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Elastohydrodynamics of Elliptical Contacts for Materials of Low Elastic Modulus

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ABS: The influence of the ellipticity parameter k and the dimensionless speed U, load W, and materials G parameters on minimum film thickness for materials of low elastic modulus was investigated. The ellipticity parameter was varied from 1 (a ball-on-plane configuration) to 12 (a configuration approaching a line contact); U and W were each varied by one order of magnitude. Seventeen cases were used to generate the minimum- and central-film-thickness relations. The influence of lubricant starvation on minimum film thickness in starved elliptical, elastohydrodynamic configurations was also investigated for materials of low elastic modulus. Lubricant starvation was studied simply by moving the inlet boundary

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closer to the center of the conjunction in the numerical solutions. Contour plots of pressure and film thickness in and around the contact were presented for both fully flooded and starved lubrication conditions. It is evident from these figures that the inlet pressure contours become less circular and closer to the edge of the Hertzian contact zone and that the film thickness decreases substantially as the serverity of starvation increases. The results presented reveal the essential features of both fully flooded and starved, elliptical, elastohydrodynamic conjunctions for materials of low elastic modulus.

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ELASTOHYDRODYNAMICS OF ELLIPTICAL CONTACTS FOR MATERIALS OF LOW ELASTIC MODULUS*

The work presented in the previous chapters related to materials of high elastic modulus, like metals. In this chapter the analysis is extended to materials of low elastic modulus, like rubber. For these materials the elastic distortions are large, even with light loads. Another feature of the elastohydrodynamics of low-elastic-modulus materials is the negligible effect of the relatively low pressures on the viscosity of the lubricating fluid. Engineering applications in which elastohydrodynamic lubrication is important for low-elastic-modulus materials include seals, human joints, tires, and a number of lubricated elastomeric-machine elements.

The problem of line contacts, where side leakage of the fluid can be ignored, has been solved theoretically for low-elastic-modulus materials by Herrebrugh (1968), Dowson and Swales (1969), and Baglin and Archard (1972). The solutions presented in the first two references were obtained numerically and are based on simultaneous solutions of the hydrodynamic and elasticity equations. The approximate analytical solution of Baglin and Archard (1972) relied on the assumption of a simplified form for the film shape in the central region to solve the point-contact (ball on plane) situation. The work presented in

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this chapter makes extensive use of the approach adopted by Hamrock and Dowson (1978) (1979a) and represents, to the best of the authors' knowledge, the first attempt at a complete numerical solution of the problem of isothermal elastohydrodynamic lubrication of elliptical contacts for low-elastic-modulus materials. No initial assumptions are made as to the pressure or film thickness within the contact, and lubricant compressibility and pressure-viscosity effects are considered.

The basic elastohydrodynamic theory presented in Chapter 7 has been used in which the conjunction was divided into a large number of equal rectangular areas and the elastic deformation at any point in the field was determined on the assumption that the pressure over each of the rectangles was constant. It was assumed that the solids could be considered to be semi-infinite bodies and hence that the normal restrictions imposed on a Hertzian analysis applied. The elastic deformation in the conjunction was represented by a double summation series of the pressure multiplied by an influence coefficient.

The elastohydrodynamic analysis of low-elastic-modulus contacts presented in this chapter is intended to extend the studies outlined in the previous chapters, but the normal safeguard of restricting the application to situations in which the conjunction dimensions are small compared with the principal radii of the solids should normally be observed. In addition, an alternative approach to the elasticity calculation would be necessary for situations in which thin layers of low-elastic-modulus material were supported on a rigid backing of considerable extent, as represented by compliant surface bearings.

11.1 Theoretical Formulation

The basic theory developed in Chapter 7 is used here with some minor modifications. It was discovered that numerical convergence was considerably better if the dimensionless pressure was written as

$$P = P_{Hz} + P_D \tag{11.1}$$

where

P_{Hz} = dimensionless Hertzian pressure

P_D = dimensionless pressure difference, representing the difference between the hydrodynamic and Hertzian pressures at a given location

By making use of equations (7.2) and (11.1) the Reynolds equation can be written as

$$\frac{\partial}{\partial X}\left(\frac{\bar{\rho}H^3}{\bar{\eta}}\frac{\partial P_D}{\partial X}\right) + \frac{1}{k^2}\frac{\partial}{\partial Y}\left(\frac{\bar{\rho}H^3}{\bar{\eta}}\frac{\partial P_D}{\partial Y}\right) + \frac{\partial}{\partial X}\left(\frac{\bar{\rho}H^3}{\bar{\eta}}\frac{\partial P_{Hz}}{\partial X}\right)$$

$$+ \frac{1}{k^2}\frac{\partial}{\partial Y}\left(\frac{\bar{\rho}H^3}{\bar{\eta}}\frac{\partial P_{Hz}}{\partial Y}\right) = 12U\left(\frac{b}{R_x}\right)\frac{\partial(\bar{\rho}H)}{\partial X} \tag{11.2}$$

Since the Hertzian pressure is known and can be obtained from equations (7.9) and (7.13), the solution of the Reynolds equation is concerned with the pressure difference P_D. In equation (11.2) compressibility and viscous terms are retained as variables even though it was mentioned earlier that the effect of the relatively low pressures on viscosity and density is normally negligible.

The equation for the dimensionless film thickness can be written as

$$H = H_0 + \frac{S + \delta_{Hz} + \delta_{P_D}}{R_x} \tag{11.3}$$

where

H_0 = constant (initially guessed)

S = geometric separation of rigid elliptical solids

δ_{Hz} = elastic deformation due to Hertzian pressure distribution

δ_{P_D} = elastic deformation due to pressure difference

The elastic deformation is evaluated in exactly the same way as outlined in Section 5.7.

The nodal structure used in the numerical analysis that yielded the results presented in this chapter is shown in Figure 11.1. This structure is different from that adopted for materials of high elastic modulus shown in Figure 7.3. Because of the dimensionless representation of the coordinates in Figure 11.1, the actual Hertzian contact ellipse becomes a circle regardless of the value of the ellipticity parameter k. The nodal structure shown in Figure 11.1 was arrived at after much exploration in which the number of nodes in the semimajor and semiminor axes, as well as the distance from the center of the contact to the edges of the computing zone, was varied until the optimum arrangement was achieved.

Apart from these points the theoretical formulation of the elastohydrodynamic lubrication problem for materials of low elastic modulus is the same as that presented in Chapter 7, provided that the appropriate modulus of elasticity is selected.

11.2 Minimum- and Central-Film-Thickness Formulas

The dimensionless grouping found to be useful in defining the elastohydrodynamic problem for high-elastic-modulus materials is also valid for materials of low elastic modulus. The dimensionless film thickness can therefore be written as

$$H = f(k,U,W,G)$$

By varying each of these parameters separately and keeping the remaining parameters constant, as outlined in Chapter 8, the numerical results shown in Table 11.1 were obtained.

The minimum film thickness shown in this table has been determined for 17 sets of input data from coupled solutions of the Reynolds and elasticity equations. From the information presented in Table 11.1 a least-squares-fit, minimum-film-thickness formula for a fully flooded, isothermal, elastohydrodynamic elliptical contact for low-elastic-modulus materials can be written as

$$\tilde{H}_{min} = 7.43(1 - 0.85\, e^{-0.31k})U^{0.65}W^{-0.21} \qquad (11.4)$$

Table 11.1 gives the values for minimum film thickness obtained from the least-squares fit as defined by equation (11.4). The percentage difference between the minimum film thickness obtained from the elastohydrodynamic elliptical-contact theory H_{min} and the minimum film thickness obtained from the least-squares fit equation $\tilde{H}_{min}$ is expressed as E_1 in equation (8.13). In Table 11.1 the values of E_1 are within the range -8 to +3 percent.

It is interesting to compare the dimensionless minimum-film-thickness equation for materials of low elastic modulus equation (11.4) with the corresponding equation generated in Chapter 8 for materials of high elastic modulus

$$\tilde{H}_{min} = 3.63(1 - e^{-0.68k})U^{0.68}W^{-0.073}G^{0.49} \qquad (8.23)$$

The powers of U in equations (11.4) and (8.23) are quite similar, but the power of W is much more significant for low-elastic-modulus materials. The expression showing the effect of the ellipticity parameter is of exponential form in both equations, but with quite different constants.

