PERFORMANCE OF A QUANTITATIVE STUDY OF INSTABILITY-RELATED DELAMINATION GROWTH

R.L. RAMKUMAR

NORTHROP CORPORATION
AIRCRAFT DIVISION
HAWTHORNE, CA 90250

CONTRACT NAS1-16727
MARCH 1983

NASA
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia 23665
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<td>3.10</td>
<td></td>
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<tr>
<td>3.11</td>
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NOMENCLATURE

a  delamination length in DCB specimens, or half-length of delamination in specimens with ITTW delamination
A  cross-sectional area of specimens
C  compliance
CLS  abbreviation for "cracked lap shear"
da/dN  delamination growth rate
dC/da  rate of change of compliance with delamination growth
DCB  abbreviation for "double cantilever beam"
E  Young's modulus in the loading direction
E_{11}  Young's modulus in the fiber direction
E_{22}  Young's modulus perpendicular to the fiber direction
G  total strain energy release rate = P^2(da/dN)/2w
G_I  mode I strain energy release rate (opening mode)
G_{II}  mode II strain energy release rate (shear mode)
G_{III}  mode III strain energy release rate (out-of-plane shear mode)
G_{12}  Shear modulus of an orthotropic lamina
G_c  critical value of the total strain energy release rate
G_{IC}, G_{IIC}, G_{IIIC}  critical strain energy release rates under modes I, II, and III, respectively
ITTW  abbreviation for "imbedded through the width"
L  half gage length of specimens with ITTW delaminations
L_1  length of thinner load-carrying portion of CLS specimens
L_2  length of thicker load-carrying portion of CLS specimens
N  number of cycles of constant amplitude fatigue loading at specified \( \omega, R, S \) values
P  load applied on DCB specimens
P_T  total tensile load applied on CLS specimens
P_{cr}  load at which delamination starts to propagate
R  algebraic minimum-to-maximum cyclic load ratio
S  ratio of the absolute maximum cyclic stress to the absolute static strength
$u_1$  axial elongation of the thinner load-carrying portion of CLS specimens
$u_2$  axial elongation of the thicker load-carrying portion of CLS specimens
w  specimen width
$X_{\text{max}}$  distance from the tab edge to the location of maximum out-of-plane displacement of delaminated plies in specimens with ITTW delaminations
$\delta$  total opening displacement in DCB specimens
$\delta_L$  out-of-plane displacement of delaminated plies at mid-length of specimens with ITTW delaminations
$\varepsilon_L$  axial strain in delaminated plies at mid-length of specimens with ITTW delaminations
$\nu_{12}$  Poisson's ratio for an orthoropic lamina
$\omega$  frequency of constant amplitude fatigue loading.
SECTION 1
INTRODUCTION AND SUMMARY

1.1 INTRODUCTION

Delaminations are precipitated in laminated composites under many process and service conditions. Their presence and growth have been shown to induce significant strength losses under static compressive loading, and have proven to be life-limiting under constant amplitude fatigue loading conditions (References 1, 2). The severity of the problem is a result of local instability induced in the delaminated region by a compressive load. The onset of instability is manifested by the local buckling of the delaminated region. Static compression and constant-amplitude compression fatigue tests on specimens containing realistic or idealized (imbedded) delaminations have identified delamination propagation as the primary failure mechanism, and have demonstrated the deleterious effect of these flaws on the strength and lifetime of the laminate (References 1, 2, 3).

In a realistic situation, delaminations of various planforms exist alongside many intraply cracks. Failure under compressive loading is induced by a simultaneous progression of these local defects. Analysis of this complex behavior requires an understanding of the interaction of various failure modes, and the availability of failure criteria that appropriately quantify this interaction. A quantitative assessment of the effect of delaminations on the strength and lifetime of a laminate is difficult. Consequently, analytical efforts to date have only attempted to quantify the effect of idealized (imbedded) delaminations (References 4-12). In every case, the delamination is also assumed to propagate along the same interfacial surface.

Delamination growth may be predicted based on point stresses, average stresses or strain energy release rates at the delamination boundary. In this report, predictions are based on strain energy release rates, $G_I$ and $G_{II}$.
corresponding to the opening or peeling mode (mode I) and the inplane shear mode (mode II), respectively. \( G_{III} \), the strain energy release rate corresponding to out-of-plane shear (mode III), is assumed to be negligible in the test specimens.

1.2 SUMMARY

The objectives of the reported program were: (a) to characterize a pure mode I delamination growth in T300/5208 graphite/epoxy laminates, under static and fatigue loading conditions; (b) to monitor delamination growth in T300/5208 laminates in a mixed mode (modes I and II) situation, under static and fatigue loading conditions; (c) to perform a finite element analysis of the mixed mode test specimen to compute the \( G_I \) and \( G_{II} \) values corresponding to various delamination sizes and load levels; (d) to incorporate the computed \( G_I \) and \( G_{II} \) values, mode I test results, and mixed mode test results into assumed static failure criteria and delamination growth rate equations, to separate the effects of \( G_I \) and \( G_{II} \) on delamination growth in T300/5208 laminates under static and fatigue loading conditions; and (e) to generate delamination growth data on T300/5208 specimens with imbedded through-the-width (ITTW) delaminations, under static compression and compression fatigue loading conditions.

Double cantilever beam (DCB) specimens were used for pure mode I tests. Cracked lap shear (CLS) specimens were used for mixed mode tests. DCB and CLS tests were conducted in a displacement-controlled mode to obtain stable delamination growth. Delaminations were introduced during fabrication by inserting folded Kapton films of specified sizes between selected plies. Delaminations were placed between two 0\(^{\circ}\) plies in all the specimens, to minimize \( G_{III} \) (out-of-plane shear) effects, and to obtain "clean" delamination growths by inhibiting failure propagation across plies into adjacent interfaces.

The critical mode I strain energy release rate, \( G_{IC} \), was obtained directly from static DCB tests, conducted in accordance with ASTM standards (Reference 13). \( G_{IC} \) for the T300/5208 material system was compared with results from similar tests on other materials (References 14-17). Static tests on the mixed mode CLS specimens measured the critical value of the total strain energy release rate (\( G_C \)) at which delamination
growth occurred. A geometrically nonlinear finite element analysis of the CLS specimen was carried out at NASA, Langley Research Center to determine the $G_I$ and $G_{II}$ components of the total strain energy release rate at various load levels for different delamination lengths. Incorporating the finite element results and DCB measurements into assumed failure criteria, the critical mode II strain energy release rate ($G_{IIC}$) was computed.

A similar procedure was used to characterize fatigue-induced delamination growth. Constant amplitude fatigue tests on DCB specimens were conducted to derive a power law relationship between delamination growth rate (da/dN) and $G_I$, for a pure mode I delamination growth. Similar tests on CLS specimens provided mixed mode delamination growth data. Because the fatigue tests were conducted in a displacement-controlled mode, the total strain energy release rate decreased with delamination growth. During the tests, the total strain energy release rate and the delamination size were monitored. The effects of $G_I$ and $G_{II}$ components on mixed mode delamination growth were assumed to be additive. Hence, the power law for a pure mode II delamination growth was derived from CLS test results by subtracting out the contribution due to $G_I$, as determined using the power law derived from the DCB tests.

Finally, specimens with imbedded through-the-width (ITTW) delaminations were subjected to static compression and constant amplitude compression fatigue loading conditions. Delaminations were imbedded between adjacent $0^\circ$ plies and different delamination locations in the thickness direction were considered. The out-of-plane deflection of the delaminated set of plies was monitored during static compression tests. The load corresponding to delamination growth to the tab region, and the load at total failure, were recorded. Delamination growth rates under constant amplitude compression fatigue loading conditions were also measured.

A description of the experimental procedure is presented in Section 2. Results are discussed in Section 3. Conclusions and recommendations are presented in Section 4.
SECTION 2

DETAILS OF THE EXPERIMENTAL PROGRAM

Details of the experimental program and a description of the various tests are presented below:

2.1 TEST MATERIAL

T300/5208 graphite/epoxy was selected to be the test material from which all the specimens were fabricated. The material was purchased in prepreg form to conform to Lockheed specification LAC-C-22-1379/114, with a nominal ply thickness of 0.132 mm (0.0052 in.) in the cured laminate.

Quality control (QC) tests were conducted on the procured material to qualify it for use in the program. Prepreg QC data, shown in Table 1, and \([0]_{16T}\) laminate QC data, shown in Table 2, indicate that the material met purchase specifications. Vendor-supplied data on T300/5208 graphite/epoxy are also presented in Table 3 for comparison.

2.2 FABRICATION OF TEST PANELS

Test panels were fabricated to yield the required number of test specimens for the program. Panels were laid up as specified in fabrication drawings. Imbedded folded Kapton films (0.0254 mm thick unfolded) introduced the desired initial delaminations in the test specimens. Thick laminates were debulked under vacuum in sets of 8 to 12 plies. No bleeder ply was used because of the low resin content (35% by weight uncured) in the T300/5208 prepreg. Test panels were cured in accordance with the following cure cycle:

1. Apply full vacuum
2. Heat to 408°C at 1-2°C per minute.
3. Dwell at 408°C for 45 minutes (starting at 403°C).
4. Apply 689.5 kPa, venting vacuum at 137.9 kPa.
5. Heat to 453°C at 1-2°C per minute.
TABLE 1. AVERAGE T300/5208 PREPREG QUALITY CONTROL (QC) DATA

<table>
<thead>
<tr>
<th>Property</th>
<th>Measurement*</th>
<th>Requirement**</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resin Content (% by weight)</td>
<td>34.7</td>
<td>(34±3)</td>
</tr>
<tr>
<td>% Volatiles</td>
<td>0.30</td>
<td>(3.0 maximum)</td>
</tr>
<tr>
<td>Areal fiber weight (gm/m²)</td>
<td>142.5⁺</td>
<td>(144±5)</td>
</tr>
<tr>
<td>Gel Time</td>
<td>17'30&quot;</td>
<td></td>
</tr>
<tr>
<td>% Flow</td>
<td>8.64</td>
<td>Acceptable</td>
</tr>
<tr>
<td>Tack</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Tests conducted per Northrop specification number NAI-1371
**Requirements per Lockheed specification number C-22-1379/114, to which the prepreg was purchased.
+Vendor (NARMCO) - supplied data.

TABLE 2. AVERAGE LAMINATE QC DATA ON T300/5208*

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Property</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transverse Tension Tests</td>
<td>Failure stress (MPa)</td>
<td>62.40 (44.8 minimum)**</td>
</tr>
<tr>
<td></td>
<td>Failure strain (μm/mm)</td>
<td>6680 (4000 minimum)</td>
</tr>
<tr>
<td></td>
<td>Modulus (GPa)</td>
<td>9.65 (9.65 minimum)</td>
</tr>
<tr>
<td>Longitudinal Flexure Tests</td>
<td>Failure Stress (MPa)</td>
<td>1630.1 (1448 minimum)</td>
</tr>
<tr>
<td></td>
<td>Modulus (GPa)</td>
<td>132.4 (124.1 minimum)</td>
</tr>
<tr>
<td>Short Beam Shear Tests</td>
<td>Shear Strength (MPa)</td>
<td>130.0 (89.6 minimum)</td>
</tr>
</tbody>
</table>

*Tests were conducted on [0]₁₆T laminates.
**Numbers within parentheses are requirements per Lockheed specification number C-22-1379/114 to which the prepreg was purchased.
TABLE 3. AVERAGE VENDOR (NARMCO) DATA ON T300/5208

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resin Content (by weight)</td>
<td>35% (uncured)</td>
</tr>
<tr>
<td>Areal Fiber Weight</td>
<td>144 gm/m²</td>
</tr>
<tr>
<td>Volatile Content</td>
<td>0.3%</td>
</tr>
<tr>
<td>Flow</td>
<td>14/12%</td>
</tr>
<tr>
<td>Gel Time</td>
<td>22'47&quot;</td>
</tr>
<tr>
<td>Tack</td>
<td>Acceptable</td>
</tr>
<tr>
<td>Specific Gravity</td>
<td>1.59</td>
</tr>
<tr>
<td>Fiber Volume</td>
<td>67%</td>
</tr>
<tr>
<td>Cured Ply Thickness</td>
<td></td>
</tr>
<tr>
<td>Longitudinal (0°) Flexural</td>
<td></td>
</tr>
<tr>
<td>Strength (RT)</td>
<td>2004 MPa (291 ksi)</td>
</tr>
<tr>
<td>Longitudinal (0°) Flexural</td>
<td></td>
</tr>
<tr>
<td>Modulus (RT)</td>
<td>136.7 GPa (19.83 Msi)</td>
</tr>
<tr>
<td>0° Tensile Strength (RT)</td>
<td>1458 MPa (211.5 ksi)</td>
</tr>
<tr>
<td>0° Tensile Modulus (RT)</td>
<td>146 GPa (21.2 Msi)</td>
</tr>
<tr>
<td>0° Flex. Strength (355°K)</td>
<td>1924 MPa (279 ksi)</td>
</tr>
<tr>
<td>0° Flex. Modulus (355°K)</td>
<td>131.1 GPa (19.02 Msi)</td>
</tr>
<tr>
<td>Short Beam Shear Strength (RT)</td>
<td>147.1 MPa (21.34 ksi)</td>
</tr>
<tr>
<td>Short Beam Shear Strength (355°K)</td>
<td>130.0 MPa (18.85 ksi)</td>
</tr>
</tbody>
</table>
(6) Cure at 453°K for 120 +10, -0 minutes.
(7) Cool to 350°K under pressure.

Fabricated panels were inspected visually and via ultrasonic through-transmission to ensure acceptable quality. Accepted panels were block-machined and pre-cured tabs were bonded to them secondarily, if required. The tabs contained ten 0° plies of 1581/3203 glass/epoxy, and were bonded to selected blocks using FM73 adhesive. The blocks were subsequently machined to extract the various test specimens.

Layup errors in the initial cracked lap shear test panel were not detected until most of the specimens from this panel were tested. Instead of the specified [(02/±45)s]s layup, which forms symmetric layups on either side of the imbedded delamination (at the midplane), a [02/±45s]s layup was fabricated. The [02/±45s]s laminate resulted in unsymmetric layups on either side of the imbedded delamination. Nevertheless, tests on the [02/±45s]s specimens were completed, and were then repeated on specimens from a [(02/±45)s]s panel that was subsequently fabricated.

2.3 DESCRIPTION OF TEST SPECIMENS

Three types of specimens were tested in the program—double cantilever beam (DCB) specimens, cracked lap shear (CLS) specimens, and specimens with imbedded through-the-width (ITTW) delaminations. Schematic drawings of the three specimens are presented in Figures 1 to 3.

Two 0.51 mm thick aluminum foils were bonded to the DCB specimens using a structural adhesive, along with two 1.27 mm thick aluminum pieces (see Figure 1). Loads were introduced through the flexible aluminum foils, ensuring a pure mode I delamination growth over a length of 76 to 102 mm.

0.51 mm thick stainless steel strips, with large axial stiffness and low bending rigidity, were pin-connected to secondarily bonded aluminum pieces on selected CLS specimens and specimens with ITTW delaminations (see Figures 2, 3). These strips, called flexures, provided constraints against specimen lateral deflection at the locations where they were present. In selected CLS specimens flexures were located in the vicinity of the
Figure 1. Double Cantilever Beam (DCB) Test Specimen.
Figures 1 and 2, and Table 1 are included on the next page.

Figure 1. Details of the Cracked Lap Shear Test Specimen.

Figure 2. Details of the Cracked Lap Shear Test Specimen.
These aluminum pieces are bonded to the specimen. 0.635 stainless steel

0.635

6.35

19.1

Pin Connection

All dimensions in mm

NOTE: Tabs contain ten 0° plies of 1581/3203 glass/epoxy material, and were bonded to the specimen using FM73 adhesive.

Figure 3. Geometry of Test Specimens with ITTW Delaminations.
delaminated step region to constrain possible large rotations. In specimens with ITTW delaminations, the flexures precluded gross (Euler) buckling of the 11.4 cm long test section under compression loading.

Two DCB laminate configurations were tested — $[0]_{24T}$ and $[(0/±45/3)/0]_S$. Both layups had a 2.54 cm (1.0 in.) long initial delamination at the mid-plane (see Figure 1).

Cracked lap shear (CLS) tests were conducted on $[0_2/±45_2/0_2]_S$ and $[(0,±45)]_T$ T300/5208 specimens with a 2.54 cm (1 in.) long initial delamination at the step location (see Figure 2). Half of the $[0_2/±45_2/0_2]_S$ CLS specimens were provided lateral constraints through flexures on the longer surface, near the step region (see Figure 2).

