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PERIODIC MOTIONS (CLOSE TO STATIONARY) OF AN AXISYMMETRIC SATELLITE WITH MAGNETIC DAMPING

M.Yu. Ovchinnikov

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16. Abstract

Close to stationary periodic motions of an axisymmetric satellite in a circular orbit are considered. The satellite was equipped with a spherical magnetic damper. The investigation was conducted on the assumption that a strong magnet was installed on the damper float. Stationary rotations of the satellite around the axis of symmetry are selected as the generating solutions. The solutions are constructed in the form of power series of the small parameter, and they are extended numerically to the region of random values of the damping coefficient. The stability of the resulting solutions was investigated.

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ANNOTATION

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M.Yu. Ovchinnikov

1. Introduction

Periodic motions of an axisymmetric satellite equipped with a spherical magnetic damper, in orbits of random declination, were investigated in [1]. With $\lambda > 4/7$ (λ is the ratio of the axial moment of inertia of the satellite to its equatorial moment of inertia), instability of motion of the satellite in the plane of the polar orbit was found with respect to spatial perturbations. An example of a transition process was presented there which resulted in stable motion, which was characterized by deviation of the axis of symmetry of the satellite from the plane of the polar orbit by "small tremors" about this position and rotation of the satellite around the axis, with a period close to the period of rotation of its center of mass around the orbit. It is shown in the present study that, with a magnetic damper aboard the satellite, its stationary rotations change to forced periodic motions. In the $(\sin^2 i, \lambda)$ plane (i is the declination of the orbit of the center of mass of the satellite to the plane of the equator), regions of existence of stationary rotations of the satellite are constructed, which are selected as generating motions. Motions of the satellite which are close to stationary rotations were constructed in the form of a power series of the small parameter. The ratio of the characteristic values of the damping and gravitational moments acting on the satellite was used as the small parameter. The orbit of the center of mass of the satellite is considered circular. The geomagnetic field is approximated by the field of a dipole which coincides with the axis of rotation of the earth. The resulting motions can be used as nominal (operating) motions of an axisymmetric /3*

*Numbers in the margin indicate pagination in the foreign text.

satellite with $\lambda \lesssim 1$.

The stationary rotational motions of an axisymmetric satellite with model damping were constructed and studied in [2].

V.A. Sarychev and Yu.A. Sadov are thanked for attention to the work.

2. Equations of Motion and Formulation of the Problem

We consider that the satellite is a solid body, the moments of inertia of the damper float are negligibly small compared with the moments of inertia of the satellite, and the center of mass of the float is fixed relative to the satellite. The motion of the float about its center of mass then does not affect the inertial characteristics of the satellite, and the float is replaced by an equivalent /4 point mass in their determination.

To write the equations of motion of the satellite and float relative to the center of mass, we introduce the following clockwise rectangular coordinate systems:

$Ox_1x_2x_3$ bound to the satellite coordinate system; its axes are the principal central axes of inertia of the satellite; point 0 is the center of mass of the satellite;

$OX_1X_2X_3$ is the orbital coordinate system; the OX_3 axis is directed along the radius vector of point 0 relative to the center of mass of the earth; the OX_1 axis coincides with the transversal, and the OX_2 axis coincides with the normal to the plane of the orbit; $\vec{E}_a$ is the unit vector of the OX_2 axis;

$OZ_1Z_2Z_3$ is the magnetic coordinate system; the OZ_1 axis is directed along vector $\vec{H}$ of the geomagnetic field strength at point 0; unit vectors

$$\vec{\mathfrak{z}}_1=\frac{\vec{H}}{|\vec{H}|},\quad \vec{\mathfrak{z}}_3=\frac{\vec{\mathfrak{z}}_1\times\vec{E}_2}{|\vec{\mathfrak{z}}_1\times\vec{E}_2|},\quad \vec{\mathfrak{z}}_2=\vec{\mathfrak{z}}_3\times\vec{\mathfrak{z}}_1$$

determine the corresponding axes of the magnetic coordinate system.

We assign the position of the $Ox_1x_2x_3$ coordinate system relative to the orbital coordinate system by means of angles α, β, γ (Fig. 1). The transition matrix and its elements have the form

	x_1	x_2	x_3
X_1	a_{11}	a_{12}	a_{13}
X_2	a_{21}	a_{22}	a_{23}
X_3	a_{31}	a_{32}	a_{33}

$$a_{11}=\cos\alpha\cos\beta,\quad a_{12}=-\cos\alpha\sin\beta,\quad a_{13}=\sin\alpha,$$

$$a_{21}=\cos\gamma\sin\beta+\sin\gamma\sin\alpha\cos\beta,$$
$$a_{22}=\cos\gamma\cos\beta-\sin\gamma\sin\alpha\sin\beta,$$
$$a_{23}=-\sin\gamma\cos\alpha,$$

$$a_{31}=\sin\gamma\sin\beta-\cos\gamma\sin\alpha\cos\beta,$$
$$a_{32}=\sin\gamma\cos\beta+\cos\gamma\sin\alpha\sin\beta,$$
$$a_{33}=\cos\gamma\cos\alpha.$$

We assign the position of vector $\vec{I}$ of the magnetic moment of the magnet installed in the float relative to the magnetic coordinate system by means of angles α_1 and β_1. The corresponding directing cosines have the form

$$c_1=\cos\alpha_1\cos\beta_1,\quad c_2=\sin\beta_1,\quad c_3=-\sin\alpha_1\cos\beta_1.$$

Let arbitrary vector $\vec{q}$ be assigned by projections q_1, q_2, q_3 on the axes of any of the coordinate systems introduced, $Ox_1x_2x_3$ for example. We will then write $\vec{q}=(q_1,q_2,q_3)x$, etc. As needed, we will employ summation over the recurrent indices and free indices. The indices run through the values 1, 2 and 3. /5

We approximate the geomagnetic field by the field of a magnetic dipole placed at the center of the earth, the axis of which is directed along its axis of rotation. The projections of vector $\vec{H}$ at point 0 on the orbital coordinate system axes, referred to the quantity $H_\nu=\mu m/\rho^3$, have the form

$$H_1=\sin i\cos u,\quad H_2=\cos i,\quad H_3=-2\sin i\sin u,$$

where $\mu m = 8.06 \cdot 10^{25}$ Oe·cm^3 is the magnetic moment of the dipole, ρ is the radius of the satellite orbit, i is its declination to the plane of the equator, and u is the argument of the latitude. We assign the position of the magnetic coordinate system relative to the orbital coordinate system by means of the transition matrix

	Z_1	Z_2	Z_3
X_1	h_{11}	h_{12}	h_{13}
X_2	h_{21}	h_{22}	h_{23}
X_3	h_{31}	h_{32}	h_{33}

$$h_{11} = \frac{\sin i \cos u}{H}, \quad h_{12} = -\frac{\cos i \cos u}{H H_u}, \quad h_{13} = \frac{2 \sin u}{H_u},$$

$$h_{21} = \frac{\cos i}{H}, \quad h_{22} = \frac{\sin i H_u}{H}, \quad h_{23} = 0,$$

$$h_{31} = -\frac{2 \sin i \sin u}{H}, \quad h_{32} = \frac{2 \cos i \sin u}{H H_u}, \quad h_{33} = \frac{\cos u}{H_u},$$

where

$$H = \sqrt{1 + 3\sin^2 i \sin^2 u}, \qquad H_u = \sqrt{1 + 3\sin^2 u}.$$

Of the external moments which act on the satellite, we will take only the gravitational moment into account. Of the external moments which act on the damper float, we will take into account only the magnetic moment. The interaction of the satellite and the float is due to eddy currents induced in the outer shell of the damper by the magnetic field of the float. The hypothesis was introduced above that the moments of inertia of the float are significantly less than the moments of inertia of the satellite. Therefore, with the exception of the small time interval after freeing the float, its motion is determined by the dynamic equilibrium of the magnetic moment and the moment of the induced eddy currents [3]. The motion of the axisymmetric satellite and the damper float are described by the system of equations [1]

$$\begin{aligned}
&\dot W_1 = -\left(\lambda\Omega - W_1 \operatorname{tg}\alpha + \frac{\sin\gamma}{\cos\alpha}\right) W_2 - 3(1-\lambda)\sin\gamma\cos\gamma\cos\alpha + \eta H[(h_{12}c_3 - \\
&\quad - h_{13}c_2)\cos\alpha + (h_{32}c_3 - h_{33}c_2)\sin\gamma\sin\alpha - (h_{22}c_3 - h_{23}c_2)\cos\gamma\sin\alpha], \\
&\dot W_2 = \left(\lambda\Omega - W_1 \operatorname{tg}\alpha + \frac{\sin\gamma}{\cos\alpha}\right) W_1 - 3(1-\lambda)\cos^2\gamma\sin\alpha\cos\alpha + \eta H[(h_{22}c_3 - \\
&\quad - h_{23}c_2)\cos\gamma + (h_{32}c_3 - h_{33}c_2)\sin\gamma], \\
&\dot\Omega = \frac{\eta}{\lambda} H a_{13}(h_{12}c_3 - h_{13}c_2), \\
&\dot\gamma = \frac{W_1}{\cos\alpha} - \sin\gamma \operatorname{tg}\alpha, \quad \dot\alpha = W_2 - \cos\gamma, \quad \dot\beta = \Omega - W_1 \operatorname{tg}\alpha + \frac{\sin\gamma}{\cos\alpha}, \\
&\dot\alpha_1 = \omega_2^d - (\omega_1^d \cos\alpha_1 - \omega_3^d \sin\alpha_1)\operatorname{tg}\beta_1, \quad \dot\beta_1 = \omega_1^d \sin\alpha_1 + \omega_3^d \cos\alpha_1.
\end{aligned} \tag{1}$$

