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NASA Technical Memorandum 81700

NASA-TM-81700 19830020178

Film Thickness for Different Regimes of Fluid-Film Lubrication

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May 1983

NASA

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12 1 1 RN/NASA-TM-81700
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83N28449*# ISSUE 17 PAGE 2761 CATEGORY 37 RPT#: NASA-TM-81700
E-209 NAS 1.15:81700 83/05/00 55 PAGES UNCLASSIFIED DOCUMENT
UTTL: Film thickness for different regimes of fluid-film lubrication --- elliptical contacts
AUTH: A/HAMROCK, B. J.; B/DOWSON, D. PAA: B/(Leeds Univ.)
CORP: National Aeronautics and Space Administration. Lewis Research Center, Cleveland, Ohio. AVAIL.NTIS SAP: HC A04/MF A01
MAJS: /*ELLIPTICITY/*FILM THICKNESS/*FLUID FILMS/*FORMULAS (MATHEMATICS)/* LUBRICATION
MINS: / ELASTIC PROPERTIES/ ELASTOHYDRODYNAMICS/ PRESSURE EFFECTS/ VISCOSITY
ABA: A.R.H.
ABS: Mathematical formulas are presented which express the dimensionless minimum film thickness for the four lubrication regimes found in elliptical contacts: isoviscous-rigid regime; piezoviscous-rigid regime; isoviscous-elastic regime; and piezoviscous-elastic regime. The relative importance of pressure on elastic distortion and lubricant viscosity is the factor that distinguishes these regimes for a given conjunction geometry. In addition, these equations were used to develop maps of the lubrication regimes by plotting film thickness contours on a log-log grid of the dimensionless viscosity and elasticity parameters for three values of the ellipticity parameter. These results present a complete theoretical film thickness parameter solution for elliptical contancts in the four
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FILM THICKNESS FOR DIFFERENT REGIMES OF FLUID-FILM LUBRICATION*

It is evident from the discussion in previous chapters that there are a number of reasonably well-defined regimes within the full range of conditions of fluid-film lubrication of elliptical contacts. Each regime has characteristics determined by the operating conditions and the properties of the material.

The type of lubrication for a particular contact is influenced by two major physical effects: the elastic deformation of the solids under an applied load, and the increase in fluid viscosity with pressure. Therefore it is possible to have four main regimes of fluid-film lubrication, depending on the magnitude of these effects and on their importance. These four regimes are defined as

(1) <u>Isoviscous-rigid</u>: In this regime the magnitude of the elastic deformation of the surfaces is such an insignificant part of the thickness of the fluid film separating them that it can be neglected, and the maximum pressure in the contact is too low to increase fluid viscosity significantly. This form of lubrication is typically encountered in circular-arc thrust bearing pads; in industrial coating processes in which paint, emulsion, or protective coatings are applied to sheet or film

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*Published as Chapter 12 in Ball Bearing Lubrication by Bernard J. Hamrock and Duncan Dowson, John Wiley & Sons, Inc., Sept. 1981.

materials passing between rollers; and in very lightly loaded rolling bearings.

(2) Piezo-viscous-rigid: If the pressure within the contact is sufficiently high to increase the fluid viscosity within the conjunction significantly, it may be necessary to consider the pressure-viscosity characteristics of the lubricant while assuming that the solids remain rigid. For the latter part of this assumption to be valid it is necessary that the deformations of the surfaces remain an insignificant part of the fluid-film thickness. This form of lubrication may be encountered on roller end-guide flanges, in contacts in moderately loaded cylindrical tapered rollers, and between some piston rings and cylinder liners.

(3) Isoviscous-elastic: In this regime the elastic deformation of the solids is a significant part of the thickness of the fluid film separating them, but the pressure within the contact is quite low and insufficient to cause any substantial increase in viscosity. This situation arises with materials of low elastic modulus, and it is a form of lubrication that may be encountered in seals, human joints, tires, and elastomeric-material machine elements.

(4) Piezo-viscous-elastic: In fully developed elastohydrodynamic lubrication the elastic deformation of the solids is often a significant part of the thickness of the fluid film separating them, and the pressure within the contact is high enough to cause a significant increase in the viscosity of the lubri-

cant. This form of lubrication is typically encountered in ball and roller bearings, gears, and cams.

Several authors - Moes (1965-66), Theyse (1966), Archard (1968), Greenwood (1969), Johnson (1970), and Hooke (1977) - have contributed solutions for the film thickness in the four lubrication regimes, but their results have been confined largely to nominal line or rectangular contacts. The essential difference between these contributions is the way in which the dominant parameters were made dimensionless. In this chapter the film thickness is defined for the four fluid-film lubrication regimes just described for conjunctions ranging from circular to rectangular. The film thickness equations for the respective lubrication regimes come from theoretical studies reported in previous chapters on elastohydrodynamic and hydrodynamic lubrication of elliptical conjunctions. The results are valid for isothermal, fully flooded conjunctions. In addition to the film thickness equations for the various conditions a map is presented of the lubrication regimes, with film thickness contours being represented on a log-log grid of viscosity and elasticity parameters for five values of the ellipticity parameter. This chapter draws extensively on the work of Hamrock and Dowson (1979b).

12.1 Dimensionless Grouping

The representation of results of elastohydrodynamic theory for elliptical contacts in this book in terms of the dimensionless groups (H, U, W, G, k) has been particularly helpful since the physical explanation of conjunction behavior can readily be associated with each set of numerical results. However, several authors have noted that this set of dimensionless groups can be reduced by one parameter, without any loss of generality, by using dimensionless analysis. The film thickness contours for the four fluid-film lubrication regimes can then be conveniently represented graphically by the smallest number of parameters, even though the physical meaning of each composite parameter requires careful consideration.

