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National Aeronautics
and Space Administration

**Scientific and Technical
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INTRODUCTION

The prospect of extensive space operations in the era of the Space Transportation System and possibly a permanent manned space laboratory accentuates the need for understanding the interaction of energetic heavy ions with extended matter (ref. 1). In addition to the space program, high-altitude aircraft operations (ref. 2) and radiotherapy (ref. 3) are areas requiring better understanding of heavy ion beam transport. In the present report, previous calculations (refs. 4 and 5) are extended to arrive at a more complete model for heavy ion beam transport.

The solution of the heavy ion transport equations is considered in the straight ahead approximation. The equations are solved by methods of characteristics using a perturbation expansion. After the transport coefficients are reviewed, results for neon beams in water are presented along with conclusions based on these results.

HEAVY ION TRANSPORT

In moving through extended matter, heavy ions lose energy through interaction with atomic electrons along their trajectories. On occasion, they interact violently with nuclei of the matter producing ion fragments moving in the forward direction and low energy fragments of the struck target nucleus. The transport equations for the short range target fragments can be solved in closed form in terms of collision density (ref. 4). Hence, the projectile fragment transport is the interesting unsolved problem. In previous work, the projectile ion fragments were treated as if all went straight forward (ref. 5).

With the straight ahead approximation and the target secondary fragments neglected (refs. 4 and 5), the transport equation may be written as

$$\left[\frac{\partial}{\partial x} - \frac{\partial}{\partial E}\tilde{S}_j(E) + \sigma_j(E)\right]\Phi_j(x,E) = \sum_{k>j} m_{jk}(E)\,\sigma_k(E)\,\Phi_k(x,E) \tag{1}$$

where $\Phi_j(x,E)$ is the flux of ions of type j with atomic mass A_j at x moving along the x-axis at energy E in units of MeV/amu, $\sigma_j(E)$ is the corresponding macroscopic nuclear absorption cross section, $\tilde{S}_j(E)$ is the change in E per unit distance, and $m_{jk}(E)$ is the multiplicity of ion j produced in collision by ion k. (A list of symbols appears after the references in this report.) The range of the ion is given as

$$R_j(E) = \int_0^E \frac{dE'}{\tilde{S}_j(E')} \tag{2}$$

The solution to equation (1) is to be found subject to boundary specification at $x = 0$ and arbitrary E as

$$\Phi_j(0,E) = F_j(E) \tag{3}$$

Usually $F_j(E)$ is called the incident beam spectrum.

It follows from Bethe's theory (ref. 6) that

$$\tilde{S}_j(E) = \frac{A_p Z_j^2}{A_j Z_p^2} \tilde{S}_p(E) \tag{4}$$

for which

$$\frac{Z_j^2}{A_j} R_j(E) = \frac{Z_p^2}{A_p} R_p(E) \tag{5a}$$

The subscript p refers to proton. Equation (5a) is quite accurate at high energy and only approximately true at low energy because of electron capture by the ion which effectively reduces its charge (ref. 7), higher order Born corrections to Bethe's theory (ref. 8), and nuclear stopping at the lowest energies (ref. 9). Herein, the parameter ν_j is defined as

$$\nu_j R_j(E) = \nu_k R_k(E) \tag{5b}$$

so that

$$\nu_j = Z_j^2/A_j \tag{6}$$

Equations (5) and (6) are used in the subsequent development and the energy variation in ν_j is neglected. The inverse function of $R_j(E)$ is defined as

$$E = R_j^{-1}\left[R_j(E)\right] \tag{7}$$

and plays a fundamental role subsequently. For the purpose of solving equation (1), define the coordinate transformation (refs. 4 and 10),

$$\left.\begin{aligned} \eta_j &\equiv x - R_j(E) \\ \xi_j &\equiv x + R_j(E) \end{aligned}\right\} \tag{8}$$

and new functions

$$\left.\begin{aligned} \chi_j(\eta_j,\xi_j) &\equiv \tilde{S}_j(E)\ \Phi_j(x,E) \\ \bar{\chi}_k(\eta_j,\xi_j) &\equiv \chi_k(\eta_k,\xi_k) \end{aligned}\right\} \tag{9}$$

where

$$\left.\begin{aligned} \xi_j + \eta_j &= \xi_k + \eta_k \\ \eta_j - \xi_j &= \frac{\nu_k}{\nu_j}(\eta_k - \xi_k) \end{aligned}\right\} \tag{10}$$

for which equation (1) becomes

$$\left(2\frac{\partial}{\partial\eta_j} + \sigma_j\right)\chi_j(\eta_j,\xi_j) = \sum_k m_{jk}\sigma_k \frac{\nu_j}{\nu_k}\bar{\chi}_k(\eta_j,\xi_j) \tag{11}$$

where the σ_j are assumed to be energy independent. Solving equation (11) by using line integration with an integrating factor,

$$\mu_j(\eta_j,\xi_j) = \exp\left[\frac{1}{2}\sigma_j(\xi_j + \eta_j)\right] \tag{12}$$