Three laminate configurations were chosen for tests on specimens with ITTW delaminations: (1) a $[0_4/(0/45/90/−45)_7]_S$ layup with a 1.91 cm (0.75 in.) long delamination between plies 3 and 4 (0/0 interface); (2) a $[0/45_2/0/(0/45/90/−45)_7]_S$ layup with a 2.54 cm (1 in.) long delamination between plies 4 and 5 (0/0 interface); and (3) a $[0/45/90_2/45/0_3/(0/45/90/−45)_6]_S$ layup with a 3.18 cm (1.25 in.) long delamination between plies 6 and 7 (0/0 interface).

### 2.4 TEST MATRICES

The various tests conducted on DCB specimens are listed in Table 4. Five static and ten constant amplitude fatigue tests were conducted on each laminate configuration. DCB tests were conducted in a displacement-controlled mode to obtain stable delamination growth under static loading. Fatigue tests were conducted at a frequency ($\omega$) of 10 Hertz, with a minimum to maximum cyclic displacement ratio ($R$) of 0.05.

CLS specimens were subjected to static tension and constant amplitude fatigue loading at $\omega = 10$ Hertz and $R = 0.05$ as shown in Table 5. CLS tests were also displacement-controlled to obtain stable delamination growth under static loading.
TABLE 4. TESTS ON DOUBLE CANTILEVER BEAM (DCB) SPECIMENS

<table>
<thead>
<tr>
<th>Test Case</th>
<th>Laminate</th>
<th>Delamination Between Plies</th>
<th>Loading</th>
<th>No. of Specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>D</td>
<td>12 and 13</td>
<td>Static **</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>D</td>
<td>12 and 13</td>
<td>Fatigue</td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>E</td>
<td>13 and 14</td>
<td>Static **</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>E</td>
<td>13 and 14</td>
<td>Fatigue</td>
<td>10</td>
</tr>
</tbody>
</table>

*Laminates D and E had $[0]_{24T}$ and $[(0_2/\pm 45)_3/0_s]$ layups, respectively.

**These displacement-controlled tests were conducted at $R = 0.05$ and $\omega = 10$ Hertz.

TABLE 5. TESTS ON CRACKED LAP SHEAR (CLS) SPECIMENS

<table>
<thead>
<tr>
<th>Test Case</th>
<th>Laminate</th>
<th>Lateral Supports</th>
<th>Loading</th>
<th>No. of Specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>F</td>
<td>None</td>
<td>Static Tension **</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>F</td>
<td>None</td>
<td>Tension Fatigue</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>F</td>
<td>Flexures</td>
<td>Static Tension **</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>F</td>
<td>Flexures</td>
<td>Tension Fatigue</td>
<td>11</td>
</tr>
<tr>
<td>9</td>
<td>H</td>
<td>None</td>
<td>Static Tension **</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>H</td>
<td>None</td>
<td>Tension Fatigue</td>
<td>6</td>
</tr>
</tbody>
</table>

*All the laminates had a delamination at the midplane (between plies 8 and 9). A thickness change from 16 plies to 8 plies occurred 2.54 cm (1 in.) from the delamination boundary.

Laminates F and H had $[0_2/\pm 45_2/0_s]$ and $[(0_2/\pm 45)_s]_s$ layups, respectively.

**These were conducted at $R = 0.05$ and $\omega = 10$ Hertz.
The various tests on specimens with ITTW delaminations are listed in Table 6. Static compression and constant amplitude compression fatigue tests were conducted in a load-controlled mode. Fatigue tests were conducted at $\omega = 10$ Hertz and $R = 10$. The ratio $(S)$ of the minimum cyclic stress to the static stress at which the ITTW delamination propagated to the tab region was varied to obtain different delamination growth rates.

2.5 DESCRIPTION OF DCB TESTS

DCB tests were conducted in a displacement-controlled mode in the setup shown in Figure 4. Stable delamination growth was achieved by controlling the tip displacement (see Section 3.1). If these tests had been conducted in a load-controlled mode, delamination growth would have been unstable and limited data would have been gathered from each test (see Section 3.1). Prior to mounting the specimen as shown in Figure 4, its free edges were coated with a typewriter correction fluid. After the fluid dried out, fine visible marks were made on these edges, at 2.54 mm (0.10 in.) intervals on either side, to aid in the measurement of the extent of delamination. Prior to recording test data, the DCB specimen was loaded to cause the folded Kapton imbeddment to debond and introduce a sharply defined, visible delamination. The initial delamination length was recorded with the help of a microscope mounted adjacent to the specimen (see Figure 4).

Static tests were initiated at slow crosshead speeds (approximately 0.51 mm/minute) to induce slow delamination growth. The crosshead speed was increased to approximately 5.1 mm/minute when the delamination extended beyond 76.2 mm. The load ($P$) corresponding to the applied displacement ($\delta$) was also monitored. $P$ increased linearly with $\delta$ when the delamination length ($a$) remained constant. This continued until a critical value ($P_{cr}$, $\delta_{cr}$) was reached. When the tip displacement exceeded $\delta_{cr}$, a delamination growth ($\Delta a$) was observed, accompanied by a reduction in the load from the $P_{cr}$ value (Section 3.1). The applied displacement was increased until the delamination growth was observed to be 1.27 cm (0.5 in). The applied displacement was then decreased until a zero load reading was observed. It should be noted that a zero load reading did not correspond
### Table 6. Tests on Coupons with Imbedded Through-the-Width (ITTW) Delaminations

<table>
<thead>
<tr>
<th>Test Series</th>
<th>Laminate</th>
<th>$N_D$</th>
<th>$2a^*$ mm (in.)</th>
<th>Load Type $++$</th>
<th>$S^*$</th>
<th>No. of Specimens</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>A</td>
<td>3</td>
<td>19.1 (0.75)</td>
<td>SC</td>
<td>---</td>
<td>4</td>
</tr>
<tr>
<td>12</td>
<td>A</td>
<td>3</td>
<td>19.1 (0.75)</td>
<td>CF</td>
<td>0.41, 0.46</td>
<td>10</td>
</tr>
<tr>
<td>13</td>
<td>B</td>
<td>4</td>
<td>25.4 (1.00)</td>
<td>SC</td>
<td>---</td>
<td>4</td>
</tr>
<tr>
<td>14</td>
<td>B</td>
<td>4</td>
<td>25.4 (1.00)</td>
<td>CF</td>
<td>0.67</td>
<td>6</td>
</tr>
<tr>
<td>15</td>
<td>C</td>
<td>6</td>
<td>31.8 (1.25)</td>
<td>SC</td>
<td>---</td>
<td>4</td>
</tr>
<tr>
<td>16</td>
<td>C</td>
<td>6</td>
<td>31.8 (1.25)</td>
<td>CF</td>
<td>0.75</td>
<td>6</td>
</tr>
</tbody>
</table>

$^+$ A, B and C are 64-ply, T300/5208 laminates with $[0_4/(0/45/90/-45)_7]_S$, $[0/45_{2}/0/(0/45/90/-45)_7]_S$, and $[0/45/90_{2}/45/0_3/(0/45/90/-45)_6]_S$ layups, respectively.

$^+SC$ denotes static compression

$CF$ denotes constant amplitude, compression fatigue loading at $\omega = 10$ Hertz and $R = 10$.

$^*N_D$ is the number of plies that are delaminated (smaller number).

$2a$ is the initial length of the through-the-width delamination.

$S$ is the ratio of the minimum cyclic stress to the static strength.
Figure 4. Double Cantilever Beam Test Setup.
to a zero displacement reading (Section 3.1). This has been observed by other investigators (Reference 15). The loading/unloading procedure was repeated for every 1.27 cm (0.5 in.) growth of delamination. The slopes of the load-deflection plots indicated an increase in compliance with an increase in delamination size. The critical load and deflection values, and the compliance measurements corresponding to delamination sizes that were 1.27 cm (0.5 in.) apart, were recorded during each static test. Plots of critical loads and compliances as a function of the delamination size were obtained, and the slopes of the curves were used to compute $G_{IC}$, the mode I critical strain energy release rate. The computational procedure is explained in Section 3.

Constant amplitude fatigue tests on DCB specimens were conducted at $\omega = 10$ Hertz and $R = 0.05$. Based on the static test data, the maximum cyclic displacement value was selected to cause a desired initial rate of delamination growth. Fatigue test specimens had their free edges marked at 2.54 mm (0.1 in.) intervals. A microscope was used to monitor delamination growth (see Figure 4). Since the tests were displacement-controlled, delamination growth rates reduced significantly as the delamination propagated in a stable manner. The slowest rate monitored corresponded to 250,000 cycles for a delamination growth of 1.27 cm (0.5 in.). Consequently, the maximum cyclic displacement was increased after each 12.7 mm delamination growth was measured. In a limited number of specimens, the initial displacement amplitude was unaltered and delamination growth was monitored for approximately 1.5 million cycles. Denoting delamination growth rate by $da/dN$, its variation with delamination size ($a$) was obtained for each applied maximum displacement level. Using available static test data, these were converted to $da/dN$ versus $G_I$ plots that are conventional delamination growth rate records.

2.6 DESCRIPTION OF CLS TESTS

Static tension and constant amplitude tension fatigue tests were conducted on $[0_2/\pm 45_2/0_2]$ and $[(0_2/\pm 45)_s]$ CLS specimens. Half of the $[0_2/\pm 45_2/0_2]$ specimens were constrained laterally by flexures at two locations (see Figure 5). The remaining specimens were laterally
Figure 5. Cracked Lap Shear Test Setup Showing a Laterally Constrained $[0_2/\pm45_2/0_2]_s$ Specimen.
Figure 5. Cracked Lap Shear Test Setup Showing a Laterally Constrained $[0_2/\pm45_2/0_2]_8$ Specimen (Concluded).
unconstrained during testing. Photographs of an unconstrained [(0₂/±45)ₜ]ₜ
CLS specimen in the test fixture are presented in Figure 6.

The applied displacements on [(0₂/+45₂/₀₂)ₜ]ₜ specimens corresponded to
the relative displacement of one grip fixture (attached to the loading
cylinder) with respect to the other. As explained later (Section 3.3),
the recorded stroke (displacement) was effectively between the centers of
the holes at either end of the specimen (See Figure 2). Therefore, the
effective gage length for the [(0₂/+45₂/₀₂)ₜ]ₜ CLS specimen was 21.6 cm
(8.5 in.). The [(₀₂/+45)ₜ]ₜ specimens, on the other hand, had extensometers
mounted on them (see Figure 6). The extensometer gage length was chosen
to be large enough (14 cm or 5.5 in.) to record useful compliance measure­
ments until the imbedded delamination propagated over approximately 10 cm
(see Figure 7).

Static tension tests on CLS specimens were conducted in a displacement­
controlled mode to ensure stable delamination growth (see Section 3.3).
Prior to testing, specimen edges were marked to monitor delamination
growth (see Section 2.5). The applied displacement was increased slowly,
and the corresponding load varied linearly with displacement until the
delamination started to propagate. Delamination growth occurred only along
the imbeddment 0/0 interface (see Figure 8), and was accompanied by an
unloading (see Section 3.3). The applied displacement was slowly
increased during this propagation phase until a 1.27 cm (0.5 in.)
delamination growth was recorded. At this point, the applied displacement
was reduced until a zero load reading was obtained. This procedure was
repeated for every 1.27 cm (0.5 in.) growth of the delamination, and
the corresponding critical load, critical deflection and compliance measure­
ments were recorded. These data were used, as explained in Section 3, to
compute the critical total strain energy release rate (Gₗ) for the CLS
specimen.

Constant amplitude tension fatigue tests on CLS specimens were con­
ducted in a displacement-controlled mode at ω = 10 Hertz and R = 0.05
(see Table 5). The maximum cyclic displacement was selected, based on
static test data, to induce delamination growth rates that were
large (10 to 60 mm growth in 1000 cycles) at the beginning. As the delami­
Figure 6. Cracked Lap Shear Test Setup, Showing a Laterally Unconstrained $[(0_2/\pm 45)_s]$ Specimen.
Figure 7. Extensometer Locations on the \([(0_{2}/+45)_{s}s]\) CLS Test Specimens.
Figure 8. Photographs Showing Delamination Growth Along the Imbeddment 0/0 Interface in $[0_2/\pm 45_2/0_2]_9$ CLS Specimens.
nation propagated in a stable manner, its growth rate reduced sharply. In most of the tests, the maximum cyclic displacement was increased after every 12.7 mm delamination growth to reset the growth rate to be large. A few specimens were tested for approximately 1.5 million cycles without altering the maximum cyclic displacement to obtain slow growth rate data.

2.7 DESCRIPTION OF TESTS ON SPECIMENS WITH ITTW DELAMINATIONS

Table 6 lists the various tests--static compression and constant amplitude compression fatigue tests—that were conducted on specimens with ITTW delaminations (see Figure 3). Figure 9 presents the static compression test setup for these specimens, showing how the flexures provide antibuckling constraints. The compression fatigue test setup is shown in Figure 10. During fatigue testing, a microscope was mounted adjacent to the test specimen to monitor delamination growth.

Four static compression tests were conducted on each laminate (Table 6). Three strain gages were bonded to selected specimens (one out of four) to monitor the strain level at which local instability occurred. These gages included back-to-back axial gages located 2.54 cm (1 in.) from the tab edge, and an axial gage centrally located over the delaminated set of plies (Figure 3). During the static tests, the central lateral deflection on the specimen surface closest to the delaminated interface was monitored using a dial indicator. The applied load levels corresponding to delamination failure and total failure were recorded. Delamination failure was defined as the propagation of the imbedded delamination to the tab edge (boundary of the bevelled region). Total failure occurred when other delaminations followed the imbedded delamination, in quick succession, from the outer surface toward the midplane, drastically reducing the load-carrying capacity of the specimen. Static compression loads were introduced slowly and were periodically held constant while the corresponding central lateral deflection and delamination growth, if any, were measured.

Constant amplitude compression fatigue tests on specimens with ITTW delaminations were conducted at \( \omega = 10 \) Hertz and \( R = 10 \). The two free edges of the specimens were coated with a typewriter correction fluid.
Figure 9. Static Compression Test Setup for Specimens With ITTW Delaminations.
Figure 9. Static Compression Test Setup for Specimens with ITTW Delaminations. (Concluded)
Figure 10. Compression Fatigue Test Setup For Specimens With ITTW Delaminations.
and marked at 6.35 mm (0.25 in.) intervals. A microscope was mounted adjacent to the specimen to monitor upward and downward delamination growth, as seen from either free edge. That is, the movement of two points on each edge was monitored. Cyclic loading was interrupted periodically to measure delamination growth.

Prior to recording fatigue test data, the specimens were cycled at a load amplitude below the static delamination failure load, to "release" the Kapton inclusion and create a well-defined delaminated region. In laminates B and C, with delamination imbeddments below 4 and 6 plies, respectively, this posed a problem. The ITTW delamination propagated to the tab region when it was "released", resulting in a delamination failure. To preclude this, the remaining specimens were clamped tightly over a 31.8 mm (1.25 in.) length from the tab boundary, on either side, during the "releasing" phase of the test (see Figure 11). The clamps were removed at zero load after the "release" sound was heard. This procedure was only partially successful in preventing delamination failure prior to initiating the fatigue tests on laminates B and C.

The minimum cyclic load was selected to be a fraction (S) of the static strength corresponding to delamination failure (not total failure). Cycling was interrupted periodically to monitor delamination growth. Fatigue failure was assumed to occur when the ITTW delamination propagated to the tab region (delamination failure).
All dimensions in mm.

Laminates B and C were clamped over the region marked "C" in the figure prior to initiating fatigue tests. In this state, a low frequency high amplitude load was introduced to "free" the Kapton inclusion within the 51 mm length.

Figure 11. Measurement Locations on Specimens With ITTW Delaminations.
SECTION 3

DISCUSSION OF RESULTS

3.1 STATIC DCB TEST RESULTS

Static double cantilever beam (DCB) tests were conducted to compute the critical strain energy release rate \( G_{IC} \) for a pure mode I delamination growth in T300/5208 laminates. These displacement-controlled tests produced load-displacement \((P \text{ versus } \delta)\) curves similar to those in Figures 12 and 13. At the onset of delamination growth, the load and the total opening displacement are referred to as \( P_{cr} \) and \( \delta_{cr} \). The total compliance of the DCB specimen \((C = \delta/P)\), \( P_{cr} \) and \( \delta_{cr} \) were obtained from figures similar to those in Figures 12 and 13, and recorded as a function of the delamination length \( (a) \). Using the measured \( C \) and \( P_{cr} \) variations with \( a \), \( G_{IC} \) for T300/5208 laminates can be computed using:

\[
G_{IC} = \frac{P_{cr}^2 (dC/da)/(2w)}
\]

where \( w \) is the width of the DCB specimen (25.4 mm). If \( P_{cr} \) is replaced by \( P \), \( G_{IC} \) becomes \( G_I \), the mode I strain energy release rate corresponding to the load \( P \).