A major difference between equations (11.4) and (8.23) is the absence of a materials parameter G in the expression for low-elastic-modulus materials. There are two reasons for this. One is the negligible effect of the relatively low pressures on the viscosity of the lubricating fluid, and the other is the way in which the role of elasticity is simply and automatically incorporated into the prediction of conjunction behavior through an increase in the size of the Hertzian contact zone corresponding to changes in load. As a check on the validity of this, case 9 of Table 11.1 was repeated with the material properties changed from those of nitride to those of silicone rubber. The results of this change are recorded as case 17 in Table 11.1.

The dimensionless minimum film thickness calculated from the full numerical solution to the elastohydrodynamic contact theory was 181.8×10^{-6}, and the dimensionless minimum film thickness predicted from equation (11.4) turned out to be 182.5×10^{-6}. This clearly indicates a lack of dependence of the minimum film thickness for low-elastic-modulus materials on the materials parameter.

There is interest in knowing the central film thickness, in addition to the minimum film thickness, in elastohydrodynamic contacts. The procedure used to obtain an expression for the central film thickness was the same as that used to obtain the minimum-film-thickness expression. The central-film-thickness formula for low-elastic-modulus materials as obtained from Hamrock and Dowson (1978) is

$$\tilde{H}_c = 7.3(1 - 0.72\, e^{-0.28k})U^{0.64}W^{-0.22} \qquad (11.5)$$

A comparison of the central- and minimum-film-thickness equations, equations (11.5) and (11.4), reveals only slight differences. The ratio of minimum to central film thickness evident in the computed values given in Hamrock and Dowson (1978) ranged from 70 to 83 percent, the average being 77 percent.

11.3 Comparison of Different Investigators' Results

To evaluate the dimensionless minimum-film-thickness equation (11.4) developed by Hamrock and Dowson (1978), a comparison was made between its predictions, those of the numerical solution obtained by Biswas and Snidle (1976), and the recent experimental findings of Jamison, et al. (1978). The Biswas and Snidle (1976) and the Jamison, et al. (1978) results are applicable only for $k = 1$.

The Biswas and Snidle (1976) solution for the dimensionless film thickness can be written as

$$(\overline{H}_{min})_B = 1.96\ M_p^{-0.11} \tag{11.6}$$

where

$$\overline{H}_{min} = H_{min} U^{-0.5} \tag{11.7}$$

$$M_p = WU^{-0.75} \tag{11.8}$$

The dimensionless groups given in equations (11.7) and (11.8) were first used by Moes and Bosma (1972).

The central-film-thickness equation obtained from the experimental results of Jamison, et al. (1978) can be written as

$$(\overline{H}_c)_J = 2.4\ M_p^{-0.075} \tag{11.9}$$

where

$$\overline{H}_c = H_c U^{-0.5} \tag{11.10}$$

Modifying equation (11.9) to provide a minimum-film-thickness equation by using the assumption that $(H_{min})_J = 0.78\ (H_c)_J$ yields

$$(\overline{H}_{min})_J = 1.87\ M_p^{-0.075} \tag{11.11}$$

Recall again that equations (11.6) and (11.11) are for $k = 1$ only.

From equations (11.7) and (11.8) and Table 11.1, we can write equation (11.4) as

$$(\overline{H}_{min})_H = 8.53(1 - 0.85\ e^{-0.31k})M_p^{-0.21} \tag{11.12}$$

Therefore, for $k = 1$, equation (11.12) reduces to

$$\left(\overline{H}_{min}\big|_{k=1}\right)_H = 3.21\ M_p^{-0.21} \tag{11.13}$$

Note that the negative exponent in equation (11.13) is numerically larger than the exponents in equations (11.6) and (11.11).

A comparison of the different investigators' results from equations (11.6), (11.11), and (11.13) is shown in Figure 11.2. The three equations seem to agree quite well with each other numerically over the range considered, but there is quite a discrepancy in the slopes of the lines. The Hamrock and Dowson (1978) prediction of film thickness is equivalent to the Biswas and Snidle (1976) theoretical value at $M_p = 100$ and to the Jamison, et al. (1978) experimental results at $M_p = 45$. Therefore, even though the exponent on M_p for the Hamrock and Dowson (1978) results is larger than those obtained by Biswas and Snidle (1976) and by Jamison, et al. (1978), the agreement is quite good for $k = 1$.

Also shown in Figure 11.2 is the rigid isoviscous solution obtained from Kapitza (1955). The enormous difference in slopes between the Kapitza (rigid) line and the three sets of results presented by equations (11.6), (11.11), and (11.13) demonstrates the tremendous potential of elastohydrodynamic action for the generation and preservation of satisfactory lubricating films under severe operating conditions. The discrepancies between the predictions of the three relationships represented by equations (11.6), (11.11), and (11.13) are seen in better perspective if they are compared with the predictions of the Kapitza (rigid) theory at values of M_p of 100 and 1000.

The variation of the ratio $H_{min}/H_{min,r}$ is shown in Figure 11.3, where $H_{min,r}$ is the minimum film thickness for rectangular contacts, with the ellipticity parameter k for both high- and low-elastic-modulus materials. If it is assumed that the minimium film thickness obtained from the elastohydrodynamic analysis of elliptical contacts can only be obtained to an accuracy of 3 percent, we find that the ratio $H_{min}/H_{min,r}$ approaches the limiting value of unity at $k = 5$ for high-elastic-modulus materials. For low-elastic-modulus materials the ratio approaches the limiting value of unity more slowly, but it is reasonable to state that the rectangular-contact solution will give a very good prediction of the minimum film thickness for conjunctions in which k exceeds about 11.

11.4 Contour Plots of Results

Contour plots of the dimensionless pressure are shown in Figure 11.4 for two extreme values of the dimensionless speed parameter U of 0.05139×10^{-7} and 0.5139×10^{-7}. As mentioned in Chapters 8 and 9, the + symbol indicates the center of the Hertzian contact in each case. Because of the dimensionless representation of the X and Y coordinates the actual Hertzian contact ellipse becomes a circle regardless of the value of k. The Hertzian contact circle is shown in each figure by asterisks. On each figure the contour labels and each corresponding value of the dimensionless pressure are given. The inlet region is to the left and the exit region to the right in each figure.

The pressure contours shown in Figure 11.4 are nearly circular or Hertzian. In Figure 11.4(b), the high-speed case, the pressure at any point in the inlet is higher than in the low-speed case shown in Figure 11.4(a). Inside the Hertzian contact region the contour values of the dimensionless pressure for the low-speed case are higher than those for the high-speed case. The pressure spikes found when dealing with materials of high elastic modulus (Chapter 8) are not evident in these solutions for low-elastic-modulus materials. The absence of a pressure spike for low-elastic-modulus materials has been noted before for nominal line contacts and is due to the pressures generated for a given load in a contact between low-elastic-modulus materials being considerably lower than those generated in a contact between high-elastic-modulus materials.

Contour diagrams of the dimensionless film thickness are shown in Figure 11.5 for these same values of $U = 0.05139 \times 10^{-7}$ and 0.5139×10^{-7}. Figure 11.5(a) shows three regions of minimum film thickness: one close to the rear edge of the Hertzian ellipse, and two off to the side. At the higher speed ($U = 0.5139 \times 10^{-7}$) the minimum-film-thickness region lies on the midplane of the contact in the direction of rolling, between the center of the contact and the trailing edge of the Hertzian ellipse.

The variation of pressure and film thickness in the direction of rolling along a line near the midplane of the conjunction is shown in Figure 11.6 for the same two values of U considered in Figures 11.4 and 11.5. For all the solutions obtained at various speeds, the values of the dimensionless load, materials, and ellipticity parameters were held fixed. Figure 11.6 shows that the pressure at any point in the inlet region increases as the speed increases. The dominant effect of the dimensionless speed parameter on the minimum film thickness in elastohydrodynamic contacts for low-elastic-modulus materials evident in equation (11.4) is reflected in Figure 11.6. Similar results were found for high-elastic-modulus materials, as noted in Chapter 8.

Contour plots of dimensionless pressure for the two extreme values of the dimensionless load parameter W that were investigated, 0.2202×10^{-3} and 2.202×10^{-3}, are shown in Figure 11.7. Again the pressure contours are nearly circular, or Hertzian.

Contour plots of dimensionless film thickness for the same two values of W are shown in Figure 11.8. In Figure 11.8(a), for the low-load case ($W = 0.2203 \times 10^{-3}$), the minimum film thickness occurs directly behind the center of the contact. Likewise in Figure 11.8(b), for the high-load case ($W = 2.202 \times 10^{-3}$), the minimum film thickness also occurs directly behind the center of the contact but closer to the Hertzian circle. The two contours marked C in Figure 11.8(b) indicate a slight increase in film thickness before the minimum-film-thickness region is reached.

