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EVALUATION OF TRENDS IN WHEAT YIELD MODELS

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Mary C. Ferguson

Earth Resources Research Division
Lyndon B. Johnson Space Center
Houston, Texas 77058

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16. Abstract

Trend terms in models for wheat yield in the U.S. Great Plains for the years 1932-1976 are evaluated. The subset of meteorological variables yielding the largest adjusted R^2 is selected using the method of leaps and bounds. Latent root regression is used to eliminate multicollinearities. And generalized ridge regression is used to introduce bias to provide stability in the data matrix. The regression model used provides for two trends in each of two models: a "dependent" model in which the trend line is piece-wise continuous, and an "independent" model in which the trend line is discontinuous at the year of the slope change.

It was found that the trend lines best describing the wheat yields consisted of combinations of increasing, decreasing, and constant trend: four combinations for the dependent model and seven for the independent model.

17. Key Words (Suggested by Author(s))

latent root regression	trend
leaps and bounds	U.S. Great Plains
multicollinearities	weather variables
selection of variables	wheat
slope-change year	yield model

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EVALUATION OF TRENDS IN WHEAT YIELD MODELS

This report describes activities of the Supporting Research project of the AgRISTARS program.

PREPARED BY

Mary C. Ferguson

Earth Resources Research Division

Scene Analysis Branch

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
LYNDON B. JOHNSON SPACE CENTER
HOUSTON, TEXAS

December 1982

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PREFACE

The Agriculture and Resources Inventory Surveys Through Aerospace Remote Sensing is a multiyear program of research, development, evaluation, and application of aerospace remote sensing for agricultural resources, which began in fiscal year 1980. This program is a cooperative effort of the U.S. Department of Agriculture, the National Aeronautics and Space Administration, the National Oceanic and Atmospheric Administration (U.S. Department of Commerce), the Agency for International Development (U.S. Department of State), and the U.S. Department of the Interior.

The work which is the subject of this document was performed within the Earth Resources Research Division, Space and Life Sciences Directorate, at the Lyndon B. Johnson Space Center, National Aeronautics and Space Administration.

CONTENTS

TABLES

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FIGURES

Figure		Page

Figure | Page

Figure		Page

Figure		Page

Figure		Page

ACRONYMS

AgRISTARS	Agriculture and Resources Inventory Surveys Through Aerospace Remote Sensing
CCEA	Center for Climatic and Environmental Assessment, U. S. Department of Agriculture
DFN	Departure from normal
LACIE	Large Area Crop Inventory Experiment
NOAA	National Oceanic and Atmospheric Administration, U. S. Department of Commerce
PET	Potential evapotranspiration
TY	Transition Year
USGP	U. S. Great Plains

EVALUATION OF TRENDS IN WHEAT YIELD MODELS

INTRODUCTION:

The CCEA models for wheat yield in the U.S. Great Plains (Refs. 1 and 2) are multiple regression equations using year numbers and selected weather variables as independent variables The models assume that the yield can be represented by an almost exact linear model with one or more trends. For each year the yield may be expressed

$$Y = C + \sum_{i=1}^{n} b_i X_i \qquad (1)$$

where Y is the yield, C is a constant, b_i is the coefficient of the independent variable, X_i, and n is the number of independent variables. Figure 1 shows the spring and winter wheat areas to which the models apply. Figures 2 and 3 are typical curves, for spring and winter wheat, showing the measured yields and the trend lines of the regression models, about which the yields vary. Values used in plotting the curves were taken from References 1 and 2. The constant plotted in each case is the sum of the indicated constant and trend coefficients; variables used in the calculations to represent the trends have values of one until the year of the slope change, then increase by one each year until the end of that trend, at which time they become constant.

The trend lines, alone, do not appear to provide a good fit to the observed values of yield. Marquina (3) discusses the use of multiple regression in the prediction of yield as a function of meteorological variables. Correlation among the independent variables causes multicollinearities which can contribute to misleading results. The coefficients associated with the variables may be too large, and the signs may be incorrect; adding one or more new observations may change the size and the sign of one or more of the coefficients. Marquina discusses techniques which may be used to find a regression model when there are a large number of independent variables and linear relations (multicollinearities) exist among them. The method of leaps and bounds is used for finding the subset of variables constituting the best regression. Principal components and latent root regression ar sed for eliminating multicollinearities. And,

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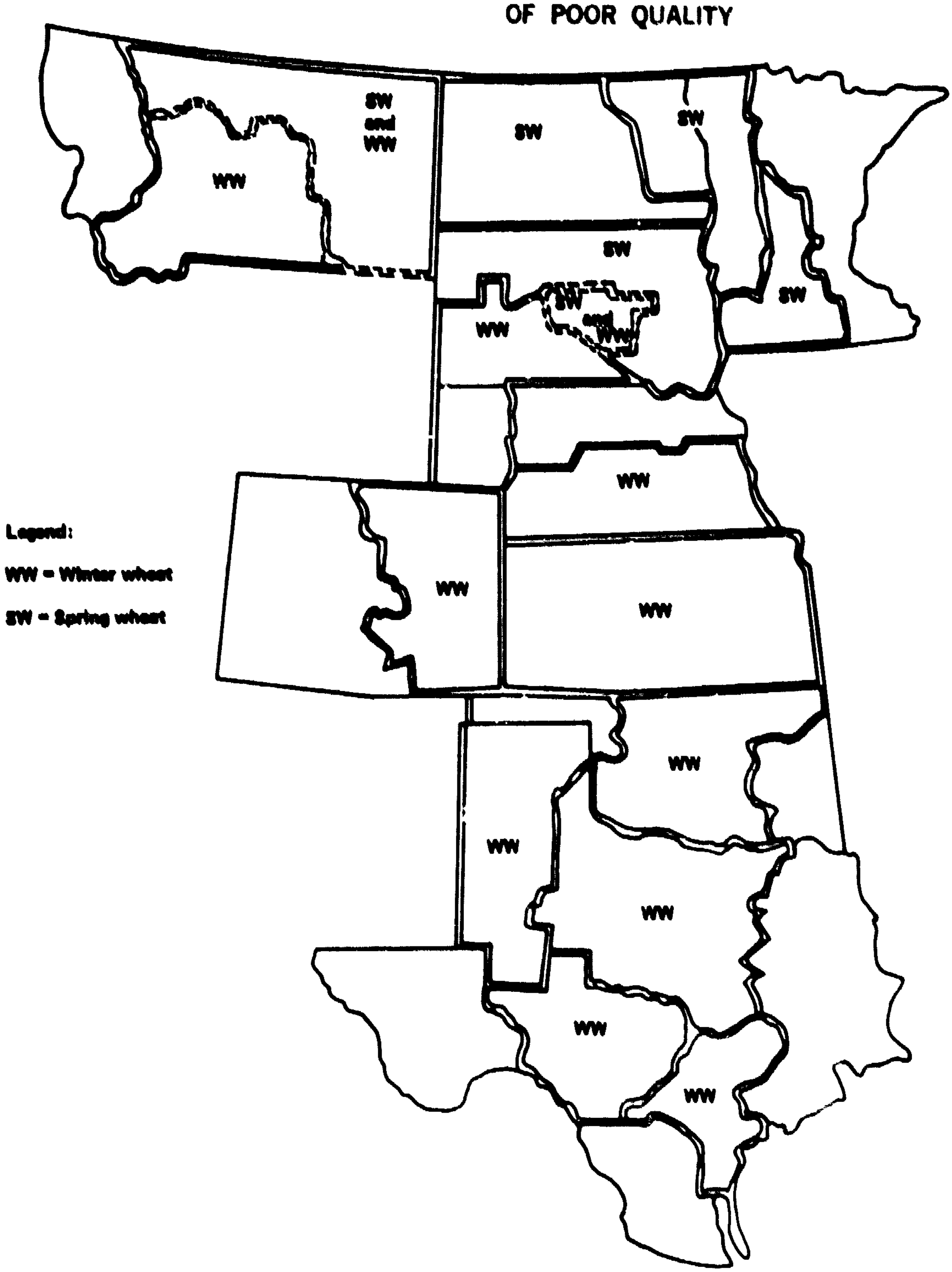

Figure 1.- Spring and winter wheat growing areas in the U.S. Great Plains.

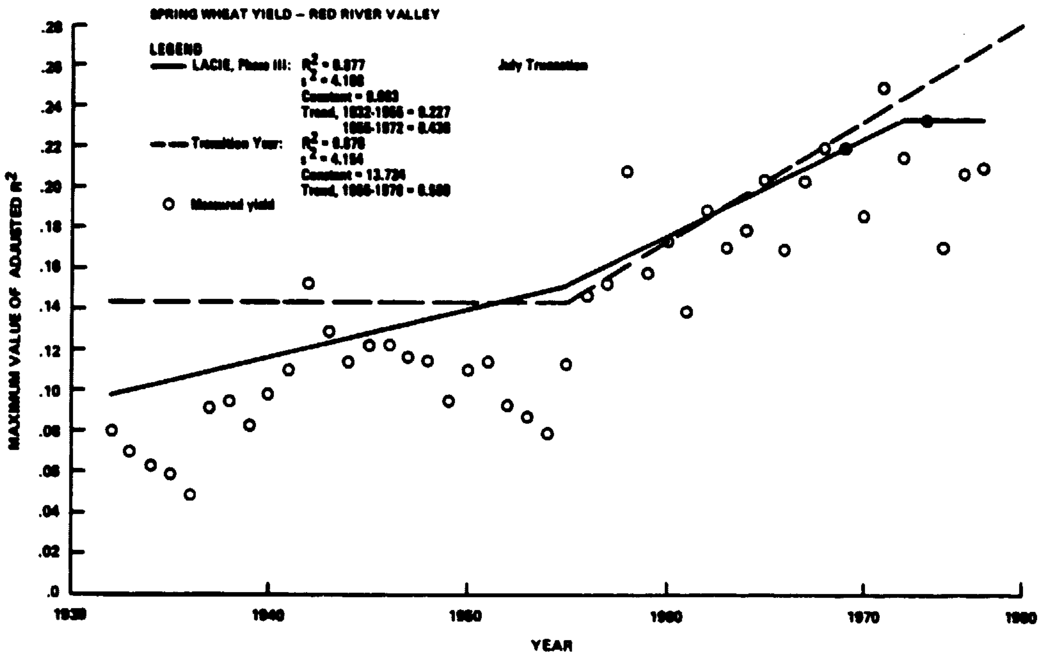

Figure 2.- Trend lines for the CCEA models for spring wheat yield in the Red River Valley.

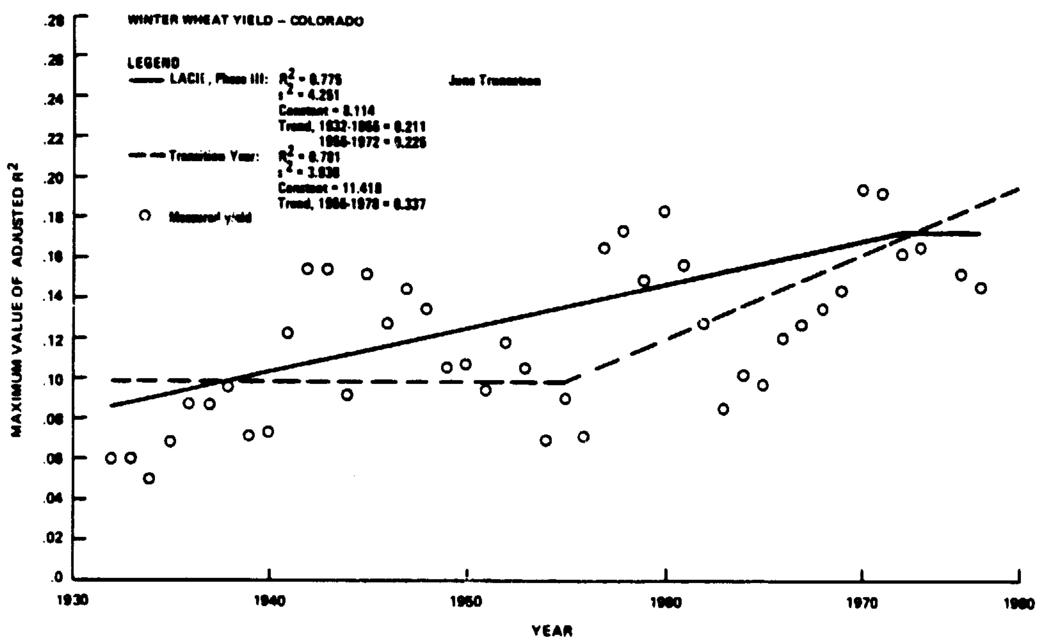

Figure 3.- Trend lines for the CCEA models for winter wheat yield in Colorado.

generalized ridge regression is used for introducing bias to provide stability in the data matrix.

Techniques discussed by Marquina (3) are implemented in programs SELECT and BEIRA, written by Marquina and modified by Johnson Space Center personnel. Program SELECT uses the method of leaps and bounds, described by Furnival and Wilson (4). Binary trees of all possible regressions are searched, and tests made, to identify the best regression for each subset size without computing all possible regressions. Program BEIRA performs the regression and provides the information for using the techniques of principal components and latent root regression to eliminate multicollinearities. Bias may be introduced by changing one or more of the eigenvalues of the correlation matrix of the independent variables.

Each program allows the user to choose the number of trends (one, two or none), the year of the slope change, and either of two models, "dependent" or "independent." In the dependent model, the yield varies about a line which is piecewise continuous, with the slope changing at the specified year. In the independent model, the line about which the yield varies is discontinuous at the year of the slope change.

Models were developed using programs SELECT and BEIRA and the data from which the CCEA models were derived, for the wheat growing areas of Figure 1. The trend lines in these curves appear to provide a better fit to the measured yields than the trend lines of the CCEA models. Table 1 lists the values of the multiple correlation coefficients, R^2, and residual variances, S^2, for the CCEA models, and for those developed using SELECT and BEIRA, for the five spring wheat and nine winter wheat growing areas. Values of R^2 and S^2, for the SELECT-BEIRA models are underlined to indicate their comparison with the CCEA models. A double underline indicates that a value of R^2 is greater, or a value of S^2 is smaller, than the corresponding values for both CCEA models for the same area. A single underline indicates that R^2 is larger, or S^2 smaller, than the corresponding value for one CCEA model. It is seen that R^2 and S^2 are both underlined at least once for 22 of the 28 SELECT-BEIRA models. Of these R^2 and S^2 are both underlined twice for 17 models, and R^2 and S^2 for both dependent and independent models are underlined twice for 6 wheat growing areas. It is

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TABLE 1. R^2 AND S^2 FOR THE CCEA MODELS AND THOSE DEVELOPED USING SELECT AND BEIRA

		CCEA MODELS		MODELS DEVELOPED USING SELECT - BEIRA	
		PHASE III	TY	Dep.	Ind.
SPRING WHEAT					
Minnesota	R^2	.940	.859	.940	.954*
	S^2	2.030*	4.798	2.344	2.017
Montana	R^2	.855	.821	.925	.937*
	S^2	2.413	3.254	1.563	1.407*
North Dakota	R^2	.883	.878	.875	.897*
	S^2	3.822	3.604	3.701	3.142*
Red River Valley	R^2	.877	.870	.915	.964*
	S^2	4.108	4.154	2.887	1.314*
South Dakota	R^2	.897*	.843	.827	.882
	S^2	2.099*	3.029	3.339	2.807
WINTER WHEAT					
Badlands	R^2	.784	.754	.822*	.822*
	S^2	8.689	9.954	7.792	7.578*
Colorado	R^2	.775	.781	.863	.872*
	S^2	4.251	3.938	3.169	2.855*
Kansas	R^2	.915	.928	.932	.944*
	S^2	2.750	2.139	2.372	2.077*
Montana	R^2	.835	.824	.856	.867*
	S^2	3.577	3.606	3.033	2.956*
Nebraska	R^2	.901	.884	.872	.943*
	S^2	4.325*	5.064	5.772	2.885
Oklahoma	R^2	.900	.868	.890	.946*
	S^2	2.273	2.901	3.145	1.517*
Texas Edwards Plateau	R^2	.831*	.760	.786	.809
	S^2	1.670*	1.905	1.908	1.701
Texas Low Plains	R^2	.837	.847	.863	.884*
	S^2	1.607	1.590	1.386	1.214*
Texas-Oklahoma Panhandle	R^2	.890	.884	.877	.903*
	S^2	2.643*	3.008	3.301	2.771

═══ R^2 larger (S^2 smaller) than both CCEA models.

——— R^2 larger (S^2 smaller) than one CCEA model.

*Smallest S^2 (largest R^2) of the four models for an area.

apparent that wheat yield models can be developed having larger values of R^2 and smaller values of S^2 than the CCEA models for Phase III and TY of LACIE.

Programs SELECT and BEIRA and the techniques available with BEIRA are described briefly in Appendix A.

METHOD:

Programs SELECT and BEIRA were used with yield and weather data, for the spring and winter wheat areas shown in Figure 1, for the years 1932 through 1976. SELECT was used to find the year of slope change and subset of independent variables with the largest value of adjusted R^2, given by

$$R^2_{adj} = 1 - [\,1 - R^2\,](\frac{n}{n-m}) \tag{2}$$

where R^2 is the coefficient of determination,

m is the number of variables included in the regression, and

n is the number of observations.

Subsets of independent variables and slope change years identified using SELECT were used as input to BEIRA to run the regressions. Programs SELECT and BEIRA are described briefly in Appendix A. Independent variables used as input to SELECT are listed in Table B-1 in Appendix B.

Program SELECT was run for different slope-change years, for each wheat growing area and model (dependent, with a piecewise continuous trend line, and independent, with a discontinuous trend line), until there were enough points to allow identification of the slope-change year yielding a regression with the largest value of R^2_{adj}. Results are shown in Figures B-1 through B-14 in Appendix B. Variables included in the regressions with peak values of R^2_{adj} are listed on the plots.

Regressions were run, using program BEIRA, for the slope change years with maximum values of R^2_{adj} on the plots and the indicated independent variables. The process of finding a "best regression" for a wheat growing model and area is described in Appendix C. In the course of running the regressions, it was found for several areas that, after the multicollinearities were removed, the values of R^2 were smaller than those for the CCEA models. More regressions were run for the slope-change years indicated by the secondary peaks in the plots of Figures B-1 through B-14 in Appendix B. When values of R^2_{adj} for two or more slope-change years were nearly equal, regressions were run for each of them.

For comparison with the CCEA models, regressions were run, using the variables chosen by SELECT, for the slope-change years of the CCEA models. For most of the winter wheat areas, the CCEA models have more than one slope change. Regressions were run for each slope-change year except when the slope changed after 1970 and the yield values didn't appear to level off. Variables chosen by SELECT for each slope-change year and model (dependent and independent) are given in Table B-2 in Appendix B.

RESULTS:

Table 2 lists the best regressions for the slope-change years determined using SELECT and for the slope-change years of the CCEA models. Quantities given for each regression include R^2, the coefficient of multiple determination which indicates goodness of fit, S^2, the residual variance, n_Δ, the difference between the number of predicted values greater than and less than the measured values, the number of variables in the regression, the variables deleted from the subset chosen by SELECT, and whether bias was introduced by increasing the value of an eigenvalue. For each wheat area regressions for the slope-change years from SELECT are followed by those for the CCEA slope-change years. When there are regressions for more than one of the slope-change years from SELECT, the one chosen as the best is underlined.

The best regressions were chosen taking into consideration the values of R^2, S^2, the distribution of the predicted values above and below the measured values as indicated by n_Δ, the uniformity of the eigenvalues of R (the matrix of correlation coefficients of the independent variables), and values of $|R|$ and $tr(R^{-1})$, which indicate stability of the data matrix. The values of R^2 and S^2 for the CCEA slope-change years are underlined to indicate the comparison with those of the CCEA models. Two underlines indicates a value of R^2 larger, or a value of S^2 smaller, than those for both CCEA models. One underline indicates a value of R^2 greater, or a value of S^2 smaller, than the value for one of the CCEA models.

TABLE 2: BEST REGRESSIONS FOR SLOPE-CHANGE YEARS WITH PEAK VALUES OF R^2_{adj} AND SLOPE-CHANGE YEARS OF THE CCEA MODELS

(C) = CCEA Models

		DEPENDENT MODEL							INDEPENDENT MODEL					
		SLOPE CHANGE YEAR	R^2	S^2	n_Δ*	NO. VARS.	VARS DELETED		SLOPE CHANGE YEAR	R^2	S^2	n_Δ*	NO. VARS.	VARS *** DELETED
a. Spr'ng Wheat Areas														
Minnesota		1953	.940	2.344	-3	13	--		1954	.954	2.017	-3	16	--
									1951	.944	2.268	-3	14	--
	(C)	1955	.933	2.722	-5	14	--	(C)	1955	.943	2.394	1	15	--
Montana		1961	.925	1.558	9	12	--		1936	.953	1.189	7	18	--
		1960	.925	1.561	9	12	--		1943	.937	1.407	1	14	18
		1959	.925	1.563	7	12	--		1956	.927	1.568	7	13	--
		1951	.923	1.600	9	12	--							
		1952	.923	1.603	9	12	--							
	(C)	1955	.922	1.638	9	12	--	(C)	1955	.922	1.683	9	13	--
North Dakota		1953	.875	3.701	1	9	--		1948	.896	3.085	3	9	--
									1961	.897	3.142	-1	10	--
									1955	.892	3.387	-3	11	--
	(C)	1955	.875	3.878	3	11	--	(C)	1955	.892	3.387	-3	11	--
	(C)	1965	.821	5.281	3	9	--	(C)	1965	.851	4.655	1	11	6
Red River Valley		1951	.915	2.887	-5	8	--		1955	.964	1.314	1	12	--e_1 + 0.1
	(C)	1955	.897	3.394	-3	7	--	(C)	1955	.964	1.314	1	12	--e_1 + 0.1
South Dakota		1953	.827	3.339	-3	8	--		1966	.882	2.807	-1	15	--
		1973	.831	3.566	-7	11	--		1970	.834	3.127	-1	7	--
									1954	.846	3.158	-5	10	--
	(C)	1955	.820	3.482	-7	8	--	(C)	1955	.843	3.313	-5	11	--

TABLE 2 (continued)

		DEPENDENT MODEL							INDEPENDENT MODEL					
		SLOPE CHANGE YEAR	R^2	S^2	n_Δ^*	NO. VARS.	VARS. DELETED		SLOPE CHANGE YEAR	R^2	S^2	n_Δ^*	NO. VARS.	VARS. DELETED
b. Winter Wheat Areas														
Badlands		1975	.822	7.792	-1	10	19		1975	.822	7.578	1	9	19
		1972	.809	8.604	-1	11	19		1970	.816	8.557	-1	12	19
		1943	.770	10.083	5	10	21		1939	.772	9.999	3	10	17
									1972	.812	8.745	1	12	19
									1967	.818	8.727	1	13	19
									1946	.779	9.982	9	11	21
	(C)	1955	.760	10.498	5	10	21	(C)	1955	.766	10.227	7	10	21
	(C)	1972	.809	8.604	-1	11	19	(C)	1972	.812	8.745	1	12	19
Colorado		1965	.863	3.169	1	16	17,16,14		1967	.774	5.214	7	16	16,17
									1962	.873	3.033	5	17	17,16,14
									1950	.853	3.403	1	16	22,17
									1944	.872	2.855	1	15	22,23
	(C)	1955	.861	3.106	3	15	22,17	(C)	1955	.827	3.625	1	13	16,17
Kansas		1955	.932	2.372	-1	12	20,4		1946	.944	2.077	-1	14	15,20
	(C)	1955	.932	2.372	-1	12	20,4	(C)	1955	.933	2.399	5	13	20,4
	(C)	1943	.912	2.960	-3	11	20	(C)	1943	.911	3.097	7	12	15,20,4
Montana		1932	.856	3.030	-3	7	--		1953	.875	2.781	-7	9	--
		1933	.856	3.026	-3	7	--		1944	.867	2.956	-1	9	--
		1934	.856	3.033	-1	7	--		1937	.863	2.967	-5	8	--
		1975	.856	3.040	-3	7	--		1959	.863	3.124	-5	10	20
		1958	.852	3.110	-1	8	--e_1 + 0.1							
		1957	.854	3.105	-1	8	--e_1 + 0.1							
	(C)	1943	.856	3.112	-3	8	--	(C)	1943	.865	3.012	-3	9	--
	(C)	1955	.857	3.095	-5	8	--	(C)	1955	.864	3.206	-5	11	20

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TABLE 2 (continued)

		DEPENDENT							INDEPENDENT					
		SLOPE CHANGE YEAR	R^2	S^2	n_Δ*	NO. VARS.	VARS. DELETED		SLOPE CHANGE YEAR	R^2	S^2	n_Δ*	NO. VARS.	VARS. DELETED
Nebraska		1963	.867	5.990	7	10	11,9		1969	.943	2.885	3	14	17,19
		1962	.867	5.989	7	10	11,9		1946	.864	6.469	3	12	11,9**
		1961	.866	6.006	9	10	11,9		1940	.902	5.335	1	16	17,15
		1951	.856	6.285	9	9	11,8							
		1952	.855	6.315	5	9	11,9							
		1954	.856	6.292	9	9	11,8							
		1955	.855	6.335	9	9	11,8							
		1938	.872	5.772	5	10	17,9							
		1939	.871	5.792	5	10	17,9							
	(C)	1955	.855	6.335	9	9	11,8	(C)	1955	.857	6.429	7	10	11,8
Oklahoma		1963	.890	3.145	-1	17	15,20 e_1 + 0.1		1958	.946	1.517	-1	16	20
		1964	.889	3.070	-3	16	15,20 e_1 + 0.1		1955	.941	1.700	3	17	15,16
		1962	.887	3.236	-1	17	15,20 e_1 + 0.1		1953	.902	2.574	1	14	15
									1965	.887	3.208	-3	17	15,20 e_1 + 0.1
	(C)	1943	.868	3.151	3	11	15,23,20,21**	(C)	1943	.895	3.172	3	18	15,20
	(C)	1955	.889	3.338	-3	18	15,20	(C)	1955	.941	1.700	3	17	15,16
	(C)	1960	.879	3.462	-1	17	15,20 e_1 + 0.1	(C)	1960	.894	2.898	-1	16	15,20,2 ** e_1 + 0.1
	(C)	1962	.887	3.326	-1	17	15,20 e_1 + 0.1	(C)	1962	.901	2.863	-5	17	15,20
Texas Edwards Plateau		1955	.758	2.098	-1	12	22,20,19		1957	.809	1.703	3	13	22,20
		1954	.759	2.087	-1	12	22,20,19		1959	.799	1.912	3	15	22,20,19
		1949	.786	1.908	3	13	22,20,13		1946	.741	2.314	1	13	22,21,19,18
		1948	.785	1.923	3	13	22,20,19		1973	.764	2.177	1	14	22,21,19,18
		1974	.757	2.172	1	13	22,21,19,18							
	(C)	1955	.758	2.098	-1	12	22,20,19	(C)	1955	.790	1.933	3	14	22,20,19
	(C)	1960	.694	2.816	5	14	22,15,19,12	(C)	1960	.759	2.147	3	13	22,20,19,6**
	(C)	1965	.688	2.874	1	14	22,15,19,12	(C)	1965	.667	2.977	1	13	22,15,19,12,6**
	(C)	1975	.797	1.707	-1	11	22,19	(C)	1975	.797	1.707	-1	11	22,19
Texas Low Plains		1948	.863	1.386	-5	8	23,20,18		1955	.887	1.250	-3	11	23,18
									1956	.884	1.214	-1	10	18 e_1 + 0.1
	(C)	1955	.831	1.711	-5	8	20	(C)	1955	.887	1.250	-3	11	23,18
	(C)	1962	.807	1.958	-9	8	20	(C)	1962	.850	1.650	-5	11	20

TABLE 2 (continued)

		DEPENDENT MODEL							INDEPENDENT MODEL					
		SLOPE CHANGE YEAR	R^2	S^2	n_Δ*	NO. VARS.	VARS. DELETED		SLOPE CHANGE YEAR	R^2	S^2	n_Δ*	NO. VARS.	VARS. DELETED
Texas-Oklahoma Panhandle		1965	.858	3.810	-1	12	17,21,13		1955	.893	2.790	1	11	17,20,13,23**
		1964	.866	3.701	3	13	23,21		1957	.891	2.765	1	10	17,22,20,13
		1948	.877	3.299	-3	12	23,13		1959	.895	2.809	3	12	17,13
		1950	.874	3.269	-3	11	17,13		1946	.903	2.771	-1	14	23
		1949	.877	3.301	-1	12	17,13							
	(C)	1955	.872	3.327	-1	11	13	(C)	1955	.893	2.790	1	11	17,20,13,23**
	(C)	1960	.848	4.075	-3	12	17,21,13	(C)	1960	.902	2.636	5	12	23,21,13
	(C)	1962	.849	4.044	-3	12	17,22,21, 13,7**	(C)	1962	.886	3.486	5	16	21,22,17,13

* n_Δ = Difference between the number of predicted values greater than and less than the measured value.

** Deleted because $\beta_1 < .001$. Generally the value of S^2 decreased; other quantities remained the same.

*** e_1+0.1 indicates that the first eigenvalue was increased by 0.1.

The trend line curves are found to take seven different forms, depending on the values of the two trend coefficients, β_{T1} and β_{T2}, the relation between the coefficient of the constant variable, β_c, and the quantity $\beta_{T1}(T_c-T_o)$ where T_c is the year of the slope change and T_o is the first year for which data are given. The models with trend lines of each form are listed in Table 3 by kind of model (dependent or independent), wheat growing area (S or W to indicate spring or winter wheat), and slope-change year. Figures 4 and 5 show the forms of the models and the geographical distribution of the different types.

Plots of the yield values and the trend lines for the 28 models are given in Figures 6 through 19, on the same scale as Figures 2 and 3. Computer-generated plots, which show the predicted and measured values, are given in Appendix D. Regression coefficients are listed in Table 4. Plots of the CCEA models, other than those shown in Figures 2 and 3, are given in Appendix E, for comparison; a list of variables and regression coefficients follows the plots.

CONCLUSIONS:

It is concluded, as stated in the introduction, that yield models can be developed that have larger values of R^2 and smaller values of S^2 than the CCEA models of References 1 and 2. And, that programs SELECT and BEIRA provide an objective method for developing yield models based on the variables which best describe the yield of a given area. It is evident from Figures 4 and 5 that a model of one form will not serve to describe the wheat yield for all of the wheat growing areas in the U.S. Great Plains. Nor will one slope-change year provide a good description of the change in wheat yield with time. Slope-change years for the "best fit" dependent models varied between 1934 and 1965, except for the one with slope change in 1975. The slope-change years for independent models ranged from 1943 to 1969, and 1975. Table 1 shows that the independent models, in which the trend lines are discontinuous at the slope-change year, have consistently larger values of R^2 and smaller values of S^2 than the dependent models, with trend lines that are piecewise continuous.

This leads to the conclusion that a model with a "jump" in yield, or with three trends, may provide the best description of yield as a function of time.

TABLE 3: TYPES OF WHEAT YIELD MODELS

TYPE 1. A constant followed by an increasing trend: $\beta_{T1} = 0$, $\beta_{T2} > 0$. Eight of the 14 dependent models, and 2 of the 14 independent models have this form.

Dependent: Minnesota (S), 1953
North Dakota (S), 1953
Red River Valley (S), 1951
South Dakota (S), 1953
Montana (W), 1934
Nebraska (W), 1938
Texas Edwards Plateau (W), 1949
Texas Low Plains (W), 1948

Independent: For both these models $\beta_c > 0$; the trend lines describe a wheat yield which remains constant until the slope-change year then increases from a yield value higher than the previous constant value.
Red River Valley (S), 1955
Texas Low Plains (W), 1956

TYPE 2. Two increasing trends: $\beta_{T1} > 0$, $\beta_{T2} > 0$. Four dependent models and four independent models are of this form.

Dependent: Montana (S), 1959
Colorado (W), 1965
Kansas (W), 1955
Texas-Oklahoma Panhandle (W), 1949

Independent: $\beta_c = 0$, the second trend starts at the overall constant, the same yield value as at the start of the first trend.
Kansas (W), 1946

$\beta_c < \beta_{T1}(T_c - T_o)$, the second trend starts at a lower yield value than the end of the first trend.
Montana (S), 1943
Montana (W), 1944
Texas-Oklahoma Panhandle (W), 1946

TABLE 3 (continued)

TYPE 3. An increasing trend followed by a constant $\beta_{T1} > 0$, $\beta_{T2} = 0$.

Dependent: Oklahoma (W), 1963

Independent: $\beta_c > \beta_{T1}(T_c - T_o)$. The second trend starts at a higher yield value than the end of the first trend.

North Dakota (S), 1961

Oklahoma (W), 1958

TYPE 4. An increasing trend followed by a decreasing trend. $\beta_{T1} > 0$, $\beta_{T2} < 0$.

Independent: $\beta_c > \beta_{T1}(T_c - T_o)$. The second trend starts at a higher yield value than the end of the first trend.

South Dakota (S), 1966

Nebraska (W), 1969

TYPE 5. A decreasing trend followed by an increasing trend: $\beta_{T1} < 0$, $\beta_{T2} > 0$.

Independent: Because of the initial decreasing trend both of these models have a second trend which starts at a yield value higher than the end of the first trend.

Minnesota (S), 1954, $\beta_c > 0$.

Colorado (W), 1944, $\beta_c = 0$.

TYPE 6. A constant value followed by a constant value: $\beta_{T1} = 0$, $\beta_{T2} = 0$, $\beta_c > 0$.

Independent: Texas Edwards Plateau (W), 1957.

TYPE 7. An increasing trend until 1975 followed by a low value in 1976. The trend lines for the two models are almost identical: The dependent model has a large negative slope from 1975 to 1976. The independent model has $\beta_{T2} = 0$, $\beta_c = 0$; the value of the trend line curve for 1976 is the same as the overall constant.

Dependent: Badlands (W), 1975.

Independent: Badlands (W), 1975.

TABLE 4.- COEFFICIENTS FOR BEST FIT REGRESSIONS

(a) Spring wheat

Variable	MINNESOTA		MONTANA		NORTH DAKOTA		RED RIVER VALLEY		SOUTH DAKOTA	
	Depend. model, 13 variables	Independ. model, 16 variables	Depend. model, 12 variables	Independ. model, 14 variables	Depend. model, 9 variables	Independ. model, 10 variables	Depend. model, 8 variables	Independ. model, 12 variables	Depend. model, 8 variables	Independ. model, 15 variables
Precipitation, mm:										
1. January	-0.044	-0.028	0.099	0.076			-0.033	-0.028	-0.051	-0.060
2. February				-.086	0.065			.060		
3. March			-.070			0.072		.056		.066
4. April	-.032	-.028	.032	.033					.032	.030
5. May	-.022	-.022			.033					.023
6. June	-.021	-.017	.051	.043	.013		.018	.019		
7. July			.021	.034	-.009					-.022
8. August	.027	.018		.020						
Precipitation of the year before, mm:										
9. August		-.012		.011			.011	.009		.019
10. September	-.021	-.026	.030	.024	.041	.020	.011		.019	.046
11. October	-.031	-.032		.044		.026		.015		
12. November									.067	.082
13. December			-.116	-.092		-.137			.060	
Average temperature in °C:										
14. April	.347	.333				.186	.157	.361		
15. May	-.395	-.476	.237	.292		.268				.470
16. June	-.967	-.990			-.455	-.494	-.402	-.456	.540	-.257
17. July	-1.090	-.992	-.566	-.662	-.638	-.567	-1.156	-1.010	-.802	-.519
18. August	.422	.359	-.369		-.270			.196		.214
Year of slope change	1953	1954	1959	1943	1953	1961	1951	1965	1953	1966
First trend		-0.066	0.119	0.392		0.150				0.125
Second trend	0.639	.517	.365	.254	0.511		0.523	0.317	0.291	-.644
Constant		1.892		1.426		10.628		5.537		11.304
Overall constant:	10.919	11.377	7.680	6.357	7.622	5.767	10.245	9.814	7.403	5.838
R^2	.940	.954	.925	.937	.875	.897	.915	.964	.827	.882
s^2	2.344	2.017	1.563	1.407	3.701	3.142	2.887	1.315	3.339	2.807
Variables deleted from the set chosen by SELECT	--	--	--	18	--	--	--	--	--	--
Eigenvalues increased								$e_1 + 0.1$		
n (Difference between number of predicted values which are greater and less than the measured values.)	-3	-3	7	1	1	-1	-5	1	-3	-1

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TABLE 4.- Continued.

(b) Winter wheat

[Numbers in parentheses are variable numbers.]

Variable	BADLANDS		COLORADO		KANSAS		MONTANA		NEBRASKA	
	Depend. model, 10 variables	Independ. model, 9 variables	Depend. model, 16 variables	Independ. model, 15 variables	Depend. model, 12 variables	Independ. model, 14 variables	Depend. model, 7 variables	Independ. model, 9 variables	Depend. model, 10 variables	Independ. model, 14 variables
Precipitation, mm:										
1. January			0.120	0.185	-0.022	-0.056				-0.048
2. February			.112	.077	.043	.070			0.040	
3. March			.040	.041	.037	.026	0.103	0.113		
4. April	0.025	0.025	.039	.052			.031	.029	-.024	
5. May			.037	.034	-.004		.022	.021	-.043	-.033
6. June					.010		.045	.039	.018	-.034
Precipitation of the year before, mm:										
7. August					-.007					.014
8. September			.021	.028			.048	.046	.005	.020
9. October	.124	.125	.11ᐢ	.124	.031	.034				-.015
10. November	.091	.091				.034	.077	.083		.015
11. December			-.070	-.056	-.039	-.069			-.025	
Average temperature in °C:										
12. January	.256	.257	-.042							
13. February	-.143	-.148	.298	.478		.322				-.402
14. March	.378	.380			-.201	-.246				-.173
15. April	.245	.244	-.146						.274	
16. May				.080					-.876	
17. June				.005		.502				
PET (Potential evapotranspiration):										
18. January					-.323	-.778				
19. February				-.148		-.204			(18)-.295	
20. March			.092	.082						
21. April	-.042	-.042				-.002				
22. May			.024	.024						(21)-.148
23. June			-.003	-.023					(22)-.082	(22)-.088
Year of slope change	1975	1975	1965	1944	1955	1946	1934	1944	1938	1969
First trend	0.367	0.368	0.168	-0.221	0.125	0.406		0.436		(23) 0.283
Second trend	-16.129		.350	.203	.497	.468	(24)0.270	(24) .298	(24)0.385	(24)-1.552
Constant						3.205		3.205		22.090
Overall constant	6.605	6.578	8.123	10.091	8.596	7.477	9.628	8.511	9.048	8.498
R^2	.822	.822	.863	.872	.932	.944	.856	.867	.872	.943
s^2	7.792	7.578	3.169	2.855	2.372	2.077	3.033	2.956	5.772	2.885
Variables deleted from the set chosen by SELECT	19	19	17, 16, 14	22, 23	20, 4	15, 20	--	--	17, 9	17, 19
n (Difference between number of predicted values which are greater and less than the measured values.)	-1	1	1	1	-1	-1	-1	-1	5	3

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TABLE 4.- Concluded

(b) Winter wheat

Variable	[illegible]		[illegible]		[illegible]		[illegible]	
	Independ. model, [illegible] variables	Independ. model, [illegible] variables	Independ. model, [illegible] variables	Independ. model, [illegible] variables	Independ. model, [illegible] variables	Independ. model, [illegible] variables	Independ. model, [illegible] variables	Independ. model, [illegible] variables
Precipitation, mm								
1. January	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		
2. February	[illegible]		[illegible]	[illegible]		[illegible]	[illegible]	[illegible]
3. March	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	
4. April	[illegible]				[illegible]	[illegible]		
5. May	[illegible]	[illegible]				[illegible]	[illegible]	[illegible]
6. June	[illegible]	[illegible]						
Precipitation of the year before, mm								
7. August	[illegible]	[illegible]	[illegible]	[illegible]				
8. September			[illegible]	[illegible]				[illegible]
9. October	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]		[illegible]
10. November	[illegible]	[illegible]	[illegible]	[illegible]			[illegible]	[illegible]
11. December		[illegible]					[illegible]	[illegible]
Average temperature in °C								
12. January	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-.192		[illegible]
13. February	[illegible]							
14. March	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	-.242		
15. April							[illegible]	[illegible]
16. May		[illegible]	[illegible]	[illegible]				[illegible]
17. June	[illegible]				[illegible]			
PET (Potential evapotranspiration)								
18. January	[illegible]	[illegible]					-.227	
19. February			[illegible]	[illegible]			-.197	[illegible]
20. March							[illegible]	[illegible]
21. April	[illegible]	[illegible]						[illegible]
22. May	[illegible]						[illegible]	
23. June		[illegible]	[illegible]	[illegible]				
Year of slope change	[illegible]	1958	[illegible]	1957	1948	1956	[illegible]	[illegible]
First trend	[illegible]	[illegible]					[illegible]	[illegible]
Second trend			[illegible]		[illegible]	.128	[illegible]	[illegible]
Constant		[illegible]		[illegible]		2.967		[illegible]
Overall constant	[illegible]	8.229	[illegible]	6.577	7.622	7.750	4.951	[illegible]
R^2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.877	[illegible]
S^2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.214	[illegible]	2.771
Variable deleted from the set chosen by "SELECT"	15, 20	20	22, 20, 13	22, 20	23, 20, 18	18	17, 13	23
Eigenvalue created						e_1 = 9.1		
ε (Difference between number of predicted values which are greater and less than the measured values.)	[illegible]	-1	1	1	-5	-1	-1	-1

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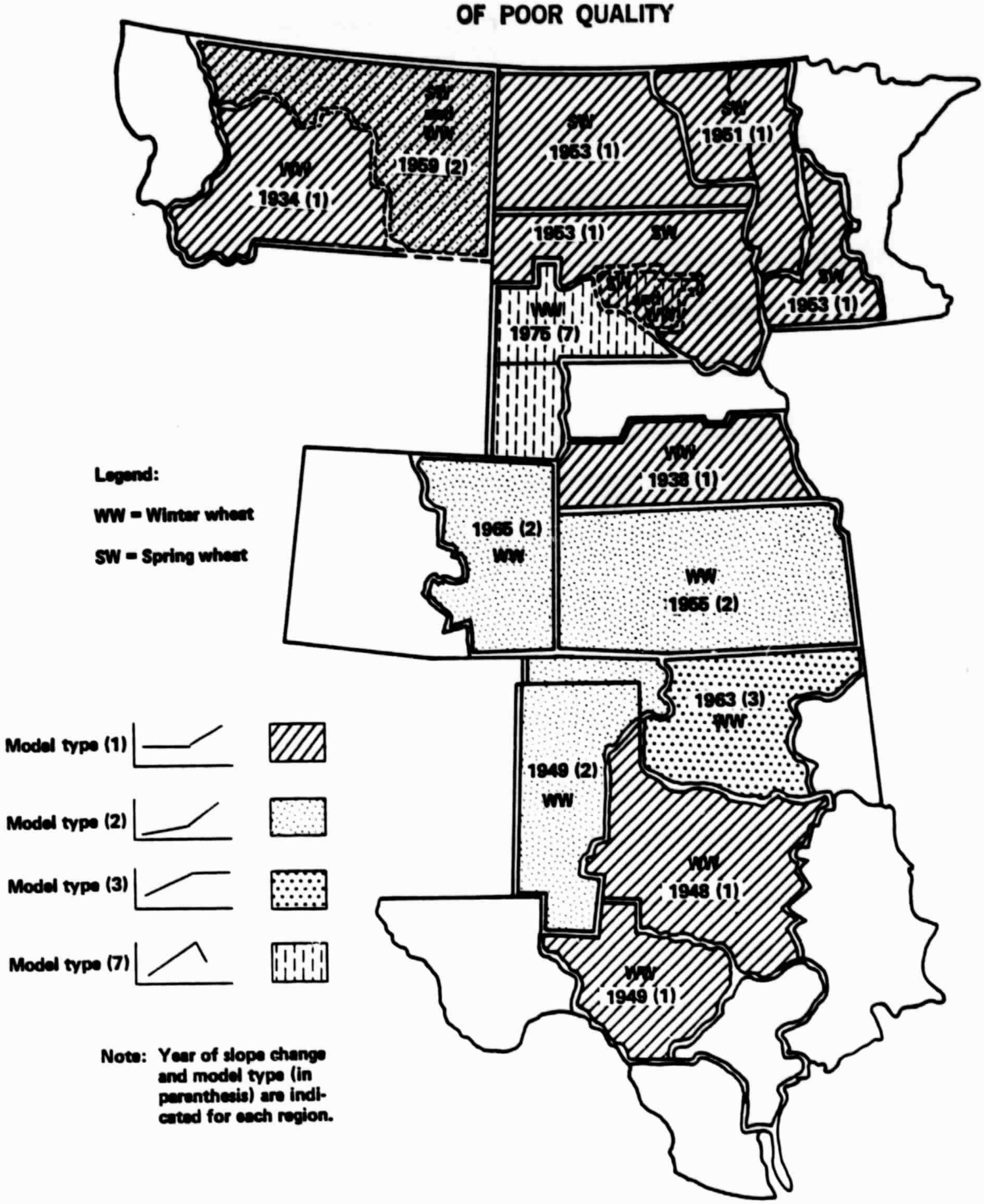

Figure 4.- Geographical distribution of the four types of dependent models.