Here /6

$$\omega_i^d = W_1(h_{1i}\cos\alpha + h_{2i}\sin\gamma\sin\alpha - h_{3i}\cos\gamma\sin\alpha) + W_2(h_{2i}\cos\gamma + h_{3i}\sin\gamma) + h_{ji}a_{j3}\Omega - \omega_i^H - h_{2i} + m_i;$$
$$m_1 = 0,\quad m_2 = \varepsilon H c_3,\quad m_3 = -\varepsilon H c_2;\quad \lambda = \frac{C}{A},\quad p = \frac{IH_0}{C\omega_0^2},\quad \varepsilon = \frac{IH_0}{k_\varphi\omega_0}$$
$$\omega_1^H = \frac{2\cos i}{HH_0^2},\quad \omega_2^H = \frac{2\sin i}{HH_0},\quad \omega_3^H = -\frac{3\sin i\cos i\sin u\cos u}{H^3H_0};$$

A, C are the equatorial and axial moments of inertia of the satellite, ω_0 is the angular velocity of orbital motion of the satellite center of mass, k_ϕ is the damping coefficient, W_1, W_2, Ω are the projections, related to ω_0, of the absolute angular velocity of the satellite on the Rezal' axes, which coincide with the axes of coordinate system $Ox_1x_2x_3$ at $\beta=0$. The Ox_3 axis is the axis of symmetry of the satellite. The point is designated by differentiation over u. The derivation of system of Eq. (1) is described in [1], where another sequence of flight angles α, β, γ is introduced.

Let $I/\varepsilon \ll I$. The motion of the satellite is then [1] described to within $O(I/\varepsilon)$ by the following equations:

$$\begin{aligned}
\dot W_1 = &-\left(\lambda\Omega - W_1\,tg\,\alpha + \frac{\sin\gamma}{\cos\alpha}\right)W_2 - 3(1-\lambda)\sin\gamma\cos\gamma\cos\alpha + k_\varphi\big[Q(h_{11}\cos\alpha + \\
&+ h_{21}\sin\gamma\sin\alpha - h_{31}\cos\gamma\sin\alpha) + \omega_2^H(h_{12}\cos\alpha + h_{22}\sin\gamma\sin\alpha - h_{32}\cos\gamma\sin\alpha) + \\
&\omega_3^H(h_{13}\cos\alpha + h_{23}\sin\gamma\sin\alpha - h_{33}\cos\gamma\sin\alpha) - W_1\big], \qquad (2)\\
\dot W_2 = &\left(\lambda\Omega - W_1\,tg\,\alpha + \frac{\sin\gamma}{\cos\gamma}\right)W_1 - 3(1-\lambda)\cos^2\gamma\sin\alpha\cos\alpha + k_\varphi\big[Q(h_{21}\cos\gamma + \\
&+ h_{31}\sin\gamma) + \omega_2^H(h_{22}\cos\gamma + h_{32}\sin\gamma) + \omega_3^H(h_{23}\cos\gamma + h_{33}\sin\gamma) + \cos\gamma - W_2\big].\\
\dot\Omega = &\frac{k_\varphi}{\lambda}\left[a_{j3}(Qh_{j1} + \omega_2^H h_{j2} + \omega_3^H h_{j3}) + a_{23} - \Omega\right],\\
\dot\gamma = &\frac{W_1}{\cos\alpha} - \sin\gamma\,tg\,\alpha,\quad \dot\alpha = W_2 - \cos\gamma
\end{aligned}$$

Here, /7

$$Q = -h_{21} + W_1(h_{11}\cos\alpha + h_{21}\sin\gamma\sin\alpha - h_{31}\cos\gamma\sin\alpha) + W_2(h_{21}\cos\gamma + h_{31}\sin\gamma) + \Omega h_{k1}a_{k3},\quad k_\varphi = \frac{p}{\varepsilon}.$$

We will next investigate system (2). Angle β, which describes the rotation of the satellite around its axis of symmetry, is determined by the equation

$$\dot{\beta} = \Omega - W_1 \operatorname{tg}\alpha + \frac{\sin\gamma}{\cos\alpha}.$$

System of Eq. (2) with 2π periodic clockwise segments with respect to u contains three parameters i, λ, kg. The latter two satisfy the inequalities $0<\lambda\leqslant 2$, kg>0. Equations (2) do not change their form in the following substitutions of the phase variables and parameter i:

$$\begin{array}{l} W_1 \to -W_1,\ \Omega \to -\Omega,\ \gamma \to -\gamma,\ \begin{cases} i \to \pi - i \\ i \to -i \end{cases}; \\ W_1 \to -W_1,\ W_2 \to -W_2,\ \gamma \to \pi - \gamma,\ \alpha \to -\alpha,\ \begin{cases} i \to \pi - i \\ i \to -i \end{cases} \end{array} \qquad (3)$$

In accordance with this, it is sufficient to investigate the solutions of Eq. (2) in the interval $0\leqslant i\leqslant \pi/2$.

With kg=0, system (2) permits solutions which correspond to stationary rotations of the satellite. With kg>0, we obtained and investigate the 2π periodic with respect to u solutions of system (2) generated from them.

3. Stationary Rotations of a Satellite with kg=0

If kg=0, system (2) has the generalized energy integral

$$T_0 = \frac{1}{2}(W_1^2 + W_2^2) + \frac{\lambda}{2}\Omega^2 - W_1 \sin\alpha \sin\gamma - W_2 \cos\gamma + \lambda\Omega\cos\alpha\sin\gamma - \frac{3}{2}(1-\lambda)\cos^2\alpha\cos^2\gamma \qquad (4)$$

and stationary solutions $\gamma=\gamma_0$, $\alpha=\alpha_0$ (γ_0, α_0=const), which are determined by the system of equations

$$\begin{array}{l} \cos\gamma_0 [\lambda\Omega_0 + (4-3\lambda)\sin\gamma_0 \cos\alpha_0] = 0, \\ \sin\alpha_0 \{\lambda\Omega_0 \sin\gamma_0 + [1-(4-3\lambda)\cos^2\gamma_0]\cos\alpha_0\} = 0, \end{array} \qquad (5)$$

where $\Omega_0 = \dot{\beta} + \sin\gamma_0 \cos\alpha_0$=const is an integral which corresponds to cyclic coordinate β. We consider that $\lambda\neq 1$. System (5) permits the following solutions:

$$\sin\alpha_0=0,\quad \lambda\Omega_0+(4-3\lambda)\sin\gamma_0\cos\alpha_0=0; \qquad (6)$$

$$\cos\gamma_0=0,\quad \lambda\Omega_0\sin\gamma_0+[1-(4-3\lambda)\cos^2\gamma_0]\cos\alpha_0=0; \qquad (7)$$

$$\cos\gamma_0=0,\quad \sin\alpha_0=0 \qquad (8)$$

Angle v between the $0x_3$ axis of the natural rotation of the satellite and the current radius vector of the satellite center of mass relative to the center of mass of the earth is determined by the relationship /8

$$\cos\vartheta=\cos\gamma_0\cos\alpha_0,$$

the angular velocity of the natural rotation is determined by the equation

$$\dot{\rho}=\Omega_0+\sin\gamma_0\cos\alpha_0.$$

By using integral (4) and the motions of the satellite linearized in the vicinity of the stationary solutions of the equations, both sufficient and necessary conditions of stability of these solutions can be obtained [4].

