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ABSTRACT

The recently discovered millisecond pulsar[1] (PSR1937+214) is observed to be rotating close to the limit of dynamical instability for a neutron star. In spite of its extremely rapid rotation, the present measurements of the period derivative[2,3] put a stringent upper limit on the energy loss from gravitational radiation, thus requiring that the quadrupole moment be quite small. The pulsar must also be rotating below the critical frequency at which its equilibrium configuration would become non-axisymmetric, since the lifetime of this configuration against decay by gravitational radiation is very short. This critical frequency, given by the theory of rotating ellipsoids[4], imposes a restriction on the rotation rate more severe than the break-up frequency and may be used to set a lower limit, $\langle\rho\rangle > 2 \times 10^{14}$ g cm^{-3}, on the density of the star. If the mass is 0.5-1.5 $M_{\odot}$, several of the stiffer neutron star equations of state may be ruled out, and the radius should be less than 16 km. The condition for axisymmetry also imposes an upper limit on the rotation rate to which neutron stars may be spun up by accretion disks in binary systems, a model recently proposed for the evolution of the millisecond pulsar[5].

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Uniformly rotating masses deform into non-spherical equilibrium shapes, whose geometry becomes increasingly complex at rapid rotation rates. A homogeneous, uniformly rotating body will deform (in the Newtonian approximation) into an oblate spheroid with eccentricity dependent only on rotation rate Ω and density ρ. At high rotation rates, the equilibrium figure can become triaxial at the point where the Jacobi ellipsoids, having lower Ω at a given eccentricity, bifurcate from the axisymmetric Maclaurin sequence. This bifurcation point, for a homogeneous body, is[4]

$$\frac{\Omega_{crit}^2}{\pi G \rho} = 0.374 \tag{1}$$

Above this point, and below the bifurcation points of more complex ellipsoids, there are actually two possible equilibrium configurations with different Ω at a given eccentricity. It has been shown, however, that the Maclaurin spheroids are secularly unstable from either viscosity or gravitational radiation reaction at the bifurcation point.[6] Therefore, masses rotating with $\Omega > \Omega_{crit}$ are expected to become Jacobi ellipsoids. Because of its quadrupole moment, though, a Jacobi ellipsoid will relax by gravitational radiation to a Maclaurin spheroid with $\Omega = \Omega_{crit}$ by spinning up (but losing angular momentum by decreasing its moment of inertia) on a time scale of ~0.1 s.[7]

There is reason to believe that rapidly rotating neutron stars may deform into triaxial ellipsoids. Their rigidity is high and, with the exception of possible (small) slippage between the superfluid core and the crust, the star is thought to be uniformly rotating. Homogeneity is probably a good approximation for neutron stars with masses greater than around 0.5 $M_{\odot}$, whose density is fairly uniform from the center out to the last few meters of crust. Low mass neutron stars have large radii, thick crusts and density profiles which are better modeled by polytropes of index $n > 1$. Equilibrium figures for polytropic models have been investigated[8], and it is shown that Jacobi ellipsoids exist for a polytropic index $n < 0.8$, which would correspond to neutron stars of higher mass. By assigning polytropic indices to neutron star models, Tsuruta and Cameron[9] conclude that neutron stars with $M \gtrsim 0.5\ M_\odot$ can deform into triaxial ellipsoids. If, at some stage in its evolution, a pulsar is spinning rapidly enough to become non-axisymmetric, however, it would not have much chance of being observed, because of the short lifetime of the Jacobi ellipsoid phase. Therefore, Ω_{crit} may be considered an upper limit to observable pulsar rotation rates, unless pulsars are very low mass neutron stars.