results in

$$\begin{aligned} \chi_j(\eta_j,\xi_j) = \exp\left[-\frac{1}{2}\sigma_j(\xi_j + \eta_j)\right]\ &\chi_j(-\xi_j,\xi_j) \\ + \frac{1}{2}\int_{-\xi_j}^{\eta_j} \exp\left[\frac{1}{2}\sigma_j(\eta' - \eta_j)\right] &\sum_k m_{jk}\sigma_k \frac{\nu_j}{\nu_k}\chi_k(\eta_k',\xi_k')\,d\eta' \end{aligned} \tag{13}$$

where

$$\left.\begin{aligned} \eta_k' &= \frac{\nu_k + \nu_j}{2\nu_k}\,\eta' + \frac{\nu_k - \nu_j}{2\nu_k}\,\xi_j \\ \xi_k' &= \frac{\nu_k - \nu_j}{2\nu_k}\,\eta' + \frac{\nu_k + \nu_j}{2\nu_k}\,\xi_j \end{aligned}\right\} \tag{14}$$

and the boundary condition (eq. (3)) is written as

$$\chi_j(-\xi_j,\xi_j) = \tilde{S}_j\left[R_j^{-1}(\xi_j)\right]\, F_j\left[R_j^{-1}(\xi_j)\right]$$

Consider a Neumann series for equation (13) for which the first term is

$$\chi_j^{(0)}(\eta_j,\xi_j) = \exp\left[-\tfrac{1}{2}\,\sigma_j(\eta_j + \xi_j)\right]\, \tilde{S}_j\left[R_j^{-1}(\xi_j)\right]\, F_j\left[R_j^{-1}(\xi_j)\right] \tag{15}$$

and the second term is

$$\chi_j^{(1)}(\eta_j,\xi_j) = \frac{1}{2}\int_{-\xi_j}^{\eta_j} \exp\left[\tfrac{1}{2}\,\sigma_j(\eta' - \eta_j)\right] \sum_k m_{jk}\sigma_k\,\frac{\nu_j}{\nu_k}\exp\left[-\tfrac{1}{2}\,\sigma_k(\eta_k' + \xi_k')\right]$$

$$\times\, \tilde{S}_k\left[R_k^{-1}(\xi_k')\right]\, F_k\left[R_k^{-1}(\xi_k')\right]\, d\eta' \tag{16}$$

An expression for $\chi_j^{(2)}(\eta_j,\xi_j)$ is derived once equation (16) is reduced and higher order terms can be found by continued iteration of equation (13). These expressions (eqs. (15) and (16)) are now simplified for a monoenergetic beam of type M ions.

The boundary condition is now taken as

$$F_j(E) = \delta_{jM}\,\delta(E - E_0) \tag{17}$$

where δ_{jM} is the Kronecker delta, $\delta(\ )$ is the Dirac delta, and E_0 is the incident beam energy. Thus,

$$\chi_j(-\xi_j,\xi_j) = \tilde{S}_j\left[R_j^{-1}(\xi_j)\right]\,\delta_{jM}\,\delta\left[R_j^{-1}(\xi_j) - E_0\right] = \delta_{jM}\,\delta\left[\xi_j - R_j(E_0)\right] \tag{18}$$

for which $\chi_j^{(0)}$ becomes

$$\chi_j^{(0)}(\eta_j,\xi_j) = \delta_{jM} \exp\left[-\frac{1}{2}\sigma_j(\eta_j + \xi_j)\right] \delta\left[\xi_j - R_j(E_0)\right] \tag{19}$$

and $\chi_j^{(1)}$ becomes

$$\chi_j^{(1)}(\eta_j,\xi_j) = \frac{1}{2}\int_{-\xi_j}^{\eta_j} m_{jM}\,\sigma_M\,\frac{\nu_j}{\nu_M}\exp\left[\frac{1}{2}\sigma_j(\eta' - \eta_j) - \frac{1}{2}\sigma_M(\eta_M' + \xi_M')\right]$$

$$\times\,\delta\left[\xi_M' - R_M(E_0)\right]\,d\eta' \tag{20}$$

The contribution to the integral (eq. (20)) occurs at

$$\eta' = \frac{2\nu_M}{\nu_M - \nu_j} R_M(E_0) - \frac{\nu_M + \nu_j}{\nu_M - \nu_j}\xi_j \tag{21}$$

provided that η' lies on the interval $-\xi_j \leq \eta' \leq \eta_j$, so that

$$\chi_j^{(1)}(\eta_j,\xi_j) = \frac{m_{jM}\sigma_M\nu_j}{\nu_M - \nu_j}\exp\left[-\frac{1}{2}\sigma_M(\xi_j + \eta') - \frac{1}{2}\sigma_j(\eta_j - \eta')\right] \tag{22}$$