Johnson (1970) has pointed out that the behavior distinguishing the four lubrication regimes can be characterized by three quantities, each having the dimensions of pressure:

(1) The reduced pressure parameter q_f, a measure of the fluid pressure generated by an isoviscous lubricant when elastic deformation is neglected

(2) The inverse pressure-viscosity coefficient $1/\alpha$, a measure of the change of viscosity with pressure

(3) The maximum Hertzian pressure p_{max}, the maximum pressure in a dry elastic contact

Although Johnson (1970) did not consider elliptical contacts, he did state what the nondimensional parameters for such configurations should be:

Dimensionless film parameter:

$$\hat{H} = H\left(\frac{W}{U}\right)^2 \tag{12.1}$$

Dimensionless viscosity parameter:

$$g_V = \alpha q_f = \frac{GW^3}{U^2} \tag{12.2}$$

Dimensionless elasticity parameter:

$$g_E = \frac{1}{\pi}\left(\frac{3}{2}\right)^{1/3}\left(\frac{q_f}{p_{max}}\right) = \frac{W^{8/3}}{U^2} \tag{12.3}$$

The ellipticity parameter k remains as discussed in Chapter 3, equation (3.28). Therefore the reduced dimensionless group is $(\hat{H}, g_V, g_E, k)$.

12.2 Isoviscous-Rigid Regime

The influence of conjunction geometry on the isothermal hydrodynamic film separating two rigid solids was investigated in Chapter 6 for fully flooded, isoviscous conditions. The effect of geometry on the film thickness was determined by varying the radius ratio R_y/R_x from 1 (a circular configuration) to 36 (a configuration approaching a rectangular contact). The film thickness was varied over two orders of magnitude for conditions representative of steel solids separated by a paraffinic mineral oil. It was found that the computed minimum film thickness had the same speed, viscosity, and load dependence as the classical Kapitza (1955) solution. However, when the Reynolds cavitation boundary condition ($\partial p/\partial n = 0$ and $p = 0$) was introduced at the cavitation boundary, where n represents the normal coordinate to the interface between the full film and the cavitation region, an additional geometrical effect emerged. Therefore from Chapter 6 the dimensionless minimum, or central, film thickness parameter for the isoviscous-rigid lubrication regime can be written as

$$\left(\hat{H}_{min}\right)_{IR} = \left(\hat{H}_c\right)_{IR} = 128\ \alpha_a \lambda_b^2 \left[0.131\ \tan^{-1}\left(\frac{\alpha_a}{2}\right) + 1.683\right]^2 \qquad (12.4)$$

where

$$\alpha_a = \frac{R_y}{R_x} \approx \left(\frac{k}{1.03}\right)^{1/0.64} \qquad (12.5)$$

$$\lambda_b = \left(1 + \frac{2}{3\alpha_a}\right)^{-1} \qquad (12.6)$$

In equation (12.4) the dimensionless film thickness parameter $\hat{H}$ is shown to be strictly a function of the geometry of the contact R_y/R_x.

12.3 Piezo-Viscous-Rigid Regime

Blok (1952) has shown that the minimum film thickness for the piezo-viscous-rigid lubrication regime in a rectangular contact can be expressed as

$$h_{min} = h_c = 1.66 \left(\alpha^2 \eta_0^2 u^2 R_x\right)^{1/3} \qquad (12.7)$$

By taking account of the ellipticity of the conjunction under consideration equation (12.7) can be rewritten as

$$h_{min} = h_c = 1.66 \left(\alpha^2 \eta_0^2 u^2 R_x\right)^{1/3} \left(1 - e^{-0.68k}\right) \qquad (12.8)$$

The absence of an applied-load term in (12.8) should be noted. When expressed in terms of the dimensionless parameters introduced in equations (12.1) and (12.2), this can be written as

$$\left(\hat{H}_{min}\right)_{PVR} = \left(\hat{H}_c\right)_{PVR} = 1.66\, g_V^{2/3} \left(1 - e^{-0.68k}\right) \qquad (12.9)$$

Note the absence of the dimensionless elasticity parameter g_E in equation (12.9).

12.4 Isoviscous-Elastic Regime

The influence of the ellipticity parameter k and the dimensionless speed U, load W, and materials G parameters on the minimum, or central, film thicknesses was investigated theoretically for the isoviscous-elastic regime, and the results have been presented in Chapter 11. The ellipticity parameter was varied from 1 (a circular configuration) to 12 (a configuration approaching a rectangular contact). The dimensionless speed and load parameters were each varied by one order of magnitude. Seventeen cases were considered in obtaining the dimensionless minimum-film-thickness equation

$$H_{min} = 7.43\ U^{0.65} W^{-0.21} (1 - 0.85\ e^{-0.31k}) \tag{12.10}$$

From equations (12.1) and (12.3) the general form of the dimensionless minimum-film-thickness parameter for the isoviscous-elastic lubrication regime can be expressed as

$$\hat{H}_{min} = A g_E^c (1 - 0.85\ e^{-0.31k}) \tag{12.11}$$

where A and c are constants to be determined. From equation (12.1) and equation (12.3) we can write equation (12.11) as

$$H_{min} = AU^{2-2c} W^{(8/3c)-2} (1 - 0.85\ e^{-0.31k}) \tag{12.12}$$

Comparing equation (12.10) with equation (12.12) gives c = 0.67. Substituting this into equation (12.11) while solving for A gives

$$A = \frac{\hat{H}_{min}}{g_E^{0.67} (1 - 0.85\ e^{-0.31k})} \tag{12.13}$$

The arithmetic mean for A based on the 17 cases considered in Chapter 11 is 8.70, with a standard deviation of ± 0.05. Therefore the dimensionless minimum-film-thickness parameter for the isoviscous-elastic lubrication regime can be written as

$$\left(\hat{H}_{min}\right)_{IE} = 8.70\ g_E^{0.67} (1 - 0.85\ e^{-0.31k}) \tag{12.14}$$

With a similar approach the dimensionless central-film-thickness parameter for the isoviscous-elastic lubrication regime can be written as

$$\left(\hat{H}_c\right)_{IE} = 11.15\ g_E^{0.67} (1 - 0.72\ e^{-0.28k}) \tag{12.15}$$