The variation of pressure and film thickness in the rolling direction along a line close to the midplane of the conjunction is shown in Figure 11.9 for the same two values of the dimensionless load parameter. Once again the essential features of the minimum-film-thickness equation are reflected in this figure since a change in the dimensionless load parameter of one order of magnitude produces a considerable change in the pressure profile but not such a significant change in the film thickness. The small effect of load on minimum film thickness is a reasonable and most important feature of elastohydrodynamic lubrication.

11.5 Effect of Lubricant Starvation

By using the theory and numerical procedures outlined earlier in this chapter, we can investigate the influence of lubricant starvation on minimum film thickness in elliptical elastohydrodynamic conjunctions formed by low-elastic-modulus materials. Lubricant starvation is studied by simply moving the inlet boundary closer to the center of the conjunction, as described in Chapter 9.

Table 11.2 shows how the dimensionless inlet distance affects the dimensionless film thickness for three groups of dimensionless load and speed parameters. For all the results presented in this section the dimensionless materials parameter G was fixed at 0.4276, and the ellipticity parameter k was fixed at 6. The results shown in Table 11.2 clearly indicate the adverse effect of lubricant starvation in the sense that, as the dimensionless inlet distance $\tilde{m}$ decreases, the dimensionless minimum film thickness H_{min} also decreases.

Table 11.3 shows how the three groups of dimensionless speed and load parameters affect the limiting location of the dimensionless critical inlet boundary distance m^* at which starvation starts to influence film thickness. Also given in this table are corresponding values of the dimensionless minimum film thickness for the fully flooded condition, as obtained by interpolating the numerical values. By making use of Table 11.2 and following the procedure outlined in Chapter 9, we can write the critical dimensionless inlet boundary distance at which starvation becomes important for low-elastic-modulus materials as

$$m^* = 1 + 1.07\left[\left(\frac{R_x}{b}\right)^2 \tilde{H}_{min}\right]^{0.16} \qquad (11.14)$$

where $\tilde{H}_{min}$ is obtained from equation (11.4).

Table 11.4 shows how m^* affects the ratio of minimum film thickness in the starved and fully flooded conditions $\tilde{H}_{min,s}/H_{min}$. The dimensionless minimum film thickness for a starved condition for low-elastic-modulus materials can thus be written as

$$\tilde{H}_{min,s} = H_{min}\left(\frac{\tilde{m} - 1}{m^* - 1}\right)^{0.22} \tag{11.15}$$

Therefore, whenever $\tilde{m} < m^*$, where m^* is defined by equation (11.14), a lubricant starvation condition exists. When this is true, the dimensionless minimum film thickness is expressed by equation (11.15). If $\tilde{m} \geq m^*$, a fully flooded condition exists and equation (11.4) can be used to predict the minimum film thickness.

In Figure 11.10 contour plots of the dimensionless pressure ($P = p/E'$) are shown for the group 3 conditions recorded in Table 11.2 and for dimensiosnless inlet distances of 1.967, 1.333, and 1.033. Note that the contour levels and intervals are identical in all parts of Figure 11.10. In Figure 11.10(a), with $\tilde{m} = 1.967$, an essentially fully flooded condition exists. The contours are almost circular and extend further into the inlet region than into the exit region. In Figure 11.10(b), with $\tilde{m} = 1.333$, starvation is influencing the distribution of pressure and the inlet contours are slightly less circular than those shown in Figure 11.10(a). By the time $\tilde{m}$ falls to 1.033 (Figure 11.10(c)) the conjunction is quite severely starved and the inlet contours are even less circular.

In Figure 11.11 contour plots of the dimensionless film thickness ($H = h/R_x$) are shown, also for the group 3 conditions recorded in Table 11.2 and for dimensionless inlet distances of 1.967, 1.333, and 1.033. These film thickness contours correspond to the pressure results shown in Figure 11.10. The central portion of the film thickness contours becomes more parallel as starvation increases and the minimum-film-thickness area moves to the exit region. The values of the film thickness contours for the most starved condition (Figure 11.11(c)) are much lower than those for the fully flooded condition (Figure 11.11(a)).

Figure 11.12 more clearly describes these film thickness results. It shows the variation of the dimensionless film thickness in the rolling direction for four values of the dimensionless inlet distance. The value of Y was held fixed near the axial center of the contact, and the dimensionless parameters U and W were held constant as shown in group 3 of Table 11.2 for these calculations. This figure clearly shows that the central region becomes flatter as starvation becomes more severe. Also, in going from a fully flooded condition to a starved condition the film thickness decreases substantially and the location of the minimum film thickness moves closer to the exit region.

The variation of the dimensionless film thickness perpendicular to the rolling direction is shown in Figure 11.13 for four values of the dimensionless inlet distance. The value of

X was held constant near the axial center of the contact, and the dimensionless parameters U and W were held constant at the values recorded for the conditions corresponding to group 3 of Table 11.2. The results shown in this figure clearly indicate that the central region of the conjunction is quite flat in the fully flooded situation but that the curvature of the profile increases as the severity of starvation increases.

11.6 Closure

By modifying the procedures outlined in Chapter 7 we have investigated the influence of the ellipticity parameter k and the dimensionless speed U, load W, and materials G parameters on minimum film thickness for materials of low elastic modulus. The ellipticity parameter was varied from 1 (a ball-on-plane configuration) to 12 (a configuration approaching a line contact). The dimensionless speed and load parameters were each varied by one order of magnitude. Seventeen cases were used to generate the minimum- and central-film-thickness relations:

$$\tilde{H}_{min} = 7.43(1 - 0.85\ e^{-0.31k})U^{0.65}W^{-0.21}$$

$$\tilde{H}_{c} = 7.32(1 - 0.72\ e^{-0.28k})U^{0.64}W^{-0.22}$$

The influence of lubricant starvation on minimum film thickness in starved elliptical elastohydrodynamic conjunctions has also been investigated for materials of low elastic modulus. Lubricant starvation was studied simply by moving the inlet boundary closer to the center of the conjunction in the numerical solutions. The results show that the location of the dimensionless critical inlet boundary distance m^* denoting the location between the fully flooded and starved conditions can be expressed simply as

$$m^* = 1 + 1.07\left[\left(\frac{R_x}{b}\right)^2 \tilde{H}_{min}\right]^{0.16}$$

That is, for a dimensionless inlet distance $\tilde{m}$ less than m^*, starvation occurs; and for $\tilde{m} \geq m^*$, a fully flooded condition exists. Furthermore it has been possible to express the minimum film thickness for a starved condition as

$$H_{min,s} = H_{min}\left(\frac{\tilde{m} - 1}{m^* - 1}\right)^{0.22}$$

Contour plots of pressure and film thickness in and around the contact have been presented for both fully flooded and starved lubrication conditions. It is evident from these figures that the inlet pressure contours become less circular and closer to the edge of the Hertzian contact zone and that the film thickness decreases substantially as the severity of starvation increases.

The results presented in this chapter have revealed the essential features of both fully flooded and starved, elliptical, elastohydrodynamic conjunctions for materials of low elastic modulus.

SYMBOLS

A	constant used in equation (3.113)
$A^*, B^*, C^*, D^*, L^*, M^*$	relaxation coefficients
A_v	drag area of ball, m^2
a	semimajor axis of contact ellipse, m
$\bar{a}$	$a/2\bar{m}$
B	total conformity of bearing
b	semiminor axis of contact ellipse, m
$\bar{b}$	$b/2\bar{m}$
C	dynamic load capacity, N
C_v	drag coefficient
$C_1,\ldots,C_8$	constants
c	19,609 N/cm^2 (28,440 lbf/in^2)
$\bar{c}$	number of equal divisions of semimajor axis
D	distance between race curvature centers, m
$\tilde{D}$	material factor
$\bar{D}$	defined by equation (5.63)
De	Deborah number
d	ball diameter, m
$\bar{d}$	number of divisions in semiminor axis
d_a	overall diameter of bearing (Figure 2.13), m
d_b	bore diameter, m
d_e	pitch diameter, m
d_e'	pitch diameter after dynamic effects have acted on ball, m
d_i	inner-race diameter, m
d_o	outer-race diameter, m