The advantage gained by conducting the tests in a displacement-controlled mode may be understood by invoking a strength of materials expression for the total compliance \((C)\) of the DCB specimen:

\[
C = \frac{\delta}{P} = \frac{2a^3}{(3D_x)}
\]

where \( D_x \) is the longitudinal flexural stiffness of the delaminated set of plies, assuming bending about its transverse centroidal axis. The expression in equation (2) assumes a clamped condition at the delamination boundary. The validity of this assumption is subsequently discussed. Taking the derivative of \( C \) with respect to the delamination length \((a)\),

\[
dC/da = 2a^2/D_x
\]
Figure 12. A Typical Load-Deflection Plot From a Static DCB Test on a $[0]_{24T}$ Specimen.
Figure 13. A Typical Load-Deflection Plot Obtained From a Static DCB Test on a \([(0_2/\pm 45)_3/0]\) Specimen.
It is seen that \( \frac{dC}{da} \) increases with \( a \). Since \( G_{IC} \) is a material constant (by hypothesis), it is inferred from equation (1) that \( P_{cr} \) will decrease when \( a \) increases (see Figures 12 and 13). Therefore, if the tests were load-controlled, an unstable growth of the delamination will result at the \( P_{cr} \) value corresponding to the initial delamination length. If the tests were displacement-controlled, substituting \( C = \frac{\delta_{cr}}{P_{cr}} \) and equations (2) and (3) into equation (1), it is seen that

\[
G_{IC} = \frac{\delta_{cr}^2}{2wC^2} \frac{dC}{da}
\]

or

\[
G_{IC} \propto \frac{\delta_{cr}^2}{a^4}
\]

Since \( G_{IC} \) is a material constant, \( \delta_{cr} \) will increase with delamination growth (\( a \)). Therefore, a stable delamination growth can be achieved by conducting the tests in a displacement-controlled mode.

Static DCB test results from figures similar to those in Figures 12 and 13 yielded critical loads and compliances that varied with delamination size as shown in Figures 14 and 15. The presented plots are on a logarithmic scale, and linear approximations to the plotted data were obtained. The data in Figures 14 and 15 are averages of all the respective static test data. The measured compliances were subsequently compared with strength of materials predictions (equation 2) to assess the validity of the assumed clamped constraint conditions at the delamination boundary in equation (2). Assuming \( a \) to be expressed in mm, and \( E_{11} = 137.9 \) GPa (20 Msi), \( E_{22} = 14.5 \) GPa (2.1 Msi), \( \nu_{12} = 0.21 \), \( G_{12} = 5.9 \) GPa (0.85 Msi), and the nominal cured ply thickness = 0.014 cm (0.0055 in) for T300/5208 graphite/epoxy, equation (2) yielded:

\[
C = 0.4850 \times 10^{-6} a^3 \text{ mm/N for the } [9]_{24T} \text{ specimen, and}
\]

\[
C = 0.6133 \times 10^{-6} a^3 \text{ mm/N for the } [(0_2/\pm 45)_3/0]_s \text{ specimen}
\]

Referring to Figures 14 and 15, the measured compliances are seen to be:

\[
C = 0.6705 \times 10^{-6} a^3 \text{ mm/N for the } [0]_{24T} \text{ specimen, and}
\]

\[
C = 0.8494 \times 10^{-6} a^3 \text{ mm/N for the } [(0_2/\pm 45)_3/0]_s \text{ specimens}
\]
Static DCB tests on $[0]_{24T}$ T300/5208 specimens with a delamination at the midplane. If $a$ is expressed in mm, $C$ in mm/N and $P_{cr}$ in N, $C=0.6705 \times 10^{-6} a^3$.

\[
P_{cr} = 1610.0 a^{-1} \frac{P^2}{(dC/da)} = \frac{G_{IC}}{2w}
\]

$G_{IC} = 102.6 \text{ J/m}^2$

Figure 14. Static DCB Test Data From $[0]_{24T}$ T300/5208 Specimens.
Static DCB tests on $[(0_2/\pm 45)_3/0]_s$ T300/5208 specimens with a delamination at the midplane. If $a$ is expressed in mm, $C$ in mm/N, and $P_{cr}$ in N,

$$C = 0.8494 \times 10^{-6} \ a^3 \text{ mm/N}$$

$$P_{cr} = 1412 \ a^{-1} \text{ N}$$

$$G_{IC} = \frac{P_{cr}^2 (dC/da)}{2w}$$

$$G_{IC} = 100.0 \text{ J/m}^2$$

Figure 15. Static DCB Test Data From $[(0_2/\pm 45)_3/0]_s$ T300/5208 Specimens.
Compliance predictions based on equation (2) are 28% lower than the measured values. The difference is due to the actual constraint condition at the delamination boundary being less stringent than the clamped condition assumed in equation (2).

It is noted that the log $C$ versus log $a$ results in Figures 14 and 15 were deliberately approximated by straight lines with a slope of three, to conform to the strength of materials expression in equation (2). Likewise, test data corresponding to the log $P_{cr}$ versus log $a$ plots in Figures 14 and 15 were approximated by straight lines with a slope of negative unity:

$$P_{cr} = 1610 \ a^{-1} \ N \text{ for the } [0]_{24T} \text{ specimen, and}$$

$$P_{cr} = 1412 \ a^{-1} \ N \text{ for the } [(0_2/\pm45)_3/0]_{s} \text{ specimen}$$

where $a$ is expressed in mm. This is a result of the premise that $G_{IC}$ is a material constant that is independent of $a$, the compliance expressions in equation (6), and the definition of $G_{IC}$ in equation (1). Figures 14 and 15 indicate that the experimental data are adequately represented by the approximations in equations (6) and (7).

Substituting the expressions in equations (6) and (7) into equation (1), the following $G_{IC}$ values were computed:

$$G_{IC} = 102.6 \ J/m^2 \text{ using results from tests on } [0]_{24T} \text{ specimens, and}$$

$$G_{IC} = 100.0 \ J/m^2 \text{ using results from tests on } [(0_2/\pm45)_3/0]_{s} \text{ specimens}$$

It is noted that the lack of midplane symmetry in the delaminated portion of the $[(0_2/\pm45)_3/0]_{s}$ DCB specimens caused them to twist slightly when the load was applied. Nevertheless, a pure mode I propagation of the delamination resulted. The twist probably caused the interlaminar normal stress distribution in the widthwise direction at the delamination boundary, to be non-uniform in these specimens. Referring to equation (8), it is seen that $G_{IC}$ was not adversely affected by this behavior.

$G_{IC}$ based on results from $[0]_{24T}$ specimens ($102.6 \ J/m^2$) will henceforth be referred to as the $G_{IC}$ for the T300/5208 material system. In
Reference 14, a $G_{IC}$ value of 87.6 J/m$^2$ (15% lower than 102.6 J/m$^2$) was obtained for the T300/5208 material using a similar test procedure.

3.2 DELAMINATION GROWTH RATES IN DCB SPECIMENS

Constant amplitude fatigue tests on $[0]_{24T}$ and $[(0_2/\pm 45)_3/0]_S$ DCB specimens were conducted at $R = 0.05$ and $\omega = 10$ Hertz. Delamination growth was monitored as explained in Section 2.5. The imposed maximum cyclic displacements ($\delta_{max}$) and $dC/da$ expressions from static test results were used to compute the maximum cyclic strain energy release rates ($G_{I_{max}}$) associated with in situ delamination sizes ($a$) as follows:

$$G_{I_{max}} = \frac{P_{max}^2 (dC/da)/(2w)}{\delta_{max}}$$  \hspace{1cm} (9)

where $P_{max} = \delta_{max}/C$  \hspace{1cm} (10)

Therefore, $G_{I_{max}} = \delta_{max}^2 (dC/da)/(2wC^2)$  \hspace{1cm} (11)

Recalling that $\delta_{max}$ was held constant during fatigue, substitution of equation (6) into (11) implies that $G_{I_{max}}$ was inversely proportional to $a^4$:

$$G_{I_{max}} \propto a^{-4}$$  \hspace{1cm} (12)

Also, the constant amplitude fatigue tests were run at an $R$ ratio ($\delta_{min}/\delta_{max}$) of 0.05. Therefore, referring to equation (11), the minimum cyclic strain energy release rate ($G_{I_{min}}$) was always:

$$G_{I_{min}} = (0.05)^2 G_{I_{max}} = 0.0025 G_{I_{max}}$$  \hspace{1cm} (13)

During fatigue, the incremental number of cycles ($\Delta N$) corresponding to a 2.54 mm change in the delamination size ($\Delta a$) was recorded. The rate of delamination growth ($da/dN$) was thus monitored as a function of in situ delamination size ($a$). Substituting the $a$ values and the expressions in (6) into equation (11), relationships between $da/dN$ and $G_{I_{max}}$ were obtained for the two test laminates.
Plots of delamination growth rate (da/dN) as a function of $G_{I_{\text{max}}}$ for the two DCB laminates are presented in Figures 16 and 17. Power law fits to the data on these logarithmic plots were obtained using a least squares analysis, and yielded the following relationships:

For $[0]_{24T}$ DCB specimens,

$$\frac{da}{dN} = 0.0283 \left(\frac{G_{I_{\text{max}}}}{G_{IC}}\right)^{8.02} \text{ mm/cycle}$$  \hspace{1cm} (14)

For $[(0_2/\pm 45)_3/0]_s$ DCB specimens,

$$\frac{da}{dN} = 0.0791 \left(\frac{G_{I_{\text{max}}}}{G_{IC}}\right)^{10.08} \text{ mm/cycle}$$  \hspace{1cm} (15)

Considerable scatter is noticed in the $da/dN$ versus $G_{I_{\text{max}}}$ data presented in Figures 16 and 17. Consequently, the exponents in equations (14) and (15) bear a poor correlation to the value of 26 obtained in Reference 14.

Equations (14) and (15) quantify the effect of $G_{I_{\text{max}}}/G_{IC}$ on delamination growth rate, for an R ratio of 0.05 and for $\omega = 10$ Hertz. The differences in the growth rate equations for the two specimens are due to the reasons explained in Section 3.1. The delaminated set of plies in the $[(0_2/\pm 45)_3/0]_s$ specimens lacked midplane symmetry, and twisted slightly when a tip load was applied. Consequently, a non-uniform distribution of the interlaminar normal stress, in the widthwise direction, existed at the delamination boundary. Hence the difference between equations (14) and (15).

Henceforth, the following approximation of equations (14) will be assumed to quantify the effect of $G_{I_{\text{max}}}/G_{IC}$ on mode I delamination growth rate in T300/5208 laminates, at $R = 0.05$ and $\omega = 10$ Hertz:

$$\frac{da}{dN} = 0.0283 \left(\frac{G_{I_{\text{max}}}}{G_{IC}}\right)^{8} \text{ mm/cycle}$$  \hspace{1cm} (16)

Substituting equation (12) into the above equation, it is seen that:
Figure 16. Variation of Delamination growth Rate with Strain Energy Release Rate in [0]_{24T} DCB Specimens.
\[ \frac{d a}{d N} = 0.0791 \left( \frac{G_{I \text{max}}}{G_{IC}} \right)^{10.08} \]

Figure 17. Variation of Delamination Growth Rate with Strain Energy Release Rate in \([0_2/\pm 45)_3/0]_s\) DCB Specimens.
when \( \Delta_{\text{max}} \) is held constant during fatigue. Pure mode I delamination growth rate, therefore, decreases drastically as the delamination size increases, in a displacement-controlled fatigue test.

3.3 STATIC CLS TEST RESULTS

Static tensile tests on \([0/2/\pm 45]_s\) and \([0/2/\pm 45_2/0]_s\) cracked lap shear (CLS) specimens were conducted in a displacement-controlled mode to obtain compliance measurements at various crack (delamination) lengths. The geometry of the CLS specimens is defined in Figure 2. Extensometers were used on \([0/2/\pm 45]_s\) specimens, and the displacement between test grips was monitored on \([0/2/\pm 45_2/0]_s\) specimens (see Figures 2 and 7). Half the \([0/2/\pm 45_2/0]_s\) specimens were laterally constrained by flexures, placed 50.8 mm apart on the tool surface, as shown in Figures 2 and 5. The flexures were placed directly below the step location (see Figures 2 and 5), and were originally assumed to be adequate to prevent out-of-plane displacement of the tool surface of these CLS specimens. But, a nonlinear finite element analysis of the CLS specimen, accounting for these lateral constraint (flexure) locations indicated that this assumption was incorrect. The lateral constraints were not sufficient to prevent all lateral displacements. Hence the constrained CLS specimen was not representative of the behavior of one half of a symmetrical double cracked lap shear specimen.

Typical load-displacement records from static tests on \([0/2/\pm 45]_s\) and \([0_2/\pm 45_2/0]_s\) CLS specimens are presented in Figures 18 and 19. In many specimens—especially the \([0_2/\pm 45_2/0]_s\) specimens with flexure constraints—delamination growth at \(P_{\text{cr}}\) could not be controlled to within the 12.7 mm increment shown in Figure 19. A growth (\(\Delta a\)) of 25 mm at \(P_{\text{cr}}\) was not uncommon during these displacement-controlled tests. The critical loads \(P_{\text{cr}}\) and displacements \(u_{\text{cr}}\), and the compliances \(C\) corresponding to various delamination sizes \(a\), were recorded during each static test. \(dC/da\) was computed based on a least squares linear fit through the compliance data (see Figures 20 to 22). Table 7 presents the measurements.
Static Tension Loading – $a$ is the delamination length (see Figure 2)

Extensometer gage length = 140 mm. (see Figure 7)

Figure 18. Typical Load-Deflection Curves for a $[(0_2/±45)_s]_s$ CLS Specimen Subjected to Static Tension.
Figure 19. Typical Load-Deflection Curves for a $[0_2/\pm 45_2/0_2]$ s CLS Specimen Subjected to Static Tension.
Figure 20. Compliance Variation with Delamination Size in [(0_2/45)_s]_s CLS Specimens.
Figure 21. Compliance Variation with Delamination Size in $[0_2/\pm 45_2/0_2]$ CLS Specimens With Flexure Constraints.
Figure 22. Compliance Variation with Delamination Size in Laterally Unconstrained $[0_2/\pm45_2/0_2]_s$ CLS Specimens.
TABLE 7. STATIC TENSION TEST DATA FROM $[(0_2/\pm45)_s]_s$ CLS SPECIMENS

<table>
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<th>Delamination Length, $a$ (mm)</th>
<th>Critical Load, $P_{cr}$ (N)</th>
<th>Critical Displacement $u_{cr}$ (mm)</th>
<th>Compliance $C^*$ ($\mu$ mm/N)</th>
<th>Critical Total Strain Energy Release Rate, $G_c$ ** ($J/m^2$)</th>
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<td>89.2</td>
<td>7980</td>
<td>0.4877</td>
<td>61.27</td>
<td>424.1</td>
</tr>
<tr>
<td>104.1</td>
<td>8042</td>
<td>0.5334</td>
<td>66.35</td>
<td>430.8</td>
</tr>
<tr>
<td>111.8</td>
<td>8060</td>
<td>0.5715</td>
<td>70.75</td>
<td>432.7</td>
</tr>
<tr>
<td>127.0</td>
<td>8309</td>
<td>0.6172</td>
<td>74.40</td>
<td>459.8</td>
</tr>
</tbody>
</table>

* Based on an extensometer gage length of 140 mm (see Figure 7)

** Refer to Figure 20 for $dC/da$ values used in the computation of $G_c$. 

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on \([\{(0/2/-45)\}_s]_s\) specimens. Tables 8 and 9 present corresponding data on 
[\(0/2/-45/0/2\)]_s specimens, with and without flexure constraints, respectively.

The critical total strain energy release rates \((G_c)\) in the mixed-mode 
CLS specimens were obtained using:

\[
G_c = \frac{P^2}{c_{cr}} \left( \frac{dC}{da} \right) / (2w)
\]  
(18)

where \(w = 2.54\) cm (1 in.) for the specimens. Computed \(G_c\) values are also 
listed in Tables 7 to 9.

3.4 STRENGTH OF MATERIALS ANALYSIS OF CLS SPECIMENS

The following strength of materials analysis was carried out to approxi-
mately compute \(C\), \(dC/da\) and \(G\) for the CLS test specimens (see Refer-
ence 18). The CLS specimen was modeled as shown in Figure 23, and bending 
effects due to load eccentricity in the 16-ply specimens were ignored.