The simplified form in equation (22) may now be used to calculate the next iteration of equation (13):

$$\chi_j^{(2)}(\eta_j,\xi_j) = \frac{1}{2}\sum_{jk} m_{jk}\sigma_k m_{kM}\sigma_M \frac{\nu_j}{\nu_M - \nu_k}\int_{-\xi_j}^{\eta_j}\exp\left[-\frac{1}{2}\sigma_M(\xi_k'' + \tilde{\eta})\right.$$

$$\left. - \frac{1}{2}\sigma_k(\eta_k'' - \tilde{\eta}) - \frac{1}{2}\sigma_j(\eta_j - \eta'')\right] d\eta'' \tag{23}$$

where

$$\left.\begin{aligned} \eta_k'' &= \frac{\nu_k + \nu_j}{2\nu_k}\,\eta'' + \frac{\nu_k - \nu_j}{2\nu_k}\,\xi_j \\ \xi_k'' &= \frac{\nu_k - \nu_j}{2\nu_k}\,\eta'' + \frac{\nu_k + \nu_j}{2\nu_k}\,\xi_j \end{aligned}\right\} \tag{24}$$

and

$$\tilde{\eta} = \frac{2\nu_M}{\nu_M - \nu_k}\,R_M(E_0) - \frac{\nu_M + \nu_k}{\nu_M - \nu_k}\,\xi_k'' \tag{25}$$

with the requirement that $-\xi_k'' \leq \tilde{\eta} \leq \eta_k$. The inverse of the transformation is now applied to obtain from equation (19)

$$\Phi_j^{(0)}(x,E) = \frac{1}{\tilde{S}_j(E)}\exp(-\sigma_j x)\;\delta_{jM}\;\delta\left[x+R_j(E)-R_M(E_0)\right] \tag{26}$$

and from equation (22)

$$\Phi_j^{(1)}(x,E) = \frac{1}{\tilde{S}_j(E)}\,m_{jM}\sigma_M\,\frac{\nu_j}{\nu_M - \nu_j}\exp\left\{-\frac{1}{2}\,\sigma_j\left[x - R_j(E) - \eta'\right] - \frac{1}{2}\,\sigma_M\left[x + R_j(E) + \eta'\right]\right\} \tag{27}$$

so long as

$$\frac{\nu_M}{\nu_j}\left[R_M(E_0) - x\right] \leq R_j(E) \leq \frac{\nu_M}{\nu_j}\,R_M(E_0) - x \tag{28}$$

Otherwise $\Phi_j^{(1)}(x,E)$ is zero. After a complicated but straightforward manipulation, a similar result may be obtained from equation (23) for $\Phi_j^{(2)}(x,E)$.

The dose as a function of depth is given as

$$D(x) = \sum_j \int_0^\infty S_j(E)\ \Phi_j(x,E)\ dE \tag{29}$$

which is evaluated from equations (26) and (27) as follows:

$$D^{(0)}(x) = A_M \exp(-\sigma_M x) \int_0^\infty \delta\left[R_M(E)+x-R_M(E_0)\right] dE$$

$$= A_M\ \tilde{S}_M\left\{R_M^{-1}\left[R_M(E_0)-x\right]\right\} \exp(-\sigma_M x) \tag{30}$$

and

$$D^{(1)}(x) = \sum_j m_{jM}\sigma_M \frac{A_j \nu_j}{\nu_M - \nu_j} \int_{E_{\ell_j}}^{E_{u_j}} \exp\left\{-\frac{1}{2}\ \sigma_j\left[x - R_j(E) - \eta'\right]\right.$$

$$\left. - \sigma_M\left[x + R_j(E) + \eta'\right]\right\} dE \tag{31}$$

where E_{u_j} and E_{ℓ_j} span the energy limits associated with equation (28). The values of the integrand of equation (31) at the limits of integration are

$$\left.\begin{array}{ll} \exp(-\sigma_M x) & \left(E = E_{\ell_j}\right) \\ \exp(-\sigma_j x) & \left(E = E_{u_j}\right) \end{array}\right\} \tag{32}$$

With a linear approximation of the exponent, equation (31) becomes

$$D^{(1)}(x) \approx \sum_j m_{jM}\sigma_M \frac{A_j \nu_j}{\nu_M - \nu_j} \frac{E_{u_j} - E_{\ell_j}}{(\sigma_M - \sigma_j)x}\left[\exp(-\sigma_j x) - \exp(-\sigma_M x)\right] \tag{33}$$

Note that assuming the stopping power to be negligible results in an energy-independent solution for the secondary flux:

$$\Phi_j^{(1)}(x) = \sum_j m_{jM}\sigma_M \frac{\exp(-\sigma_j x) - \exp(-\sigma_M x)}{\sigma_M - \sigma_j} \tag{34}$$

which provides some confidence in the above approximations (see also ref. 5).