E	modulus of elasticity, N/m^2
E'	effective elastic modulus, $2\Big/\left(\frac{1-\nu_a^2}{E_a}+\frac{1-\nu_b^2}{E_b}\right)$, N/m^2
E_a	internal energy, m^2/s^2
$\tilde{E}$	processing factor
E_1	$[(\tilde{H}_{min} - H_{min})/H_{min}] \times 100$
$\mathscr{E}$	elliptic integral of second kind with modulus $(1 - 1/k^2)^{1/2}$
$\bar{\mathscr{E}}$	approximate elliptic integral of second kind
e	dispersion exponent
F	normal applied load, N
F^*	normal applied load per unit length, N/m
$\tilde{F}$	lubrication factor
$\bar{F}$	integrated normal applied load, N
F_c	centrifugal force, N
F_{max}	maximum normal applied load (at $\psi = 0$), N
F_r	applied radial load, N
F_t	applied thrust load, N
F_ψ	normal applied load at angle ψ, N
$\mathscr{F}$	elliptic integral of first kind with modulus $(1 - 1/k^2)^{1/2}$
$\bar{\mathscr{F}}$	approximate elliptic integral of first kind
f	race conformity ratio
f_b	rms surface finish of ball, m
f_r	rms surface finish of race, m
G	dimensionless materials parameter, αE
G^*	fluid shear modulus, N/m^2
$\tilde{G}$	hardness factor
g	gravitational constant, m/s^2

g_E	dimensionless elasticity parameter, $W^{8/3}/U^2$
g_V	dimensionless viscosity parameter, GW^3/U^2
H	dimensionless film thickness, h/R_x
$\hat{H}$	dimensionless film thickness, $H(W/U)^2 = F^2h/u^2\eta_0^2R_x^3$
H_c	dimensionless central film thickness, h_c/R_x
$H_{c,s}$	dimensionless central film thickness for starved lubrication condition
H_f	frictional heat, N m/s
H_{min}	dimensionless minimum film thickness obtained from EHL elliptical-contact theory
$H_{min,r}$	dimensionless minimum film thickness for a rectangular contact
$H_{min,s}$	dimensionless minimum film thickness for starved lubrication condition
$\tilde{H}_c$	dimensionless central film thickness obtained from least-squares fit of data
$\tilde{H}_{min}$	dimensionless minimum film thickness obtained from least-squares fit of data
$\bar{H}_c$	dimensionless central-film-thickness - speed parameter, $H_cU^{-0.5}$
$\bar{H}_{min}$	dimensionless minimum-film-thickness - speed parameter, $H_{min}U^{-0.5}$
$\bar{H}_0$	new estimate of constant in film thickness equation
h	film thickness, m
h_c	central film thickness, m
h_i	inlet film thickness, m

h_m	film thickness at point of maximum pressure, where $dp/dx = 0$, m
h_{min}	minimum film thickness, m
h_0	constant, m
I_d	diametral interference, m
I_p	ball mass moment of inertia, m N s^2
I_r	integral defined by equation (3.76)
I_t	integral defined by equation (3.75)
J	function of k defined by equation (3.8)
J^*	mechanical equivalent of heat
$\bar{J}$	polar moment of inertia, m N s^2
K	load-deflection constant
k	ellipticity parameter, a/b
$\bar{k}$	approximate ellipticity parameter
$\tilde{k}$	thermal conductivity, N/s °C
k_f	lubricant thermal conductivity, N/s °C
L	fatigue life
L_a	adjusted fatigue life
L_t	reduced hydrodynamic lift, from equation (6.21)
$L_1,\ldots,L_4$	lengths defined in Figure 3.11, m
L_{10}	fatigue life where 90 percent of bearing population will endure
L_{50}	fatigue life where 50 percent of bearing population will endure
ℓ	bearing length, m
$\bar{\ell}$	constant used to determine width of side-leakage region
M	moment, Nm

M_g	gyroscopic moment, Nm
M_p	dimensionless load-speed parameter, $WU^{-0.75}$
M_s	torque required to produce spin, N m
m	mass of ball, N s^2/m
m^*	dimensionless inlet distance at boundary between fully flooded and starved conditions
$\tilde{m}$	dimensionless inlet distance (Figures 7.1 and 9.1)
$\bar{m}$	number of divisions of semimajor or semiminor axis
m_W	dimensionless inlet distance boundary as obtained from Wedeven, et al. (1971)
N	rotational speed, rpm
n	number of balls
n^*	refractive index
$\bar{n}$	constant used to determine length of outlet region
P	dimensionless pressure
P_D	dimensionless pressure difference
P_d	diametral clearance, m
P_e	free endplay, m
P_{Hz}	dimensionless Hertzian pressure, N/m^2
p	pressure, N/m^2
p_{max}	maximum pressure within contact, $3F/2\pi ab$, N/m^2
$p_{iv,as}$	isoviscous asymptotic pressure, N/m^2
Q	solution to homogeneous Reynolds equation
Q_m	thermal loading parameter
$\bar{Q}$	dimensionless mass flow rate per unit width, $q\eta_0/\rho_0 E'R^2$
q_f	reduced pressure parameter
q_x	volume flow rate per unit width in x direction, m^2/s

q_y	volume flow rate per unit width in y direction, m^2/s
R	curvature sum, m
R_a	arithmetical mean deviation defined in equation (4.1), m
R_c	operational hardness of bearing material
R_x	effective radius in x direction, m
R_y	effective radius in y direction, m
r	race curvature radius, m
r_{ax}, r_{bx}, r_{ay}, r_{by}	radii of curvature, m
r_c, ϕ_c, z	cylindrical polar coordinates
r_s, θ_s, ϕ_s	spherical polar coordinates
$\bar{r}$	defined in Figure 5.4
S	geometric separation, m
S^*	geometric separation for line contact, m
S_0	empirical constant
s	shoulder height, m
T	τ_0/p_{max}
$\tilde{T}$	tangential (traction) force, N
T_m	temperature, °C
T_b^*	ball surface temperature, °C
T_f^*	average lubricant temperature, °C
ΔT^*	ball surface temperature rise, °C
T_1	$(\tau_0/p_{max})_{k=1}$
T_v	viscous drag force, N
t	time, s
t_a	auxiliary parameter
u_B	velocity of ball-race contact, m/s

u_c	velocity of ball center, m/s
U	dimensionless speed parameter, $\eta_0 u/E'R_x$
u	surface velocity in direction of motion, $(u_a + u_b)/2$, m/s
$\bar{u}$	number of stress cycles per revolution
Δu	sliding velocity, $u_a - u_b$, m/s
v	surface velocity in transverse direction, m/s
W	dimensionless load parameter, $F/E'R^2$
w	surface velocity in direction of film, m/s
X	dimensionless coordinate, x/R_x
Y	dimensionless coordinate, y/R_x
X_t, Y_t	dimensionless grouping from equation (6.14)
X_a, Y_a, Z_a	external forces, N
Z	constant defined by equation (3.48)
Z_1	viscosity pressure index, a dimensionless constant
$x, \tilde{x}, \bar{x}, \bar{x}_1$; $y, \tilde{y}, \bar{y}, \bar{y}_1$; $z, \tilde{z}, \bar{z}, \bar{z}_1$	coordinate system
α	pressure-viscosity coefficient of lubrication, m^2/N
α_a	radius ratio, R_y/R_x
β	contact angle, rad
β_f	free or initial contact angle, rad
β'	iterated value of contact angle, rad
Γ	curvature difference
γ	viscous dissipation, N/m^2 s
$\dot{\gamma}$	total strain rate, s^{-1}
$\dot{\gamma}_e$	elastic strain rate, s^{-1}
$\dot{\gamma}_v$	viscous strain rate, s^{-1}

γ_a	flow angle, deg
δ	total elastic deformation, m
δ^*	lubricant viscosity temperature coefficient, $^{\circ}C^{-1}$
δ_D	elastic deformation due to pressure difference, m
δ_r	radial displacement, m
δ_t	axial displacement, m
δ_x	displacement at some location x, m
$\bar{\delta}$	approximate elastic deformation, m
$\tilde{\delta}$	elastic deformation of rectangular area, m
ε	coefficient of determination
ε_1	strain in axial direction
ε_2	strain in transverse direction
ζ	angle between ball rotational axis and bearing centerline (Figure 3.10)
ζ_a	probability of survival
η	absolute viscosity at gauge pressure, N s/m^2
$\bar{\eta}$	dimensionless viscosity, η/η_0
η_0	viscosity at atmospheric pressure, N s/m^2
η_∞	6.31×10^{-5} N s/m^2(0.0631 cP)
θ	angle used to define shoulder height
Λ	film parameter (ratio of film thickness to composite surface roughness)
λ	equals 1 for outer-race control and 0 for inner-race control
λ_a	second coefficient of viscosity
λ_b	Archard-Cowking side-leakage factor, $(1 + 2/3\ \alpha_a)^{-1}$
λ_c	relaxation factor

