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STUDY OF AN AUTOMATIC TRAJECTORY
FOLLOWING CONTROL SYSTEM

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PREFACE

Initial funding for this study began in October 1976 under the NASA Terminal Configured Vehicle (ATOPS) program. The main effort was directed toward the application of a variation of a partitioned adaptive control algorithm to the dynamics of a B-737 aircraft. The special control law was one which could accommodate nonlinearities in the aircraft dynamics. Particular emphasis was placed on the problem of guiding the craft on optimum landing trajectories to provide quicker and safer landing approaches in adverse weather conditions. The details of this control law and its development were reported in interim unpublished reports dated 10/77 and 5/78. This work was done with the help of P. E. Zwicke.

Subsequently, with some shifts in personnel the project was continued with some refinements in the main control law. Some inefficiency was incurred through the natural difficulty of transferring computer code from one user to another. The emphasis was on simulation of nonlinear measurements, incorporation of a realistic wind shear and gust model and model reduction using a known correlated input as a pseudo-input to each configuration model. These aspects were reported on 2/79.

The project then suffered some delay and redirection. Taking over from William Lucas was Joel Brinkley. Joel's interest lay in studying the detection of failures in the control system. This last phase of the project is reported herein and provides a completion of the project (8/82).

All of the investigators would like to take this opportunity to thank the associated NASA personnel for their support and encouragement; in particular, Dr. T. M. Walsh for his confidence in getting our project started, Dr. J. R. Creedon for the bulk of the technical guidance and patience with our staffing problems and R. M. Hueschen and Nesim Halyo for their assistance and work on the technical aspects.

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1. INTRODUCTION

One of the major goals of the NASA Terminal Configured Vehicle (TCV) project is automatic control of aircraft in adverse weather for quicker, safer landings. The ATOPS B-737 was chosen for the program because of its nonlinear flight envelope. In addition to the nonlinear plant, some of the aerodynamic parameters are only partially known which makes it a difficult control problem. This part of the problem has been solved by Zwicke and Lucas (14,15,16); however, the effect of actuator and sensor failures was not explored. Sensor and actuator failure detection and compensation is an extremely important part of the overall aircraft control because if a failure is not detected and the controller is not properly compensated, the flight may end disastrously. In this thesis only the failure detection problem is undertaken except to show that the estimator part of the control algorithm used by Zwicke (14,16) can adapt to properly modeled sensor failures.

The first method to be used for failure detection is the configuration detection algorithm from the Modified Partitioned Adaptive Controller (14). This method uses a bank of parallel Kalman filters to predict the state of the various configurations and determines the correct configuration by the whiteness of the noise in the filter residue. The failure detection technique is tested to determine the types of failures it will detect and under what conditions this detection occurs. The most important criterion is the detection technique's ability to accurately and quickly determine actuator and sensor failures in the presence of sensor noise with inaccuracies in the plant model.

The second method to be used for failure detection is the failure detection filter, FDF, technique developed by Beard (1) and Jones (2). Since this failure detection technique was developed, little work has been done on estimation theory to determine how the noisy plant output is associated with the line or plane characteristic of a failure. A solution to this problem is presented and the FDF technique is tested with the same failures and conditions as the first method. Again the most important criterion is the FDF technique's ability to detect failures in the présence of noise with inaccuracies in the plant model.

In the past a number of different techniques have been used to detect actuator and sensor failures. The primary emphasis has been to detect sensor failures. The simplest schemes involve redundant hardware; but with the development of the computer, more and more schemes have been developed which depend on computational algorithms rather than redundant hardware to detect failures. Only a brief summary of the work done in the past on failure detection is presented here. A detailed summary of the different failure detection techniques can be found in (3).

Voting schemes are the simplest sensor failure detection scheme. In most voting schemes the output of three or more redundant sensors is compared. If the output of one of the sensors differs from the others, it is declared failed and removed from future consideration. This method is easy to implement, but can not detect soft failures or small biases and does not take advantage of information available from non-identical sensors. The following papers give examples of this method

(4,5,6,7). A modification of this method involves two sensors comparing outputs and if a difference arises, an analytical method is used to determine which sensor has failed. This reduces redundant hardware but at the cost of increased complexity. Work in this area has been done by Deckert, et al. (8). Another modification of the straight voting procedure is the method developed by Chien (9). This method uses a time weighted average of measurement errors to detect failures that memoryless voting schemes would miss.

In addition to voting schemes, a large number of failure detection techniques have been developed based on the likelihood ratio developed by Van Trees (10). The extension of this, the Generalized Likelihood Ratio or GLR, has been used in many applications because it is optimal in a statistical sense; however, it can become extremely complex and this is its major limitation. Caglayan and Montgomery (11,12) developed a technique using a bank of parallel Kalman filters to predict the state of all possible failure modes and used the GLR to find the probability of each configuration. Various other methods have been employed to reduce the complexity of the full GLR method and a good synopsis of these is done by Willsky (3).

Another method to detect actuator and sensor failures is the Failure Detection Filter, FDF, technique developed by Beard (1). This failure detection method can detect a wide range of failures in actuators and sensors or dynamic changes in one element of the system dynamics matrix or the input coefficient matrix. The filter is designed so that in the no failed state the filter estimator tracks the state estimator. When a failure occurs, the filter is designed to accentuate the failure in a predictable manner in the filter residual. A design algorithm was developed for continuous, linear, time-invariant, deterministic systems. Jones (2) extended the FDF method to stochastic and discrete systems. He also demonstrated that the FDF could be used as a suboptimal state estimator.

2. KALMAN FILTERS FOR FAILURE DETECTION

2.1 Introduction

The Kalman filter has been used for a number of years for a variety of applications. Its wide use is due primarily to its prediction correction structure for state estimation and its noise filtering capabilities. The Kalman filter is optimal in the sense that it minimizes the mean square error. In this chapter the Kalman filter is used to predict the state of various plant models in the presence of white noise. The whiteness of the filter residue, the plant output minus the output predicted by the Kalman filter, is used to determine the actual configuration.

The decision theory used in this chapter was presented by Moose and Wang in (13) and was developed for randomly switching plant configurations. These randomly switching plant configurations may be actuator or sensor failure models as long as the system remains observable and controllable after the failure.

One of the purposes of this chapter is to review the B737 project sponsored by NASA and show that the algorithm developed by Zwicke (14) can adequately control the aircraft after a sensor failure. The other purpose of this chapter is to show that the method developed by Zwicke, Moose, Wang, et al. (13,14) can be modified to detect both sensor and actuator failures and present an example for simulation.

2.2 The Structure of the Kalman Filter

The discrete time description of a linear system corrupted by noise is

$$x(k+1) = Ax(k) + Bu(k) + Gw(k) \tag{2.2.1}$$

$$y(k) = Cx(k) + Hv(k) \tag{2.2.2}$$

where,

- x is an $n\times1$ state vector
- u is an $m\times1$ input vector
- y is an $r\times1$ output vector
- w is an $n\times1$ input disturbance vector
- v is an $r\times1$ measurement noise vector
- A is an $n\times n$ system dynamics matrix
- B is an $n\times m$ input coefficient matrix
- C is an $r\times n$ measurement coefficient matrix

The vectors w and v are zero mean white Gaussian processes with variances Q and R respectively.

The Kalman filter is described by equations,

$$z(k+1/k) = Az(k) + Bu(k) \tag{2.2.3}$$

$$z(k+1) = z(k+1/k) + K(k+1)[y(k+1) - Cz(k+1/k)] \tag{2.2.4}$$

The Kalman filter gain K is calculated as follows

$$M(k+1) = AP(k)A^T + GQG^T \tag{2.2.5}$$

$$K(k+1) = M(k+1)C^T[CM(k+1)C^T + HRH^T]^{-1} \quad (2.2.6)$$

$$P(k+1) = [I - K(k+1)C]M(k+1)[I - K(k+1)C]^T + K(k+1)HRH^TK^T(k+1) \quad (2.2.7)$$

2.3 The B737 Project

A major goal of the NASA Terminal Configured Vehicle (TCV) project is automatic landing of aircraft in adverse weather on a predetermined glideslope with a linear decrease in both velocity and altitude. The automatic control is complicated by the nonlinear dynamics of the aircraft and the fact that many of these dynamics are not precisely known. In addition many aerodynamic coefficients are in tabular form and some of these are only estimated. The nonlinear measurement model and measurement errors further complicate the problem.

In the past, work has been done by Moose, VanLandingham, Zwicke and Lucas (14,15,16) on the control of the longitudinal dynamics of the B737 ignoring the possibility of a sensor failure. The goal of this part of the project is to show that satisfactory control of the aircraft can be maintained after a sensor failure.

In this section there is a brief review of the work done by Zwicke (14) and a development of the sensor failure model.

2.3.1 The B737 Model

The B737 is a highly nonlinear plant and is linearized about ten operating points using Taylor Series Expansion. From these ten plant models the longitudinal dynamics of the airplane can be adequately represented for the adaptive control algorithm. The assumptions that are made to further simplify the problem are:

1) Control inputs are elevator perturbation and throttle rate.

2) All lateral variables and their rates are small such as yaw, roll and sideslip.

3) The pitch angle is small.

4) All equations are linearized around

   u_o steady state inertial speed x-direction

   w_o steady state inertial speed z-direction

   θ_o steady state pitch angle

5) Perturbations of the pitch angle, θ, are small so $\cos\theta \simeq 1$ and $\sin\theta \simeq \theta$.

The B737 plant model is described by equations (2.2.1) and (2.2.2). The system dynamics is represented by fourteen state variables. Nine of these correspond to plant states and the other five state variables were added for the simulation of correlated wind gusts. The input coefficient, B, matrix has two rows: one indicating the effect of the throttle rate on the system and the other the effect of the elevator perturbation on the system. The measurement coefficient, C, matrix is a nonlinear combination of the states. It has been linearized about an operating point and is accurate for small values of noise.

2.3.2 The B737 Controller

The control technique used in the B737 project is the Modified Partitioned Adaptive Controller or MPAC developed by Zwicke (14). The controller computes a system input based on the plant configuration S_i and the reference input. A bank of parallel Kalman filters operates independently on noisy measurements. Each filter produces a state estimate $Z_i(k+1)$. This state estimate is multiplied by a feedback gain calculated for the i^{th} configuration. In addition to the state variable feedback, two other terms must be added to the input for each configuration. The reference input, q, is scaled so that zero steady state error can be achieved, and a linearization constant from the Taylor Series Expansion associated with each model must be added. Therefore, the input is of the form

$$U_i(k) = F_i(k)Z_i(k) + H_i q + T_i \tag{2.3.1}$$

where

$U_i(k)$	is the input calculated for the i^{th} configuration
$F_i(k)$	is the time varying feedback gain for the i^{th} configuration
$Z_i(k)$	is the Kalman estimate for the i^{th} configuration
H_i	is the scaling constant for zero steady state error
q	is the reference input
T_i	is the Taylor Series Expansion constant for the i^{th} configuration

In order to calculate the overall system input a weighted sum of the individual inputs U_i is computed. The weighting coefficients Π_i are calculated as follows,

$$\Pi(k+1) = C(k+1)\begin{bmatrix} p_1(r(k+1)) & & 0 \\ \vdots & \ddots & \vdots \\ 0 & & p_m(r(k+1)) \end{bmatrix}\begin{bmatrix} \theta_{11} & \cdots & \theta_{1m} \\ \vdots & \ddots & \vdots \\ \theta_{m1} & \cdots & \theta_{mm} \end{bmatrix}\Pi(k) \tag{2.3.2}$$

where

$$p_i(r(k+1)) = p_i(y(k+1)/S(k+1) = S_i(k),\ \bar{y}(k)) \tag{2.3.3}$$

and

θ_{ji} is a semi-Markov transition parameter

$C(\bar{k}+1)$ is a normalizing constant

$\Pi(k+1)$ is a vector containing the probability of each plant configuration

$r_i(k+1)$ is the plant output minus the i^{th} filter output

The probability density $p(y(k+1)/S(k+1) = S_i, \bar{y}(k))$ is approximated by a Gaussian density for the cases where the probability of transition between adjacent samples is small and is given by

$$p_i(r_i(k+1)) = D_i \exp\{-1/2 \; r_i^T(k+1)V_i^{-1} \; r_i(k+1)\} \qquad (2.3.4)$$

where,

V_i is the measurement residual covariance for the i^{th} filter

$$D_i = (2\Pi)^{-M/2} \; |V_i|^{-1/2} \qquad (2.3.5)$$

The semi-Markov transition probabilities θ_{ji} in (2.3.2) represent the probability of changing from configuration S_i to S_j. An excellent discussion of how to choose the semi-Markov matrix can be found in (14). When the largest weighting coefficient $\Pi_j(k+1)$ falls below a predetermined threshold, the Kalman gains for all the configurations are reset in order to improve the convergence of the filters when the plant moves from one configuration to another. This threshold is found by trial and error and is dependent primarily on the signal to noise ratio.

2.3.3 The Sensor Failure Model for B737

A sensor failure is modeled as zero output from the failed sensor which is equivalent to a zero row in the C matrix in equation (2.2.2). The sensor failures that were included as possible configurations were those which (A, C) from equations (2.2.1) and (2.2.2) remained an observable pair after the sensor failure.

2.4 Actuator and Sensor Failure Detection using Kalman Filters

The Kalman filter approach to detecting failures has many similarities to the estimator part of the MPAC developed in the last section. The Kalman filter approach to detecting failures developed in this thesis is a slight modification of the work done by Moose and Wang (13).

The method developed in this section can detect sensor or actuator failures in a system which is observable after the failure. For each failure a model is developed. The actuator and sensor failure models are developed in sections 3.4.1 and 3.4.2 respectively. A bank of parallel Kalman filters, one filter for each possible failure mode, is run with each filter independent of the others. The filter residue from each filter is then used to compute the probability of that state, using equation (2.3.2). As in the MPAC when the largest probability $\Pi_i(k+1)$ falls below a predetermined threshold, the Kalman gains are reset to improve convergence.

2.4.1 Actuator Failure Model

A failure in the i^{th} actuator is modeled for the Kalman filter detection technique by modifying the i^{th} column of the B matrix, b_i. A constant bias is modeled by multiplying b_i by a constant not equal to one. The magnitude of the bias is one minus the constant multiplier. A zero failure is modeled by replacing b_i by a vector with all zero entries.

2.4.2 Sensor Failure Model

A failure in the i^{th} sensor is modeled for the Kalman filter detection technique by modifying the i^{th} row of the C matrix, C_i. A constant bias is modeled by multiplying C_i by a constant. The magnitude of the bias is found exactly as it was in section 2.4.1. A zero failure is modeled by replacing C_i with a vector containing all zeros and a soft failure in the i^{th} sensor is modeled by increasing the i^{th} diagonal element H which increases the noise on the i^{th} sensor.

2.4.3 Boiler System Example E1

This is an example of a boiler system that might be used on board ship and is adapted from an example in (17). There are two boilers on board but only one will be modeled. It is assumed that the other is operating at steady state and at the desired operating point. There are three states of interest: 1) pressure deviation, P, around 20 bars; 2) temperature deviation, T, around 300°C; 3) the integral of the energy/KG of the superheated steam about the desired operating point (20 bars and 300°C). The control implemented would try to keep P and T at the operating point and keep the integral of the steam energy zero; however, in this thesis only detection of an actuator or sensor failure is considered.

The three inputs to the system are fuel, superheated steam and saturated steam. The steam can be considered an input when it is used to preheat the boiler or to meet the operating conditions when there is a zero or low fuel rate to the boiler.