Let segments 1 and 2 have \(L_1\) and \(L_2\) as their lengths, \(A_1\) and \(A_2\) as their 
cross-sectional areas, and \(E_1\) and \(E_2\) as their longitudinal moduli. Both 
segments are subjected to the applied tensile load \(P_T\). If \(u_1\) and \(u_2\) are 
the axial elongations of sections 1 and 2, the overall compliance \((C)\) 
of the CLS specimen is given by:

\[
C = \frac{(u_1 + u_2)}{P_T} = \frac{L_1}{A_1 E_1} + \frac{L_2}{A_2 E_2}
\]  
(19)

When the delamination propagates over a differential distance of "da", 
\(dL_1 = da\) and \(dL_2 = -da\). Using this in equation (19), the following 
relationship may be obtained:

\[
dC/da = \frac{(A_2 E_2 - A_1 E_1)}{A_1 A_2 E_1 E_2}
\]  
(20)

For the CLS specimens tested in this program \(A_2 = 2A_1\) and \(E_1 = E_2\). Substituting 
these into equations (19) and (20), and noting that \(L = L_1 + L_2\) (Figure 23), 
\[
C = \frac{(2L_1 + L_2)}{(2A_1 E_1)} = \frac{(L + L_1)}{A_2 E_2}
\]  
(21)

\[
dC/da = \left(\frac{2A_1 E_1}{A_2 E_2}\right)^{-1} = (A_2 E_2)^{-1}
\]  
(22)

Substituting the \(dC/da\) expression in equation (22) into equation (18), 
the critical total strain energy release rate \((G_c)\) is expressed as:

\[
G_c = \frac{P^2}{c_{cr}} / (4WA_1 E_1) = \frac{P^2}{c_{cr}} / (2WA_2 E_2)
\]  
(23)
TABLE 8. STATIC TENSION TEST DATA ON \([0_2/\pm45_2/0_2]_s\) CLS SPECIMENS WITH LATERAL (FLEXURE) CONSTRAINTS

<table>
<thead>
<tr>
<th>Specimen</th>
<th>a (mm)</th>
<th>P&lt;sub&gt;cr&lt;/sub&gt; (N)</th>
<th>u&lt;sub&gt;cr&lt;/sub&gt; (mm)</th>
<th>C (μm/N)</th>
<th>G&lt;sub&gt;c&lt;/sub&gt;&lt;sup&gt;*&lt;/sup&gt; (J/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-17</td>
<td>50.8</td>
<td>8096</td>
<td>0.6604</td>
<td>81.60</td>
<td>433.4</td>
</tr>
<tr>
<td></td>
<td>82.6</td>
<td>7651</td>
<td>0.6858</td>
<td>89.65</td>
<td>387.2</td>
</tr>
<tr>
<td></td>
<td>95.3</td>
<td>7437</td>
<td>0.7061</td>
<td>94.96</td>
<td>365.8</td>
</tr>
<tr>
<td></td>
<td>101.6</td>
<td>7473</td>
<td>0.7341</td>
<td>98.21</td>
<td>369.3</td>
</tr>
<tr>
<td></td>
<td>114.3</td>
<td>7651</td>
<td>0.7874</td>
<td>102.90</td>
<td>387.2</td>
</tr>
<tr>
<td>F-8</td>
<td>50.8</td>
<td>8896</td>
<td>0.7239</td>
<td>81.37</td>
<td>546.6</td>
</tr>
<tr>
<td></td>
<td>101.6</td>
<td>7775</td>
<td>0.7569</td>
<td>97.36</td>
<td>417.5</td>
</tr>
<tr>
<td></td>
<td>125.7</td>
<td>7775</td>
<td>0.8407</td>
<td>108.15</td>
<td>417.5</td>
</tr>
<tr>
<td>F-11</td>
<td>99.1</td>
<td>7259</td>
<td>0.7442</td>
<td>102.50</td>
<td>453.6</td>
</tr>
<tr>
<td></td>
<td>114.3</td>
<td>7669</td>
<td>0.8458</td>
<td>110.32</td>
<td>506.1</td>
</tr>
<tr>
<td></td>
<td>133.4</td>
<td>8274</td>
<td>0.9728</td>
<td>117.57</td>
<td>589.1</td>
</tr>
<tr>
<td>F-26</td>
<td>114.3</td>
<td>7962</td>
<td>0.8255</td>
<td>103.70</td>
<td>521.9</td>
</tr>
<tr>
<td></td>
<td>127.0</td>
<td>8274</td>
<td>0.9017</td>
<td>109.01</td>
<td>563.4</td>
</tr>
<tr>
<td></td>
<td>139.7</td>
<td>9074</td>
<td>1.0465</td>
<td>114.32</td>
<td>677.7</td>
</tr>
</tbody>
</table>

* Refer to Figure 21 for dC/da values used in the computation of G<sub>c</sub>.
TABLE 9. STATIC TENSION TEST DATA FROM UNCONSTRAINED $\left[0_2/\pm45_2/0_2\right]_8$ CLS SPECIMENS

<table>
<thead>
<tr>
<th>$a$ (mm)</th>
<th>$P_{cr}$ (N)</th>
<th>$u_{cr}$ (mm)</th>
<th>$C$ ($\mu$ mm/N)</th>
<th>$G_c^*$ ($J/m^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specimen F-10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40.6</td>
<td>7929</td>
<td>0.5588</td>
<td>71.38</td>
<td>400.5</td>
</tr>
<tr>
<td>53.3</td>
<td>7740</td>
<td>0.5791</td>
<td>74.63</td>
<td>391.6</td>
</tr>
<tr>
<td>66.0</td>
<td>7651</td>
<td>0.6071</td>
<td>79.26</td>
<td>382.7</td>
</tr>
<tr>
<td>76.2</td>
<td>7740</td>
<td>0.6350</td>
<td>82.05</td>
<td>391.6</td>
</tr>
<tr>
<td>95.3</td>
<td>7740</td>
<td>0.6858</td>
<td>88.62</td>
<td>391.6</td>
</tr>
<tr>
<td>104.1</td>
<td>7953</td>
<td>0.7366</td>
<td>92.62</td>
<td>413.5</td>
</tr>
<tr>
<td>114.3</td>
<td>8007</td>
<td>0.7620</td>
<td>95.19</td>
<td>419.1</td>
</tr>
<tr>
<td>Specimen F-6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38.1</td>
<td>8274</td>
<td>0.6655</td>
<td>80.46</td>
<td>391.8</td>
</tr>
<tr>
<td>50.8</td>
<td>8274</td>
<td>0.6858</td>
<td>82.91</td>
<td>391.8</td>
</tr>
<tr>
<td>66.0</td>
<td>7989</td>
<td>0.6985</td>
<td>87.42</td>
<td>365.3</td>
</tr>
<tr>
<td>76.2</td>
<td>8007</td>
<td>0.7239</td>
<td>90.39</td>
<td>366.9</td>
</tr>
<tr>
<td>89.2</td>
<td>7918</td>
<td>0.7366</td>
<td>93.02</td>
<td>358.8</td>
</tr>
<tr>
<td>101.6</td>
<td>7829</td>
<td>0.7874</td>
<td>100.56</td>
<td>350.8</td>
</tr>
<tr>
<td>119.4</td>
<td>7793</td>
<td>0.8001</td>
<td>102.67</td>
<td>347.6</td>
</tr>
<tr>
<td>Specimen F-16</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38.1</td>
<td>8398</td>
<td>0.6045</td>
<td>72.00</td>
<td>436.6</td>
</tr>
<tr>
<td>55.9</td>
<td>8398</td>
<td>0.6299</td>
<td>75.03</td>
<td>436.6</td>
</tr>
<tr>
<td>63.5</td>
<td>8274</td>
<td>0.6401</td>
<td>77.37</td>
<td>423.8</td>
</tr>
<tr>
<td>76.2</td>
<td>8345</td>
<td>0.6934</td>
<td>83.08</td>
<td>431.2</td>
</tr>
<tr>
<td>91.4</td>
<td>8185</td>
<td>0.7137</td>
<td>87.19</td>
<td>414.7</td>
</tr>
<tr>
<td>104.1</td>
<td>8007</td>
<td>0.7366</td>
<td>91.99</td>
<td>396.8</td>
</tr>
<tr>
<td>Specimen F-1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38.1</td>
<td>9074</td>
<td>0.6985</td>
<td>76.97</td>
<td>407.7</td>
</tr>
<tr>
<td>55.9</td>
<td>9039</td>
<td>0.7239</td>
<td>80.11</td>
<td>404.5</td>
</tr>
<tr>
<td>68.6</td>
<td>8985</td>
<td>0.7620</td>
<td>84.80</td>
<td>399.8</td>
</tr>
</tbody>
</table>

* Refer to Figure 22 for $dC/da$ values used in the computation of $G_c^*$. 
Figure 23. A Strength of Materials Model of the CLS Specimen

TABLE 10. AVERAGE STATIC TENSION TEST DATA ON 
\[ ((0_2/\pm 45)_s)_s \] CLS SPECIMENS

<table>
<thead>
<tr>
<th>Delamination Length, a (mm)</th>
<th>Average P_{cr} (N)</th>
<th>Average u_{cr} (mm)</th>
<th>Average C (\text{mm}/N)</th>
<th>Average G_{c} (*) (J/m^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.8</td>
<td>7962</td>
<td>0.4115</td>
<td>51.56</td>
<td>457.3</td>
</tr>
<tr>
<td>63.5</td>
<td>7918</td>
<td>0.4369</td>
<td>55.05</td>
<td>452.2</td>
</tr>
<tr>
<td>76.2</td>
<td>8007</td>
<td>0.4826</td>
<td>60.07</td>
<td>462.2</td>
</tr>
<tr>
<td>88.9</td>
<td>8047</td>
<td>0.5156</td>
<td>64.07</td>
<td>467.1</td>
</tr>
<tr>
<td>101.6</td>
<td>8025</td>
<td>0.5486</td>
<td>68.35</td>
<td>464.5</td>
</tr>
<tr>
<td>114.3</td>
<td>7949</td>
<td>0.5867</td>
<td>73.83</td>
<td>455.8</td>
</tr>
<tr>
<td>127.0</td>
<td>7989</td>
<td>0.6350</td>
<td>79.71</td>
<td>460.4</td>
</tr>
</tbody>
</table>

*This is computed using \( \frac{dC}{da} = 0.3664\mu N^{-1} \), based on a least squares linear fit through the average compliance data listed above.
The average $A_2$ value for the fabricated CLS specimens was measured to be 57.03 mm$^2$, yielding a ply thickness of 0.14 mm (0.0055 in) in the cured laminate. Back-to-back axial strain gages bonded to a $[(0_2/\pm45)_s]_s$ test specimen (H-3) measured a Young's modulus of $E_1=E_2 = 74.05$ GPa. Likewise, back-to-back axial gages on a $[0_2/\pm45_2/0_2]_s$ specimen (F-28) measured an $E$ value of 70.05 GPa.

The average static test data on $[(0_2/\pm45)_s]_s$ specimens are presented in Table 10. These were obtained from the data listed in Table 7. For the $[(0_2/\pm45)_s]_s$ specimens, the variation in $P_{cr}$ (and hence in $G_c$) with delamination size ($a$) is negligible—within 1% from the average value (see Table 10). Therefore $P_{cr}$ and $G_c$ may be assumed to be independent of $a$.

Substituting the measured areas, Young's modulus and $P_{cr}$ into equations (21), (22) and (23), a strength of materials estimate for $C$, $dC/da$ and $G_c$ may be obtained. A comparison between experimental measurements and strength of materials estimates is presented in Table 11. It is seen that the measured compliances are 10 to 20% larger than the strength of materials prediction. $dC/da$ and $G_c$ values based on experimental data are approximately 35% higher than the strength of materials predictions.

Tests on $[0_2/\pm45_2/0_2]_s$ CLS specimens were not conducted with extensometers. The gage length for these specimens corresponded to the distance between the centers of the two holes (see Figure 2). This was established by comparing the midplane strain based on this gage length with the direct strain gage measurement on specimen F-28. Table 12 presents a comparison between measurements and strength of materials estimates for laterally unconstrained $[0_2/\pm45/0_2]_s$ specimens. The average measurements in Table 12 were obtained using the data presented in Table 9. Though $P_{cr}$ (and $G_c$) differed by approximately 10% from the average value corresponding to the various delamination sizes in Table 9, they were again assumed to be independent of $a$. Measured average compliances, $dC/da$ based on average $C$ values, and $G_c$ are approximately 6%, 20% and 17% larger than predictions, respectively.
<table>
<thead>
<tr>
<th>a (mm)</th>
<th>L₁ (mm)</th>
<th>C (μm/N)</th>
<th>dC/da (μN⁻¹)</th>
<th>Gc (J/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50.8</td>
<td>53.34</td>
<td>45.73</td>
<td>51.56</td>
<td>297.2</td>
</tr>
<tr>
<td>63.5</td>
<td>66.04</td>
<td>48.71</td>
<td>55.05</td>
<td>297.2</td>
</tr>
<tr>
<td>76.2</td>
<td>78.74</td>
<td>51.73</td>
<td>60.07</td>
<td>297.2</td>
</tr>
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<td>88.9</td>
<td>91.44</td>
<td>54.70</td>
<td>64.07</td>
<td>297.2</td>
</tr>
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<td>101.6</td>
<td>104.14</td>
<td>57.73</td>
<td>68.35</td>
<td>297.2</td>
</tr>
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<td>114.3</td>
<td>116.84</td>
<td>60.76</td>
<td>73.83</td>
<td>297.2</td>
</tr>
<tr>
<td>127.0</td>
<td>129.54</td>
<td>63.73</td>
<td>79.71</td>
<td>297.2</td>
</tr>
</tbody>
</table>

+ L is the extensometer gage length
L = 139.7 mm (see Figure 7)
A₂ = 57.03 mm²
E₂ = E₁ = 74.05 GPa
L₁ = (a + 2.54) mm (See Figures 7 and 23)
* S.O.M. = strength of materials (equations 21, 22 and 23)
** Average Values from Table 10.
Average Pcr = 7985 N and average Gc = 460.0 J/m² from Table 10
TABLE 12. COMPARISON BETWEEN STRENGTH OF MATERIALS ESTIMATES AND ACTUAL MEASUREMENTS ON LATERALLY UNCONSTRAINED \([\theta_2/\pm 45_2/\theta_2]_s\) CLS SPECIMENS

<table>
<thead>
<tr>
<th>a (mm)</th>
<th>(L_1) (mm)</th>
<th>(C) ((\mu\text{mm/N}))</th>
<th>(dC/da) ((\mu\text{N}^{-1}))</th>
<th>(G_c) (J/m(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>S.O.M.*</td>
<td>Measurement**</td>
<td>S.O.M. Measurement</td>
</tr>
<tr>
<td>38.1</td>
<td>76.2</td>
<td>73.09</td>
<td>75.03</td>
<td>0.2495</td>
</tr>
<tr>
<td>50.8</td>
<td>88.9</td>
<td>76.29</td>
<td>77.54</td>
<td>0.2495</td>
</tr>
<tr>
<td>63.5</td>
<td>101.6</td>
<td>79.49</td>
<td>81.21</td>
<td>0.2495</td>
</tr>
<tr>
<td>76.2</td>
<td>114.3</td>
<td>82.63</td>
<td>85.20</td>
<td>0.2495</td>
</tr>
<tr>
<td>88.9</td>
<td>127.0</td>
<td>85.82</td>
<td>88.68</td>
<td>0.2495</td>
</tr>
<tr>
<td>101.6</td>
<td>139.7</td>
<td>89.02</td>
<td>94.33</td>
<td>0.2495</td>
</tr>
<tr>
<td>114.3</td>
<td>152.4</td>
<td>92.16</td>
<td>98.61</td>
<td>0.2495</td>
</tr>
</tbody>
</table>

+ \(L\) is the distance between the centers of the holes in the CLS specimens, and is assumed to be the distance over which the recorded displacement occurred.

\[ L = 215.9 \text{ mm (see Figure 2)} \]

\[ A_2 = 57.03 \text{ mm}^2 \]

\[ E_2 = E_1 = 70.05 \text{ GPa} \]

\[ L_1 = (38.1 + a) \text{ mm (see Figures 2 and 23)} \]

* S.O.M. = strength of materials (equations 21, 22 and 23)

** Average Values Obtained from Data in Table 9.

Average \(P_{cr} = 8149\) N and average \(G_c = 396.3\) J/m\(^2\) from Table 9.
3.5 **NONLINEAR FINITE ELEMENT ANALYSIS (NFEA) OF CLS SPECIMENS**

**G_1**, **G_II**, and **G_c** **COMPUTATIONS**

Delamination growth in mixed mode CLS specimens is in general induced by an interaction among modes I, II and III. For the CLS specimens tested in this program, mode III effects were assumed to be negligible -- an assumption that has to be verified through a three-dimensional numerical analysis that was beyond the scope of the reported program. Therefore, the measured **G_c** values were assumed to have only **G_I** and **G_II** components which were computed as explained below.

Laterally unconstrained \([(0\_2/\pm45)s]\_s) CLS test specimens were analyzed using an appropriate finite element model. Finite element analysis was performed by the program project engineer at NASA, Langley Research Center. The geometry of the specimen was taken from Figure 2. Assuming the x and y coordinates to be along the loading and thickness directions, respectively, the \(0^\circ\) plies were assumed to have the following properties: \(E_x = 137.9\) GPa, \(E_y = 14.5\) GPa, \(\nu_{yx} = 0.022\), and \(G_{xy} = 5.86\) GPa. The \(\pm45^\circ\) plies were assumed to have the following properties: \(E_x = 20.4\) GPa, \(E_y = 15.0\) GPa, \(\nu_{yx} = 0.044\), and \(G_{xy} = 5.86\) GPa. The performed finite element analysis accounted for geometric nonlinearity, and a reduced integration scheme was employed. **G_I** and **G_II** were computed at the delamination boundary using a virtual crack closure technique (References 9 and 10). For each of three initial crack lengths (a = 51, 76 and 102 mm), **G_I** and **G_II** were computed for a range of loads that extended from below to above the measured \(P_{cr}\) values (see Table 13).