μ	coefficient of sliding friction
μ^*	$\bar{\rho}/\bar{\eta}$
ν	Poisson's ratio
ξ	divergence of velocity vector, $(\partial u/\partial x) + (\partial v/\partial y) + (\partial w/\partial z)$, s^{-1}
ρ	lubricant density, N s^2/m^4
$\bar{\rho}$	dimensionless density, ρ/ρ_0
ρ_0	density at atmospheric pressure, N s^2/m^4
σ	normal stress, N/m^2
σ_1	stress in axial direction, N/m^2
τ	shear stress, N/m^2
τ_0	maximum subsurface shear stress, N/m^2
$\tilde{\tau}$	shear stress, N/m^2
$\tilde{\tau}_e$	equivalent stress, N/m^2
$\tilde{\tau}_L$	limiting shear stress, N/m^2
Φ	ratio of depth of maximum shear stress to semiminor axis of contact ellipse
Φ^*	$PH^{3/2}$
Φ_1	$(\Phi)_{k=1}$
ϕ	auxiliary angle
ϕ_T	thermal reduction factor
ψ	angular location
ψ_ℓ	limiting value of ψ
Ω_i	absolute angular velocity of inner race, rad/s
Ω_o	absolute angular velocity of outer race, rad/s
ω	angular velocity, rad/s
ω_B	angular velocity of ball-race contact, rad/s
ω_b	angular velocity of ball about its own center, rad/s

ω_c	angular velocity of ball around shaft center, rad/s
ω_s	ball spin rotational velocity, rad/s

Subscripts:

a	solid a
b	solid b
c	central
bc	ball center
IE	isoviscous-elastic regime
IR	isoviscous-rigid regime
i	inner race
K	Kapitza
min	minimum
n	iteration
o	outer race
PVE	piezoviscous-elastic regime
PVR	piezoviscous-rigid regime
r	for rectangular area
s	for starved conditions
x,y,z	coordinate system

Superscript:

$(\bar{\ })$	approximate

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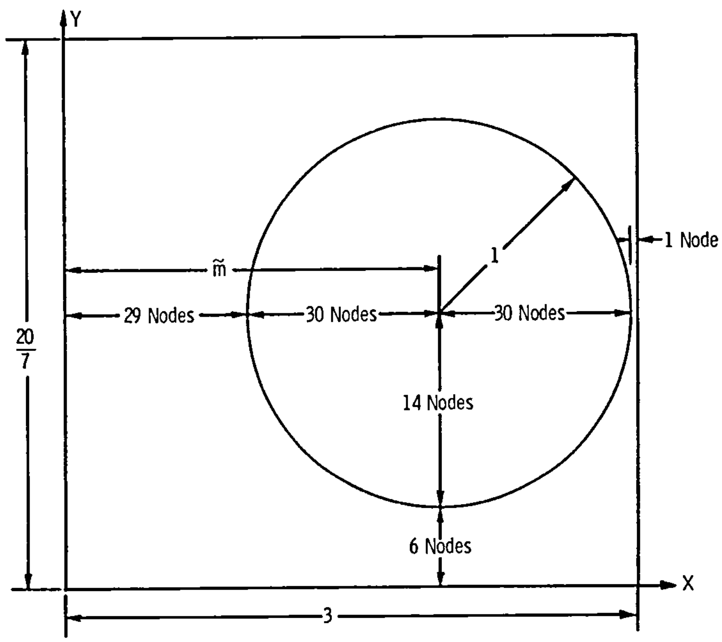


Figure 11.1 - Nodal structure used for fully flooded numerical calculations.

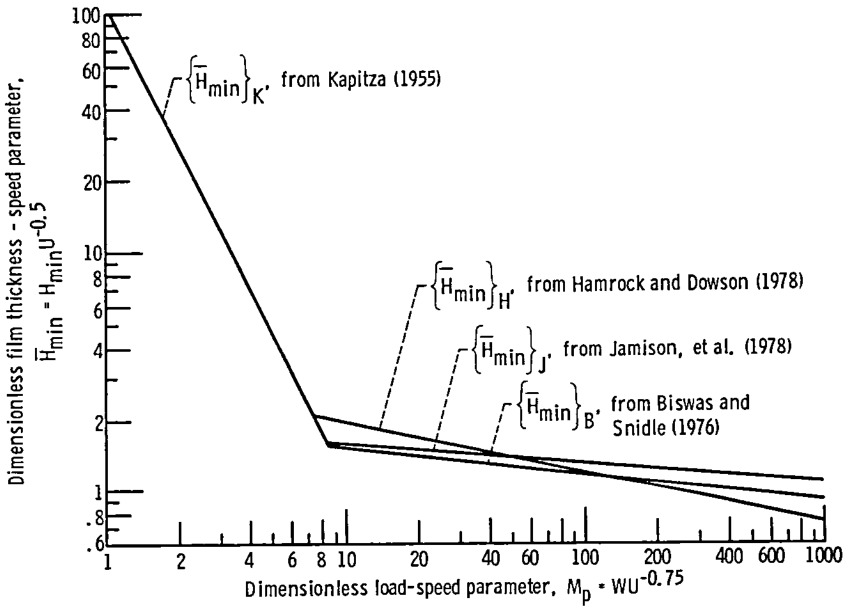


Figure 11.2. - Comparison of different investigators' results.

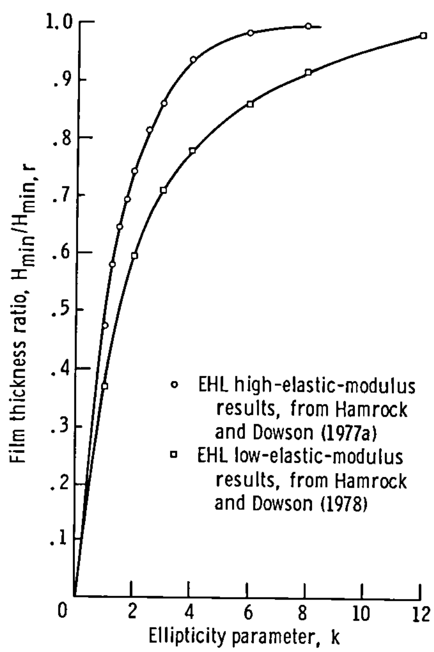
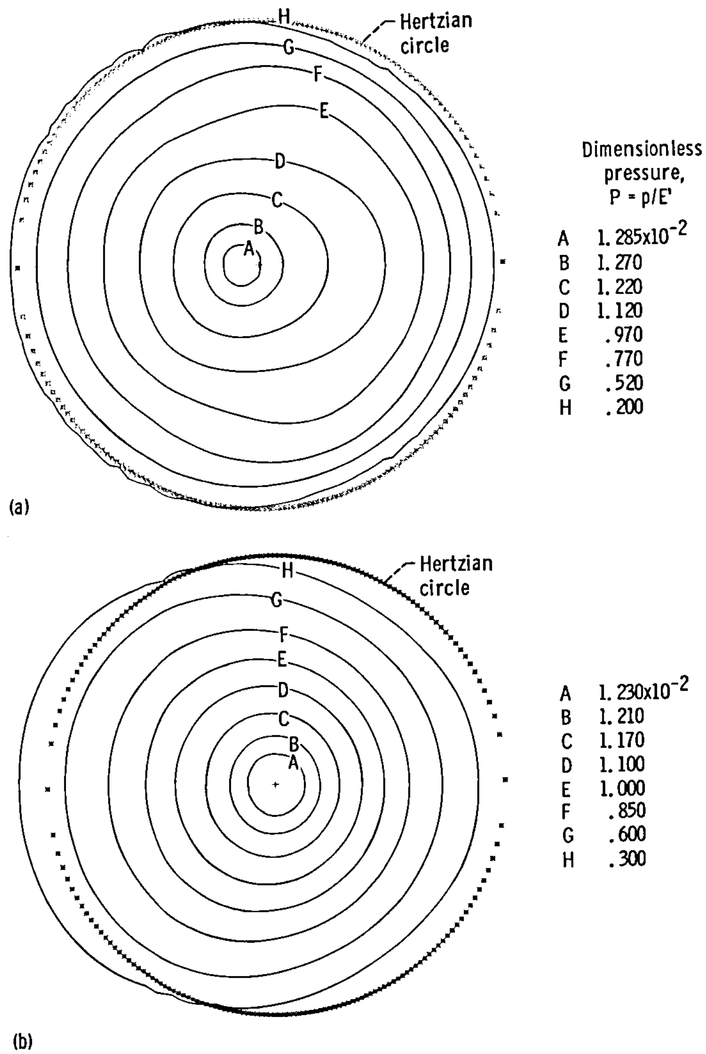
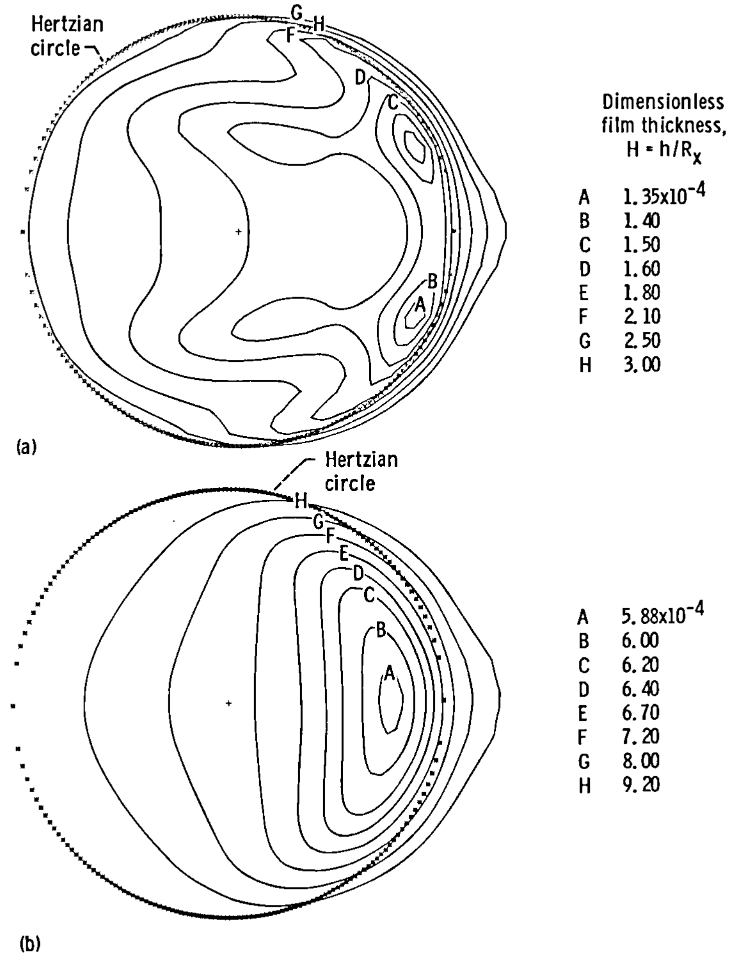