12.5 Piezo-Viscous-Elastic Regime

In Chapter 8 the influence of the ellipticity parameter and the dimensionless speed, load, and materials parameters on the minimum and central film thicknesses was investigated theoretically for the piezo-viscous-elastic regime. The ellipticity parameter was varied from 1 to 8, the dimensionless speed parameter over nearly two orders of magnitude, and the dimensionless load parameter over one order of magnitude. Conditions corresponding to the use of solid materials of bronze, steel, and silicon nitride and lubricants of paraffinic and naphthenic oils were considered in obtaining the exponent on the dimensionless materials parameter. Thirty-four cases were used in obtaining the following dimensionless minimum-film-thickness formula:

$$H_{min} = 3.63\, U^{0.68} G^{0.49} W^{-0.073} (1 - e^{-0.68k}) \qquad (12.16)$$

The general form of the dimensionless minimum-film-thickness parameter for the piezo-viscous-elastic lubrication regime can be written as

$$\hat{H}_{min} = B g_V^d g_E^f (1 - e^{-0.68k}) \qquad (12.17)$$

where B, d and f are constants to be determined. From equations (12.1), (12.2), and (12.3) we can write equation (12.17) as

$$H_{min} = B G^d U^{2-2d-2f} W^{-2+3d+(8f/3)} (1 - e^{-0.68k}) \qquad (12.18)$$

Comparing equation (12.16) with equation (12.18) gives $d = 0.49$ and $f = 0.17$. Substituting these values into equation (12.17) while solving for B gives

$$B = \frac{\hat{H}_{min}}{g_V^{0.49} g_E^{0.17} (1 - e^{-0.68k})} \tag{12.19}$$

For the 34 cases considered in Chapter 8 for the derivation of equation (12.16) the arithmetic mean for B was 3.42, with a standard deviation of ± 0.03. Therefore the dimensionless minimum-film-thickness parameter for the piezo-viscous-elastic lubrication regime can be written as

$$\left(\hat{H}_{min}\right)_{PVE} = 3.42\, g_V^{0.49} g_E^{0.17} (1 - e^{-0.68k}) \tag{12.20}$$

An interesting observation to make in comparing equations (12.9), (12.14), and (12.20) is that in each case the sum of the exponents on g_V and g_E is close to the value of 2/3 required for complete dimensional representation of these three lubrication regimes: piezo-viscous-rigid, isoviscous-elastic, and piezo-viscous-elastic.

By adopting a similar approach to that outlined here the dimensionless central-film-thickness parameter for the piezo-viscous-elastic lubrication regime can be written as

$$\left(\hat{H}_c\right)_{PVE} = 3.61\, g_V^{0.53} g_E^{0.13} (1 - 0.61\, e^{-0.73k}) \tag{12.21}$$

12.6 Procedure for Mapping the Different Lubrication Regimes

Having expressed the dimensionless minimum-film-thickness parameters for the four fluid-film lubrication regimes in equations (12.4), (12.9), (12.14), and (12.20) we used these relationships to develop a map of the lubrication regimes in the form of dimensionless minimum-film-thickness-parameter contours. These maps are shown in Figures 12.1 to 12.3 on a log-log grid of the dimensionless viscosity and elasticity parameters for ellipticity parameters of 1, 3, and 6, respectively. The procedure used to obtain these figures was as follows:

(1) For a given value of the ellipticity parameter, $(\hat{H}_{min})_{IR}$ as calculated from equation (12.4).

(2) For a value of $\hat{H}_{min} > (\hat{H}_{min})_{IR}$ and the value of k chosen in step 1, the dimensionless viscosity parameter was calculated from equation (12.9) as

$$g_V = \left[\frac{\hat{H}_{min}}{1.66(1 - e^{-0.68k})}\right]^{3/2} \qquad (12.22)$$

This established the dimensionless minimum-film-thickness-parameter contours $\hat{H}_{min}$ as a function of g_V for a given value of k in the piezo-viscous-rigid regime.

(3) For the values of k selected in step 1, $\hat{H}_{min}$ selected in step 2, and g_V obtained from equation (12.22), the dimensionless elasticity parameter was calculated from the following

equation, which was derived from equation (12.20):

$$g_E = \left[\frac{\hat{H}_{min}}{3.42\ g_V^{0.49}(1 - e^{-0.68k})} \right]^{1/0.17} \tag{12.23}$$

This established the boundary between the piezo-viscous-rigid and piezo-viscous-elastic regimes and enabled contours of $\hat{H}_{min}$ to be drawn in the piezo-viscous-elastic regime as functions of g_V and g_E for given values of k.

(4) For the values of k and $\hat{H}_{min}$ chosen in steps 1 and 2 the dimensionless elasticity parameter was calculated from the following equation, obtained by rearranging equation (12.14):

$$g_E = \left[\frac{\hat{H}_{min}}{8.70(1 - 0.85\ e^{-0.31k})} \right]^{1/0.67} \tag{12.24}$$

This established the dimensionless minimum-film-thickness-parameter contour $\hat{H}_{min}$ as a function of g_E for a given value of k in the isoviscous-elastic lubrication regime.

(5) For the values of k and $\hat{H}_{min}$ selected in steps 1 and 2 and the value of g_E obtained from equation (12.24), the viscosity parameter was calculated from the following equation:

$$g_V = \left[\frac{\hat{H}_{min}}{3.42\ g_E^{0.17}(1 - e^{-0.68k})} \right]^{1/0.49} \tag{12.25}$$

This established the isoviscous-elastic and piezo-viscous-elastic boundaries for the particular values of k and $\hat{H}_{min}$ chosen in steps 1 and 2.

(6) At this point, for particular values of k and $\hat{H}_{min}$, the contours were drawn, through the piezo-viscous-rigid, piezo-viscous-elastic, and isoviscous-elastic regimes. A new value of $\hat{H}_{min}$ was then selected, and the new contour was constructed by returning to step 2. This procedure was continued until an adequate number of contours had been generated. A similar procedure was followed for the range of ellipticity ratios considered.