NFEA results in Table 13 indicate that **G_I** and **G_II** are practically unaffected by the delamination size (a) in the laterally unconstrained \([(0\_2/\pm45)s]\_s) CLS specimens. A least squares curve fit analysis of the average NFEA **G_I** and **G_II** values (corresponding to a=51, 76 and 102mm) provided the following expressions:

\[
G_I = 0.72 \, P^{2.15} \, \text{J/m}^2 \quad (24)
\]

\[
G_{II} = 3.77 \, P^{1.95} \, \text{J/m}^2 \quad (25)
\]
**TABLE 13.** $G_I$, $G_{II}$ COMPUTATIONS BASED ON NFEA OF LATERALLY UNCONSTRAINED $[(0_2/\pm 45)_s]_s$ CLS SPECIMENS

<table>
<thead>
<tr>
<th>Load, P (kN)</th>
<th>Delamination Length (a) (mm)</th>
<th>$G_I^*$ (J/m²)</th>
<th>$G_{II}^*$ (J/m²)</th>
<th>$G_I/G_{II}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.448</td>
<td>50.8</td>
<td>17.69</td>
<td>68.82</td>
<td>0.257</td>
</tr>
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<td></td>
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<td>17.86</td>
<td>69.00</td>
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<td></td>
<td>101.6</td>
<td>18.04</td>
<td>69.18</td>
<td>0.261</td>
</tr>
<tr>
<td>6.672</td>
<td>50.8</td>
<td>42.03</td>
<td>152.36</td>
<td>0.276</td>
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<td></td>
<td>76.2</td>
<td>42.03</td>
<td>152.54</td>
<td>0.276</td>
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<td></td>
<td>101.6</td>
<td>42.38</td>
<td>152.89</td>
<td>0.277</td>
</tr>
<tr>
<td>8.896</td>
<td>50.8</td>
<td>78.28</td>
<td>266.19</td>
<td>0.294</td>
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<tr>
<td></td>
<td>76.2</td>
<td>78.28</td>
<td>266.19</td>
<td>0.294</td>
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<tr>
<td></td>
<td>101.6</td>
<td>78.81</td>
<td>267.94</td>
<td>0.294</td>
</tr>
<tr>
<td>11.121</td>
<td>50.8</td>
<td>127.84</td>
<td>411.55</td>
<td>0.311</td>
</tr>
<tr>
<td></td>
<td>76.2</td>
<td>128.02</td>
<td>411.55</td>
<td>0.311</td>
</tr>
<tr>
<td></td>
<td>101.6</td>
<td>128.54</td>
<td>411.55</td>
<td>0.312</td>
</tr>
</tbody>
</table>

* $G_I$, $G_{II}$ were computed at the delamination boundary.
In the above equations, P is expressed in kN, and $G_I$ and $G_{II}$ are independent of $a$.

Referring to Table 10, it is seen that the critical loads corresponding to $a=51, 76$ and 102 mm are relatively unaltered. Incorporating the average critical load value (7.985 kN) from Table 11 into equations (24) to (26), one obtains:

$$G_I = 62.69 \text{ J/m}^2, \quad G_{II} = 216.66 \text{ J/m}^2, \quad \frac{G_I}{G_{II}} = 0.289$$

(27)

corresponding to delamination growth in $[(0_2/\pm 45)_s]_s$ CLS specimens, for any delamination size ($a$). The sum of $G_I$ and $G_{II}$ yields the total strain energy release rate at any load level. Therefore, the critical total strain energy release rate ($G_C$) for the $[(0_2/\pm 45)_s]_s$ CLS specimen, based on NFEA results, is (see equation 27):

$$G_C = 62.69 + 216.66 = 279.35 \text{ J/m}^2$$

(28)

Referring to Table 11, it is seen that $G_C$ based on NFEA results agrees well with the strength of materials prediction, but is 39% lower than the measured value.

3.6 ANALYTICAL PREDICTION OF $G_{IIC}$

Static DCB tests measured $G_{IC}$ for T300/5208 to be 102.6 J/m$^2$. Finite element analysis of the $[(0_2/\pm 45)_s]_s$ CLS specimens estimated $G_I$ and $G_{II}$ corresponding to static delamination growth to be 62.7 and 216.7 J/m$^2$, respectively. An estimation of $G_{IIC}$ may be obtained by incorporating the mentioned $G_{IC}$, $G_I$ and $G_{II}$ values into a failure criterion. Since none has yet been established to be valid, the following three failure criteria were selected:

$$\frac{G_I}{G_{IC}} + \frac{G_{II}}{G_{IIC}} = 1$$

(29)

$$\left(\frac{G_I}{G_{IC}}\right)^2 + \left(\frac{G_{II}}{G_{IIC}}\right)^2 = 1$$

(30)

$$\left(\frac{G_I}{G_{IC}}\right)^2 + \left(\frac{G_{II}}{G_{IIC}}\right)^2 + \left(\frac{G_I}{G_{IC}}\right)\left(\frac{G_{II}}{G_{IIC}}\right) = 1$$

(31)

Substituting $G_{IC} = 102.6$ J/m$^2$, $G_I = 62.7$ J/m$^2$ and $G_{II} = 216.7$ J/m$^2$ into equations (29), (30) and (31), $G_{IIC}$ values of 587.0, 279.0 and 414.9
J/m$^2$, respectively, were obtained. Only a pure shear test and the concomitant $G_{\text{IIC}}$ measurement will establish the best choice among equations (29), (30) and (31).

In section 3.5, $G_c$ based on NFEA was shown to be 39% lower than the $G_c$ based on test results. If the NFEA estimations of $G_I$ and $G_{II}$, used in $G_{\text{IIC}}$ computation above, are scaled up (by a factor of 1.64) to estimate $G_I$ and $G_{II}$ corresponding to test results, $G_I$ is seen to be equal to $G_{IC}$ (103 J/m$^2$). This indicates that static delamination growth in the $[(0_2/±45)_s]_3$ CLS specimens is influenced largely by the opening mode (mode I). Also, $G_{\text{IIC}}$ will be computed to be larger than the reported values if the scaled-up $G_I$ and $G_{II}$ are substituted into the selected failure criteria ($G_{II}/G_{\text{IIC}} \rightarrow 0$ as $G_I/G_{IC} \rightarrow 1$).

3.7 CLS FATIGUE TEST RESULTS

Constant amplitude tension fatigue tests on CLS specimens were conducted at $R = 0.05$ and $\omega = 10$ Hertz. The tests were displacement-controlled and the maximum cyclic displacements ($u_{\text{max}}$) were selected, based on static test data, to yield initial delamination growth rates ($da/dN$) in the neighborhood of 2.54 mm (0.1 in.) per 200 cycles. These maximum cyclic displacement values were reset in most cases to a higher value after a 12.7 mm (0.5 in.) growth in the delamination (see Section 2.6). Compliance measurements and cycle counts were recorded at $\Delta a$ intervals of 2.54 mm (0.1 in.). Figure 24 shows a typical static compliance variation with delamination growth. Recorded data also yielded $da/dN$ values for various $a$, $u_{\text{max}}$ values (see Tables 14 to 16).

The maximum cyclic value of the total strain energy release rate ($G_{\text{max}}$) in CLS specimens can be expressed in a form similar to equation (11):

$$G_{\text{max}} = \frac{u_{\text{max}}^2}{u_{\text{max}}} \left(\frac{dC}{da}\right)/(2wC^2)$$

where $w = 25.4$ mm (1 in) for the tested CLS specimens. Recalling that compliance is a linear function of the delamination length (see Figures 20 to 22), it is seen that $G_{\text{max}}$ is proportional to $a^{-2}$ in CLS specimens:

$$G_{\text{max}} \propto a^{-2}$$

Static Compliance Measurements During Fatigue Testing (R = 0.05, ω = 10 Hertz)

No lateral support through flexures

a is the delamination length (see Figure 2)

Figure 24. Typical Static Compliance Measurements During Fatigue Testing of CLS Specimens.
### TABLE 14. TENSION FATIGUE TEST DATA ON \([ (0_2/\pm 45)_s ]_s\)

**T300/5208 CLS SPECIMENS AT R=0.05 AND \(\omega=10\) HERTZ**

<table>
<thead>
<tr>
<th>Specimen H-17</th>
<th>Specimen H-12</th>
<th>Specimen H-11</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) (mm)</td>
<td>(da/dN) ((\mu m)/cycle)</td>
<td>(G_{max}) (J/m(^2))</td>
</tr>
<tr>
<td>28.4</td>
<td>7493</td>
<td>353</td>
</tr>
<tr>
<td>38.1</td>
<td>3505</td>
<td>304</td>
</tr>
<tr>
<td>50.8</td>
<td>2718</td>
<td>253</td>
</tr>
<tr>
<td>63.5</td>
<td>879</td>
<td>215</td>
</tr>
<tr>
<td>76.2</td>
<td>869</td>
<td>184</td>
</tr>
<tr>
<td>88.9</td>
<td>177</td>
<td>160</td>
</tr>
<tr>
<td>101.6</td>
<td>85</td>
<td>140</td>
</tr>
<tr>
<td>114.3</td>
<td>17</td>
<td>123</td>
</tr>
<tr>
<td>119.4</td>
<td>12</td>
<td>118</td>
</tr>
</tbody>
</table>

A least squares curve-fit analysis of the presented data yielded the following equation:

\[
\frac{da}{dN} = 7.7279 \times 10^{-12} G_{max}^{0.0660} \text{ mm/cycle where } G_{max} \text{ is expressed in J/m}^2
\]

* Same units as shown for specimen H-17.
TABLE 15. TENSION FATIGUE TEST DATA ON \([0_2/45_2/0_2]_s\) T300/5208 CLS SPECIMENS WITH LATERAL FLEXURE CONSTRAINTS (R=0.05, \(\omega=10\) Hz)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Specimen F-29</th>
<th>Specimen F-31</th>
<th>Specimen F-35</th>
<th>Specimen F-23</th>
</tr>
</thead>
<tbody>
<tr>
<td>a (mm)</td>
<td>da/dN ((\mu)mm/cycle)</td>
<td>G(_{max}) (J/m(^2))</td>
<td>a*</td>
<td>da/dN*</td>
</tr>
<tr>
<td>50.8</td>
<td>1681</td>
<td>262</td>
<td>38.1</td>
<td>21166</td>
</tr>
<tr>
<td>63.5</td>
<td>1090</td>
<td>237</td>
<td>43.2</td>
<td>10584</td>
</tr>
<tr>
<td>76.2</td>
<td>1450</td>
<td>220</td>
<td>48.3</td>
<td>9769</td>
</tr>
<tr>
<td>88.9</td>
<td>752</td>
<td>205</td>
<td>63.5</td>
<td>8758</td>
</tr>
<tr>
<td>101.6</td>
<td>330</td>
<td>209</td>
<td>78.7</td>
<td>640</td>
</tr>
<tr>
<td>114.3</td>
<td>249</td>
<td>193</td>
<td>86.4</td>
<td>259</td>
</tr>
<tr>
<td>121.9</td>
<td>183</td>
<td>187</td>
<td>91.4</td>
<td>836</td>
</tr>
</tbody>
</table>

A least squares curve-fit analysis of the presented data yielded the following equation:

\[
da/dN = 2.7536 \times 10^{-11} G_{\max}^{5.6873} \text{ \(\mu\)mm/cycle where } G_{\max} \text{ is expressed in J/m}^2\]

* Same units as shown for specimen F-29.
TABLE 16. TENSION FATIGUE TEST DATA ON LATERALLY UNCONSTRAINED $[0_{2}/\pm 45_{2}/0_{2}]_s$
T300/5208 CLS SPECIMENS (R=0.05, $\omega=10$ HZ)

<table>
<thead>
<tr>
<th>Specimen</th>
<th>$a$ (mm)</th>
<th>da/dN ((\mu)m/cycle)</th>
<th>$G_{\text{max}}$ (in-lb/in$^2$)</th>
<th>da/dN*</th>
<th>$G^{*}_{\text{max}}$</th>
<th>da/dN*</th>
<th>$G^{*}_{\text{max}}$</th>
<th>da/dN*</th>
<th>$G^{*}_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-33</td>
<td>66.0</td>
<td>8458</td>
<td>353</td>
<td>38.1</td>
<td>3785</td>
<td>335</td>
<td>38.1</td>
<td>22479</td>
<td>362</td>
</tr>
<tr>
<td></td>
<td>71.1</td>
<td>6045</td>
<td>337</td>
<td>40.6</td>
<td>2896</td>
<td>327</td>
<td>40.6</td>
<td>14427</td>
<td>359</td>
</tr>
<tr>
<td></td>
<td>76.2</td>
<td>5080</td>
<td>316</td>
<td>43.2</td>
<td>2540</td>
<td>323</td>
<td>43.2</td>
<td>16383</td>
<td>350</td>
</tr>
<tr>
<td></td>
<td>78.7</td>
<td>12700</td>
<td>358</td>
<td>45.7</td>
<td>2289</td>
<td>317</td>
<td>58.4</td>
<td>5410</td>
<td>356</td>
</tr>
<tr>
<td></td>
<td>81.3</td>
<td>9398</td>
<td>365</td>
<td>53.3</td>
<td>12090</td>
<td>338</td>
<td>61.0</td>
<td>4801</td>
<td>344</td>
</tr>
<tr>
<td></td>
<td>86.4</td>
<td>7468</td>
<td>344</td>
<td>58.4</td>
<td>8458</td>
<td>326</td>
<td>63.5</td>
<td>4089</td>
<td>341</td>
</tr>
<tr>
<td></td>
<td>104.1</td>
<td>25400</td>
<td>376</td>
<td>63.5</td>
<td>7950</td>
<td>312</td>
<td>66.0</td>
<td>5766</td>
<td>342</td>
</tr>
<tr>
<td></td>
<td>106.7</td>
<td>15875</td>
<td>374</td>
<td>68.6</td>
<td>6350</td>
<td>313</td>
<td>73.7</td>
<td>4039</td>
<td>315</td>
</tr>
<tr>
<td></td>
<td>109.2</td>
<td>9779</td>
<td>371</td>
<td>71.1</td>
<td>4699</td>
<td>305</td>
<td>97.0</td>
<td>40</td>
<td>124</td>
</tr>
</tbody>
</table>

A least squares curve-fit analysis of the presented data yielded the following equation:

$$\frac{\text{da}}{\text{dN}} = 4.3942 \times 10^{-9} G_{\text{max}}^{4.8273} \text{\(\mu\)m/cycle where } G_{\text{max}} \text{ is expressed in J/m}^2$$

* Same units as shown for specimen F-33.
The average $dC/da$ value was taken from Figures 20 to 22, and $C$ was computed from records similar to Figure 24. $G_{\text{max}}$ was computed by substituting $C$, $dC/da$ and the imposed $u_{\text{max}}$ values into equation (32). Subsequently, plots of $da/dN$ versus $G_{\text{max}}$ were generated for the $(0_2/45_2)_s$ and $0_2/45_2/0_2_s$ CLS specimens (see Figures 25 to 27).

A least squares curve-fit analysis of the delamination growth data presented in Tables 14 to 16 (Figures 25 to 27) yielded the following expressions:

For the $(0_2/45_2)_s$ CLS specimens,

$$\frac{da}{dN} = 7.73 \times 10^{-12} G_{\text{max}}^{6.07} \text{ mm/mm/cycle} \quad (34)$$

For the $[0_2/45_2/0_2_s$ CLS specimens with lateral (flexure) constraints,

$$\frac{da}{dN} = 2.75 \times 10^{-11} G_{\text{max}}^{5.69} \text{ mm/mm/cycle} \quad (35)$$

For the laterally unconstrained $[0_2/45_2/0_2_s$ CLS specimens,

$$\frac{da}{dN} = 4.39 \times 10^{-9} G_{\text{max}}^{4.83} \text{ mm/mm/cycle} \quad (36)$$

In the above equations, $G_{\text{max}}$ is expressed in $J/m^2$. It is also noted that these delamination growth relationships were obtained from constant amplitude fatigue tests run at a frequency of 10 Hertz with the minimum cyclic total strain energy release rate ($G_{\text{min}}$) maintained to be equal to 0.0025 $G_{\text{max}}$. Equations (34) to (36) are therefore valid only for $\omega = 10$ Hz and $R = 0.05$. The differences in the coefficient and exponent values in the three equations is due to different $G_I/G_{II}$ ratios in the corresponding test cases. $G_I/G_{II}$ will vary with the laminate layup, and for the same layup, will vary with the specimen support conditions (lateral constraints).