The continuous time system is modeled

$$\dot{x} = Ex + Fu \tag{2.4.1}$$

$$y = Cx \tag{2.4.2}$$

where the state variables in the vector x are

x_1 is the deviation of pressure around 20 bars in the boiler

x_2 is the deviation of the output temperature around 300°C

x_3 is an intermediate variable

x_4 is MJ/kg of energy in the steam integrated over time

and the inputs u are defined

u_1 is the flow rate of fuel (Mg/hr)

u_2 is the flow rate of superheated steam (Mg/hr)

u_3 is the flow rate of saturated steam (Mg/hr)

The continuous model matrices are

$$E = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1/154 & -1/154 & 0 \\ 0 & 0 & -1/183 & 0 \\ .00338 & .02363 & 0 & 0 \end{bmatrix} \tag{2.4.3}$$

$$F = \begin{bmatrix} 396\times10^{-3} & 325\times10^{-4} & -325\times10^{-4} \\ 1250/154 & 1.7 & -629.35\times10^{-3} \\ 0 & 1375/83 & -662.65 \\ 0 & 0 & 0 \end{bmatrix} \tag{2.4.4}$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 3 & 1 & 0 & 1 \\ 2 & 1 & 1 & 1 \end{bmatrix} \qquad (2.4.5)$$

The continuous time system is converted to a discrete time system with a forty-second sampling time. The discrete time model is given by

$$x(k+1) = Ax(k) + Bu(k) + Gw(k) \qquad (2.2.1)$$

$$y(k) = Cx(k) + Hu(k) \qquad (2.2.2)$$

where

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & .77125 & -.1796 & 0 \\ 0 & 0 & .61759 & 0 \\ .1352 & .832419 & -.09633 & 1 \end{bmatrix} \qquad (2.4.6)$$

$$B = \begin{bmatrix} 15.84 & 1.3 & -1.3 \\ 285.935 & -7.65 & 2679.275 \\ 0 & 525.812 & -21032.44 \\ 140.98 & 6.998 & 893.619 \end{bmatrix} \qquad (2.4.7)$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 3 & 1 & 0 & 1 \\ 2 & 1 & 1 & 1 \end{bmatrix} \qquad (2.4.8)$$

Below are examples of the system with a 10% bias failure for both an actuator and sensor failure. Other failures are modeled similarly following the rules in section 2.4.1 or 2.4.2. A 10% bias failure in the fuel flow rate, actuator one, is modeled by letting B become B' where B' is

$$B' = \begin{bmatrix} 16.94 & 1.3 & -1.3 \\ 314.53 & -7.65 & 2679.275 \\ 0 & 525.812 & -21032.44 \\ 155.078 & 6.998 & 893.619 \end{bmatrix} \tag{2.4.9}$$

A 10% bias in the first sensor is modeled by letting C become C' where C' is

$$C' = \begin{bmatrix} 1.1 & 1.1 & 0 & 1.1 \\ 3 & 1 & 0 & 1 \\ 2 & 1 & 1 & 1 \end{bmatrix} \tag{2.4.10}$$

3. FAILURE DETECTION FILTERS FOR FAILURE DETECTION

The failure detection filter, FDF, was first developed by Beard (1) for continuous, deterministic, time-invariant, linear systems. Jones (2) extended the FDF theory to include discrete and stochastic systems. However, since this work was done, very little research has been done either on further development or application of FDF theory. In this chapter the basic concepts and design algorithms are presented so that a reader will have the tools to design and use FDF's to detect actuator or sensor failures. In addition a configuration estimation technique is developed that gives the probability that a noisy plant output is associated with a line or plane characteristic of an actuator or sensor failure.

3.1 Development of Actuator and Sensor Failure for Rank of C Greater Than or Equal to N

The purpose of this section is to give an introduction to the failure detection filter. The case demonstrated in this section, the rank of the measurement coefficient matrix greater than or equal to the rank of the system dynamics matrix, demonstrates many of the basic concepts in failure detection filters. A more complete development of the topic may be found in (1,2). It should be noted that the failure models shown in this section are valid for C of any rank.

The plant model used in this section is found in equations (2.2.1) and (2.2.2); however, v(k) and w(k) will be identically zero.

The failure detection filter equation is

$$z(k+1) = Gz(k) + Dy(k) + BFuD(k) \tag{3.1.1}$$

where (3.1.1) is valid for C of any rank and (3.1.2-6) are valid for Rank[C] greater than or equal to n. The parameter G is defined

$$G = A - DC \tag{3.1.2}$$

$$G = \sigma I \tag{3.1.3}$$

where $|\sigma|$ must be less than one for a stable filter. The detector gain D is defined for m=n

$$D = (A - G)C^{-1} \tag{3.1.4}$$

or for $m > n$

$$D = (A - G)(C^T C)^{-1} C^T \tag{3.1.5}$$

and BF is defined

$$BF = B \tag{3.1.6}$$

It is important to note that, for the rank of the C matrix greater than or equal to n, equations (3.1.1-6) yield a filter that will detect any single actuator of sensor failure.

3.1.1 Actuator Failure

A failure in the i^{th} actuator is modeled

$$u(k) = uD(k) + er_i\, n(k) \tag{3.1.7}$$

where

uD(k) is the desired control input

n(k) is an arbitrary function of time sampled at $t = kT$

and

$$er_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{3.1.8}$$

where the 1 is in the i^{th} position.

Example E2 is an example of an actuator failure with the rank of C equal to n. Assume,

$$C = I \tag{3.1.9}$$

and the FDF is defined by equation (3.1.1)

$$z(k+1) = Gz(k) + Dy(k) + BFuD(k) \tag{3.1.1}$$

where

$$y(k) = x(k) \tag{3.1.10}$$

$$G = \sigma I \tag{3.1.11}$$

$$D = A - \sigma I \tag{3.1.12}$$

$$BF = B \tag{3.1.13}$$

$$u(k) = uD(k) + er_i\ n(k) \tag{3.1.14}$$

The FDF residue is

$$\varepsilon(k+1) = x(k+1) - z(k+1) \tag{3.1.15}$$

where this can be expanded to

$$\varepsilon(k+1) = Ax(k) + Bu(k) - Gz(k) - Dx(k) - BFuD(k) \tag{3.1.16}$$

$$\varepsilon(k+1) = Ax(k) + BuD(k) + Ber_i\ n(k) - \sigma Iz(k) - (A - \sigma I)x(k) - BuD(k) \tag{3.1.17}$$

$$= \sigma Ix(k) + Ber_i\ n(k) - \sigma Iz(k) \tag{3.1.18}$$

$$= \sigma I(x(k) - z(k)) + b_i\ n(k) \tag{3.1.19}$$

where

$$b_i = Ber_i \tag{3.1.20}$$

The solution to equation (3.1.19) is

$$\varepsilon(k) = (\sigma I)^k\ \varepsilon(0) + \sum_{j=0}^{k-1} (\sigma I)^{k-j-1}\ b_i\ n(j) \tag{3.1.21}$$

$$= (\sigma)^k\ \varepsilon(0) + \sum_{j=0}^{k-1} (\sigma I)^{k-j-1}\ b_i\ n(j) \tag{3.1.22}$$

$$= \sigma^k\ \varepsilon(0) + b_i \sum_{j=0}^{k-1} (\sigma)^{k-j-1}\ n(j) \tag{3.1.23}$$

The steady state solution to (3.1.23) is

$$\varepsilon(k) \simeq b_i \sum_{j=0}^{k-1} (\sigma I)^{k-j-1}\ n(j) \tag{3.1.24}$$

It should be noted that b_i is a vector and the rest of equation (3.1.24) is a scalar and that if n(k) is a constant then

$$\lim_{k\to\infty} \sum_{j=0}^{k-1} \sigma^{k-j-1}\, n(j) = \text{constant} \tag{3.1.25}$$

Also, it can be shown that in equation (3.1.22), $\varepsilon(k)$ does not depend on the rank of C and is valid for any C if σI is replaced by (A-DC) in that equation.

However, $\varepsilon(k)$ is not an accessible signal. The accessible signal is

$$\varepsilon'(k) = C\varepsilon(k) \tag{3.1.26}$$

$$= y(k) - Cz(k) \tag{3.1.27}$$

$$= Cb_i \sum_{j=0}^{k-1} {}^{k-j-1}\, n(j) \tag{3.1.28}$$

Therefore, the error vector Cb_i is indicative of a failure in the i^{th} actuator for C of any rank and the magnitude of the failure may give information about the nature of the failure. It can easily be shown that in order to identify a failure in the i^{th} actuator, Cb_i must be independent of Cb_j for all $j \neq i$. If there are two dependent vectors Cb_i and Cb_j, i not equal to j, then the direction alone will not supply enough information to determine which actuator has failed.

3.1.2 Sensor Failure

A failure in the i^{th} sensor is modeled

$$y(k) = Cx(k) + em_i\, n(k) \tag{3.1.29}$$

where n(k) is an arbitrary function of time sampled at t=kT and

$$em_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{3.1.30}$$

where 1 is in the i^{th} position.

Example E3 is an example of the i^{th} sensor failing with the rank of C equal to n. Assume

$$C = I \tag{3.1.31}$$

The FDF is defined by equation (3.1.1)

$$z(k+1) = Gz(k) + Dy(k) + BFu(k) \tag{3.1.1}$$

where G, D and BF are defined by equations (3.1.11-13) and where u(k) is the desired input defined

$$u(k) = uD(k) \tag{3.1.32}$$

The sensor outputs are defined by (3.1.29). The FDF residue is

$$\begin{aligned} x(k+1) - z(k+1) = {} & Ax(k) + Bu(k) - \\ & [\sigma Iz(k) + D(x(k) + em_i\ n(k)) + Bu(k)] \end{aligned} \tag{3.1.33}$$

$$\varepsilon(k+1) = \sigma I(x(k) - z(k)) - Dem_i\ n(k) \tag{3.1.34}$$

$$\varepsilon(k+1) = \sigma I \ \varepsilon(k) - Dem_i \ n(k) \tag{3.1.35}$$

The solution to equation (3.1.35) is

$$\varepsilon(k) = \sigma^k \ \varepsilon(0) - \sum_{j=0}^{k-1} \sigma^{k-j-1} \ Dem_i \ n(j) \tag{3.1.36}$$

The settled out signal is

$$\varepsilon(k) \simeq - Dem_i \sum_{j=0}^{k-1} \sigma^{k-j-1} \ n(j) \tag{3.1.37}$$

As in the actuator failure case, $\varepsilon(k)$ is not accessible. The accessible signal is

$$\varepsilon'(k) = y(k) - Cz(k) \tag{3.1.38}$$

$$\varepsilon'(k) = C\varepsilon(k) - em_i \ n(k) \tag{3.1.39}$$

$$\simeq -CDem_i \sum_{j=0}^{k-1} (\sigma I)^{k-j-1} \ n(j) + em_i \ n(k) \tag{3.1.40}$$

since

$$D = A - \sigma I \tag{3.1.12}$$

then,

$$Dem_i = (A - \sigma I)em_i \tag{3.1.41}$$

and letting

$$Aem_i = a_i \tag{3.1.42}$$

$$\varepsilon'(k) = a_i \sum_{j=0}^{k-1} \sigma^{k-j-1} \ n(j) + \left[n(k) + \sum_{j=0}^{k-1} \sigma^{k-j} \ n(j) \right] em_i \tag{3.1.43}$$

It can be shown that equation (3.1.40) is valid for any C if σI is replaced by A-DC.

The error vector associated with a sensor failure can only be constrained to a plane determined by $CDem_i$ and em_i and it can be shown that this plane is characteristic of a failure in the i^{th} sensor for C of any rank. The sensor failures are detectable even when two or more error planes intersect except when one of the two following cases occurs: 1) the error planes are coincident; or 2) the error signal maintains a fixed direction coincident with the intersection of the two planes.

3.2 Fundamental Background for Rank C Less Than N

This section is intended to present the basic concepts of the work done by Beard (1) and Jones (2) on failure detection filters. The proofs and underlying theory will not be presented and can be found in (1,2). The goal of this section is to develop the background material needed for designing FDF for either actuator or sensor failures where the rank of C is less than n. The actual design algorithms for actuator and sensor failures are presented in sections 3.3 and 3.4.

The background material starts with the definition of the error vector $\varepsilon(k+1)$ and f the failure vector associated with a particular failure:

$$\varepsilon(k+1) = G\varepsilon(k) + fn(k) \qquad (3.2.1)$$

where

$$G = A - DC \qquad (3.1.2)$$

is the filter dynamics matrix and n(k) is defined in equation (3.1.7) or (3.1.29) and f is given by one of the two following cases:

$$1) \quad f = b_i \quad \text{for effector failures} \tag{3.2.2}$$

$$2) \quad f = De_i \quad \text{for sensor failures} \tag{3.2.3}$$

An event associated with the vector f in equation (3.2.1) is defined to be detectable if there exists a matrix D, the filter gain, such that

1) $C\varepsilon(k)$ maintains a fixed direction (actuator failure) or plane (sensor failure) in the output space.

2) All eigenvalues of (A-DC) can be specified almost arbitrarily.

Condition 1) should be self-explanatory and condition 2) is necessary in order to insure a stable filter and to give the designer freedom to change eigenvalues of the filter in order to improve the filter's performance in a noisy environment.

The detection space of f is defined as the controllable subspace of G with respect to f. The dimension of the controllable subspace of f with respect to G is v.

The n-vector g is defined to be the k^{th} order detection generator for f if the following conditions are met:

1) $$\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{k-2} \end{bmatrix} g = 0 \tag{3.2.4}$$

2) $$CA^{k-1}\, g \neq 0 \tag{3.2.5}$$

3) $$f = \alpha_1 g + \ldots + \alpha_{k-1} A^{k-2} g + A^{k-1} g \tag{3.2.6}$$

if $Cf \neq 0$ and where $\alpha_1, \alpha_2, \ldots, \alpha_{k-1}$ are scalars

If g is a k^{th} order detection generator and k eigenvalues of A-DC are associated with the controllable space of f, then any solution of

$$DCf = P_1 g + P_2 Ag + \ldots + P_k A^{k-1} g + A^k g \tag{3.2.7}$$

will be a detector gain for f. The eigenvalues associated with the controllable space of A-DC are given by

$$z^k + P_k z^{k-1} + \ldots + P_2 z + P_1 = 0 \tag{3.2.8}$$

where P_i are scalars and z is the z-transform variable.

The general solution to equation (3.2.7) is

$$D = QD[(Cf)^T Cf]^{-1} (Cf)^T + D'\{I - Cf[(Cf)^T Cf]^{-1}(Cf)^T\} \tag{3.2.9}$$

where

$$QD = P_1 g + P_2 Ag + \ldots + P_k A^{k-1} g + A^k g \tag{3.2.10}$$

and D' is the freedom left in D after D has been constrained to be a detector gain for f. It can be shown that the number of eigenvalues of A-DC which can be changed arbitrarily with D' is n-v and the number of eigenvalues specified by QD is k. Therefore, v-k eigenvalues are not under the control of the designer. This result will become important later.