Figure 11.3. - Effect of ellipticity parameter on ratio of dimensionless minimum film thickness for rectangular contact, for EHL high- and low-elastic-modulus analyses.



(a) $U = 0.05139 \times 10^{-7}$.
(b) $U = 0.5139 \times 10^{-7}$.

Figure 11.4. - Contour plots of dimensionless pressure for dimensionless speed parameters U of 0.05139×10^{-7} and 0.5139×10^{-7}.



(a) $U = 0.05139 \times 10^{-7}$.
(b) $U = 0.5139 \times 10^{-7}$.

Figure 11.5. - Contour plots of dimensionless film thickness for dimensionless speed parameters U of 0.05139×10^{-7} and 0.5139×10^{-7}.

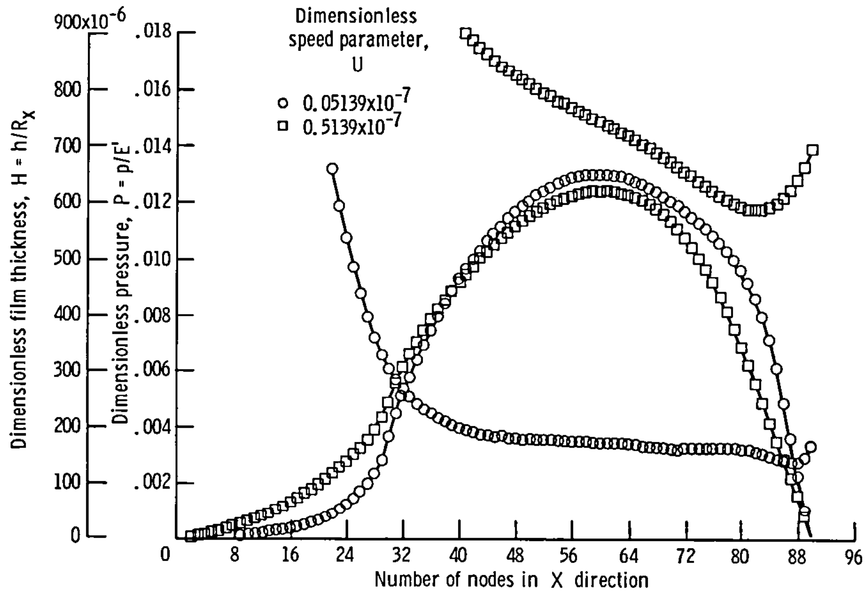
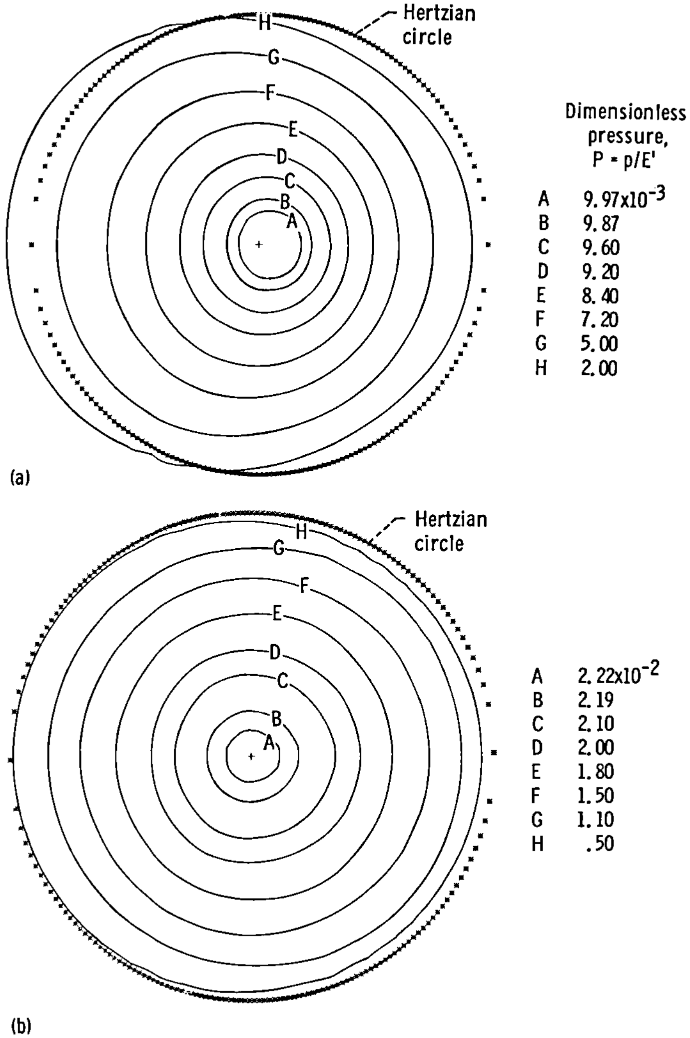
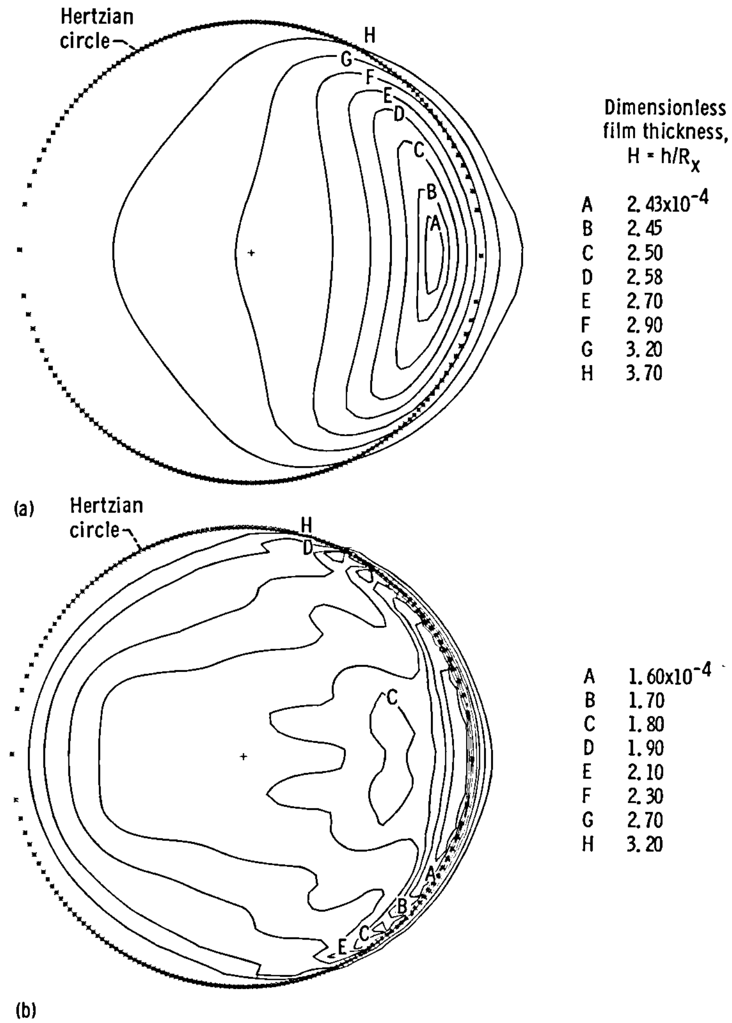