In similar fashion, the inverse transformation can be applied to equation (23) and $D^{(2)}(x)$ can be evaluated as

$$D^{(2)}(x) = \sum_{jk} m_{jk}\sigma_k m_{kM}\sigma_M \frac{A_j \nu_j \left(E'_{u_j} - E'_{\ell_j}\right)}{(\nu_M - \nu_k)(\sigma_M - \sigma_j)x}$$

$$\times \left[\frac{\exp(-\sigma_j x) - \exp(-\sigma_M x)}{\sigma_M - \sigma_j} - \frac{\exp(-\sigma_j x) - \exp(-\sigma_k x)}{\sigma_k - \sigma_j}\right] \tag{35}$$

where

$$R_j\left(E'_{u_j}\right) = \frac{\nu_M}{\nu_j} R_M(E_0) - x \tag{36}$$

and

$$R_j\left(E'_{\ell_j}\right) = \frac{\nu_M}{\nu_j}\left[R_M(E_0) - x\right] \tag{37}$$

The results of equations (36) and (37) are understood to be zero whenever the right-hand sides are negative. The above expressions can be applied to various shield materials of uniform composition. Each specific application requires knowledge of the appropriate transport coefficients $S_j(E)$, σ_j, and m_{jk}. The next section presents the transport coefficients for water. Water is particularly important since it resembles organic materials, is under extensive study at heavy ion facilities, and is related to an astronaut's self-shielding factors.

TRANSPORT COEFFICIENTS OF WATER

Stopping Power

In passing through a material, an ion loses the larger fraction of its energy to electronic excitation of the material. Although a satisfactory theory of high-energy ion-electron interaction is available in the form of Bethe's theory utilizing the Born approximation, an equally satisfactory theory for low energies is not available. Bethe's high-energy approximation to the energy loss per unit path is given as

$$S_e = \frac{4\pi N Z_1^2 Z_2 e^4}{mv^2}\left[\ln \frac{2mv^2}{(1-\beta^2)I_2} - \beta^2 - \frac{C}{Z_2}\right] \tag{38}$$

where Z_1 is the projectile charge, N is the number of targets per unit volume, Z_2 is the number of electrons per target, m is the electron mass, v is the projectile velocity, $\beta = v/c$, c is the velocity of light, C is the velocity-dependent shell correction term (ref. 11), and I_2 is the mean excitation energy given by

$$Z_2 \ln I_2 = \sum_n f_n \ln E_n \tag{39}$$

where the f_n are the electric dipole oscillator strengths of the target and the E_n the corresponding excitation energies. Note that the sum in equation (39) includes discrete and continuum levels. Empirically it has been observed that molecular stopping power is reasonably approximated by the sum of the corresponding empirically derived "atomic" stopping powers for which equations (38) and (39) imply

$$Z \ln I = \sum_j n_j Z_j \ln I_j \tag{40}$$

where Z and I pertain to the molecule, Z_j and I_j are the corresponding atomic values, and n_j are the stoichiometric coefficients. This additivity rule (eq. (40)) is usually called Bragg's rule (ref. 12).

Sources of deviations from Bragg's additivity rule for molecules and for the condensed phase are discussed by Platzman (ref. 13). Aside from shifts in excitation energies and adjustments in line strengths as a result of molecular bonding, new terms in the stopping power appear due to coupling between vibrational and rotational modes. Additionally, in condensed phase, some discrete transitions are moved into the continuum, and collective modes among valence electrons in adjacent atoms produce new terms in the absorption spectrum to be dealt with. Platzman proposed that the experimentally observed additivity rule may not show that molecular stopping power is the sum of atomic processes but rather demonstrate that molecular bond shifts for covalent bonded molecules are relatively independent of the molecular combination. On the basis of such arguments, Platzman suggested that ionic bonded substances

should be studied as a rigid test of the additivity rule because of the radical difference in bonding type. He further estimated that ionic bond shifts could change the stopping power by as much as 50 percent. Recent results on molecular bond shifts on mean excitation energies are discussed in references 14 to 16. Effects of the physical state have likewise been studied (ref. 17).

The electronic stopping power for protons is adequately described by equation (38) for energies above 500 keV for which the shell or "tight binding" correction C makes an important contribution below 10 MeV (ref. 18). For proton energies below 500 keV, it is well-known that charge exchange (electron transfer) reactions alter the proton charge over much of its path, so that equation (38) is to be understood in terms of an average over the proton charge states. It is normal to introduce into equation (38) an effective charge that is found by averaging equation (38) over the charge states so that the effective charge is the root-mean-square ion charge and not the average ion charge. At any ion energy, charge equilibrium is established very quickly in all materials. Utilizing the effective charge in equation (38) appears to make only modest improvement below 500 keV, an indication presumably of the failure of this theory based on the Born approximation (ref. 7). Customarily data below 500 keV are treated on an empirical basis (refs. 7 and 18). The resultant stopping power for protons in water are shown in comparison to the evaluated data of Bichsel (ref. 19) in figure 1.