Solution (6)

The sufficient condition of stability is satisfied if

$$\lambda-1<0.$$

The necessary conditions of stability are satisfied in the following two regions:

$$\lambda-1\leq 0.$$

$$\lambda-\tfrac{4}{3}>0,\quad 81\lambda^2(\lambda-1)\Omega_0^4-6\lambda^2(\lambda-1)(10\lambda^2-27\lambda+9)(3\lambda-4)\Omega_0^2+(3\lambda-4)^4>0$$

The condition of existence of solution (6) has the form

$$|\lambda\Omega_0|\leq|4-3\lambda|$$

Solution (7)

The sufficient and necessary conditions of stability of the solutions coincide and have the form

$$\lambda-1>0, \ \sin\alpha_0\neq 0.$$

To these conditions must be added the condition of existence of solution (7)

$$|\lambda\Omega_0|\leq 1.$$

Solution (8)

Let $\gamma_0=\pi/2$. The sufficient conditions of its stability have the form

$$\lambda\Omega_0-1>0, \ \lambda\Omega_0+3\lambda-4>0,$$

and the necessary conditions of stability are 79

$$(\lambda\Omega_0-1)(\lambda\Omega_0+3\lambda-4)>0,$$
$$[(\lambda\Omega_0-1)^2+3\lambda-2]^2+4(\lambda\Omega_0-1)(\lambda\Omega_0+3\lambda-4)>0.$$

4. Satellite Motion Close to Stationary Rotation (6)

We will investigate the forced 2π periodic solution of system (2) which satisfies the boundary conditions

$$W_1(2\pi)=W_1(0), \ W_2(2\pi)=W_2(0), \ \Omega(2\pi)=\Omega(0), \ \gamma(2\pi)=\gamma(0), \ \alpha(2\pi)=\alpha(0), \qquad (9)$$

by solving boundary value problem (2), (9). We investigate boundary value problem (2), (9) by the Poincare small parameter method [5]. We use solution (6) as the generating solution. For the other generating solutions examined in this work, such an investigation is carried out similarly. We use vector notation to shorten the writing [9]. We introduce vector $Z=(W_1,W_2,\Omega,\gamma,\alpha)^T$, and we define function

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$F(u, z, i, k_g) \in R^5$ so that system (2) and boundary conditions (9) could be written in the form

$$z = F(u, z, i, k_g) \tag{2'}$$

and

$$z(2\pi) - z(0) = 0 \tag{9'}$$

respectively. Let $\bar{z}(u, a, i, k_g) = (\bar{w}_1(u, a, i, k_g), \bar{w}_2(u, a, i, k_g), \bar{\Omega}(u, a, i, k_g), \bar{\gamma}(u, a, i, k_g), \bar{\lambda}(u, a, i, k_g))^T$ be the solution of system (2') with the initial conditions $\bar{Z}(0, a, i, kg) = a = (a_1, \ldots, a_5)^T$. Boundary value problem (2'), (9') can then be written

$$g(a, i, k_g) = \bar{z}(2\pi, a, i, k_g) - \bar{z}(0, a, i, k_g) = 0. \tag{10}$$

We will consider relationship (10) as an equation relative to a. If kg=0, this equation permits the solution $\bar{a} = (\bar{a}_1, \ldots, \bar{a}_5)^T$ where

$$\bar{a}_1 = 0,\ \bar{a}_2 = \cos\gamma_0,\ \bar{a}_3 = -\frac{\nu - \lambda\lambda}{\lambda}\sin\gamma_0,\ \bar{a}_4 = \gamma_0,\ \bar{a}_5 = 0. \tag{11}$$

Because of the analytical nature of the right side of system (2') with respect to Z and kg, with sufficiently small kg and $|Z - \bar{a}|$, function g(a,i,kg) analytically depends on kg, a in the vicinity of point kg=0 and $a = \bar{a}$. If /10

$$J = \det\left|\frac{\partial g(\bar{a}, i, 0)}{\partial a}\right| \neq 0, \tag{12}$$

according to the theorem of the implicit function, with sufficiently small kg, Eq. (10) has the unique solution $a = \tilde{a}(kg, i)$, which depends analytically on kg and satisfies the condition $\tilde{a}(0, i) = \bar{a}$. In this case, boundary value problem (2'), (9') has the unique solution

$$z = \bar{z}(u, \tilde{a}(k_g, i), i, k_g), \tag{13}$$

which depends analytically on kg in the vicinity of point kg=0 and coincides at this point with stationary rotation (6) with $\alpha_0 = 0$.

We will investigate solution (13) in the form of an integral power series of parameter kg

$$W_1 = \bar{a}_1 + k_g W_{1,1}(u) + \ldots,\quad W_2 = \bar{a}_2 + k_g W_{2,1}(u) + \ldots,\quad \Omega = \bar{a}_3 + k_g \Omega_1(u) + \ldots,$$
$$\gamma = \bar{a}_4 + k_g \gamma_1(u) + \ldots,\quad \alpha = \bar{a}_5 + k_g \alpha_1(u) + \ldots \tag{14}$$

with 2π periodic coefficients with respect to u. The equations in the variations for stationary solution $\bar{a}$ have the form

$$\Delta\dot{W}_1 = 3(1-\lambda)\sin\gamma_0\,\Delta W_2 - \lambda\cos\gamma_0\,\Delta\Omega + [3(1-\lambda) - (7-6\lambda)\cos^2\gamma_0]\Delta\gamma,$$
$$\Delta\dot{W}_2 = -3(1-\lambda)\sin\gamma_0\,\Delta W_1 - 3(1-\lambda)\cos^2\gamma_0\,\Delta\alpha,$$
$$\Delta\dot{\Omega} = 0,\quad \Delta\dot{\gamma} = \Delta W_1 - \sin\gamma_0\,\Delta\alpha,\quad \Delta\dot{\alpha} = \Delta W_2 + \sin\gamma_0\,\Delta\gamma. \tag{15}$$

The characteristic equation of system (15) is separated into the equations p=0 and

$$p^4 + ap^2 + \beta = 0, \tag{16}$$

where $a = 7 - 6\lambda - 9\lambda(1-\lambda)\sin^2\gamma_0$, $\beta = 3(1-\lambda)(4-3\lambda)\cos^2\gamma_0$. For determination of the forced solution of system (2), we substitute series (14) in system (2), and we equate the terms with the same powers of kg. We obtain a series of systems, the general form of which is the following

$$\dot{W}_{1,k} = 3(1-\lambda)\sin\gamma_0\,W_{2,k} - \lambda\cos\gamma_0\,\Omega_k + [3(1-\lambda) - (7-6\lambda)\cos^2\gamma_0]\gamma_k + S_{1,k},$$
$$\dot{W}_{2,k} = -3(1-\lambda)\sin\gamma_0\,W_{1,k} - 3(1-\lambda)\cos^2\gamma_0\,\alpha_k + S_{2,k},$$
$$\dot{\Omega}_k = \frac{1}{\lambda}S_{3,k},\quad \dot{\gamma}_k = W_{1,k} - \sin\gamma_0\,\alpha_k + S_{4,k},\quad \dot{\alpha}_k = W_{2,k} + \sin\gamma_0\,\gamma_k + S_{5,k}. \tag{17}$$

Here, $S_{i,k}$ (i=1, . . . 5; k=1,2, . . .) are some functions $W_{1,0}$, . . ., $W_{1,k-1}$, . . .; α_0, . . ., α_{k-1}; u, $S_{1,0}$=0, and system (17), to within the designation of the variables with k=0, coincides with system (15). In the solution of system (17), we will use the results of [6]. Let the solution of system (17) be found up to k-1 inclusive. This solution depends on integration constant Ω_{k-1}. The equation for Ω_k is separated out, and it can be integrated. Then, /11

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$$\Omega_k(u) = \frac{1}{\lambda}\int_0^u S_{3k}(\tau)d\tau + \Omega_k^0 .$$

The condition of 2π periodicity of the function

$$\int_0^{2\pi} S_{3k}(\tau)d\tau = 0 \tag{18}$$

permits determination of the value of Ω^0_{k-1}. In particular, to find function $\Omega_1(u)$, we have the equation

$$\dot{\Omega}_1 = \frac{1}{\lambda}\Big\{-\frac{4(1-\lambda)}{\lambda}\sin\gamma_0\left[(h_{31}^2-h_{21}^2)\cos^2\gamma_0 - h_{21}h_{31}\sin 2\gamma_0 - (h_{22}^2+h_{23}^2)\right] - \\ -(\omega_2'' h_{22}+\omega_3'' h_{23})\sin\gamma_0 + (\omega_2'' h_{32}+\omega_3'' h_{33})\cos\gamma_0\Big\}.$$

According to Eq. (18), function $\Omega_1(u)$ is 2π periodic if $\sin\gamma_0=0$ or

$$\cos^2\gamma_0 = 3\,\frac{2(1-\lambda)(\sqrt{1+3\sin^2 i}-1)+(2-3\lambda)\sin^2 i}{2(1-\lambda)(4\sqrt{1+3\sin^2 i}-7+3\sin^2 i)} . \tag{19}$$

To within O(kg), the first condition corresponds to the motion of the satellite in the orbital plane. From the second condition, we find the region of existence of generating solution (6) in the plane ($\sin^2 i$, λ). The boundaries of this region are determined by curves $\lambda_1(i)$, $\lambda_2(i)$, which are assigned by the expressions

$$\lambda_1(i) = 2\,\frac{\sqrt{1+3\sin^2 i}-\cos i}{2(\sqrt{1+3\sin^2 i}-1)+3\sin^2 i}\,, \quad \lambda_2(i) = 2\,\frac{4-\sqrt{1+3\sin^2 i}}{2(4-\sqrt{1+3\sin^2 i})+3\sin^2 i}.$$

The equality $\cos^2\gamma_0=0$ ($\cos^2\gamma_0=1$) is satisfied along the curve $\lambda_1(i)$ ($\lambda_2(i)$). The region within which the inequality $0<\cos^2\gamma_0<1$ is satisfied is crosshatched in Fig. 2. Curves $\lambda_1(i)$ and λ_2 (i) intersect at point P with coordinates

$$\sin^2 i = \frac{15-8\sqrt{3}}{3} \approx 0.381, \quad \lambda = 4\frac{2-\sqrt{2(2-\sqrt{3})}}{4(2-\sqrt{2(2-\sqrt{3})})+15-8\sqrt{3}} \approx 0.816.$$

