin(tp(6))))
```

$$\frac{\partial f5}{\partial tb5} =$$

```
     sin(jl(2)) ((sin(tb(3)) sin(tb(5)) - cos(tb(3)) cos(tb(4)) cos(tb(5)))
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ (- cos(tb(3)) cos(tb(4)) sin(tb(5)) - sin(tb(3)) cos(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6))))
+ cos(jl(2)) ((- cos(tb(3)) sin(tb(5)) - sin(tb(3)) cos(tb(4)) cos(tb(5)))
```

```
((cos(t(4)) cos(t(6)) - sin(t(4)) cos(t(5)) sin(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) sin(t(6)) + sin(t(4)) cos(t(5)) cos(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
+ (cos(tb(3)) cos(tb(5)) - sin(tb(3)) cos(tb(4)) sin(tb(5)))
((- cos(t(4)) cos(t(5)) sin(t(6)) - sin(t(4)) cos(t(6)))
(cos(tp(4)) cos(tp(6)) - sin(tp(4)) cos(tp(5)) sin(tp(6)))
+ (cos(t(4)) cos(t(5)) cos(t(6)) - sin(t(4)) sin(t(6)))
(- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
+ cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6))))
```

$$\frac{\partial f6}{\partial tb1} = \frac{\partial f6}{\partial tb2} = \frac{\partial f6}{\partial tb3} = 0$$

$\frac{\partial f6}{\partial tb4} =$

```
     cos(tb(4)) sin(tb(5)) ((cos(t(4)) cos(t(6))
 - sin(t(4)) cos(t(5)) sin(t(6))) (cos(tp(4)) cos(tp(6))
 - sin(tp(4)) cos(tp(5)) sin(tp(6))) + (cos(t(4)) sin(t(6))
 + sin(t(4)) cos(t(5)) cos(t(6))) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
 - sin(tp(4)) cos(tp(6))) + sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
 - cos(tb(4)) cos(tb(5)) ((- cos(t(4)) cos(t(5)) sin(t(6))
 - sin(t(4)) cos(t(6))) (cos(tp(4)) cos(tp(6))
 - sin(tp(4)) cos(tp(5)) sin(tp(6))) + (cos(t(4)) cos(t(5)) cos(t(6))
 - sin(t(4)) sin(t(6))) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
 - sin(tp(4)) cos(tp(6))) + cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
 - sin(tb(4)) (sin(t(5)) sin(t(6)) (cos(tp(4)) cos(tp(6))
 - sin(tp(4)) cos(tp(5)) sin(tp(6))) - sin(t(5)) cos(t(6))
 (- cos(tp(4)) cos(tp(5)) sin(tp(6)) - sin(tp(4)) cos(tp(6)))
 + cos(t(5)) sin(tp(5)) sin(tp(6)))
```

$$\frac{\partial f6}{\partial tb5} =$$

```
     sin(tb(4)) cos(tb(5)) ((cos(t(4)) cos(t(6))
   - sin(t(4)) cos(t(5)) sin(t(6))) (cos(tp(4)) cos(tp(6))
   - sin(tp(4)) cos(tp(5)) sin(tp(6))) + (cos(t(4)) sin(t(6))
   + sin(t(4)) cos(t(5)) cos(t(6))) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
   - sin(tp(4)) cos(tp(6))) + sin(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
   + sin(tb(4)) sin(tb(5)) ((- cos(t(4)) cos(t(5)) sin(t(6))
   - sin(t(4)) cos(t(6))) (cos(tp(4)) cos(tp(6))
   - sin(tp(4)) cos(tp(5)) sin(tp(6))) + (cos(t(4)) cos(t(5)) cos(t(6))
   - sin(t(4)) sin(t(6))) (- cos(tp(4)) cos(tp(5)) sin(tp(6))
   - sin(tp(4)) cos(tp(6))) + cos(t(4)) sin(t(5)) sin(tp(5)) sin(tp(6)))
```

LIST OF REFERENCES

1. A. Gelb, ed. Applied Optimal Estimation. MIT Press, Cambridge, MA, 1974.

2. R. Paul. Robot Manipulators; Mathematics, Programming and Control. MIT Press, Cambridge, MA, 1981.

3. F. Hildebrand. Advanced Calculus for Applications. Prentice-Hall, Englewood Cliffs, NJ, 1976, Ch. 7.

4. R. Sturges. Teleoperator Arm Design Program (TOAD). CSDL Report E-2746. February 1973.

5. E. Junkel. MS Thesis, MIT Department of Mechanical Engineering. June 1983.