Figure 11.6. - Variation of dimensionless pressure and film thickness on X axis for dimensionless speed parameters U of 0.05139×10^{-7} and 0.5139×10^{-7}. The value of Y is held fixed near axial center of contact.



(a) $W = 0.2202 \times 10^{-3}$.
(b) $W = 2.202 \times 10^{-3}$.

Figure 11.7. - Contour plots of dimensionless pressure for dimensionless load parameters W of 0.2202×10^{-3} and 2.202×10^{-3}.



(a) $W = 0.2202 \times 10^{-3}$.
(b) $W = 2.202 \times 10^{-3}$.

Figure 11.8. - Contour plots of dimensionless film thickness for dimensionless load parameters W of 0.2202×10^{-3} and 2.202×10^{-3}.

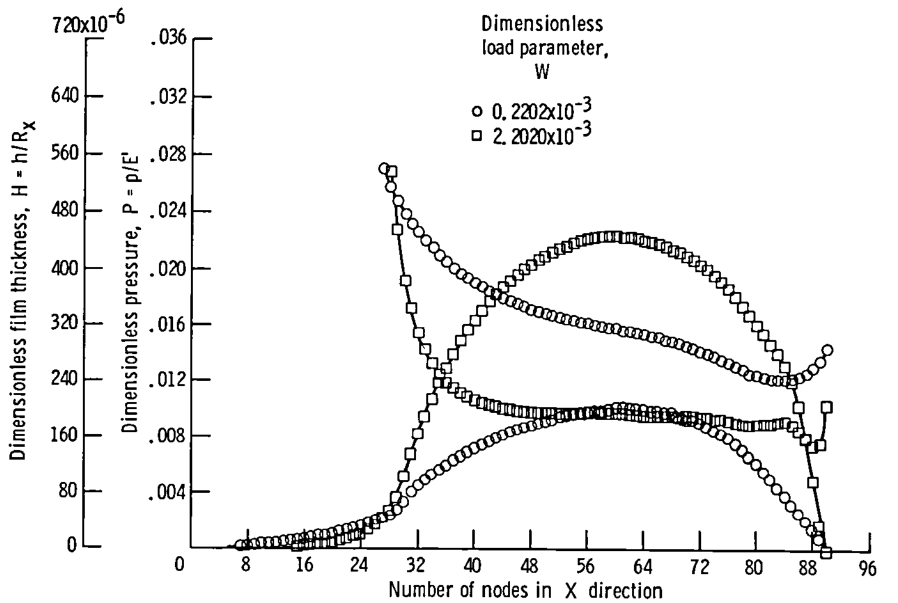
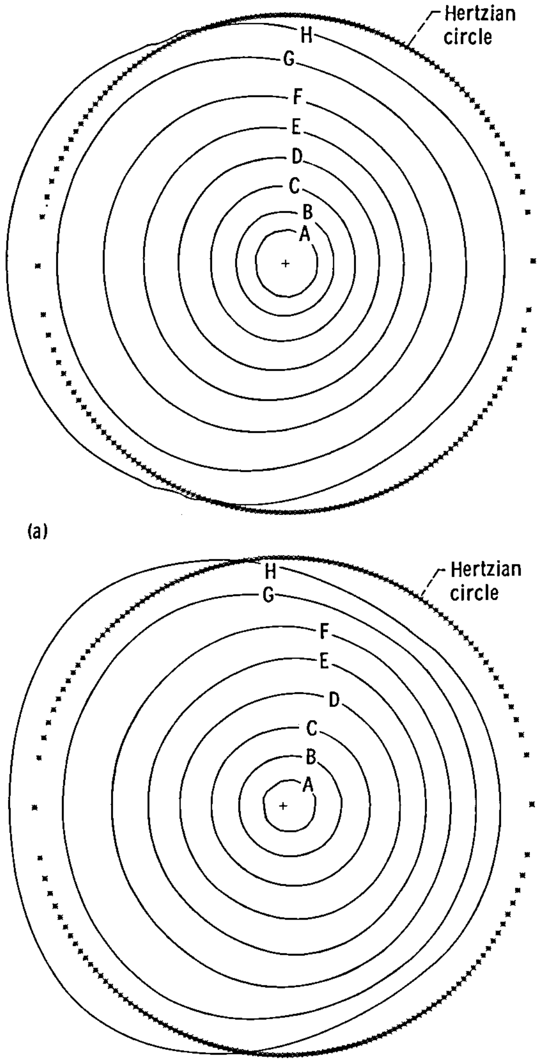


Figure 11.9. - Variation of dimensionless pressure and film thickness on X axis for dimensionless load parameters W of 0.2202×10^{-3} and 2.202×10^{-3}. The value of Y is held fixed near axial center of contact.



(a)

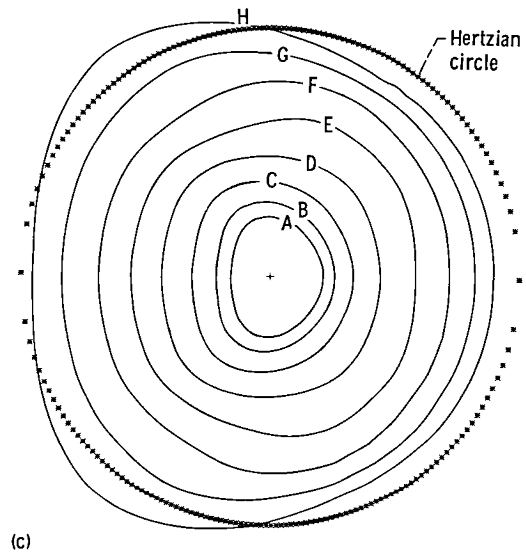
(b)

Dimensionless pressure, $P = p/E'$

A	0.0123
B	.0121
C	.0117
D	.0110
E	.0100
F	.0085
G	.0060
H	.0030

(a) $\tilde{m} = 1.967$.
(b) $\tilde{m} = 1.333$.

Figure 11.10. - Contour plots of dimensionless pressure for dimensionless inlet distances $\tilde{m}$ of 1.967, 1.333, and 1.033 and for group 3 of Table 11.2.