Subsequently in this section, we consider that γ_0 and consequently Ω_0 are determined by Eq. (19). /12

Expansion of the following functions in a Fourier series is subsequently required:

$$\frac{1}{H^2} = \frac{1}{\sqrt{1+3\sin^2 i}}\left(1+2\sum_{n=1}^{\infty} \varkappa^{2n}\cos 2nu\right),$$

$$\frac{\sin u}{H^2} = \frac{2}{\sqrt{3}\sin i\sqrt{1+3\sin^2 i}}\sum_{n=0}^{\infty} \varkappa^{2n+1}\sin(2n+1)u,$$

$$\frac{\sin^2 u}{H^2} = \frac{1}{3\sin^2 i\sqrt{1+3\sin^2 i}}\left(\sqrt{1+3\sin^2 i}-1-2\sum_{n=1}^{\infty} \varkappa^{2n}\cos 2nu\right),$$

$$\frac{\cos u}{H^2} = \frac{2}{\sqrt{3}\sin i}\sum_{n=0}^{\infty} \varkappa^{2n+1}\cos(2n+1)u,$$

$$\frac{\cos^2 u}{H^2} = \frac{\sqrt{1+3\sin^2 i}-1}{3\sin^2 i} + \frac{2\sqrt{1+3\sin^2 i}}{3\sin^2 i}\sum_{n=1}^{\infty} \varkappa^{2n}\cos 2nu.$$

Here is introduced the notation

$$\varkappa = \frac{\sqrt{1+3\sin^2 i}-1}{\sqrt{3}\sin i}$$

The expression for Ω_1 can now be written. It has the form

$$\Omega_1(u) = d_1\sum_{n=1}^{\infty}\frac{\varkappa^{2n}}{2n}\sin 2nu - d_2\sum_{n=0}^{\infty}\frac{\varkappa^{2n+1}}{2n+1}\cos(2n+1)u + \Omega_1^0,$$

where

$$d_1 = \frac{4\sin 2\gamma_0}{\lambda^2\sqrt{1+3\sin^2 i}}\left[-2(1-\lambda)\sin^2\gamma_0\cos^2 i - \lambda\sin^2 i + \frac{32}{3}(1-\lambda)\cos^2\gamma_0\right],$$

$$d_2 = -\frac{2\left[16(1-\lambda)\sin^2\gamma_0 - \lambda\right]\cos i\cos\gamma_0}{\sqrt{3}\sqrt{1+3\sin^2 i}}.$$

The value of Ω_1^0 will be determined below. By substituting the expression found for $\Omega_1(u)$ in the equations of system (17) for $W_{1,1}$, $W_{2,1}$, γ_1, α_1, we obtain a system of linear heterogeneous equations with periodic free terms. The corresponding uniform equations coincide with the equations of system (15), in which the equation $\Delta\dot{\Omega}=0$ should be excluded, $\Delta\Omega=0$ should be set, and the variables should be ignated. We assume that Eq. (16) does not have a root of the p=k$\sqrt{-1}$ with any whole k, i.e.,

$$k^4-k^2a+b\neq 0, \quad k=0,1,2,\ldots \qquad (20)$$

eterogeneous system in question then has the unique 2π periodic /13
ion $W_{1,1}(u)$, $W_{2,1}(i)$, $\gamma_1(u)$, $\alpha_1(u)$. This solution can be found e form of trigonometric series of the form

$$\sum_{n=0}^{\infty}(b_n \cos nu + b_n' \sin nu) \qquad (21)$$

so can be shown that, upon satisfaction of condition (20), such s exist and are unique. From the condition of 2π periodicity of

ion $\Omega_2(u)$, we find $\Omega_2^0 = \sum$ [illegible]. An example of construction of the

ion in the form of trigonometric series in explicit form will be nted in Section 6. The solution for arbitrary k is constructed similar manner, if all solutions to some k-1 inclusive are found.

We find the values of $\lambda=\lambda(\sin^2 i)$ at which condition (12) is ted. It can be shown that J=0 when and only when system (15) ts a nontrivial solution which satisfies the boundary conditions

$$\Delta W_1(2\pi)=\Delta W_1(0), \Delta W_2(2\pi)=\Delta W_2(0), \Delta\Omega(2\pi)=\Delta\Omega(0), \Delta\gamma(2\pi)=\Delta\gamma(0), \Delta\alpha(2\pi)=\Delta\alpha(0).$$

by writing out the general solution of system (15), we find that nen and only when solution (16) has the root p=k$\sqrt{-1}$ with some k. Thus, condition (12) is equivalent to condition (20). Calions have shown that condition (20) is violated at k=0 on the curve (Fig. 2). At k=1, condition (20) is violated on the rese curves designated by the dashed lines. If k>1, the resonance s pass through point p, but they lie outside the region of existof generating solution $\bar{a}$.

ical Study of Satellite Motion Generated from Solution (6)

For arbitrary values of parameter kg, we construct solution (13) numerically, by solving boundary value problem (2), (9), and we investigate its dependence on parameters kg, λ. The solution of this boundary value problem is reduced to solution of system (10). Here and subsequently, system (10) is considered a system of equations which defines the curve in space $R^6(a, kg)$ with λ=const [7]. System (10) was solved numerically by the method of Newton, in which both a and kg were refined at each step. For calculation of functions $g(a, kg)$, $\frac{\partial g(a, kg)}{\partial a}$, which are used in the method of Newton, system (2) and the system in variations corresponding to it were integrated in the interval $0 \leq u \leq 2\pi$. The solutions of boundary value problem (2), (9) presented in Fig. 5 were found by this method. Figure 5 also presents the dependence of $a_1, \ldots, a_5$ on kg with λ=const, $i=\pi/2$. It is easy to obtain the other curves by means of substitution (3). Here and subsequently, the number beside the curve in the figures designates the value of fixed parameter λ. With $i=\pi/2$, the right sides of the equations in system (2) are π periodic functions of the true anomaly, which permitted the substitution $2\pi \to \pi$ to be performed in boundary conditions (9) and permitted restriction to the integration interval $0 \leq u \leq \pi$. /14

For convenience, we will call this method of construction of curves in space $R^6(a, kg)$ extension by parameter kg. Three types of solution of stationary rotation (6), obtained by extension by kg, can be distinguished. In the interval $4/7<\lambda<2/3$, solution (13) is extended right up to merger with the plane solution which describes the motion of the axis of symmetry of the satellite in the orbital plane. Branching of the solution within the region of its existence does not occur. In the interval $2/3<\lambda<8/11$, solution (13) also is extended right up to merger with the plane solution, but branching of this solution within the region of its existence occurs. In the interval $8/11<\lambda<4/5$, merger of solution (13) with the plane solution does not occur. This solution "escapes" to the region of larger values of kg. Interval $4/7<\lambda<4/5$ of existence of π periodic solutions is broken down into the intervals indicated by points $\lambda=2/3$ and $\lambda=8/11$, from which the resonance curves for k=1 originate.

Amplitude curves of the solutions obtained are presented in Fig. 5. We understand amplitude here to be the quantity

$$\theta_m = \max_{0 \le u \le T} \arccos(\vec{e}_z, \vec{r}), \tag{22}$$

where $\vec{e}_z$, $\vec{r}$ are unit vectors along the Oz axis and the axis of stationary rotation of the satellite respectively. The position of the latter in space is determined by relationships (19) and $\alpha_0=0$.

The stability of the solutions obtained was investigated in the following manner. The system in variations along solution (13) which corresponds to system (2) was integrated. Roots $\rho_1, \ldots, \rho_5$ of the characteristic equation for the system in variations were calculated. Degree of stability λ_s of the resulting periodic solution, which determines the response speed of the system, was calculated by the equation /15

$$\lambda_s = -\frac{1}{T} \ln \max_{1 \le i \le 5} |\rho_i|$$

The condition $\lambda_s>0$ $(\lambda_s<0)$ corresponds to a stable (unstable) solution. Sections of the λ_s curves which $\lambda_s>0$ are presented in Fig. 6 with various λ. The nature of the roots which determine λ_s changes at the break points of the curves. The curves marked with hachures in Fig. 5 [sic] correspond to stable solutions.

We extend the periodic motions of the satellite plotted with $i=\pi/2$ by parameter i for $kg=kg^*>0$. It can be proved by the Poincare small parameter method that, because of analytical nature of the right side of system (2) with respect to Z and i, with sufficiently small

$|i-\pi/2|<<1, |a-\tilde{z}|<<1$, function $g(a,i,kg)$ depends analytically on i, a in the vicinity of point $i=\pi/2$, $a=\tilde{Z}_0$, $kg=kg^*$. Here, $\tilde{Z}_0$ is solution (13) with $i=\pi/2$, $u=0$. If

$$J = \det \left\| \frac{\partial g(\tilde{z}, \pi/2, kg^*)}{\partial a} \right\| \neq 0,$$

according to the implicit function theorem, with sufficiently small $|i-\pi/2|$, Eq. (10) has the unique solution

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$$z = \bar{z}(u, \bar{z}(k_g^*, i), i, k_g^*), \qquad (23)$$

which depends analytically on i in the vicinity of point $i=\pi/2$ and coincides at this point with solution (13) with $i=\pi/2$, which was constructed above. This same solution can be obtained by extension of solution (13) by parameter kg with $i=i^*$. This method of extension of periodic solutions in the (i, kg) plane was described in detail in [8].