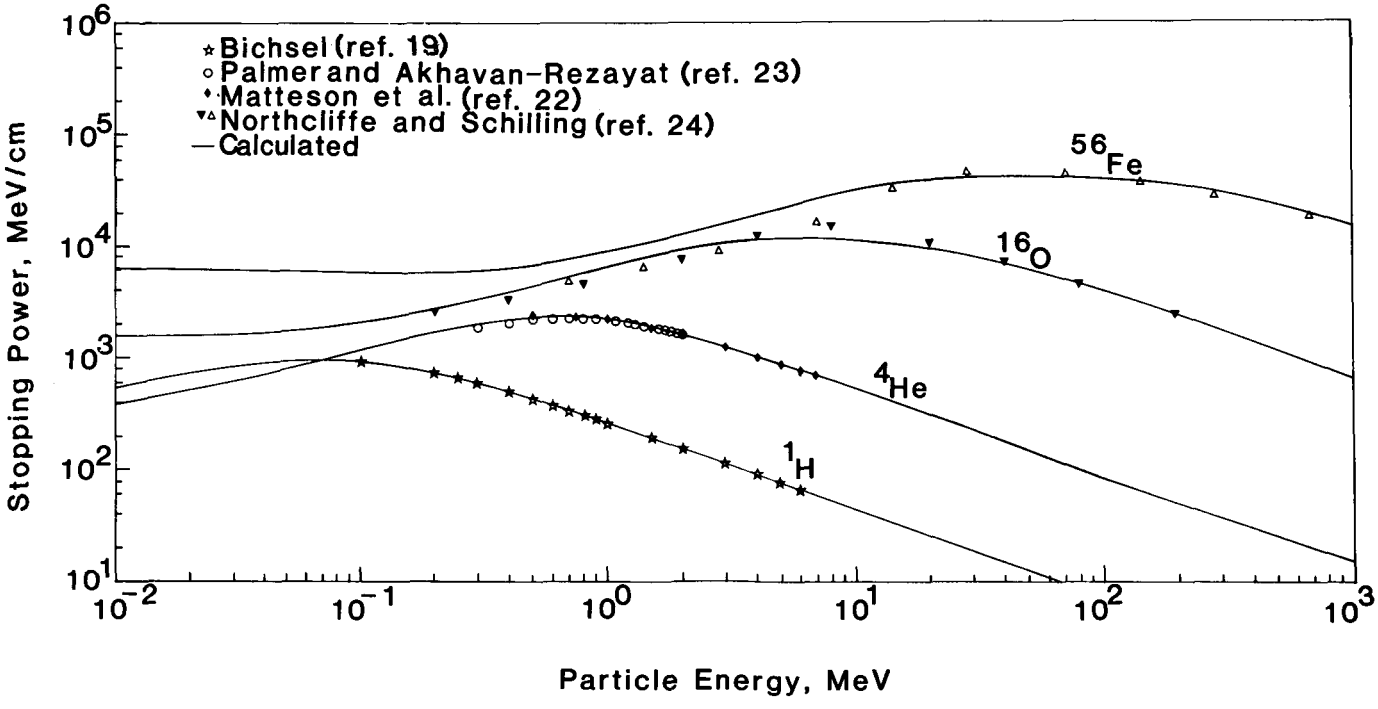

Figure 1.- Calculated and experimental stopping powers in water for typical cosmic ray ions as a function of kinetic energy.

The electronic stopping power for alpha particles requires terms in equation (38) of higher order in the projectile charge Z_1 resulting from corrections to the Born approximation. The alpha stopping power cannot be related to the proton stopping power through their effective charges. Parametric fits to experimental data are given by Ziegler in reference 20 for all elements in both the gaseous and condensed phase.

The electronic stopping powers for heavier ions are related to the alpha stopping power through their corresponding effective charges. The effective charge suggested by Barkas (ref. 21) is used:

$$Z^* = Z\left[1 - \exp\left(-125\beta/Z^{2/3}\right)\right] \tag{41}$$

where Z in equation (41) is atomic number of the ion.

At sufficiently low energies, the energy lost by an ion in nuclear collision becomes important. The nuclear stopping theory used in this paper is a modification of the theory of Lindhard, Scharff, and Schiott (ref. 9). The reduced energy is given as

$$\varepsilon = \frac{32.53A_1A_2E}{Z_1Z_2(A_1 + A_2)\left(Z_1^{2/3} + Z_2^{2/3}\right)^{1/2}} \tag{42}$$

where E is in units of keV/amu and A_1 and A_2 are the atomic masses of the projectile and target. The nuclear stopping power in reduced units (ref. 20) is

$$S_n = \begin{cases} 1.593\varepsilon^{1/2} & (\varepsilon < 0.01) \\ \dfrac{1.7\varepsilon^{1/2}\ln(\varepsilon + \exp 1)}{1 + 6.8\varepsilon + 3.4\varepsilon^{3/2}} & (0.01 < \varepsilon < 10) \\ \dfrac{\ln(0.47\varepsilon)}{2\varepsilon} & (10 < \varepsilon) \end{cases} \tag{43}$$

and the conversion factor to units of eV/(10^{15} atoms/cm^2) is

$$f = \frac{8.426Z_1Z_2A_1}{(A_1 + A_2)\left(Z_1^{2/3} + Z_2^{2/3}\right)^{1/2}} \tag{44}$$

The total stopping power S_j is obtained by summing the electronic and nuclear contributions. Other processes of energy transfer such as Bremsstrahlung and pair production are unimportant.

