NASA TM.77075

NASA TECHNICAL MEMORANDUM NASA TM-77075

NASA-TM-77075 19830023222

SOLITONS IN SEMICONDUCTORS WITH A SUPERLATTICE

A. P. Tetervov

Translation of "Solitony v poluprovodnikakh so sverkhreshetkoy". Ukrainskiy Fizicheskiy Zhurnal, Vol. 23, No. 7, July 1978, pp 1182-1187.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 MAY 1983

STANDARD TITLE PAGE

1. Report No. NASA TM-77075	2. Government Accession No.	3. Recipient's Catalog No.
4. Title and Subtitle SOLITONS IN SEMICONDUCTORS WITH A SUPPERLATTICE		5. Report Date May 1983
		6. Performing Organization Code
7. Author(s) A. P. Tetervov		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address SCITRAN Box 5456 Santa Barbara, CA 93108		11. Contract or Grant No. NASw- 3542
		13. Type of Report and Period Covered Translation
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of "Solitony v poluprovodnikakh so sverkhreshetkoy". Ukrainskiy Fizicheskiy Zhurnal, Vol. 23, No. 7, July 1978, pp. 1182-1187.

16. Abstract

The nonlinear parabolic equation describing the propagation of of the electromagnetic wave in a semiconductor with the superlattice is analyzed. The possibility of the existence of the solitary waves is proved both for a small amplitude of the electrical field and the latter moderate values.

17. Key Words (Selected by Author(s))	18. Distribution Statement Unclassified - Unlimited

19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 11	22. Price

A78-51656 (ORIGINAL)
N83-31493#
N-151,919

SOLITONS IN SEMICONDUCTORS WITH A SUPERLATTICE

A. P. Tetervov

/1182*

Several studies [1-4] have recently appeared which investigated the properties of semiconductors with superlattices (SL) in a variable electromagnetic field. The presence of a SL leads to very specific nonlinear effects, which are particularly apparent in the millimeter and submillimeter ranges. However, these studies devoted basic attention to calculating the nonlinear high frequency current.

The propagation of strong electromagnetic waves in semiconductors with SL is of definite interest. This process may be described by a system of Maxwell equations with a known conductivity current. The nonlinear nature of this current leads to the formation of nonlinear effects during the propagation. This article studies the possibility of nonlinear electromagnetic waves, namely solitons, existing in SL.

We use the following wave equation as the initial equation

$$\left[\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\varepsilon(\hat{\omega})\right]E = \frac{4\pi}{c^2}\frac{\partial j}{\partial t}, \qquad (1)$$

where E is the wave electric field (the field vector directed along the SL period parallel to the 0x axis); $j = j(E)$ — the functional of the field, which describes the conductivity current; $\varepsilon(\hat{\omega})$ — linear operator determining the frequency dispersion of the lattice, existing in the millimeter and submillimeter ranges [5] $\hat{\omega} = \omega - i\frac{\partial}{\partial t}$.

* Numbers in margin indicate pagination in foreign text.

We shall assume that the conductivity current is small as compared with the displacement current, i.e., the right side of Equation (1) contains a small parameter (the corresponding estimates will be given below). Then we shall look for the solution of the wave equation in the form

$$E = \{\psi(\vec{r}, t)\exp[i(\omega t - kx)] + w(\vec{r}, t)\} + \text{к. с.} \qquad (2)$$

Here $\psi(\vec{r}, t) = u(\vec{r}, t)\exp[i\varphi(\vec{r}, t)]$; u and ϕ are the amplitude and phase of the basic harmonics which slowly change in time and space; ω and k— frequency and wave vector of the waves; $w(\vec{r}, t)$ — value of the first order of smallness with respect to the conductivity current which describes the higher harmonics.

Let us expand the current in Fourier series

$$j = \sum_{l=-\infty}^{\infty} j_l \exp(iS_l), \qquad (3)$$

where $S_l = l[\omega t - kx + \varphi(\vec{r}, t]$, and we place the expansion (3) in the wave equation. Considering the slow change in the amplitude and phase of the wave we obtain the following equation /1183

$$\left\{\left[\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\varepsilon(\omega)\right]w(\vec{r}, t) - \frac{4\pi i\omega}{c^2}\sum_{l \neq 1} l j_l \exp(iS_l)\right\} =$$
$$= -\left\{\Delta_\perp\psi - 2ik\left(\psi_x + \frac{1}{v_r}\psi_t\right) + k\frac{v_r'}{v_r}\psi_{xx} - \right.$$
$$\left. - \frac{4\pi i\omega}{c^2} j_1 \exp(i\varphi)\right\}\exp[i(\omega t - kx)]. \qquad (4)$$

Here $\Delta_\perp = \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $v_r = \frac{\partial\omega}{\partial k}$ — group velocity of the wave; $v_r' = \frac{\partial^2\omega}{\partial k^2}$, and the lower index of the variable ψ designates differentiation with respect to the corresponding argument. Considering the slow change in the amplitude and phase of the wave, we may set $\psi_{tt} = v_r^2\psi_{xx}$. The component which is proportional to ψ_{xx}, describes the dispersion of the wave responsible for the formation of the soliton.