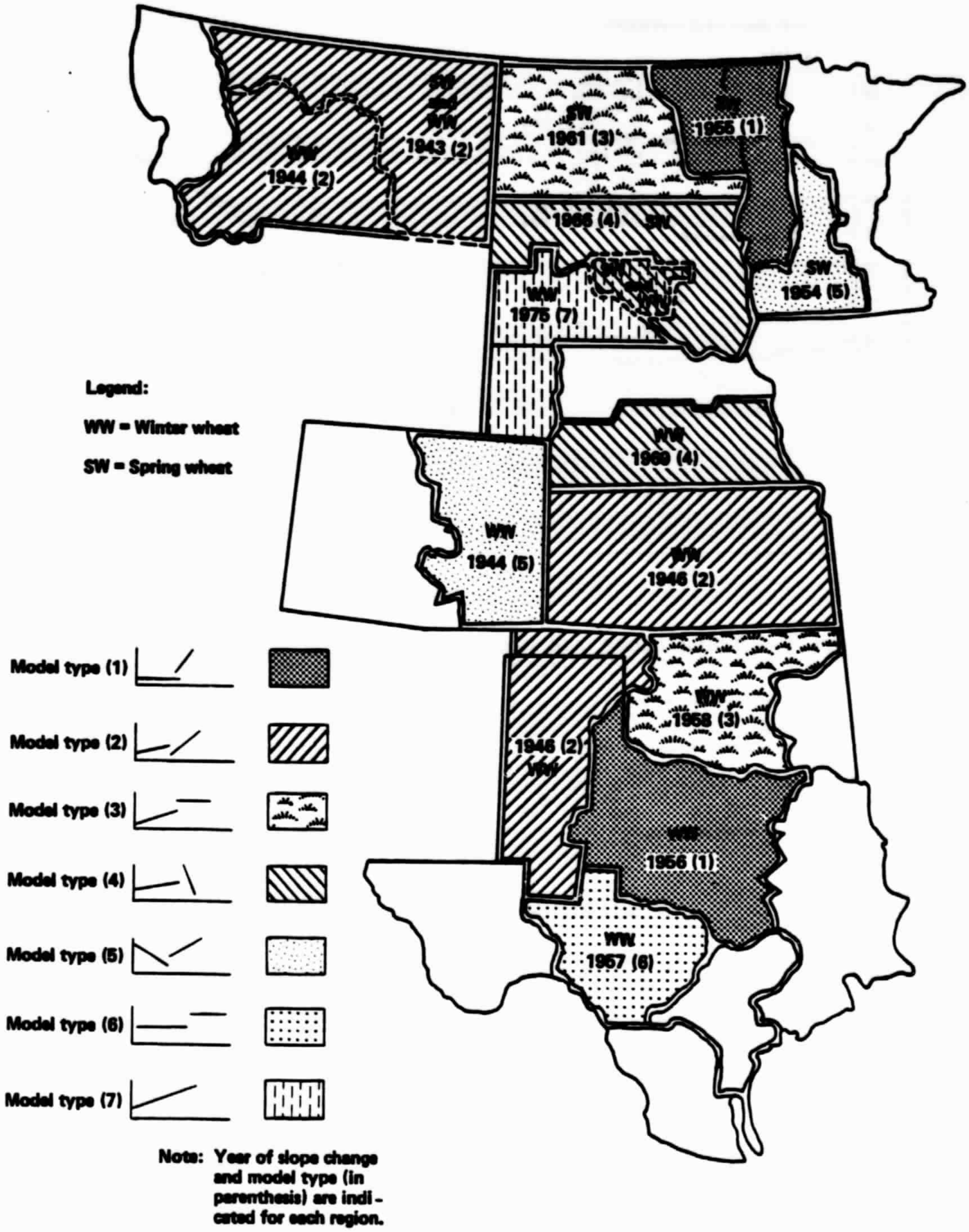

Figure 5.- Geographical distribution of the seven types of independent models.

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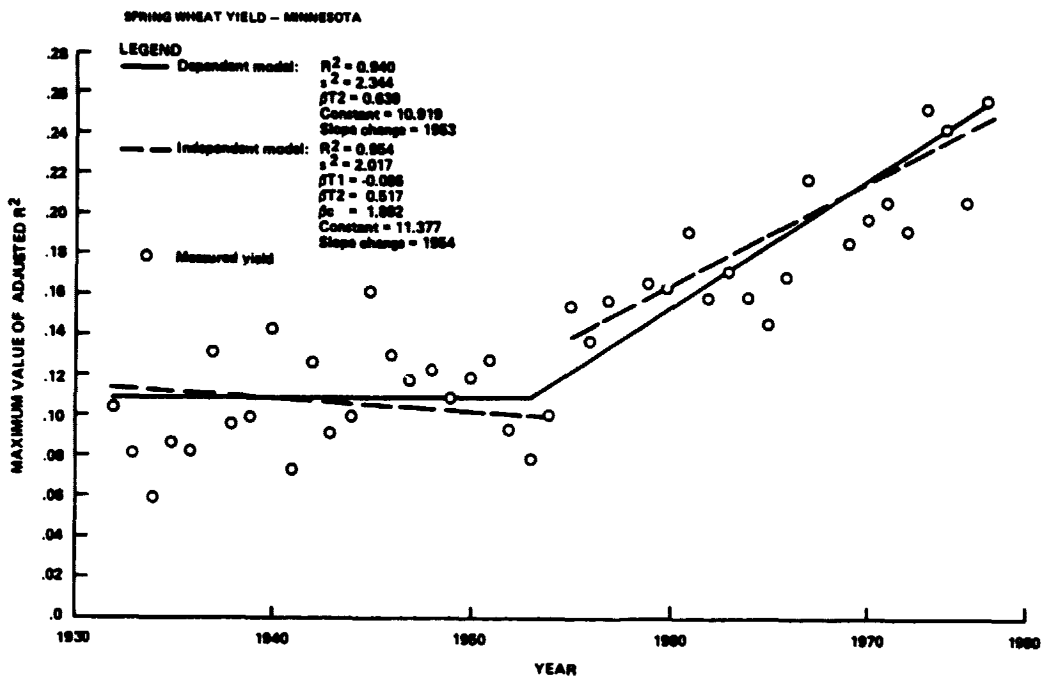

Figure 6.- Trend Lines for models from SELECT-BEIRA of spring wheat yield in Minnesota.

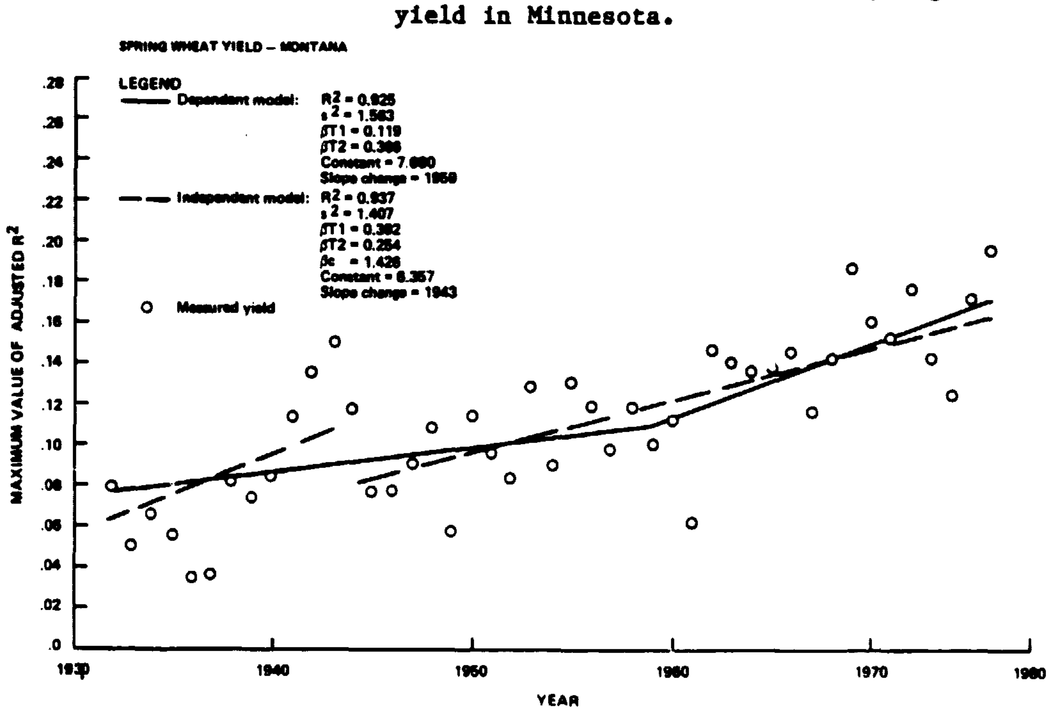

Figure 7.- Trend lines for models from SELECT-BEIRA of spring wheat yield in Montana.

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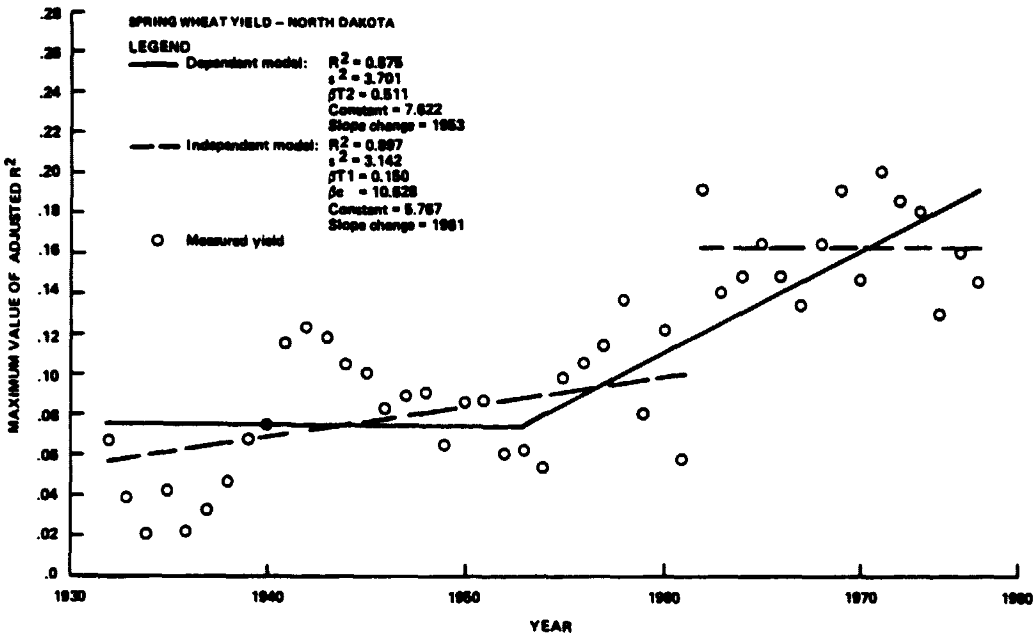

Figure 8.- Trend lines for models from SELECT-BEIRA of spring wheat yield in North Dakota.

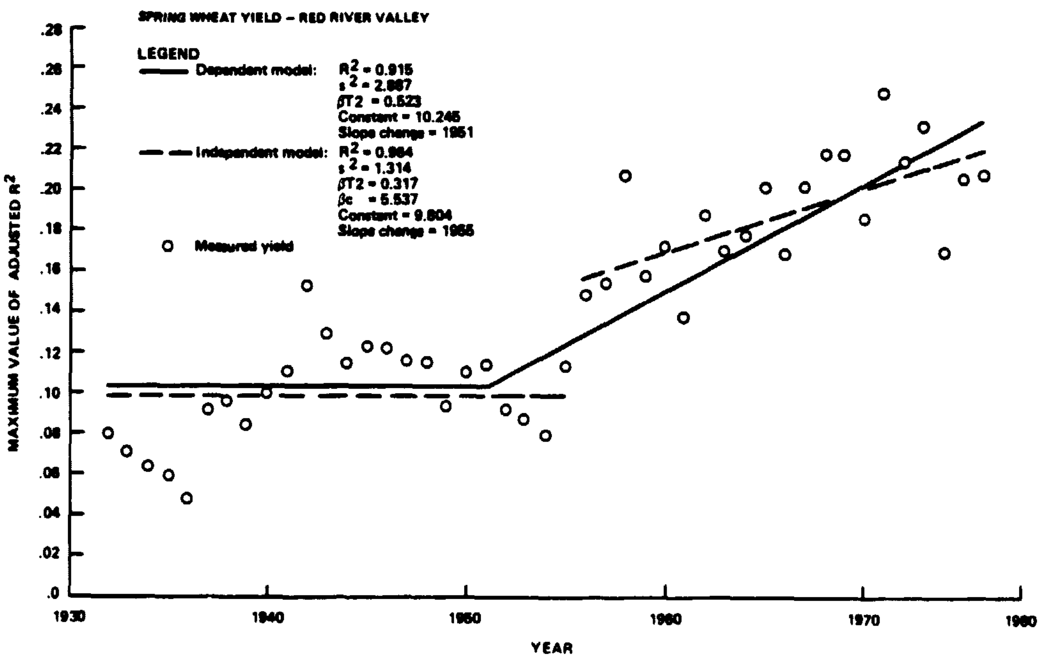

Figure 9.- Trend lines for models from SELECT-BEIRA of spring wheat yield in the Red River Valley.

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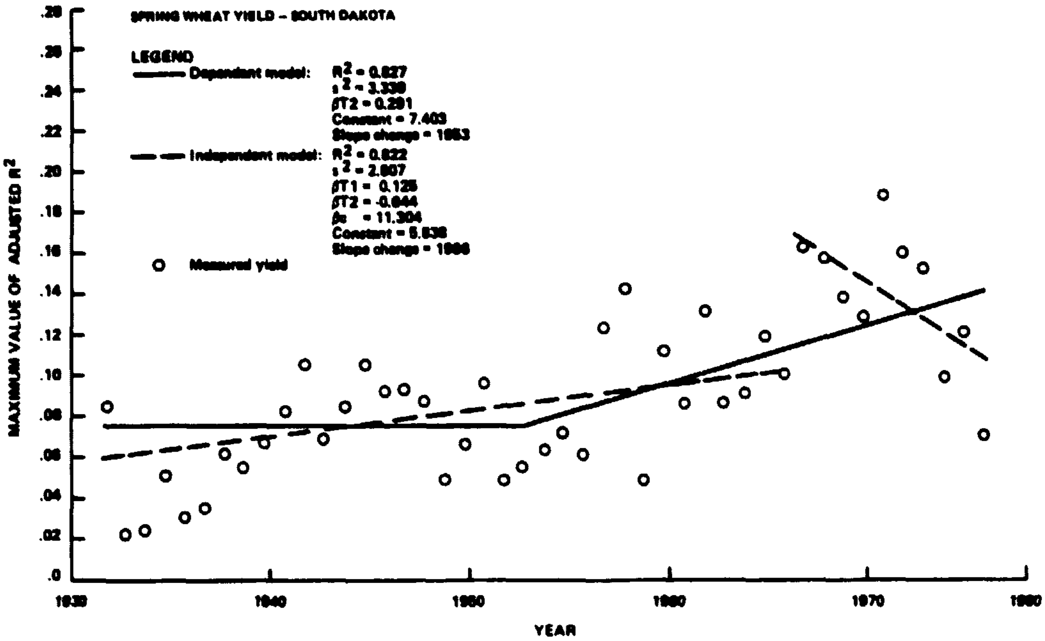

Figure 10.- Trend lines for models from SELECT-BEIRA of spring wheat yield in South Dakota.

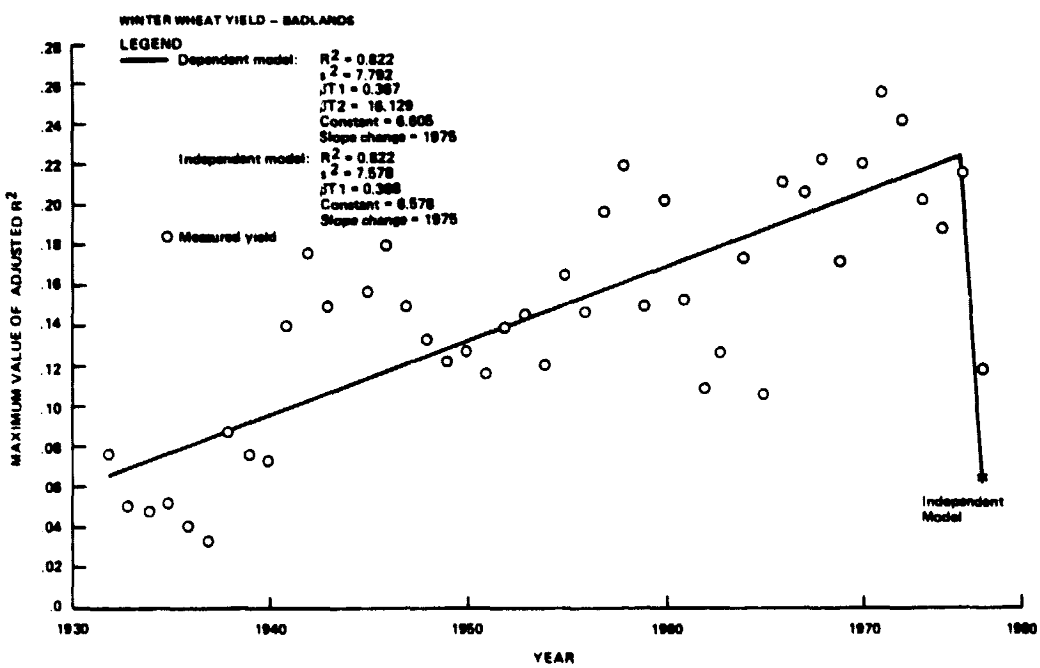

Figure 11.- Trend lines for models from SELECT-BEIRA of winter wheat yield in the Badlands.

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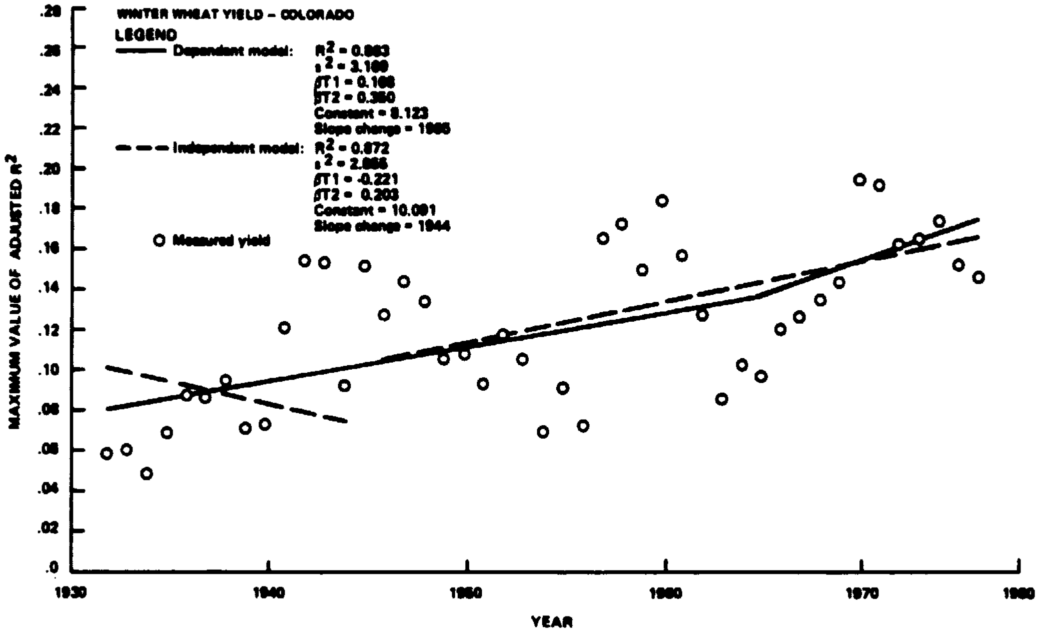

Figure 12.- Trend lines of models from SELECT-BEIRA of winter wheat yield in Colorado.

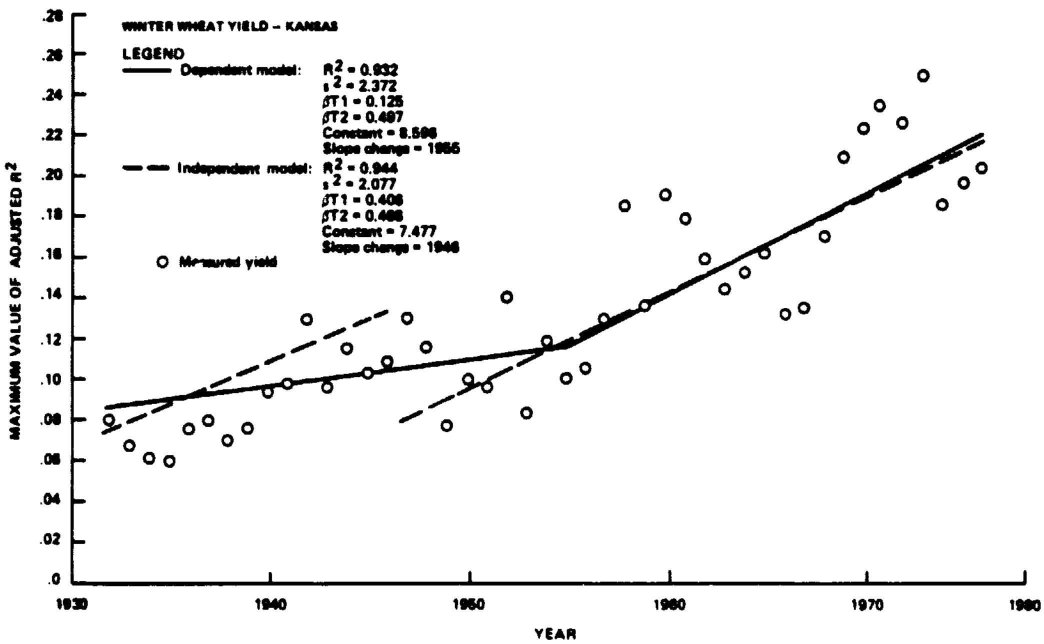

Figure 13.- Trend lines of models from SELECT-BEIRA of winter wheat yield in Kansas.

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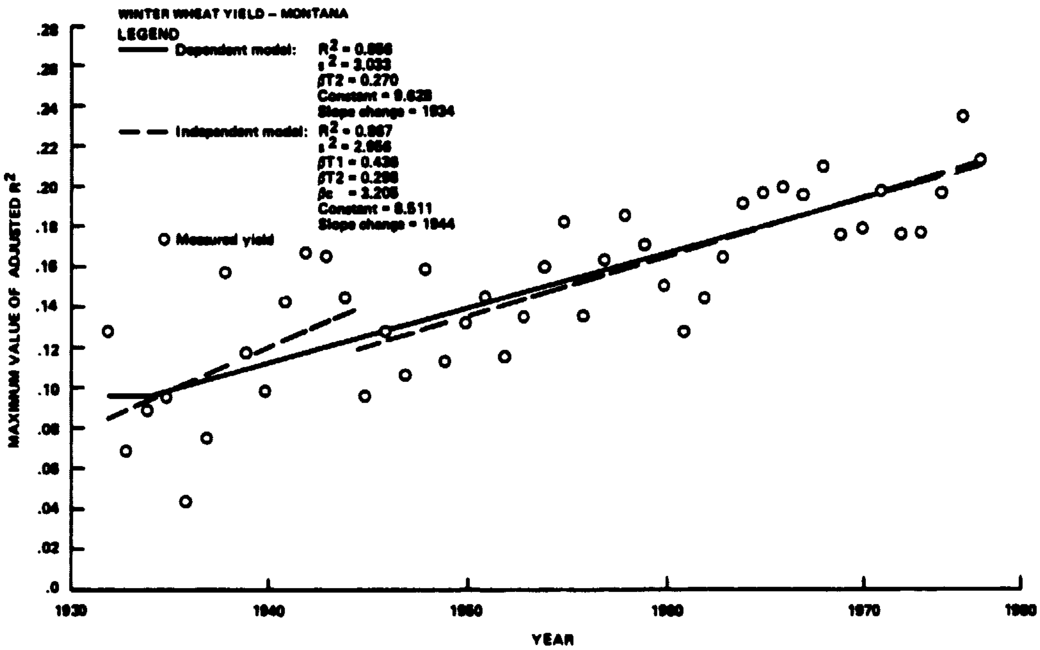

Figure 14.- Trend lines of models from SELECT-BEIRA of winter wheat yield in Montana.

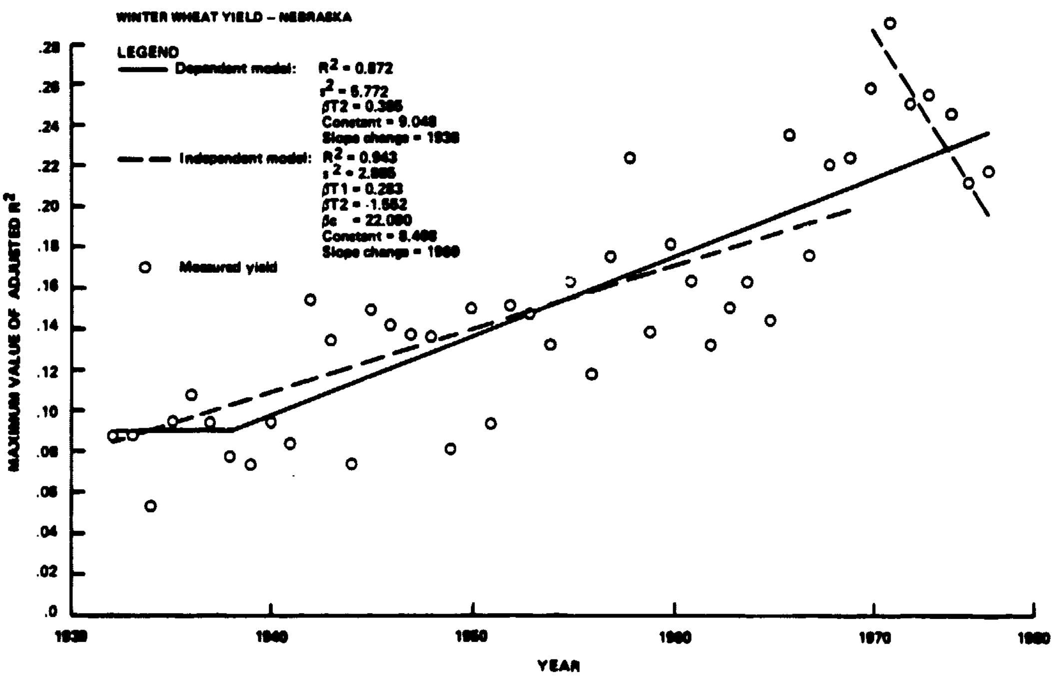

Figure 15.- Trend lines for models from SELECT-BEIRA of winter wheat yield in Nebraska.

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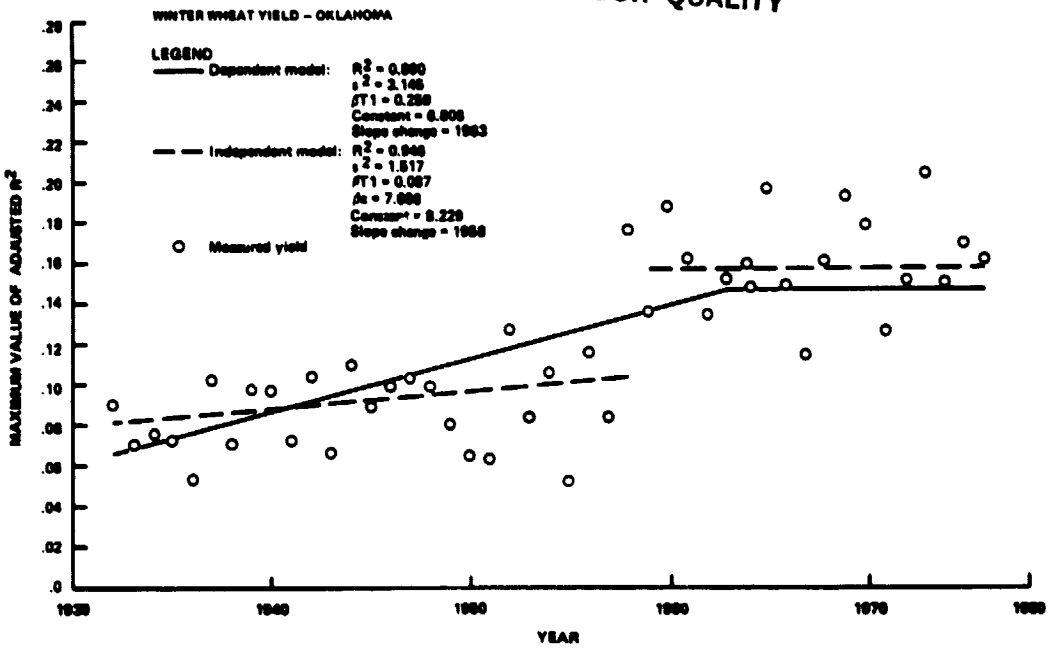

Figure 16.- Trend lines for models from SELECT-BEIRA of winter wheat yield in Oklahoma.

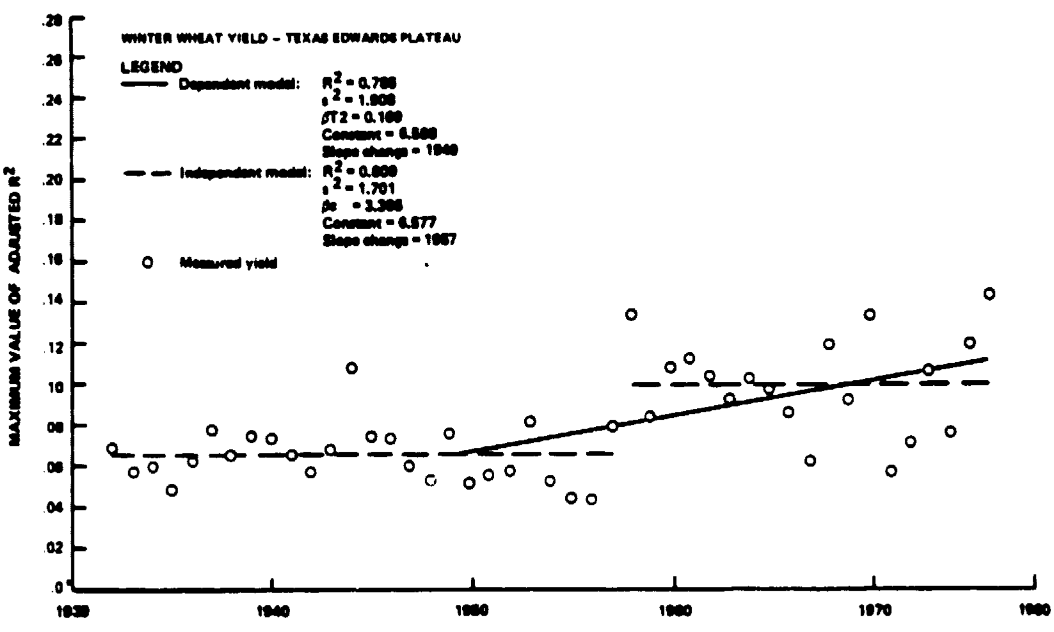

Figure 17.- Trend lines for models from SELECT-BEIRA of winter wheat yield in the Texas Edwards Plateau.

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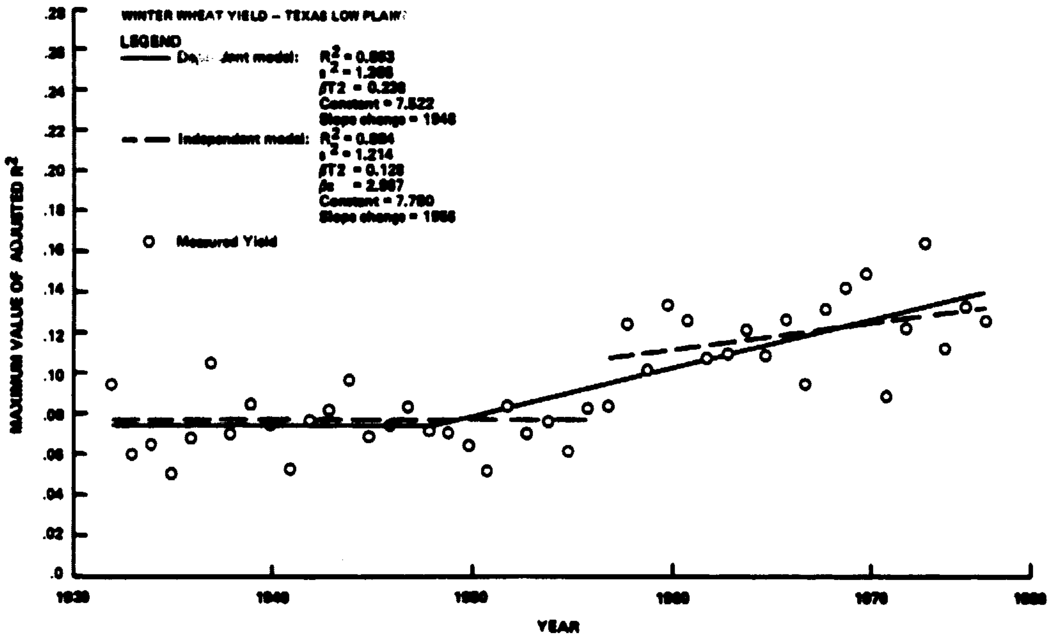


Figure 18.- Trend lines for models from SELECT-BEIRA of winter wheat yield in the Texas Low Plains.

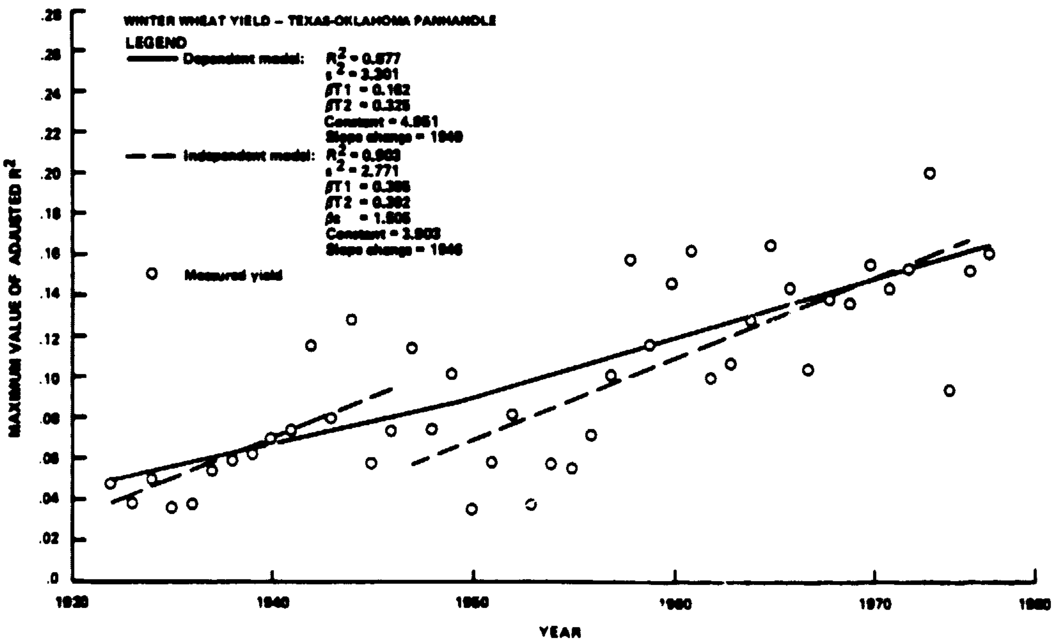


Figure 19.- Trend lines for models from SELECT-BEIRA of winter wheat yield in the Texas-Oklahoma Panhandle.

REFERENCES:

1. Wheat Yield Models for the United States, CCEA Technical Note 77-1, LACIE-00431 (Rev. A), June 1977.

2. Sakamoto, C. M., Reanalysis of CCEA I U.S. Great Plains Wheat Yield Models, FY78-79 LACIE Transition Project Report, LACIE-00472, JSC-13740, June 1978.

3. Marquina, N., Regressions by Leaps and Bounds and Biased Estimation Techniques in Yield Modeling, Technical Memorandum JSC-12700, February 1979.

4. Furnival, G. M., and Wilson, R. W., Regression by Leaps and Bounds, Technometrics, Vol. 16, 1974, pp. 499-511.

5. Draper, N. R., and Smith, H., Applied Regression Analysis, John Wiley & Sons, New York, 1966.

6. Massey, William F., Principal Components in Exploratory Statistical Research, American Statistical Association Journal, Vol. 60, March 1965, pp. 234-256.

7. Webster, J. A., Gunst, R. F., and Mason, R. L., Latent Root Regression Analysis, Technometrics, Vol. 16, No. 4., November 1974, pp. 513-522.

8. Hoerl, Arthur E., and Kennard, Robert W., Ridge Regression: Biased Estimation for Nonorthogonal Problems, Technometrics, Vol. 12, No. 1, February 1970, pp. 55-68.

9. Hoerl, Arthur E., and Kennard, Robert W., Ridge Regression: Application to Nonorthogonal Problems, Technometrics, Vol. 12, No. 1, February 1970, pp. 69-82.

10. Hocking, R. R., The Analysis and Selection of Variables in Linear Regression, Biometrics, 32, March 1976, pp. 1-49.

APPENDIX A

PROGRAMS SELECT AND BEIRA AND TECHNIQUES FOR ELIMINATING MULTICOLLINEARITIES

APPENDIX A

PROGRAMS SELECT AND BEIRA, AND TECHNIQUES FOR ELIMINATING MULTICOLLINEARITIES

Programs SELECT and BEIRA were written by Marquina (3) and modified by JSC personnel. Both programs use the same input variables: in the present analysis, the year, selected meteorological variables, and the yield. Both programs perform calculations for a multiple linear regression varying about either a constant followed by an upward trend, or two upward trends. The year of the slope change is specified by the user. SELECT finds the best regressions for subsets of each size from one through the number of independent variables. BEIRA performs the regressions and provides information for use in identifying and removing multicollinearities.

The yield for a given year may be expressed

$$y = C + \beta_{T1}x_{T1} + \beta_{T2}x_{T2} + \beta_c x_c + \sum_{i=1}^{m} \beta_i x_i + \varepsilon \qquad \text{(A-1)}$$

where y is the yield,

C is an overall constant,

β_{T1} and β_{T2} are the coefficients for the two trend variables, x_{T1} and x_{T2},

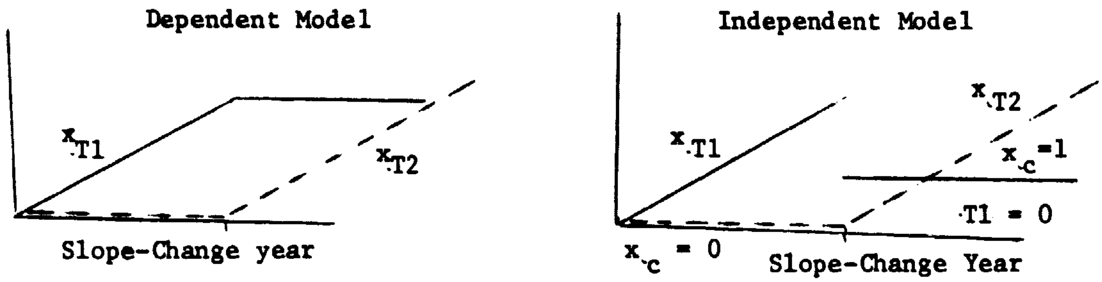
β_c is the coefficient for the constant variable, x_c,

m is the number of independent variables in addition to the trend and constant variables, and

β_i is the coefficient of the ith variable, x_i.

ε is the random error.

Trend and constant variables generated by the program are illustrated in the diagram. Both trend variables start at zero and increase by one each year. x_{T1} increases from the beginning of the data set. x_{T2} is zero through the slope change year, then increases by one each year. In the dependent model x_{T1} remains constant after the year of the slope change; the constant variable is not used. In the independent model x_{T1} becomes zero after the slope change; x_c is zero through the slope change year and one afterwards. In the models with one trend, x_{T1} becomes the constant variable for the independent model, and takes the form of x_c in the diagram. x_{T2}, the trend variable, is 1 before the slope change for for the dependent model. For the independent model x_{T2} is as shown. x_c is not used.