For energies above a few MeV per nucleon, Bethe's equation is adequate provided that appropriate corrections to Bragg's rule (refs. 14 to 16), shell corrections (refs. 7, 11, and 18), and effective charge are included. Electronic stopping power for protons is calculated from the parametric formulas of Andersen and Ziegler (ref. 18). The calculated stopping power for protons in water is shown in figure 1 in comparison with data given by Bichsel (ref. 19).

Because alpha stopping power is not derivable from the proton stopping power formula using the effective charge at low energy, the parametric fits to empirical alpha stopping powers given by Ziegler (ref. 20) are used. Applying his results for condensed phase water poorly represented the data of references 22 and 23. Considering that physical state and molecular binding effects are most important for hydrogen (ref. 14), the water stopping power was approximated by using the condensed phase parameters for hydrogen and the gas phase parameters for oxygen (which are known experimentally). These results are compared with experimental data for condensed phase water (refs. 22 and 23) in figure 1. It appears that Ziegler overestimated the condensed phase effects for oxygen.

Electronic stopping powers for ions of charge greater than 2 are related to the alpha stopping power through the effective charge given by equation (41). For water, the condensed phase formula of Ziegler for alpha particles gives probably the best stopping powers for heavier ions. Calculated results for ^{16}O and ^{56}Fe ions in water are shown in figure 1 in comparison with the Northcliffe and Schilling data (ref. 24). Good agreement with Northcliffe and Schilling for ^{56}Fe ions is especially important since their data seem to agree with the range experiments of J. H. Chan in Lexan[1] (ref. 25). The stopping powers in Lexan and tissue equivalent material can be calculated in a way similar to the procedure given above for calculating stopping powers in water.

Nuclear Absorption Cross Section

The nuclear absorption cross section σ_k is calculated from a quantum mechanical model of the heavy ion reaction. Approximate solution of the coupled-channel equations for high-energy composite particle scattering is used to calculate the elastic scattered amplitude from which total and absorption cross sections are derived (ref. 26). Nucleon-nucleus cross sections are better than 5 percent accurate with respect to absorption mean free paths. More accurate nuclear density functions are now being incorporated into the model (refs. 27 to 29) and improved calculational methods will result in more accurate cross sections at all energies from 20 MeV/amu to 30 GeV/amu. The present results derived from reference 26 are shown in figure 2.

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[1]Lexan: Trade name of the General Electric Co., Polymers Product Dept.

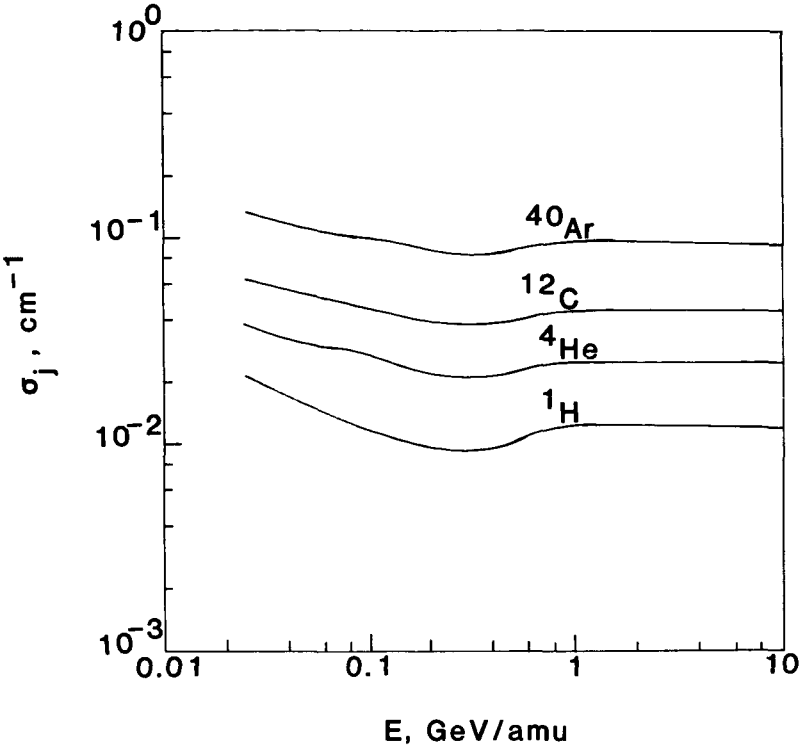

Figure 2.- Calculated (ref. 26) macroscopic absorption cross section for typical cosmic ray ions in water.

Nuclear Fragmentation Parameters

Nuclear fragmentation by proton beams has been studied by many researchers, and approximate formulas for the production cross sections of any fragment with $Z > 2$ for proton beams and arbitrary target nuclei have been derived (ref. 30). Basic fragmentation studies for carbon ion beams allow extension of the production cross sections for $Z > 2$ and hydrogen beams to any arbitrary nucleus (ref. 30). Given the production cross section $\sigma(Z_k,A_k,A_T,Z_j,A_j,E)$ for a fragment of type A_j and Z_j produced by an ion of type A_k and Z_k colliding with a target A_T, then the fragmentation parameter, or multiplicity, required for the present method is

$$m_{jk}(E) = \frac{\sigma(Z_k,A_k,A_T,Z_j,A_j,E)}{\sigma_k(A_T,E)} \tag{45}$$

In the present calculation the production cross sections are averaged over the shield material constituents. Fragmentation parameters for ^{50}V at 1 GeV/amu in water are shown in figure 3.