12.7 Contour Plots

The maps of the lubrication regimes shown in Figures 12.1 to 12.3 were generated by following the procedure outlined in the previous section. The contours of the dimensionless minimum-film-thickness parameter were plotted on a log-log grid of the dimensionless viscosity parameter and the dimensionless elasticity parameter for ellipticity parameters of 1, 3, and 6. The four lubrication regimes are clearly shown in these figures. The smallest contour of $\hat{H}_{min}$ considered in each case represents the values obtained from equation (12.4), and this forms a boundary to the isoviscous-rigid region. The value of $\hat{H}_{min}$ on the isoviscous-rigid boundary increases as the ellipticity ratio k increases.

By using Figures 12.1 to 12.3 for given values of the parameters k, g_V, and g_E, the fluid-film lubrication regime in which any elliptical conjunction is operating can be ascertained and the approximate value of $\hat{H}_{min}$ determined. When the lubrication regime is known, a more accurate value of $\hat{H}_{min}$ can be obtained by using the appropriate dimensionless minimum-film-thickness-parameter equation.

A three-dimensional view of the surfaces developed by using constant values of $\hat{H}_{min}$ of 500, 2000, and 6000 is shown in Figure 12.4. The coordinates in this figure are g_E, g_V, and k. The four fluid-film lubrication regimes are clearly shown. This figure not only defines the regimes of fluid-film lubrication clearly for elliptical contacts, but it also indicates in a single illustration how the parameters g_V, g_E, and k influence the dimensionless minimum-film-thickness parameter.

12.8 Closure

Relationships for the dimensionless minimum film thickness for the four lubrication regimes found in elliptical contacts have been developed and expressed as

(1) <u>Isoviscous-rigid regime</u>:

$$\left(\hat{H}_{min}\right)_{IR} = \left(\hat{H}_c\right)_{IR} = 128\ \alpha_a \lambda_b^2 \left[0.131\ \tan^{-1}\left(\frac{\alpha_a}{2}\right) + 1.683\right]^2 \quad (12.4)$$

where

$$\alpha_a = \frac{R_y}{R_x} \cdot \left(\frac{k}{1.03}\right)^{1/0.64} \tag{12.5}$$

$$\lambda_b = \left(1 + \frac{2}{3\alpha_a}\right)^{-1} \tag{12.6}$$

(2) <u>Piezo-viscous-rigid regime</u>:

$$\left(\hat{H}_{min}\right)_{PVR} = 1.66\ g_V^{2/3}\left(1 - e^{-0.68k}\right) \tag{12.9}$$

(3) <u>Isoviscous-elastic regime</u>:

$$\left(\hat{H}_{min}\right)_{IE} = 8.70\ g_E^{0.67}\left(1 - 0.85\ e^{-0.31k}\right) \tag{12.14}$$

(4) <u>Piezo-viscous-elastic regime</u>:

$$\left(\hat{H}_{min}\right)_{PVE} = 3.42\ g_V^{0.49} g_E^{0.17}\left(1 - e^{-0.68k}\right) \tag{12.20}$$

The relative importance of the influence of pressure on elastic distortion and lubricant viscosity is the factor that distinguishes these regimes for a given conjunction geometry.

In addition, these equations have been used to develop maps of the lubrication regimes by plotting film thickness contours on a log-log grid of the dimensionless viscosity and elasticity parameters for three values of the ellipticity parameter. These results present a complete theoretical film-thickness-parameter solution for elliptical contacts in the four lubrication regimes. The results are particularly useful in initial investigations of many practical lubrication problems involving elliptical conjunctions.

SYMBOLS

A	constant used in equation (3.113)
A^*, B^*, C^*, D^*, L^*, M^*	relaxation coefficients
A_v	drag area of ball, m^2
a	semimajor axis of contact ellipse, m
$\bar{a}$	$a/2\bar{m}$
B	total conformity of bearing
b	semiminor axis of contact ellipse, m
$\bar{b}$	$b/2\bar{m}$
C	dynamic load capacity, N
C_v	drag coefficient
$C_1,\ldots,C_8$	constants
c	19,609 N/cm^2 (28,440 lbf/in^2)
$\bar{c}$	number of equal divisions of semimajor axis
D	distance between race curvature centers, m
$\tilde{D}$	material factor
$\bar{D}$	defined by equation (5.63)
De	Deborah number
d	ball diameter, m
$\bar{d}$	number of divisions in semiminor axis
d_a	overall diameter of bearing (Figure 2.13), m
d_b	bore diameter, m
d_e	pitch diameter, m
d_e'	pitch diameter after dynamic effects have acted on ball, m
d_i	inner-race diameter, m
d_o	outer-race diameter, m

E	modulus of elasticity, N/m^2
E'	effective elastic modulus, $2\Big/\left(\frac{1-\nu_a^2}{E_a}+\frac{1-\nu_b^2}{E_b}\right)$, N/m^2
E_a	internal energy, m^2/s^2
$\widetilde{E}$	processing factor
E_1	$[(\widetilde{H}_{min} - H_{min})/H_{min}] \times 100$
$\mathcal{E}$	elliptic integral of second kind with modulus $(1 - 1/k^2)^{1/2}$
$\bar{\mathcal{E}}$	approximate elliptic integral of second kind
e	dispersion exponent
F	normal applied load, N
F^*	normal applied load per unit length, N/m
$\widetilde{F}$	lubrication factor
$\bar{F}$	integrated normal applied load, N
F_c	centrifugal force, N
F_{max}	maximum normal applied load (at $\psi = 0$), N
F_r	applied radial load, N
F_t	applied thrust load, N
F_ψ	normal applied load at angle ψ, N
$\mathcal{F}$	elliptic integral of first kind with modulus $(1 - 1/k^2)^{1/2}$
$\bar{\mathcal{F}}$	approximate elliptic integral of first kind
f	race conformity ratio
f_b	rms surface finish of ball, m
f_r	rms surface finish of race, m
G	dimensionless materials parameter, αE
G^*	fluid shear modulus, N/m^2
$\widetilde{G}$	hardness factor
g	gravitational constant, m/s^2