The following definitions are made, in part, so that it can be determined how many eigenvalues of A-DC are free after constraining D to be a detector gain for f.

$$A' = A - QD[(Cf)^T Cf]^{-1}(Cf)^T \tag{3.2.11}$$

$$C' = [I - Cf[(Cf)^T Cf]^{-1}(Cf)^T]C \tag{3.2.12}$$

$$K = A - Af[(Cf)^T Cf]^{-1}(Cf)^T C \tag{3.2.13}$$

$$M' = \begin{bmatrix} C' \\ C'A' \\ \vdots \\ C'A'^{n-1} \end{bmatrix} \tag{3.2.14}$$

or

$$M' = \begin{bmatrix} C' \\ C'K \\ \vdots \\ C'K^{n-1} \end{bmatrix} \tag{3.2.15}$$

$$q' = \text{rank } [M'] \tag{3.2.16}$$

$$v = n - q' \tag{3.2.17}$$

where v is the detection order of f.

It has been shown that

$$A' - D'C' = A - DC \tag{3.2.18}$$

where D' is the arbitrary matrix of equation (3.2.9). The unobservable space of (A', C') is associated with the eigenvalues which were constrained by making D a detector gain for f. Therefore it follows that the detection space of f is the null space of M'. The dimension of the detection space of f, v, is defined to be the detection order of f.

It was noted earlier that if the detection order of f, v, is not equal to the order of the detection generator, k, then some of the eigenvalues of A-DC will not be under the control of the designer. Therefore a generator of order v, called a maximal detection generator or just a maximal generator, must be found in order to have full control over the eigenvalues. It has been shown that if (A, C) is an observable pair, then a maximal generator exists and is unique. Appendix A describes the procedure for finding the maximal generator. If (A, C) is not an observable pair and the error residue does not lie in the unobservable space, then a detection filter gain D for f may be found but n-q'-k eigenvalues of the filter will be uncontrollable.

The theory developed so far applies to detection of a single failure f. The background material covered in the rest of this section is for multiple failure detection with a single filter.

The vectors $\{f_1, f_2, \ldots, f_r\}$ are defined to be output separable if

$$\text{Rank}[CF] = r \tag{3.2.19}$$

where F is the n×r matrix

$$F = [A^{u_1}f_1, \ldots, A^{u_r}f_r] \tag{3.2.20}$$

with u_i for each f_i defined by

$$CA^j f_i = 0 \qquad j = 1, \ldots, u-1 \tag{3.2.21}$$

$$CA^{u_i} f_i \neq 0 \tag{3.2.22}$$

If two vectors are not output separable, then one of two cases must hold:

1) The two vectors have the same detection space and a detector gain for one is a detector gain for both.

2) The two vectors do not have the same detection space and can not be detected by the same filter.

The requirements for detection of more than one failure with a single filter are the same as for a single failure detection filter. The vectors $\{f_1, \ldots, f_r\}$ are defined to be mutually detectable if there exists a D which satisfies the following conditions.

1) $C\varepsilon(t)$ maintains a fixed direction in the output space (actuator failure) or stays in a fixed plane (sensor failure).

2) All eigenvalues of A-DC can be specified almost arbitrarily.

The detector gain D that will constrain the error residue to a direction or plane can be found by solving the following equation

$$D = QD[(CF)^T CF]^{-1}(CF)^T + D'\{I - CF[(CF)^T CF]^{-1}(CF)^T\} \tag{3.2.23}$$

where the eigenvalues of A-DC associated with constraining D to be a detector gain for F are fixed by the following equation

$$QD = [wd_1, \ldots, wd_r] \tag{3.2.24}$$

where

$$wd_i = P_{i1}g_i + \ldots + P_{ik_i}A^{k_i-1}g_i + A^{k_i}g_i \tag{3.2.25}$$

and g_i is the maximal generator. The scalars $[P_{i1}, \ldots, P_{ik_i}]$ are defined by

$$z^{k_i} + P_{ik_i}z^{k_i-1} + \ldots + P_{i1}z + P_{i2} = 0 \tag{3.2.26}$$

where z is the z-transform variable. As in the single failure detection case D' is the freedom left in D after D has been constrained to be a detector gain for F.

Definitions similar to equations (3.2.11-16) are needed to determine how much freedom will be left in the eigenvalues of A-DC after constraining D to be a detector gain for F.

$$C' = I - CF[(CF)^T CF]^{-1}(CF)^T C \tag{3.2.27}$$

$$K = A - AF[(CF)^T CF]^{-1}(CF)^T C \tag{3.2.28}$$

$$MG' = \begin{bmatrix} C' \\ C'K \\ \vdots \\ C'K^{n-1} \end{bmatrix} \tag{3.2.29}$$

$$qg' = \text{rank}\,[MG'] \tag{3.2.30}$$

The group detection order, gdo, of $\{f_1, \ldots, f_r\}$ is

$$gdo = n - qg' \qquad (3.2.31)$$

In the equation (3.2.23) it can be shown that the number of eigenvalues which are fixed when QD is found is $\sum_{i=1}^{r} v_i$. Similar to the single filter case, qg' is the number of eigenvalues which can be specified by arbitrary choice of D'. Therefore, $n - \sum_{i=1}^{r} v_i - qg'$ is the number of eigenvalues not under the control of the designer. These eigenvalues can be found. The algorithm for determining them may be found in (1). The designer will have control over all the eigenvalues if and only if the group detection order, gdo, is equal to the sum of the individual detection orders, $\sum_{i=1}^{r} v_i$.

3.3 Failure Detection Filter Design for Actuator Failures

This section is a step by step guide to designing FDF of the form of equation (3.1.1) for multiple actuator failures and is derived from (1). The design of single failure filters is similar except that steps 2, 3, 5 and 7 are not needed.

The failure of the i^{th} actuator is associated with b_i, the i^{th} column of B, where

$$B = [b_1, \ldots, b_r] \qquad (3.3.1)$$

The following steps are a guide to designing the one or more FDF's needed to detect an actuator failure.

1) If any

$$Cb_i = 0 \qquad (3.3.2)$$

replace b_i by $A^u b_i$ where u is defined by

$$CA^j b_i = 0 \qquad j = 0, \ldots, u-1 \qquad (3.3.3)$$

$$CA^u b_i \neq 0 \qquad (3.3.4)$$

2) Form F

$$F = [A^{u_1} b_1, A^{u_2} b_2, \ldots, A^{u_r} b_r] \qquad (3.3.5)$$

3) Find the rank of CF and if it is less than r divide the set into two or more subsets until the rank of each set is the same as the number of vectors in it. If the rank of CF equals r, then f is output separable.

4) Determine the maximal generator and the detection order, v_i, for each f_i in F. If two or more f_i have the same detection spaces, only one of them needs to be considered in the following steps. Any detection filter for one such vector will be a detection filter for all.

5) Determine C', K, MG' and qg' for F using equations (3.2.27-3.2.30).

6) Determine C', K and M' for each f_i using equations (3.2.12-3.2.14).

7) If the group detection order n-qg' is not equal to $\sum_{i=1}^{r} v_i$,

i) Divide the set into two or more sets and find C', K, MG' and qg' for the new set F.

ii) See if the group detection order equals $\sum_{i=1}^{r} v_i$.

iii) Repeat the process if the group detection order of the new set F is not equal to $\sum_{i=1}^{r} v_i$.

8) A detector gain for each set of vectors from step 7 may be found by solving equation (3.2.23).

9) From

$$G = A - DC \tag{3.2.2}$$

10) Let

$$BF = B \tag{3.3.6}$$

The line characteristic of a failure in the i^{th} actuator is given by Cb_i.

If there are other detection filters, they will produce error signals if a failure in the i^{th} actuator occurs, but they will not lie in a fixed direction, or they will not lie in a direction for which the filter was designed.

This example E4 will demonstrate the design of a FDF for multiple actuator failures. The system will be the boiler system in example E1. The eigenvalues of all the filters will be placed at ±.5. The system model is described by the matrices (2.4.6-8).

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & .77125 & -.1796 & 0 \\ 0 & 0 & .61759 & 0 \\ .1352 & .832419 & -.09633 & 1 \end{bmatrix} \tag{2.4.6}$$

$$B = \begin{bmatrix} 15.84 & 1.3 & -1.3 \\ 285.935 & -7.65 & 2679.275 \\ 0 & 525.812 & -21032.44 \\ 140.98 & 6.998 & 893.619 \end{bmatrix} \tag{2.4.7}$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 3 & 1 & 0 & 1 \\ 2 & 1 & 1 & 1 \end{bmatrix} \tag{2.4.8}$$

The filter design is below.

1) Check all Cb_i and

$$Cb_i \neq 0 \tag{3.3.7}$$

so go to next step.

2) Form F

$$F = \begin{bmatrix} 15.84 & 1.3 & -1.3 \\ 285.935 & -7.65 & 2679.275 \\ 0 & 525.812 & -21032.44 \\ 140.98 & 6.998 & 813.619 \end{bmatrix} \tag{3.3.8}$$

3) The rank of CF is 3 therefore F is output separable.

4) Determine the maximal generators for f_1, f_2 and f_3 and they are

$$g_1 = f_1 = \begin{bmatrix} 15.84 \\ 285.935 \\ 0 \\ 140.98 \end{bmatrix} \tag{3.3.9}$$

$$g_2 = f_2 = \begin{bmatrix} 1.3 \\ -7.65 \\ 525.812 \\ 6.998 \end{bmatrix} \tag{3.3.10}$$

$$g_3 = f_3 = \begin{bmatrix} -1.3 \\ 2679.275 \\ -21032.44 \\ 893.619 \end{bmatrix} \tag{3.3.11}$$

The detection order, v_i, of all three vectors is one. Steps 5) and 6) are simple equation evaluations and will not be shown here; however, the main result of steps 5) and 6) is

$$qg' = 0 \tag{3.3.12}$$

7) The group detection order is four and the sum of the individual detection orders is three. Therefore, one eigenvalue will not be under the control of the designer if the FDF is designed with F. So the set is subdivided into two sets

$$F_1 = \{f_1, f_2\} \tag{3.3.13}$$

$$F_2 = \{f_3\} \tag{3.3.14}$$

The group detection order of F_1 is two and the sum of the individual detection orders is two. Therefore, all the eigenvalues of A-DC are under the control of the designer. For the rest of the design, only F_1 will be considered.

8) Find the detector gain for F_1 realizing that two eigenvalues must be constrained to make D a detector gain for F_1 and two eigenvalues will be arbitrary.

The values of P_{11} and P_{12} in w_{d1} and w_{d2} are -.5, where wd_i is given by equation (3.2.25).

The solution is

$$D = \begin{bmatrix} 7.92 & .65 \\ 77.5612 & -92.36 \\ 0 & 61.83 \\ 310.65 & -40.61 \end{bmatrix} \begin{bmatrix} .001088 & .001162 & -.00007164 \\ -.000921 & -.000979 & .0019019 \end{bmatrix} +$$

$$\begin{bmatrix} -.42 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} .53316 & -.4989 & .00138 \\ -.4989 & .46683 & -.00129 \\ .00138 & -.00129 & 0 \end{bmatrix} \tag{3.3.15}$$

$$D = \begin{bmatrix} -.21591 & .2181 & 8.8367\times10^{-3} \\ .16941 & .1805 & -.18121 \\ -.05694 & -.06053 & .11759 \\ .37525 & .040075 & -.099485 \end{bmatrix} \tag{3.3.16}$$

9) Form G

$$G = \begin{bmatrix} .56143 & -.00279 & -.00009 & -.00229 \\ -.348617 & .60251 & .001616 & -.16874 \\ .00334 & -.00012 & .49999 & -.000124 \\ -1.2433 & .1559 & .0031556 & .32349 \end{bmatrix} \tag{3.3.17}$$

The eigenvalues of G are .4937, .4937, .5 and .5.

10) Let

$$BF = B \tag{3.3.6}$$

11) The line characteristic of a failure in actuator 1 is given by the vector

$$Cb_1 = \begin{bmatrix} 442.775 \\ 474.435 \\ 458.595 \end{bmatrix} \tag{3.3.18}$$

$$Cb_1 \simeq \begin{bmatrix} 1 \\ 1.07 \\ 1.04 \end{bmatrix} \tag{3.3.19}$$

The characteristic vector for f_2 is found similarly.

The FDF that will detect a failure in actuator one or two is given by

$$Z_1(k+1) = GZ_1(k) + Dy(k) + BuD(k) \tag{3.3.20}$$

where G is given by equation (3.3.17), D is given by equation (3.3.16) and B is given by equation (2.4.7).

A failure in actuator one will cause, in the noiseless case,

$$y(k) - C\,Z_1(k) = \varepsilon_1(k) \tag{3.3.21}$$

after all transients have settled out to lie on a line given by (3.3.19). Similar results can be derived for a failure in actuator two.

3.4 Failure Detection Filter Design for Sensor Failures

This section is a step by step guide for designing FDF. The design algorithm is for a single sensor failure and is derived from (1). The design of a filter for detecting multiple sensor failures will not be covered in this thesis because of the complexity of the algorithm. The detection of more than one sensor failure with one filter is developed in (1).

A FDF for a sensor failure is designed by the following steps.

1) Find f such that

$$Cf = em_i \tag{3.4.1}$$

where em_i is defined by equation (3.1.30).

2) Find g the maximal generator for f using the algorithm in Appendix A.

3) Find A', C' and K using equations (3.2.11-13).

4) Find

$$f' = Af \tag{3.4.2}$$

5) One of the following must hold

   i) Af lies in the observable space of C' with respect to A'.

   ii) Af lies in the unobservable space of C' with respect to A' and any detection gain satisfying

$$DCf = QD \tag{3.4.3}$$

   is also a detector gain for Af.

6) If case ii holds, solve for D using equation (3.2.9).

7) If case i holds, find g', A" by repeating steps two and three replacing f with Af, C with C', and A with A'.

8) Solve for QD'. Where QD' is formed by equation (3.2.10) with A replaced by A' and g replaced by g'.

9) Solve for D' using the equation

$$D' = QD'[(C'Af)^T C'Af]^{-1} C'Af + D''\{I-C'Af[(C'Af)^T C'Af]^{-1} C'Af\} \tag{3.4.4}$$

where D" is an arbitrary matrix.

10) Solve for A'-D'C' noting that

$$A-DC = A'-D'C' \tag{3.4.5}$$

and

$$G = A'-D'C' \tag{3.4.6}$$

11) Solve for D

$$D = (A-G)(CC^T)^{-1}C^T \tag{3.4.7}$$

12) Let

$$BF = B \tag{3.4.8}$$

The plane characteristic of the failure of the i^{th} sensor is given by the vectors em_i and C'Af.

This example, E5, will demonstrate the design of a FDF for a single sensor failure. The system will be the boiler system of example E1.

The eigenvalues of the filter will be placed at ±.5. The system model is described by the matrices (2.4.6-8).

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & .77125 & -.1796 & 0 \\ 0 & 0 & .61759 & 0 \\ .1352 & .832419 & -.09633 & 1 \end{bmatrix} \tag{2.4.6}$$

$$B = \begin{bmatrix} 15.84 & 1.3 & -1.3 \\ 285.935 & -7.65 & 2679.275 \\ 0 & 525.812 & -21032.44 \\ 140.98 & 6.998 & 813.419 \end{bmatrix} \tag{2.4.7}$$

$$C = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 3 & 1 & 0 & 1 \\ 2 & 1 & 1 & 1 \end{bmatrix} \tag{2.4.8}$$

The filter design is as follows:

1) Solve for f in (3.4.1)

$$Cf = em_1 \tag{3.4.1}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 3 & 1 & 0 & 1 \\ 2 & 1 & 1 & 1 \end{bmatrix} f = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{3.4.9}$$

$$f = \begin{bmatrix} -.5 \\ 1.5 \\ -.5 \\ 0 \end{bmatrix} \tag{3.4.10}$$

2) The maximal generator, g, for f is

$$g = \begin{bmatrix} -.5 \\ 1.5 \\ -.5 \\ 0 \end{bmatrix} \tag{3.4.11}$$

The detection order of f is one.