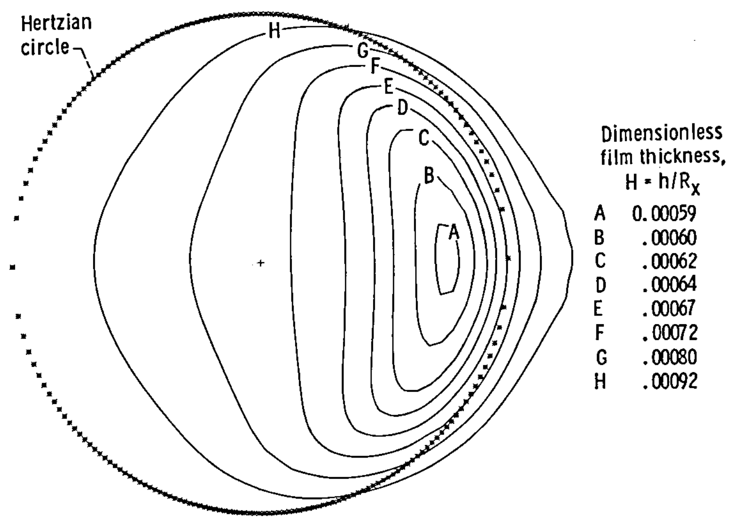


Dimensionless pressure, $P = p/E'$

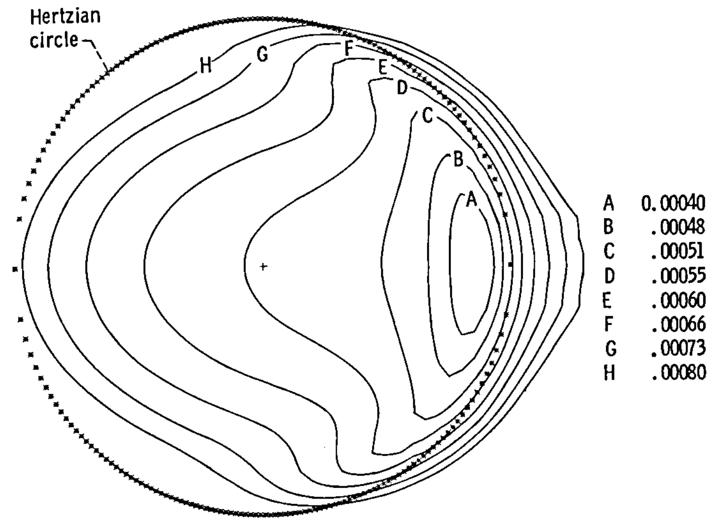
A	0.0123
B	.0121
C	.0117
D	.0110
E	.0100
F	.0085
G	.0060
H	.0030

(c) $\tilde{m} = 1.033$.

Figure 11.10. - Concluded.



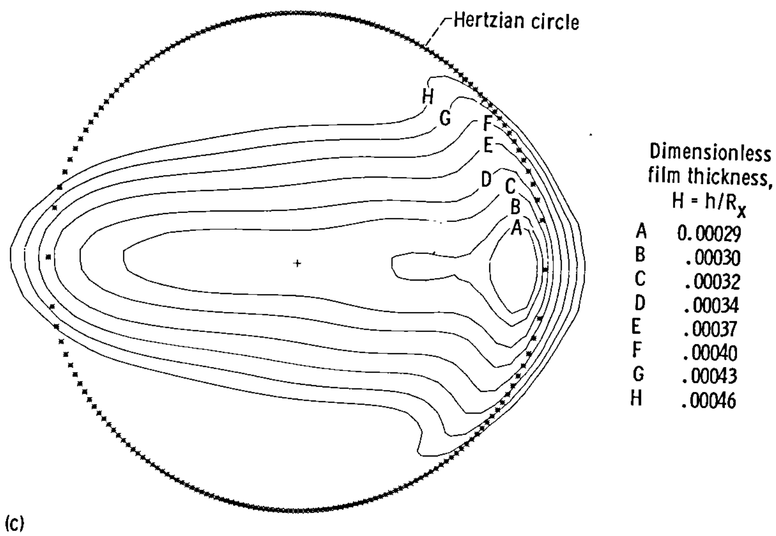
(a)



(b)

(a) $\tilde{m} = 1.967$.
(b) $\tilde{m} = 1.333$.

Figure 11.11. - Contour plots of dimensionless film thickness for dimensionless inlet distances $\tilde{m}$ of 1.967, 1.333, and 1.033 and for group 3 of Table 11.2.



(c) $\tilde{m}$ = 1.033.

Figure 11.11. - Concluded.

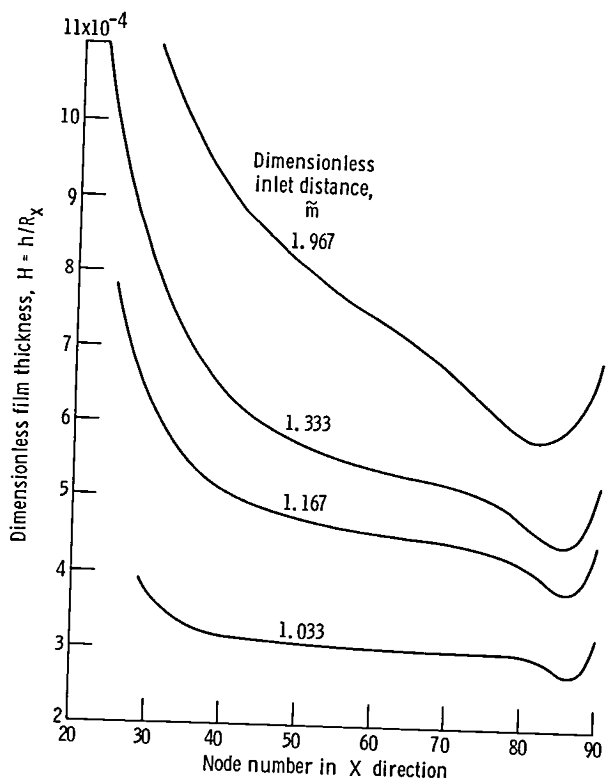


Figure 11.12. - Variation of dimensionless film thickness on X axis for four dimensionless inlet distances and for group 3 of Table 11.2. The value of Y is held fixed near axial center of contact.

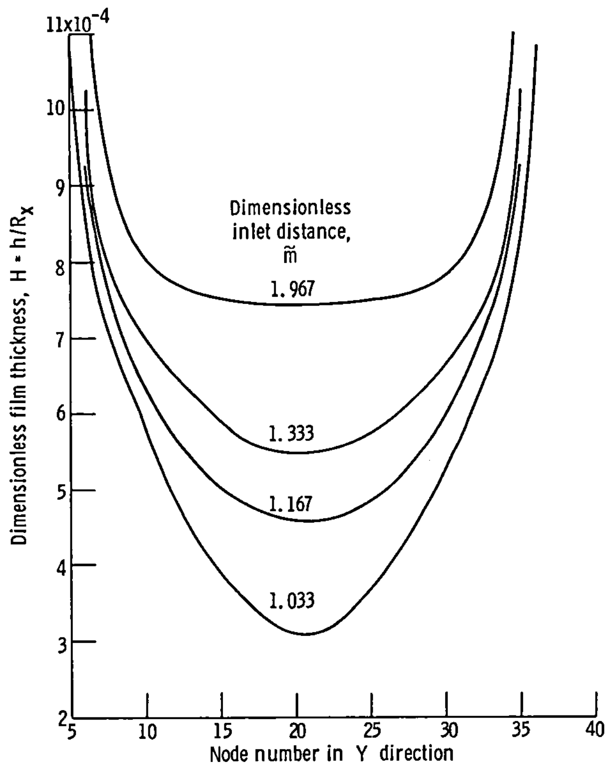


Figure 11.13. - Variation of dimensionless film thickness on Y axis for four dimensionless inlet distances and for group 3 of Table 11.2. The value of X is held fixed near axial center of contact.

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16. Abstract

The influence of the ellipticity parameter k and the dimensionless speed U, load W, and materials G parameters on minimum film thickness for materials of low elastic modulus has been investigated. The ellipticity parameter was varied from 1 (a ball-on-plane configuration) to 12 (a configuration approaching a line contact); U and W were each varied by one order of magnitude. Seventeen cases were used to generate the miniumum- and central-film-thickness relations

$$\tilde{H}_{min} = 7.43(1 - 0.85\ e^{-0.31k})U^{0.65}W^{-0.21}$$

$$\tilde{H}_c = 7.32(1 - 0.72\ e^{-0.28k})U^{0.64}W^{-0.22}$$

The influence of lubricant starvation on minimum film thickness in starved elliptical, elastohydrodynamic conjunctions has also been investigated for materials of low elastic modulus. Lubricant starvation was studied simply by moving the inlet boundary closer to the center of the conjunction in the numerical solutions. The results show that the location of the dimensionless critical inlet boundary distance m^* denoting the location between the fully flooded and starved conditions can be expressed simply as

$$m^* = 1 + \left[1.07 \left(\frac{R_x}{b}\right)^2 \tilde{H}_{min}\right]^{0.16}$$

That is, for $\tilde{m} < m^*$, starvation occurs; for $\tilde{m} \geq m^*$, a fully flooded condition exists. Furthermore, it has been possible to express the minimum film thickness for a starved condition as

$$H_{min,\ s} = H_{min}\left(\frac{\tilde{m} - 1}{m^* - 1}\right)^{0.22}$$

Contour plots of pressure and film thickness in and around the contact have been presented for both fully flooded and starved lubrication conditions. It is evident from these figures that the inlet pressure contours become less circular and closer to the edge of the Hertzian contact zone and that the film thickness decreases substantially as the severity of starvation increases. The results presented reveal the essential features of both fully flooded and starved, elliptical, elastohydrodynamic conjunctions for materials of low elastic modulus.

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