g_E	dimensionless elasticity parameter, $W^{8/3}/U^2$
g_V	dimensionless viscosity parameter, GW^3/U^2
H	dimensionless film thickness, h/R_x
$\hat{H}$	dimensionless film thickness, $H(W/U)^2 = F^2h/u^2\eta_0^2R_x^3$
H_c	dimensionless central film thickness, h_c/R_x
$H_{c,s}$	dimensionless central film thickness for starved lubrication condition
H_f	frictional heat, N m/s
H_{min}	dimensionless minimum film thickness obtained from EHL elliptical-contact theory
$H_{min,r}$	dimensionless minimum film thickness for a rectangular contact
$H_{min,s}$	dimensionless minimum film thickness for starved lubrication condition
$\tilde{H}_c$	dimensionless central film thickness obtained from least-squares fit of data
$\tilde{H}_{min}$	dimensionless minimum film thickness obtained from least-squares fit of data
$\overline{H}_c$	dimensionless central-film-thickness - speed parameter, $H_cU^{-0.5}$
$\overline{H}_{min}$	dimensionless minimum-film-thickness - speed parameter, $H_{min}U^{-0.5}$
$\overline{H}_0$	new estimate of constant in film thickness equation
h	film thickness, m
h_c	central film thickness, m
h_i	inlet film thickness, m

h_m	film thickness at point of maximum pressure, where $dp/dx = 0$, m
h_{min}	minimum film thickness, m
h_0	constant, m
I_d	diametral interference, m
I_p	ball mass moment of inertia, m N s^2
I_r	integral defined by equation (3.76)
I_t	integral defined by equation (3.75)
J	function of k defined by equation (3.8)
J^*	mechanical equivalent of heat
$\bar{J}$	polar moment of inertia, m N s^2
K	load-deflection constant
k	ellipticity parameter, a/b
$\bar{k}$	approximate ellipticity parameter
$\tilde{k}$	thermal conductivity, N/s °C
k_f	lubricant thermal conductivity, N/s °C
L	fatigue life
L_a	adjusted fatigue life
L_t	reduced hydrodynamic lift, from equation (6.21)
$L_1,\ldots,L_4$	lengths defined in Figure 3.11, m
L_{10}	fatigue life where 90 percent of bearing population will endure
L_{50}	fatigue life where 50 percent of bearing population will endure
ℓ	bearing length, m
$\bar{\ell}$	constant used to determine width of side-leakage region
M	moment, Nm

M_g	gyroscopic moment, Nm
M_p	dimensionless load-speed parameter, $WU^{-0.75}$
M_s	torque required to produce spin, N m
m	mass of ball, N s^2/m
m^*	dimensionless inlet distance at boundary between fully flooded and starved conditions
$\tilde{m}$	dimensionless inlet distance (Figures 7.1 and 9.1)
$\bar{m}$	number of divisions of semimajor or semiminor axis
m_W	dimensionless inlet distance boundary as obtained from Wedeven, et al. (1971)
N	rotational speed, rpm
n	number of balls
n^*	refractive index
$\bar{n}$	constant used to determine length of outlet region
P	dimensionless pressure
P_D	dimensionless pressure difference
P_d	diametral clearance, m
P_e	free endplay, m
P_{Hz}	dimensionless Hertzian pressure, N/m^2
p	pressure, N/m^2
p_{max}	maximum pressure within contact, $3F/2\pi ab$, N/m^2
$p_{iv,as}$	isoviscous asymptotic pressure, N/m^2
Q	solution to homogeneous Reynolds equation
Q_m	thermal loading parameter
$\bar{Q}$	dimensionless mass flow rate per unit width, $q\eta_0/\rho_0 E'R^2$
q_f	reduced pressure parameter
q_x	volume flow rate per unit width in x direction, m^2/s

q_y	volume flow rate per unit width in y direction, m^2/s
R	curvature sum, m
R_a	arithmetical mean deviation defined in equation (4.1), m
R_c	operational hardness of bearing material
R_x	effective radius in x direction, m
R_y	effective radius in y direction, m
r	race curvature radius, m
r_{ax}, r_{bx}, r_{ay}, r_{by}	radii of curvature, m
r_c, ϕ_c, z	cylindrical polar coordinates
r_s, θ_s, ϕ_s	spherical polar coordinates
$\bar{r}$	defined in Figure 5.4
S	geometric separation, m
S^*	geometric separation for line contact, m
S_0	empirical constant
s	shoulder height, m
T	τ_0/p_{max}
$\tilde{T}$	tangential (traction) force, N
T_m	temperature, °C
T_b^*	ball surface temperature, °C
T_f^*	average lubricant temperature, °C
ΔT^*	ball surface temperature rise, °C
T_1	$(\tau_0/p_{max})_{k=1}$
T_v	viscous drag force, N
t	time, s
t_a	auxiliary parameter
u_B	velocity of ball-race contact, m/s

u_c	velocity of ball center, m/s
U	dimensionless speed parameter, $\eta_0 u/E'R_x$
u	surface velocity in direction of motion, $(u_a + u_b)/2$, m/s
$\overline{u}$	number of stress cycles per revolution
Δu	sliding velocity, $u_a - u_b$, m/s
v	surface velocity in transverse direction, m/s
W	dimensionless load parameter, $F/E'R^2$
w	surface velocity in direction of film, m/s
X	dimensionless coordinate, x/R_x
Y	dimensionless coordinate, y/R_x
X_t, Y_t	dimensionless grouping from equation (6.14)
X_a, Y_a, Z_a	external forces, N
Z	constant defined by equation (3.48)
Z_1	viscosity pressure index, a dimensionless constant
$x, \tilde{x}, \overline{x}, \overline{x}_1$; $y, \tilde{y}, \overline{y}, \overline{y}_1$; $z, \tilde{z}, \overline{z}, \overline{z}_1$	coordinate system
α	pressure-viscosity coefficient of lubrication, m^2/N
α_a	radius ratio, R_y/R_x
β	contact angle, rad
β_f	free or initial contact angle, rad
β'	iterated value of contact angle, rad
Γ	curvature difference
γ	viscous dissipation, N/m^2 s
$\dot{\gamma}$	total strain rate, s^{-1}
$\dot{\gamma}_e$	elastic strain rate, s^{-1}
$\dot{\gamma}_v$	viscous strain rate, s^{-1}