### 3.8 $G_I$, $G_{II}$ Contributions to Delamination Growth Rate in CLS Specimens

CLS specimens are mixed mode specimens in which delamination growth is potentially influenced by all the strain energy release rate components ($G_I$, $G_{II}$ and $G_{III}$). In the preceding section, a least squares curve fit analysis of the experimental data on $(0_2/45_2)_s$ CLS specimens was per-
Figure 25. Variation of Delamination Growth Rate With Maximum Cyclic Strain Energy Release Rate in $[(0_{2}/\pm 45)^{s}_{s}]_{s}$ T300/5208 CLS Specimens.
Figure 26. Variation of Delamination Growth Rate with Strain Energy Release Rate in $[0_2/\pm45_2/0_2]_s$ CLS Specimens with Lateral Constraints.
Figure 27. Variation of Delamination Growth Rate with Strain Energy Release Rate in $[0_2/\pm 45_2/0_2]_s$ CLS Specimens Without Lateral Constraints.
formed to obtain a power law approximation (equation 34) relating delamination growth rate \((da/dN)\) to the maximum cyclic total strain energy release rate \((G_{\text{max}})\). Individual mode contributions to \(da/dN\) are implicitly included in this relationship, and useful information regarding the CLS specimen behavior can be obtained by separating these values. Assuming that \(G_{\text{III}}\) effects are negligible in the \([(0_{2}/±45)_{s}]_{s}\) CLS specimens, modes I and II were assumed to contribute to delamination growth rate according to the following equation:

\[
da/dN = C_1 \left( G_{I_{\text{max}}} / G_{\text{IC}} \right)^{n_1} + C_2 \left( G_{II_{\text{max}}} / G_{\text{IIC}} \right)^{n_2} \tag{37}
\]

where \(G_{I_{\text{max}}} + G_{II_{\text{max}}} = G_{\text{max}}\) \(\tag{38}\)

In section 3.2, it was shown that the mode I contribution to delamination growth was adequately represented by \(C_1 = 0.0283 \text{ mm/cycle}\) and \(n_1 = 8\), when \(da/dN\) is expressed in \(\text{mm/cycle}\) (see equation 16). This was based on fatigue test data on DCB specimens in which \(G_{II}\) (and \(G_{\text{III}}\)) effects were absent. Also, static tests on DCB specimens yielded a \(G_{\text{IC}}\) value of 102.6 \(\text{J/m}^2\). Substituting these values into equation (37) yields:

\[
da/dN = 0.0283 \left( G_{I_{\text{max}}} /103 \right)^{8} + C_2 \left( G_{II_{\text{max}}} / G_{\text{IIC}} \right)^{n_2} \tag{39}
\]

In the above equation, \(da/dN\) and \(C_2\) are expressed in \(\text{mm/cycle}\), and \(G_{I_{\text{max}}}, G_{II_{\text{max}}}, G_{\text{IC}}\) and \(G_{\text{IIC}}\) are expressed in \(\text{J/m}^2\). In section 3.6, \(G_{\text{IIC}}\) was computed to be 587, 279, and 415 \(\text{J/m}^2\), assuming the failure criteria in equations (29), (30) and (31), respectively. The choice of \(G_{\text{IIC}}\) will only affect \(C_2\) in equation (39), and not \(n_2\). The values of \(C_2\) and \(n_2\) are computed below using finite element results and tension fatigue test data on the \([(0_{2}/±45)_{s}]_{s}\) CLS specimen.

NFEA of the \([(0_{2}/±45)_{s}]_{s}\) CLS specimen provided the expressions in equations (24) to (26) for \(G_I, G_{II}\) and \(G_{I}/G_{II}\) as a function of the applied load, \(P\). These expressions are independent of the delamination size \((a)\). During fatigue, \(u_{\text{max}}\) was set to be a specified value. The compliance variation with \(a\) and the delamination growth rate \((da/dN)\) were measured. The maximum cyclic load \((P_{\text{max}})\) corresponding to any delamination size \((a)\)
was computed using the C versus a records and the imposed \( u_{\text{max}} \) value. Substituting \( P_{\text{max}} \) into equation (26), \( G_{I_{\text{max}}}/G_{II_{\text{max}}} \) was computed. The average \( dC/da \) from Figure 20 and the computed \( P_{\text{max}} \) yielded \( G_{\text{max}} \) expressed as \( P_{\text{max}}^2 \frac{(dC/da)}{(2\omega)} \). \( G_{\text{max}} \) was subsequently divided into \( G_{I_{\text{max}}} \) and \( G_{II_{\text{max}}} \) components using the calculated \( G_{I_{\text{max}}}/G_{II_{\text{max}}} \) ratios and equation (38).

In equation (39), the mode I contribution to \( da/dN \) was assumed to be \( 0.0283 \left( \frac{G_{I_{\text{max}}}}{103} \right)^8 \) mm/cycle, when \( G_{I_{\text{max}}} \) is expressed in J/m². Computed \( G_{I_{\text{max}}} \) values were substituted into this expression, and the results were subtracted from the measured \( da/dN \) values to obtain the mode II contribution to \( da/dN \), expressed as \( C_2 \left( \frac{G_{II_{\text{max}}}}{G_{IIC}} \right)^n_2 \) in equation (39). Table 17 presents the relevant delamination growth data for \([(0/±45)_{s} s]_s CLS \) specimens. A least squares analysis of the computed mode II contributions to \( da/dN \) yielded the following results:

\[
n_2 = 5.7865
\]

\[
C_2 = 1.1237, \ 0.0152 \text{ or } 0.1511 \text{ mm/cycle if } G_{IIC} = 587, 279 \text{ or } 415 \text{ J/m², respectively.}
\]

It is interesting to note that \( n_2 \) in equation (40) is approximately equal to the exponent in equation (34). Also, Table 17 reveals that the mode I contribution to \( da/dN \) is very small in comparison to the mode II contribution to \( da/dN \), especially for low \( G_{\text{max}} \) values. Delamination growth in the \([(0_2/±45)_{s} s]_s CLS \) specimens is, therefore, predominantly influenced by mode II.

In summary, delamination growth rate in T300/5208 laminates may be quantified by:

\[
da/dN = 0.0283 \left( \frac{G_{I_{\text{max}}}}{103} \right)^8 + C_2 \left( \frac{G_{II_{\text{max}}}}{G_{IIC}} \right)^n_2
\]

(42)

where \( C_2 \) for three \( G_{IIC} \) values are given by equation (41). The expression in equation (42) assumes negligible \( G_{III} \) effects, and is restricted to constant amplitude fatigue loading at \( R=0.05 \) and \( \omega = 10 \text{ Hertz.} \)
TABLE 17. MODE II CONTRIBUTION TO DELAMINATION GROWTH IN \([(0_2 / ±45)_s]_s\) CLS SPECIMENS

<table>
<thead>
<tr>
<th>Specimen</th>
<th>(a) (mm)</th>
<th>(u_{\text{max}}) (mm)</th>
<th>(C) ((\mu m/N))</th>
<th>(P_{\text{max}}) (kN)</th>
<th>(G_{\text{max}}) ((J/m^2))</th>
<th>(\frac{da}{dN}) ((\mu m/\text{cycle}))</th>
<th>(G_{I_{\text{max}}}) ((J/m^2))</th>
<th>(G_{II_{\text{max}}}) ((J/m^2))</th>
<th>(C_2\left(\frac{G_{II_{\text{max}}}}{G_{IIC}}\right)^n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H-12</td>
<td>30.5</td>
<td>0.3226</td>
<td>38.69</td>
<td>8.340</td>
<td>464</td>
<td>60477</td>
<td>109</td>
<td>375</td>
<td>32177</td>
</tr>
<tr>
<td>H-12</td>
<td>38.1</td>
<td>0.3226</td>
<td>41.28</td>
<td>7.816</td>
<td>425</td>
<td>39065</td>
<td>95</td>
<td>330</td>
<td>24244</td>
</tr>
<tr>
<td>H-12</td>
<td>50.6</td>
<td>0.3226</td>
<td>45.60</td>
<td>7.073</td>
<td>348</td>
<td>27305</td>
<td>76</td>
<td>272</td>
<td>24818</td>
</tr>
<tr>
<td>H-12</td>
<td>63.5</td>
<td>0.3226</td>
<td>49.92</td>
<td>6.463</td>
<td>291</td>
<td>14122</td>
<td>63</td>
<td>228</td>
<td>13568</td>
</tr>
<tr>
<td>H-12</td>
<td>76.2</td>
<td>0.3226</td>
<td>54.25</td>
<td>5.947</td>
<td>246</td>
<td>3454</td>
<td>53</td>
<td>193</td>
<td>3315</td>
</tr>
<tr>
<td>H-12</td>
<td>88.9</td>
<td>0.3226</td>
<td>58.57</td>
<td>5.507</td>
<td>211</td>
<td>1026</td>
<td>44</td>
<td>167</td>
<td>995</td>
</tr>
<tr>
<td>H-12</td>
<td>101.6</td>
<td>0.3226</td>
<td>62.89</td>
<td>5.129</td>
<td>183</td>
<td>518</td>
<td>38</td>
<td>145</td>
<td>508</td>
</tr>
<tr>
<td>H-12</td>
<td>114.3</td>
<td>0.3226</td>
<td>67.21</td>
<td>4.800</td>
<td>160</td>
<td>84</td>
<td>33</td>
<td>127</td>
<td>81</td>
</tr>
<tr>
<td>H-12</td>
<td>124.5</td>
<td>0.3226</td>
<td>70.67</td>
<td>4.564</td>
<td>145</td>
<td>57</td>
<td>30</td>
<td>115</td>
<td>56</td>
</tr>
<tr>
<td>H-11</td>
<td>50.8</td>
<td>0.3531</td>
<td>52.93</td>
<td>6.672</td>
<td>310</td>
<td>9548</td>
<td>67</td>
<td>243</td>
<td>8641</td>
</tr>
<tr>
<td>H-11</td>
<td>63.5</td>
<td>0.3531</td>
<td>57.42</td>
<td>6.147</td>
<td>263</td>
<td>5570</td>
<td>56</td>
<td>207</td>
<td>5354</td>
</tr>
<tr>
<td>H-11</td>
<td>76.2</td>
<td>0.3531</td>
<td>61.90</td>
<td>5.703</td>
<td>226</td>
<td>1857</td>
<td>48</td>
<td>178</td>
<td>1794</td>
</tr>
<tr>
<td>H-11</td>
<td>88.9</td>
<td>0.3531</td>
<td>66.39</td>
<td>5.316</td>
<td>197</td>
<td>737</td>
<td>41</td>
<td>156</td>
<td>719</td>
</tr>
<tr>
<td>H-11</td>
<td>101.6</td>
<td>0.3531</td>
<td>70.88</td>
<td>4.982</td>
<td>173</td>
<td>368</td>
<td>36</td>
<td>137</td>
<td>362</td>
</tr>
<tr>
<td>H-11</td>
<td>114.3</td>
<td>0.3531</td>
<td>75.37</td>
<td>4.684</td>
<td>153</td>
<td>155</td>
<td>31</td>
<td>122</td>
<td>153</td>
</tr>
<tr>
<td>H-11</td>
<td>124.5</td>
<td>0.3531</td>
<td>78.96</td>
<td>4.471</td>
<td>139</td>
<td>56</td>
<td>28</td>
<td>111</td>
<td>55</td>
</tr>
</tbody>
</table>

+ Refer to equation (39). (Continued)
TABLE 17. MODE II CONTRIBUTION TO DELAMINATION GROWTH in \([(0_2/\pm45)_s]\) _s CLS SPECIMENS (CONCLUDED)

| Specimen | \(a\) (mm) | \(u_{\text{max}}\) (mm) | \(C\) (\(\mu\) mm/N) | \(P_{\text{max}}\) (kN) | \(G_{\text{max}}\) (J/m\(^2\)) | da/dN (\(\mu\) mm/cycle) | \(G_{I_{\text{max}}}\) (J/m\(^2\)) | \(G_{II_{\text{max}}}\) (J/m\(^2\)) | \(C_2 \left( \frac{G_{II_{\text{max}}}}{G_{IIC}} \right)^{n_2}\) |
|----------|------------|-----------------|----------------|----------------|----------------|-----------------|----------------|----------------|----------------|----------------|
| H-17     | 28.4       | 0.3048          | 42.80          | 7.122          | 353            | 7493            | 78             | 275            | 4432          |
| H-17     | 38.1       | 0.3048          | 46.13          | 6.610          | 304            | 3505            | 66             | 238            | 2701          |
| H-17     | 50.8       | 0.3048          | 50.49          | 6.036          | 253            | 2718            | 54             | 199            | 2556          |
| H-17     | 63.5       | 0.3048          | 54.86          | 5.556          | 215            | 879             | 45             | 170            | 841           |
| H-17     | 76.2       | 0.3048          | 59.22          | 5.147          | 184            | 869             | 38             | 146            | 859           |
| H-17     | 88.9       | 0.3048          | 63.59          | 4.795          | 160            | 177             | 33             | 127            | 174           |
| H-17     | 101.6      | 0.3048          | 67.95          | 4.484          | 140            | 85              | 29             | 111            | 84            |
| H-17     | 114.3      | 0.3048          | 72.32          | 4.212          | 123            | 17              | 25             | 98             | 17            |
| H-17     | 119.4      | 0.3048          | 74.06          | 4.115          | 118            | 12              | 24             | 94             | 12            |

+ Refer to equation (39).
3.9 **STATIC COMPRESSION TEST RESULTS ON SPECIMENS WITH ITTW DELAMINATIONS**

Specimens with imbedded through-the-width (ITTW) delaminations had the geometry shown in Figure 3, and were tested under static compression as shown in Figure 9. Specimens with three laminate configurations were tested. Laminate A had a \([0_4/(0/45/90/-45)]_7\) layup with a 19 mm (0.75 in.) long initial delamination between plies 3 and 4. Laminate B had a \([0/45_2/0/(0/45/90/-45)]_7\) layup with a 25 mm (1 in.) long initial delamination between plies 4 and 5. Laminate C had a \([0/45/90_2/45/0_3/(0/45/90/-45)]_6\) layup with a 32 mm (1.25 in.) long initial delamination between plies 6 and 7. In every case, the delamination was imbedded at a 0/0 interface.

Static compression loads were introduced at a slow rate, and the transverse deflection of the delaminated set of plies at midlength was monitored. The far-field axial strain and the strain in the delaminated region at midlength were measured in selected specimens. If the ITTW delamination propagated over only a short distance, the new length and the corresponding location of the maximum transverse deflection were recorded as shown in Figure 11.

Static compression test results on laminate A specimens are presented in Tables 18 to 21. With the exception of one specimen (A-4), the ITTW delamination propagated in an unstable manner, initially to one tab edge, and subsequently to the other. In specimen A-4, a stable delamination growth was observed initially prior to its abrupt propagation to the tab region. In one specimen (A-3), the 25.4 mm (1 in.) wide delaminated \([0]_3T\) plies split between fibers, 19.1 mm (0.75 in.) from one edge. The 19.1 mm (0.75 in.) wide delamination propagated to the tab region first, and the 6.4 mm (0.25 in.) wide delamination followed it at a higher load level.

Static compression test results on laminates B and C with ITTW delaminations are presented in Table 22. Delamination failure -- propagation of the ITTW delamination to the tab region -- was abrupt (without any stable growth) during these load-controlled tests. In laminate B, the delaminated \([0/45]_7\) segment exhibited fiber failures and splitting.
### TABLE 18. STATIC COMPRESSION TEST RESULTS ON SPECIMEN A-1+

<table>
<thead>
<tr>
<th>Applied Stress ksi (MPa)</th>
<th>Event</th>
<th>$2a^*$ in (mm)</th>
<th>Measured Strain</th>
<th>$\varepsilon^*$ L $\mu$mm/mm</th>
<th>$\delta^*$ L in (mm)</th>
<th>$X_{max}^*$ in (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-26.26 (-181.1)</td>
<td>Kapton Separation</td>
<td>0.75 (19.1)</td>
<td>-3330</td>
<td>-2796</td>
<td>0.125 (3.18)</td>
<td>2.25 (57.15)</td>
</tr>
<tr>
<td>-29.18 (-201.2)</td>
<td>--</td>
<td>0.75 (19.1)</td>
<td>-3700</td>
<td>-2695</td>
<td>0.130 (3.30)</td>
<td>2.25 (57.15)</td>
</tr>
<tr>
<td>-35.02 (-241.4)</td>
<td>--</td>
<td>0.75 (19.1)</td>
<td>-4405</td>
<td>-2617</td>
<td>0.135 (3.43)</td>
<td>2.25 (57.15)</td>
</tr>
<tr>
<td>-39.39 (-271.6)</td>
<td>Propagation of delamination to one tab edge</td>
<td>2.63 (66.8)</td>
<td>-4995</td>
<td>364</td>
<td>0.239 (6.07)</td>
<td>1.32 (33.4)</td>
</tr>
<tr>
<td>-44.35 (-305.8)</td>
<td>Propagation of delamination to the other tab edge (Delam. Failure)</td>
<td>4.50 (114.3)</td>
<td>-5624</td>
<td>1001</td>
<td>--</td>
<td>2.25 (57.15)</td>
</tr>
<tr>
<td>-621.5 (-428.5)</td>
<td>Total Failure</td>
<td>---</td>
<td>-7781</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

+ $[0_4/(0/45/90/-45)_7]_s$ T300/5208 laminate. The top $[0_3]$ layers were initially delaminated over 19.1 mm (0.75 in.) through the entire width (25.4 mm).