3) Find A' and C'. K will not be found since it is unnecessary for this part.

$$A' = \begin{bmatrix} 1.25 & .25 & 0 & .25 \\ -.496675 & .274575 & -.1796 & -.496675 \\ .058795 & .058795 & .61759 & .058795 \\ -1.09399 & -.39667 & -.09633 & -.22919 \end{bmatrix} \tag{3.4.12}$$

where P_1 from equation (3.2.10) is -.5 and

$$C' = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 3 & 1 & 0 & 1 \\ 2 & 1 & 1 & 1 \end{bmatrix} \tag{3.4.13}$$

4) Find Af which is f'

$$Af = \begin{bmatrix} -.5 \\ 1.24668 \\ -.308795 \\ 1.22919 \end{bmatrix} \tag{3.4.14}$$

5) Since

$$C'Af \neq 0 \tag{3.4.15}$$

then Af lies in the observable space of A' with respect to C'.

6) Since case i holds, step 6 is skipped.

7) Find g'

$$g' = \begin{bmatrix} -.08647 \\ .006089 \\ .104733 \\ 1.2292 \end{bmatrix} \tag{3.4.16}$$

8) Find QD'

$$QD' = \begin{bmatrix} .243968 \\ -.587745 \\ .079686 \\ -.814229 \end{bmatrix} \tag{3.4.17}$$

9) Solve for D' using (3.4.4).

$$D' = \begin{bmatrix} .243968 \\ -.587745 \\ .079686 \\ -.814229 \end{bmatrix} \begin{bmatrix} 0 & .421659 & .50427 \end{bmatrix} +$$

$$\begin{bmatrix} 0 & 1.65 & 0 \\ 0 & 3.3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & .58852 & -.4921 \\ 0 & -.4921 & .41148 \end{bmatrix} \qquad (3.4.18)$$

It should be noted that it takes two eigenvalues of (A-DC) to constrain the residue to a plane and therefore two are left arbitrary and were specified by D".

10) Find A'-D'C'

$$A'-D'C' = \begin{bmatrix} -.5939 & -.1350 & .6889 & -.1350 \\ -1.7389 & .5 & 1.7407 & -.2706 \\ -.1228 & -.0151 & .5773 & -.0151 \\ .7572 & .3571 & .3143 & .5247 \end{bmatrix} \qquad (3.4.19)$$

11) Solve for D

$$D = \begin{bmatrix} -.25 & 1.0739 & -.6889 \\ .49709 & 1.694 & -1.9203 \\ -.0589 & .0337 & .04029 \\ 1.229 & -.3433 & -.41063 \end{bmatrix} \qquad (3.4.20)$$

12) Let

$$BF = B \tag{3.4.21}$$

The plane characteristic of a failure in sensor one is given by the vectors em_i and $C'Af$

$$em_i = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{3.4.22}$$

$$C'Af = \begin{bmatrix} 0 \\ .9757 \\ 1.167075 \end{bmatrix} \tag{3.4.23}$$

The FDF that will detect a failure in sensor one is given by

$$Z_2(k+1) = GZ_2(k) + Dy(k) + Bu(k) \tag{3.4.24}$$

where G and D are given by (3.4.19) and (3.4.20).

A failure in sensor one will cause, in the noiseless case,

$$y(k) = CZ_2(k) = \varepsilon_2(k) \tag{3.4.25}$$

after transients have settled out to lie in a plane given by the vectors (3.4.22) and (3.4.23). Failure detection filters to detect a failure in sensors two and three are found similarly.

3.5 Configuration Estimation Technique

The first part of the chapter is concerned with developing a FDF that will constrain the filter residue to a predictable line or plane if there is a failure which the filter is designed to detect. In this section a decision technique is developed which gives the probability that a noisy plant output is associated with a particular line or plane.

Moose and Wang (13) demonstrated a decision technique that detects random configuration changes in the presence of noise. The decision algorithm is given below.

$$\Pi(k+1) = \begin{bmatrix} p_1(r_1(k+1)) \dots & 0 \\ \vdots & \vdots \\ 0 & \dots\ p_m(r_m(k+1)) \end{bmatrix} \begin{bmatrix} \theta_{11} & \cdots & \theta_{1m} \\ \vdots & & \vdots \\ \theta_{m1} & \cdots & \theta_{mm} \end{bmatrix} \Pi(k) \quad (3.5.1)$$

where θ_{ji} is a semi-Markov transition probability and where

$$p_i(r_i(k+1)) = p_i(y(k+1))|S(k+1) = S_i(k),\bar{y}(k)) \qquad (3.5.2)$$

where $p_i(y(k+1))|S(k+1) = S_i(k),\bar{y}(k))$ is approximated to be a Gaussian density function and where $S_i(k)$ is the i^{th} configuration at time t=k. Since $p_i(r_i(k+1))$ is a Gaussian density it is given by

$$p_i(r_i(k+1)) = D_i \exp\{-1/2\ r_i^T(k+1)V_i^{-1}\ r_i(k+1)\} \qquad (3.5.3)$$

where V_i is the covariance of a filter perfectly matched to the plant and where the constant D_i is defined by

$$D_i = (2\Pi)^{-m/2} |V_i|^{-1/2} \tag{3.5.4}$$

It is assumed for this development that equation (3.5.2) can be approximated as a Gaussian density as it was in (13). Using this assumption, the only unknown parameters of (3.5.1) and (3.5.3) are θ_{ji}, the semi-Markov transition probabilities, $r_i(k+1)$, the plant output minus the Kalman filter output in (13), and V_i, the covariance of the residue $r_i(k+1)$ when the filter is perfectly matched to the plant configuration.

The semi-Markov transition probability θ_{ji} is the probability that the plant will change from configuration S_i to configuration S_j. An excellent discussion of how to choose the semi-Markov parameters is in (14).

The residue $r_i(k+1)$, as stated above, is in (13) the plant output minus the Kalman filter output. In three dimensional output space, the residue $r_i(k+1)$ is the minimum distance between two points in output space. Therefore, the algorithm is actually dependent on the minimum distance between two points in the output space.

Since the algorithm developed in (13) is minimum distance dependent, it follows that the algorithm can be extended to be the minimum distance between the filter residue point and a line or a plane, and the algorithm can be used to compute the probability that a filter residue is associated with a line for actuator failures or a plane for sensor failures. The modified algorithm is called the line or plane detection algorithm or LOPDA.

The value $r_i(k+1)$ must be the minimum distance between the plant output and the predicted line or plane. The minimum distance is computed by least squares using the formula

$$S^T S\ x = S^T b \tag{3.5.5}$$

where

S is the set of vectors $[S_{1i}, S_{2i}]$ needed to describe the plane associated with the i^{th} sensor failure or the vector $[S_{i1}]$ needed to describe the line associated with the i^{th} actuator failure.

b is the residue $\varepsilon_i(k+1)$

x is the solution to (3.5.5)

The minimum distance is calculated

$$r_i(k+1) = \left| X_1 * S_{1i} + X_2 * S_{2i} - \varepsilon_i(k+1) \right| \tag{3.5.6}$$

where $\varepsilon_i(k+1)$ is defined

$$\varepsilon_i(k+1) = y(k+1) - CZ_i(k+1) \tag{3.5.7}$$

The final unknown in equation (3.5.1) and (3.5.3) is the covariance V_i. The mean is also calculated to show that the predicted line or plane is the mean of a FDF residue where the plant is in a failure state the FDF is designed to detect. The plant is described by equations (2.2.1-2) and the i^{th} FDF is given by

$$z_i(k+1) = Gz_i(k) + Dy(k) + Bu(k) \tag{3.5.8}$$

where

$$G = A\text{-}DC \tag{3.1.2}$$

The following definitions are made to simplify the mean and variance calculations.

$$xd(k) = Ax(k) + Bu(k) \tag{3.5.9}$$

$$yd(k) = Cxd(k) \tag{3.5.10}$$

$$zd(k+1) = (A\text{-}DC)zd(k) + Dyd(k) + Bu(k) \tag{3:5.11}$$

$$d(k+1) = yd(k+1) - Czd(k+1) \tag{3.5.12}$$

The vector G is defined as any vector which lies in the predicted direction or plane for an actuator or sensor failure respectively.

The expected value of $\varepsilon(k+1)$ is

$$E\{\varepsilon(k+1)\} = E\{y(k+1) - Cz(k+1)\} \tag{3.5.13}$$

$$= E\{C[Ax(k) + Bu(k) + Gu(k)] + Hw(k) - C[A\text{-}DC)z(k) + Dy(k) + Bu(k)]\} \tag{3.5.14}$$

$$= E\{yd(k+1) - C[A\text{-}DC)z(k) + Dyd(k) + Bu(k)] + CGv(k) + Hw(k) - CD[CGv(k-1) + Hw(k-1)]\} \tag{3.5.15}$$

$$= E\{\varepsilon d(k+1)\} + E\{CGv(k) + Hw(k) - CD[Gv(k-1) + Hw(k-1)]\} \tag{3.5.16}$$

$$= E\{\varepsilon d(k+1)\} + 0 \tag{3.5.17}$$

The term $\varepsilon d(k+1)$ is the FDF residue with no noise. Therefore, it can be concluded that

$$E\{\varepsilon(k+1)\} = \Gamma \tag{3.5.18}$$

The steady state variance of $\varepsilon(k+1)$ is

$$E\{\varepsilon(k+1)\varepsilon^T(k+1)\} = E\{[y(k+1) - Cz(k+1)][y(k+1) - Cz(k+1)]^T\} \tag{3.5.19}$$

$$\begin{aligned} = E\{&(C[Ax(k) + Bu(k) + Gv(k)] + \\ &Hw(k) - C[(A-DC)z(k) + \\ &D[yd(k) + CGv(k-1) + Hw(k-1)] + Bu(k)]) \\ &(C[Ax(k) + Bu(k) + Gv(k)] + \\ &Hw(k) - C[(A-DC)Z(k) + \\ &D[yd(k) + CGv(k-1) + Hw(k-1) + Bu(k)])^T\} \end{aligned} \tag{3.5.20}$$

$$\begin{aligned} = E\{&([yd(k+1) - Czd(k+1)] + CGv(k) + Hw(k) - \\ &CD[CGv(k-1) + Hw(k)])([yd(k+1) - Czd(k+1)] + \\ &CGv(k) + Hw(k) - CD[CGv(k-1) + Hw(k)])^T\} \end{aligned} \tag{3.5.21}$$

$$\begin{aligned} = E\{&\varepsilon d(k+1)\varepsilon d^T(k+1)\} + E\{CGv(k)v^T(k)G^TC^T + \\ &Hw(k)w^T(k)H^T - CD(CGv(k-1)v^T(k-1)G^TC^T + \\ &Hw(k-1)w^T(k-1)H^T)\} \end{aligned} \tag{3.5.22}$$

At steady state if $n(k)$ in equation (3.2.1) is constant or slowly time-varying, then the variance will be approximately

$$E[\varepsilon(k+1)\varepsilon^T(k+1)] \simeq CGQG^TC^T + HRH^T - CD(CGQG^TC^T + HRH^T)D^TC^T \tag{3.5.23}$$

where Q and R are the covariance of $v(k)$ and $w(k)$ respectively.

The failure detection part of the algorithm is complete and a failure can be detected using an equation of the form (3.5.1); however, the no failure case has not been discussed. For failure detection filters the no failure state is characterized by a zero residue because all FDF's track the plant state. Therefore, to detect the no failure state in the presence of noise, the distance from the residue point to zero $r_i(k+1)$ is $\varepsilon_i(k+1)$. It can easily be shown that if

$$\varepsilon_i(k+1) = 0 \tag{3.5.24}$$

the least squares algorithm in equation (3.5.5) will give

$$\bar{x} = 0 \tag{3.4.25}$$

Therefore, $r_i(k+1)$ will be the same for all values of i. To prevent this a failure threshold is set and the magnitude of $\bar{x}$ is not allowed to be less than the threshold. It can be shown that a FDF will always have a residue when a failure occurs even though it may not be in a fixed direction or plane. The probability of the no failed state is computed by letting any residue

$$r_1(k+1) = \varepsilon_i(k+1) \tag{3.5.25}$$

and where $r_1(k+1)$ is placed in equation (3.5.1) and $\Pi_1(k+1)$ is the probability of the no failed state.

To summarize the results of this section, the steps to determine if a failure has occurred in actuator one or two are given where Γ_i

is the direction of the line characteristic of a failure in the i^{th} actuator.

1) Find the semi-Markov transition matrix, set the threshold and find V_i the covariance for the i^{th} residue and D_i.

The following steps must be completed each iteration.

2) Find the plant output $y(k+1)$.

3) Find the FDF outputs $z_i(k+1)$.

4) Find the FDF residue $\varepsilon_i(k+1)$.

5) Calculate $r_i(k+1)$, the minimum distance between the plant output and Γ_i or, for the no failure case, $P_i(k+1)$ is $\varepsilon_i(k+1)$.

6) Calculate $p_i(r_i(k+1))$.

7) Calculate $\Pi(k+1)$ the probability of a failure in actuator i or a no failure state.

4. SIMULATION AND RESULTS

The purpose of this chapter is to exercise the theory developed in the previous two chapters and show their merit. This will be done by computer simulation. First, the B737 aircraft is simulated using the model developed by Zwicke, et al., to show that the estimator part of the control algorithm can adapt to a sensor failure, and the results are discussed in Section 4.1. Then the boiler system developed in example E1 is simulated with actuator and sensor failures using the two detection techniques developed, the Kalman filter technique developed in Chapter 2 and the FDF technique developed in Chapter 3.

The boiler system simulations were designed to find what types of failures the failure detection techniques can detect and how they respond to modeling errors and sensor noise. The following types of actuator and/or sensor failures were considered.

1) Biases on the actuator or sensor.
2) Zero failures, i.e. no input from an actuator or no output from a sensor.
3) Soft failures, i.e. increased noise on sensors.

The results of these simulations are discussed in Sections 4.2 and 4.3.

4.1 B737 Simulation and Results

The B737 model with the linearized measurement model was simulated in the presence of sensor noise. The sensor noise covariance is gotten from (14). The only types of failures simulated are zero failures of a sensor.

The aircraft is started below the glideslope and below the target speed. The failure of sensor one occurs at T=6 seconds. Figure 1 shows that the estimator did determine with a high degree of certainty that a failure did occur. Figure 2 shows that the plane does stay on the glideslope even after the sensor failure and the aircraft performance is very good.

As a result of the simulations, it has been shown that the control algorithm adapts quickly to a sensor failure as long as (A,C) remains an observable pair. The simulations also show that the estimator part of the controller has good noise rejection; but as expected, performance is degraded as sensor noise increases. A major point that should not be overlooked is the fact that all possible failures must be modeled.

4.2 Kalman Filter Technique Simulation and Results

The boiler system developed in example E1 was simulated for both actuator and sensor failures using the Kalman filter technique developed in Chapter 2.