The millisecond pulsar, with a period of $P = 1.558$ ms. ($\Omega = 4033\ s^{-1}$), is the first observed example of a neutron star spinning rapidly enough to approach the Jacobi bifurcation point and thus affords the possibility of constraining neutron star physics. From the presently measured value of the period derivative, $\dot{P} = 1.2 \times 10^{-19}\ s\ s^{-1}$, [2,3] the pulsar is losing energy at a rate $\dot{E} = 1.2 \times 10^{36}$ erg s^{-1}, only 2×10^{-3} times that of the Crab pulsar. Even if all of this energy loss is gravitational radiation, the star must be close to axisymmetry, with an eccentricity $e = D_\perp / 17\ I_{45} < 4 \times 10^{-9}$, where $D_\perp$ is the quadrupole moment perpendicular to Ω and I_{45} is the moment of inertia in units of 10^{45} g cm ^{2}. Because quadrupole moments can also result from anisotropic pressure and magnetic field stresses[10], this is only an upper limit to rotationally induced axisymmetry. Regardless of the origin of the rapid rotation rate of the pulsar, it must presently have $\Omega < \Omega_{crit} = (.374\ \pi\rho G)^{1/2}$, from Eqn (1). Its mean density must therefore satisfy $\langle\rho\rangle > 2 \times 10^{14}$ g cm^{-3}, in the approximation of homogeneity. Some neutron star equations of state predict mean densities below this value for certain values of the mass. Figure 1 shows models for neutron stars calculated from a variety of equations of state, ranging from very soft to very stiff dependences of pressure on density. The soft equations of state, including the pure neutron (R) and pion condensate cores (π,π'), require higher densities to provide the pressure necessary to hold the star up against collapse. The stiff equations of state, however, allow lower densities which appear to be incompatible with the condition above for masses below 1.5 $M_\odot$. For neutron stars with $M \lesssim 0.5\ M_\odot$, which may not deform into triaxial ellipsoids, the limit on Ω would not apply. If the millisecond pulsar has a mass in the range 0.5-1.5 $M_\odot$, then, the stiff equations of state can be ruled out. It would of course be interesting to investigate the equilibrium figures of rotating neutron stars with these equations of state to verify the above result.

Several evolutionary scenarios have been proposed for this pulsar, and these models predict a whole class of short period pulsars with low magnetic fields. Alpar et. al.[5] suggest that neutron stars in binary systems may be spun up by Keplerian accretion disks. The minimum period is the period of the Keplerian orbit at the Alfven radius, so that

$$P \gtrsim 6 \times 10^{-4}\ B_8^{6/7} R_6^{18/7}\ (M/M_\odot)^{-5/7}\ \dot{M}_{17}^{-3/7}\ \text{s.}\ , \qquad (2)$$

where B_8 is the surface dipole field in units of 10^8 G, R_6 is the stellar radius in units of 10^6 cm., M is the mass and $\dot{M}_{17}$ is the accretion rate in units of 10^{17} g s^{-1}. Neutron stars with weak magnetic fields should therefore have the shortest periods. However, the star can only be spun up to the point of becoming non-axisymmetric, $P_{eq} = 1 \times 10^{-3}\ (M/M_\odot)^{-1/2}\ R_6^{3/2}$ s. [using Eqn (1)], beyond which the excess kinetic energy of rotation will be released as gravitational radiation. This will be the case for neutron stars with magnetic fields,

$$B_8 < 1.8\ R_6^{-5/4}\ (M/M_\odot)^{1/4}\ \dot{M}_{17}^{1/2}. \tag{3}$$

As the accretion continues, the star will remain at the equilibrium period P_{eq}, emitting gravitational radiation at a rate equal to the rate of transfer of rotational kinetic energy from the disk,

$$\dot{E}_{GRAV} = I\Omega_{eq}\dot{\Omega}$$
$$\simeq 4.4 \times 10^{36}\ I_{45}\ (M/M_\odot)^{1/2} R_6^{-3/2}\ B_8^{2/7}\ \dot{M}_{17}^{6/7}\ \text{erg s}^{-1} \tag{4}$$

where I_{45} is the moment of inertia, and the rate of spin-up $\dot{\Omega}$ is that given by Ghosh and Lamb[11].

Alternatively, the pulsar could have been born in isolation with a very weak magnetic field, as has been proposed by Brecher and Chanmugam[12]. In this case, if the initial spin were beyond the critical bifurcation frequency, the star would have quickly relaxed back to Ω_{crit} through gravitational radiation.

Discovery of a shorter period pulsar could place further constraints on neutron star structure. For example, the two stiffest equations of state (MF and TI) could be eliminated completely for $M \gtrsim 0.5\ M_\odot$ by the existence of a pulsar with a period less than ~ 0.9 ms, and the Bethe-Johnson equation (BJ) would be restricted to either very low or very high masses. Since the BJ equation of state is currently believed to be the most realistic, the discovery of a pulsar with $P \lesssim 1$ ms. would have very interesting implications for neutron star structure. The constraint on certain equations of state may have consequences for calculations, such as neutron star cooling times, which have been found to depend on the structure of the star.[13]

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FIGURE CAPTION

Figure 1 - Gravitational mass versus radius (from Baym and Pethick[14]) for a variety of neutron star equations of state. References are, beginning with the softest equations, π',π (Ref. 15), R (Ref. 16), BJ (Ref. 17), MF (Ref. 18), TI (Ref. 19). The hatched line denotes the mean density cutoff at $\langle\rho\rangle = 2 \times 10^{14}$ g cm^{-3}, imposed by the rotation of the millisecond pulsar. The dashed lines are density lower limits which would be set by shorter period pulsars.