E = 54.38 GPa (7.89 Msi); Cross-sectional area = 221.1 mm$^2$ (0.3427 in$^2$)

* See Figure 11
TABLE 19. STATIC COMPRESSION TEST RESULTS ON SPECIMEN A-2+

<table>
<thead>
<tr>
<th>Applied Stress ksi (MPa)</th>
<th>Event</th>
<th>2a * in (mm)</th>
<th>δL * in (mm)</th>
<th>Xmax * in (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-13.99 (-96.5)</td>
<td>Kapton Separation</td>
<td>0.88 (22.2)</td>
<td>0.007 (0.18)</td>
<td>2.13 (53.98)</td>
</tr>
<tr>
<td>-22.38 (-154.3)</td>
<td>--</td>
<td>0.88 (22.2)</td>
<td>0.014 (0.36)</td>
<td>2.13 (53.98)</td>
</tr>
<tr>
<td>-29.3 (-202.6)</td>
<td>Propagation of delamination to one tab edge</td>
<td>2.63 (66.7)</td>
<td>0.049 (1.24)</td>
<td>1.31 (33.34)</td>
</tr>
<tr>
<td>-33.58 (-231.5)</td>
<td>--</td>
<td>2.63 (66.7)</td>
<td>0.051 (1.30)</td>
<td>1.31 (33.34)</td>
</tr>
<tr>
<td>-39.17 (-270.1)</td>
<td>--</td>
<td>2.63 (66.7)</td>
<td>0.100 (2.54)</td>
<td>1.31 (33.34)</td>
</tr>
<tr>
<td>-44.49 (-306.7)</td>
<td>Propagation of delamination to the other tab edge</td>
<td>4.5 (114.3)</td>
<td>0.245 (6.22)</td>
<td>2.25 (57.15)</td>
</tr>
<tr>
<td>-62.95 (-434.1)</td>
<td>Total Failure</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

+ [04/045/090/-45] 7 T300/5208 laminate. The top [03] layers were initially delaminated over 9.53 mm (0.75 in.) through the entire width (25.4 mm)

Cross-sectional area = 230.6 mm² (0.3574 in²)

* See Figure 11
TABLE 20. STATIC COMPRESSION TEST RESULTS ON SPECIMEN A-3+

<table>
<thead>
<tr>
<th>Applied Stress ksi (MPa)</th>
<th>Event</th>
<th>$2a^*$ in (mm)</th>
<th>$\delta_L^*$ in (mm)</th>
<th>$X_{max}^*$ in (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-21.93 (-151.2)</td>
<td>Kapton Separation</td>
<td>0.75 (19.1)</td>
<td>0.010 (0.25)</td>
<td>2.25 (57.15)</td>
</tr>
<tr>
<td>-27.41 (-189.0)</td>
<td>--</td>
<td>0.75 (19.1)</td>
<td>0.014 (0.36)</td>
<td>2.25 (57.15)</td>
</tr>
<tr>
<td>-31.25 (-215.5)</td>
<td>Delamination propagated on one side</td>
<td>1.75 (44.5)</td>
<td>0.015 (0.37)</td>
<td>2.13 (53.98)</td>
</tr>
<tr>
<td>-32.89 (-226.8)</td>
<td>--</td>
<td>1.75 (44.5)</td>
<td>0.015 (0.38)</td>
<td>2.13 (53.98)</td>
</tr>
<tr>
<td>-39.47 (-272.2)</td>
<td>Delamination propagated to tab edge on either side over a 3/4&quot; width</td>
<td>4.5 (114.3)</td>
<td>0.175 (4.45)</td>
<td>2.25 (57.15)</td>
</tr>
<tr>
<td>-46.60 (-321.3)</td>
<td>Delamination propagated to tab edge on either side over a 1/4&quot; width</td>
<td>4.5 (114.3)</td>
<td>---</td>
<td>2.25 (57.15)</td>
</tr>
<tr>
<td>-74.00 (-510.3)</td>
<td>Total Failure</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

$^+[0_4/(0/45/90/-45)]_7s$ T300/5208. The top $[0_3]$ layers were initially delaminated over 19.1 mm (0.75 in.) through the entire width (25.4 mm)

Cross-sectional area = 235.3 mm$^2$ (0.3648 in$^2$)

* See Figure 11.
### TABLE 21. STATIC COMPRESSION TEST RESULTS ON SPECIMEN A-4

<table>
<thead>
<tr>
<th>Applied Stress ksi (MPa)</th>
<th>Event</th>
<th>$2a^*$ in (mm)</th>
<th>$\delta L^*$ in (mm)</th>
<th>$X_{max}$ in (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-163.39 (-113.0)</td>
<td>Kapton separation</td>
<td>0.75 (19.1)</td>
<td>0.005 (0.13)</td>
<td>2.25 (57.15)</td>
</tr>
<tr>
<td>-21.86 (-150.7)</td>
<td></td>
<td>0.75 (19.1)</td>
<td>0.013 (0.33)</td>
<td>2.25 (57.15)</td>
</tr>
<tr>
<td>-27.32 (-188.4)</td>
<td></td>
<td>0.75 (19.1)</td>
<td>0.020 (0.51)</td>
<td>2.25 (57.15)</td>
</tr>
<tr>
<td>-30.60 (-211.0)</td>
<td>Delamination propagated 25.4 mm upward</td>
<td>1.75 (44.5)</td>
<td>0.023 (0.58)</td>
<td>2.13 (53.98)</td>
</tr>
<tr>
<td>-32.79 (-226.1)</td>
<td></td>
<td>1.75 (44.5)</td>
<td>0.024 (0.61)</td>
<td>2.13 (53.98)</td>
</tr>
<tr>
<td>-39.21 (-270.3)</td>
<td>Delamination propagated to the upper tab</td>
<td>2.63 (66.8)</td>
<td>0.020 (0.51)</td>
<td>1.32 (33.4)</td>
</tr>
<tr>
<td>-47.81 (-329.7)</td>
<td>Delamination propagated to the lower tab</td>
<td>4.50 (114.3)</td>
<td>0.210 (5.33)</td>
<td>2.25 (57.15)</td>
</tr>
<tr>
<td>-73.50 (-506.7)</td>
<td>Total Failure</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

+ $[0_3/(0/45/90/-45)]_7$, T300/5208. The top $[0_3]$ layers were initially delaminated over 19.1 cm (0.75 in.) through the entire width (25.4 mm)
Cross-sectional area = 230.6 mm$^2$ (0.3574 in$^2$ )

* See Figure 11.
<table>
<thead>
<tr>
<th>Specimen</th>
<th>Cross-Sectional Area $cm^2$</th>
<th>Applied Stress in MPa (ksi) Corresponding to Delamination Failure</th>
<th>Total Failure</th>
<th>Modulus GPa (Msi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-2</td>
<td>2.257 (0.3499)</td>
<td>321.2 (46.585)</td>
<td>419.7 (60.875)</td>
<td>---</td>
</tr>
<tr>
<td>B-3</td>
<td>2.309 (0.3579)</td>
<td>321.7 (46.661)</td>
<td>454.6 (65.940)</td>
<td>---</td>
</tr>
<tr>
<td>B-4</td>
<td>2.337 (0.3623)</td>
<td>348.3 (50.511)</td>
<td>443.4 (64.311)</td>
<td>43.50 (6.31)</td>
</tr>
<tr>
<td>B-5</td>
<td>2.345 (0.3635)</td>
<td>322.5 (46.768)</td>
<td>383.1 (55.571)</td>
<td>---</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>328.4 (47.631)</td>
<td>425.2 (61.674)</td>
</tr>
<tr>
<td>C-1</td>
<td>2.108 (0.3268)</td>
<td>379.8 (55.080)</td>
<td>451.5 (65.483)</td>
<td>49.64 (7.20)</td>
</tr>
<tr>
<td>C-2</td>
<td>2.245 (0.3480)</td>
<td>299.2 (43.391)</td>
<td>400.2 (58.046)</td>
<td>---</td>
</tr>
<tr>
<td>C-3</td>
<td>2.315 (0.3589)</td>
<td>337.5 (48.947)</td>
<td>418.8 (60.741)</td>
<td>---</td>
</tr>
<tr>
<td>C-4</td>
<td>2.348 (0.3640)</td>
<td>284.1 (41.209)</td>
<td>394.0 (57.143)</td>
<td>---</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>325.1 (47.157)</td>
<td>416.1 (60.353)</td>
</tr>
</tbody>
</table>

* Laminate B has a $[0/45_2/0/(0/45/90/-45)]_7$ layup, with a 25.4 mm long Kapton inclusion between plies 4 and 5, across the 25.4 mm width. Laminate C has a $[0/45/90_2/45/0_3/(0/45/90/-45)]_6$ layup with a 31.8 mm long Kapton inclusion between plies 6 and 7, across the 25.4 mm width.

+ This modulus is lower than the expected value (49.6 MPa<E<54.4 MPa)
between fibers in the 0° and 45° plies at delamination failure. Laminate C exhibited similar local failures in the delaminated [0/45/90]S portion. All the laminates were loaded beyond delamination failure to measure strengths corresponding to total failure.

The static compression test results presented above can, and should, be used to identify the appropriate delamination failure criterion among equations (29), (30) and (31). This would require the computation of \( G_I \) and \( G_{II} \) values for the test laminates for various delaminations sizes \( a \) and load levels. A rigorous analysis or a numerical approximation of the analysis (Reference 5) may be employed for this purpose. Computed \( G_I \) and \( G_{II} \) values corresponding to delamination failure loads should then be incorporated into equations (29), (30), and (31) to identify a suitable delamination failure criterion.

3.10 COMPRESSION FATIGUE TEST RESULTS ON SPECIMENS WITH ITTW DELAMINATIONS

Constant amplitude compression fatigue tests were conducted at \( R = 10 \) and \( \omega = 10 \) Hertz (see Figure 10). The Kapton inclusions (ITTW delaminations) were initially "released" by cycling the specimens at a compressive stress amplitude below the static delamination failure value. Fatigue tests were restarted after this to record the presented data. Cyclic delamination growth was monitored by constantly observing the marked free edges under a microscope. Table 23 presents the delamination growth rate data for laminate A specimens. Presented results are averages of data from three tests, at each \( \sigma_{\text{min}} \) value, on specimens that exhibited "clean" delamination growth. In a few specimens, the delaminated \([0]_3 \) plies exhibited splitting between fibers, and the split regions propagated to the tab region at different rates. Results from these tests are not included in Table 23. It is evident from the presented results that a higher absolute cyclic stress amplitude induces faster delamination growth. Also, the delamination growth rate reduces sharply as the delamination size \( (2a) \) increases.

Delamination growth rate results for laminate A (Table 23) were subsequently analyzed, using a least squares curve fit algorithm, to yield the following results:
### TABLE 23. SUMMARY OF DELAMINATION GROWTH RATES AT TWO STRESS AMPLITUDES IN LAMINATE A SPECIMENS

<table>
<thead>
<tr>
<th>a (mm)</th>
<th>Average da/dN (μm/cycle) **</th>
<th>( \sigma_{\text{min}} = -144.2 \text{ MPa}^+ )</th>
<th>( \sigma_{\text{min}} = -129.8 \text{ MPa}^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.525</td>
<td>14631</td>
<td>8675</td>
<td></td>
</tr>
<tr>
<td>15.875</td>
<td>4428</td>
<td>2472</td>
<td></td>
</tr>
<tr>
<td>22.225</td>
<td>1123</td>
<td>661</td>
<td></td>
</tr>
<tr>
<td>28.575</td>
<td>377</td>
<td>202</td>
<td></td>
</tr>
<tr>
<td>34.925</td>
<td>187</td>
<td>92</td>
<td></td>
</tr>
<tr>
<td>41.275</td>
<td>110</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>47.625</td>
<td>79</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>53.975</td>
<td>18</td>
<td>13</td>
<td></td>
</tr>
</tbody>
</table>

* \([0_4/(0/45/90/-45)_7]_s\) T300/5208 Laminate.

The top three 0° plies were initially delaminated over a 19.1 mm (0.75 in.) length, through the 25.4 mm (1 in.) width. 
2a is the total delamination length.

** Presented results are averages of data from three tests, at each \( \sigma_{\text{min}} \) value, on specimens that exhibited "clean" delamination growth.

+ Average applied stress at which the imbedded delamination propagated to the tab edges during the static tests = -315.9 MPa. The listed stress amplitudes therefore correspond to stress ratios of 0.46 and 0.41, respectively, with respect to the applied static stress value at delamination failure.
Fatigue tests on laminates B and C posed a special problem (see Section 2.7). These specimens suffered delamination failures when they were cycled to "release" the imbedded Kapton film, prior to initiating fatigue tests. Subsequent specimens were tightly clamped over 37.1 from either tab edge (see Figure 11) to preclude delamination failures when Kapton inclusions were "released". While this procedure did not succeed in every case, it contained the imbedded delamination over at least 37.1 mm in most of the specimens. After the Kapton inclusion was "released", fatigue loading at \( R = 10 \) and \( \omega = 10 \) Hertz was introduced to record delamination growth rate data. Tables 24 and 25 present \( da/dN \) data for laminates B and C. Again, a higher absolute cyclic stress amplitude induces faster delamination growth, and the growth rate reduces substantially as the delamination size (2a) increases.

Using a least squares curve fit algorithm, the results in Tables 24 and 25 were analyzed to yield delamination growth rate equations. For laminate B,

\[
da/dN = 23.27 a^{-2.81} \text{ mm/cycle for } \sigma_{\text{min}} = -138 \text{ MPa} \tag{45}
\]

Only the above relationship is presented because insufficient results were obtained for the other \( \sigma_{\text{min}} \) cases. Laminate C results were also too limited in number, and the range of a values, to yield reliable delamination growth rate expressions.

The results presented in equations (43), (44) and (45) for laminates with ITTW delaminations can, and should, be analyzed further to verify the delamination growth rate expression in equation (42). The \( G_I, G_{II} \) values for these laminates, for various delamination lengths and load levels, should be determined using a rigorous analysis or a numerical approximation of the analysis (Reference 5). Computed \( G_I \) and \( G_{II} \) values should then be incorporated into equation (42) to obtain \( da/dN \) values. The expres-
**TABLE 24. SUMMARY OF DELAMINATION GROWTH RATES IN LAMINATE B SPECIMENS**

<table>
<thead>
<tr>
<th>( a ) (mm)</th>
<th>( \frac{da}{dN} ) (( \mu )mm/cycle) at ( \sigma_{\text{min}} = -197 ) MPa</th>
<th>( a ) (mm)</th>
<th>( \frac{da}{dN} ) (( \mu )mm/cycle) at ( \sigma_{\text{min}} = -138 ) MPa</th>
<th>( a ) (mm)</th>
<th>( \frac{da}{dN} ) (( \mu )mm/cycle) at ( \sigma_{\text{min}} = -99 ) MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>41.3</td>
<td>12700</td>
<td>15.2</td>
<td>6960</td>
<td>26.4</td>
<td>762</td>
</tr>
<tr>
<td>44.5</td>
<td>5080</td>
<td>25.0</td>
<td>4641</td>
<td>27.1</td>
<td>381</td>
</tr>
<tr>
<td>47.6</td>
<td>2311</td>
<td>30.2</td>
<td>2159</td>
<td>28.8</td>
<td>216</td>
</tr>
<tr>
<td>50.8</td>
<td>584</td>
<td>32.0</td>
<td>1506</td>
<td>29.9</td>
<td>152</td>
</tr>
<tr>
<td></td>
<td></td>
<td>42.0</td>
<td>931</td>
<td>31.4</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>48.8</td>
<td>203</td>
<td>32.4</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>33.7</td>
<td>13</td>
</tr>
</tbody>
</table>

* Laminate B has a \([0/45_2/0/(0/45/90)-45)\_7\] layup and a 25.4 mm long Kapton inclusion between plies 4 and 5, across the entire (25.4 mm) width.

Average applied stress corresponding to delamination failure in laminate B = -328.4 MPa

\( R = 20, \ \omega = 10 \) Hertz.