Solution (23) was constructed numerically. The results of the calculations, which were performed for kg=0.2 and several values of λ, are presented in Fig. 7. Curves of the initial values of phase variables θ_m and λ_s are represented here by the solid lines. The curves of the tabulated values for the solutions which are not characterized by rotation, but by oscillations of the satellite around its axis of symmetry, are represented by the dashed lines. Such solutions were constructed in [1]. Solution (23) exists right up to the point of merger with the solution, the curves of which are designated by the dashed lines.

The explicit form of periodic motion at λ=0.63, kg=0.2, i=1.37 is presented in Fig. 8. Curves of the phase variables, angle θ and the trace of the Oz axis on a unit sphere which surrounds point 0 are presented here for $0 \leq n \leq 2$, n is the number of orbits. Angle θ is determined by the expression (see Eq. (22))

$$\vartheta = \arccos(\bar{e}_z, \bar{z}).$$

Where possible, the curves of the corresponding stationary solution $\bar{a}$ are designated by dashed lines and the symbol (*). The arrow on the curve in the (α,γ) plane indicates the direction of motion of the trace of the Oz axis with increase in u. The points on the curve are 0.1 orbit apart. The values of γ_0 and Ω_0 were determined from Eq. (19) and (6). /16

5. Satellite Motion Close to Stationary Rotation (7)

By using the algorithm and notation of Section 4, we will seek a solution of system (2) in the form of integer power series (14) of parameter kg. As solution $\bar{a}$ of system (10) with kg=0, we select stationary rotation (7):

$$\bar{a}_1 = \sin\alpha_0,\ \bar{a}_2 = 0,\ \bar{a}_3 = -\cos\alpha_0/\lambda,\ \bar{a}_4 = \pi/2,\ \bar{a}_5 = \alpha_0. \qquad (24)$$

The equations in variations for the stationary solution selected have the form

$$\begin{gathered}\Delta\dot{W}_1 = 3(1-\lambda)\cos\alpha_0\Delta\gamma,\ \Delta\dot{W}_2 = -\frac{\sin^2\alpha_0}{\cos\alpha_0}\Delta W_1 + \lambda\sin\alpha_0\Delta\Omega,\\ \Delta\dot{\Omega} = 0,\ \Delta\dot{\gamma} = \frac{\Delta W_1}{\cos\alpha_0} - \Delta\alpha,\ \Delta\dot{\alpha} = \Delta W_2 + \Delta\gamma.\end{gathered} \qquad (25)$$

The characteristic equation of system of equations (25) is broken down into the equation p=0 and

$$p^4 + (3\lambda-2)p^2 - 3(1-\lambda)\sin^2\alpha_0 = 0. \qquad (26)$$

For determination of the forced solution of system (2), we substitute series (14), where the first terms are determined by Eq. (24), in Eq. (2), and we equate the terms for identical powers of kg. We obtain a series of systems, the general form of which is the following

$$\begin{aligned}&\dot{W}_{1,k} = 3(1-\lambda)\cos\alpha_0\gamma_k + T_{1,k},\\ &\dot{W}_{2,k} = -\frac{\sin^2\alpha_0}{\cos\alpha_0}W_{1,k} + \lambda\sin\alpha_0\cdot\Omega_k + T_{2,k},\\ &\dot{\Omega}_k = \frac{1}{\lambda}T_{3,k},\ \dot{\gamma}_k = \frac{W_{1,k}}{\cos\alpha_0} - \alpha_k + T_{4,k},\ \dot{\alpha}_k = W_{2,k} + \gamma_k + T_{5,k}.\end{aligned}$$

Here, $T_{i,k}$ ($i=1, \ldots, 5$) are some functions $W_{1,0}, \ldots, W_{1,k-1}, \ldots, \alpha_0, \ldots, \alpha_{k-1}, u$, $T_{i,0}=0$. Similarly to the way it was done in Section 4, for determination of the constant value of Ω_0 and consequently, the value of $\cos\alpha_0$, we write the equation for $\Omega_1(u)$

$$\begin{gathered}\dot{\Omega}_1(u) = \frac{1}{\lambda}\Big[-\frac{1-\lambda}{\lambda}\cos\alpha_0(h_{11}^2\sin^2\alpha_0 - 2h_{11}h_{21}\sin\alpha_0\cos\alpha_0 + h_{21}^2\cos^2\alpha_0 - 1) +\\ + \sin\alpha_0(\omega_2^N h_{12} + \omega_3^N h_{13}) - \cos\alpha_0(\omega_2^N h_{22} + \omega_3^N h_{23})\Big].\end{gathered} \qquad (27)$$

The condition of 2π periodicity of function $\Omega_1(u)$

$$\int_0^{2\pi} T_{4,2}(\tau)\,d\tau = 0$$

is reduced to the equalities $\cos\alpha_0=0$ or /17

$$\cos^2\alpha_0 = \frac{(1-\lambda)\sqrt{1+3\sin^2 i}\,\left(\sqrt{1+3\sin^2 i}-4\right)+6\lambda\sin^2 i}{(1-\lambda)\sqrt{1+3\sin^2 i}\,\left(\sqrt{1+3\sin^2 i}-1\right)-3\cos^2 i} \qquad (28)$$

The equality $\cos\alpha_0=0$ is satisfied with any permissible λ and i. It corresponds to orientation of the axis of symmetry of the satellite along the velocity vector of its center of mass. Investigation of this generating solution can be carried out within the framework of another sequence of rotations α, β, γ, that presented in [1] for example. With $\lambda>1$, such a solution is stable. From the condition $0\leq\cos\alpha^2\leq1$, we find the region of existence of generating solution α (24) determined by equality (28) in the $(\sin^2 i, \lambda)$ plane. The boundaries of the region are fixed by the $\bar{\lambda}_1(i)$, $\bar{\lambda}_2(i)$ curves, which are determined by the expressions

$$\bar{\lambda}_1(i) = \frac{\sqrt{1+3\sin^2 i}\,\left(4-\sqrt{1+3\sin^2 i}\right)}{\sqrt{1+3\sin^2 i}\,\left(4-\sqrt{1+3\sin^2 i}\right)+6\sin^2 i}, \quad \bar{\lambda}_2(i) = \frac{\sqrt{1+3\sin i}-\cos^2 i}{\sqrt{1+3\sin^2 i}+3\sin^2 i-1}.$$

The equality $\cos^2\alpha_0=0$ $(\cos^2\alpha_0=1)$ is fulfilled along the $\bar{\lambda}_1(i)$ $(\bar{\lambda}_2(i))$ curve. The region bounded by the $\bar{\lambda}_1(i)$, $\bar{\lambda}_2(i)$ curves is crosshatched in Fig. 3. The $\bar{\lambda}_1(i)$ and $\bar{\lambda}_2(i)$ curves intersect at point $\bar{p}$, with coordinates $\sin^2 i = (9+\sqrt{33})/24 \approx 0{,}615$,

$$\lambda = \frac{6\sqrt{2}\sqrt{17+\sqrt{33}}+\sqrt{33}-15}{3\left(2\sqrt{2}\sqrt{17+\sqrt{33}}+\sqrt{33}+1\right)} \approx 0{,}514.$$

By using the expansion of the functions included in the right side of Eq. (27) in a Fourier series, we write the solution of Eq. (27) in the form

$$\Omega_1(u) = \bar{a}_1\sum_{n=1}^{\infty}\frac{x^{2n}}{2n}\sin 2nu + \bar{a}_2\sum_{n=0}^{\infty}\frac{x^{2n+1}}{2n+1}\sin(2n+1)u,$$

where

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$$\bar{\alpha}_1 = -\frac{2\cos\alpha_0}{3\lambda^2\sqrt{1+3\sin^2 i}}\left[(1-\lambda)(\sin^2\alpha_0 + 3\sin^2\alpha_0 \sin^2 i + 3\cos^2\alpha_0 \cos^2 i) + \lambda\sin^2 i\right]$$

$$\bar{\alpha}_2 = \frac{8\sin\alpha_0\left[(1-\lambda)\cos^2\alpha_0 - \lambda\right]\cos i}{\sqrt{3}\lambda^2}$$

For determination of functions $W_{1,1}$, $W_{2,1}$, γ_1, α_1, we obtain a system of linear heterogeneous equations with periodic free terms. The corresponding uniform equations coincide with Eq. (25), where the equation $\Delta\dot{\Omega}=0$ should be excluded and $\Delta\Omega=0$ should be placed in the remaining equations. If Eq. (26) does not have the root $p=k\sqrt{-1}$ at any integer k, i.e.,

$$k^4-(3\lambda-2)k^2-3(1-\lambda)\sin^2\alpha_0 \neq 0, \tag{29}$$

the resulting system of heterogeneous equations then has the unique 2π periodic solution $W_{1,1}(u)$, $W_{2,1}(u)$, $\gamma_1(u)$, $\alpha_1(u)$. It can be found in the form of a trigonometric series of the type of Eq. (21). A series for arbitrary k is constructed in a similar manner, if all solutions to some k-1 inclusive are found. /18

The calculations showed that condition (29) is violated at k=0 (k=1) on the $\bar{\lambda}_2(i)$ $(\bar{\lambda}_1(i))$ curve. If k>1, the resonance curves pass through point $\bar{p}$ and lie outside the region of existence of generating solution $\bar{a}$ (24).