The different types of actuator failures simulated include large and small bias and a zero failure under a variety of sensor noise levels. The failures were simulated for two types of desired inputs. They were a step input and a ramp input. This was done to see if the type of input affected the failure detection. For all these conditions the Kalman filter technique quickly and accurately identified the failure even in the presence of sensor noise. Figure 3 is typical

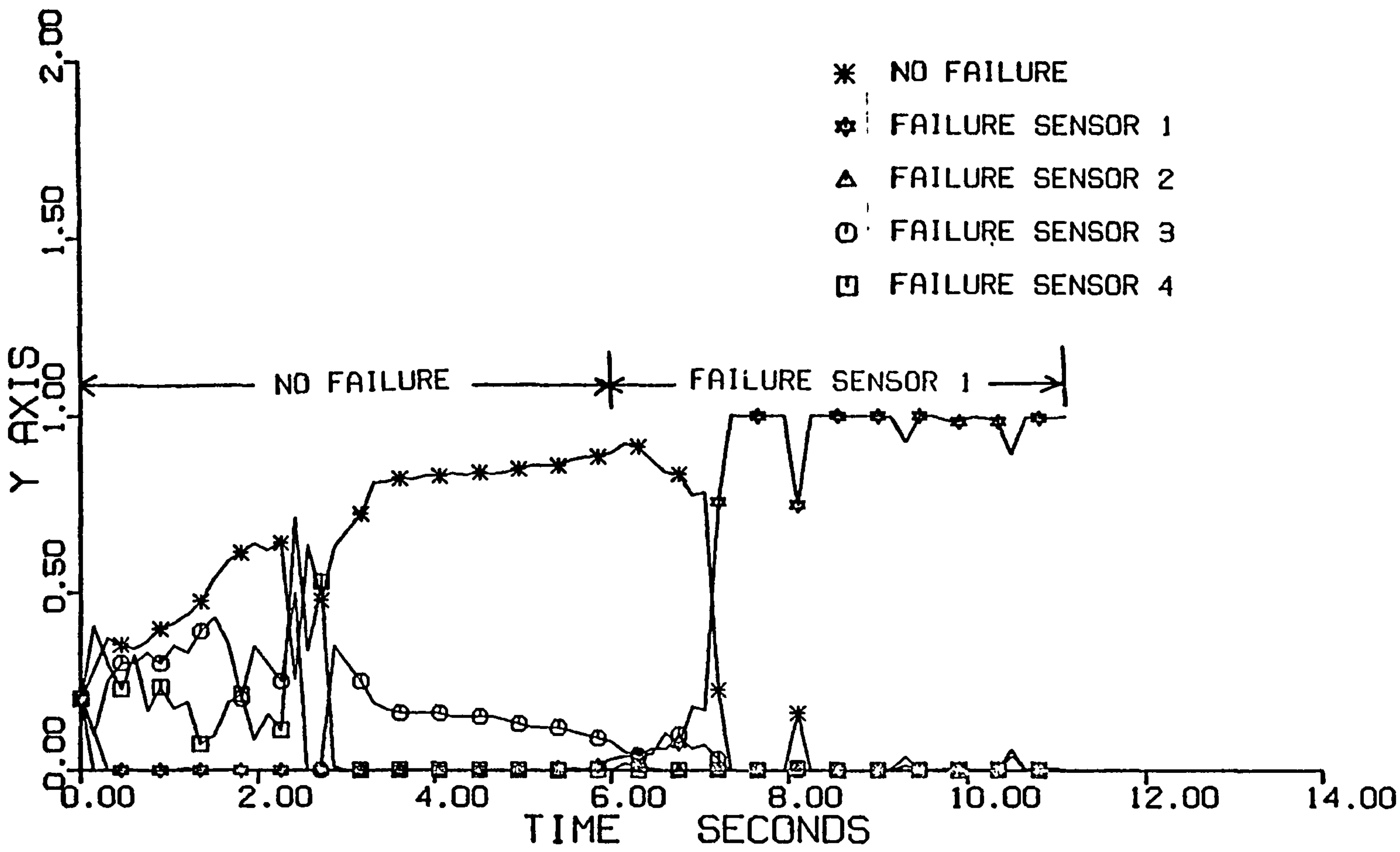

Figure 1. Configuration probabilities for B737.

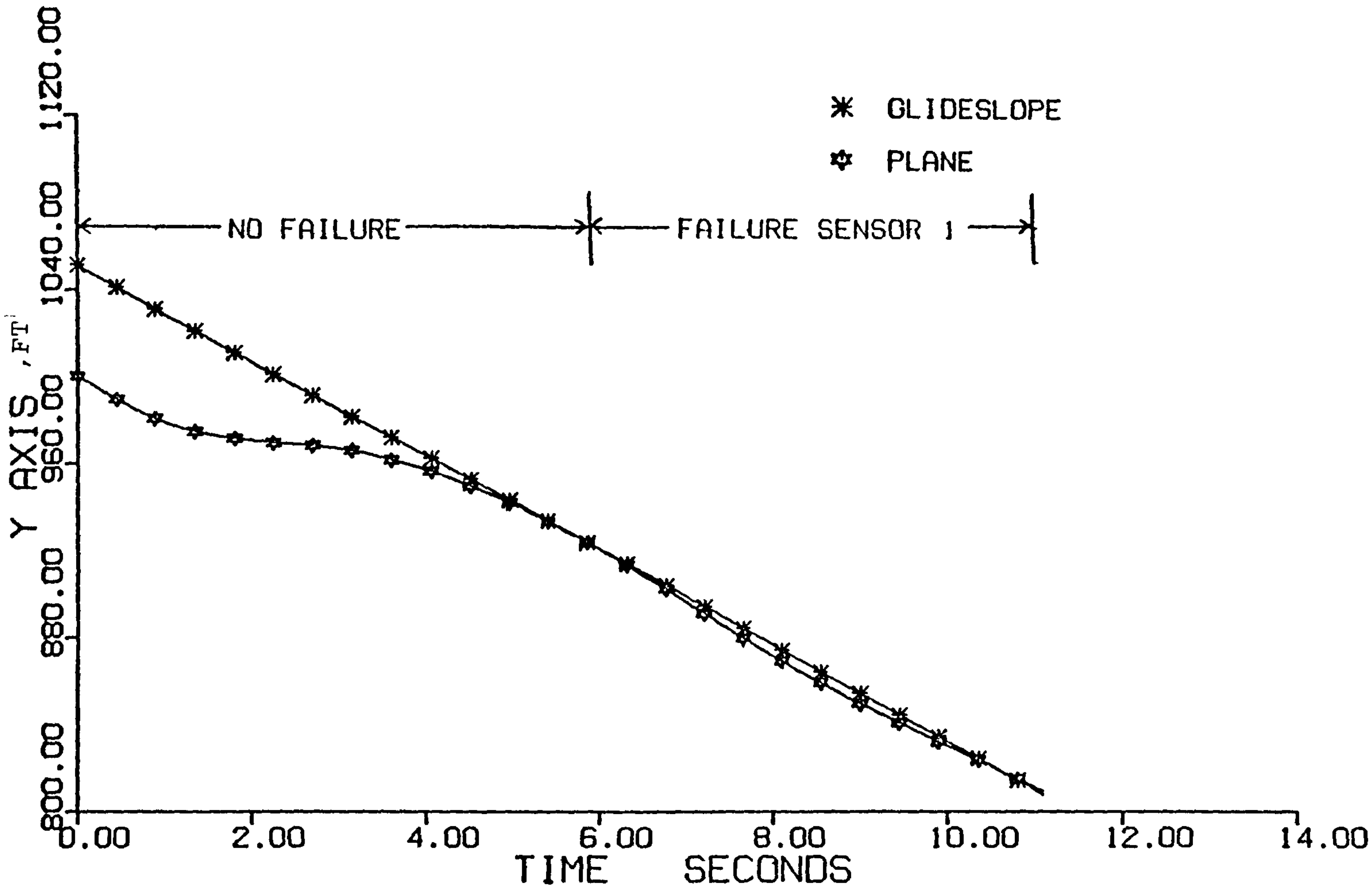

Figure 2. Performance of B737 on the Glide Slope.

showing a correct identification of the no failed state and the failed actuator (actuator 2).

Actuator failures were also simulated in the presence of modeling error. This was done by changing one or all of the eigenvalues of the plant by a known value from the model the Kalman filter was using. In this, too, the Kalman filter approach did extremely well. Figure 4 is typical showing the identification of a failure in actuator one when there is a 5% change of all the eigenvalues. This method showed good tolerance for modeling error but the performance of the decision algorithm was degraded as the number of changed eigenvalues increased and as the percent error increased.

The different types of sensor failures simulated were large and small biases on a sensor and a zero output, all of these under a variety of sensor noise levels. The failures were simulated for two types of inputs, a step and a ramp. The failure detection was not affected by a change in inputs. For all the different types of failures, the failure detection technique worked extremely well even in the presence of sensor noise; however, this technique will not detect soft failures in sensors. For all other cases, the no failed state was quickly identified and the failure state was identified soon after the failure occurred even in the presence of sensor noise. Figure 5 is typical showing the identification of the no failed state and the identification of the sensor failure after a short false identification.

Sensor failures were also simulated in the presence of modeling error just as the actuator failures were. In this case too, the

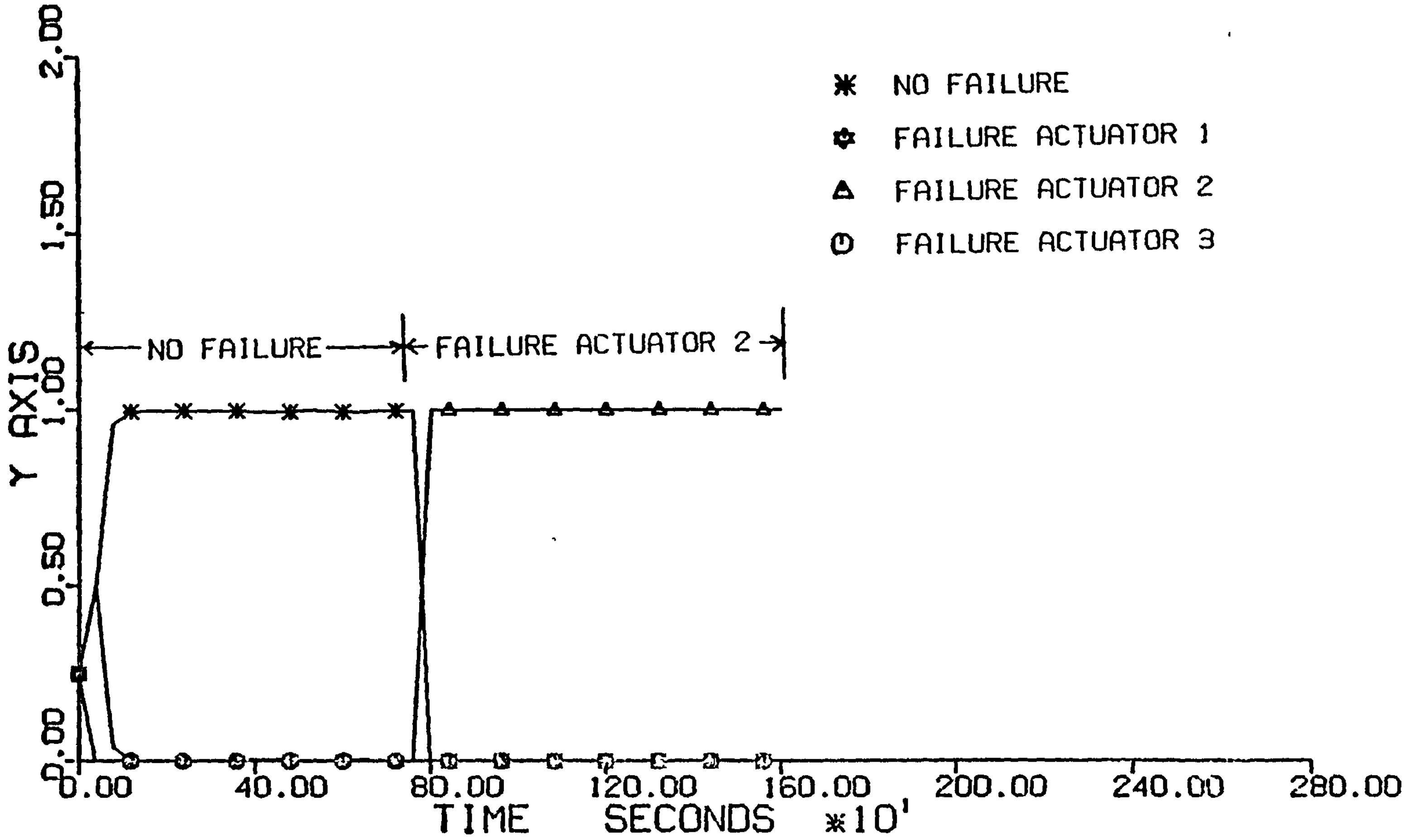

Figure 3. Actuator failure detection with sensor noise by Kalman filter technique.

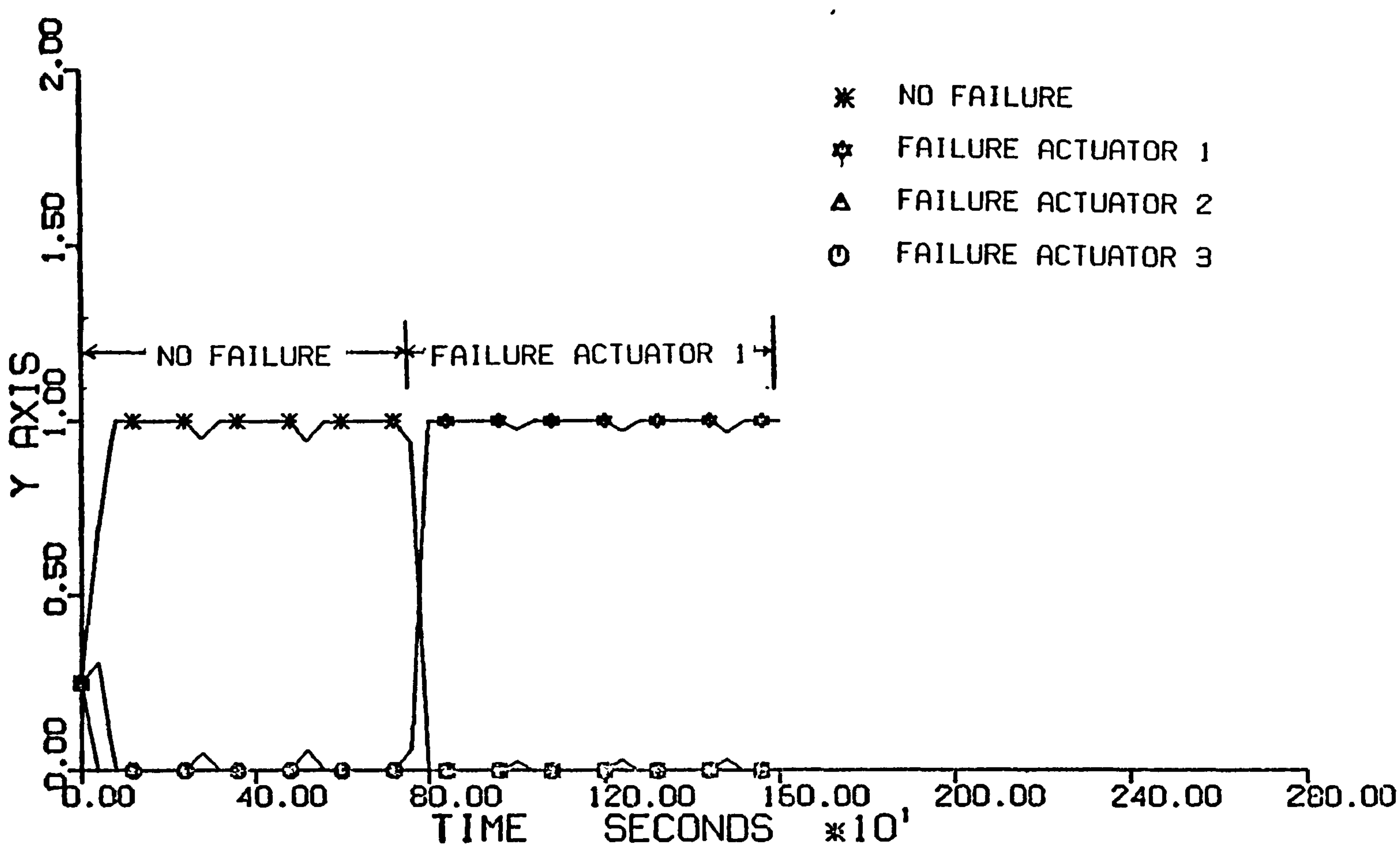

Figure 4. Actuator failure detection with sensor noise by Kalman filter technique.