The condition that $w(\vec{r}, t)$ does not contain the first harmonics leads to the nonlinear parabolic equation [6], which describes the behavior of the complex amplitude of the basic harmonics

$$i\left(\psi_t + v_r \psi_x\right) - \frac{1}{2}\left(v_r' \psi_{xx} + \frac{v_r}{k}\Delta_\perp \psi\right) + i\frac{2\pi\omega v_r}{c^2 k} j_1(u)\, e^{i\varphi} = 0. \qquad (5)$$

As was shown in [3], for the quasiclassical case ($\Delta_s \gg \hbar\omega$, $\hbar\tau_{\mathrm{eff}}^{-1}$, eEd, where d— the SL period; Δ_s — width of the sth permitted minizone, and τ_{eff}— does not depend on the field and energy of the carriers of the pulse relaxation time) in the single minizone approximation, the expression for $j_i(E)$ has the form

$$j_1 = j_{-1}^* = -ij_{\mathrm{st}} \sum_{m=-\infty}^{\infty} J_m(a)\, J_{m+1}(a) \frac{1}{1 - im\omega\tau_{\mathrm{eff}}}, \qquad (6)$$

where $j_{\mathrm{st}} = \frac{\hbar\sigma_0}{ed\tau_{\mathrm{eff}}}$ — static current. $\sigma_0 = \frac{\omega_0^2 \tau_{\mathrm{eff}}}{4\pi}$, $\omega_0 = \left[\frac{2\pi e^2 n d^2 \Delta_1}{\hbar^2} \times I_1\left(\frac{\Delta_1}{T}\right) I_0^{-1}\left(\frac{\Delta_1}{T}\right)\right]^{1/2}$ — generalized plasma frequency in a narrow zone semiconductor; n— concentration of the carriers; $I_{0,1}$— modified Bessel functions with the corresponding index; $J_m(a)$— Bessel function of the real argument $a = \frac{edu}{\hbar\omega}$. At the high frequency limit, when $\omega\tau_{\mathrm{eff}} \gg \gg 1$, it follows from (6) that

$$A(a) = \frac{\mathrm{Re}\, j_1}{j_{\mathrm{st}}} = \frac{2}{a}\frac{1}{\omega\tau_{\mathrm{eff}}}\left[1 - J_0^2(a)\right], \qquad (7)$$

$$B(a) = \frac{\mathrm{Im}\, j_1}{j_{\mathrm{st}}} \approx -J_0(a)\, J_1(a). \qquad (8)$$

The quantity A(a) is the dissipative component of the variable current and describes the absorption of the electromagnetic wave by the free carriers and, as follows from the definition, $A(a) \ll 1$ for any type a. B(a) makes a small addition to the displacement current. /1184

Substituting (7), (8) into the parabolic equation, we obtain the following system of nonlinear equations with respect to the dimensionless amplitude a and the phase ϕ

$$(a^2)_t + v_r (a^2)_x + \left[v_r' (a^2\varphi_x)_x + \frac{v_r}{k} \vec{\nabla}_\perp (a^2 \vec{\nabla}_\perp \varphi)\right] + \mu a A(a) = 0,$$

$$\varphi_t + v_r\varphi_x + \frac{1}{2}\left[v_r'\varphi_x^2 + \frac{v_r}{k} (\vec{\nabla}_\perp \varphi)^2\right] - \frac{1}{2a}\left[v_r' a_{xx} + \frac{v_r}{k} \Delta_\perp a\right] - \mu \frac{B(a)}{2a} = 0, \qquad (9)$$

where $\mu = \frac{v_r \omega_0^2}{c^2 k}$.

We assume that the solution of the nonlinear parabolic equation with small damping has the form of a stationary plane wave with the amplitude a = $a^{(0)}$, and the phase $\varphi = \varphi^{(0)}$. Let us study the stability of the solution obtained with respect to the small oscillations. The solution of the linearized system (9) can be written in the form of waves $[i(\vec{\varkappa}\vec{r} - \delta t)]$, then

$$\delta = \varkappa_\parallel v_r \pm \frac{1}{2}\left(v_r'\varkappa_\parallel^2 + \frac{v_r}{k}\varkappa_\perp^2\right)\left[1 + 4v_r' s_0^2 \left(v_r'\varkappa_\parallel^2 + \frac{v_r}{k}\varkappa_\perp^2\right)^{-1}\right]^{1/2}, \qquad (10)$$

where $\varkappa_\parallel$ and $\varkappa_\perp$ are the longitudinal and transverse components of the perturbation wave vector and $s_0 = \left[-\frac{\mu}{2v_r'}\frac{d}{da}\left(\frac{B(a)}{a}\right)\right]^{1/2}\Big|_{a=a^{(0)}}$.