\( 2a \) is the total delamination length.
TABLE 25. SUMMARY OF DELAMINATION GROWTH RATES IN LAMINATE C SPECIMENS

<table>
<thead>
<tr>
<th>a (mm)</th>
<th>da/dN ((\mu\text{mm}/\text{cycle})) at (\sigma_{\text{min}}) = -179 MPa</th>
<th>a (mm)</th>
<th>da/dN ((\mu\text{mm}/\text{cycle})) at (\sigma_{\text{min}}) = -130 MPa</th>
<th>a (mm)</th>
<th>da/dN ((\mu\text{mm}/\text{cycle})) at (\sigma_{\text{min}}) = -98 MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>42.9</td>
<td>8026</td>
<td>46.4</td>
<td>1270</td>
<td>42.7</td>
<td>4877</td>
</tr>
<tr>
<td>46.0</td>
<td>3886</td>
<td>47.6</td>
<td>846</td>
<td>46.0</td>
<td>218</td>
</tr>
<tr>
<td>49.2</td>
<td>2362</td>
<td>48.9</td>
<td>254</td>
<td>49.2</td>
<td>79</td>
</tr>
<tr>
<td>52.4</td>
<td>838</td>
<td>49.5</td>
<td>32</td>
<td>52.4</td>
<td>48</td>
</tr>
</tbody>
</table>

* Laminate C has a \([0/45/90_2, 45/0_3, (0/45/90/-45)_6\)\(_{s}\) layup and a 31.8 mm long Kapton inclusion between plies 6 and 7, across the entire (25.4 mm) width.

Average applied stress corresponding to delamination failure in laminate C = -325.1 MPa.

\(R = 20, \omega = 10\) Hertz.

2\(a\) is the total delamination length.
sion in equation (42) is based on test results obtained at R = 0.05 and \( \omega = 10 \) Hertz. If the predictions based on equation (42) agree well with the results expressed by equations (43) to (45), the difference in the R values can be established to have a minimal effect on delamination growth rate. Also, equation (42) can be established to be a reliable delamination growth rate equation for \( \omega = 10 \) Hertz, when \( G_{III} \) effects are negligible.

3.11 SUMMARY OF RESULTS

The results discussed in the preceding sections are summarized below:

(1) \( G_{IC} = 103 \) J/m\(^2\) for the T300/5208 graphite/epoxy material system. \( G_{IC} \) was computed using static DCB test data and compares well with the result in Reference 14 (see Table 26). \( G_{IC} \) computations based on width-tapered DCB test results (Reference 3) yielded a \( G_{IC} \) value (205 J/m\(^2\)) that is twice the value obtained in this program and in Reference 14. A comparison of \( G_{IC} \) for T300/5208 with \( G_{IC} \) for other material systems is presented in Table 26.

(2) The contribution of mode I alone to delamination growth rate in T300/5208 laminates may be expressed by equation (16):

\[
da/dN = C_1 \left( G_{I_{max}} / G_{IC} \right)^8 \text{ for } R = 0.05, \omega = 10 \text{ Hertz.}
\]

where \( C_1 = 0.0283 \) mm/cycle, \( G_{IC} = 103 \) J/m\(^2\), and \( da/dN \) is expressed in mm/cycle:

(3) In displacement-controlled, constant amplitude fatigue tests on DCB specimens (see equations 12 and 16):

\[
G_{I_{max}} \propto a^{-4}, \text{ and}
\]

\[
da/dN \propto \left( G_{I_{max}} / G_{IC} \right)^8
\]

Therefore, \( da/dN \propto a^{-32} \)

Hence the drastic reduction in the delamination growth rate with an increase in \( a \), in displacement-controlled fatigue tests on DCB specimens.
## TABLE 26. COMPARISON OF AVAILABLE $G_{IC}$, $G_{IIC}$ DATA WITH GENERATED RESULTS

<table>
<thead>
<tr>
<th>Material</th>
<th>$G_{IC}$ J/m$^2$</th>
<th>$G_{IIC}$ J/m$^2$</th>
<th>Reference, Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>T300/5208 (Narmco)</td>
<td>103</td>
<td>279-587</td>
<td>This report</td>
</tr>
<tr>
<td>T300/5208 (Narmco)</td>
<td>88</td>
<td>154</td>
<td>Ref. 14</td>
</tr>
<tr>
<td>T300/934 (Fiberite) and</td>
<td>205</td>
<td>--</td>
<td>Ref. 3—Width-tapered DCB tests</td>
</tr>
<tr>
<td>T300/5208 (Narmco)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T300/P1700 (U.S. Polymeric)</td>
<td>380</td>
<td>--</td>
<td>Ref. 3—Width-tapered DCB tests</td>
</tr>
<tr>
<td>T300/8907 (American Cyanamid)</td>
<td>306*</td>
<td>--</td>
<td>+ based on crack initiation load (Ref. 3)</td>
</tr>
<tr>
<td></td>
<td>937**</td>
<td></td>
<td>++ based on maximum load (Ref. 3)</td>
</tr>
<tr>
<td>AS/3501-6 (Hercules)</td>
<td>131</td>
<td>322</td>
<td>Ref. 17</td>
</tr>
<tr>
<td>AS/3501 (Hercules)</td>
<td>--</td>
<td>825</td>
<td>Ref. 8—Delamination at the midplane of a beam under 3-point bending</td>
</tr>
<tr>
<td>AS4/2220-3 (Hercules)</td>
<td>245</td>
<td>--</td>
<td>Ongoing Northrop IRAD Project.</td>
</tr>
<tr>
<td>AS/3502 (Hercules)</td>
<td>133</td>
<td>--</td>
<td>Ref. 16</td>
</tr>
<tr>
<td>T300/V378-A (U.S. Polymeric)</td>
<td>79*</td>
<td>193*</td>
<td>+Ref. 16;</td>
</tr>
<tr>
<td></td>
<td>149*</td>
<td></td>
<td>*Ref. 17</td>
</tr>
<tr>
<td>AS1/Polysulfone</td>
<td>655</td>
<td>--</td>
<td>Ref. 16</td>
</tr>
</tbody>
</table>
(4) Results from displacement-controlled, constant amplitude fatigue tests on DCB specimens indicate that, if \( \frac{G_{I}}{G_{IC_{max}}} \) is less than 0.5, \( \frac{da}{dN} \) is reduced to below 0.25 \( \mu \text{mm/cycle} \), approaching a "no growth" state. A \( G_{I} \) value of 0.5 \( G_{IC_{max}} \) may therefore be referred to as the "threshold" \( G_{I_{max}} \) value below which no significant mode I delamination growth will be observed.

(5) A reliable static failure criterion, accounting for the presence of \( G_{I} \) and \( G_{II} \), does not exist. Therefore, static CLS test data were analyzed by incorporating \( G_{IC} \) and nonlinear finite element results into three assumed failure criteria (equations 29 to 31). \( G_{IIC} \) for the T300/5208 material system was computed to be between 279 and 587 \( J/m^2 \), based on the assumed failure criteria. A validated failure criterion is mandatory for the computation of a reliable \( G_{IIC} \), using the generated static CLS test results. Only pure shear tests (like a torsion tube test) will provide reliable \( G_{IIC} \) values. A comparison of \( G_{IIC} \) for T300/5208 with \( G_{IIC} \) for other material systems is presented in Table 26.

(6) The contribution of mode II alone to delamination growth rate in T300/5208 laminates may be expressed by (see equation 42):
\[
\frac{da}{dN} = C_2 \left( \frac{G_{II_{max}}}{G_{IIC}} \right)^6 \quad \text{for } R = 0.05, \omega = 10 \text{ Hertz}
\]
where \( C_2 = 0.1511 \mu \text{mm/cycle} \) if \( G_{IIC} = 415 \ J/m^2 \), and \( \frac{da}{dN} \) is expressed in \( \mu \text{mm/cycle} \).

(7) In displacement-controlled, constant amplitude fatigue tests on CLS specimens (see equations 33 and 42),
\[
G_{max} = a^{-2},\quad \text{and}
\]
Therefore, $\frac{da}{dN} - 12$

Hence the drastic reduction in the delamination growth rate, with an increase in $a$, in displacement-controlled fatigue tests on CLS specimens. A comparison with the expression in item (3) indicates that the delamination growth rate reduces faster in displacement-controlled DCB specimens.

(8) Results from displacement-controlled, constant amplitude fatigue tests on $[(0_2/\pm 45)_s]$ specimens indicate that a "no growth" situation (growth rates below 0.25 mm/cycle) will be realized only when $G_{\text{max}}/G_C$ is below 0.2. Since delamination growth in these specimens is predominantly under mode II, the "threshold $G_{\text{II}_{\text{max}}}$" value is approximately $0.2 G_{\text{IIC}}$.

(9) Delamination growth rate in T300/5208 laminates with negligible $G_{\text{III}}$ effects, subjected to constant amplitude fatigue loading at $R = 0.05$ and $\omega = 10$ Hertz, may be quantified by (equation 42):

$$\frac{da}{dN} = C_1 \left( \frac{G_{\text{I}_{\text{max}}}}{G_{\text{IC}}} \right)^6 + C_2 \left( \frac{G_{\text{II}_{\text{max}}}}{G_{\text{IIC}}} \right)^6$$

where $C_1$, $C_2$, $G_{\text{IC}}$ and $G_{\text{IIC}}$ are defined in items (2) and (6). $da/dN$, $C_1$ and $C_2$ possess the same units.

(10) Specimens with ITTW delaminations generated static and fatigue test data that can be used to arrive at a reliable static delamination failure criterion, and to validate the delamination growth rate equation in item (9). This requires the development of a reliable analysis to predict $G_I$ and $G_{II}$ as a function of delamination size and load levels in these specimens.
(11) Assuming that mode III effects are similar to mode II effects, the delamination growth rate expression in equation (42) may be extended as follows for a general situation:

\[
\frac{da}{dN} = c_1 \left( \frac{G_{I_{\text{max}}}}{G_{IC}} \right)^8 + c_2 \left( \frac{G_{II_{\text{max}}}}{G_{IIIC}} \right)^6 + c_3 \left( \frac{G_{III_{\text{max}}}}{G_{IIIIC}} \right)^6
\]  

(46)

This expression must be verified through tests (at $R = 0.05$ and $\omega = 10$ Hertz) on, and analysis of, specimens with imbedded delaminations that grow in a two-dimensional manner along the imbedded interface.

(12) Quantification of the effect of a realistic delamination on the strength and lifetime of a laminate subjected to constant amplitude fatigue loading requires: (a) measurement of $G_{IC}$, $G_{IIIC}$ and $G_{IIIIC}$ for the material; (b) establishment of a valid static delamination failure criterion; (c) measurement of "threshold" values for $G_{I_{\text{max}}}$, etc; (d) establishment of a valid delamination growth rate equation; and (e) analytical (approximate) expressions for $G_I$, etc. as function of delamination size, delamination location and applied loads. If $G_{I_{\text{max}}}$, $G_{II_{\text{max}}}$, etc. corresponding to the absolute maximum cyclic load satisfy the static failure criterion, an abrupt delamination failure (not necessarily total failure) will occur. If this is not the case, $G_{I_{\text{max}}}$, etc. must be compared with the corresponding "threshold" values to determine if there will be any significant (>0.25 $\mu$m/mm/cycle) delamination growth. If it is determined that there will be a stable cyclic growth of the imbedded delamination, the growth rate can be computed using the validated delamination growth rate equation. Over $\Delta N$ cycles at this growth
rate, the delamination size increases by $\Delta a$. $G_I$ and $G_{II_{\text{max}}}$ corresponding to the enlarged $a$ value are incorporated into the static failure criterion to determine if the stable delamination growth is terminated by an abrupt delamination. This procedure is repeated for $\Delta a$ increments until the maximum permissible delamination growth is realized, or until an abrupt delamination failure occurs. The residual strength and lifetime of a delaminated specimen may thus be computed.
SECTION 4
CONCLUSIONS AND RECOMMENDATIONS

4.1 CONCLUSIONS

An experimental program was conducted to investigate the roles played by $G_I$ and $G_{II}$ in inducing delamination growth under static and fatigue loading. Double cantilever beam (DCB) specimens were used for pure mode I tests, and cracked lap shear (CLS) specimens were used for mixed mode tests. In addition, static compression and constant amplitude compression fatigue tests were also conducted on specimens with imbedded through-the-width (ITTW) delaminations. All the specimens were fabricated using T300/5208 graphite/epoxy.

$G_{IC}$ for T300/5208, computed using static DCB test results, agrees well with the value in Reference 14. The exponent in the $da/dN$ versus $G_I$ relationship, obtained using a least squares curve fit analysis of the DCB fatigue test results, is different from the value reported in Reference 14.

The computation of $G_{IIC}$ using static CLS test results requires a validated failure criterion that does not exist. Presented $G_{IIC}$ values are based on failure criteria that were selected without validation. A better estimation of $G_{IIC}$ can be made with the reported results if a validated failure criterion were to be available in the future. The $G_{IIC}$ value in Reference 14 is lower than the values computed in this report. But, in Reference 14, a linear three-dimensional finite element analysis of the CLS specimen was performed, and $G_{III}$ effects were accounted for in an invalidated failure criterion.

A least squares curve fit analysis of the CLS fatigue test data, using NFEA and DCB results, provided a delamination growth rate ($da/dN$) equation that accounts for the effects of $G_I$ and $G_{II}$ on $da/dN$. The growth rate equation is based on test results generated at $R=0.05$ and $\omega = 10$ Hertz. This equation should be validated through additional tests on other mixed mode test specimens at $R=0.05$ and $\omega = 10$ Hertz.
Static compression test results from specimens with ITTW delaminations should be analyzed to establish a valid static delamination failure criterion. This will require the development of a reliable analysis of the delaminated specimen to relate $G_I$ and $G_{II}$ to applied load levels, for various delamination sizes and locations. Constant amplitude compression fatigue test results from these specimens should also be analyzed in a similar manner to verify the applicability and adequacy of the delamination growth rate equation obtained using CLS test results.

4.2 RECOMMENDATIONS

1. Measured compliances were larger (up to 20%) than analytical predictions, and $dC/da$ and $G_c$ computations based on test results were even larger (up to 39%) than the corresponding analytical predictions, in mixed mode CLS specimens. To investigate the causes for these differences, additional testing and analysis of CLS specimens are recommended.

2. A valid delamination failure criterion must be established through additional tests on specimens with different $G_I/G_{II}$ ratios. This will provide a means for reliable $G_{IIc}$ computation. A pure shear situation is desirable, but is difficult to create in a test coupon.

3. A reliable analysis of specimens with ITTW delaminations should be developed to relate $G_I$ and $G_{II}$ to applied load levels, for various delamination sizes and locations.

4. The applicability of the developed delamination growth rate expression (equation 42) should be verified through additional tests at $R = 0.05$ and $\omega = 10$ Hertz, on specimens with ITTW delaminations (one-dimensional growth).

5. The proposed extension of the delamination growth rate expression to include $G_{III}$ effects (equation 46) should be verified through tests on, and analysis of, coupons and structural elements (like a stiffened panel) with imbedded delaminations. The coupons and elements should be subjected
to compressive loading at $R = 20$ and $\omega = 10$ Hertz to cause instability-induced delamination growth in a general (two-dimensional) manner along the imbeddment surface.

(6) In the presented delamination growth rate expressions, only the absolute maximum cyclic $G_I$ and $G_{II}$ values have been incorporated. The minimum-to-maximum cyclic load ratio ($R$) and frequency ($\omega$) will likely influence delamination growth significantly. A systematic test program, addressing the effects of $R$ and $\omega$ on imbedded delamination growth, is highly recommended.
SECTION 5
REFERENCES


16. Abstract

An experimental program was conducted to quantify instability-induced imbedded delamination growth. Static tests on double cantilever beam (DCB) specimens yielded the critical mode I strain energy release rate \( G_{IC} \) for T300/5208 graphite/epoxy. Static tests on mixed mode cracked lap shear (CLS) specimens, and a nonlinear finite element analysis (NFEA) of the CLS specimen to separate mode I and mode II effects, yielded the critical mode II strain energy release rate \( G_{IIC} \) for T300/5208. Constant amplitude fatigue tests on DCB and CLS specimens, along with the NFEA results on CLS specimens, quantified mode I and mode II contributions to delamination growth rate. Fatigue tests were conducted at a frequency (\( \omega \)) of 10 Hertz, maintaining the minimum to maximum cyclic load ratio (R) at 0.05.

Static compression and constant amplitude compression fatigue tests were also conducted on specimens with imbedded through-the-width (ITTW) delaminations. Kapton imbeddments were located below 3, 4 or 6 plies in a 64-ply laminate, during layup, to simulate ITTW delaminations. The onset of instability (local buckling), the transverse displacement of the delaminated region, and the load level corresponding to the propagation of the ITTW delamination to the tab boundary (delamination failure) were monitored during the static tests. Under cyclic compressive loading \((R=10, \omega = 10Hz)\), delamination growth rates were measured at various maximum cyclic compressive load values.
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