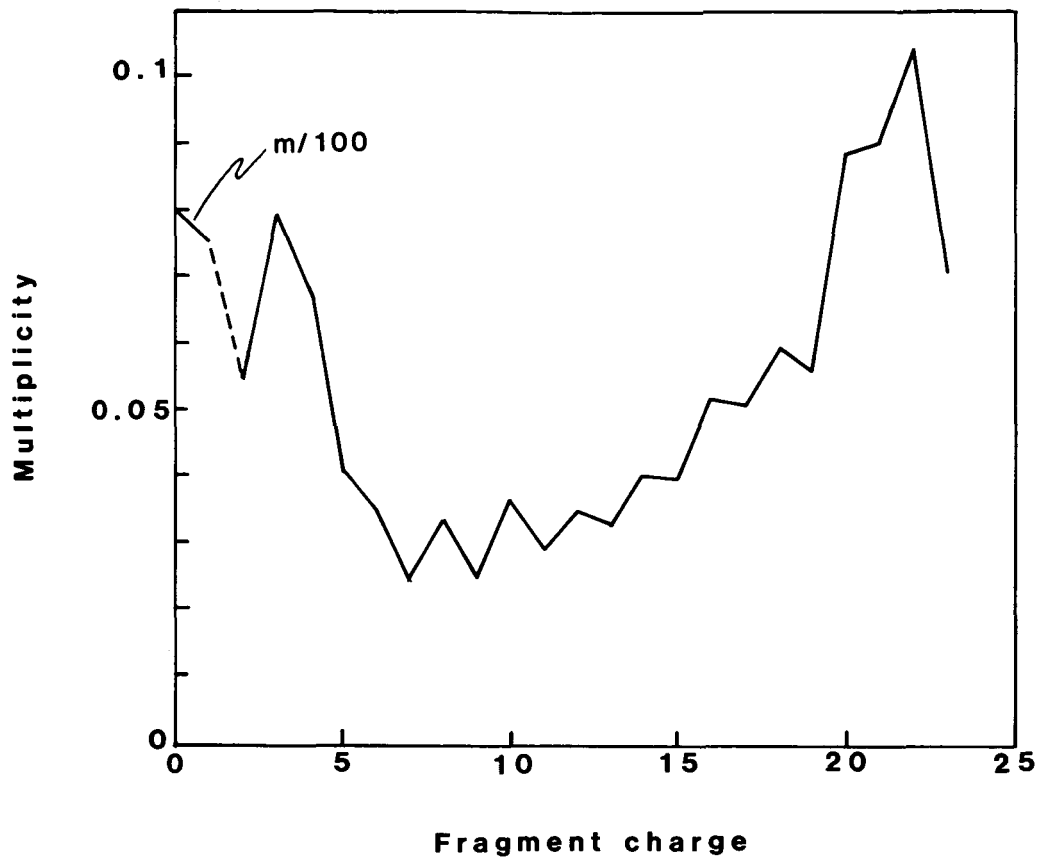

Figure 3.- Calculated fragmentation parameters (multiplicity) of various charge states for vanadium fragmentation in water. E = 1 GeV/amu.

RESULTS

The dose in MeV/g has been calculated as a function of depth in water for a ^{20}Ne ion beam of 760 MeV/amu according to equations (30), (33), and (35). The calculated dose indicates that of the incident 15.2 GeV associated with the kinetic energy of each particle, only about half would be ultimately deposited. It is clear in comparing $D^{(1)}(x)$ with $D^{(2)}(x)$ in figure 4 that the series approximation to the dose is rapidly converging, so that higher order terms are not the source of error. Testing for charge and mass conservation of the fragmentation parameters indicates that the average total charge of the first reaction products is 4.9e and the average mass is 9.2 amu. Clearly, considerable mass and charge as well as energy are lost in the Silberberg-Tsao fragmentation cross sections. Although the calculational methods developed here are converging at least in the case shown, considerable effort must be made to improve the description of the fragmentation process.