For arbitrary values of parameter kg, construction of 2π periodic motions of the satellite is reduced to numerical solution of system (10) and extension by kg of the solution constructed in form (14), which is close to stationary rotation (24). In the case of a polar orbit, the results of extension of solution (13) by kg, in the form of curves of functions a_1, . . ., a_5 with λ=const, are presented in Fig. 9. The initial conditions of the solutions obtained from the solutions found by transformation (3) can be plotted by symmetrical representation of the curves (Fig. 9) relative to the corresponding axes.

Investigation of the stability showed that all the solutions constructed are unstable. The amplitude characteristics are presented in Fig. 10. They were determined by Eq. (22), where the position $\vec{r}$ is determined by relationships (28) and $\gamma_0=\pi/2$.

6. Satellite Motion Close to Stationary Rotation (8)

By using the algorithm and notation of Section 4, we seek solution (13) of system (2), generated from stationary solution (8), in the form of series (14). We select stationary rotation (8) as solution $\bar{a}$ of system (10) with kg=0:

$$\bar{a}_1=0,\ \bar{a}_2=0,\ \bar{a}_3=\Omega_0,\ \bar{a}_4=\pi/2,\ \bar{a}_5=0. \qquad (30)$$

The value of Ω_0 is subject to determination. The equations in variations for the stationary solution selected have the form

$$\begin{aligned} &\Delta\dot{W}_1=-(1+\lambda\Omega_0)\Delta W_2+3(1-\lambda)\Delta\gamma,\\ &\Delta\dot{W}_2=(1+\lambda\Omega_0)\Delta W_1,\quad \Delta\dot{\Omega}=0,\\ &\Delta\dot{\gamma}=\Delta W_1-\Delta\alpha,\quad \Delta\dot{\alpha}=\Delta W_2+\Delta\gamma. \end{aligned} \qquad (31)$$

The characteristic equation of system (31) is broken down into the equations p=0 and

$$p^4-p^2[(\lambda\Omega_0+1)^2+3\lambda-2]+(\lambda\Omega_0+4-3\lambda)(\lambda\Omega_0+1)=0. \qquad (32)$$

For determination of the coefficients of series (14), we obtain a /19
series of systems, the general form of which is easily described from Eq. (17), (31). The condition of existence of a 2π periodic solution of the equation

$$\dot{\Omega}_1(u)=-\frac{1}{\lambda}\left[\frac{(3+\Omega_0)\sin^2 i}{H^2}+3(1+\Omega_0)\sin^2 i\,\frac{\sin^2 u}{H^3}\right] \qquad (33)$$

is reduced to the expression

$$\Omega_0=-\frac{\sqrt{1+3\sin^2 i}-1+3\sin^2 i}{\sqrt{1+3\sin^2 i}-1+\sin^2 i}.$$

The solution of Eq. (33) can be written in the form

$$\Omega_1(u) = -\frac{4\sin^2 i}{\lambda(\sqrt{1+3\sin^2 i}-1+\sin^2 i)}\sum_{m=1}^{\infty}\frac{x^{2m}}{2m}\sin 2mu+\Omega_1^0$$

If Eq. (32) does not have the root $p=k\sqrt{-1}$ at any integer k, i.e.,

$$k^4-k^2[(\lambda\Omega_0+1)^2+3\lambda-2]+(\lambda\Omega_0+4-3\lambda)(\lambda\Omega_0+1)\neq 0,\quad k=0,1,2,\ldots \tag{34}$$

Then the linear system relative to variables $W_{1,k}$, $W_{2,k}$, γ_k, α_k, with periodic free terms, has a unique 2π periodic solution. For k=1, we write this solution in the following way

$$W_{1,1}=\sum_{n=0}^{\infty}[b_n-(2n+1)d_n]\sin(2n+1)u,$$
$$W_{2,1}=\sum_{n=0}^{\infty}[b_n(2n+1)-d_n]\cos(2n+1)u,$$
$$\gamma_1=\sum_{n=0}^{\infty}d_n\cos(2n+1)u,\quad \alpha_1=\sum_{n=0}^{\infty}b_n\sin(2n+1)u.$$

where,

$$b_n=\bar b_n[(2n+1)^2+2(2n+1)(\lambda\Omega_0+2)+\lambda\Omega_0+4-3\lambda],$$
$$d_n=\bar b_n[2(2n+1)^2+(2n+1)(\lambda\Omega_0+2)+2(\lambda\Omega_0+1)],$$
$$\bar b_n=\frac{2\cos i}{\sqrt3}\cdot\frac{\sqrt{1+3\sin^2 i}-1}{\sqrt{1+3\sin^2 i}-1+\sin^2 i}\cdot\frac{x^{2n+1}}{(2n+1)^4-(2n+1)^2[(\lambda\Omega_0+1)^2+3\lambda-2]+(\lambda\Omega_0+4-3\lambda)(\lambda\Omega_0+1)}$$

From the condition of existence of 2π periodic function $\Omega_2(u)$, the equation for determination of which is not presented here because of the cumbersome form, we obtain $\Omega_1^0=0$. The solution can be constructed in the same manner, in the form of series for arbitrary k, if all solutions up to k-1 inclusive are known.

Thus, in the two preceding sections, investigation with arbitrary kg of solution (13) generated from Eq. (30) was reduced to numerical solution of system (10) with λ=const. The calculation results are presented in Fig. 11, in the form of a_3 curves for several values of λ and i=π/2. Condition (34) is violated in the curves presented in Fig. 12.

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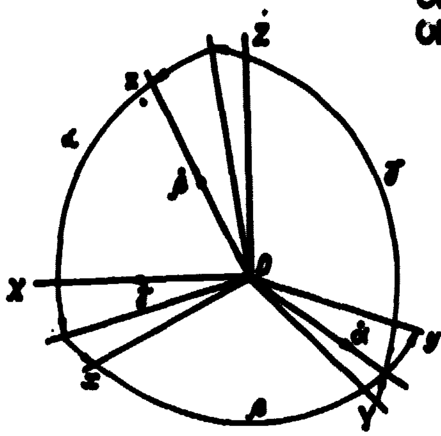

Fig. 1.

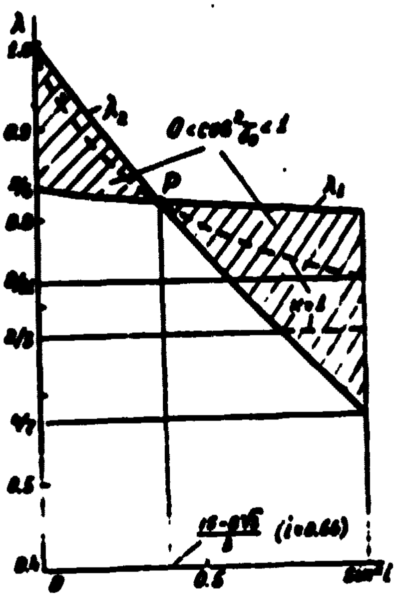

Fig. 2.

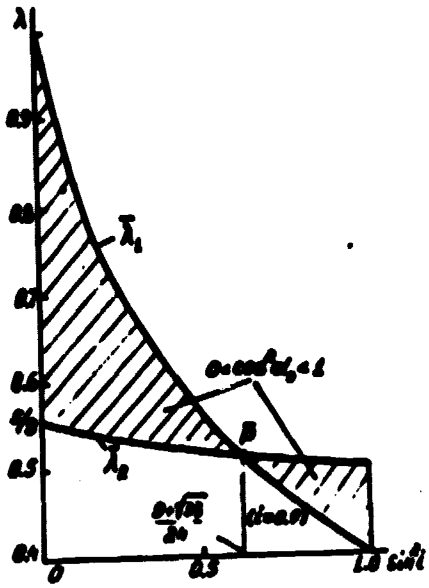

Fig. 3.