TREND AND CONSTANT VARIABLES FOR SELECT AND BEIRA

Program SELECT:

SELECT uses the method of leaps and bounds described by Furnival and Wilson (4). For each case SELECT finds the best regression for subsets of each size from one through the number of independent variables. Three options are available as criteria for determining the best regression: R^2, adjusted R^2, and Mallow's C_p. The adjusted R^2 used in this study, is given by

$$\overline{R^2_m} = 1 - [1 - R^2] \left[\frac{n}{n - m} \right] \qquad \text{(A-2)}$$

where m is the number of variables included in the regression, n is the number of observations and R^2 is given by

$$R^2 = \frac{\Sigma(\hat{y}_i - \bar{y})^2}{\Sigma (y_i - \bar{y})^2} \qquad \text{(A-3)}$$

$\hat{y}_i$ is the predicted yield for a given year, y_i is the measured yield and $\bar{y}$ is the mean.

Program BEIRA and Ordinary Least Squares:

Program BEIRA performs the regressions using the method of ordinary least squares. The yield may be described as a function of the independent variables,

$$Y = \beta_o + \sum_{i=1}^{p} \beta_i x_i + \varepsilon \qquad \text{(A-4)}$$

where Y is the yield, β_o is a constant, the β_i's are the coefficients of the p independent variables, x_i, and ε is the random error. Using vector notation the model for yield may be written

$$Y = \beta_o + X\beta + \varepsilon \tag{A-5}$$

where Y is an $n \times 1$ vector of yield measurements, β_o is a $p \times 1$ vector of equal constants, X is an $n \times p$ matrix of measurements of independent variables, β is a $p \times 1$ vector of regression coefficients and ε is an $n \times 1$ vector of random errors. The least squares estimate is found by minimizing $\varepsilon'\varepsilon$ with respect to β where ε' is the transpose of ε. The solution of the resulting expression

$$X'X\beta = X'Y$$

is given by

$$b = (X'X)^{-1}X'Y \tag{A-6}$$

where b is the vector of estimates of the true values of the β's. The matrix $X'X$ is the matrix of the simple correlation coefficients of the independent variables.

In performing the regression, program BEIRA first standardizes the variables by calculating

$$x^*_{ij} = \frac{x_{ij} - \bar{x}_j}{\sqrt{\sum_{i=1}^{n} (x_{ij} - \bar{x}_j)^2}} \tag{A-7}$$

Then the regression coefficients are calculated. The resulting regression equation describes yield as a function of the deviations of the independent variables from their means, except those representing the constant and trends. For the case with two trends it may be written

$$y_i = e_i + b_c x_c + b_{T1} x_{T1} + b_{T2} x_{T2} + \sum_{j=1}^{p} b_j (x_{ij} - \bar{x}_j) \qquad i = 1,\ldots,n \tag{A-8}$$

b_c, b_{T1}, and b_{T2} are the coefficients of the constant and trend variables x_c, x_{T1}, and x_{T2}, p is the number of independent variables, and n is the number of observations.

The constant, C, is evaluated by substituting the average values of the yield and independent variables into equation A-8. Since the average of the deviations from the means is zero, the constant becomes

$$C = \bar{y} - (b_{T1}\bar{x}_{T1} + b_{T2}\bar{x}_{T2} + b_c\bar{x}_c) \tag{A-9}$$

For the standardized variables

$$\sum_{i=1}^{n} x^*_{ij} = 0 \quad \text{and} \quad \sum_{i=1}^{n} x^{*\,2}_{ij} = 1 \quad \text{for } j = 1, 2, \ldots, p$$

R^2-goodness of fit for each regression is calculated using the vector expression

$$R^2 = \frac{b'X'Y}{Y'Y} \tag{A-10}$$

The residual variance, S^2, is calculated from

$$S^2 = \frac{Y'Y - b'X'Y}{n-p} \tag{A-11}$$

Predicted values of the yield for each year are calculated using equation A-8. Differences between the predicted and observed yield, $\Delta = \hat{y}_i - y_i$, are calculated for each observation. Plots are generated showing the observed and predicted values and the contributions of the constant and trend term to the regression equation.

BEIRA also prints out the correlation matrix for the independent and dependent variables, the LRR eigenvalues and eigenvectors, and the eigenvalues of the correlation matrix of the independent variables.

Principal Components Regression:

In principal components regression the dependent variables are transformed to their principal components by multiplying the data matrix, X, by the matrix of eigenvectors of X´X, the correlation matrix of the independent variables. The transformation is given by

$$Z = XS$$

where S is an orthogonal matrix whose columns are the eigenvectors of X´X. Then the columns of Z are the principal components of X. The regression equation is now a function of the principal components of X rather than of the original independent variables. The regression coefficients are given by

$$g = (Z'Z)^{-1}Z'Y = L^{-1}Z'Y \tag{A-12}$$

where g is the column matrix of regression coefficients, and Y is the column matrix of the values of the dependent variable. L is a diagonal matrix with the eigenvalues of $X'X$ on the diagonal. If all the components are retained in the model, transformation from g back to b will result in coefficients identical to those obtained using equation A-6.

Components are deleted from the regression to overcome the effects of multicollinearities. Then a least squares regression is performed on the remaining components. Two criteria are usually considered in deciding which components should be deleted.

1. Components associated with small eigenvalues are deleted.

2. Components are deleted which are relatively unimportant as predictors of the dependent variable.

Criterion 1 leads to deletion of variables which are relatively unimportant as predictors of the original independent variables. The remaining variables, which are important as predictors of the independent variables, are not necessarily highly correlated with the dependent variable.

Latent Root Regression:

Latent root regression examines the relations between the independent variables and the dependent variable. Latent roots (eigenvalues) and latent vectors (eigenvectors) of the extended correlation matrix (including the independent and dependent variables) are examined to

1. Identify multicollinearities among the independent variables, and

2. Determine whether the multicollinearities contribute to prediction of the dependent variable.

Let A be the matrix of the independent and dependent variables. $A'A$ is the extended correlation matrix. The jth eigenvalue of $A'A$ can be expressed

$$\lambda_j = \sum_{i=1}^{n} (y_i E_{0j} + \sum_{k=1}^{p} x_{ik} E_{kj})^2 \quad \text{(A-13)}$$

where $E_j = (E_{0j}, E_{1j}, \ldots, E_{nj})'$ is the jth eigenvector of $A'A$ corresponding to λ_j. E_{0j} is the component of E_j in the direction of the dependent variable. If $\lambda_j = 0$ for any value of j each term in equation (A-13) is equal to zero, and an exact linear relationship exists among some or all of the columns of A. If $\lambda_j = 0$ and

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$E_{0j} = 0$, equation (A-13) becomes

$$\sum_{k=1}^{p} x_{ik}E_{kj} = 0 \quad \text{for } i=1, 2, \ldots, n.$$

indicating an exact linear relationship (multicollinearity) among the columns of X, the matrix of the observed values of the independent variables. Small but nonzero eigenvalues of $A'A$ indicate near singularities. From equation (A-13)

$$y_i E_{0j} + \sum_{k=1}^{p} x_{ik}E_{kj} \approx 0 \quad \text{for } i = 1, 2, \ldots, n \qquad \text{(A-14)}$$

If E_{0j}, the component of the jth eigenvector in the direction of the dependent variable, is also small the relation becomes

$$x_{ik}E_{kj} \approx 0.$$

The dependent variable is not involved, indicating that the multicollinearity is nonpredictive. The variables involved have little or no effect on the dependent variable. Variables contributing large components to E_{0j} can be examined to determine which should be eliminated from the regression. If $\lambda_j = 0$ and E_{0j} is not small, then the multicollinearity is predictive: the dependent variable is included in the relationships indicated by the components of the eigenvector.

Program BEIRA prints the components of the first five eigenvectors of $A'A$ and the corresponding eigenvalues. When it is found that an eigenvalue and the eigenvector component in the y direction are both small, the correction coefficients for the variables with large eigenvector components may be printed. Then, of two or more variables highly correlated with each other, the one least correlated with the dependent variable can be eliminated. An example is given in Appendix C.

Ridge Regression

If $X'X$ has a nonuniform eigenvalue spectrum the regression coefficients calculated using ordinary least squares may be far removed from the true values (8). In ridge regression bias is introduced into the ordinary least squares estimator to make a nonorthogonal data matrix act more like an orthogonal data matrix. The diagonal of the normal equations is augmented by a small positive quantity, which can prevent inflation of the regression coefficients. The ordinary ridge regression estimator is given by

$$b(k) = (X'X + kI)^{-1}X'Y, \quad \text{for } k \geq 0 \qquad \text{(A-15)}$$

where X and Y are matrices of the standardized independent and dependent variables (equation (A-7)), $X'X$ is the correlation matrix of the dependent variables and k is a small positive quantity. If $k = 0$, equation (A-15) becomes the ordinary least squares estimator, given by equation (A-6).

A transformation from variable space to eigenvector space is accomplished by letting $W = XP$ where P is the orthogonal matrix of the normalized eigenvectors. The model for the dependent variable is

$$y = Wa + e$$

where $a = P'b$ and e is the random error. b is the matrix of regression coefficients of equation (A-6). The regression coefficient matrix becomes the generalized regression estimator

$$a^* = (W'W + K)^{-1}W'y \qquad \text{(A-16)}$$

where K is a diagonal matrix of the eigenvalues of $X'X$.

When all the k_i's in equation (A-16) are zero a* is the ordinary least squares estimator. Program BEIRA calculates the ordinary least squares regression coefficients using equation (A-16) with $K = 0$.

The program prints eigenvalues of $R = X'X$, (the matrix of correlation coefficients of the independent variables), the value of the determinant of R, and the trace of R inverse, for use in evaluating the stability of the data matrix. The user may introduce bias into the regression by changing one of the eigenvalues of $X'X$, normally the smallest. The regression coefficients and the quantities indicating the stability of the data matrix are calculated using the new set of eigenvalues. An example is given in Appendix C.

APPENDIX B

INDEPENDENT VARIABLES USED IN THE REGRESSIONS

APPENDIX B

INDEPENDENT VARIABLES USED IN THE REGRESSIONS

The following tables and figures are in Appendix B.

- Table B-1: Independent variables, listed by number
- Table B-2: Variables chosen by SELECT for slope-change years of the CCEA models
- Figures B-1 through B-14: Plots of R^2_{adj} from SELECT, including subsets of variables for regressions for peak values of R^2_{adj}

TABLE B-1.- INDEPENDENT VARIABLES USED TO DEVELOP YIELD MODELS

SPRING WHEAT

Precipitation in mm

1. January
2. February
3. March
4. April
5. May
6. June
7. July
8. August

Precipitation the year before, in mm

9. August
10. September
11. October
12. November
13. December

Average Temperature in degrees C

14. April
15. May
16. June
17. July
18. August

19. First Trend
20. Second Trend
21. Constant (for the independent model)

WINTER WHEAT

Precipitation in mm

1. January
2. February
3. March
4. April
5. May
6. June

Precipitation the year before, in mm

7. August
8. September
9. October
10. November
11. December

Average Temperature in degrees C

12. January
13. February
14. March
15. April
16. May
17. June

P.E.T.[a]

18. January
19. February
20. March
21. April
22. May
23. June
24. First Trend
25. Second Trend
26. Constant (for the independent model)

[a]For the Badlands and Montana, January and February PET were not given; for Nebraska, January PET was not given. Variables for these areas were renumbered omitting these quantities.

TABLE B-2: VARIABLES CHOSEN BY SELECT FOR SLOPE-CHANGE YEARS OF THE CCEA MODELS

	Slope Change Year	Model	R^2_{adj}	No. Vars.		
SPRING WHEAT						
Minnesota	1955	D	.902	14	2,3,4,5,6,8,10,11,14,15,16,17,18,20	
		I	.914	15	2,4,5,6,8,9,10,11,14,15,16,17,18,20,21	
Montana	1955	D	.892	12	1,3,4,6,7,10,13,15,17,18,19,20	
		I	.889	13	1,3.4,6,7,10,13,15,17,18,19,20,21	
North Dakota	1955	D	.835	11	2,5,7,10,13,14,16,17,18,19,20	
		I	.856	11	2,3,5,6,7,10,14,16,17,20,21	3rd peak in R^2_{adj}
Red River Valley	1955	D	.877	7	1,13,14,16,17,19,20	
		I	.953	12	1,2,3,6,9,11,14,16,17,18,20,21	Highest peak in R^2_{adj}
South Dakota	1955	D	.780	8	1,4,10,12,13,16,17,20	
		I	.790	11	1,4,7,8,12,13,14,16,17,19,20	
WINTER WHEAT						
Badlands	1955	D	.710	11	1,4,7,9,10,13,17,18,21,22,23	4th peak in R^2_{adj}
		I	.711	11	1,4,7,9,10,13,17,18,21,22,24	
	1972	D	.764	12	3,4,7,8,9,10,12,13,15,19,22,23	2nd peak in R^2_{adj}
		I	.763	13	3,4,5,8,9,10,12,13,15,19,21,22,24	4th " " "
Colorado	1955	D	.820	17	1,2,3,4,5,8,9,11,13,16,17,19,20,22,23,24,25	near 4th peak in R^2_{adj}
		I	.816	15	1,2,4,5,6,7,9,13,15,16,17,20,22,23,25	
Kansas	1955	D	.912	14	1,2,3,4,5,6,7,9,11,14,18,20,24,25	Max. peak in R^2_{adj}
		I	.910	15	1,2,3,4,5,7,9,10,11,13,18,23,24,25,26	
	1943	D	.886	12	2,3,4,5,6,7,9,11,14,18,20,25	
		I	.900	15	2,3,4,5,6,9,10,11,14,15,18,20,21,24,25	
Montana	1943	D	.824	8	3,4,5,6,8,10,22,23	
		I	.830	9	3,4,5,6,8,10,22,23,24	
	1955	D	.825	8	3,4,5,6,8,10,22,23	
		I	.825	12	3,4,5,6,8,10,16,17,20,22,23,24	

TABLE B-2: (Continued)

	Slope Change Year	Model	R^2_{adj}	No. Vars.		
Nebraska	1955	D	.858	11	2,4,5,6,8,9,11,16,22,23,24	Near peak in R^2_{adj}
		I	.855	12	2,4,5,6,8,9,11,16,22,23,24,25	
Oklahoma	1943	D	.865	15	1,2,3,5,6,7,9,14,15,17,20,21,23,24,25	
		I	.868	20	2,3,4,5,6,7,9,10,11,12,14,15,16,17,18,19,20,21,25,26	
	1955	D	.871	20	1,2,3,4,5,6,7,9,10,12,14,15,16,17,18,19,20,21,24,25	2nd peak in R^2_{adj}
		I	.928	19	1,2,3,4,5,6,7,9,10,12,14,15,16,17,18,21,22,24,26	
	1960	D	.895	19	1,2,3,4,5,6,7,9,10,12,13,14,15,18,20,21,22,23,24	
		I	.895	19	1,2,3,4,5,6,7,9,10,12,14,15,18,20,21,22,23,24,26	
	1962	D	.898	19	1,2,3,4,5,6,7,9,10,12,13,14,15,18,20,21,22,23,24	near peak
		I	.894	19	1,2,3,4,5,6,7,9,10,12,14,15,18,20,21,22,23,24,26	
Texas Edwards Plateau	1955	D	.774	15	2,3,7,8,9,10,12,13,14,16,19,20,22,23,25	Max. peak in R^2_{adj}
		I	.775	17	1,2,3,7,8,9,10,12,13,14,16,19,20,22,23,25,26	
	1960	D	.747	18	1,2,3,6,8,9,12,13,14,15,16,17,18,19,21,22,24,25	
		I	.830	17	1,2,3,6,7,8,9,10,13,14,15,16,19,20,22,23,26	
	1965	D	.744	18	1,2,3,6,8,9,12,13,14,15,16,17,18,19,21,22,24,25	
		I	.736	18	1,2,3,6,8,9,12,13,14,15,16,17,18,19,21,22,24,26	
	1975	D	.759	13	1,2,8,9,10,12,13,16,19,22,23,24,25	
		I	.759	13	1,2,8,9,10,12,13,16,19,22,23,24,25	
Texas Low Plains	1955	D	.801	9	1,3,5,9,12,14,20,24,25	
		I	.858	13	1,3,4,5,9,12,14,17,18,22,23,25,26	
	1962	D	.775	9	1,3,5,11,12,14,20,24,25	
		I	.806	12	3,4,5,7,9,11,12,14,15,20,24,26	
Texas-Oklahoma Panhandle	1955	D	.831	12	3,4,6,11,13,14,16,17,18,19,24,25	
		I	.889	15	1,4,9,10,11,13,14,17,19,20,22,23,24,25,26	Max. peak in R^2_{adj}
	1960	D	.826	15	3,4,5,10,11,13,15,16,17,19,20,21,23,24,25	
		I	.875	15	3,4,10,11,13,14,15,16,17,18,19,21,23,24,26	
	1962	D	.833	16	3,4,5,7,10,11,13,15,16,17,19,20,21,22,23,24	
		I	.855	20	1,3,4,5,9,10,11,13,15,16,17,18,19,20,21,22,23,24,25,26	

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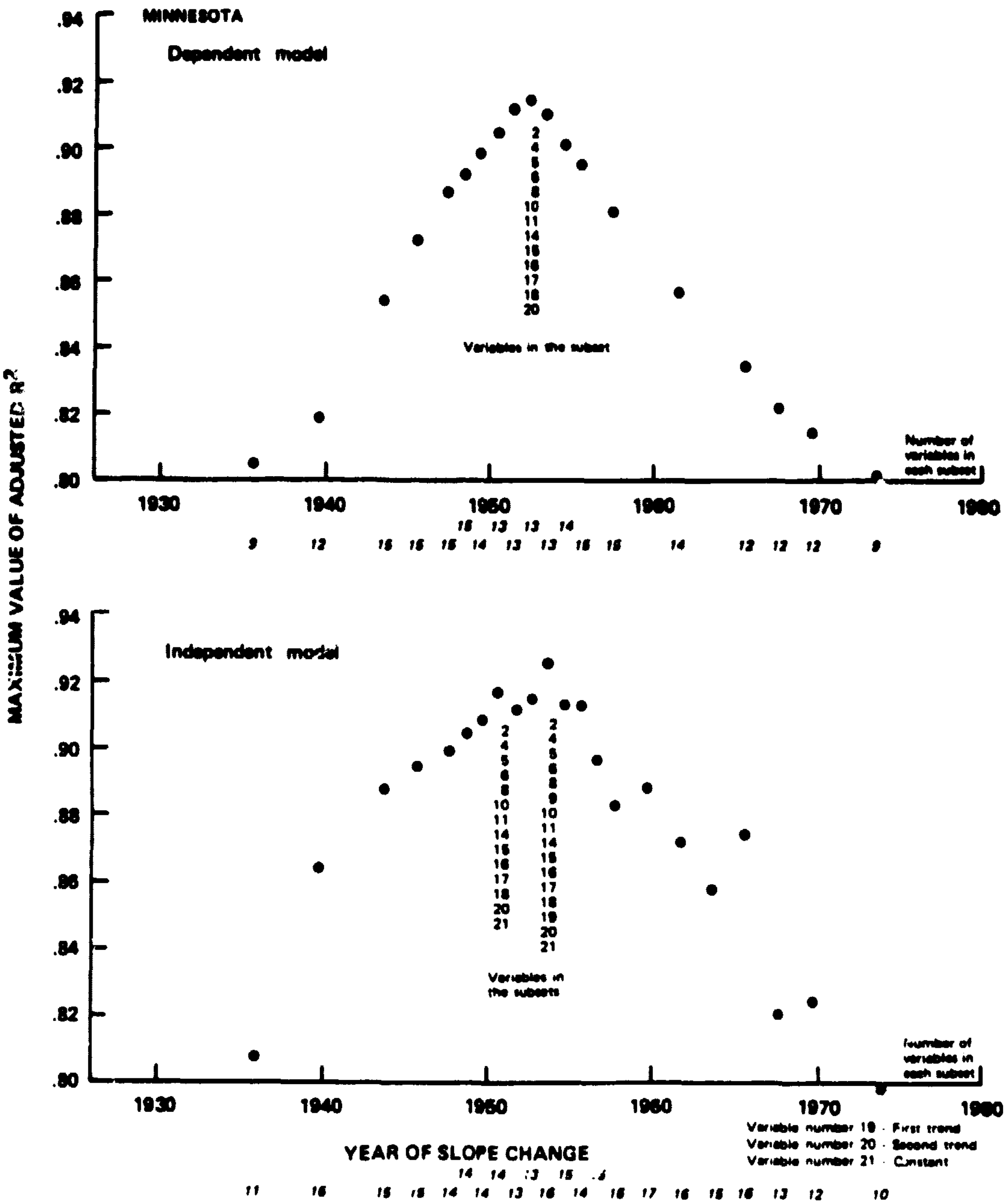

Figure B-1.- Maximum R^2_{adj} for subsets of variables from SELECT for models of spring wheat yield in Minnesota.

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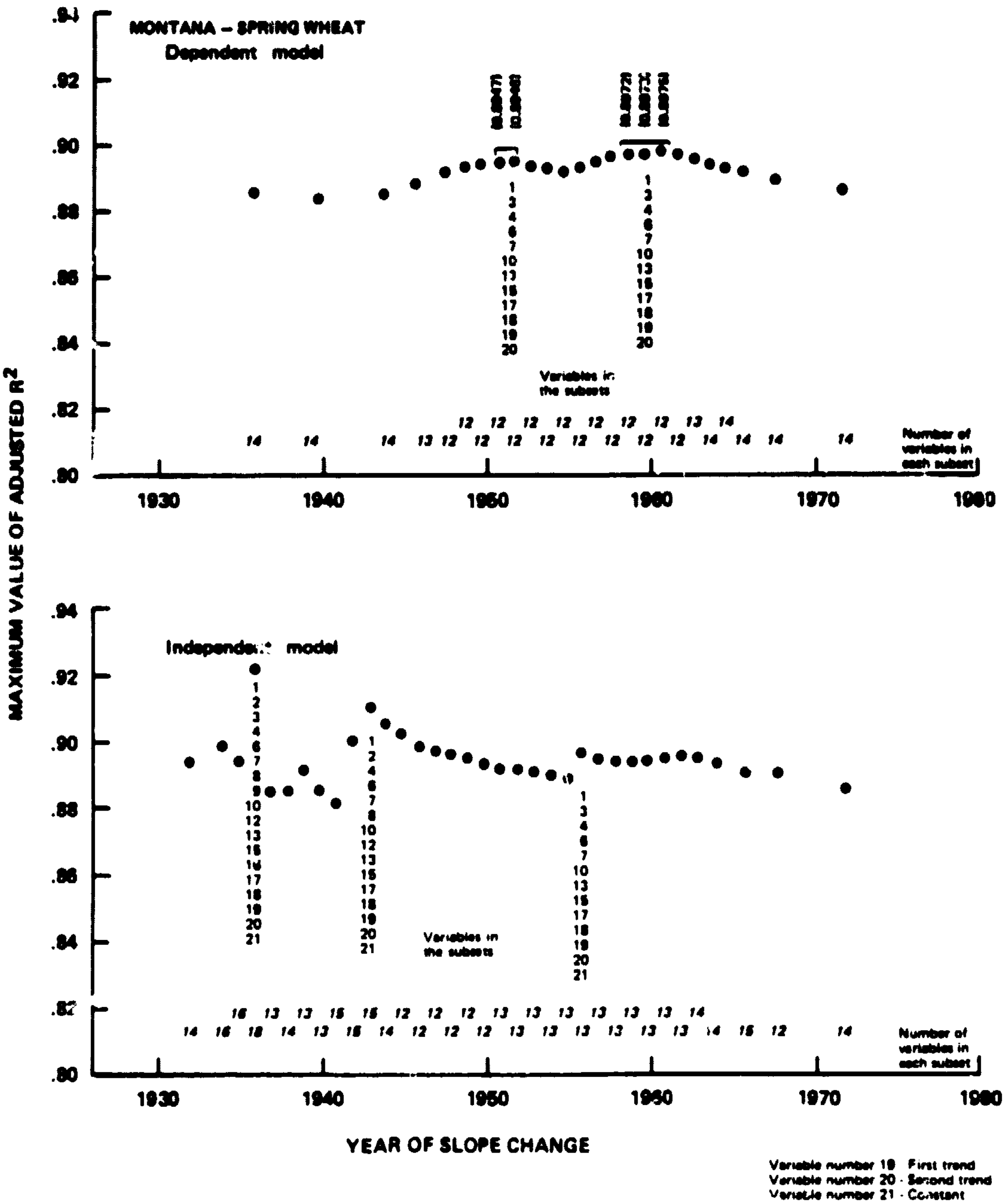

Figure B-2.– Maximum R^2_{adj} for subsets of variables from SELECT for models of spring wheat yield in Montana.

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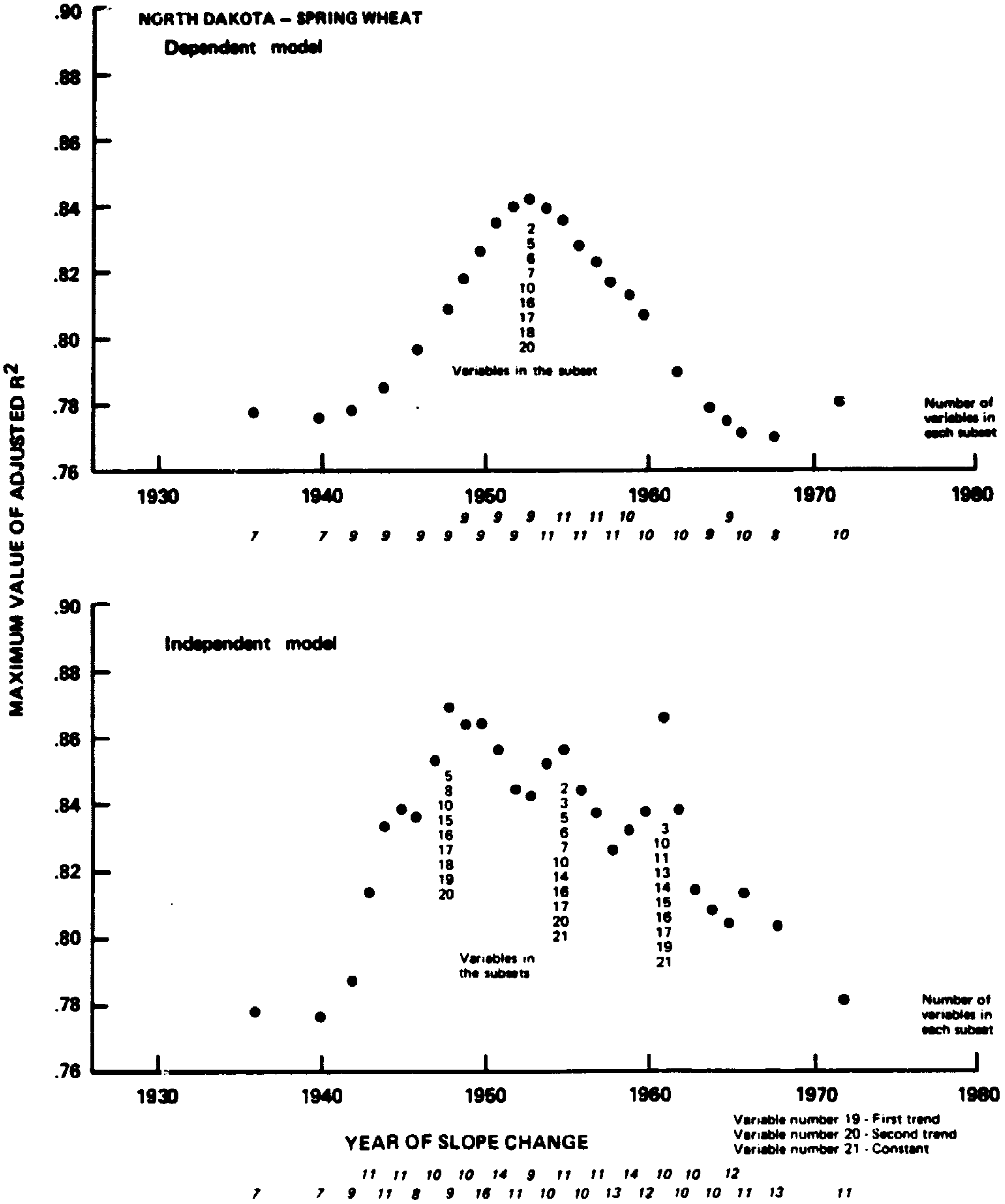

Figure B-3.- Maximum R^2_{adj} for subsets of variables from SELECT for models of spring wheat yield in North Dakota.

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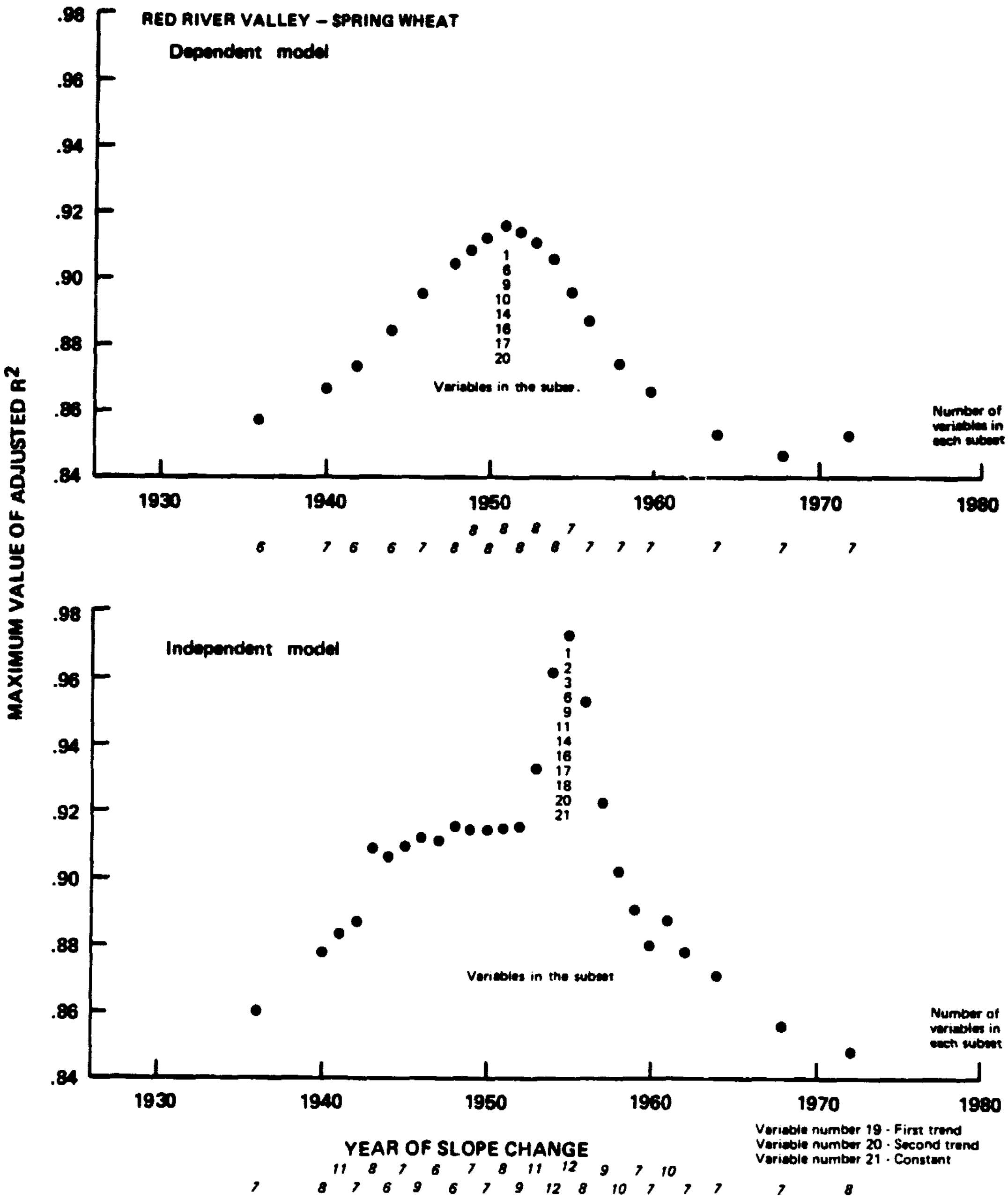

Figure B-4.- Maximum R^2_{adj} for subsets of variables from SELECT for models of spring wheat yield in the Red River Valley.

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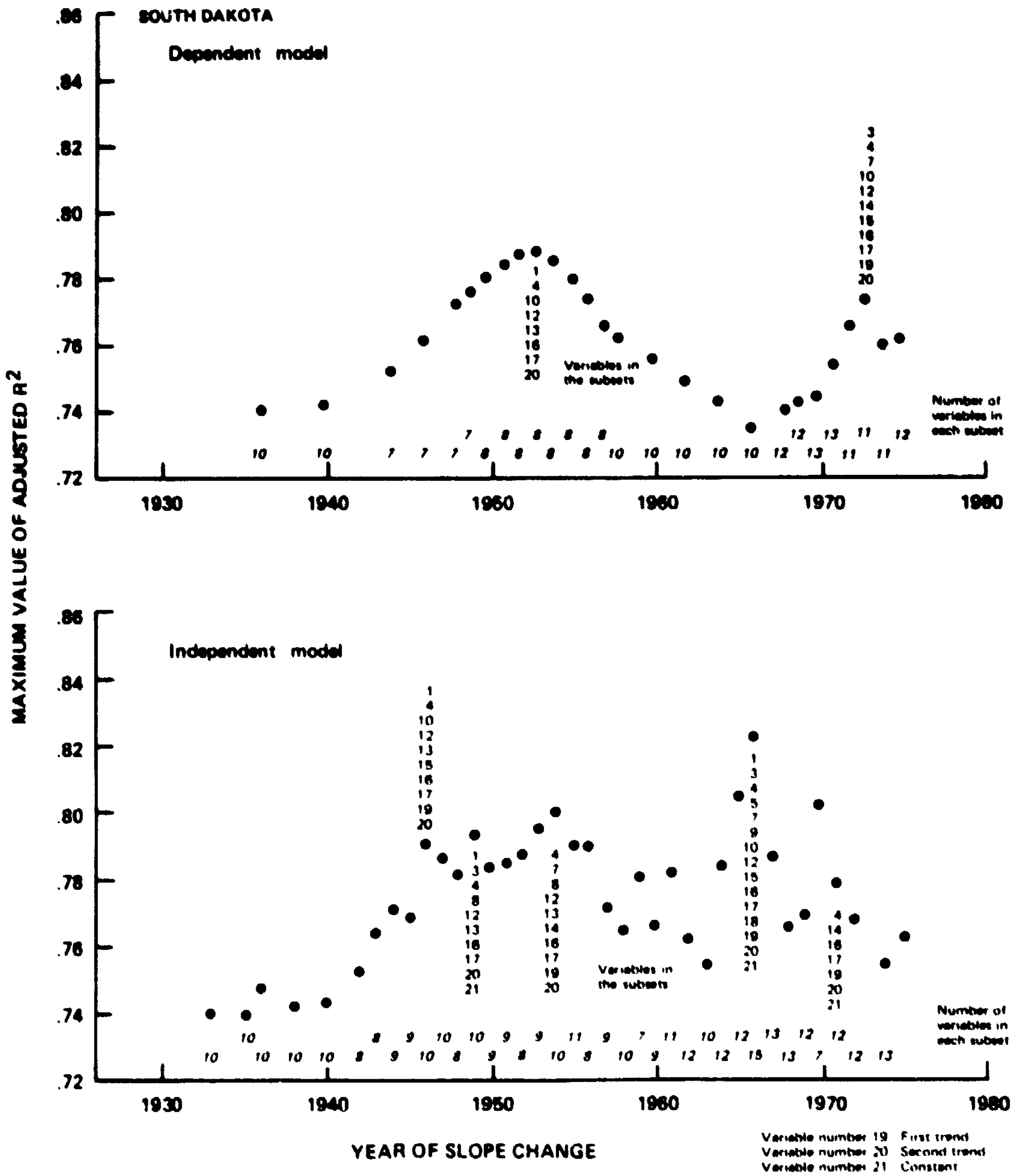

Figure B-5.- Maximum R^2_{adj} for subsets of variables from SELECT for models of spring wheat yield in South Dakota.

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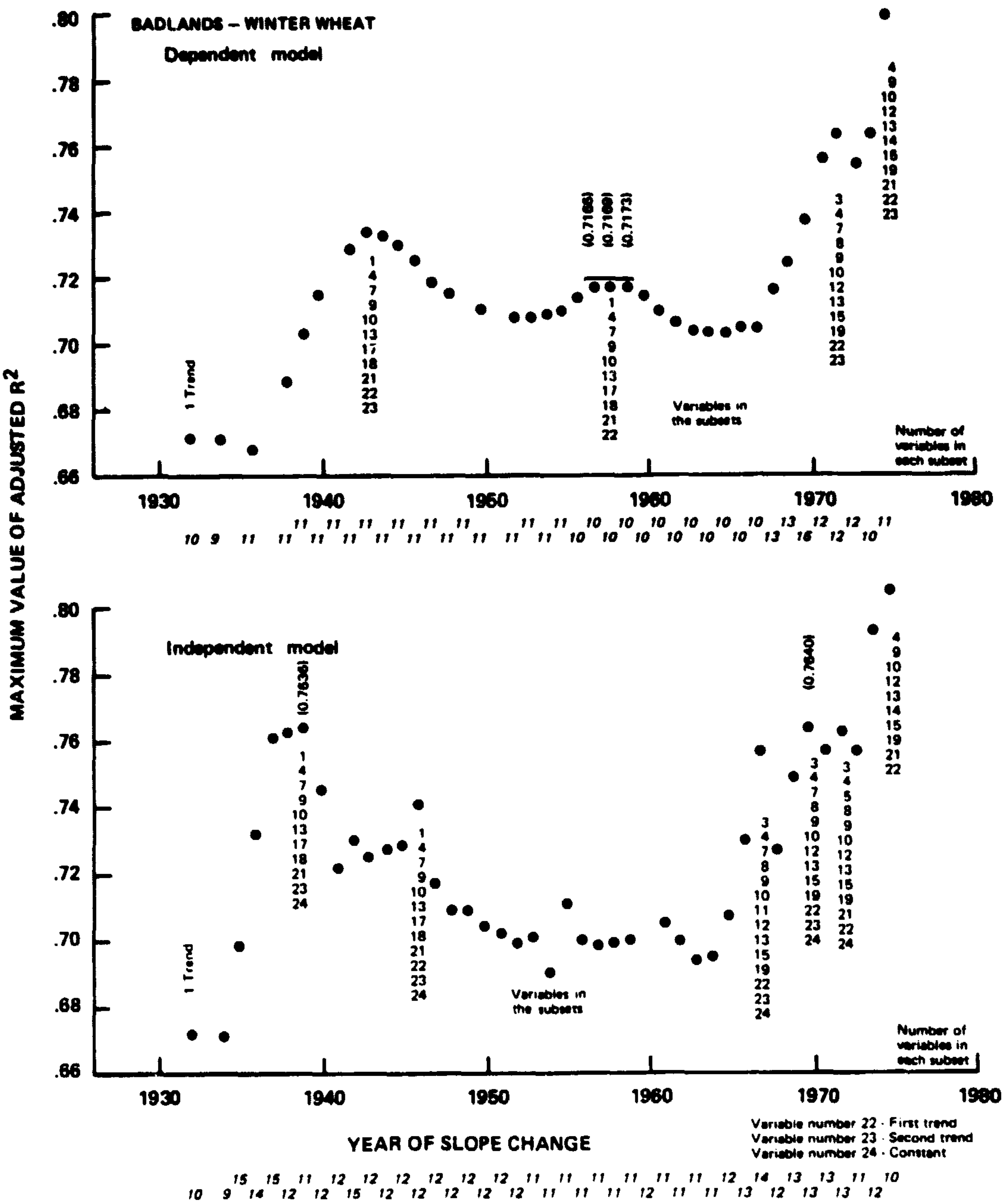

Figure B-6.- Maximum R^2_{adj} for subsets of variables from SELECT for models of winter wheat yield in the Badlands.

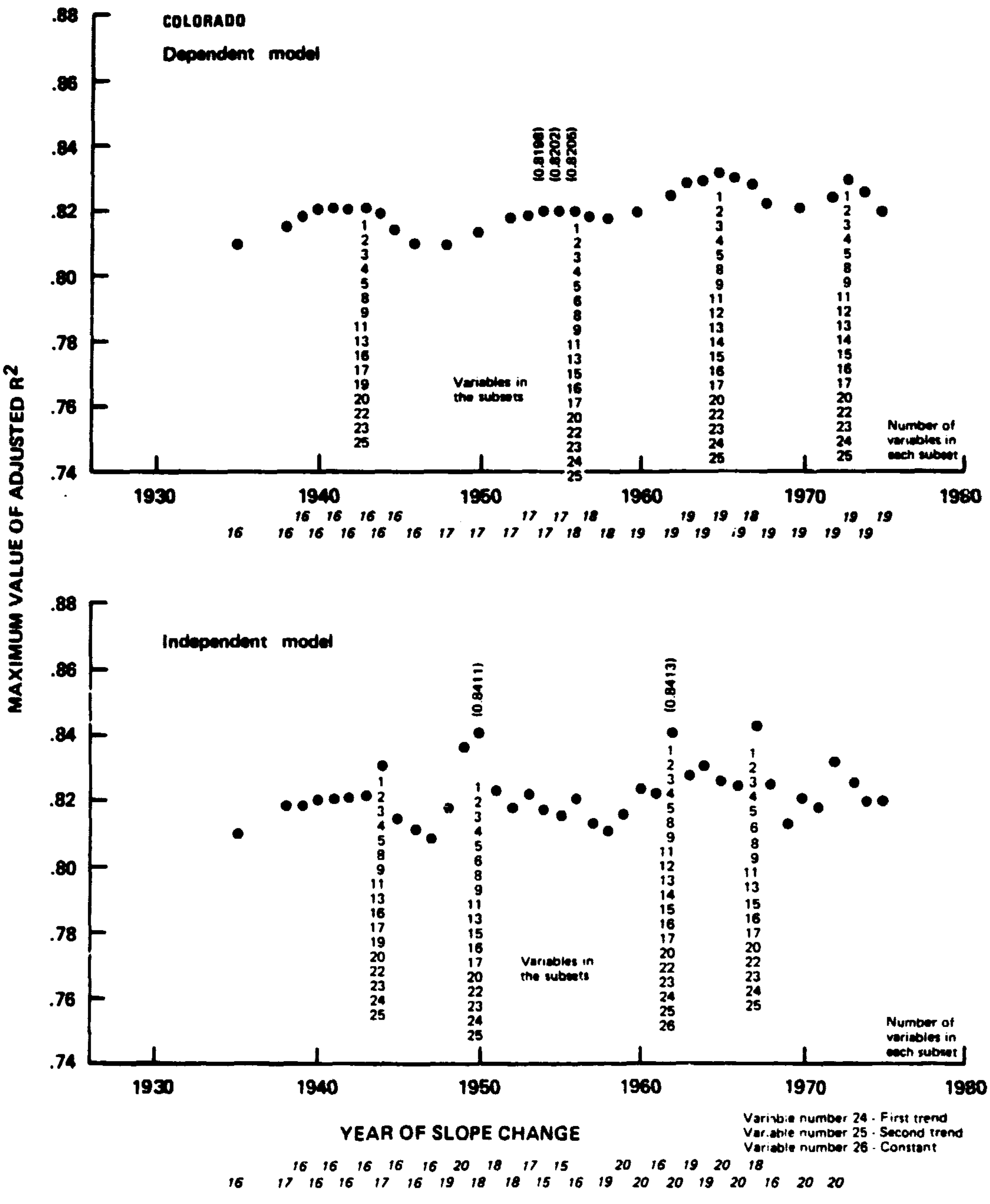

Figure B-7.- Maximum R^2_{adj} for subsets of variables from SELECT for models of winter wheat yield in Colorado.