γ_a flow angle, deg

δ total elastic deformation, m

δ^* lubricant viscosity temperature coefficient, $^oC^{-1}$

δ_D elastic deformation due to pressure difference, m

δ_r radial displacement, m

δ_t axial displacement, m

δ_x displacement at some location x, m

$\bar{\delta}$ approximate elastic deformation, m

$\tilde{\delta}$ elastic deformation of rectangular area, m

ϵ coefficient of determination

ϵ_1 strain in axial direction

ϵ_2 strain in transverse direction

ζ angle between ball rotational axis and bearing centerline (Figure 3.10)

ζ_a probability of survival

η absolute viscosity at gauge pressure, N s/m^2

$\bar{\eta}$ dimensionless viscosity, η/η_0

η_0 viscosity at atmospheric pressure, N s/m^2

η_∞ 6.31x10^{-5} N s/m^2(0.0631 cP)

θ angle used to define shoulder height

Λ film parameter (ratio of film thickness to composite surface roughness)

λ equals 1 for outer-race control and 0 for inner-race control

λ_a second coefficient of viscosity

λ_b Archard-Cowking side-leakage factor, $(1 + 2/3\ \alpha_a)^{-1}$

λ_c relaxation factor

μ	coefficient of sliding friction
μ^*	$\bar{\rho}/\bar{\eta}$
ν	Poisson's ratio
ξ	divergence of velocity vector, $(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z)$, s^{-1}
ρ	lubricant density, N s^2/m^4
$\bar{\rho}$	dimensionless density, ρ/ρ_0
ρ_0	density at atmospheric pressure, N s^2/m^4
σ	normal stress, N/m^2
σ_1	stress in axial direction, N/m^2
τ	shear stress, N/m^2
τ_0	maximum subsurface shear stress, N/m^2
$\tilde{\tau}$	shear stress, N/m^2
$\tilde{\tau}_e$	equivalent stress, N/m^2
$\tilde{\tau}_L$	limiting shear stress, N/m^2
Φ	ratio of depth of maximum shear stress to semiminor axis of contact ellipse
Φ^*	$P_H^{3/2}$
Φ_1	$(\Phi)_{k=1}$
ϕ	auxiliary angle
ϕ_T	thermal reduction factor
ψ	angular location
ψ_ℓ	limiting value of ψ
Ω_i	absolute angular velocity of inner race, rad/s
Ω_o	absolute angular velocity of outer race, rad/s
ω	angular velocity, rad/s
ω_B	angular velocity of ball-race contact, rad/s
ω_b	angular velocity of ball about its own center, rad/s

ω_c	angular velocity of ball around shaft center, rad/s
ω_s	ball spin rotational velocity, rad/s

Subscripts:

a	solid a
b	solid b
c	central
bc	ball center
IE	isoviscous-elastic regime
IR	isoviscous-rigid regime
i	inner race
K	Kapitza
min	minimum
n	iteration
o	outer race
PVE	piezoviscous-elastic regime
PVR	piezoviscous-rigid regime
r	for rectangular area
s	for starved conditions
x,y,z	coordinate system

Superscript:

$(\bar{\ })$	approximate

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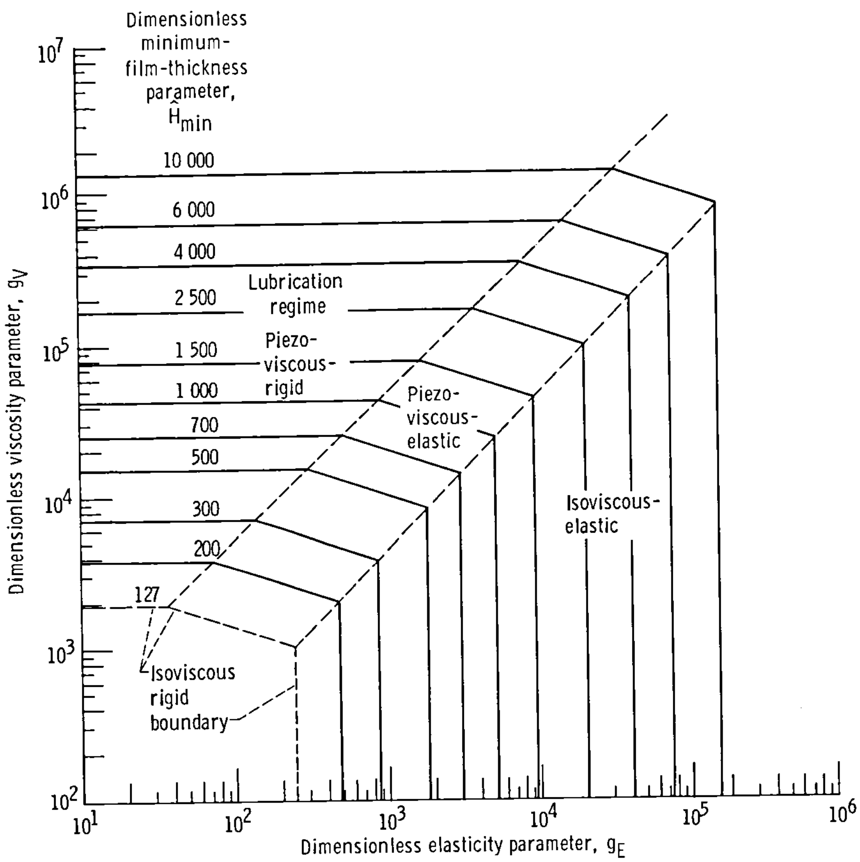

Figure 12.1. - Map of lubrication regimes for ellipticity parameter k of 1.