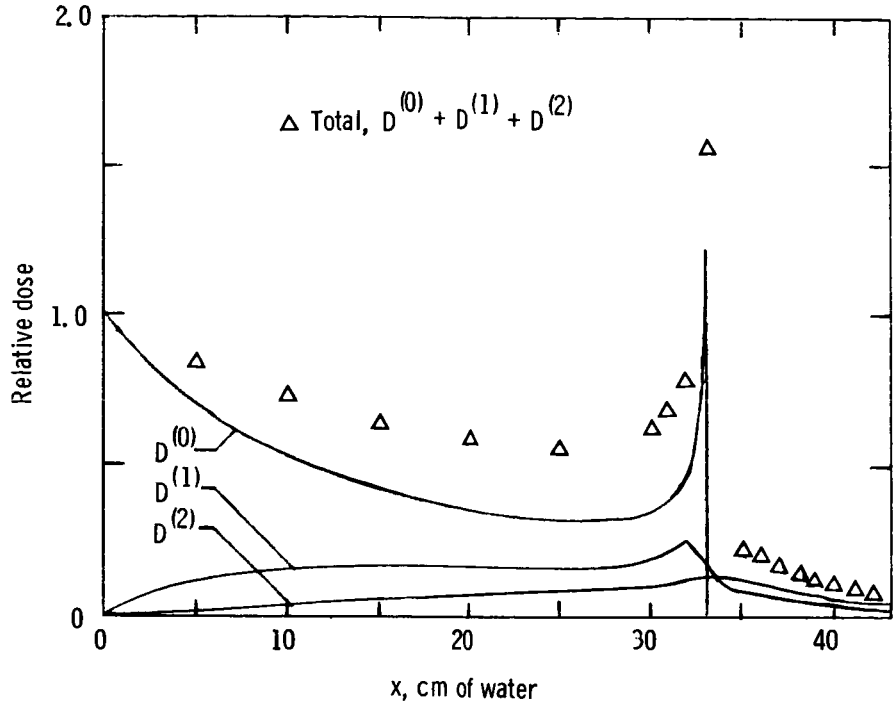

Figure 4.- First three terms of series approximation to dose as a function of penetration depth in water for a neon ion beam of 670 MeV/amu.

CONCLUDING REMARKS

A suitable approximation of heavy ion transport appears to have been found on the basis of the convergence of the series approximation of a ^{20}Ne beam in water. Inaccuracies in the transport coefficients, particularly the fragmentation parameter (multiplicity), do not yet allow meaningful comparison with experiment. Improvement in the accuracy of fragmentation parameters would allow eventual calculation of important radiation protection quantities with good accuracy.

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SYMBOLS

A_j	atomic mass of type j ion, amu
c	speed of light, cm/sec
C	shell correction
D(x)	energy absorbed per unit mass at x, MeV/g
e	electronic charge, coulumbs
E	ion kinetic energy, MeV/amu
E_0	incident beam energy, MeV/amu
E_n	dipole transition energy, eV
f_n	electric dipole oscillator strength
$F_j(E)$	incident beam flux, $(cm^2\text{-sec-MeV/amu})^{-1}$
I	mean excitation energy for stopping, eV
m	electron mass, eV/c^2
$m_{jk}(E)$	multiplicity of type j ions produced by collisions of type k ions of energy E
n_j	stoichiometric coefficient
N	density of scattering centers, cm^{-3}
$R_j(E)$	continuous slowing-down range of type j ion of energy E, cm
$R_j^{-1}[R_j(E)]$	inverse function of $R_j(E)$
S_e	electronic stopping power, MeV/cm
$S_j(E)$	stopping power of linear energy transfer (LET) due to interaction of type j ion with orbital electrons of transport medium, MeV/cm
$\tilde{S}_j(E)$	specific stopping power or linear energy transfer (LET) due to interaction of type j ion with orbital electrons of transport medium, MeV/amu-cm
S_n	nuclear stopping power, MeV/cm
v	speed of passing ion, cm/sec
x	one-dimensional position vector, g/cm^2
Z_j	atomic number of type j ion
Z^*	effective charge of a moving ion

β ratio of projectile speed to speed of light

ν_j range scale parameter for type j ion

ξ_j, η_j characteristic coordinates of type j ion, cm

$\sigma_j(E)$ macroscopic absorption cross section for type j ion of energy E, cm^{-1}

$\Phi_j(x,E)$ differential flux of type j ions at x with energy E, $(cm^2\text{-sec-MeV/amu})^{-1}$

$\chi_j(\eta_j, \xi_j)$ flux of type j ions in η_j, ξ_j characteristic space, $cm^{-3}\text{-sec}^{-1}$

$\bar{\chi}_j(\eta_k, \xi_k)$ flux of type j ions in η_k, ξ_k characteristic space, $cm^{-3}\text{-sec}^{-1}$

Subscript:

M type of ions in monoenergetic beam

Superscripts:

(0),(1),(2) terms in series approximation to equation (13)

Primes indicate a variable of integration.

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16. Abstract

An as yet unsolved problem in space radiation protection is the necessary relation between the external cosmic ray heavy ion fluence and the resultant environment within the spacecraft. Such a relation involves the transport of such ions through extended materials. Presented is a derivation of the solution of the transport equation for heavy ions in the straight ahead approximation, which is adequate for most space applications and serves as a first approximation for directed beam applications. An iterative scheme for the solution of the inhomogeneous integral transport equations is applied to a neon ion beam in water. The iterative scheme requires transport coefficients as input, which are determined from the available data on ion nuclear reactions. The iteration converged for the neon beam in water. Thus this iterative scheme appears to be a suitable approximation of heavy ion transport.

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