From an analysis of the expression obtained, we find that at $v_r' < 0$ and $s_0^2 < 0$ a plane wave is unstable with respect to the longitudinal perturbations with $\varkappa_\perp = 0$, and small values of $\varkappa_\parallel^2 < 4|s_0^2|$. As is known, this instability leads to self-modulation of the wave (its decomposition into solitons) [6].

If $a^{(0)} > 1$, , then from (8)

$$s_0^2 = \frac{\mu}{\pi v_r'} \frac{\sin 2a^{(0)}}{a^{(0)2}}, \qquad (11)$$

and then a plane wave with the amplitude ,

$$\pi m < a^{(0)} < \frac{\pi}{2}(2m+1), \quad m = 0, 1, 2, \ldots \qquad (12)$$

decomposes into solitons. We should note that the condition $a^{(0)}>1$ corresponds to a very strong field $E>\frac{\hbar\omega}{ed}$.

In the case $a^{(0)}\ll 1$ (or, which is the same thing, the wave field is very weak, i.e., $E\ll\frac{\hbar\omega}{ed}$), the sign of s_0^2, as can be readily shown, is determined by the sign of v_r'. At $v_r'>0$ the solution in the form of a plane wave is stable and at $v_r'<0$ the instability leads to self-modulation of the wave.

The sign of v_r' can be determined from the dispersion equation of linear theory, considering the lattice as a set of linear oscillators with an eigenfrequency of ω_1 and with the force Ω:

$$\frac{k^2c^2}{\omega^2}=\varepsilon(\omega)=\varepsilon_0-\frac{\Omega}{\omega^2-\omega_1^2}, \tag{13}$$

where ε_0 is the dielectric constant of the lattice. At $\omega^2\gg\omega_1^2$ /1185

$$v_r'\approx\frac{\Omega}{\omega k^2}\frac{\varepsilon(\omega)}{\varepsilon_0^2}>0, \tag{14}$$

and at $\omega^2\ll\omega_1^2$

$$v_r'\approx-2\frac{\omega^3}{\omega_1^2k^2}\frac{\Omega}{\Omega+\varepsilon_0\omega_1^2}<0. \tag{14a}$$

To determine the form of the soliton, we transformed the system (9) by changing $\xi=x-v_rt,\ \tau=v_r't$ to the form

$$(a^2)_\tau+(a^2\varphi_\xi)_\xi+\frac{\mu}{v_r'}aA(a)=0, \tag{15}$$

$$\varphi_\tau+\frac{1}{2}\varphi_\xi^2-\frac{\mu}{v_r'}\frac{B(a)}{2a}+\frac{a_{\xi\xi}}{2a}=0.$$

Disregarding the damping, the solution (15) can be sought as a function of $\eta=\xi-W\tau$, i.e., in the form of stationary waves. Here W is the "velocity" of a stationary wave, determined from the boundary conditions. It follows from the upper equation of the system (15)

$$\varphi_\eta=W\frac{a^2-a_1^2}{a^2}, \tag{16}$$

where a_1 is the integration constant. Substituting the expression obtained in the lower equation of System (15), we obtain the equation of the nonlinear oscillator:

$$a_{\eta\eta} - W^2\left(\frac{a^4 - a_1^4}{a^3}\right) - \frac{\mu}{v_r'} B(a) = 0. \tag{17}$$

Multiplying it by $2a_\eta$ and integrating, considering (8) we obtain

$$a_\eta^2 - W^2\left(a^2 + \frac{a_1^4}{a^2}\right) - \frac{\mu}{v_r'} J_0^2(a) = a_2, \tag{18}$$

where a_2 is a constant. The solution of Equation (18) can be represented in the form

$$\eta = \pm \int_{a_0}^{a(\eta)} \frac{da}{\left[a_2 + W^2\left(a^2 + \frac{a_1^4}{a^2}\right) + \frac{\mu}{v_r'} J_0^2(a)\right]^{1/2}} \tag{19}$$

(a_0 is the initial value of the wave amplitude).

If a increases with an increase in η, we must select the upper sign in (19). If it decreases, we select the lower sign. The study of the forms of stationary waves is similar to that of [7].