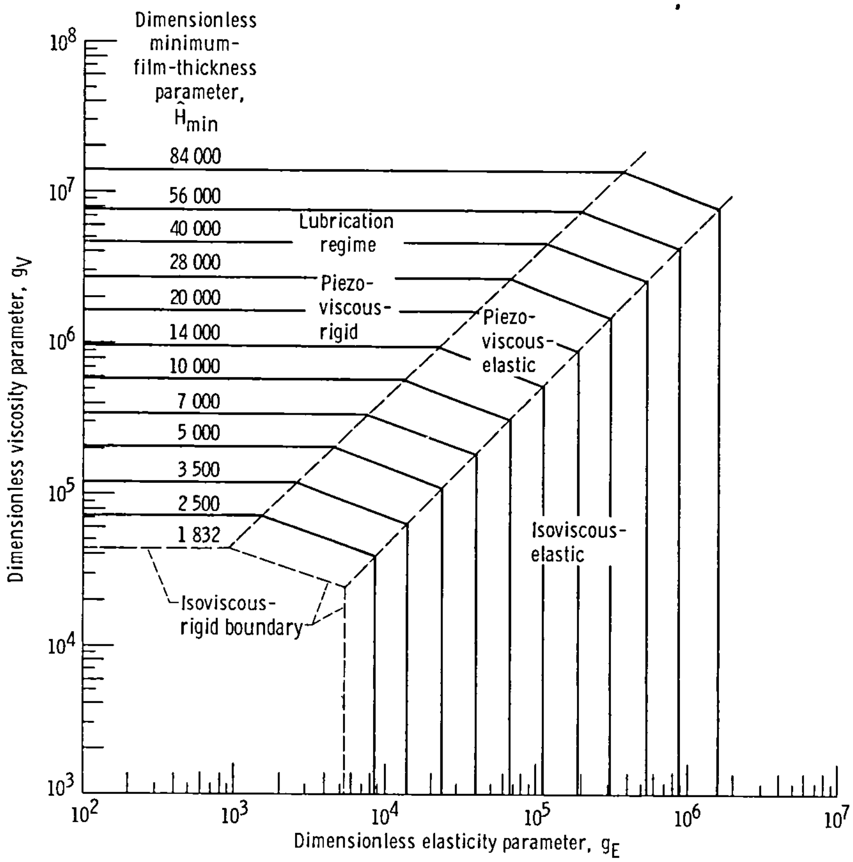

Figure 12.2. - Map of lubrication regimes for ellipticity parameter k of 3.

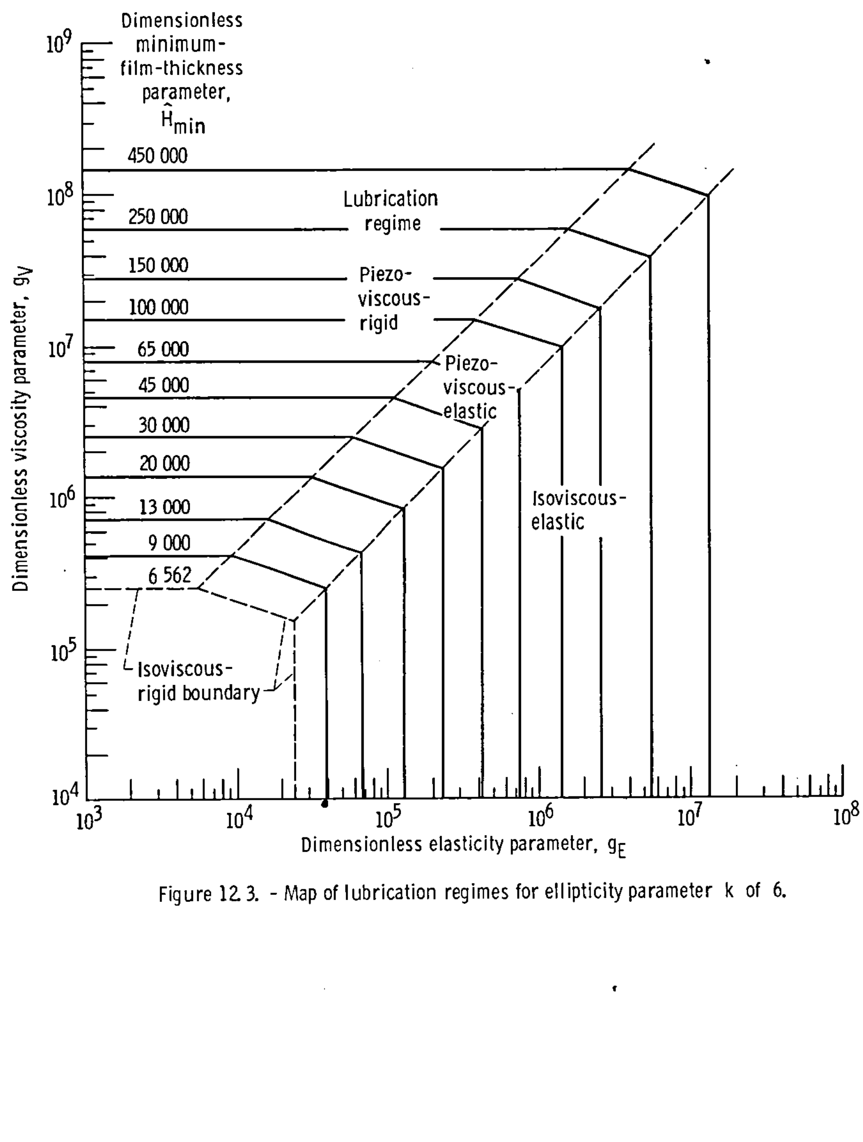

Figure 12.3. - Map of lubrication regimes for ellipticity parameter k of 6.