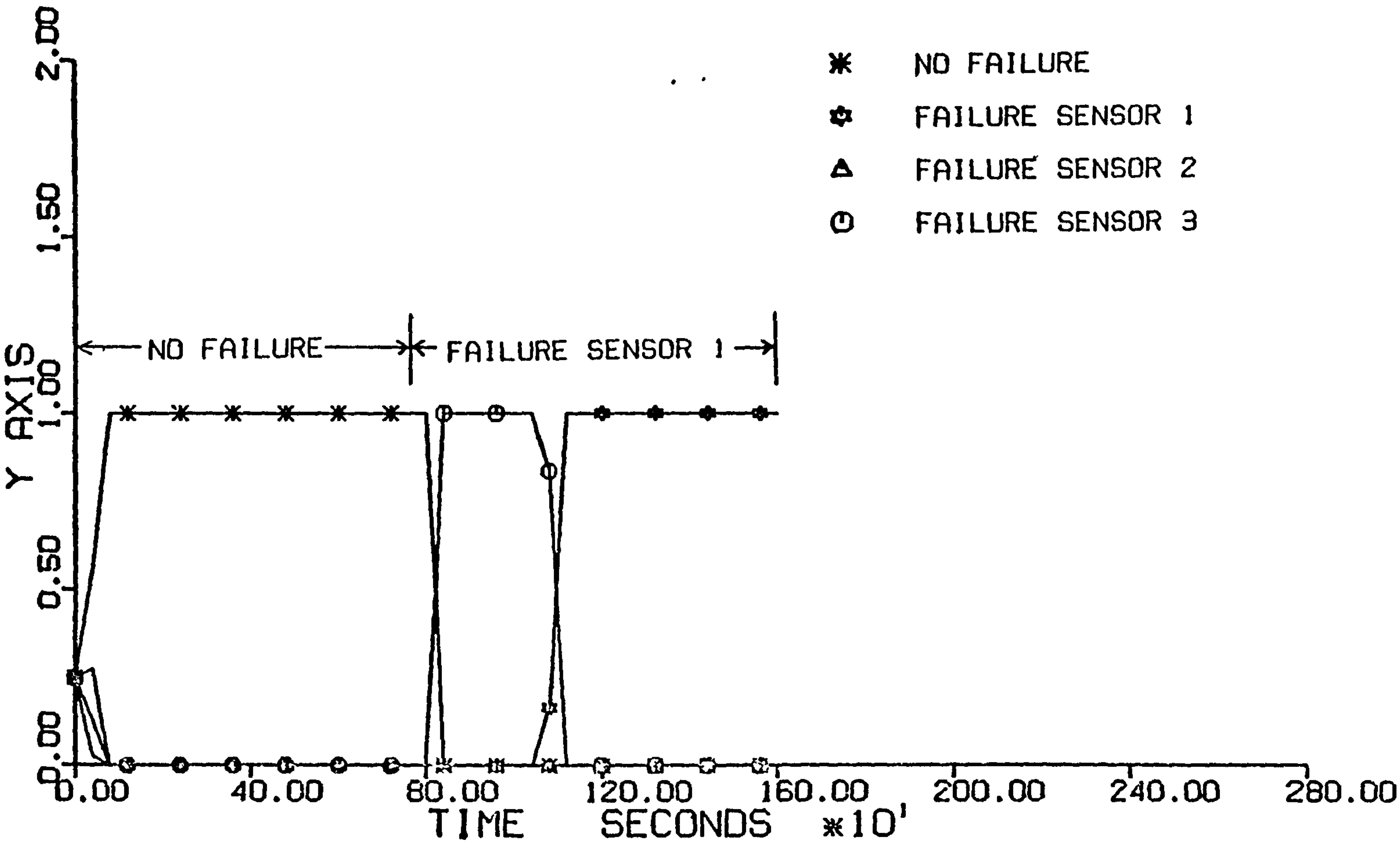

Figure 5. Sensor failure detection with sensor noise by Kalman filter technique.

failure detection technique quickly identified the failure even when there was a 10% modeling error in all the eigenvalues. Figure 6 is typical showing the identification of a failure in sensor one when all the eigenvalues of the Kalman filter are incorrectly modeled by 10%. Again as the number of eigenvalues incorrectly modeled increased and the percentage error increased, the performance was degraded but the degradation was not noticed until all the eigenvalues were changed more than 10%.

In summary, the Kalman filter detection technique developed in Chapter 2 works extremely well for a wide range of actuator and sensor failures except soft failures. Also, changing the system input seemed to have no effect on failure detection. This method also worked well in the presence of sensor noise and when there was significant modeling error. The major disadvantage of this method of failure detection is that every failure must be modeled and all the models must be run in parallel which can become overwhelming computationally.

4.3 Failure Detection Filter Technique Simulation and Results

The boiler system in Chapter 2 was also simulated for both actuator and sensor failures using the FDF technique developed in Chapter 3.

The different types of actuator failures simulated included large and small biases and zero failure of an actuator, all of these both with and without sensor noise. The failures were also simulated with the desired inputs being either a step or ramp. The no failure state and failures were identified regardless of the input, as long as the residue was large enough to exceed the threshold, for large and small biases

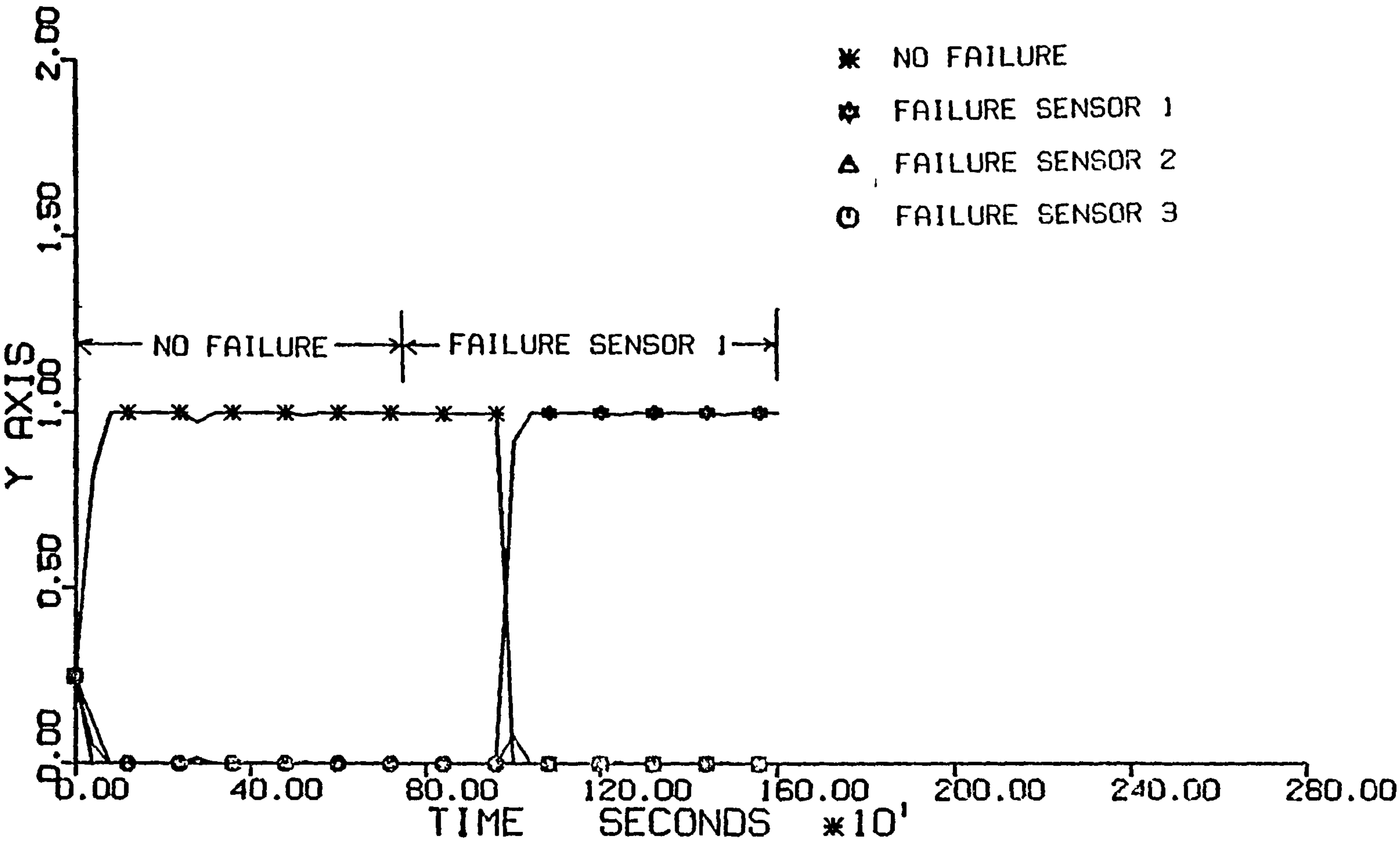

Figure 6. Sensor failure detection with 10% modeling error on all eigenvalues by Kalman filter technique.

and zero inputs when there was no sensor noise. Figure 7 is typical showing the no failure state correctly identified and a failure in actuator one being correctly identified when a small bias is applied to the actuator. However, when sensor noise is added, the performance was poor for all cases except for very low noise levels. This was not totally unexpected because the FDF was not optimized for performance in noisy environments; however, the no failure configuration is extremely unreliable. This is primarily due to the difficulty of determining the threshold which distinguishes the no failure state from the failure state. The threshold must be changed whenever the magnitude of the failure becomes small or the sensor noise level changes which makes this technique for identifying the no failure configuration very difficult to implement. Figure 8 shows a correct identification of the no failure configuration and a small bias failure for actuator two; however, the SNR was in the order of 10000:1. With noise levels greater than this, the technique could not distinguish the no failure case with any reliability.

The actuator failure was also simulated in the presence of modeling error. Unlike the Kalman filter technique, the FDF technique is very sensitive to modeling error. The largest modeling error that the FDF technique could consistently tolerate and still identify the no failure and failure configurations was about 5% in one eigenvalue. Figure 9 shows the identification of the no failure configuration and the failure of actuator one in the presence of a 5% modeling error of one eigenvalue.

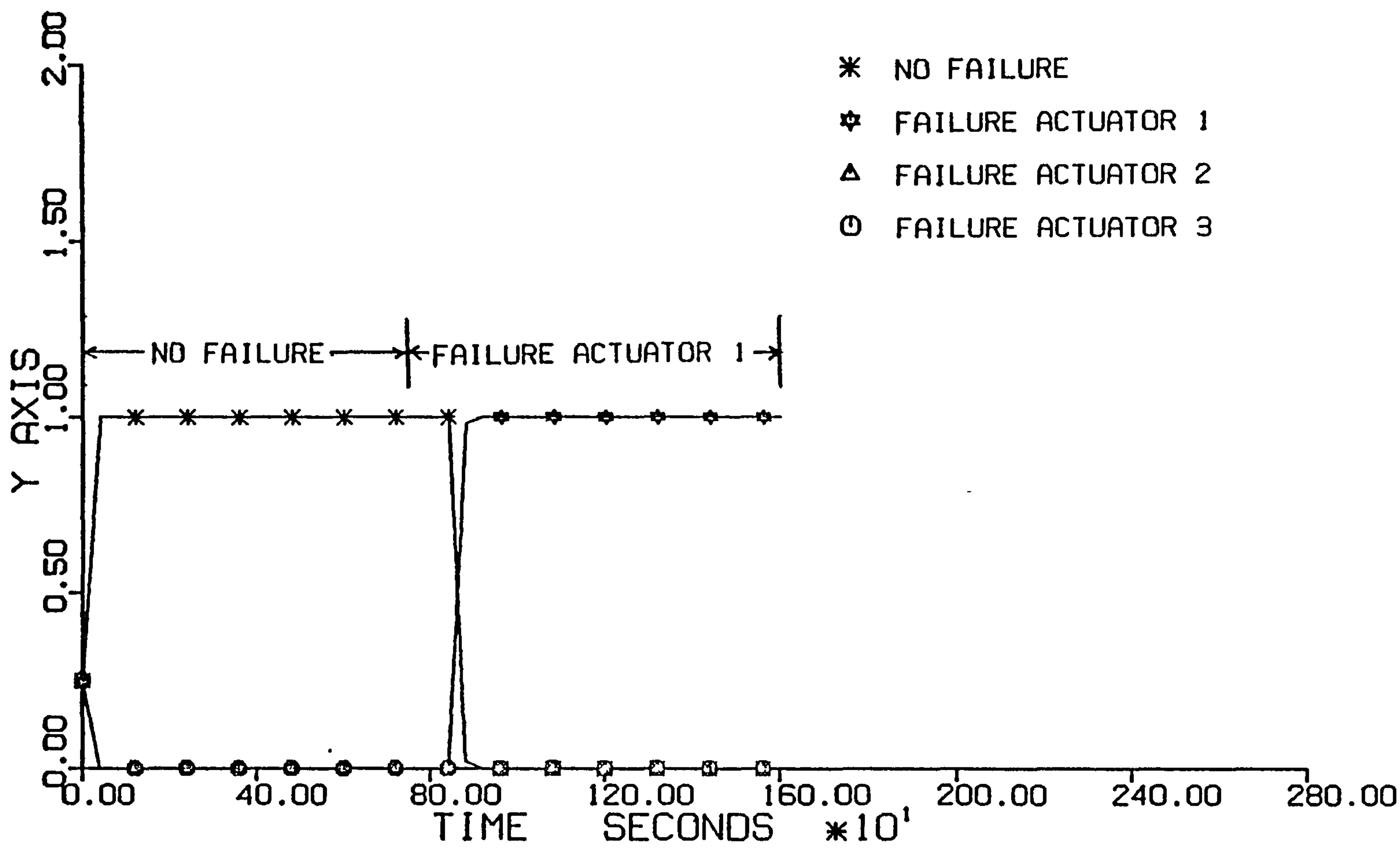

Figure 7. Actuator failure detection without noise by failure detection filter technique.