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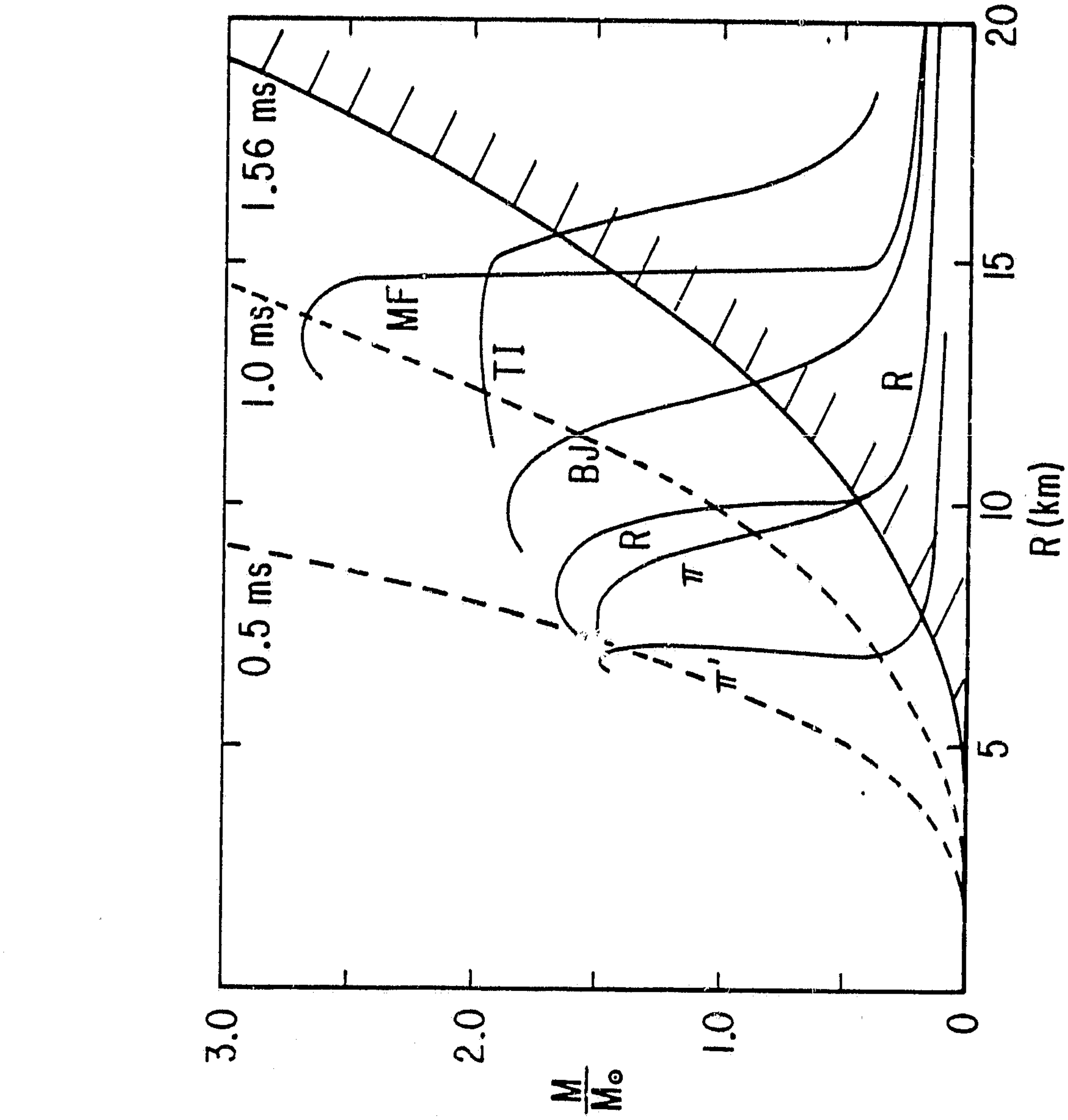

Figure 1