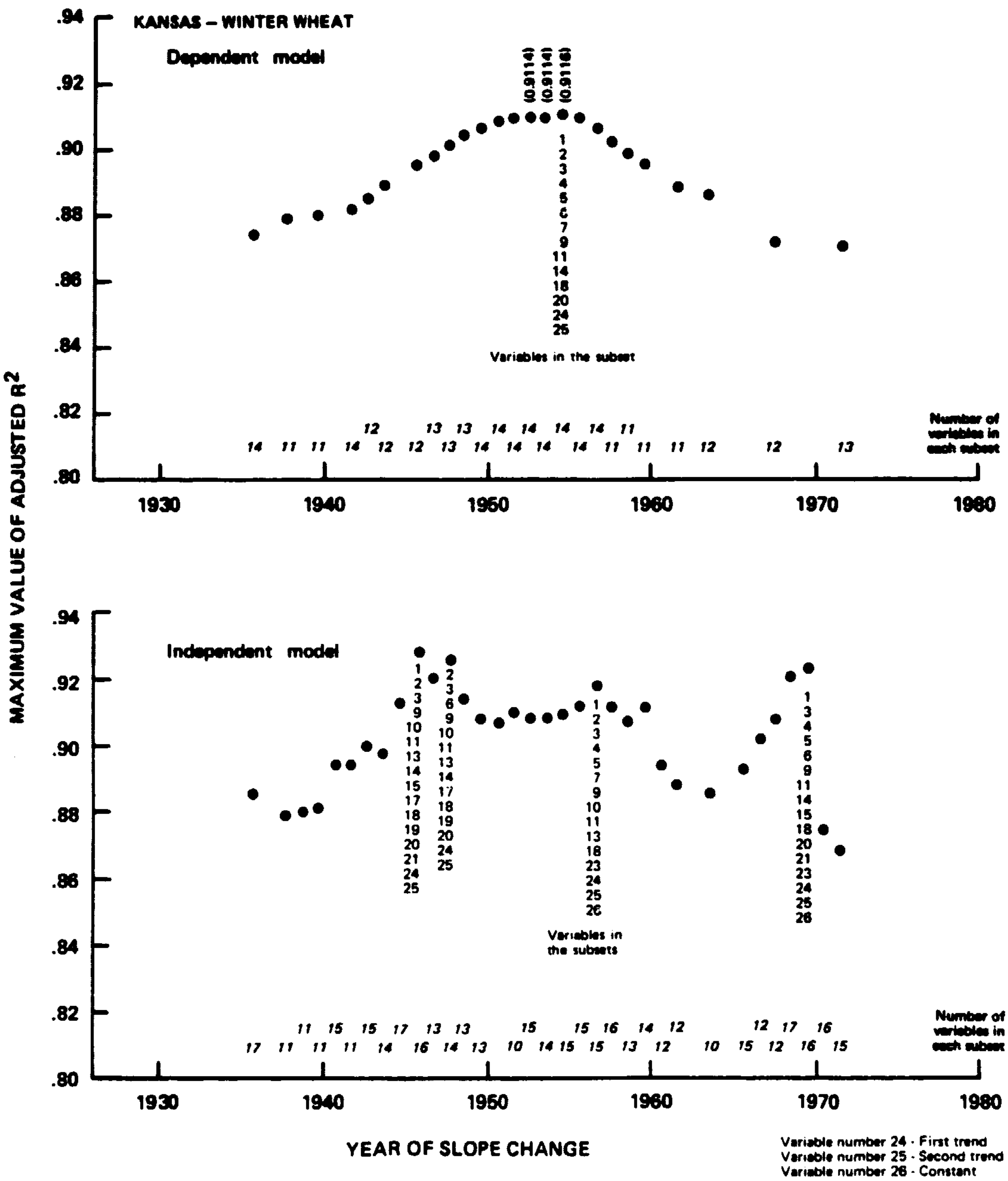

Figure B-8.- Maximum R^2_{adj} for subsets of variables from SELECT for models of winter wheat yield in Kansas.

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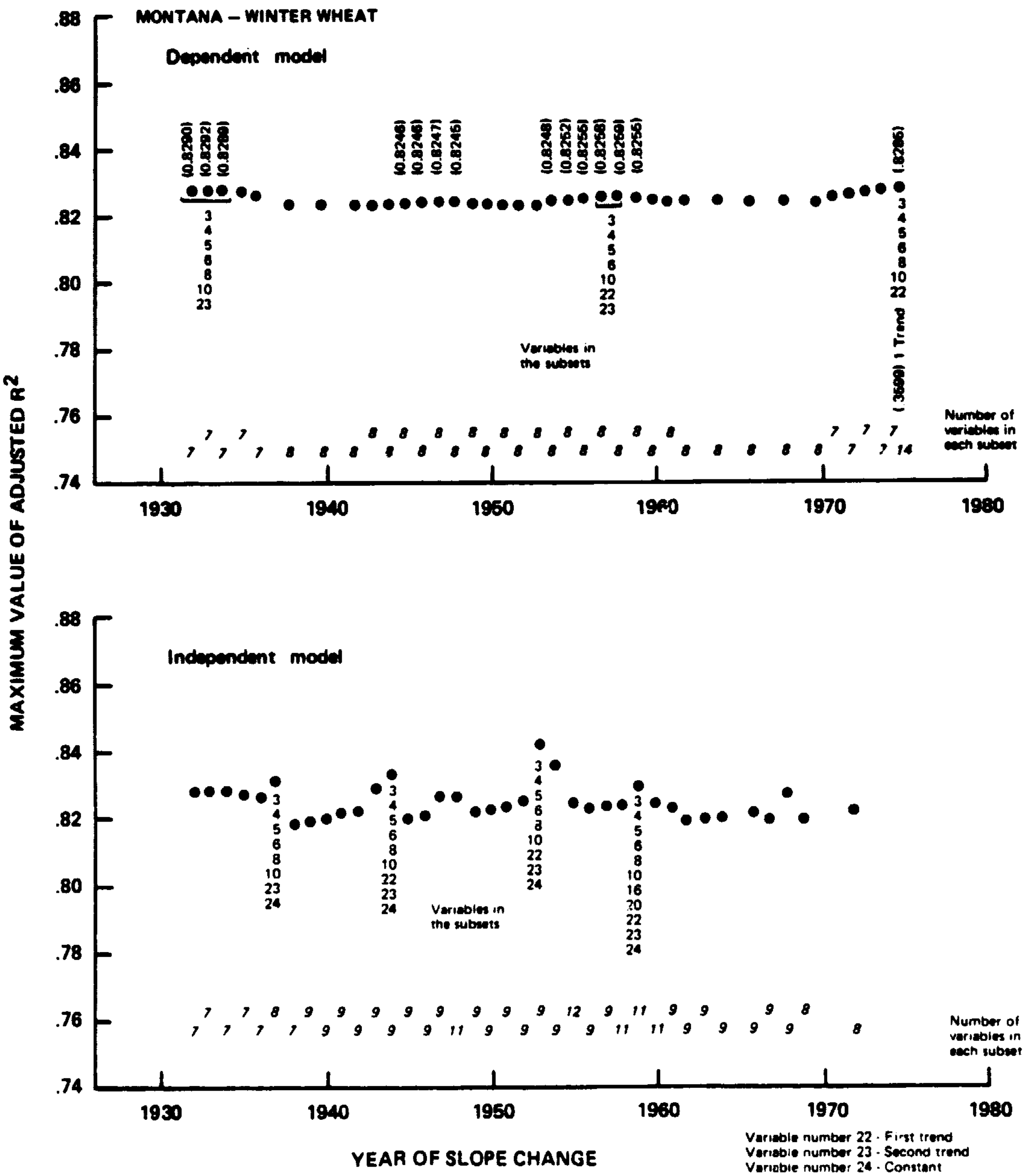

Figure B-9.- Maximum R^2_{adj} for subsets of variables from SELECT for models of winter yield in Montana.

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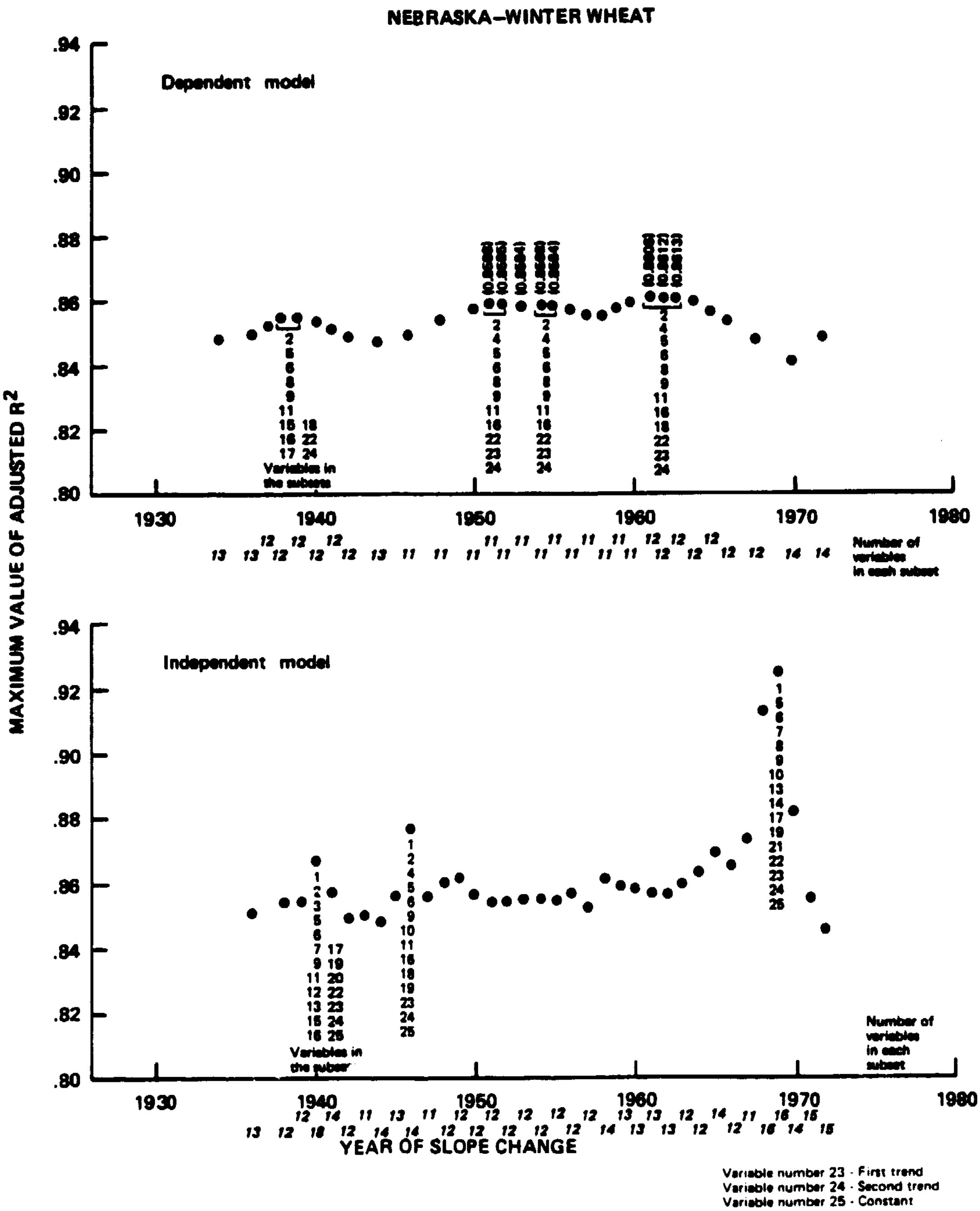

Figure B-10.- Maximum R^2_{adj} for subsets of variables from SELECT for models of winter wheat yield in Nebraska.

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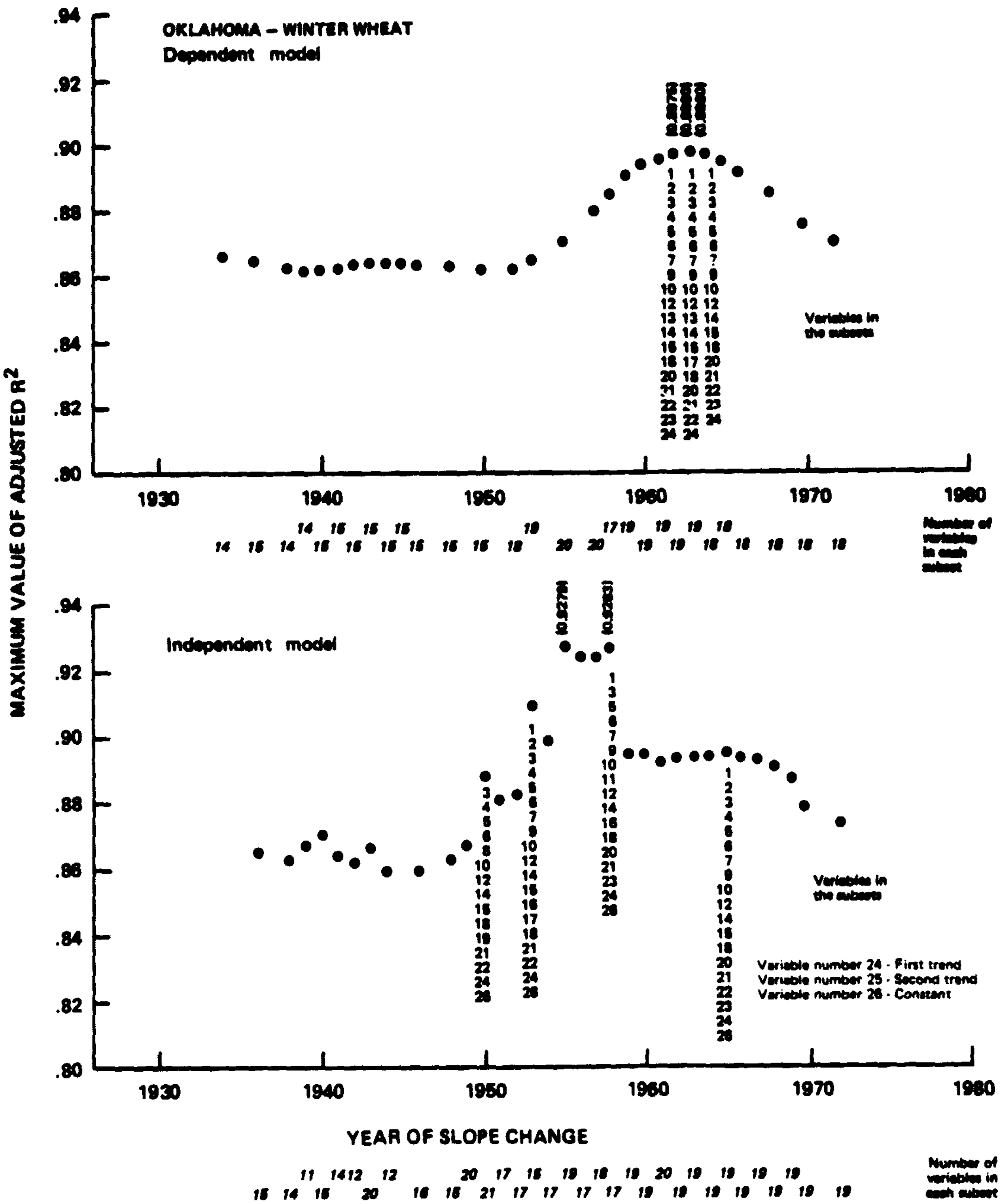

Figure B-11.- Maximum R^2_{adj} for subsets of variables from SELECT for models of winter wheat yield in Oklahoma.

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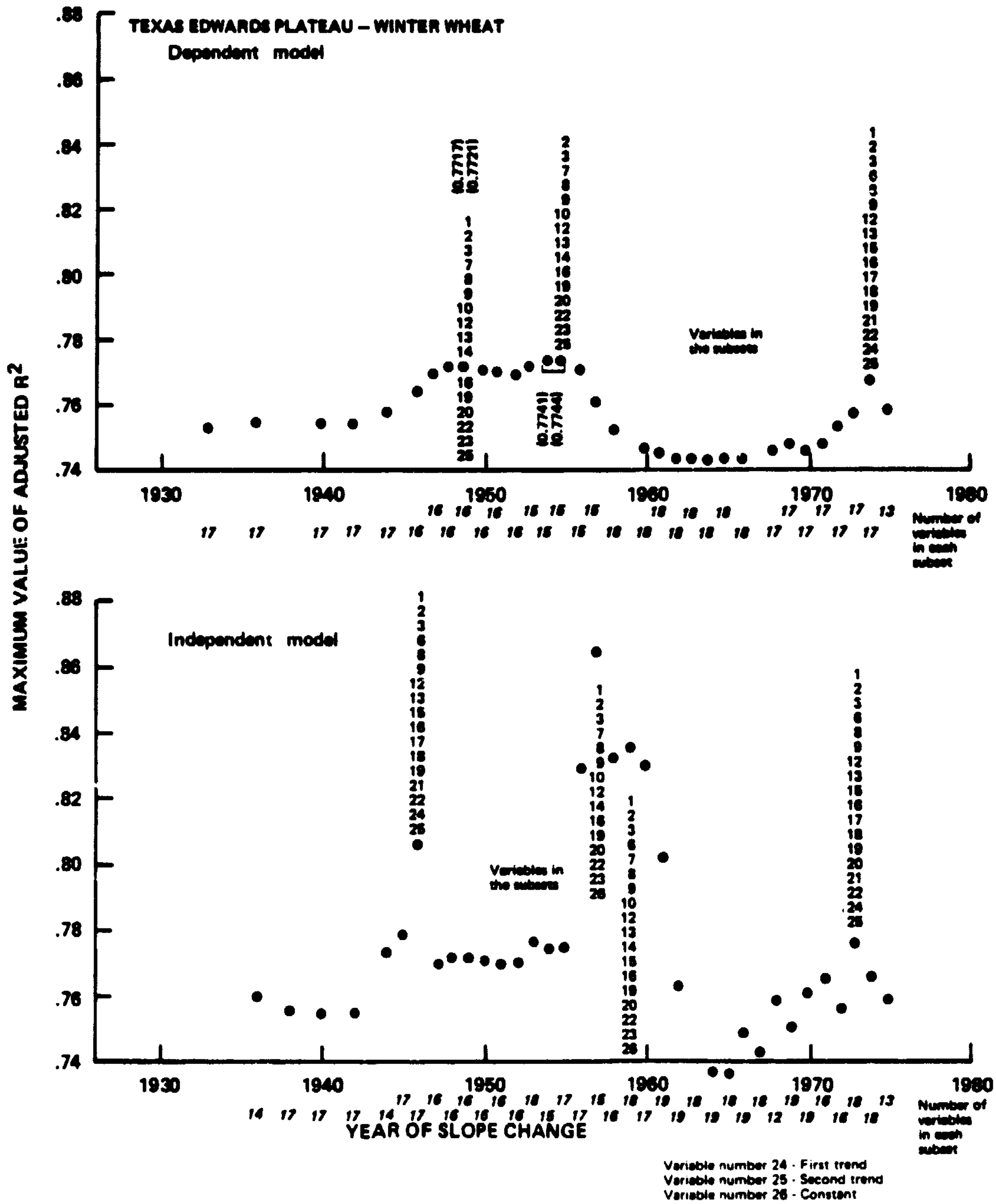

Figure B-12.- Maximum R^2_{adj} for subsets of variables from SELECT for models of winter wheat yield in the Texas Edwards Plateau.

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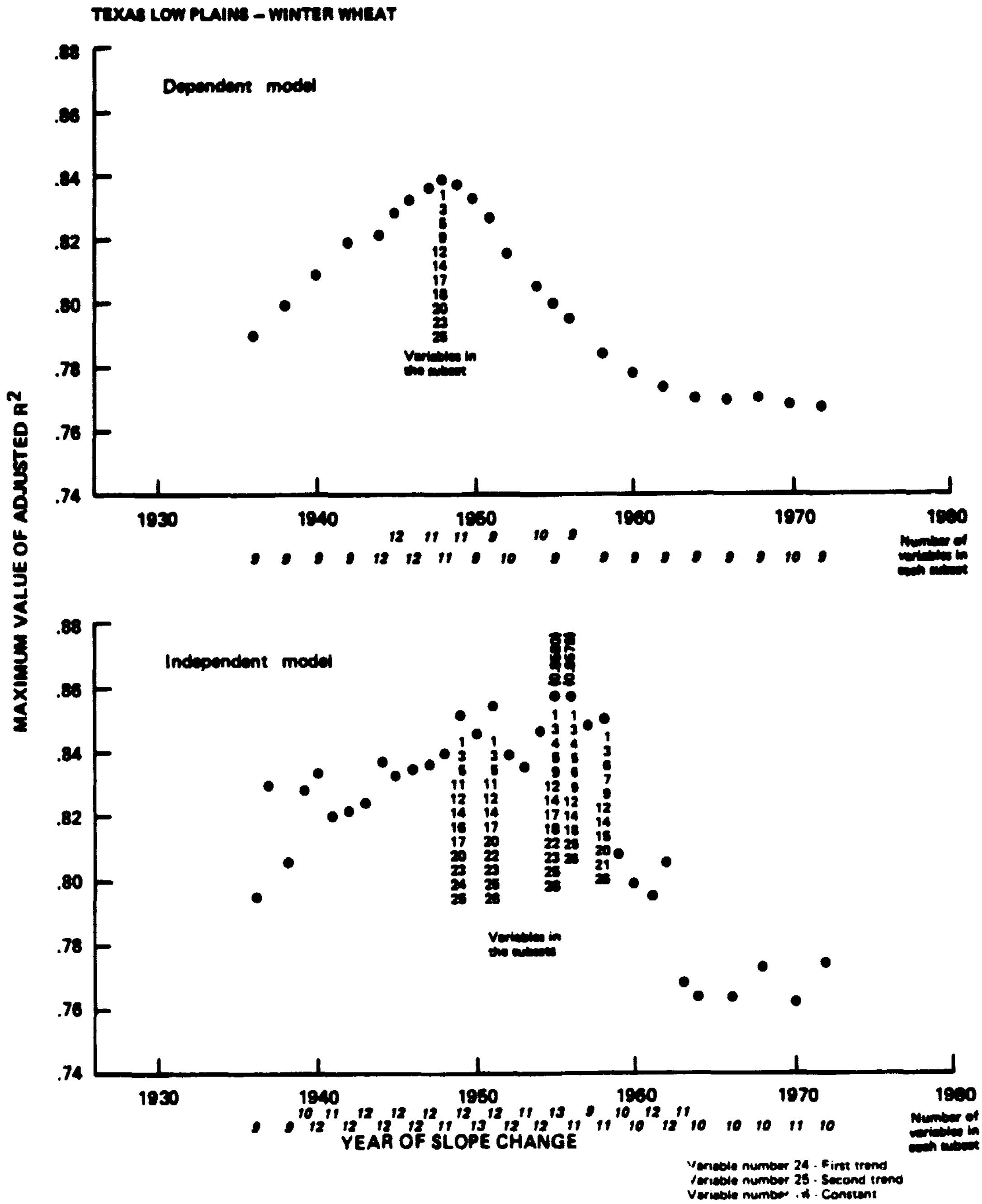

Figure B-13.- Maximum R^2_{adj} for subsets of variables from SELECT for models of winter wheat yield in the Texas Low Plains.

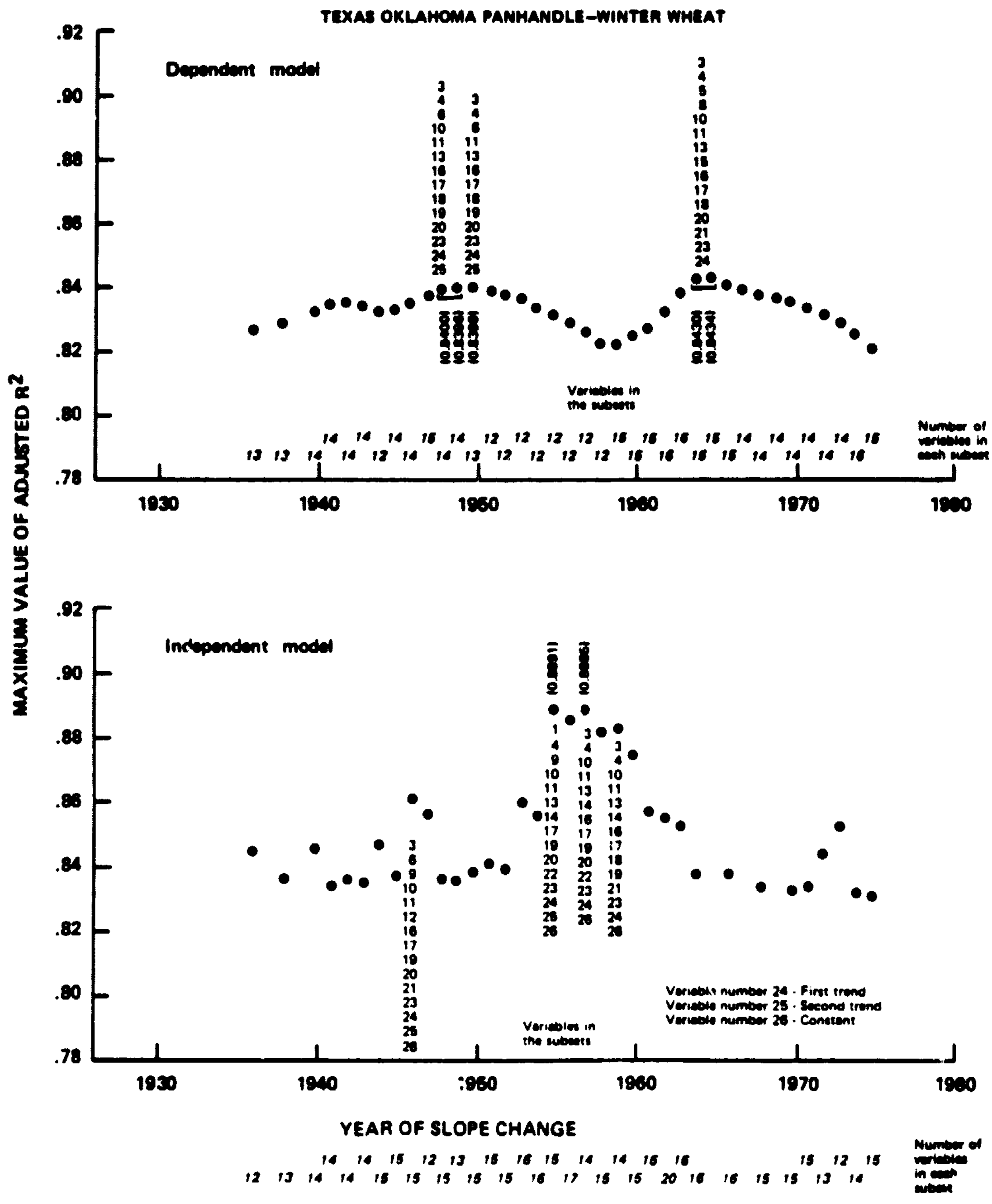

Figure B-14.- Maximum R^2_{adj} for subsets of variables from SELECT for models of winter wheat in the Texas-Oklahoma Panhandle.

APPENDIX C

RUNNING THE REGRESSIONS AND CHOOSING THE SUBSET OF INDEPENDENT VARIABLES CONSTITUTING THE BEST REGRESSIONS

APPENDIX C

RUNNING THE REGRESSIONS AND CHOOSING THE SUBSET OF INDEPENDENT VARIABLES CONSTITUTING THE BEST REGRESSIONS

Regressions were run using program BEIRA with the independent variables chosen by SELECT as input. When the LRR eigenvalues and eigenvectors indicated the existence of a nonpredictive multicollinearity the correlation coefficients were examined to determine the magnitude of the correlations between the independent variables involved, and between the independent variables and the yield. When two or more variables were highly correlated ($r > 0.85$) the variable least correlated with the yield was eliminated. When the magnitudes of the correlation between yield and two variables highly correlated with each other were nearly equal, the regression coefficients were examined to determine the variable contributing least to the dependent variable. When the regression and correlation coefficients indicated that different variables of a highly correlated pair should be deleted, regressions were run with each of the variables deleted. When the LRR eigenvectors and eigenvalues indicated that any remaining multicollinearities were predictive, the eigenvalues of X'X were examined. If they appeared not to increase uniformly, the first eigenvalue was increased by 0.1.

The "best regression" for a region was chosen taking into account the values of R^2, S^2, the uniformity of the eigenvalues, the values of the determinant of R (the matrix of the correlation coefficients) and the trace of R-inverse, and the distribution of predicted values of yield above and below the measured values, as indicated by the quantity $\Delta = y - \hat{y}$ where y is the measured value of yield, and $\hat{y}$ is the predicted value.

The two regressions for Texas Low Plains illustrate the method for deciding which variables should be eliminated and for adding bias by changing the value of an eigenvalue. Variables chosen by SELECT for the peak values of R^2_{adj} are listed in Figure B-13, in Appendix B.

Figure C-1 shows the printout, starting with the LRR eigenvectors, for the dependent model with slope change in 1944 and the 11 independent variables chosen by SELECT. The values of the first four LRR eigenvalues are small. The components of the first three LRR eigenvalues in the direction of the dependent variable

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```
LRR EIGENVECTORS?? .II. 0=NO, 1=YES
       1
    THE LRR EIGENVECTORS ARE

               1              2              3              4              5
  1       0.002695      0.014471     -0.020088     -0.140161      0.135502
  3      -0.019601     -0.098067     -0.022121     -0.115112     -0.395528
  5       0.003371     -0.006660     -0.025221      0.071906     -0.725381
  9      -0.002475      0.015707     -0.070188     -0.049539     -0.252297
 12       0.031970     -0.061563      0.597622     -0.021307      0.159262
 14       0.006082     -0.701759     -0.045602      0.054374     -0.146652
 17      -0.705580     -0.003762      0.025527     -0.039429     -0.211865
 18      -0.032353      0.044211     -0.593739      0.171685      0.219683
 20      -0.002856      0.697789      0.070199      0.117043     -0.163764
 23       0.706833      0.000178     -0.040554     -0.055116     -0.211476
 25      -0.010247      0.023161     -0.147895     -0.525602     -0.106174
 27       0.013600     -0.064392      0.041107      0.718974     -0.111953
   THE LRR EIGENVALUES ARE
          0.000415      0.003526      0.013023      0.079259      0.386382
          0.629336      0.[illegible]410      1.307656      1.402362      1.789415
          2.109694      3.[illegible]471
 THE PERFORMANCE INDEX IS           0.074
   NUMBER OF VARIABLES WITH LARGE LRR EIGENVECTOR COMPONENTS??  12
     FOR CORRELATION MATRIX.  -- 12 OR LESS -- ENTER 0 FOR NONE
     6
   ENTER INDEPENDENT VARIABLE NUMBERS.  12
  VARIABLES SELECTED ARE =
17 23 14 20 12 18

   CORRELATION MATRIX FOR VARIABLES WITH LARGE LRR EIGENVECTOR COMPONENTS
  17          23          14          20          12          18          27
1.0000      0.9994      0.0289      0.0328      0.3774      0.4002     -0.2467    17
            1.0000      0.0300      0.0339      0.3791      0.4038     -0.2479    23
                        1.0000      0.9955      0.0494      0.0824     -0.2682    14
                                    1.0000      0.0689      0.0996     -0.2484    20
                                                1.0000      0.9749     -0.3198    12
                                                            1.0000     -0.3586    18
                                                                        1.0000    27
  27
1.0000    27

         THE EIGENVALUES ARE
0.000439   0.002345   0.014763   0.376040   0.544551
0.801272   1.169574   1.316495   1.565[illegible]01   2.104185
3.074807
 THE PERFORMANCE INDEX IS        0.045
    THE DETERMINANT OF R IS   0.0000000046
   THE TRACE OF R INVERSE IS   2596.31592
    THE ESTIMATED VARIANCE IS            0.003536
    THE CHI-SQUARED VALUES IS        [illegible]2.4487
    THE STANDARDIZED BETA VECTOR IS
    0.15865        0.16221       -0.12004        0.13940       -0.55050
   -1.23435        2.75027        0.37375        0.39465       -2.67754
    0.76756
    THE STANDARDIZED LENGTH IS          4.29602

****************************************


    THE BETA VECTOR IS
    0.01698        0.02042       -0.00[illegible]       0.00417       -0.76574
   -1.53526        0.30028        0.24467        0.25295       -0.51473
    0.23336
    R-SQUARE MEASURES OF FIT IS    0.379715
    THE ESTIMATED VARIANCE IS         1.324682
```

Figure C-1.- Regression for Texas Low Plains: Dependent Model, slope change in 1948, 11 variables chosen by SELECT

(yield, variable 27) are also small. The component of the fourth eigenvector, E4, in the yield direction is large (0.718974) indicating that the eigenvector is predictive. Large components of the first three eigenvectors are underlined, and in the next step in the program the correlation coefficients are asked for. Variables 17 and 23, 14 and 20, and 12 and 18 are found to be highly correlated. Relations between the correlation coefficients for these variables and the dependent variable, yield, in the last column (headed 27) are examined. The following relations are found:

$|r_{17,y}| \approx |r_{23,y}|$ with $|r_{17,y}|$ slightly smaller

$|r_{14,y}| \approx |r_{20,y}|$ with $|r_{20,y}|$ somewhat smaller

$|r_{12,y}| \approx |r_{18,y}|$ with $|r_{12,y}|$ somewhat smaller.

The rule that the independent variables to be deleted are those least correlated with the dependent variable leads to deletion of variables 17, 20 and 12. However, examination of the regression coefficients (the beta vector) shows that $|\beta_{17}| > |\beta_{23}|$, $|\beta_{14}| > |\beta_{20}|$, and $|\beta_{12}| > |\beta_{18}|$; indicating that variables 17 and 12 have a greater influence on the dependent variables than do 23 and 18.

Figure C-2 is the printout for the regression with variables 17, 20, and 12 deleted. In Figure C-3 variables 23, 20, and 18 are deleted. For both these regressions the first LRR eigenvalue is small and the y-component of the first eigenvector is large, indicating prediction. The eigenvalues for both regressions are uniform; and the values of determinant of R and trace of R-inverse for the two regressions are nearly equal. The program calculates the predicted value of yield for each year, and subtracts the measured yield from the predicted yield. For both regressions there were 25, out of 45, values of predicted yield less than the measured yield, indicated by $25\Delta < 0$. The regression of Figure C-3 was chosen as the best regression because of the somewhat higher R^2 and smaller S^2.

Figures C-4, C-5, and C-6 show printouts for the independent model with slope change in 1956, with all the variables chosen by SELECT, with variable 12 deleted and with variable 18 deleted. The LRR eigenvectors and eigenvalues in Figure C-4 indicate that variables 12 and 18 constitute a nonpredictive multicollinearity. The correlation coefficients for these two variables and yield (variable 27) are nearly equal, with that for variable 12 and yield being somewhat smaller.

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```
LRR EIGENVECTORS?? .T1. 0=NO. 1=YES
  1
   THE LRR EIGENVECTORS ARE

             1            2            3            4            5
  1     0.140949    -0.055799     0.442930    -0.514182    -0.039927
  3     0.108631     0.258763    -0.508313     0.252630    -0.234283
  5    -0.075987     0.548322    -0.034162    -0.143835    -0.232408
  9     0.050754     0.124057    -0.535002    -0.505333     0.524021
 14    -0.181825     0.311938    -0.049627    -0.293345    -0.427429
 18    -0.167792    -0.434193    -0.235245    -0.515096    -0.237196
 23     0.094217     0.447514     0.330722    -0.113820     0.449592
 25     0.619239     0.147541     0.152773    -0.166234    -0.401574
 27    -0.71603      0.13404      0.239363    -0.061482    -0.112043
   THE LRR EIGENVALUES ARE
        0.0176        0.347999     0.570948     0.677296     0.959281
        1.0.732       1.2         1.415952     2.509059
 THE PERFORMANCE INDEX IS         0.47
  NUMBER OF VARIABLES WITH LARGE LRR EIGENVECTOR COMPONENTS??  12
   FOR CORRELATION MATRIX.  -- 12 OR LESS -- ENTER 0 FOR NONE
   0

          THE EIGENVALUES ARE
0.298190   0.51      0.573581   0.900933   0.996969
1.271119   1.41      1.423474
 THE PERFORMANCE INDEX IS           0.041
   THE DETERMINANT OF R IS   0.321517222
  THE TRACE OF R INVERSE IS     10.411712
   THE ESTIMATED VARIANCE IS         0.093837
   THE CHI-SQUARED VALUES IS        45.9555
   THE STANDARDIZED BETA VECTOR IS
   0.16410      0.14258     -0.13278      0.08574     -0.24194
  -0.17420      0.07484      0.77116
   THE STANDARDIZED LENGTH IS          0.87259
*****************************************

   THE BETA VECTOR IS
   0.01757      0.01745     -0.00932      0.00564     -0.30092
  -0.11494      0.01437      0.23445
   R-SQUARE GOODNESS OF FIT IS     0.858045
   THE ESTIMATED VARIANCE IS        1.436564
```

Figure C-2.- Regression for Texas Low Plains: Dependent Model, slope change in 1948, variables 17, 20, 12 deleted from the set chosen by SELECT

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```
LRR EIGENVECTORS?? ,I1, 0=NO, 1=YES
 1
   THE LRR EIGENVECTORS ARE

            1            2            3            4            5
   1   -0.139024     0.065716     0.513875    -0.465303    -0.023461
   3   -0.115724    -0.276298    -0.523628     0.182708    -0.258394
   5    0.056943    -0.610938    -0.023411    -0.098637    -0.228060
   9   -0.065099    -0.175119    -0.450030    -0.578173     0.524110
  12    0.164218     0.410488    -0.151147    -0.511303    -0.251089
  14    0.177016    -0.311069    -0.048294    -0.367144    -0.438523
  17   -0.106144    -0.468271     0.361539    -0.021731     0.423937
  25   -0.621568    -0.079853     0.199092    -0.095546    -0.400976
  27    0.709848    -0.156559     0.251287     0.016475    -0.100634
   THE LRR EIGENVALUES ARE
        0.072930     0.299775     0.584972     0.728255     0.953159
        1.124508     1.271122     1.414466     2.550813
 THE PERFORMANCE INDEX IS         0.078
  CORRELATION MATRIX FOR VARIABLES WITH LARGE LRR EIGENVECTOR COMPONENTS??
       ENTER NUMBER OF INDEPENDENT VARIABLES TO BE INCLUDED, I1
            MAXIMUM NUMBER IS 6  --  ENTER 0 FOR NONE
   0

        THE EIGENVALUES ARE
0.286854  0.517827  0.727941  0.917565  1.018353
1.259331  1.414466  1.857663
 THE PERFORMANCE INDEX IS         0.063
   THE DETERMINANT OF R IS  0.334329605
  THE TRACE OF R INVERSE IS     10.902164
   THE ESTIMATED VARIANCE IS         0.003701
   THE CHI-SQUARED VALUES IS      44.3729
   THE STANDARDIZED BETA VECTOR IS
   0.16530      0.14934     -0.11535      0.10041     -0.19162
  -0.24069      0.08815      0.78227
   THE STANDARDIZED LENGTH IS        0.88734

**************************************


   THE BETA VECTOR IS
   0.01769      0.01870     -0.00810      0.00661     -0.26400
  -0.29936      0.20193      0.23783
   R-SQUARE GOODNESS OF FIT IS     0.863061
   THE ESTIMATED VARIANCE IS        1.385807
```

Figure C-3.- Regression for Texas Low Plains: Dependent Model, slope change in 1948, variables 23, 20, 18 deleted from set chosen by SELECT

```
LRR EIGENVECTORS?? ,I1, 0=NO, 1=YES
THE LRR EIGENVECTORS ARE
              1              2              3              4              5
 1       0.021191      -0.124719       0.080597       0.307348      -0.348273
 3       0.028864      -0.103055      -0.053770      -0.411988       0.012477
 4      -0.039045       0.063179       0.081897       0.344338      -0.149262
 5       0.020832       0.095552      -0.112635      -0.558784      -0.384704
 6      -0.009501       0.137880       0.085017      -0.029030       0.614702
 9       0.093630      -0.091178      -0.094242      -0.404338       0.379684
12      -0.688554      -0.092349      -0.045969       0.062054       0.252707
14      -0.021719       0.114230      -0.237524      -0.239765      -0.248534
18       0.691177       0.188305      -0.009849       0.147878       0.206423
25       0.011030      -0.050502       0.756787      -0.181050      -0.117960
26       0.130559      -0.562548      -0.486125       0.131143      -0.013429
27      -0.135755       0.746511      -0.295701       0.086483      -0.062614
 THE LRR EIGENVALUES ARE
         0.016851       0.073726       0.153890       0.385816       0.596486
         0.759295       0.905600       1.066685       1.291448       1.471274
         1.902849       3.376073
THE PERFORMANCE INDEX IS        0.056
  NUMBER OF VARIABLES WITH LARGE LRR EIGENVECTOR COMPONENTS??  I2
    FOR CORRELATION MATRIX.  -- 12 OR LESS -- ENTER 0 FOR NONE
   2
  ENTER INDEPENDENT VARIABLE NUMBERS,  I2
 VARIABLES SELECTED ARE =
12 18

  CORRELATION MATRIX FOR VARIABLES WITH LARGE LRR EIGENVECTOR COMPONENTS

   12          18          27
1.0000       0.9749     -0.3198     12
             1.0000     -0.3586     18
                         1.0000     27

          THE EIGENVALUES ARE
0.018548  0.142982  0.382193  0.593086  0.758092
0.900875  1.061175  1.252568  1.408292  1.776573
2.705615
 THE PERFORMANCE INDEX IS       0.069
    THE DETERMINANT OF R IS  0.003694075
   THE TRACE OF R INVERSE IS      71.021576
    THE ESTIMATED VARIANCE IS          0.003138
    THE CHI-SQUARED VALUES IS       221.2405
    THE STANDARDIZED BETA VECTOR IS
    0.15147      0.13152      -0.08731      -0.10045      -0.13033
    0.17062     -0.50300      -0.17722       0.38515       0.21152
    0.60551
    THE STANDARDIZED LENGTH IS       0.97375

************************************

    THE BETA VECTOR IS
    0.01621      0.01647      -0.00641      -0.00705      -0.01003
    0.01122     -0.69302      -0.22042       0.25213       0.09415
    3.51505
    R-SQUARE GOODNESS OF FIT IS    0.893322
    THE ESTIMATED VARIANCE IS          1.174825
```

Figure C-4.- Regression for Texas Low Plains: Independent Model, slope change in 1956, 11 variables chosen by SELECT.

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```
LRR EIGENVECTORS?? ,I1, 0=NO, 1=YES
  1
    THE LRR EIGENVECTORS ARE

              1            2            3            4            5
   1     0.126654    -0.079770     0.294434    -0.185544    -0.659135
   3     0.105356     0.052112    -0.407250     0.043175     0.338678
   4    -0.068257    -0.082897     0.321798    -0.180665     0.046570
   5    -0.093269     0.111553    -0.570560    -0.142503    -0.294196
   6    -0.135400    -0.078455     0.038863     0.523246     0.162093
   9     0.106253     0.102581    -0.327004     0.514960    -0.282277
  14    -0.115441     0.235748    -0.248960    -0.082342    -0.400418
  18    -0.085488     0.070024     0.314440     0.598979    -0.233137
  25     0.046799    -0.754672    -0.161888     0.034627    -0.195706
  26.    0.579232     0.486017     0.131674    -0.006705    -0.030193
  27    -0.755004     0.297832     0.074311    -0.070662    -0.047585
    THE LRR EIGENVALUES ARE
         0.072668     0.153217     0.380223     0.494666     0.716817
         0.882601     1.065470     1.214314     1.395688     1.517564
         3.106772
 THE PERFORMANCE INDEX IS        0.075
   NUMBER OF VARIABLES WITH LARGE LRR EIGENVECTOR COMPONENTS??  I2
     FOR CORRELATION MATRIX.  -- 12 OR LESS -- ENTER 0 FOR NONE
     0

          THE EIGENVALUES ARE
0.142191  0.377552  0.491341  0.714258  0.879962
1.056615  1.166373  1.378069  1.504533  2.289106

 THE PERFORMANCE INDEX IS        0.097
    THE DETERMINANT OF R IS  0.096971631
   THE TRACE OF R INVERSE IS      17.884048
    THE ESTIMATED VARIANCE IS          0.003323
    THE CHI-SQUARED VALUES IS       92.9446
    THE STANDARDIZED BETA VECTOR IS
    0.15216      0.12218     -0.06626     -0.13068     -0.13413
    0.11897     -0.17875     -0.11889      0.21919      0.57557

    THE STANDARDIZED LENGTH IS        0.71911

************************************


    THE BETA VECTOR IS
    0.01629      0.01530     -0.00487     -0.00917     -0.01032
    0.00783     -0.22232     -0.07783      0.09756      3.34125

    R-SQUARE GOODNESS OF FIT IS     0.883684
    THE ESTIMATED VARIANCE IS          1.244369
```

Figure C-5.- Regression for Texas Low Plains: Independent Model, slope change in 1956, variable is deleted from the set chosen by SELECT.