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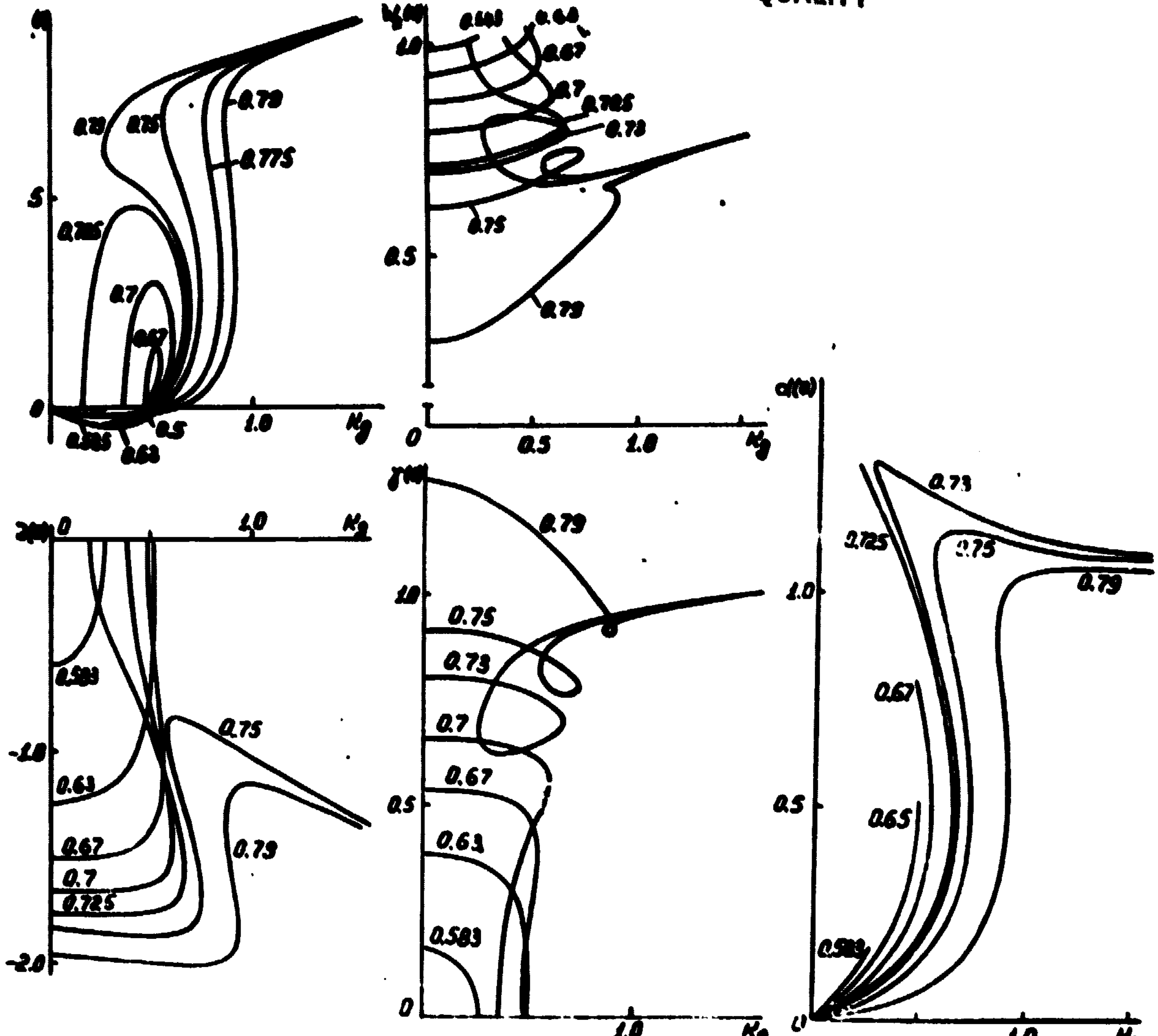

Fig. 4. (i=π/2)

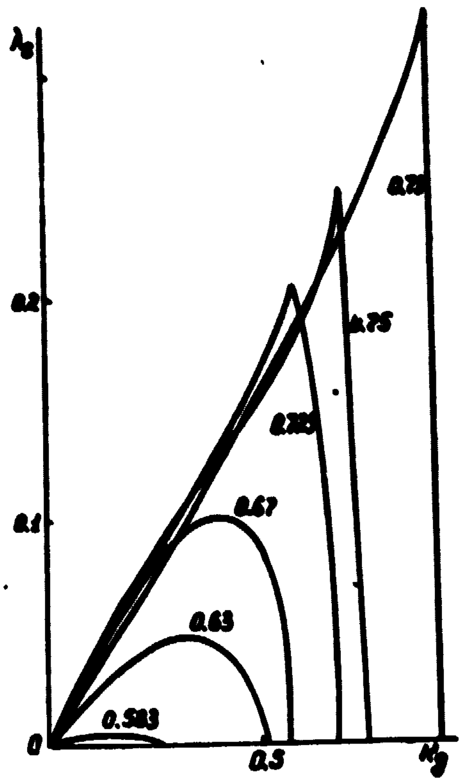

Fig. 5. (i=π/2)

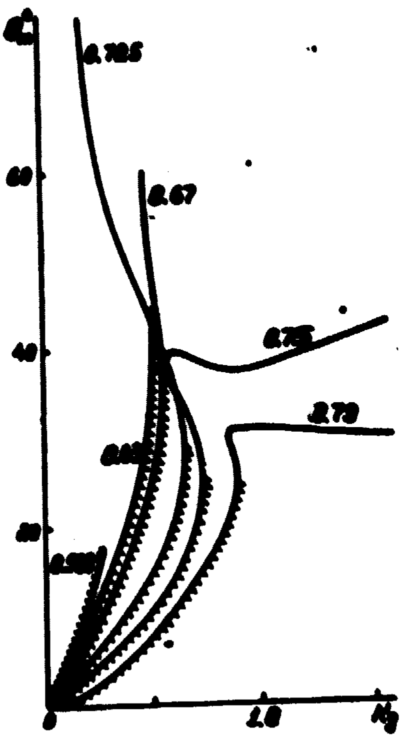

Fig. 6. (i=π/2)

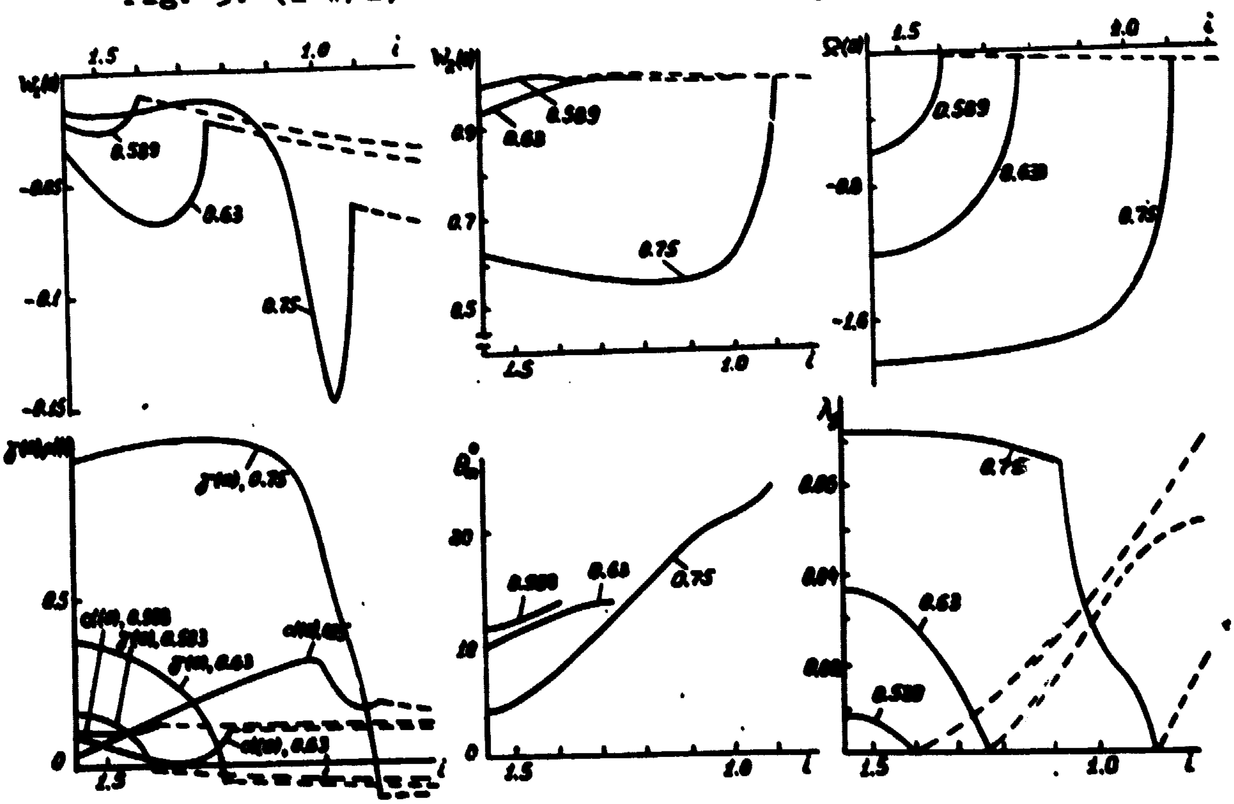

Fig. 7. (kg=0.2)