Let us consider the case of a weak field $a \ll 1$. Expanding the Bessel function in terms of the small argument, we reduce (19) to the form

$$\eta = \pm \int_{a_0}^{a(\eta)} \frac{da}{\left[\left(a_2 + \frac{\mu}{v_r'}\right) + \left(W^2 - \frac{\mu}{2v_r'}\right)a^2 + \frac{3}{32}\frac{\mu}{v_r'} a^4 + W^2 \frac{a_1^4}{a^2}\right]^{1/2}}. \tag{20}$$ /1186

At $a_2 = -\frac{\mu}{v_r'}$, $a_1 = 0$ (20) is readily integrated, as a result of which at $v_r' < 0$ we obtain the soliton equation

$$a(\eta) = \frac{4}{\sqrt{3}}\left(1 + 2W^2\frac{|v_r'|}{\mu}\right)^{1/2} \operatorname{sech}\left[\left(W^2 + \frac{\mu}{2|v_r'|}\right)^{1/2} \eta\right]. \tag{21}$$

The more general solutions of the equation (20) describe the stationary periodic waves and can be expressed in terms of elliptical functions.

There is a solution in the form of a single wave for large values of the wave amplitude. To prove this, we note that when the conditions $\frac{|a - a_0|}{a_0} \ll 1$ and $a_0 > 1$ are satisfied (18) assumes the form of the Equation of sin-Gordon [8]

$$a_\eta^2 + \frac{2\mu}{\pi v_r' a_0} \sin^2 a = C(a_0). \tag{22}$$

Here $C(a_0)$ is a constant. The solution of Equation (22) has the form

$$\eta = \pm \int_{a_0}^{a(\eta)} \frac{da}{\left[C(a_0) - \frac{2\mu}{\pi v_r' a_0} \sin^2 a\right]^{1/2}}. \tag{23}$$

In particular, assuming $a_0 = \frac{\pi}{2}(4m + 1)$, $m = 1, 2, \ldots$ at $C(a_0) = 0$,

we obtain the single equation

$$a(\eta) = 2\operatorname{arctg}\left[\exp\left(\pm\sqrt{\frac{\pi|v_r'|a_0}{2\mu}}\,\eta\right)\right] \tag{24}$$

(the derivative a_η has the usual form of the soliton:

$$a_\eta = \pm\left(\frac{\pi|v_r'|a_0}{2\mu}\right)^{1/2} \operatorname{sech}\left[\left(\frac{\pi|v_r'|a_0}{2\mu}\right)^{1/2}\eta\right]). \tag{24a}$$

It can be readily shown that the soliton solutions (21) and (24) are stable with respect to small perturbations.

In conclusion, we shall make certain numerical estimates. The condition for the smallness of the conductivity current as compared with the displacement current means the following

$$\omega^2 \varepsilon_0 \gg \frac{\omega_0^2}{a} (A^2 + B^2)^{1/2}. \tag{25}$$

Thus, the conductivity current in the semiconductor with $\varepsilon_0 \approx 10$, $n \approx 10^{16}$ см$^{-3}$, $d = 10^{-6}$ см and $\Delta_1 \approx 10^{-2}$ is small as compared with the displacement current at frequencies of $\omega \approx 5 \cdot 10^{12}$ sec^{-1}. In the region of these frequencies, the condition a = 1 corresponds to the field strength of the wave E = 3000 V/cm.

The author would like to thank F.G. Bass for his stimulating discussions.

REFERENCES

1. Ignatov, A.A., Romanov, Yu.A. Self-induced transparency in semiconductors with a superlattice. FTT, 1975, 17, No. 11, pp 3388-3389.

2. Ignatov, A.A., Romanov, Yu.A. Nonlinear electromagnetic properties of semiconductors with a superlattice. Phys. Stat. Sol. (b), 1976, 73, No. 1, pp 327-333.

3. Pavlovich, V.V., Epshteyn, E.M. Semiconductor conductivity with superlattice in strong electric fields. FTP, 1976, 10, pp 2001-2004.

4. Bass, F.G., Rubinshteyn, Ye.A. High frequency electromagnetic phenomena in semiconductors with non-quadratic dispersion law of current carriers. FTT, 1977, 19, No. 5, pp 1379-1388.

5. Plattsman, F., Vol'f, P. Volny i vzaimodeystviya v plazme tverdogo tela (Waves and interactions in the plasma of a solid). Moscow, "Mir" Press, 1975, pp 436.

6. Karpman, V.I. Nelineynyye volny v dispergiruyushchikh sredakh (Nonlinear waves in dispersing media). Moscow, "Nauka" Press, 1973, 175 pages.

7. Bass, F.G., Gurevich, Yu.G. Goryachiye elektrony i sil'nyye elektromagnitnyye volny v plazme poluprovodnika i gazovogo razryada (Hot electrons and strong electromagnetic waves in a semiconductor plasma and gas discharge). Moscow, "Nauka" Press, 1975, 399 pages.

8. Zakharov, V.Ye., Takhtadzhan, L.A., Fadeyev, L.D. Complete description of the "sin-Gordon" equation solutions. DAN SSSR, 1974, 219, pp 1334-1337.

End of Document