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LRR EIGENVECTORS?? ,I1, 0=NO, 1=YES
    1
    THE LRR EIGENVECTORS ARE

              1            2            3            4            5
    1     0.123405    -0.079597    -0.283164    -0.284175    -0.628448
    3     0.110277     0.053377     0.387025     0.092022     0.356438
    4    -0.074117    -0.081214    -0.321610    -0.186760     0.091454
    5    -0.079280     0.110389     0.578486    -0.200757    -0.238159
    6    -0.139151    -0.085097    -0.037746     0.556946     0.062591
    9     0.113022     0.093425     0.340823     0.464633    -0.357021
   12    -0.107302     0.057527    -0.327919     0.537764    -0.294030
   14    -0.113993     0.236595     0.240631    -0.103403    -0.402456
   25     0.054292    -0.756718     0.162027    -0.013960    -0.178354
   26     0.573449     0.488292    -0.142918    -0.010879    -0.023361
   27    -0.756567     0.292790    -0.047479    -0.096569    -0.027569
    THE LRR EIGENVALUES ARE
          0.069537     0.153862     0.364449     0.540704     0.735975
          0.876272     1.064629     1.246404     1.371817     1.514475
          3.061777
 THE PERFORMANCE INDEX IS         0.073
   NUMBER OF VARIABLES WITH LARGE LRR EIGENVECTOR COMPONENTS??  I2
     FOR CORRELATION MATRIX.  -- 12 OR LESS -- ENTER 0 FOR NONE
     0

          THE EIGENVALUES ARE
0.142681  0.363401  0.533525  0.735083  0.872360
1.057000  1.200549  1.362397  1.494630  2.238375

 THE PERFORMANCE INDEX IS         0.163
   THE DETERMINANT OF R IS  0.102603734
  THE TRACE OF R INVERSE IS    17.770279
   THE ESTIMATED VARIANCE IS         0.003201
   THE CHI-SQUARED VALUES IS        90.6957
   THE STANDARDIZED BETA VECTOR IS
   0.15012      0.12707     -0.07320      -0.11691      -0.13863
   0.12891     -0.13824     -0.17673       0.21866       0.57986

   THE STANDARDIZED LENGTH IS         0.72660

**************************************

   THE BETA VECTOR IS
   0.01607      0.01591     -0.00537      -0.00821      -0.01065
   0.00848     -0.19046     -0.21981       0.09733       3.36614

   R-SQUARE GOODNESS OF FIT IS     0.887581
   THE ESTIMATED VARIANCE IS         1.198393
```

Figure C-6.- Regression for Texas Low Plains: Independent Model, slope change in 1956, variable 18 deleted from the set chosen by SELECT.

Comparison of the regression coefficients in Figure C-4 indicates variable 12 makes a larger contribution to yield than variable 18. Figures C-5 and C-6 show somewhat improved values for the determinant of R and trace of R-inverse, and for R^2 and S^2, when variable 18 is deleted. Examination of the eigenvalues in Figure C-6 indicates that increasing the value of the first one by 0.1 would make them more uniform. The result is shown in Figure C-7. The values of the determinant of R and trace of R-inverse are improved, the values of R^2 and S^2 are nearly the same as before, and the number of predicted yield values greater than the measured yield is nearly the same as the number less than the measured yield (23 $\Delta < 0$ out of 45 observations). This regression was chosen as the best regression.

A number of the subsets of independent variables chosen by SELECT did not include either one or both of the trend variables, or the constant variable (for the independent model). Several regressions were run with the omitted trend or constant variable added to those chosen by SELECT. It was found that the values of R^2 remained almost the same and the values of S^2 increased somewhat. The distribution of predicted yield values above and below the measured values was more uniform without the added variables, and the values of determinant of R and trace of R-inverse indicated more stability in the data matrices for the subsets as chosen by SELECT.

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```
ENTER LOCATION OF SMALLEST EIGENVALUE,I2
 ENTER 00 TO DELETE VARIABLES OR QUIT, ENTER 01 TO PLOT AND  CHANGE EIGEN NO. 1
  1
    ENTER INCREMENT,F4.2( SMALLEST >.1 WHEN CHANGED

     0.100
    THE DETERMINANT OF R IS  0.174515069
   THE TRACE OF R INVERSE IS      14.882288
    THE CHI-SQUARED VALUES IS          69.5388
    THE STANDARDIZED BETA VECTOR IS
    0.15231      0.11811      -0.06277      -0.12424      -0.12473
    0.11632     -0.13952      -0.19450       0.28851       0.51115

    THE STANDARDIZED LENGTH IS           0.69815
 *************************************

    THE BETA VECTOR IS
    0.01630      0.01479      -0.00461      -0.00872      -0.00959
    0.00765     -0.19223      -0.24191       0.12841       2.96728

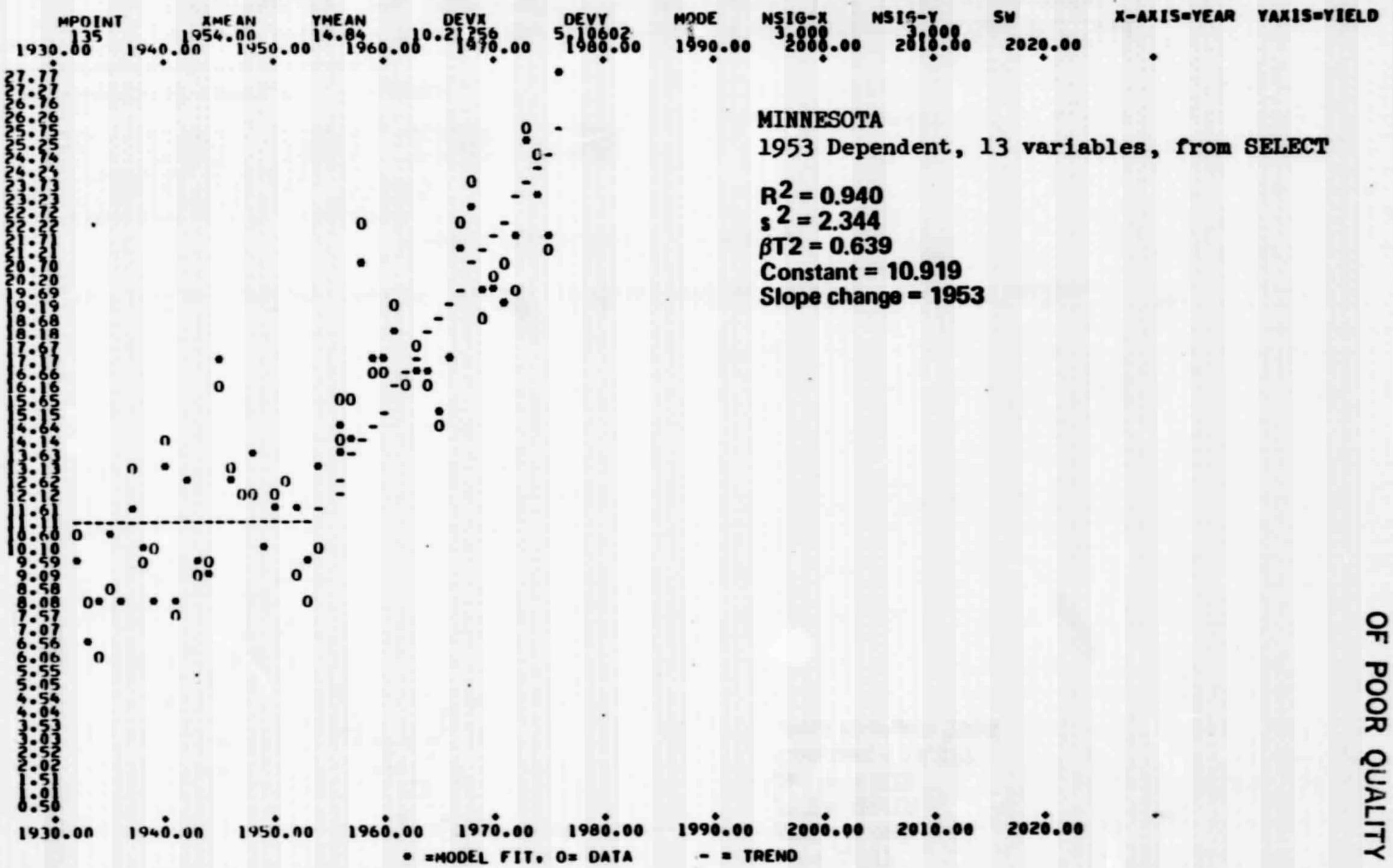
    R-SQUARE GOODNESS OF FIT IS    0.884340
 *************************************

 CONSTANT=       12.243
 CDELTA=         7.750
```

Figure C-7.- Regression for Texas Low Plains: First eigenvalue of Figure C-6 increased by 0.1.

APPENDIX D

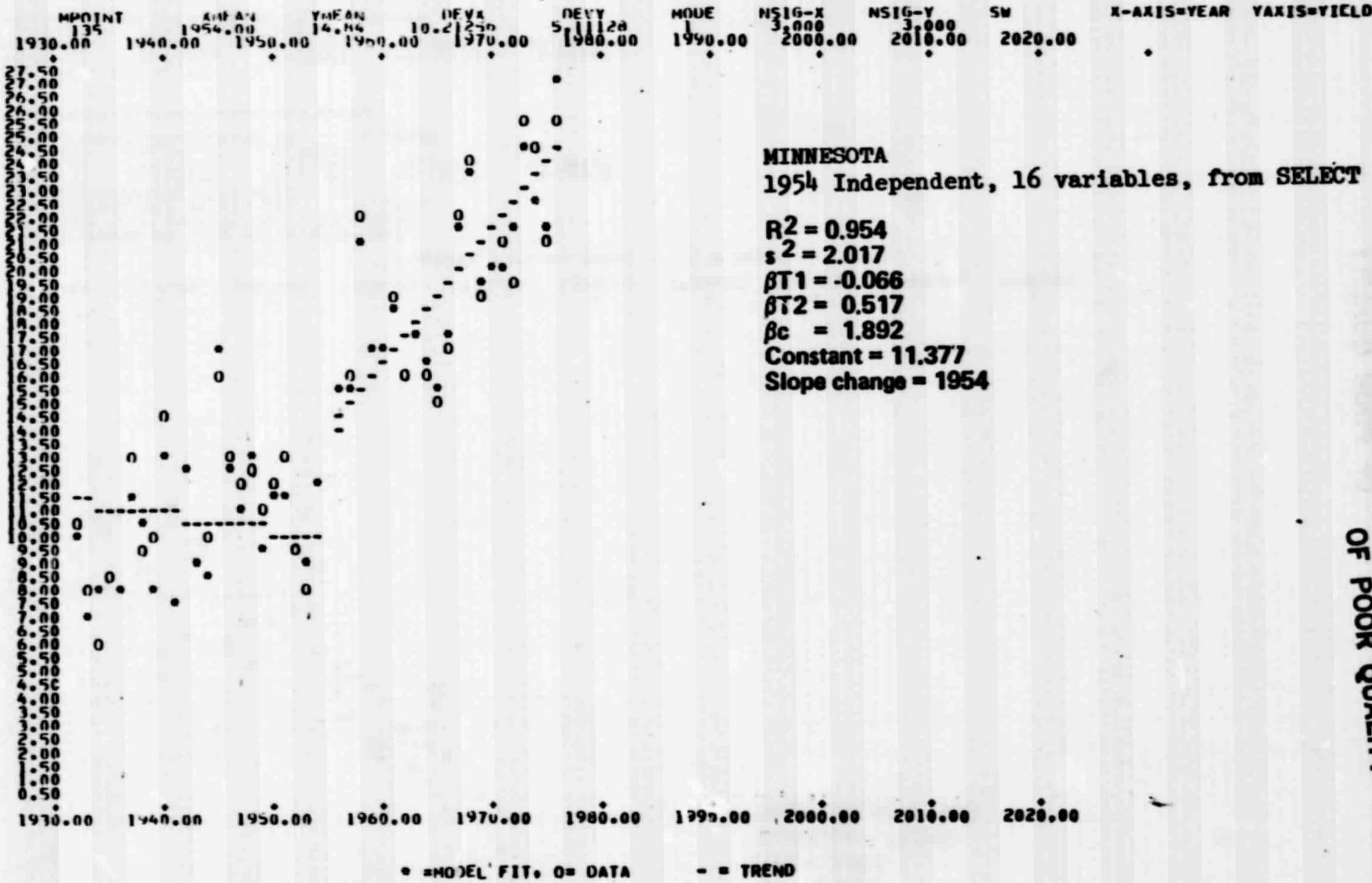
PLOTS SHOWING TREND LINES AND PREDICTED AND MEASURED YIELDS FOR BEST REGRESSIONS FROM SELECT-BEIRA




```
THE DETERMINANT OF R IS   0.159394741
THE TRACE OF R INVERSE IS      17.535431
 THE CHI-SQUARED VALUES IS       71.3124
 THE STANDARDIZED BETA VECTOR IS
-0.10278    -0.15106    -0.16302    -0.15750     0.16040
-0.14729    -0.19023     0.13349    -0.14931    -0.30105
-0.30829     0.10764     0.92278
 THE STANDARDIZED LENGTH IS        1.12129
••••••••••••••••••••••••••••••••••••••••

 THE BETA VECTOR IS
-0.04427    -0.03184    -0.02221    -0.02133     0.02662
-0.02065    -0.03059     0.34714    -0.39463    -0.96687
-1.08973     0.42190     0.63871
R-SQUARE GOODNESS OF FIT IS   0.940496
```

Figure D-1(a).- Trend lines and predicted and measured yields for best regressions from SELECT-BEIRA for Minnesota, 1953 Dependent model.




```
THE DETERMINANT OF R IS   0.006427694
THE TRACE OF R INVERSE IS     35.918198
THE CHI-SQUARED VALUES IS       190.9501
THE STANDARDIZED BETA VECTOR IS
-0.06573   -0.13128   -0.16221   -0.12724    0.10943
-0.07077   -0.18334   -0.19654    0.12814   -0.10021
-0.30818   -0.28052    0.09157   -0.09032    0.70875

0.17870
THE STANDARDIZED LENGTH IS      0.96263
```

Figure D-1(b).- Trend lines and predicted and measured yields for best regressions from SELECT-BEIRA for Minnesota, 1954 Independent model.

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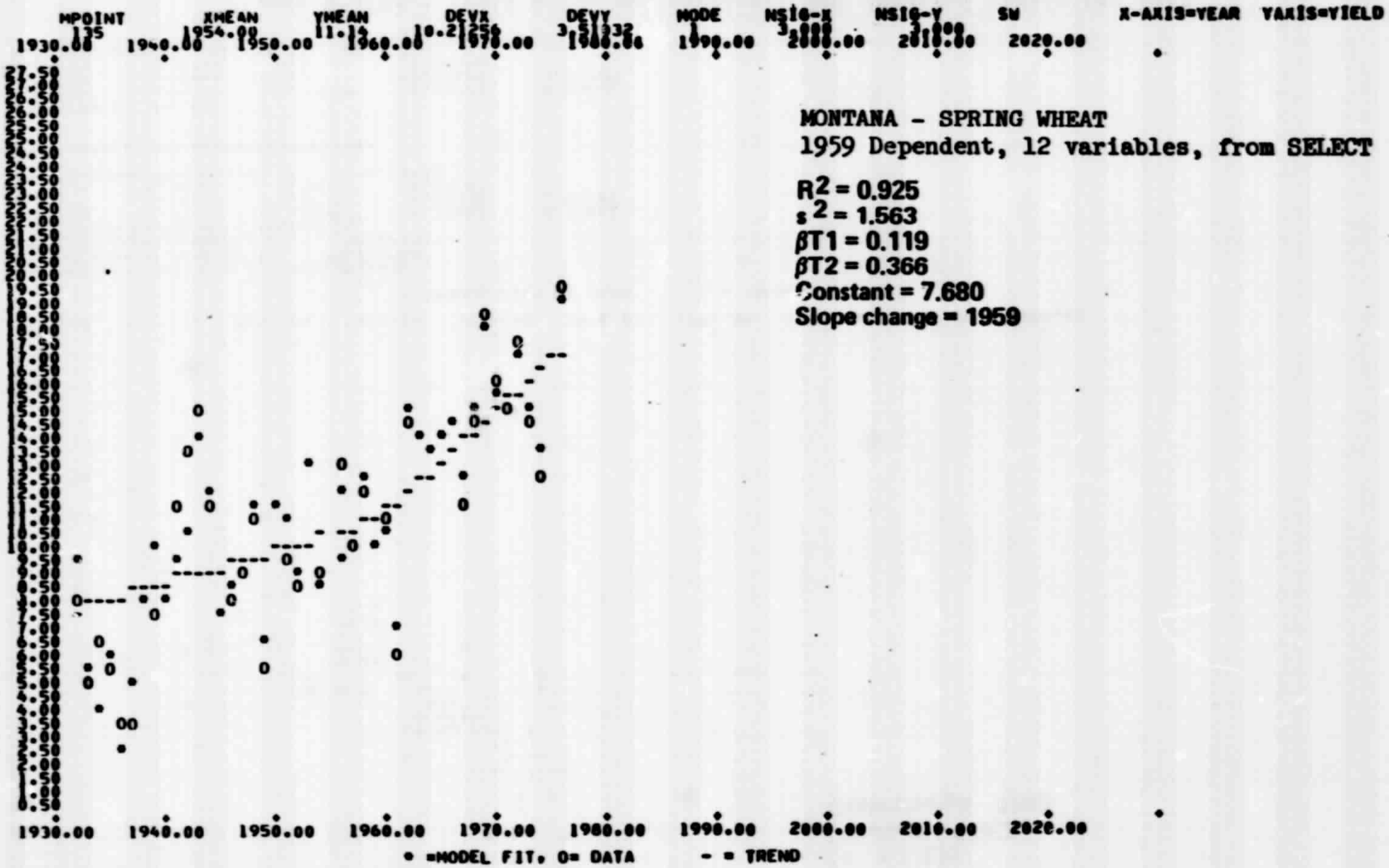
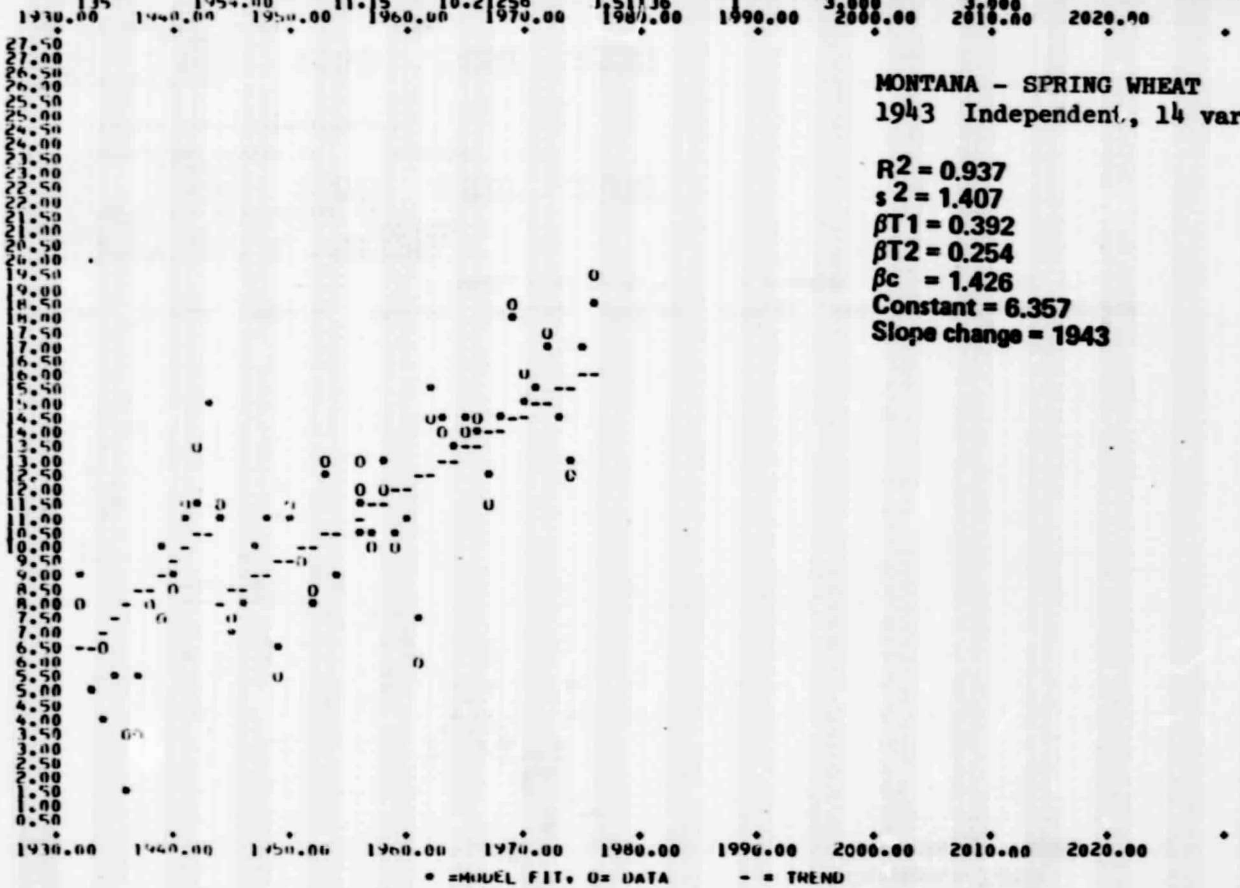



```
THE DETERMINANT OF R IS  0.070516229
THE TRACE OF R INVERSE IS     20.633728
THE CHI-SQUARED VALUES IS      103.8665
THE STANDARDIZED BETA VECTOR IS
0.1[illegible]561    -0.11302     0.13026     0.36620     0.10925
0.13605    -0.15559     0.10713    -0.21636    -0.12741
0.27048     0.49569
THE STANDARDIZED LENGTH IS       0.80252
•••••••••••••••••••••••••••••••••••••

THE BETA VECTOR IS
0.09939    -0.07018     0.03172     0.05063     0.02005
0.[illegible]2981    -0.11566     0.23742    -0.56556    -0.36860
0.[illegible]1936     0.36608
R-SQUARE GOODNESS OF FIT IS   0.925235
```

Figure D-2(a).- Trend lines and predicted and measured yields for best regressions from SELECT-BEIRA for Montana spring wheat, 1959 Dependent model.

```
MPOINT     XMEAN      YMEAN      DEVX       DEVY      MODE    NSIG-X    NSIG-Y    SW         X-AXIS=YEAR  YAXIS=YIELD
 135      1954.00     11.15     10.21256    7.51136     1      3.000     3.000
```




```
THE DETERMINANT OF R IS  0.01354747470
THE TRACE OF R INVERSE IS    29.726563
THE CHI-SQUARED VALUES IS      165.4080
THE STANDARDIZED BETA VECTOR IS
 0.12708    -0.09497     0.15597     0.31061     0.18051
 0.11445     0.11180     0.07380    -0.12330     0.13187
-0.25331     0.30225     0.71966     0.16102
THE STANDARDIZED LENGTH IS        0.97034
*************************************

THE BETA VECTOR IS
 0.07581    [illegible]  [illegible]  0.04295     0.03445
 0.02027    [illegible]  [illegible] -0.09165     0.29225
-0.66215    [illegible]  0.25400     1.42565
R-SQUARE GOODNESS OF FIT IS   0.936766
```

Figure D-2(b).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Montana spring wheat, 1943 Independent model.

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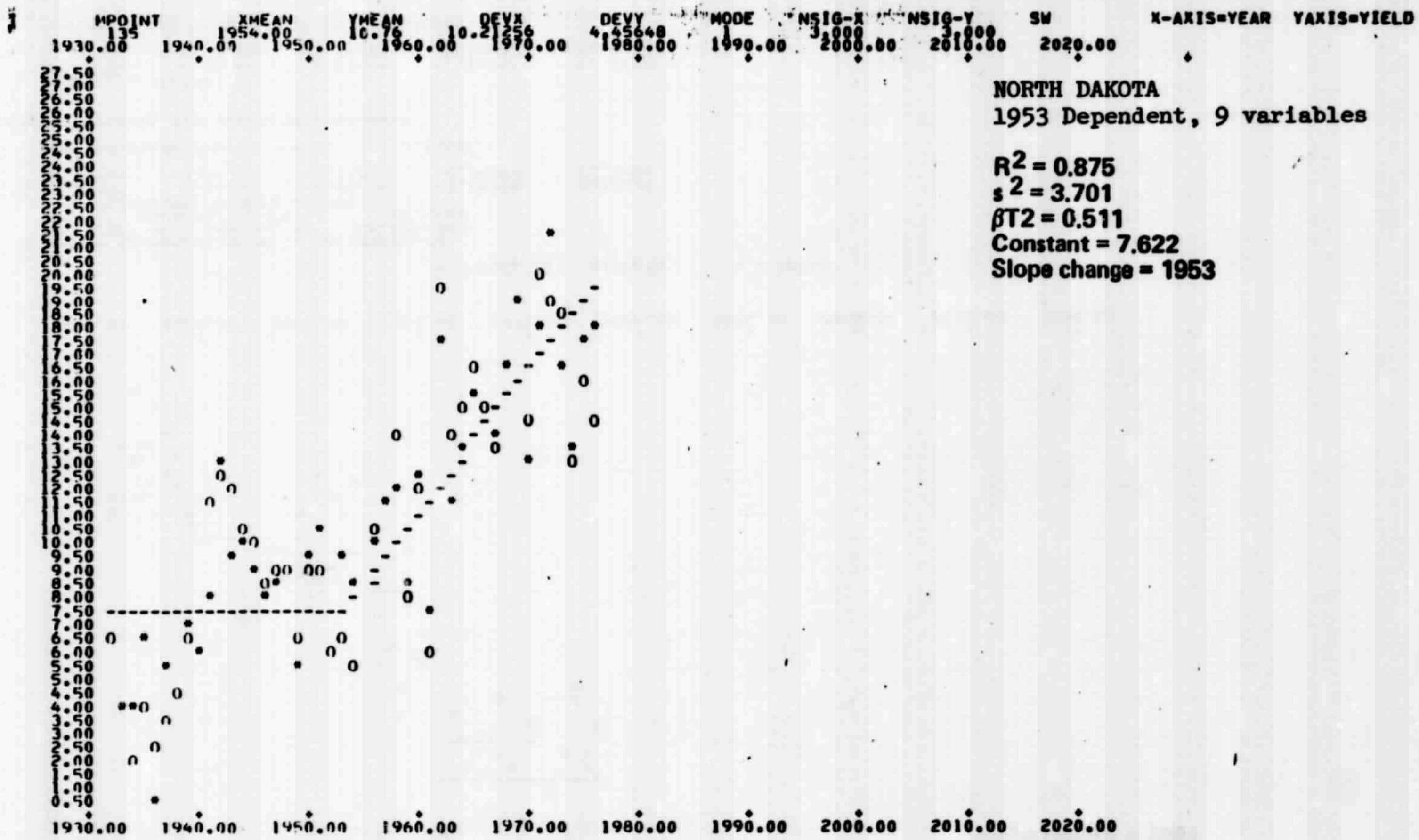


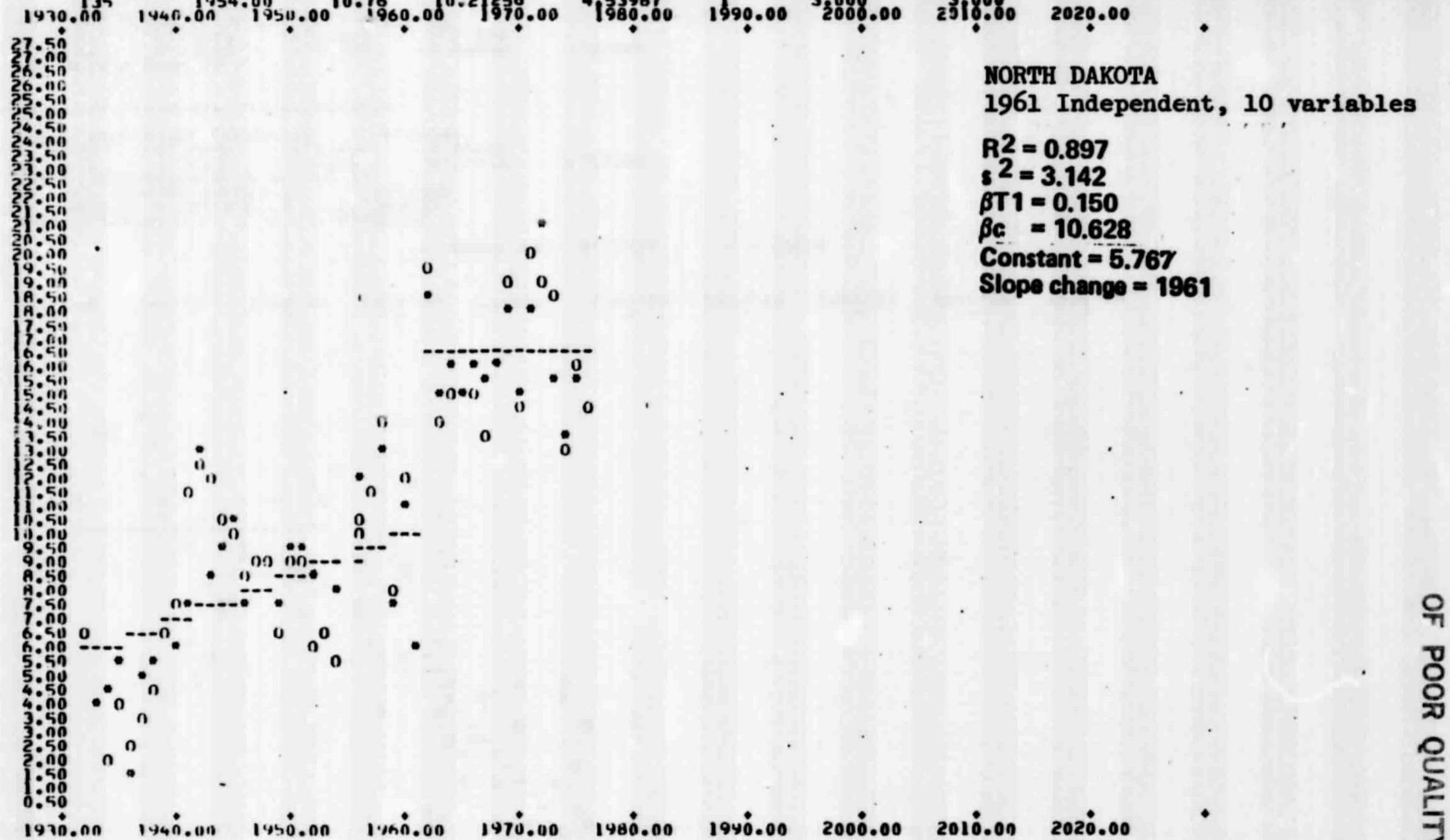
* =MODEL FIT, O= DATA   - = TREND

```
THE DETERMINANT OF R IS   0.140499175
THE TRACE OF R INVERSE IS     14.749583
 THE CHI-SQUARED VALUES IS        78.8292
 THE STANDARDIZED BETA VECTOR IS
 0.13417     0.18430     0.13799    -0.24402     0.20352
-0.15108    -0.20450    -0.07583     0.80406
 THE STANDARDIZED LENGTH IS       0.94281
************************************

 THE BETA VECTOR IS
 0.06463     0.03281     0.01304    -0.00895     0.04082
-0.45482    -0.63817    -0.26998     0.52116
 R-SQUARE GOODNESS OF FIT IS    0.87469
```

Figure D-3(a).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for North Dakota, 1953 Dependent model.

```
MPOINT      XMEAN      YMEAN      DEVX       DEVY      MODE     NSIG-X    NSIG-Y     SW        X-AXIS=YEAR  YAXIS=YIELD
  135      1954.00     10.76    10.21256   4.53987      1        3.000     3.000
```




```
THE DETERMINANT OF R IS   0.129031479
THE TRACE OF R INVERSE IS     16.180435
THE CHI-SQUARED VALUES IS        81.5667
THE STANDARDIZED BETA VECTOR IS
0.13062     0.10189     0.09164    -0.14055      0.08028
0.10569    -0.16416    -0.18173     0.30282      1.03064

THE STANDARDIZED LENGTH IS      1.13450
*****************************************

THE BETA VECTOR IS
0.07175     0.02044     0.02645    -0.13709      0.16591
0.26846    -0.49421    -0.56710     0.14972     10.62796

R-SQUARE GOODNESS OF FIT IS   0.896595
```

Figure D-3(b).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for North Dakota, 1961 Independent model.

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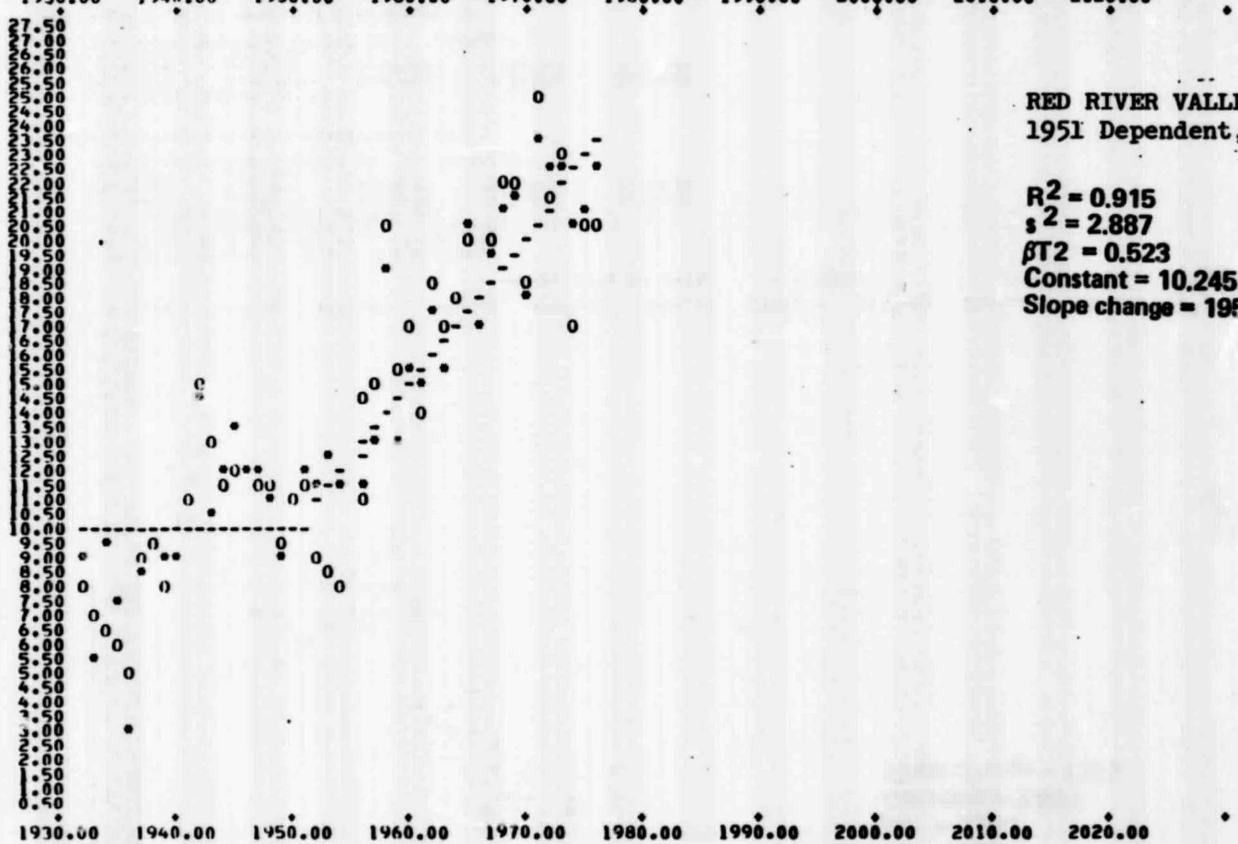
```
MPOINT         XMEAN      YMEAN     DEVX       DEVY      MODE    NSIG-X   NSIG-Y    SW         X-AXIS=YEAR  YAXIS=YIELD
  135         1954.00     14.02    10.21256   4.91704      1      3.000    3.000
1930.00  1940.00  1950.00  1960.00  1970.00  1980.00  1990.00  2000.00  2010.00  2020.00
```