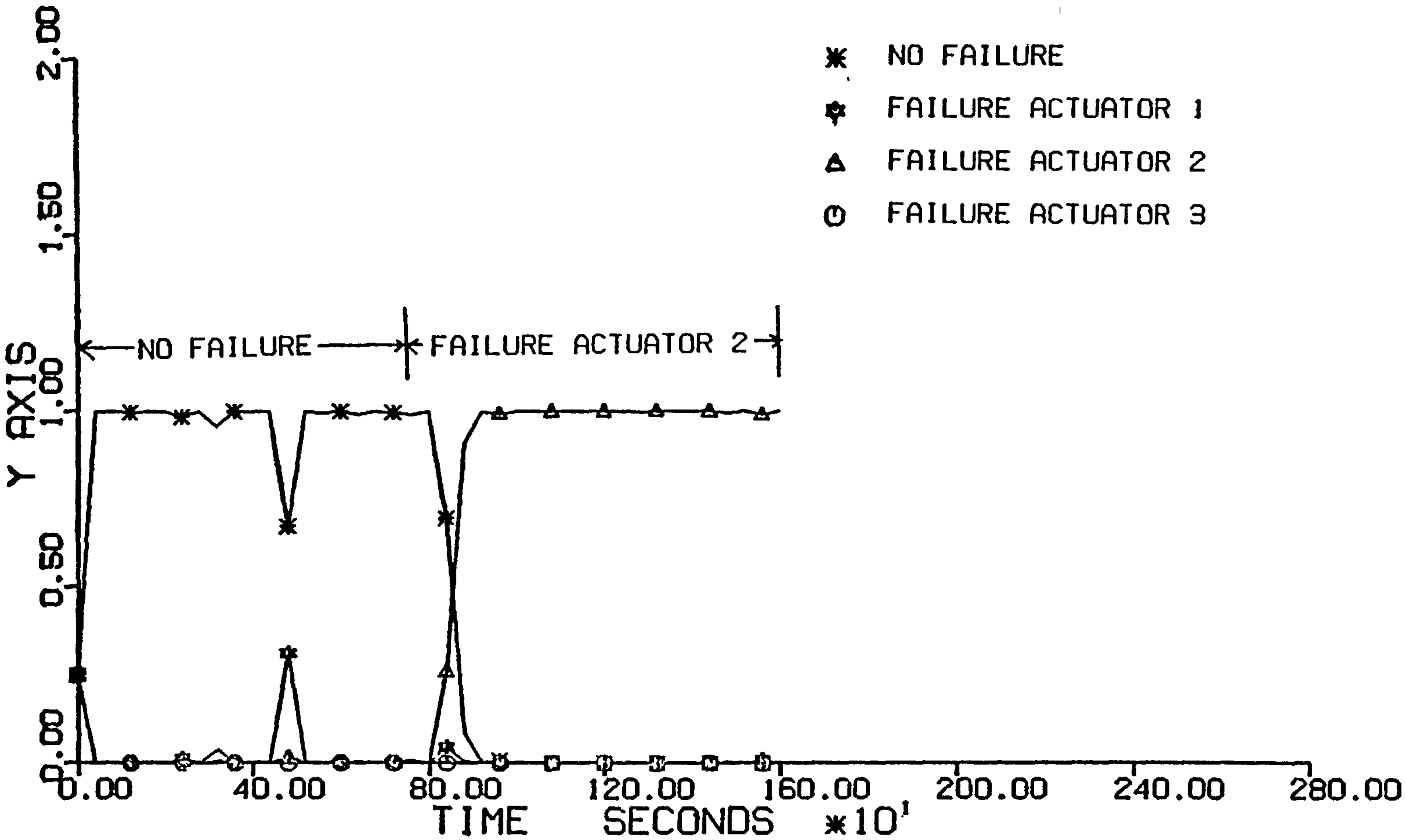

Figure 8. Actuator failure detection with sensor noise by failure detection filter technique.

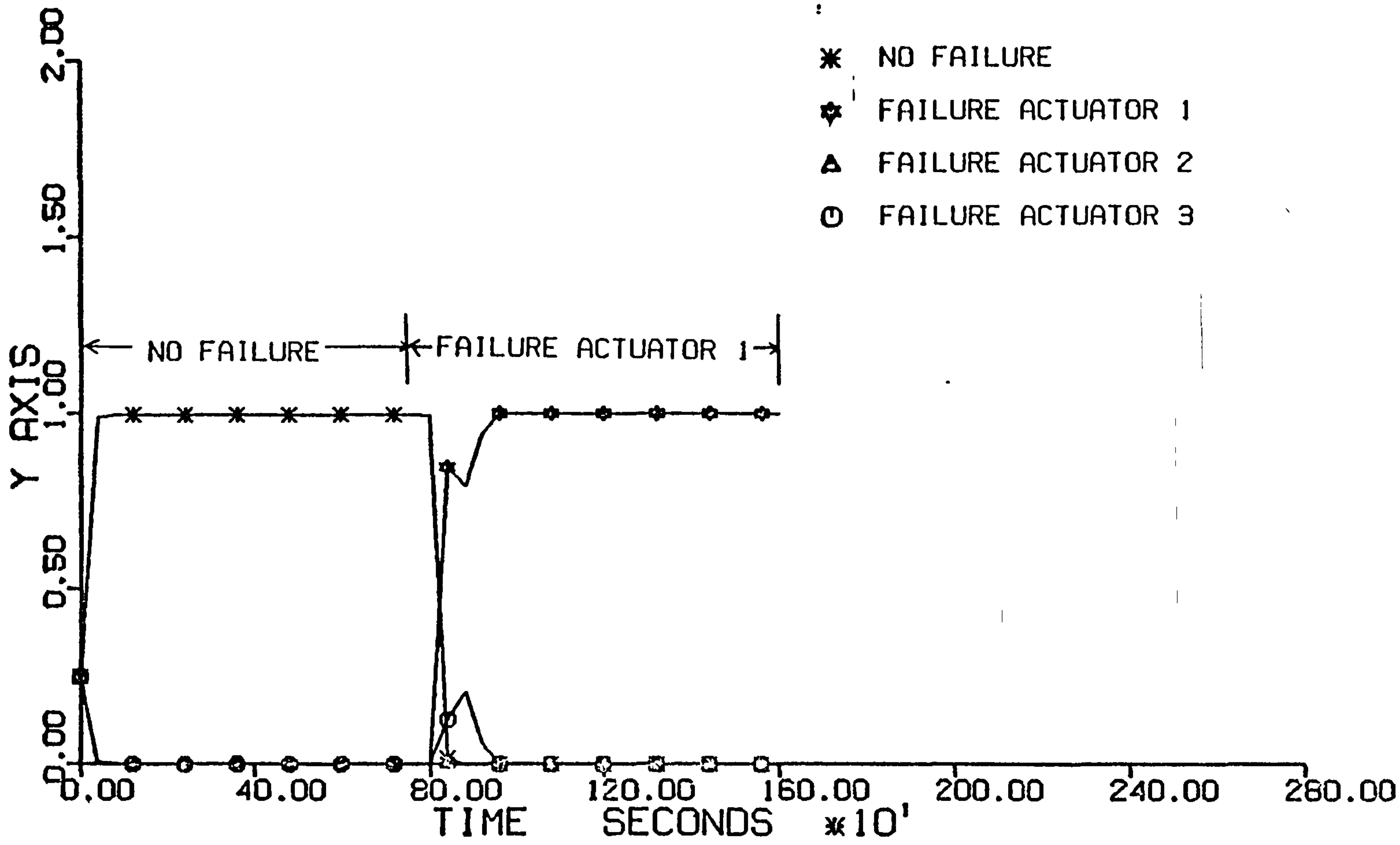

Figure 9. Actuator failure detection with 5% modeling error on one eigenvalue by failure detection filter technique.

The different types of sensor failures simulated were large and small biases of a sensor, a zero failure and increased noise on a sensor. Each of these failures were simulated with various amounts of sensor noise and both step and ramp inputs. The failure detection technique was not affected by the type of failure as long as it created a residue large enough to exceed the failure threshold. The failure detection technique was also impervious to changes in the input; however, low level sensor noise did cause the performance to degrade significantly. The no failure detection technique was very sensitive to low levels of noise which made it useless. The failure detection technique was also sensitive to sensor noise but to a much lesser degree. Figure 10 shows the detection of the no failure state and a failure of sensor 3 when there is no sensor noise. Figure 11 demonstrates that the no failure state is unreliable when sensor noise is present. The no failure configuration is identified as a failure in sensor 3. The failure configuration in Figure 11 is quickly and correctly identified. The FDF technique will detect a soft failure in a sensor as shown in Figure 12. Again note that the no failure configuration identification does not give any reliable information.

Sensor failures were also simulated in the presence of modeling error. The results were disappointing because the FDF technique shows no tolerance for any modeling error. Figure 13 shows a failure in sensor one at t=760 seconds which is incorrectly identified as a failure in sensor three when there is a 1% modeling error in one eigenvalue. This result should not be surprising because the FDF

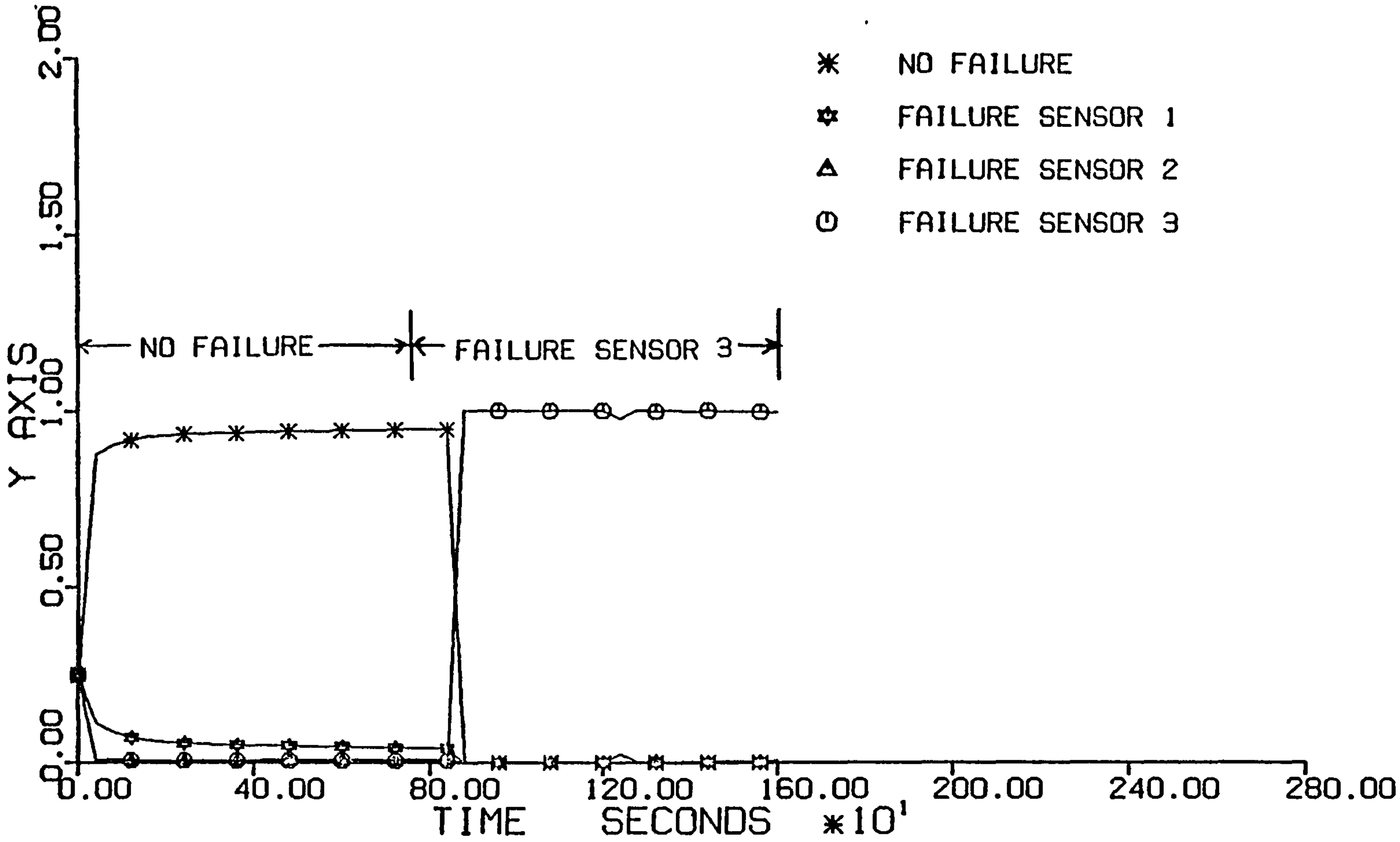

Figure 10. Sensor failure detection without noise by failure detection filter technique.

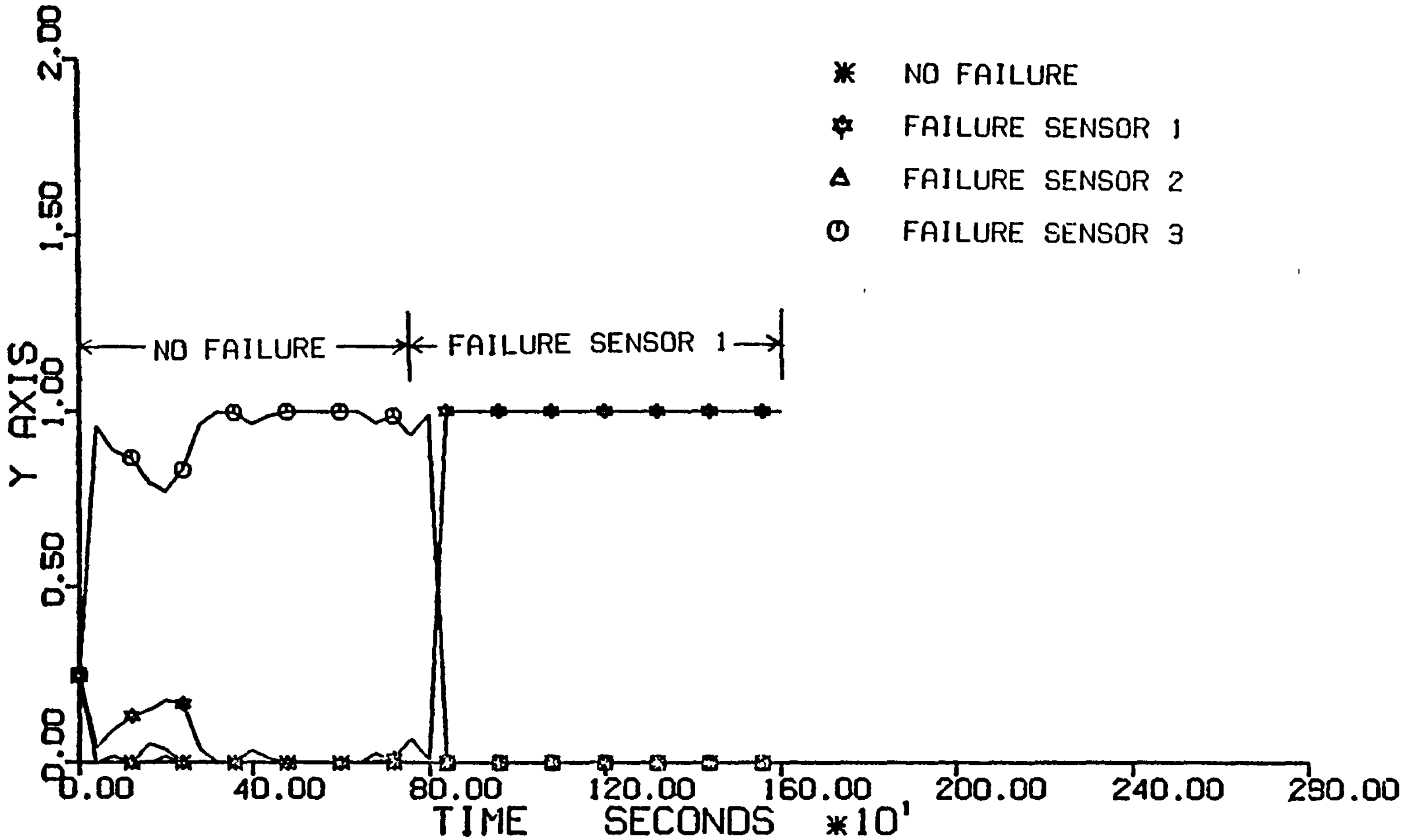

Figure 11. Sensor failure detection with sensor noise by failure detection filter technique.