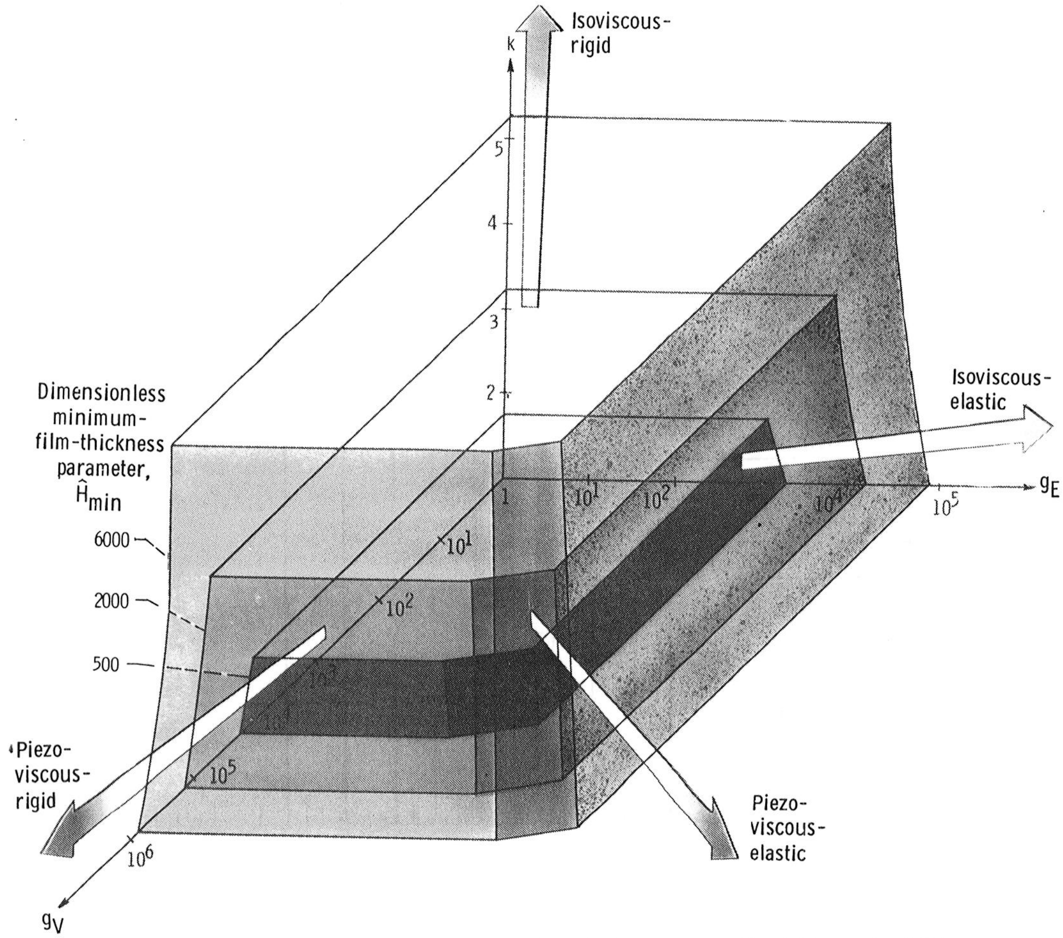

Figure 12.4. - Surfaces for constant values of dimensionless minimum-film-thickness parameter.

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-81700		

4. Title and Subtitle	5. Report Date
Film Thickness for Different Regimes of Fluid-Film Lubrication	May 1983
	6. Performing Organization Code 505-32-42
7. Author(s) Bernard J. Hamrock and Duncan Dowson	8. Performing Organization Report No. E-209
	10. Work Unit No.
9. Performing Organization Name and Address National Aeronautics and Space Administration Lewis Research Center Cleveland, Ohio 44135	11. Contract or Grant No.
	13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546	14. Sponsoring Agency Code

15. Supplementary Notes Bernard J. Hamrock, NASA Lewis Research Center, Cleveland, Ohio; Duncan Dowson, Institute of Tribology, Department of Mechanical Engineering, The University of Leeds, Leeds, England. Published as Chapter 12 in Ball Bearing Lubrication by Bernard J. Hamrock and Duncan Dowson, John Wiley & Sons, Inc., Sept. 1981.

16. Abstract

Relationships for the dimensionless minimum film thickness for the four lubrication regimes found in elliptical contacts have been developed and expressed as follows.

Isoviscous-rigid regime: $(\hat{H}_{min})_{IR} = (\hat{H}_c)_{IR}$

$$= 128\alpha_a \lambda_b^2 [0.131 \tan^{-1} (\alpha_a/2) + 1.683]^2$$

where $\alpha_a = R_y/R_x \approx (k/1.03)^{1/0.64}$ and $\lambda_b = (1 + 2/3\alpha_a)^{-1}$

Piezoviscous-rigid regime: $(\hat{H}_{min})_{PVR} = 1.66\, g_V^{2/3}(1 - e^{-0.68k})$

Isoviscous-elastic regime: $(\hat{H}_{min})_{IE} = 8.70\, g_E^{0.67}(1 - 0.85\, e^{-0.31k})$

Piezoviscous-elastic regime: $(\hat{H}_{min})_{PVE} = 3.42\, g_V^{0.49} g_E^{0.17}(1 - e^{-0.68k})$

The relative importance of the influence of pressure on elastic distortion and lubricant viscosity is the factor that distinguishes these regimes for a given conjunction geometry. In addition, these equations have been used to develop maps of the lubrication regimes by plotting film thickness contours on a log-log grid of the dimensionless viscosity and elasticity parameters for three values of the ellipticity parameter. These results present a complete theoretical film-thickness-parameter solution for elliptical contacts in the four lubrication regimes. The results are particularly useful in initial investigations of many practical lubrication problems involving elliptical conjunctions.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Fluid film lubrication Film thickness formulae Elliptical contacts Viscous and elastic effects	Unclassified - unlimited STAR Category 37

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price*
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