RED RIVER VALLEY
1951 Dependent, 8 variables

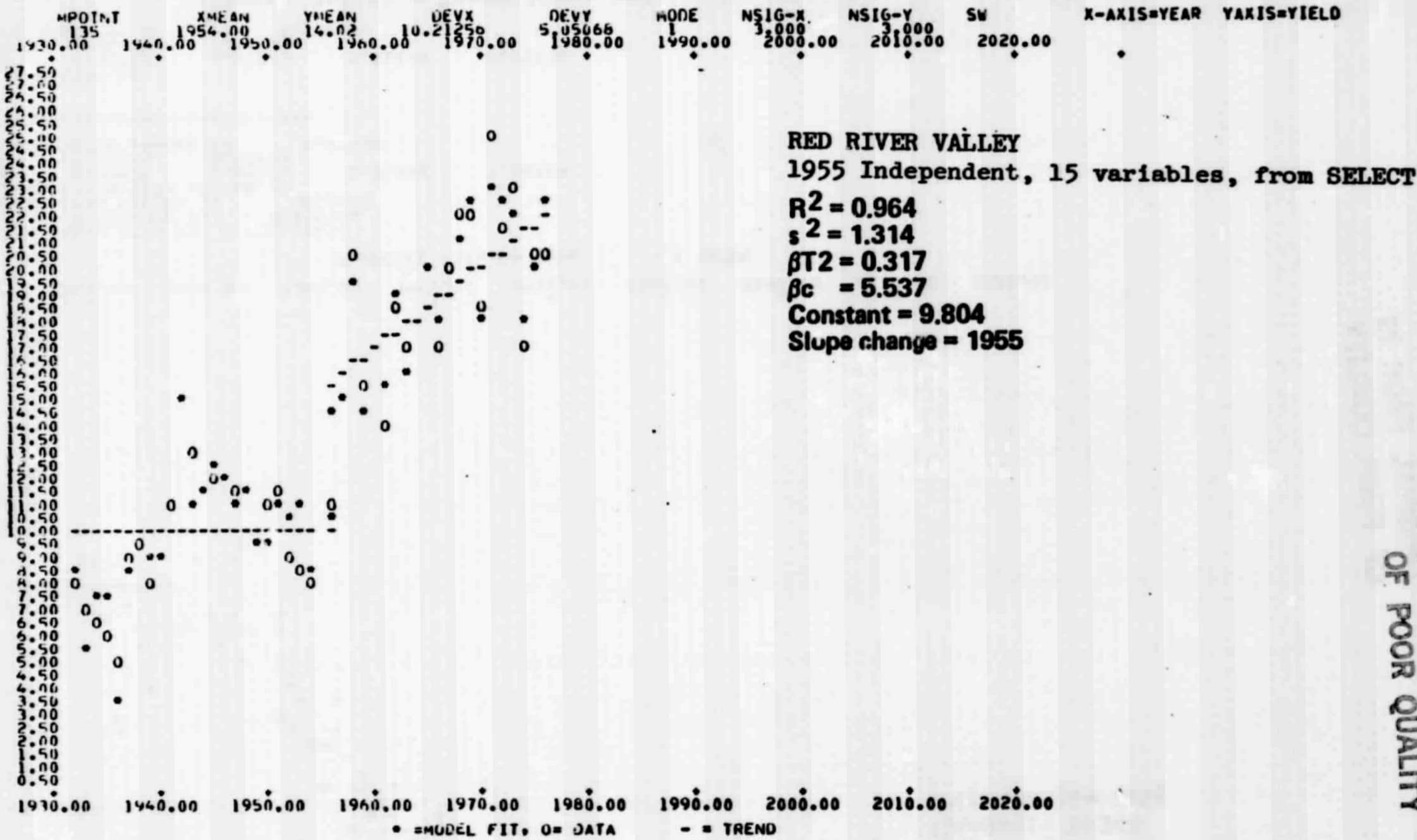
$R^2 = 0.915$
$s^2 = 2.887$
$\beta T2 = 0.523$
Constant = 10.245
Slope change = 1951

```
* =MODEL FIT, O= DATA      - = TREND

THE DETERMINANT OF R IS  0.488138437
THE TRACE OF R INVERSE IS      9.836764
THE CHI-SQUARED VALUES IS           29.0448
THE STANDARDIZED BETA VECTOR IS
-0.07082     0.09936      0.06431      0.06962      0.06209
-0.11704    -0.31546      0.83433
THE STANDARDIZED LENGTH IS          0.91490
************************************************

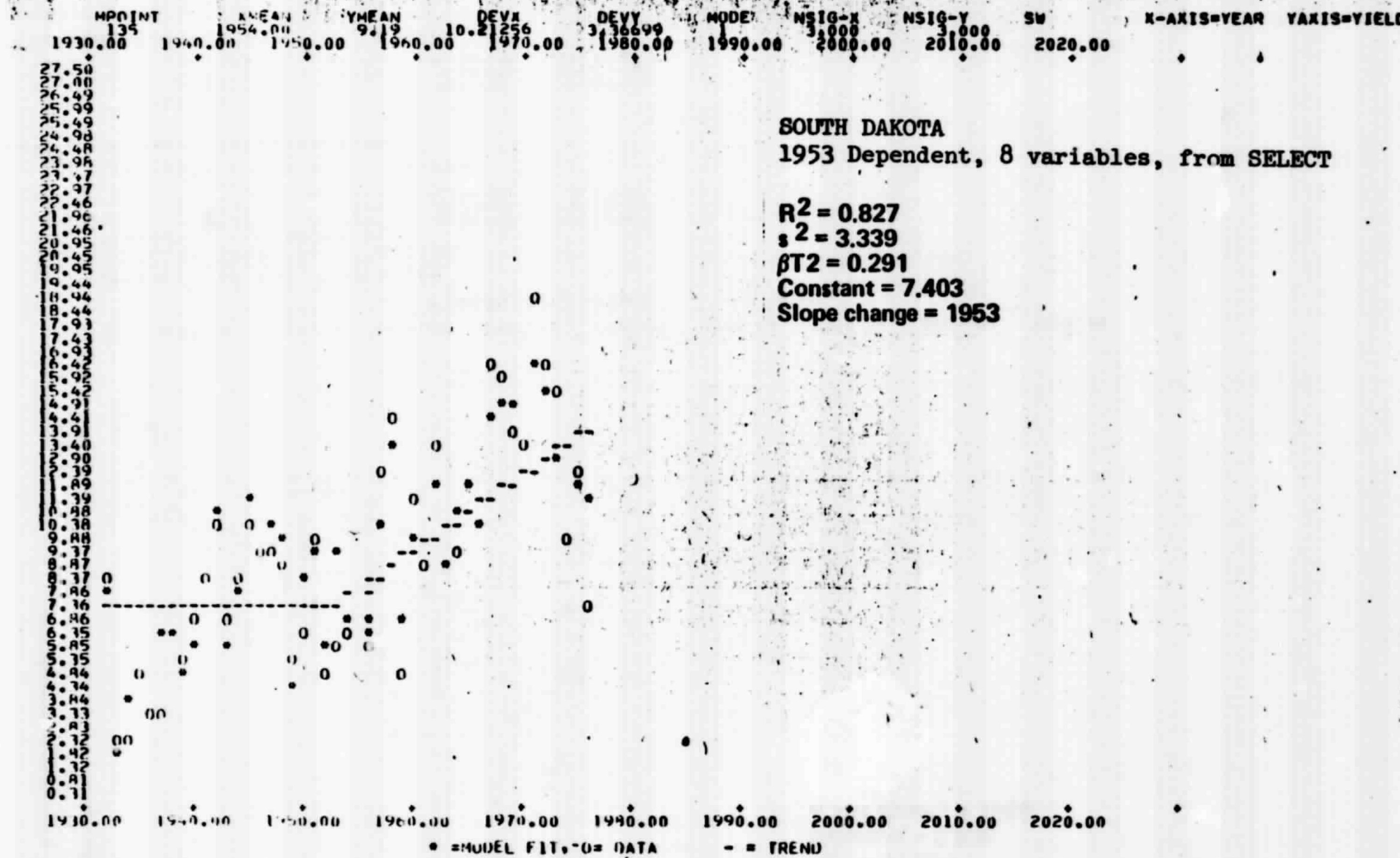
THE BETA VECTOR IS
-0.03837     0.01778      0.01073      0.01130      .15734
-0.40195    -1.15630      0.52331
R-SQUARE GOODNESS OF FIT IS    0.914560
```

Figure D-4(a).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Red River Valley, 1951 Dependent model.




```
THE DETERMINANT OF R IS   0.130133956
THE TRACE OF R INVERSE IS      17.542236
THE CHI-SQUARED VALUES IS        79.8683
THE STANDARDIZED BETA VECTOR IS
-0.05232     0.05234     0.13038     0.10581     0.05165
 0.06611     0.14259    -0.13285    -0.27544     0.04836
 0.41325     0.52407
THE STANDARDIZED LENGTH IS          0.77870
*************************************
THE BETA VECTOR IS
-0.02835     0.05959     0.05642     0.01894     0.00862
 0.01512     0.36130    -0.45625    -1.00963     0.19626
 0.31696     5.53678
R-SQUARE GOODNESS OF FIT IS    0.964289
ENTER LOCATION OF SMALLEST EIGENVALUE,I2
```

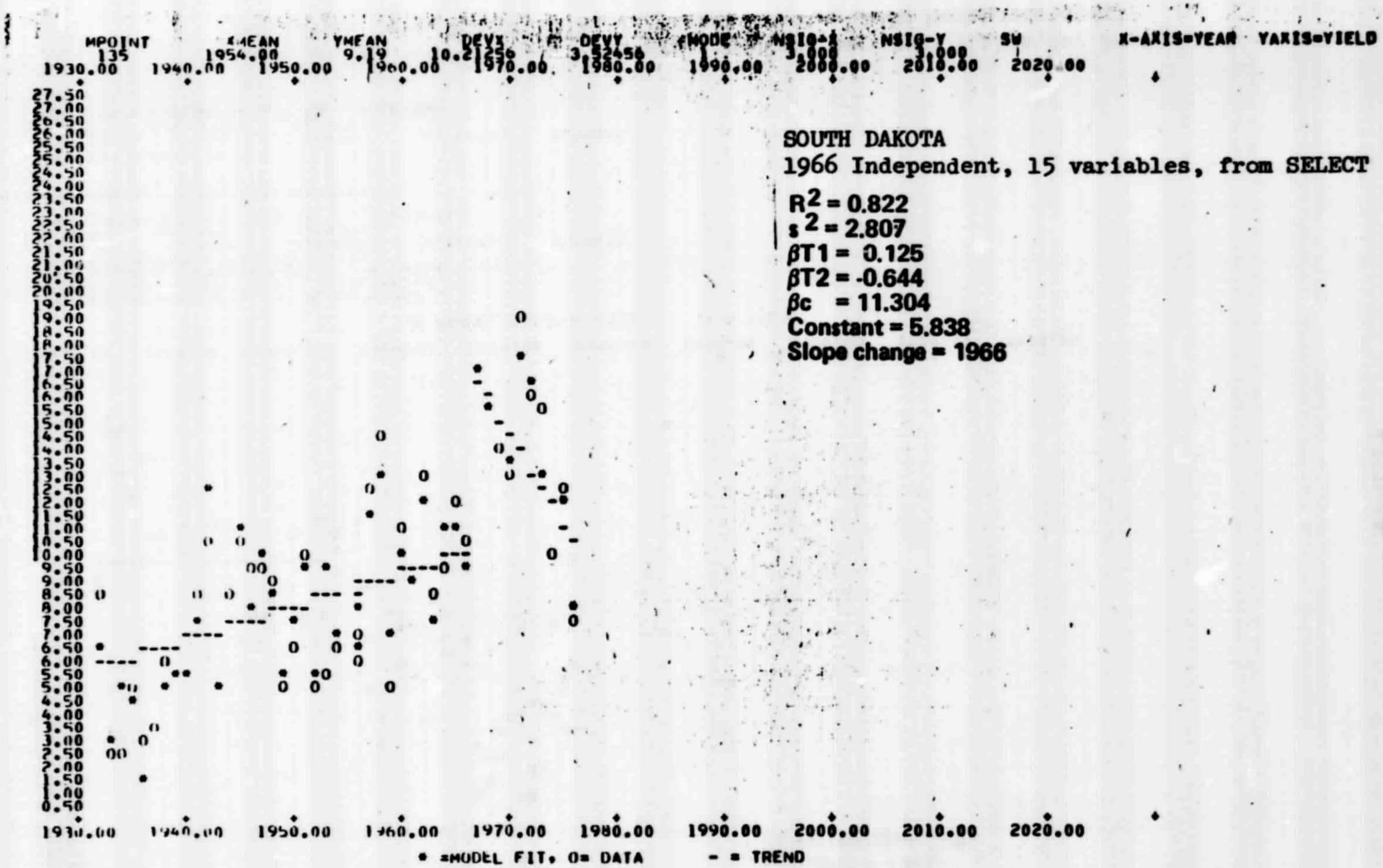
Figure D-4(b).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Red River Valley, 1955 Independent model.



THE DETERMINANT OF ... IS 0.351761997
THE TRACE OF ... INVERSE IS 10.484948
THE CHI-SQUARED VALUE IS 42.3144
THE STANDARDIZED BETA VECTOR IS
-0.09756 0.20579 0.07445 0.16390 0.10679
-0.24294 -0.15243 0.55759
THE STANDARDIZED LENGTH IS 0.77041

THE BETA VECTOR IS
-0.05111 ... ... 0.05690 0.06000
-0.53965 ... ...
R-SQUARE GOODNESS OF FIT IS 0.827366

Figure D-5(a).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for South Dakota, 1953 Dependent model.




```
THE DETERMINANT OF R IS  0.005573604
THE TRACE OF R INVERSE IS    36.321793
THE CHI-SQUARED VALUES IS      194.0740
THE STANDARDIZED BETA VECTOR IS
-0.11469     0.22917     0.19154     0.18152     -0.11902
 0.09521     0.24731     0.23656     0.24085     -0.11588
-0.22841     [illegible]  0.35542    -0.44204      1.17843

THE STANDARDIZED LENGTH IS       1.45540

*****************************************

THE BETA VECTOR IS
-0.06604     0.06526     0.03022     0.02281     -0.02194
 0.01692     0.04626     [illegible]  0.46935    -0.25741
-0.51947     0.21379     [illegible] -0.66428     11.30419

R-SQUARE GOODNESS OF FIT IS    0.882340
```

Figure D-5(b).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for South Dakota, 1966 Independent model.

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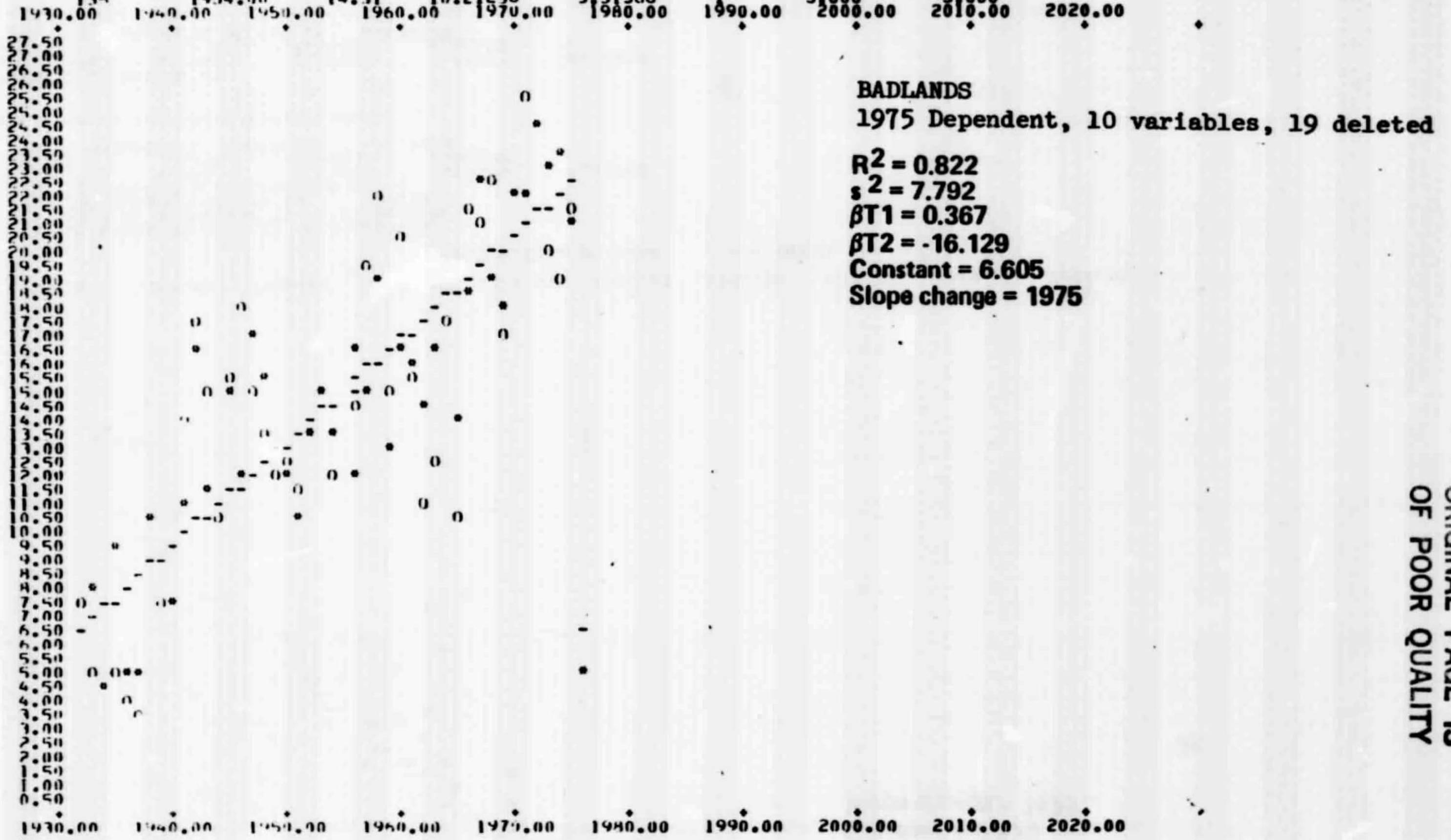
NPOINT 135 XMEAN 1954.00 YMEAN 14.31 DEVX 10.21256 DEVY 5.31566 MODE 1 NSIG-X 3.000 NSIG-Y 3.000 SW X-AXIS=YEAR YAXIS=YIELD



* = MODEL FIT. O= DATA - = TREND

THE DETERMINANT OF R IS 0.347519960
THE TRACE OF R INVERSE IS 12.644490
THE CHI-SQUARED VALUES IS 43.2523
THE STANDARDIZED BETA VECTOR IS

0.10396	0.29564	0.15185	0.13396	-0.07516
0.16433	0.67990	-0.09096	0.81433	-0.40738

THE STANDARDIZED LENGTH IS 1.00776

****

THE BETA VECTOR IS

0.02520	0.12408	0.09098	0.25621	-0.14345
0.37848	0.24525	-0.05170	0.30697	-16.12850

R-SQUARE GOODNESS OF FIT IS. 0.822063

Figure D-6(a).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Badlands, 1975 Dependent model.

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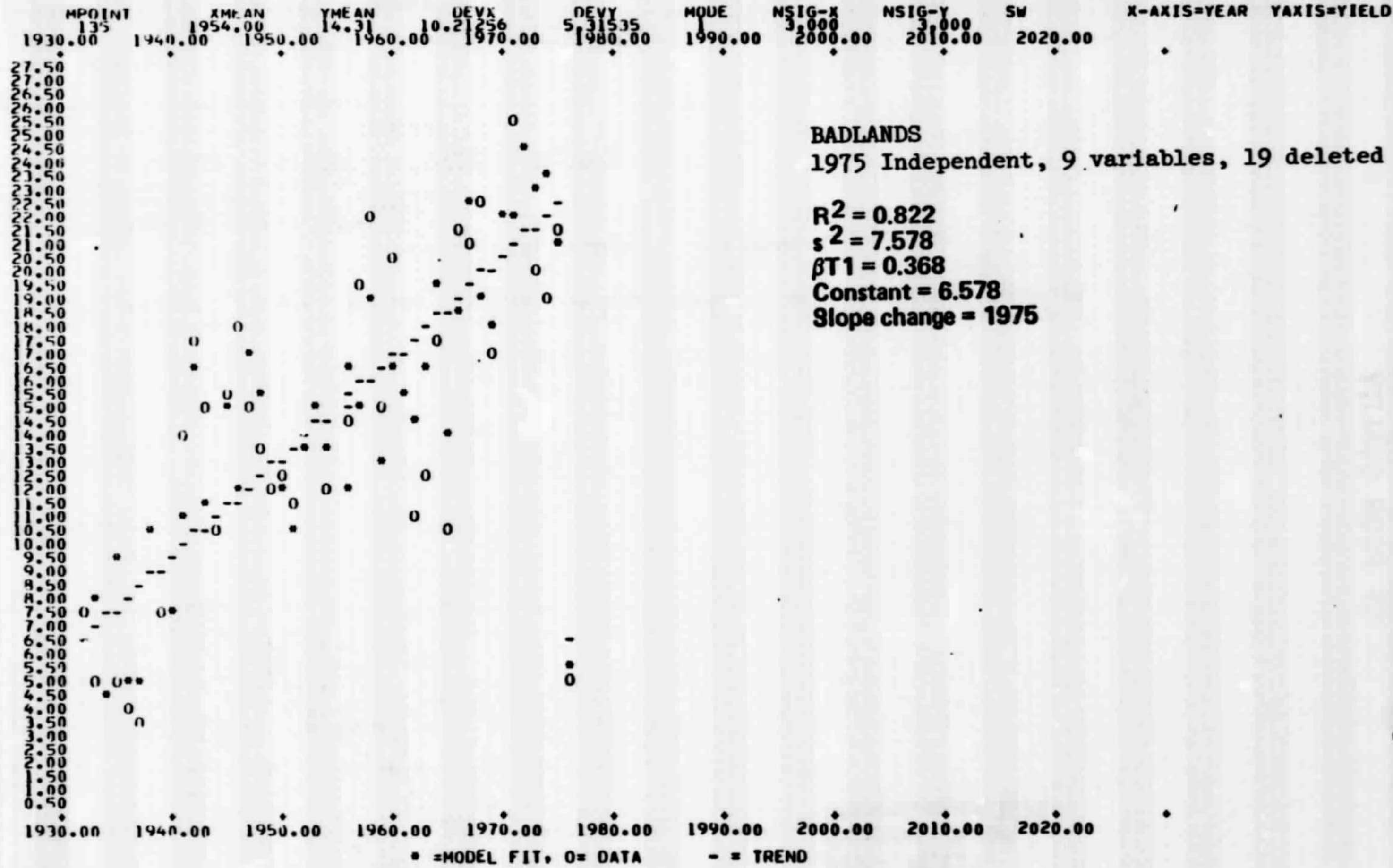


```
THE DETERMINANT OF R IS  0.39312H216
THE TRACE OF R INVERSE IS     11.263319
THE CHI-SQUARED VALUES IS        37.5004
THE STANDARDIZED BETA VECTOR IS
0.10299     0.29697     0.15220     0.13450     -0.07748
0.16489     0.07947    -0.09131     0.81639
THE STANDARDIZED LENGTH IS       0.92433
************************************

THE BETA VECTOR IS
0.02496     0.12465     0.09119     0.25743     -0.14787
0.37976     0.24393    -0.04186     0.36790
R-SQUARE GOODNESS OF FIT IS    0.821996
```

Figure D-6(b).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Badlands, 1975 Independent model.

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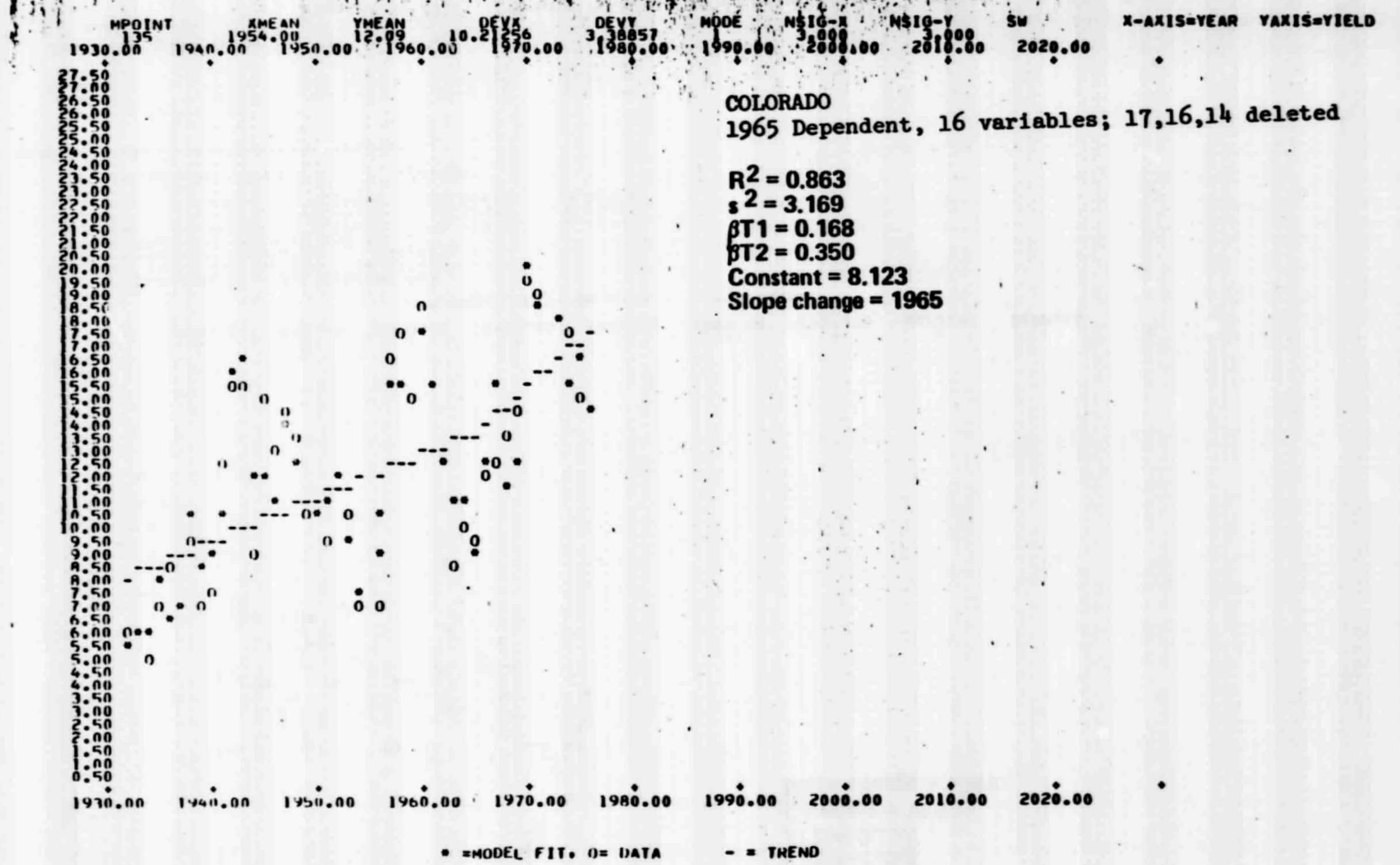



```
THE DETERMINANT OF R IS   0.011079635
THE TRACE OF R INVERSE IS     29.392761
THE CHI-SQUARED VALUES IS      170.3501
THE STANDARDIZED BETA VECTOR IS
 0.18887      0.20350      0.11718      0.23558      0.23894
 0.10936      0.55867     -0.11227     -0.02657      0.19128
-0.06435      0.18207      0.06034     -0.00827      0.48414

0.27338
THE STANDARDIZED LENGTH IS       0.96307

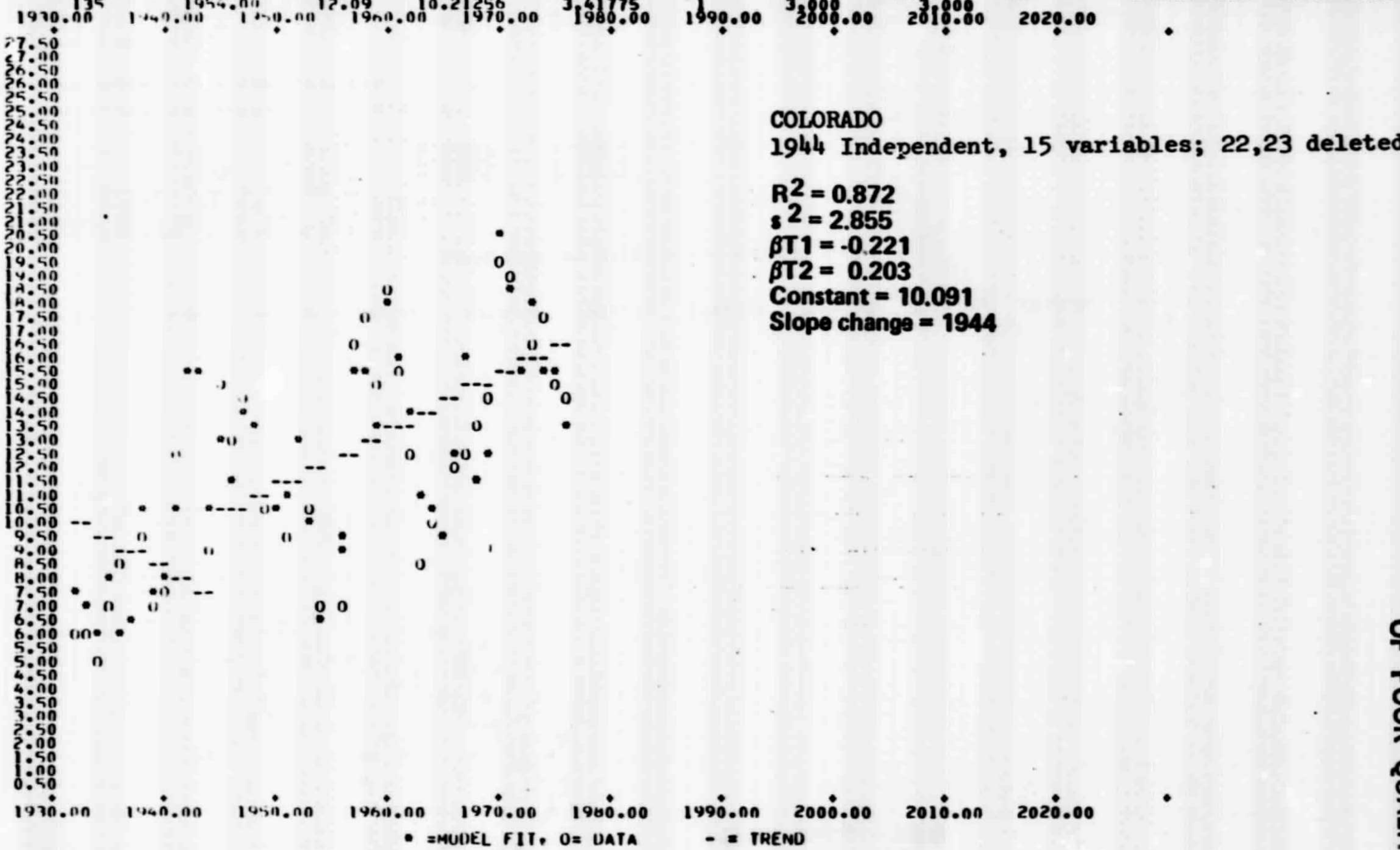
************************************

THE BETA VECTOR IS
 0.12002      0.11154      0.03974      0.03882      0.03724
 0.02061      0.10469     -0.06982     -0.04195      0.29771
-0.14601      0.09154      0.02371     -0.00308      0.16833

0.34958
R-SQUARE GOODNESS OF FIT IS   0.862639
```

Figure D-7(a).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Colorado, 1965 Dependent model.

MPOINT	XMEAN	YMEAN	DEVX	DEVY	MODE	NSIG-X	NSIG-Y	SW	
135	1954.00	12.09	10.21256	3.41775	1	3.000	3.000		X-AXIS=YEAR YAXIS=YIELD



THE DETERMINANT OF R IS 0.006429609
THE TRACE OF R INVERSE IS 30.823532
THE CHI-SQUARED VALUES IS 192.6211
THE STANDARDIZED BETA VECTOR IS

0.29180	0.14031	0.12076	0.31719	0.21793
0.14700	0.03204	-0.09062	0.30685	0.03115
0.00181	-0.13417	0.16265	-0.19419	0.56901

THE STANDARDIZED LENGTH IS 1.09457

••••••••••••••••••••••••••••••••••••••

THE BETA VECTOR IS

0.18542	0.07714	0.04045	0.05227	0.01396
0.02771	0.12419	-0.05635	0.47757	0.07952
0.00465	-0.14755	0.09174	-0.22134	0.20324

R-SQUARE GOODNESS OF FIT IS 0.872006

Figure D-7(b).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Colorado, 1944 Independent model.

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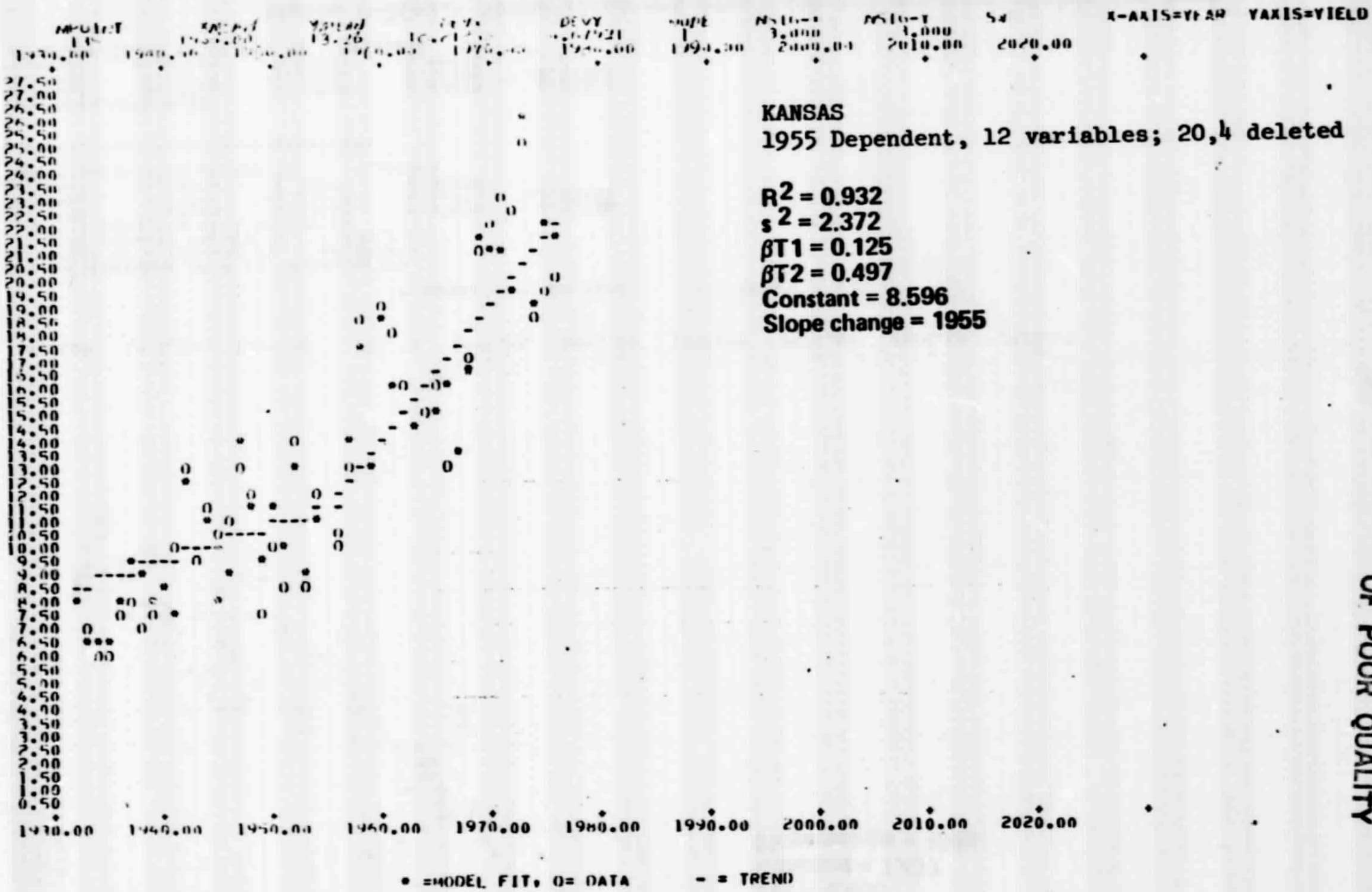


```
THE DETERMINANT OF R IS  0.000254856
THE TRACE OF R INVERSE IS    64.718857
 THE CHI-SQUARED VALUES IS     324.0967
 THE STANDARDIZED BETA VECTOR IS
-0.05327      0.10740      0.13116     -0.08154     -0.09860
-0.17569      0.61065     -0.55335     -0.10742     -0.09341
 0.18984      0.67614
 THE STANDARDIZED LENGTH IS        1.16784
****************************************

 THE BETA VECTOR IS
-0.02210      0.04295      0.03715     -0.00368     -0.01011
-0.00797      0.03086     -0.03851     -0.20086     -0.32309
 0.12530      0.49655
 R-SQUARE GOODNESS OF FIT IS    0.931696
```

Figure D-8(a).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Kansas, 1955 Dependent model.

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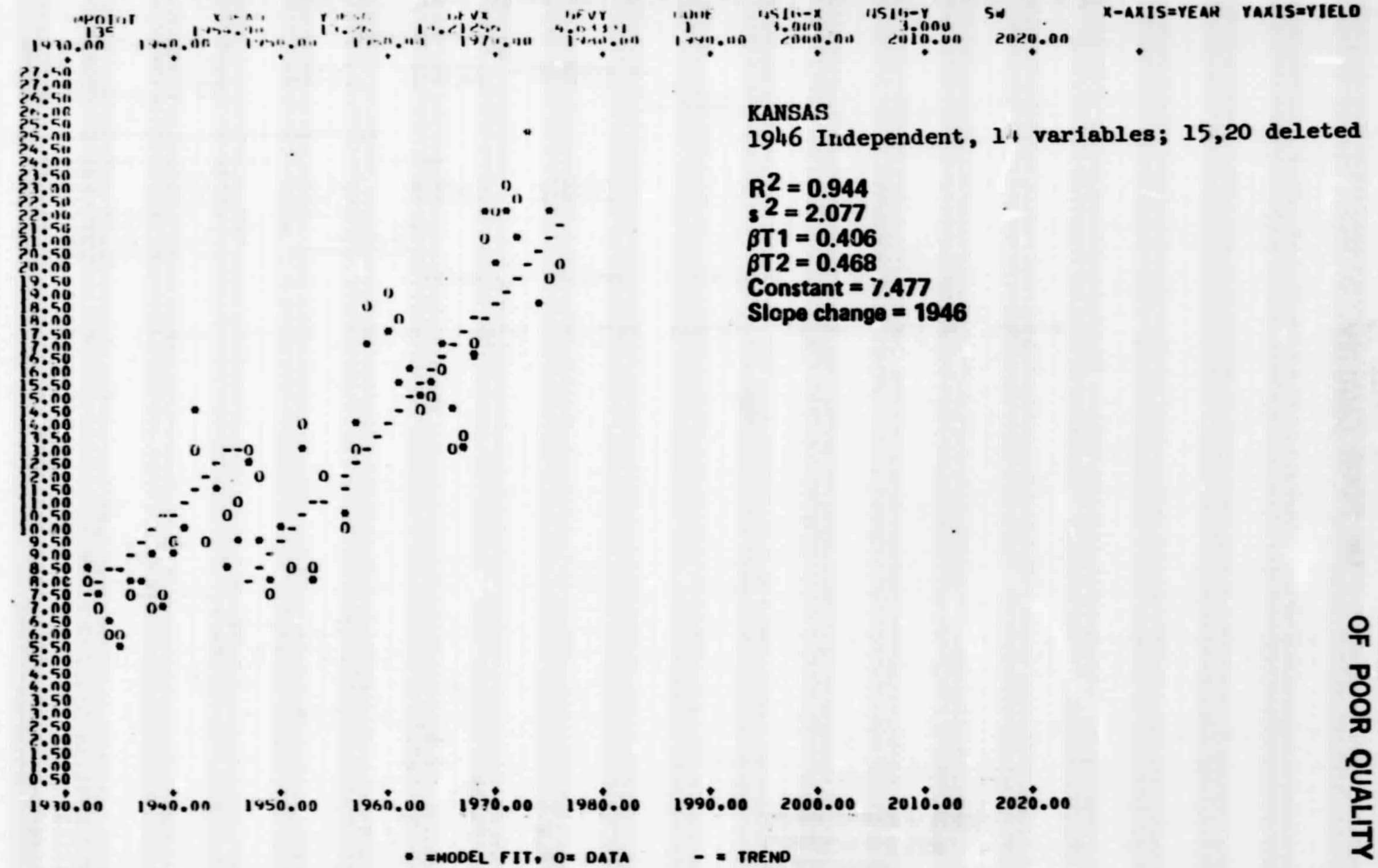
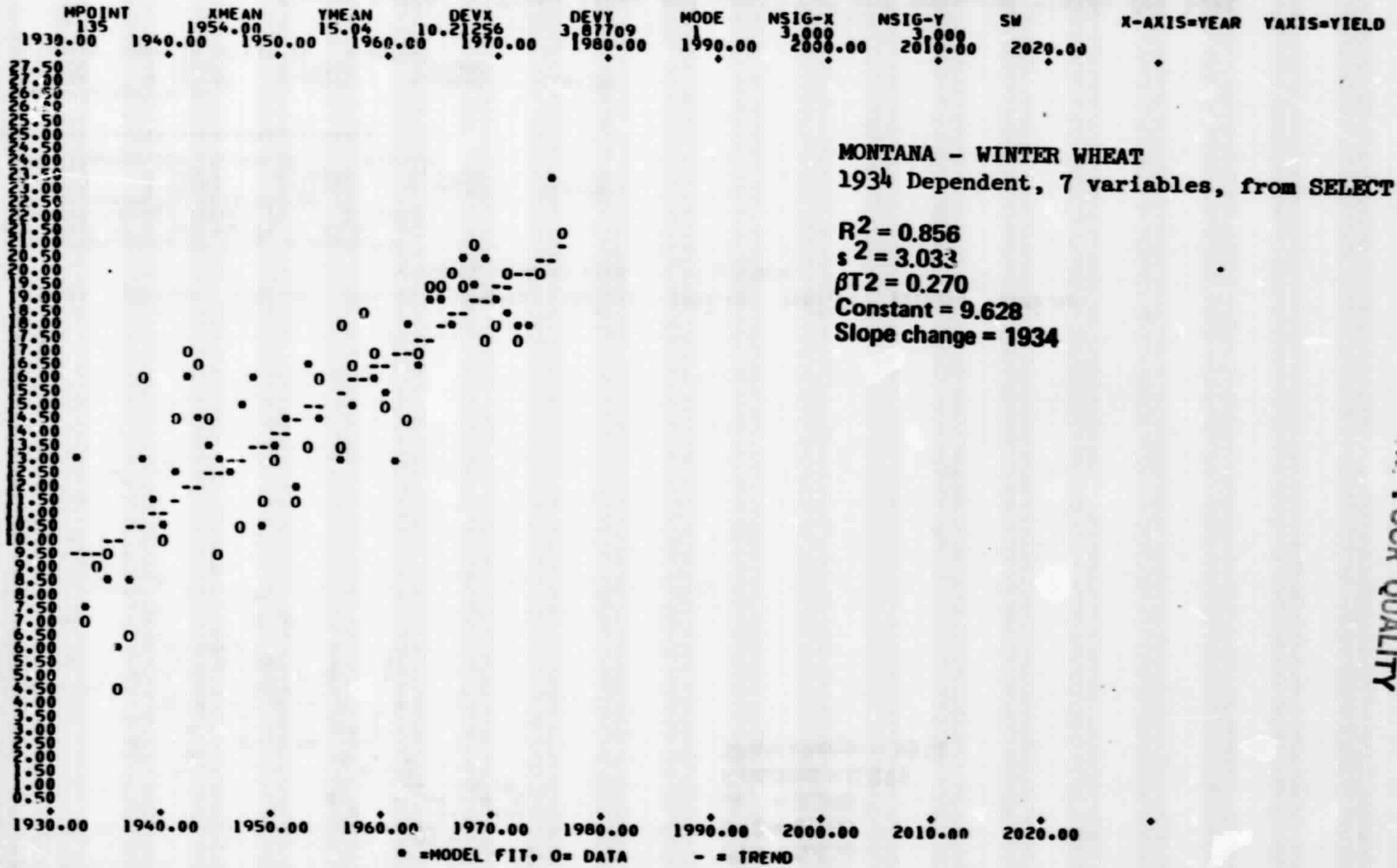



```
THE DETERMINANT OF R IS  0.000161129
THE TRACE OF R INVERSE IS    68.578979
THE CHI-SQUARED VALUES IS    336.2322
THE STANDARDIZED BETA VECTOR IS
-0.13592     0.17668     0.23142     0.67590     0.31114
-0.98984     0.15786    -0.13153     0.16773    -0.22481
-0.16543    -0.00332     0.33273     0.94229
THE STANDARDIZED LENGTH IS      1.66840
••••••••••••••••••••••••••••••••••••••••

THE BETA VECTOR IS
-0.05646     0.07049     0.02596     0.03416     0.03353
-0.06889     0.32235    -0.24594     0.50190    -0.77760
-0.20354    -0.00179     0.40595     0.46783
R-SQUARE GOODNESS OF FIT IS   0.943832
```

Figure D-8(b).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Kansas, 1946 Independent model.