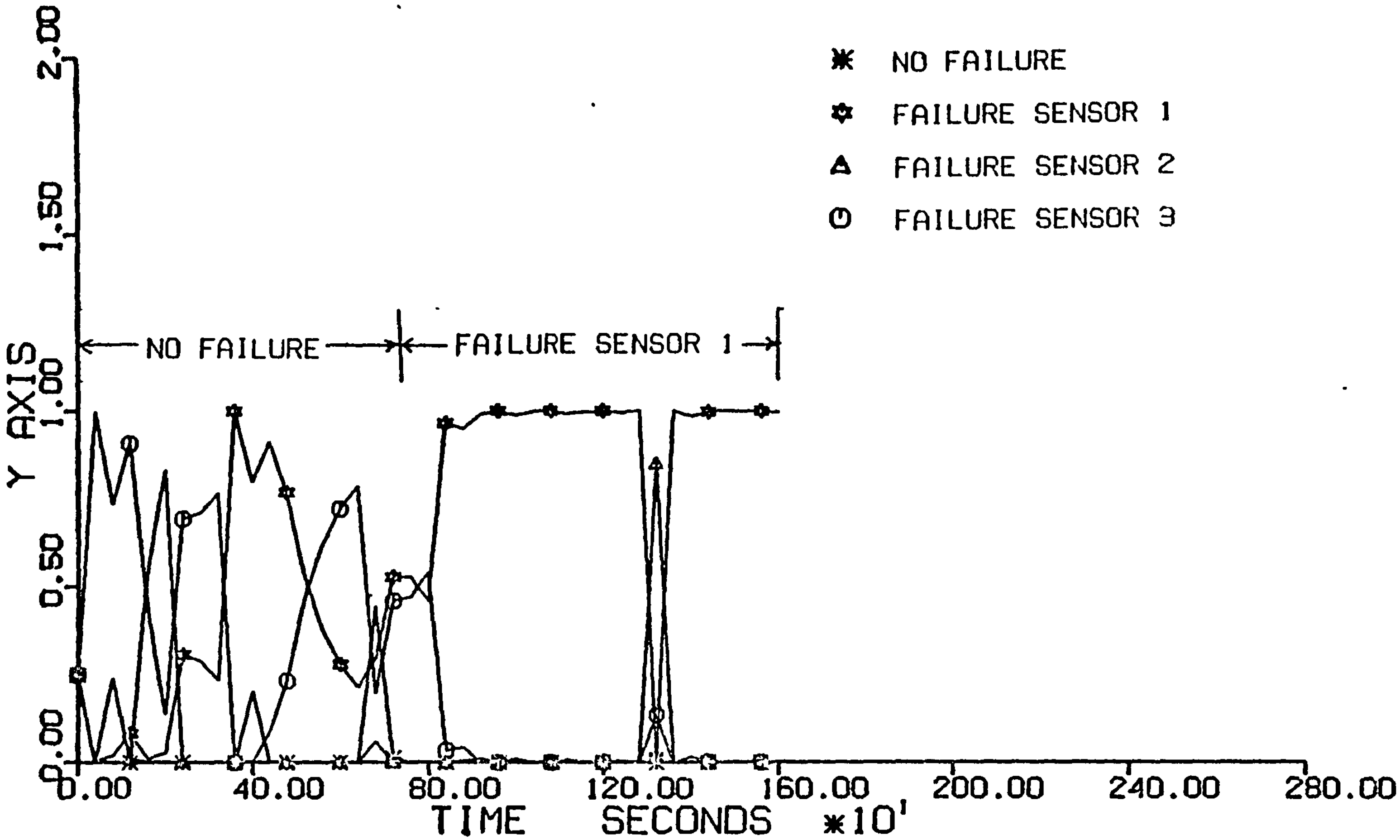

Figure 12. Soft sensor failure detection by failure detection filter technique.

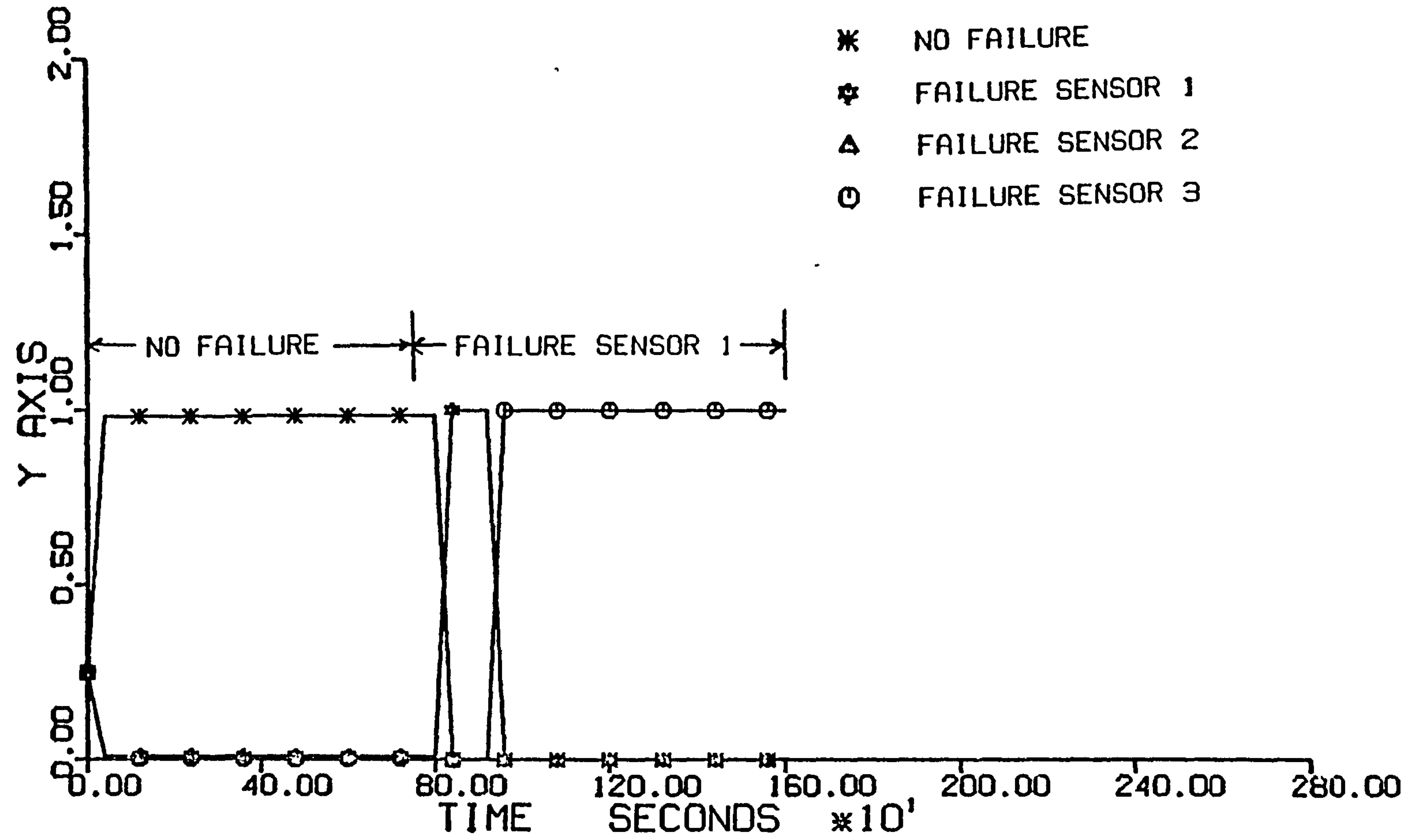

Figure 13. Sensor failure detection with 1% modeling error on one eigenvalue by failure detection filter technique.

developed by Beard can also be designed to detect changes in the A matrix. A change in the plant will cause a random error in all the filter residues in addition to any error associated with a failure and therefore may cause an incorrect identification.

In summary, the failure detection filter technique can detect a large number of failures with various inputs in the presence of noise. This technique can also detect soft failures in sensors. These benefits are overshadowed by the disadvantages. The no failure detection technique is very sensitive to sensor noise and small levels of sensor noise make it unreliable. Also, the FDF technique is also very sensitive to small changes in the eigenvalues.

5. CONCLUSIONS

This report is intended to show that the estimator part of the control algorithm developed by Zwicke, et al. for the B737 can adapt to sensor failures. In addition it is intended to explore potential usefulness of the Kalman filter identification technique adapted from the estimator of the B737 control algorithm and the FDF technique developed in this thesis for failure detection in the B737 project.

The B737 control algorithm is shown to work well during a sensor failure in the presence of sensor noise; however, the main disadvantage is that all failure modes must be modeled which can make this technique hard to implement.

The Kalman filter technique presented in Chapter 2 has some major advantages over the FDF technique present in Chapter 3. First the Kalman filter technique works extremely well in the presence of sensor noise. Also, small modeling errors do not affect the performance of the technique. This is a major concern for this particular application since the plant models are linearized and will seldom be a perfect match to the actual plant. This failure detection technique does require a model for each failure mode which can make detection unwieldy.

The FDF technique presented in Chapter 3 also has advantages but they are outweighted by the technique's limitations. The FDF technique can detect a wide range of failures including soft failures with a small number of filters. This technique can also distinguish between a zero input and the zero failure of an actuator. Failures are detected with reliability quicker than they are using the Kalman filter technique; however, the no failure configuration detection technique developed in Chapter 3 is not satisfactory because the failure threshold is difficult to set when sensor noise is present. The most serious problem with this failure detection technique is that the plant models must be too accurate to allow for the estimated parameters and the changes in the plant as the plane goes through its flight envelope.

In summary, the failure detection filter technique as it is presented in this thesis can not provide reliable failure detection for the B737 primarily because of its need for an extremely accurate plant model. The Kalman filter technique although cumbersome to implement can accurately detect actuator or sensor failures for this type of system.

6. RECOMMENDATIONS FOR FURTHER STUDY

It is hoped that this report will not be the end of research in this area, but will be a continuing part of the research effort in the flight control and failure detection areas. The following areas are recommended for further study.

First, the B737 model developed by Zwicke, et al. was not designed considering the possibility of actuator and sensor failures. Therefore, the model needs to be modified so that all the information necessary for control of the aircraft is still available after any sensor failure.

In addition to the work needed on the failure model, the Kalman filter detection technique used in the B737 project needs further study. Since the Kalman filter approach requires a failure model for each failure and there are almost an infinite number of ways actuators and sensors can fail (bias of 10%, 11%, etc.), there needs to be a determination of how accurate each failure model must be and how many similar failures can one model detect.

The failure detection filter technique developed in this thesis also needs further study. First and foremost, a better no failure configuration detector needs to be developed. A hybrid system using the Kalman filter technique for the no failure state detection and the FDF technique to detect failures appears to be a promising approach. In addition, in the FDF technique an assumption that equation (3.5.2) could be approximated by a Gaussian density function was made. This should be investigated further to see if the assumption is indeed valid and if a "better" approximation may be found.

Finally, this report has shown that the FDF technique works extremely well. However, the classes of problems for which this technique is applicable are severely restricted. Therefore, FDF theory needs to be extended so that it can be applied to nonlinear and time varying systems.

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APPENDIX A. FINDING THE MAXIMAL GENERATOR

The algorithm for finding the maximal generator is developed by Beard (1) and a brief synopsis is presented in this appendix. The algorithm depends on the orthognal reduction process and it is reviewed before the maximal generator algorithm is given.

The orthognal reduction of an n×n positive definite matrix Ω by the rows of an n×n matrix V, where

$$V = \begin{bmatrix} V_i^T \\ \vdots \\ V_n^T \end{bmatrix} \tag{A-1}$$

and V_i are n×1 arbitrary vectors, generates a matrix Ω_f whose range space coincides with the null space of V. The n×n matrix Ω_f is found by the following steps.

1) Any auxiliary vector ω_1 is defined by

$$\omega_1 = \Omega_1 V_1 \tag{A-2}$$

where Ω_1 is the positive definite matrix Ω.

2) Solve the following equation for Ω_2

$$\Omega_2 = \Omega_1 - \frac{\omega_1 \omega_1^T}{\omega_1^T V_1} \tag{A-3}$$

3) The following steps are iterative

i) With Ω_i from the previous iteration, form the auxilary vector.

ii) If $\omega_i \neq 0$, then solve the following equation for Ω_{i+1}

$$\Omega_{i+1} = \Omega_i - \frac{\omega_i \omega_i^T}{\omega_i^T V_i} \tag{A-4}$$

iii) If $\omega_i = 0$, then set

$$\Omega_{i+1} = \Omega_i \tag{A-5}$$

This orthognal reduction process is completed if all n rows of V have been processed.

The algorithm for finding the maximal generator is given by the following steps.

1) Find M', and M_{kT} where M' is given by equation (3.2.14) and where

$$M_{kT} = \begin{bmatrix} C \\ CK \\ \vdots \\ CK^{n-q'-1} \end{bmatrix} \tag{A-6}$$

where q' is defined by equation (3.2.16).

2) Form the starting matrix

$$\Omega = I_{n \times n} \tag{A-7}$$

3) Perform the orthognal reduction of Ω by the rows of M'.

4) When the reduction process is complete, Ω_f is the range scale of f.

The next four steps are for finding the maximal generator.

5) Reduce Ω_f by the rows of M_{kT}. All the auxiliary vectors of Ω_f will be zero except for one.

6) The maximal generator is formed by the last nonzero auxiliary vector before termination, given by the equation

$$\omega_i = \Omega_i (C_j K^{V-1})^T \qquad \text{(A-8)}$$

where v is the detection order of f and C_j is the j^{th} row of C.

7) The magnitude of ω_i must be adjusted to satisfy

$$CA^{v-1} g = CA^u f \qquad \text{(A-9)}$$

where u is defined by

$$CA^j f = 0 \qquad j = 0, \ldots, u-1 \qquad \text{(A-10)}$$

$$CA^u f \neq 0 \qquad \text{(A-11)}$$

8) The maximal generator g is

$$g = \left(\frac{C_j \; A^u f}{C_j K^{v-1} \omega_i} \right) \omega_i \qquad \text{(A-12)}$$

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16 Abstract

The purpose of this report is to show that the estimator part of the Modified Partitioned Adaptive Controller, (MPAC) developed for nonlinear aircraft dynamics of a small jet transport can adapt to sensor failures. In addition, an investigation is made into the potential usefulness of the configuration detection technique used in the MPAC and the failure detection filter is developed that determines how a noise plant output is associated with a line or plane characteristic of a failure.

It is shown by computer simulation that the estimator part and the configuration detection part of the MPAC can readily adapt to actuator and sensor failures and that the failure detection filter technique cannot detect actuator or sensor failures accurately for this type of system because of the plant modeling errors. In addition, it is shown that the decision technique, developed for the failure detection filter, can accurstely determine that the plant output is related to the characteristic line or plane in the presence of sensor noise.

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