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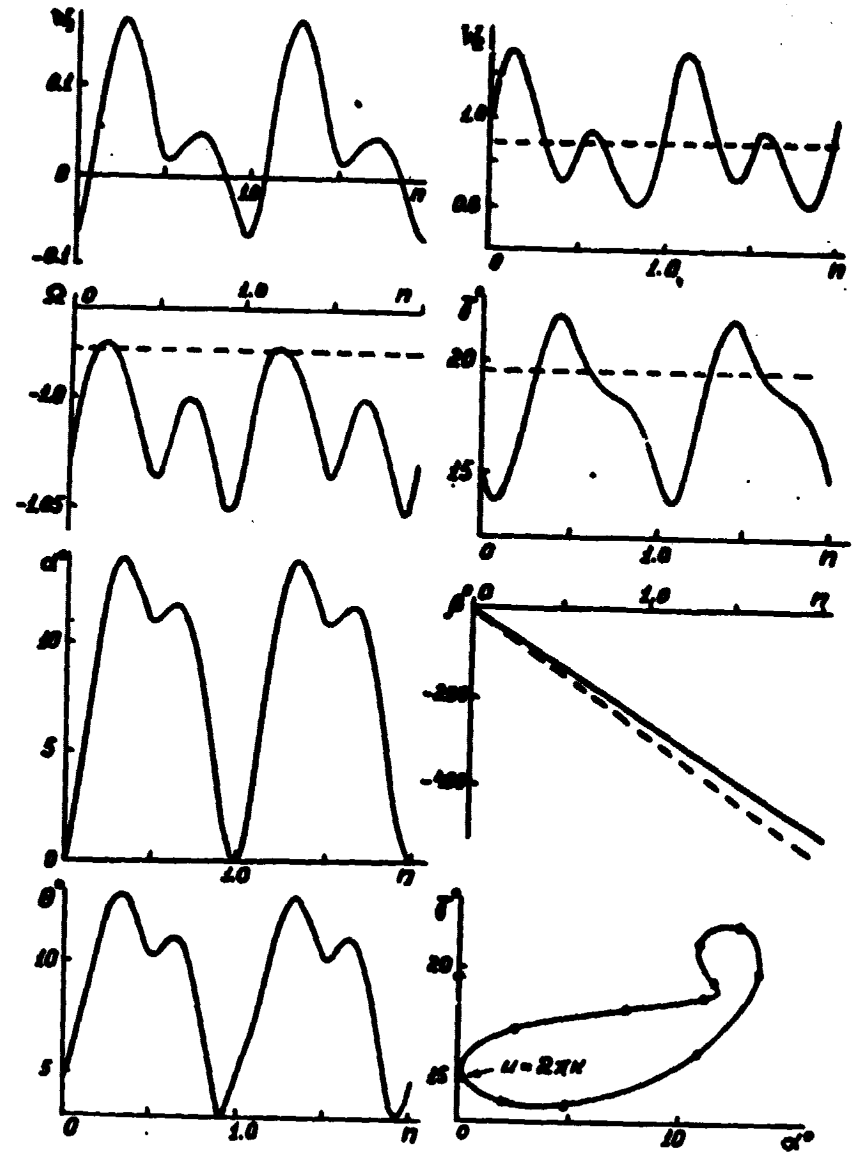

Fig. 8. (λ=0.63, kg=0.2, i=1.37)

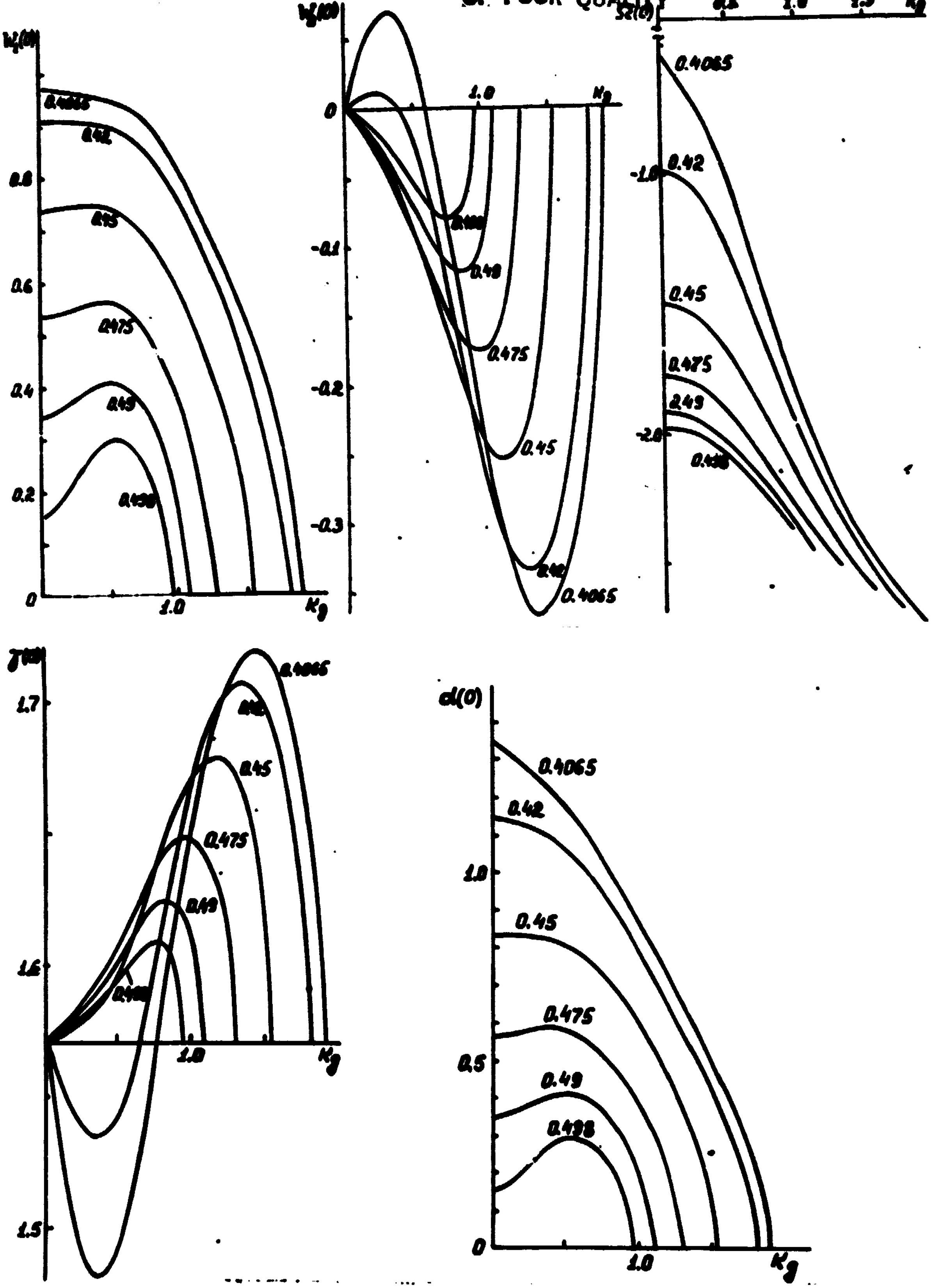
0.4065
0.42
0.45
0.475
0.49
0.498
1.0
0.5
0
-0.1
-0.2
-0.3
0.8
0.6
0.4
0.2
1.7
1.6
1.5
-1.0
-2.0
1.5
K_g
$d(0)$

Fig. 9. (1=π/2)

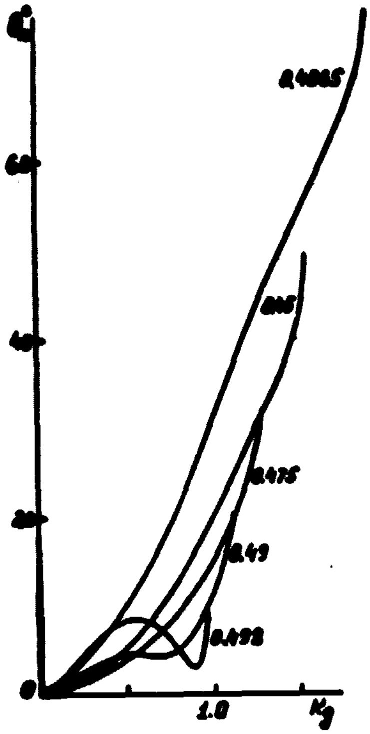

Fig. 10. (i=π/2)

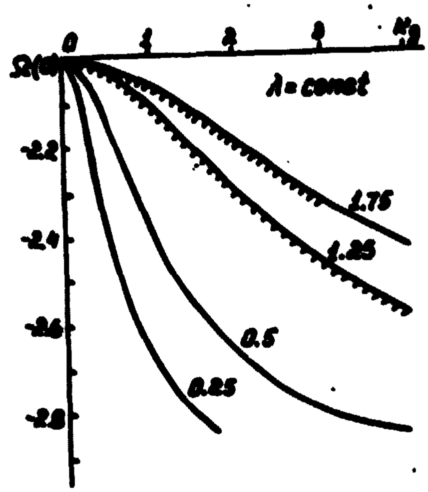

Fig. 11. (i=π/2)

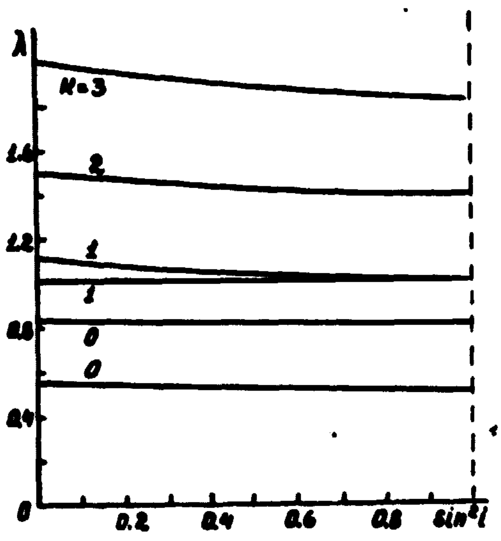

Fig. 12.