THE DETERMINANT OF R IS 0.590842187
THE TRACE OF R INVERSE IS 8.127706
THE CHI-SQUARED VALUES IS 31.4867
THE STANDARDIZED BETA VECTOR IS

0.16167	0.10227	0.12135	0.27961	0.18929
0.12853	0.82312			

THE STANDARDIZED LENGTH IS 0.92703

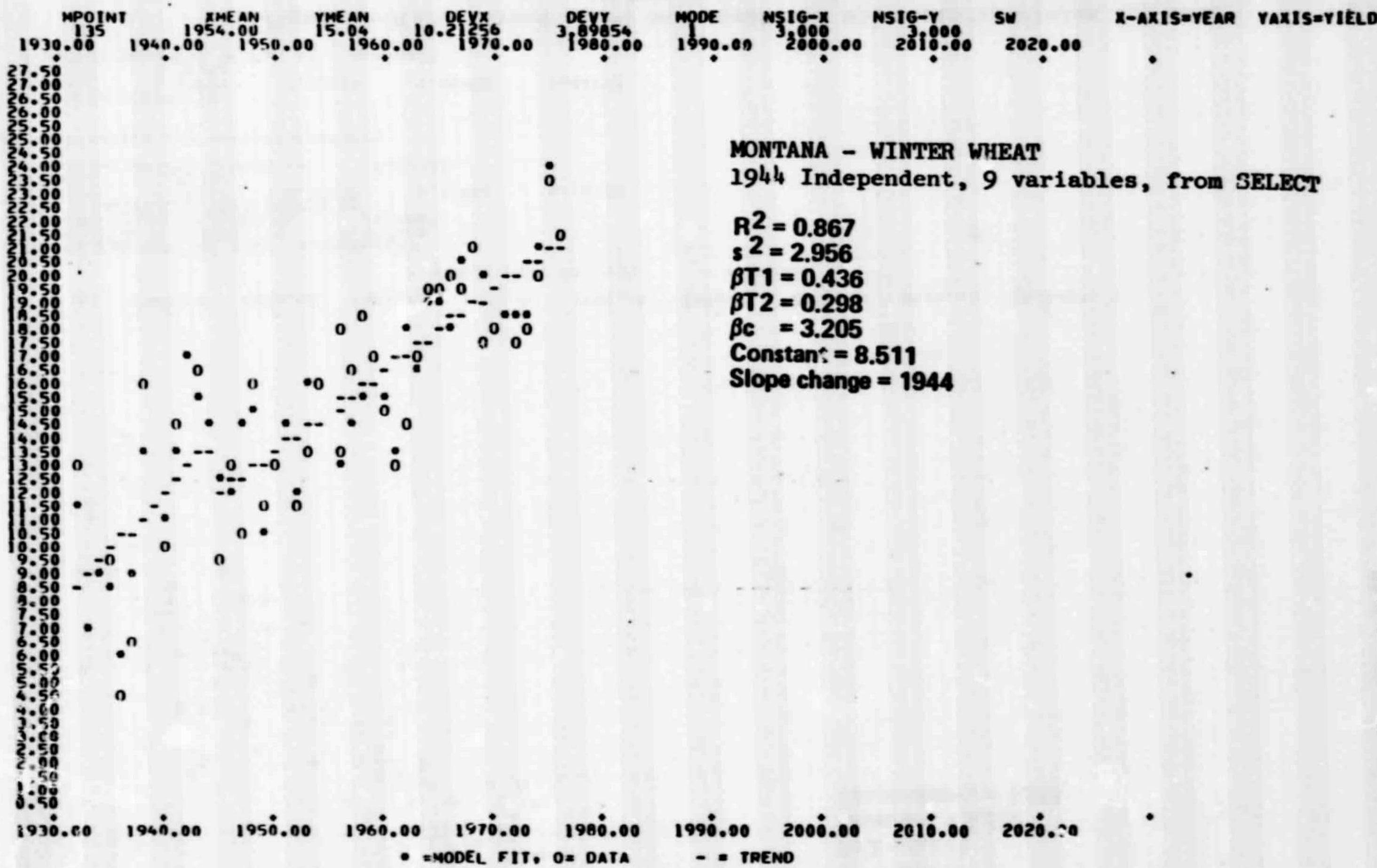
••••••c••••••••••••••••••••••••••••••••

THE BETA VECTOR IS

0.10304	0.03088	0.02205	0.04505	0.04792
0.07697	0.26961			

R-SQUARE GOODNESS OF FIT IS 0.856087

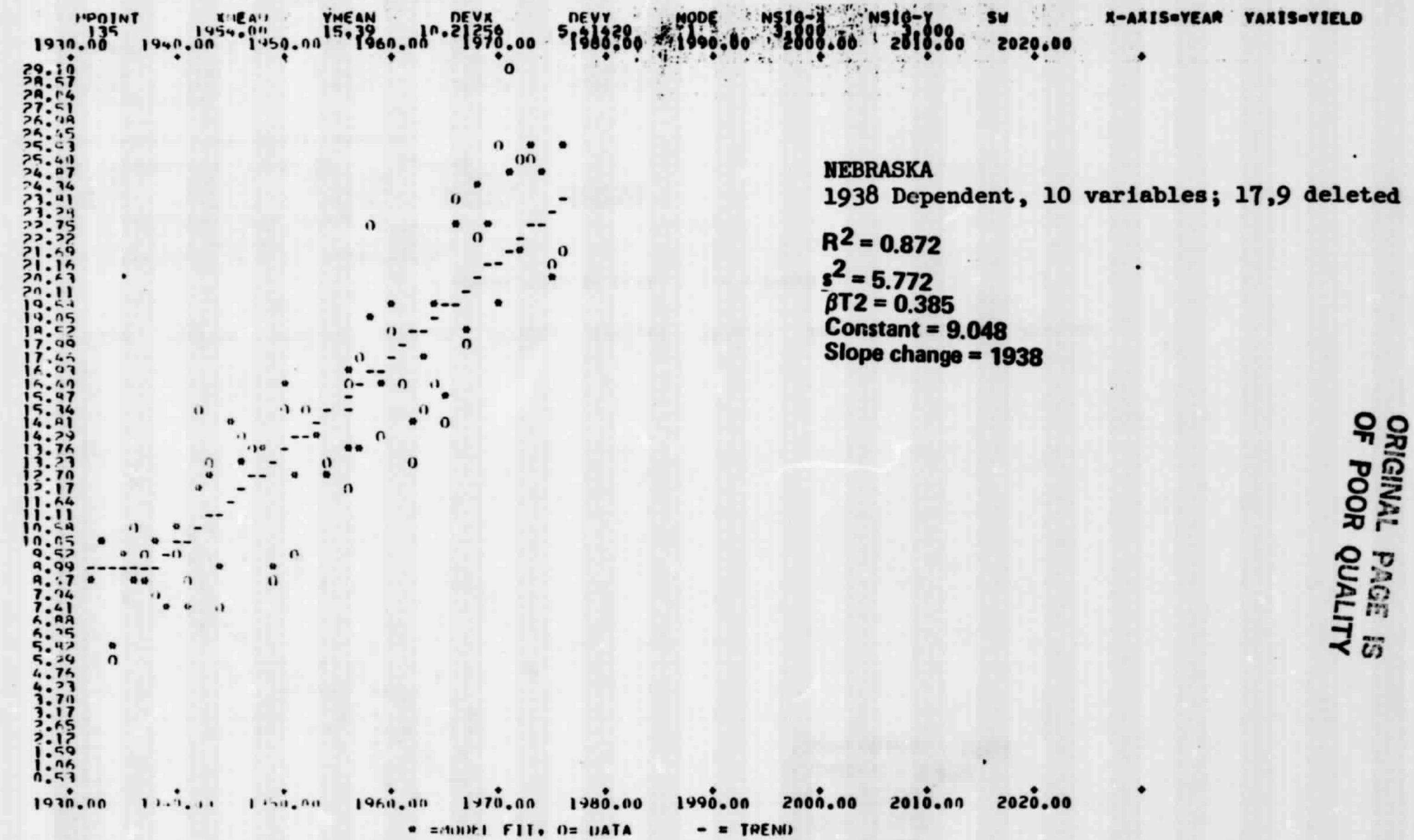
Figure D-9(a).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Montana winter wheat, 1934 Dependent model.




```
THE DETERMINANT OF R IS   0.080433249
THE TRACE OF R INVERSE IS    17.493500
THE CHI-SQUARED VALUES IS    101.2331
THE STANDARDIZED BETA VECTOR IS
0.17678     0.09761     0.11421     0.24175     0.17987
0.13817     0.34917     0.76178     6.34434
THE STANDARDIZED LENGTH IS      0.99222
**********************************

THE BETA VECTOR IS
0.11266     0.02948     0.02075     0.03895     0.04554
0.08274     0.43552     0.29768     3.20502
R-SQUARE GOODNESS OF FIT IS    0.867143
```

Figure D-9(b).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Montana winter wheat, 1944 Independent model.



THE DETERMINANT OF ... IS 0.046733837
THE TRACE OF ... IS 21.652893
THE CHI-SQUARE VALUE IS 121.8508
THE STANDARDIZED BETA VECTOR IS

0.07142	-0.14735	-0.29690	0.24132	-0.21258
0.07958	[illegible]	-0.13198	-0.19852	0.40202

THE STANDARDIZED LENGTH IS 1.01207

THE BETA VECTOR IS

0.63953	[illegible]	[illegible]	0.41581	-0.02567
0.27397	[illegible]	[illegible]	-0.00217	0.38531

R-SQUARE GOODNESS OF FIT IS 0.871605

Figure D-10(a).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Nebraska, 1938 Dependent model.

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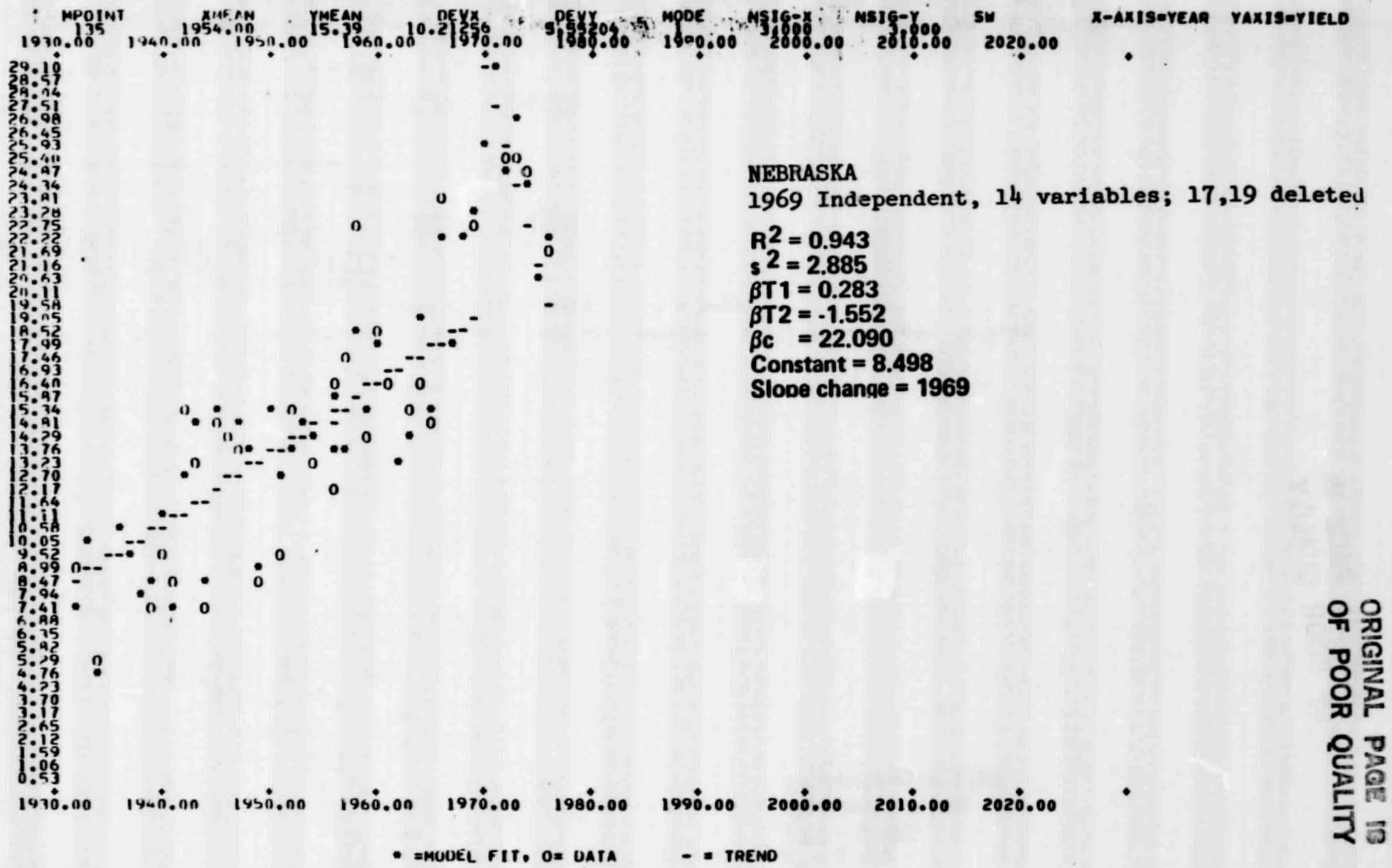
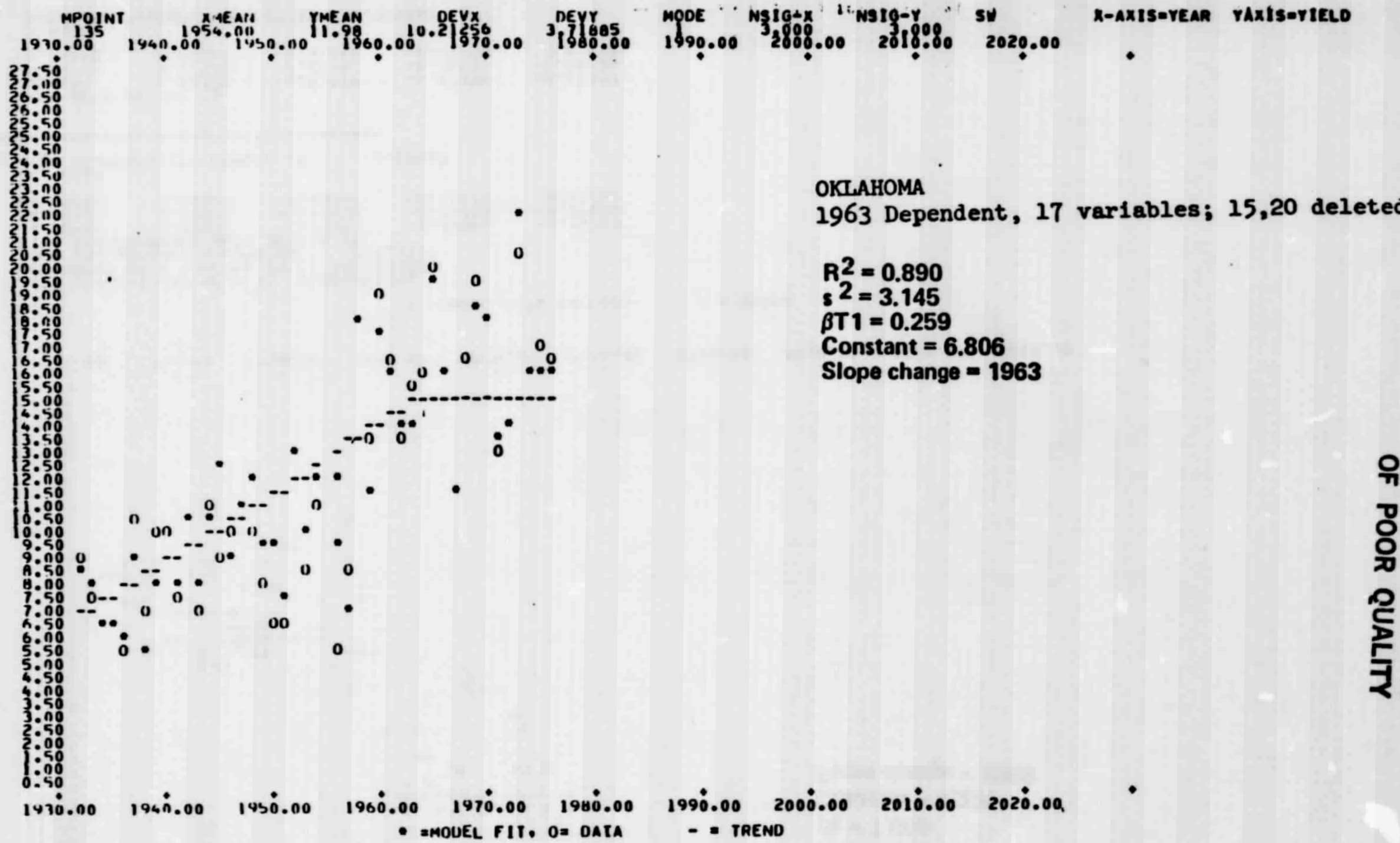



```
THE DETERMINANT OF R IS   0.000701714
THE TRACE OF R INVERSE IS    45.054810
THE CHI-SQUARED VALUES IS    279.5864
THE STANDARDIZED BETA VECTOR IS
-0.08270    -0.20035    -0.23248     0.06055     0.25681
-0.24885     0.08146    -0.18969    -0.08119    -0.34468
-0.21175     0.57975    -0.43310     1.35397
THE STANDARDIZED LENGTH IS     1.67402
************************************

THE BETA VECTOR IS
-0.04801    -0.03262    -0.03412     0.01426     0.01962
-0.01522     0.01524    -0.40179    -0.17273    -0.14819
-0.08765     0.28324    -1.55171    22.09059
R-SQUARE GOODNESS OF FIT IS    0.943155
```

Figure D-10(b).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Nebraska, 1969 Independent model.




```
THE DETERMINANT OF R IS  0.013597976
THE TRACE OF R INVERSE IS     29.657944
THE CHI-SQUARED VALUES IS     161.1688
THE STANDARDIZED BETA VECTOR IS
 0.05619    -0.03864     0.20777    -0.10649    -0.39543
-0.31797    -0.02938     0.13710     0.19669    -0.02529
-0.06604    -0.25762    -0.25509     0.05442    -0.02571

-0.06915     0.63959
THE STANDARDIZED LENGTH IS      0.96373

************************************

THE BETA VECTOR IS
 0.01006    -0.00874     0.02503    -0.01136    -0.02860
-0.03104    -0.00373     0.01342     0.02350    -0.04474
-0.12359    -0.43636    -0.79929     0.07054    -0.01057

-0.02390     0.25910
R-SQUARE GOODNESS OF FIT IS     0.890348
```

Figure D-11(a).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Oklahoma, 1963 Dependent model.

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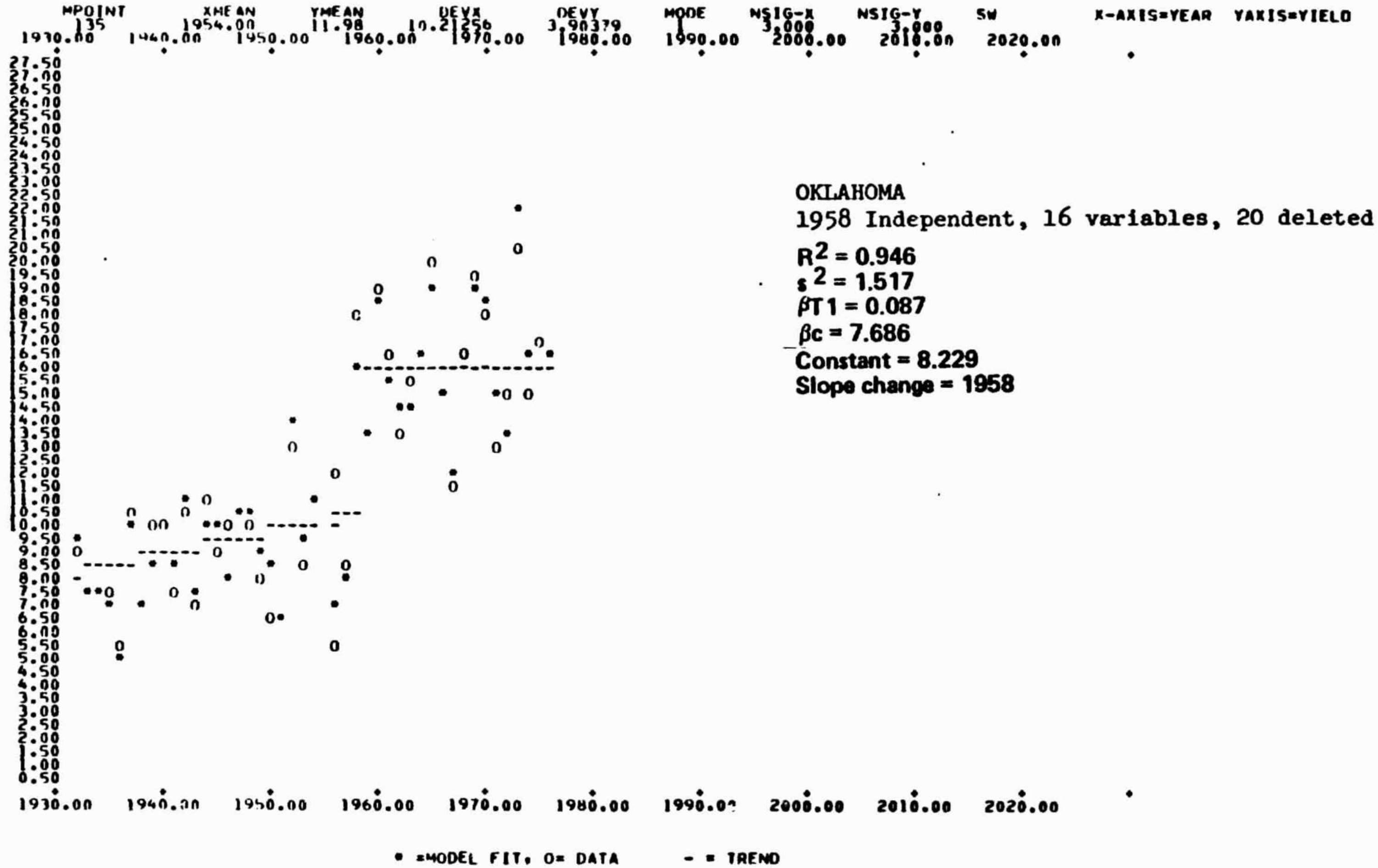


```
THE DETERMINANT OF R IS  0.002053981
THE TRACE OF R INVERSE IS    44.603485
THE CHI-SQUARED VALUES IS    234.1117
THE STANDARDIZED BETA VECTOR IS
 0.05845     0.20971    -0.25969    -0.24541    -0.10708
 0.16817     0.09375    -0.12437    -0.14715    -0.33160
-0.06662     0.21807    -0.03747    -0.11474     0.17960

0.88619
THE STANDARDIZED LENGTH IS      1.12019
************************************

THE BETA VECTOR IS
 0.01047     0.02829    -0.01878    -0.02396    -0.01359
 0.01646     0.01120    -0.02561    -0.26033    -0.56166
-0.20392     0.28269    -0.01541    -0.03385     0.08698

7.68590
R-SQUARE GOODNESS OF FIT IS   0.945862
```

Figure D-11(b).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Oklahoma, 1958 Independent model.

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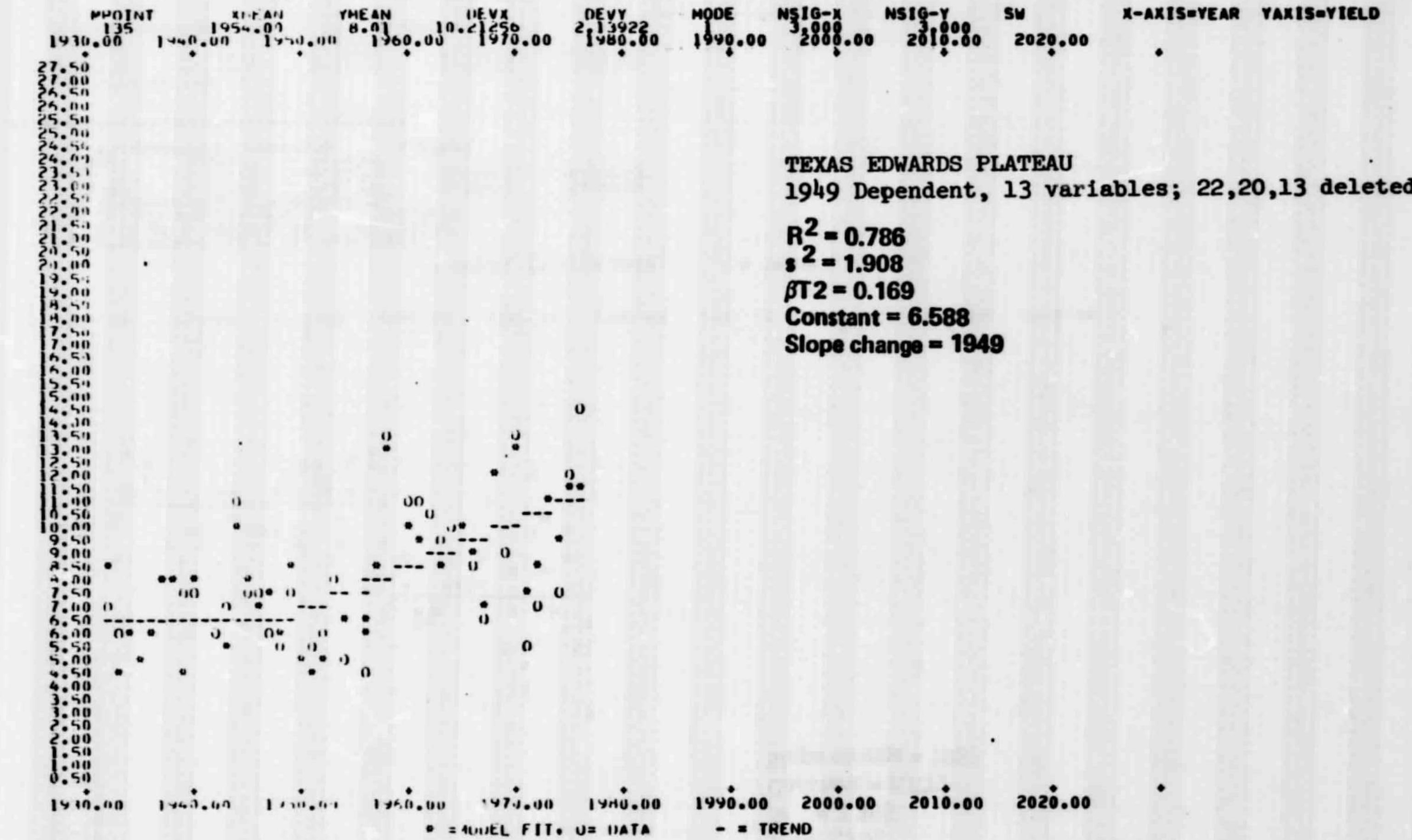


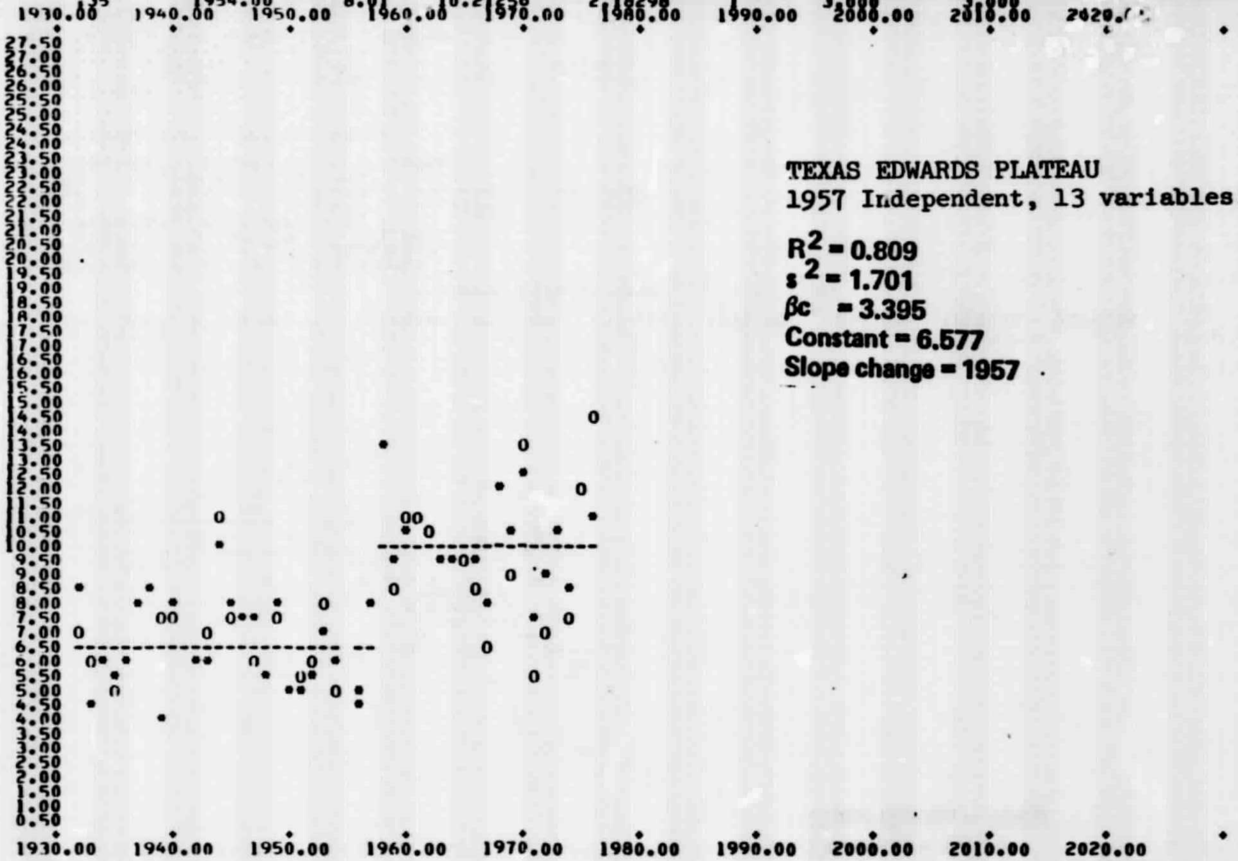
THE DETERMINANT OF R IS [illegible]
THE TRACE OF R INVERSE IS 20.936521
THE CHI-SQUARED VALUE IS 121.9510
THE STANDARDIZED BETA VECTOR IS
0.19443 [illegible] [illegible] -0.11784 0.20072
0.20840 [illegible] [illegible] -0.12420 -0.14012
0.21052 [illegible] [illegible]
THE STANDARDIZED [illegible] [illegible]

****

THE BETA VECTOR IS
0.01696 [illegible] [illegible] [illegible] [illegible]
0.01155 [illegible] [illegible] [illegible] [illegible]
0.37475 [illegible] [illegible]
R-SQUARE [illegible] FIT IS 0.786259

Figure D-12(a).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Texas Edwards Plateau, 1949 Dependent model.

MPOINT 135   XMEAN 1954.00   YMEAN 8.01   DEVX 10.21256   DEVY 2.18298   MODE 1   NSIG-X 3.000   NSIG-Y 3.000   SW   X-AXIS=YEAR   YAXIS=YIELD



TEXAS EDWARDS PLATEAU

1957 Independent, 13 variables; 22,20 deleted

R^2 = 0.809
s^2 = 1.701
βc = 3.395
Constant = 6.577
Slope change = 1957

* =MODEL FIT, 0= DATA   - = TREND

THE DETERMINANT OF R IS 0.0409502772
THE TRACE OF R INVERSE IS 20.985138
THE CHI-SQUARED VALUES IS 124.0879
THE STANDARDIZED BETA VECTOR IS

0.18202	0.17872	0.11392	-0.06892	0.18786
0.11824	0.17470	-0.22206	0.01968	-0.20663
0.25587	0.09447	0.66568		

THE STANDARDIZED LENGTH IS 0.87893

****

THE BETA VECTOR IS

0.01584	0.01880	0.01244	-0.00372	0.00810
0.00672	0.02077	-0.26682	0.02272	-0.5[illegible]70
0.09081	0.01883	3.39530		

R-SQUARE GOODNESS OF FIT IS 0.809380

Figure D-12(b).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Texas Edwards Plateau, 1957 Independent model.

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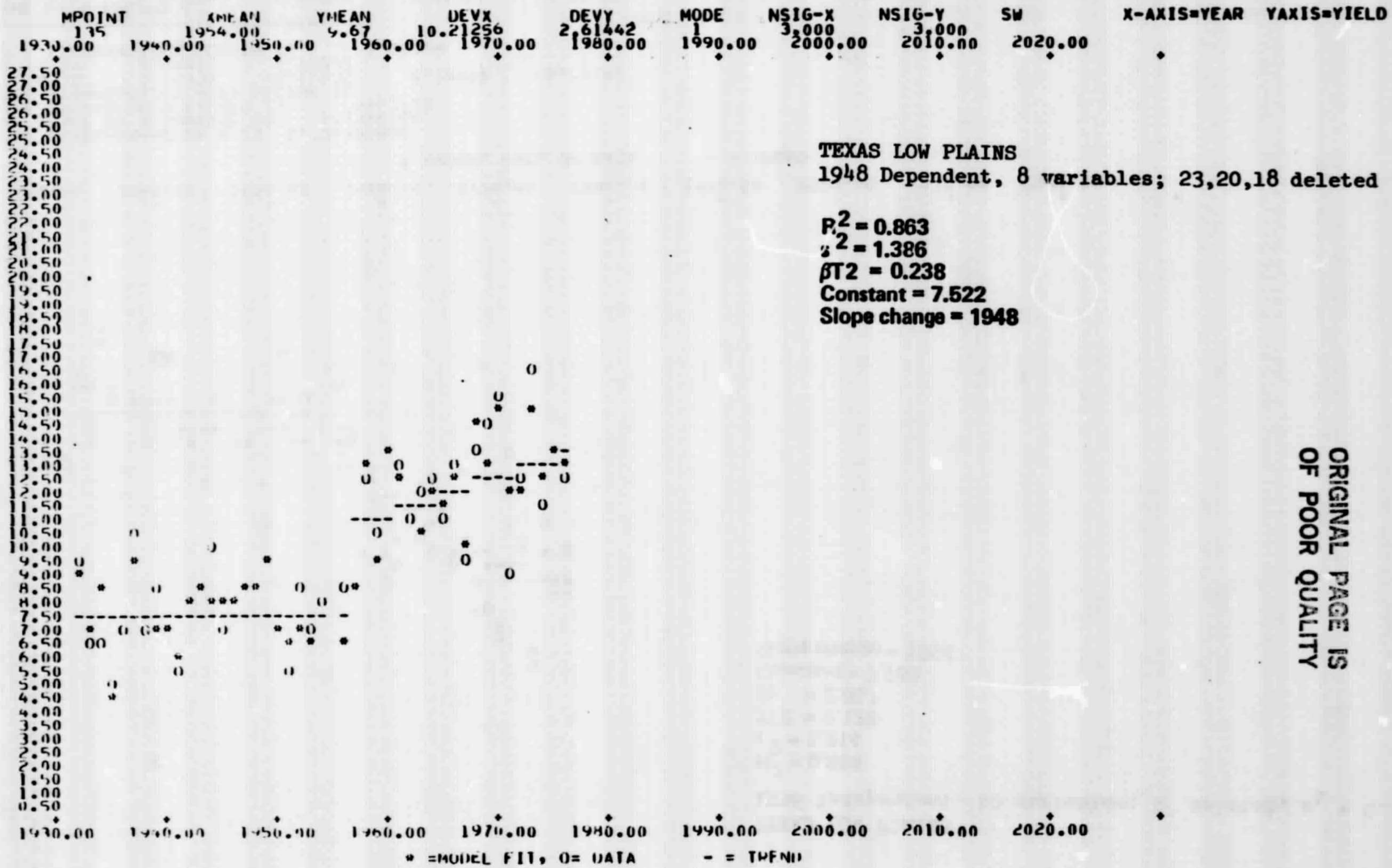


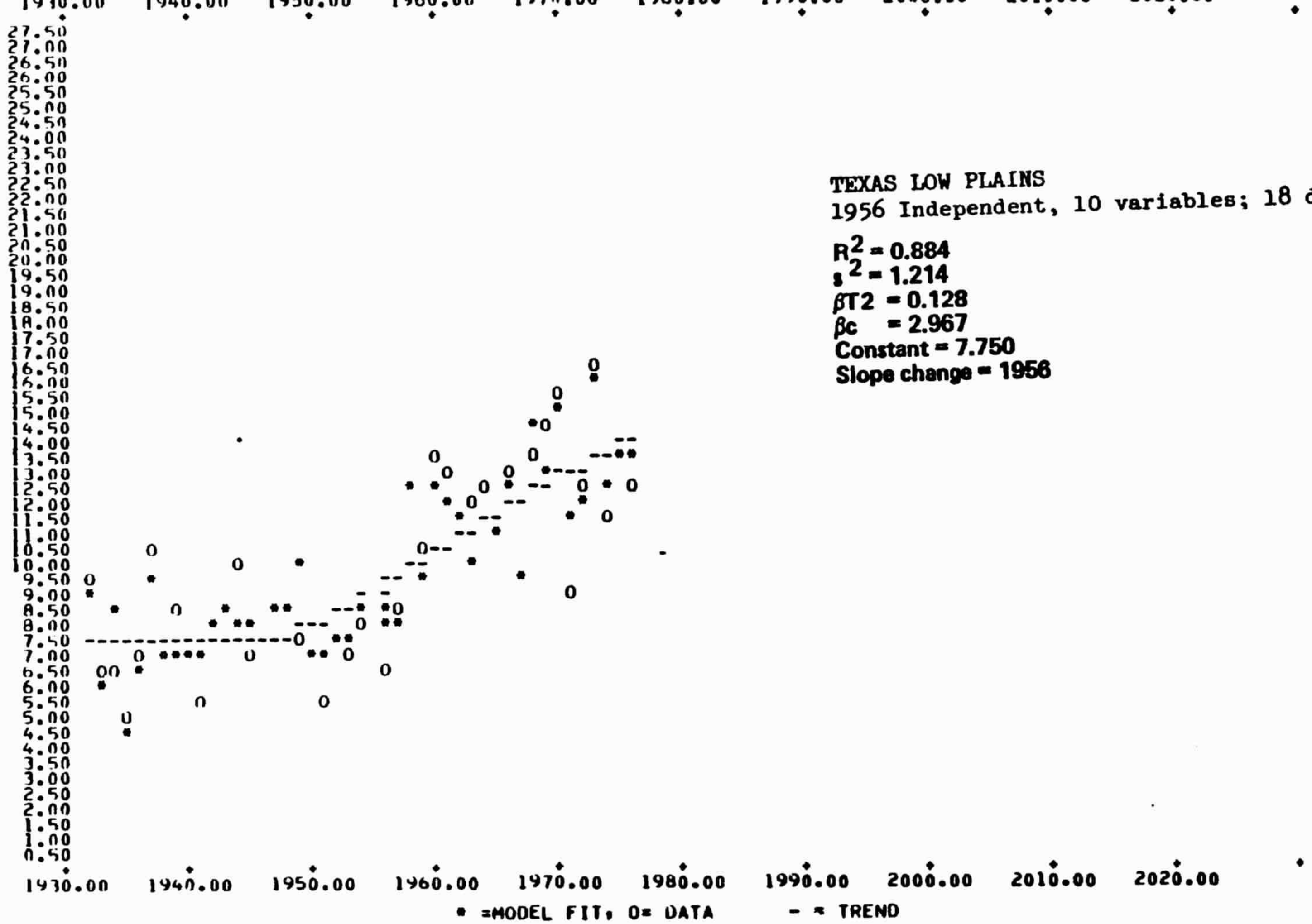
THE DETERMINANT OF R IS 0.174515069
THE TRACE OF R INVERSE IS 14.852288
THE CHI-SQUARED VALUES IS 59.5384
THE STANDARDIZED BETA VECTOR IS
0.15231 1.11811 -0.06277 -0.12424 -0.12473
0.11632 -0.13952 -0.19450 0.28851 0.51115

THE STANDARDIZED LENGTH IS 0.69815

****

Figure D-13(a).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Texas Low Plains, 1956 Independent model.

MPOINT	XMEAN	YMEAN	DEVX	DEVY	MODE	NSIG-X	NSIG-Y	SW	X-AXIS=YEAR	YAXIS=YIELD
135	1954.00	9.67	10.21256	2.62011	1	3.000	3.000			

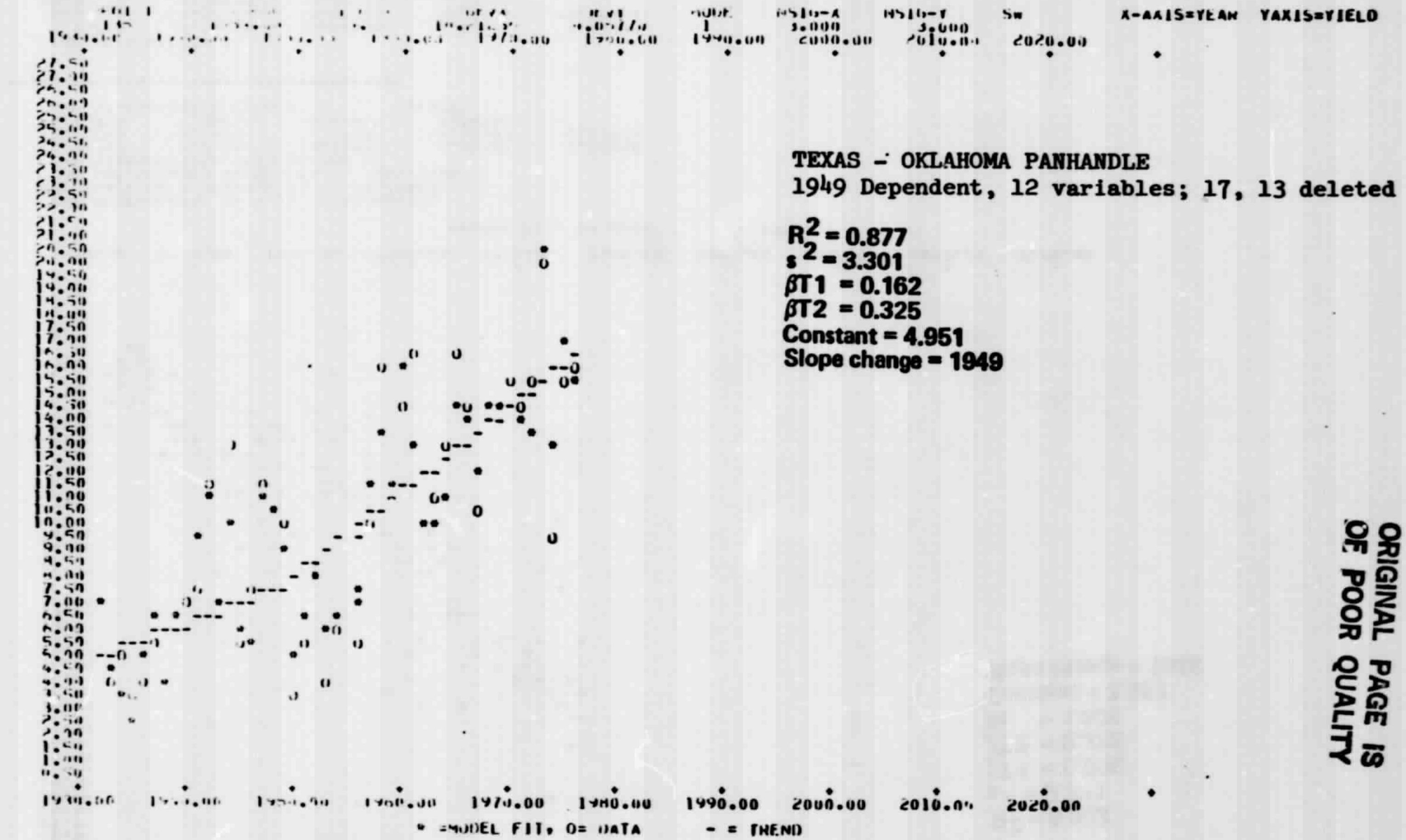


THE DETERMINANT OF R IS 0.334329605
THE TRACE OF R INVERSE IS 10.902164
THE CHI-SQUARED VALUES IS 44.3729
THE STANDARDIZED BETA VECTOR IS
0.16530 0.14934 -0.11535 0.10041 -0.19162
-0.24069 0.08815 0.78227
THE STANDARDIZED LENGTH IS 0.89734

****

THE BETA VECTOR IS
0.01769 0.01870 -0.00810 0.00661 -0.26400
-0.29936 0.20193 0.23783
R-SQUARE GOODNESS OF FIT IS 0.863061

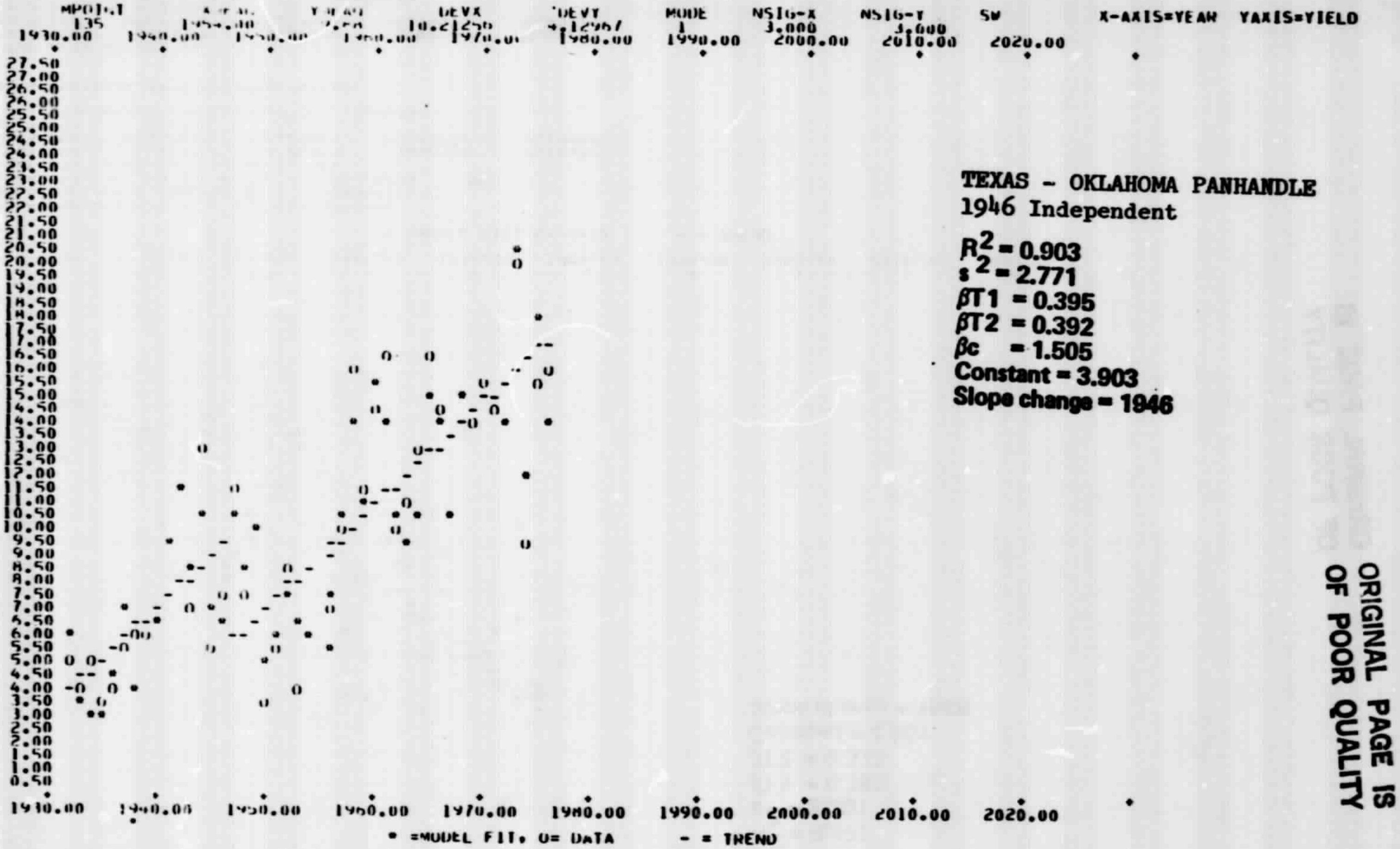
Figure D-13(b).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Texas Low Plains, 1948 Dependent model.



0.06550 0.16451
-0.13777 0.14408

THE BETA VECTOR IS
0.03720 0.03691 0.02020 0.01606 0.05291
-0.56700 -0.22679 -0.19656 -0.06650 0.05518
0.16161 0.32462
R-SQUARE GOODNESS OF FIT IS 0.876777

Figure D-14(a).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Texas-Oklahoma Panhandle, 1949 Dependent model.




```
THE DETERMINANT OF R IS  0.01362415B
THE TRACE OF R INVERSE IS    27.770874
THE CHI-SQUARED VALUES IS       165.3926
THE STANDARDIZED BETA VECTOR IS
 0.14767     0.2556?     0.11880     0.09921     0.10540
-0.19550    -0.2015?     0.27186    -0.16303    -0.19975
-0.13754     0.36855     0.5?835     0.16009
THE STANDARDIZED LENGTH IS        1.15275
****************************************

THE BETA VECTOR IS
 0.03216     0.03498     0.01540     0.02432     0.03390
-0.40144    -0.67370     0.96971    -0.13473    -0.09642
-0.06475     0.39490     0.39170     1.50524
R-SQUARE GOODNESS OF FIT IS    0.902829
```

Figure D-14(b).- Trend lines and predicted and measured yields for best regression from SELECT-BEIRA for Texas-Oklahoma Panhandle, 1946 Independent model.

APPENDIX E

CENTER FOR CLIMATIC AND ENVIRONMENTAL ASSESSMENT (CCEA) MODELS

APPENDIX E

CENTER FOR CLIMATIC AND ENVIRONMENTAL ASSESSMENT[a] (CCEA) MODELS

The following tables and figures are in Appendix E.

- Regression Coefficients

  Table E-1: The Center for Climatic and Environmental Assessment Models
  (a) spring wheat
  (b) winter wheat

- Plots showing measure yields and trend lines

  Figures E-1 through E-12

[a]This is a division of the U.S. Department of Agriculture.

(a) Spring wheat

Variable	MINNESOTA		MONTANA		NORTH DAKOTA		RED RIVER VALLEY		SOUTH DAKOTA	
	LACIE, Phase III, 9 variables	Transition year, 7 variables	LACIE, Phase III, 6 variables	Transition year, 6 variables	LACIE, Phase III, 11 variables	Transition year, 8 variables	LACIE, Phase III, 7 variables	Transition year, 5 variables	LACIE, Phase III, 8 variables	Transition year, 7 variables
Precipitation (UFR), mm:										
August-November						0.024		0.005		
August-March		-0.029	0.016	0.010	0.024		0.005		0.009	
September-November										0.011
October-March	-0.039									
April						-.014				
May	-.025	-.021		.021		.017				
June			.059	.036	.037	.017			.017	
September x June										.0003
June Precipitation (UFR)^2					-.0005					
Precipitation - PET (UFR), mm:										
April	-.041	-.022			.002					
May			.006							
July			.029							
April - PET (UFR)^2	-.0005				-.0004					
Precipitation/PET (UFR), mm:										
April									.724	.707
May					.702					
April/PET (UFR)^2									-.529	-.322
Temperature (UFR):										
April							.312	.207		
June	-.933								.396	-.609
July	-.855		-.643							
August		.293								
Temperature (UFR)^2:										
April							-.0?0			
May	-2.78									
August	.299									
Degree - days > 90° F:										
June					-.006		-1.179		-2.442	
July					-1.554		-1.979			
Number of days above 32° C:										
June		-.609		-.205		-.422		-.252		-.290
July		-.278				-.206		-.425		
Year of slope change	1955, 1975	1955, 1975	1955, 1972	1955	1955, 65, 72	1955, 65, 72	1955, 1972	1955	1955, 1972	1955
First trend			.171		.090		.227		.041	
Second trend	.757	.710	.307	.308	.697	.795	.438	.598	.422	.342
Third trend					.247	.105				
Overall constant	11.274	14.067	6.144	9.242	7.834	9.290	9.883	13.724	7.536	8.713
R^2	.90[illegible]	.859	.855	.821	.883	.870	.877	.870	.897	.843
s^2	[illegible].030	4.790	2.413	3.254	3.822	3.604	4.100	4.154	2.099	3.029
Truncation	August	August	July	July	July	July	July	July	July	July

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TABLE E-1.- Continued.

(b) Winter wheat

Variable	TEXAS EDWARDS PLATEAU		TEXAS LOW PLAINS		TEXAS-OKLAHOMA PANHANDLE	
	LACIE, Phase III, 11 variables	Transition year, 8 variables	LACIE, Phase III, 6 variables	Transition year, 8 variables	LACIE, Phase III, 8 variables	Transition year, 8 variables
Precipitation/PET (DFN), mm:						
April						2.503
Precipitation/PET (DFN)^2						
April						
Temperature in °C (DFN):						
December	-0.183					
December-January		-0.379		-0.447		
January	-.221					
January-February						-.200
May						
Temperature in °C (DFN):						
May	.303					
Degree-days >90° F:		(Constant for crop districts 81 and 82 = 2.774)				
May					-1.407	
June						
Number of days above 32° C:						
April		-.083				
May		-.054				.093
June						
Year of slope change	1975	1955, 1960, 1965	1955, 1972	1955, 1962	1955, 1960	1955, 1962
First trend	.069		0.033		.129	1.182
Second trend		.459	.820	.667	1.213	
Third trend				.021		
Fourth trend		.194				
Overall constant	6.983	7.512	6.851	7.093	4.337	6.080
R^2	.831	.760	.837	.847	.890	.884
s^2	1.670	1.905	1.607	1.590	2.643	3.008
Truncation	May	May	June	June	May	June

Symbol definition:

DFN - Departure from normal.
PET - Potential evapotranspiration.

TABLE E-1.- Continued

(b) Winter wheat

Variable	BADLANDS		COLORADO		KANSAS	
	LACIE, Phase III, 7 variables	Transition year, 8 variables	LACIE, Phase III, 8 variables	Transition year, 7 variables	LACIE, Phase III, 9 variables	Transition year, 8 variables
Precipitation (DFN), mm:						
August-November						0.019
August-February					0.009	
August-March			0.033			
September-November						
September-December						
September-February						
September-April						
October						
October-November	0.116	0.110				
October-February				0.083		
January-February						
February						
March						.057
May			.015	.023		
June	-.0003	.022	.038	.026	-.005	-.007
March x February				.0005		
Precipitation $(DFN)^2$, mm:						
August-March						
September-December						
February						
March						
May					-.0003	
June		-.0005	-.0010	-.0007	-.0001	
Precipitation - PET (DFN), mm:						
March	-.067	-.042			.042	
April	.028	.023	.027			
May						-.012
June						
Precipitation - PET $(DFN)^2$, mm:						
March					-.0001	
April						
May						-.0002
June						

Symbol definition:

DFN - Departure from normal.
PET - Potential evapotranspiration.

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TABLE E-1.- Continued.

(b) Winter wheat

Variable	BADLANDS		COLORADO		KANSAS	
	LACIE, Phase III, 7 variables	Transition year, 8 variables	LACIE, Phase III, 8 variables	Transition year, 7 variables	LACIE, Phase III, 9 variables	Transition year, 8 variables
Precipitation/PET (DFN), mm:						
April						
Precipitation/PET (DFN)^2						
April						
Temperature in °C (DFN):						
December						
December-January						
January						
January-February						
May	-0.353					
Temperature in °C (DFN):						
May						
Degree-days >90° F:						
May			-2.971		-1.137	
June	-.426					
Number of days above 32° C:						
April				-0.554		-0.301
May						
June						
Year of slope change	1955, 1972	1955, 1972	1955, 1972	1955	1955, 1972	1943, 1955, 1972
First trend	.407	0.373	.211		.173	
Second trend	.280	.246	.225	.337	.488	.243
Third trend						.528
Fourth trend						
Overall constant	6.243	6.825	8.114	11.418	8.528	9.408
R^2	.784	.754	.775	.781	.915	.928
s^2	8.689	9.954	4.251	3.938	2.750	2.139
Truncation	June	June	June	June	June	June

Symbol definition:

DFN = Departure from normal.
PET = Potential evapotranspiration.

TABLE E-1.- Continued.

(b) Winter wheat

Variable	MONTANA		NEBRASKA		OKLAHOMA	
	LACIE, Phase III, 8 variables	Transition year, 5 variables	LACIE, Phase III, 8 variables	Transition year, 7 variables	LACIE, Phase III, 8 variables	Transition year, 7 variables
Precipitation (DFN), mm:						
August-November						
August-February					0.006	
August-March	0.016					
September-November				0.023		
September-December						0.013
September-February						
September-April		0.0204				
October			0.049			
October-November						
October-February						
January-February						.014
February						
March				-.023	-.013	-.020
May						
June	.039	.040	-.033	-.043	-.008	-.016
March x February						
Precipitation $(DFN)^2$, mm:						
August-March	-.00042					
September-December						
February						
March						
May					-.0002	
June			-.0002			
Precipitation - PET (DFN), mm:						
March					.039	.033
April						
May	.029					
June		.028				
Precipitation - PET $(DFN)^2$, mm:						
March						
April	-.00027					
May						
June						

Symbol definition:

DFN = Departure from normal.
PET = Potential evapotranspiration.

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TABLE E-1.- Continued.

(b) Winter wheat

Variable	MONTANA		NEBRASKA		OKLAHOMA	
	LACIE, Phase III, 8 variables	Transition year, 5 variables	LACIE, Phase III, 8 variables	Transition year, 7 variables	LACIE, Phase III, 8 variables	Transition year, 7 variables
Precipitation/PET (DFN), mm:						
April						
Precipitation/PET $(DFN)^2$						
April			-0.151			
Temperature in °C (DFN):						
December						
December-January						
January						
January-February				-0.243		
May			-.577	-.736		
Temperature in °C (DFN):						
May						
Degree-days >90° F:						
May					-0.746	
June	-1.617		-1.004			
Number of days above 32° C:						
April						
May						-0.165
June		-0.274		-.224		
Year of slope change	1955	1943	1955, 1972	1955	1955, 1960	1943, 1962
First trend	0.229		.270	.295	.136	
Second trend	.282	.293	.480	.383	.953	.389
Third trend						
Fourth trend						
Overall constant	9.980	11.694	8.864	9.423	7.390	8.536
R^2	.835	.824	.901	.884	.900	.868
s^2	3.577	3.606	4.325	5.064	2.273	2.901
Truncation	June	June	June	June	June	June

Symbol definition:

DFN = Departure from normal.
PET = Potential evapotranspiration.

TABLE E-1.- Concluded.

(b) Winter wheat

Variable	TEXAS EDWARDS PLATEAU		TEXAS LOW PLAINS		TEXAS-OKLAHOMA PANHANDLE	
	LACIE, Phase III, 11 variables	Transition year, 8 variables	LACIE, Phase III, 6 variables	Transition year, 8 variables	LACIE, Phase III, 8 variables	Transition year, 8 variables
Precipitation (DFN), mm:						
August-November						
August-February			0.004		0.015	
August-March						
September-November				0.007		
September-December	0.009					0.014
September-February		0.006				
September-April						
October						
October-November						
October-February						
January-February				.011		.038
February	.034					
March						
May			-.008		.005	
June						-.015
March x February						
Precipitation $(DFN)^2$, mm:						
August-March						
September-December	-.00002					
February	-.00034					
March	-.0007					
May					-.0002	
June						
Precipitation - PET (DFN), mm:						
March	.028	.023	.025	.020	.022	.029
April				.004		.036
May				-.011		
June						
Precipitation - PET $(DFN)^2$, mm:						
March		-.00034				
April						
May						
June			-.00013			

Symbol definition:

DFN = Departure from normal.
PET = Potential evapotranspiration.

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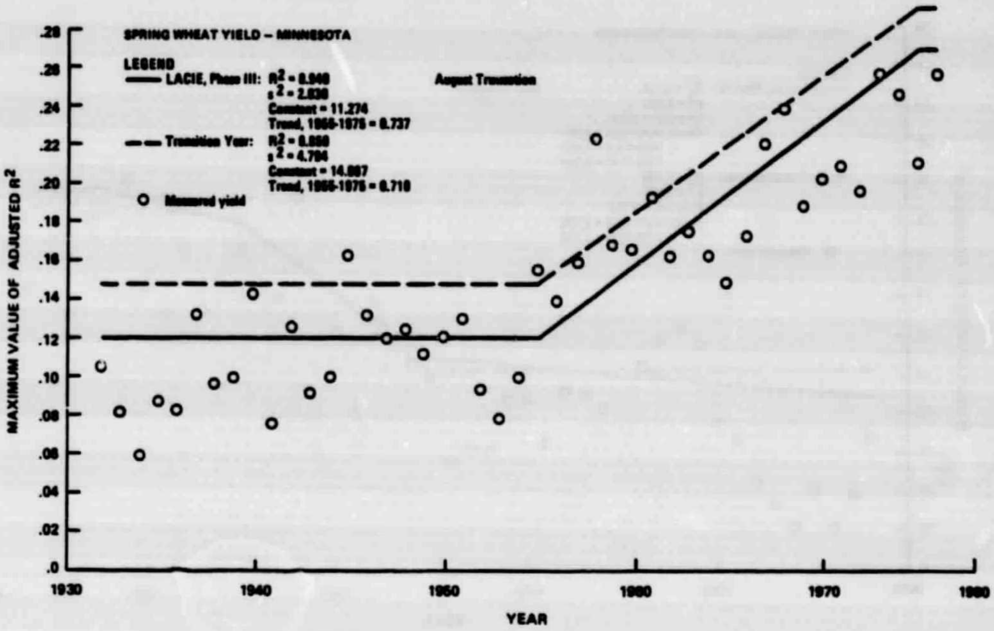

Figure E-1.- Trend lines for the CCEA models for spring wheat yield in Minnesota.

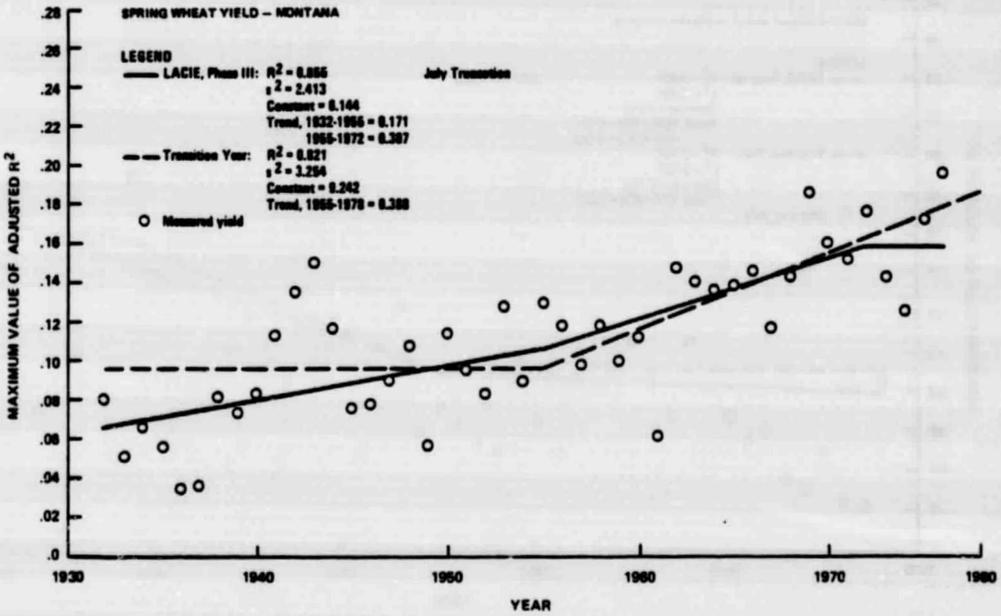

Figure E-2.- Trend lines for the CCEA models for spring wheat yield in Montana.

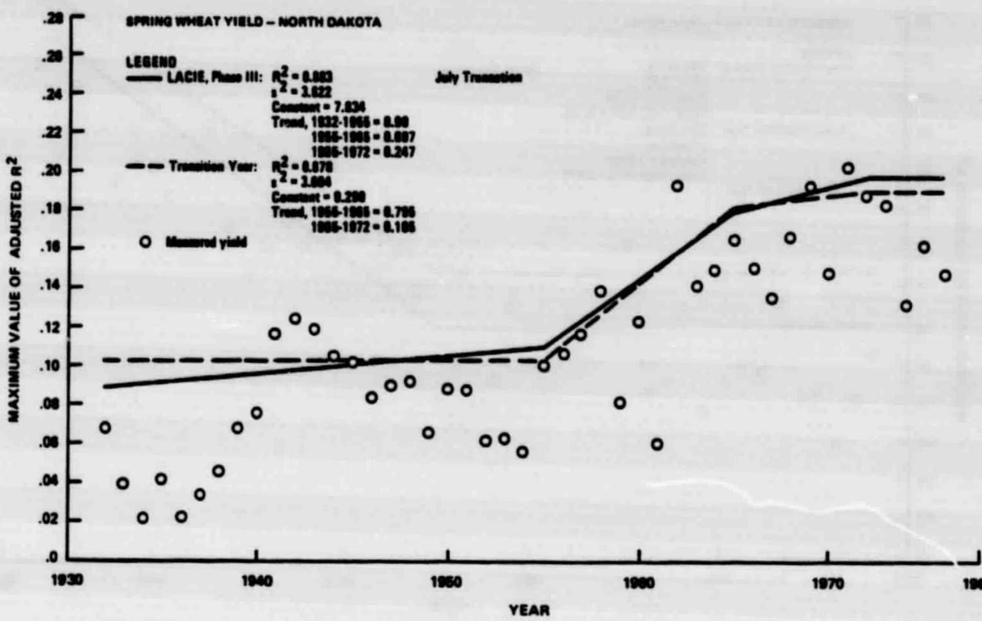

Figure E-3.- Trend lines for CCEA models for spring wheat yield in North Dakota.

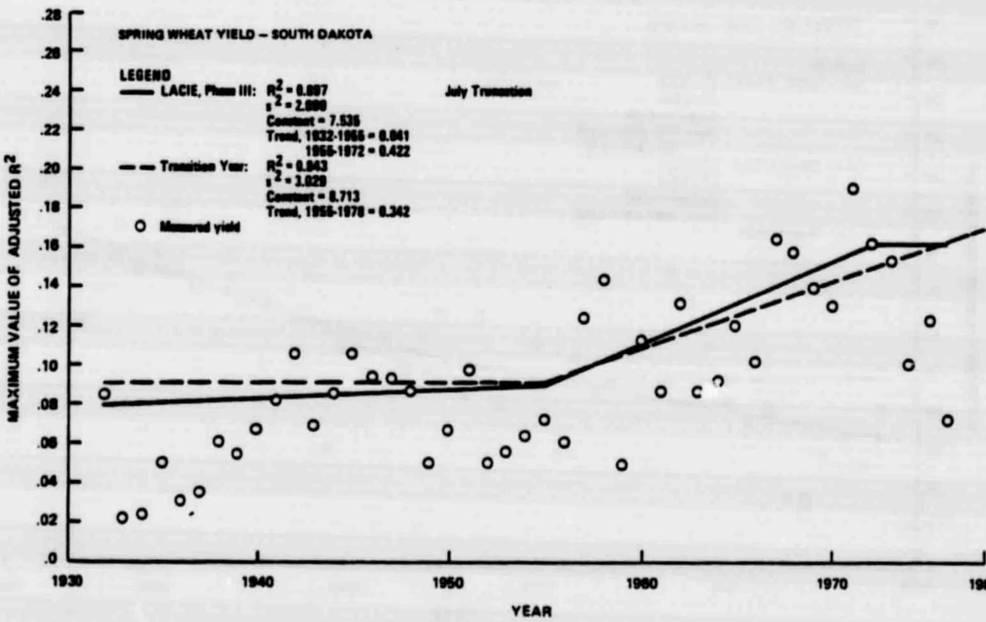

Figure E-4.- Trend lines for CCEA models for spring wheat yield in South Dakota.

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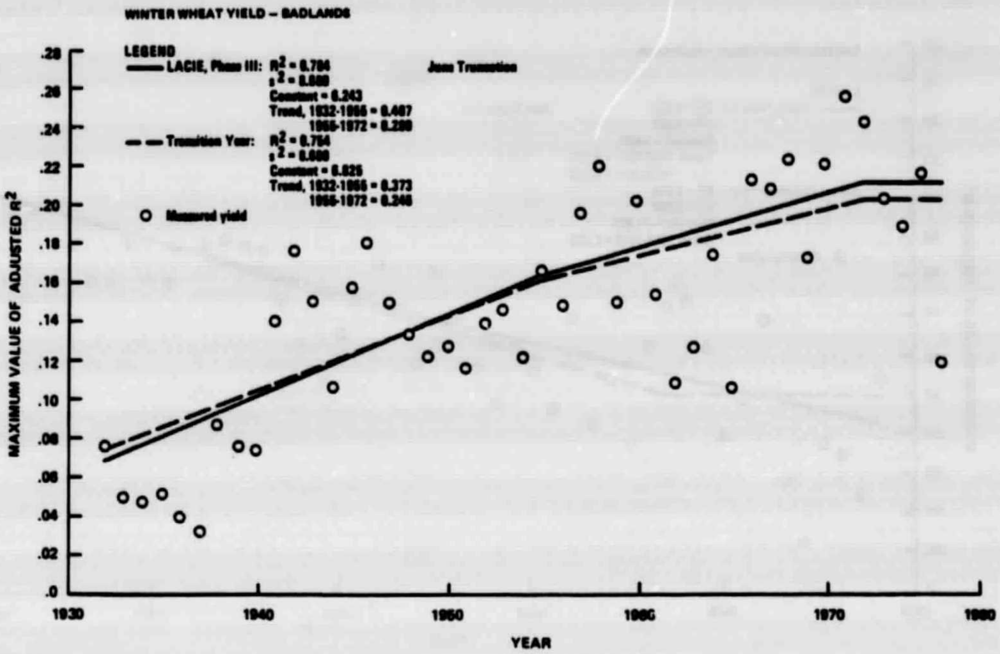

Figure E-5.- Trend lines for CCEA models for winter wheat yield in the Badlands.

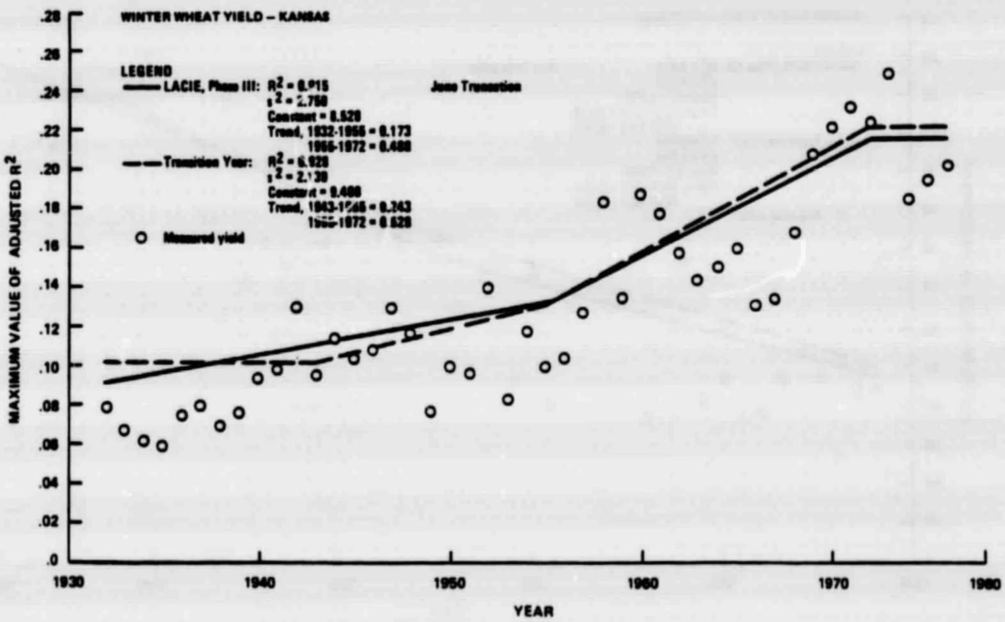

Figure E-6.- Trend lines for CCEA models for winter wheat yield in Kansas.

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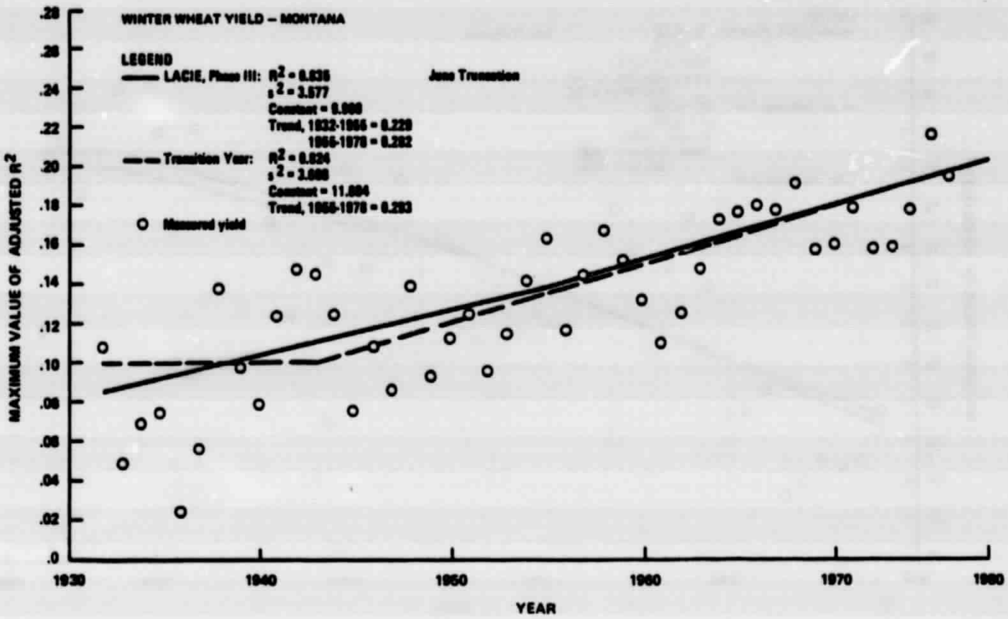

Figure E-7.- Trend lines for CCEA models for winter wheat yield in Montana.

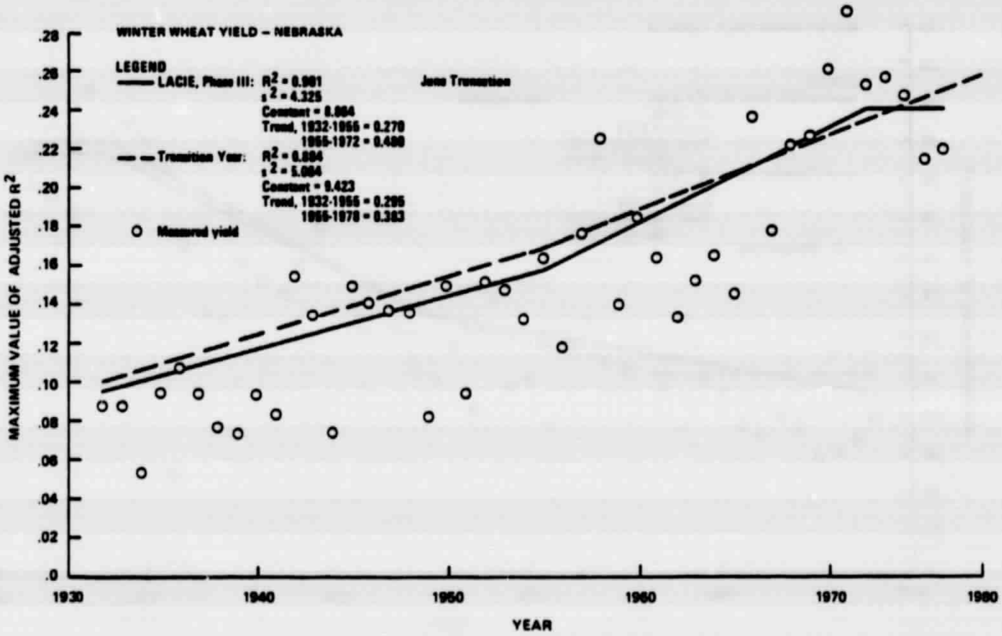

Figure F-8.- Trend lines for CCEA models for winter wheat yield in Nebraska.

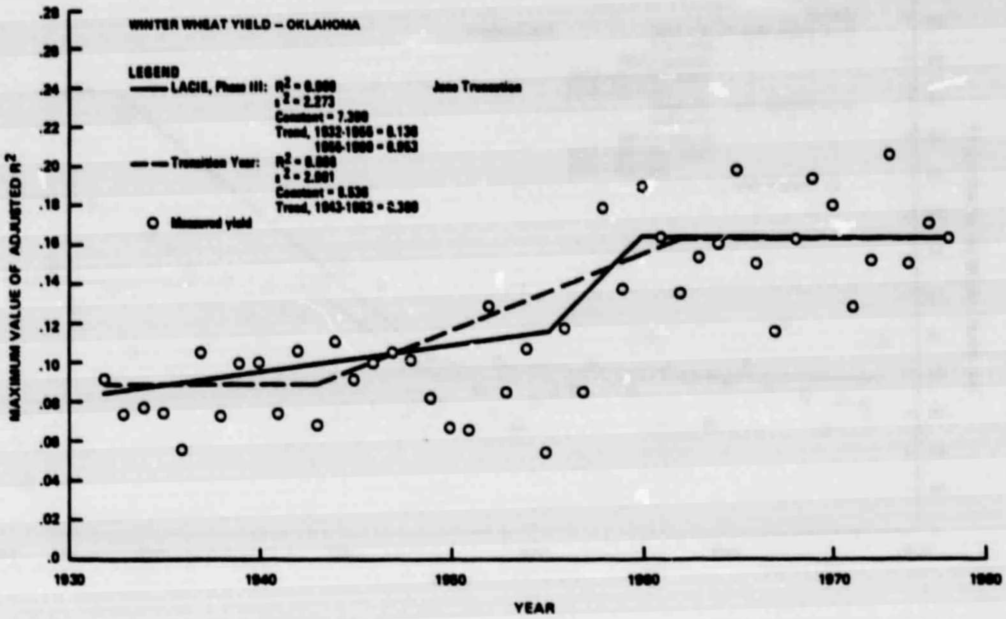

Figure E-9.- Trend lines for CCEA models for winter wheat yield in Oklahoma.

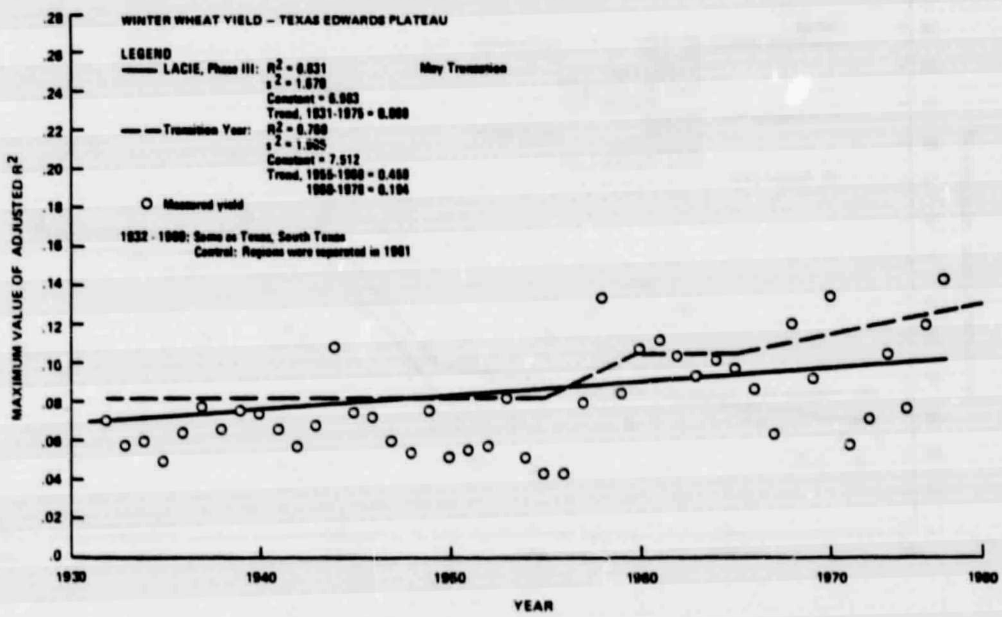

Figure E-10.- Trendlines for CCEA models for winter wheat yield in the Texas Edwards Plateau.

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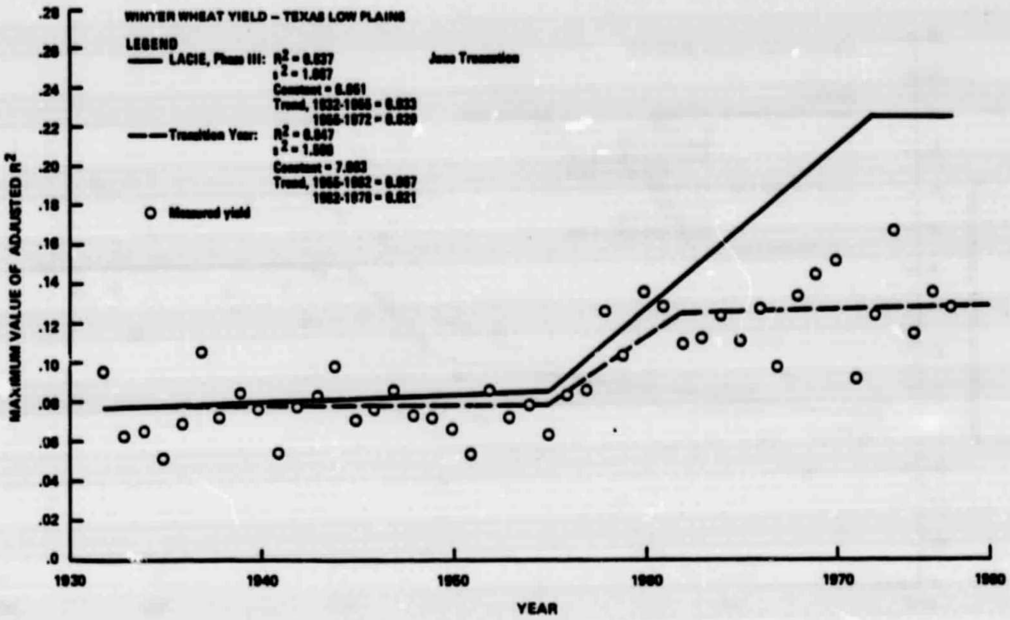

Figure E-11.- Trend lines for CCEA models for winter wheat models in the Texas Low Plains.

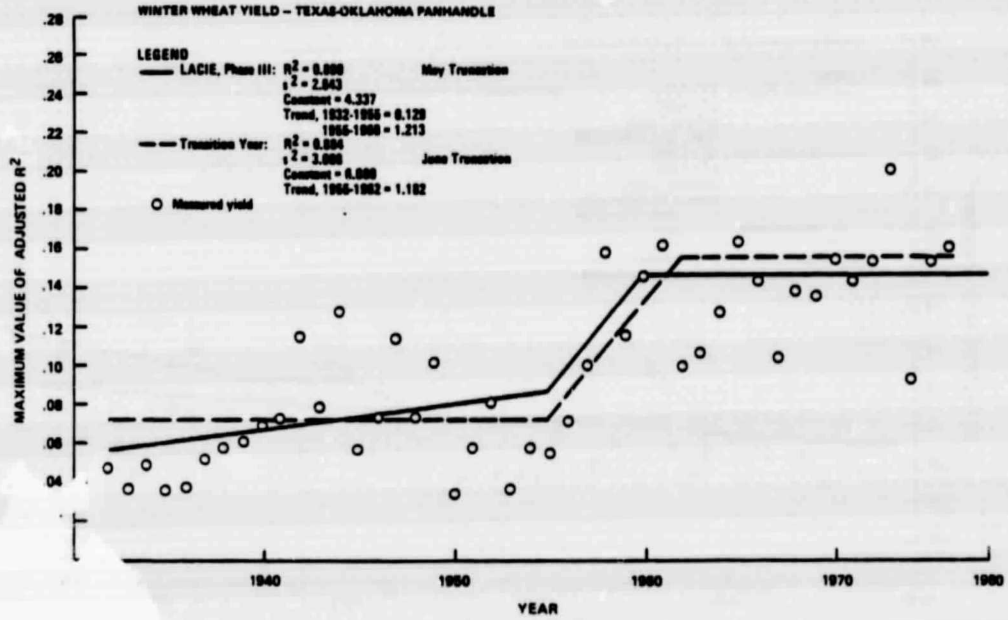

Figure E-12.- Trend lines for CCEA models for winter wheat yield in the Texas-Oklahoma Panhandle.

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