NASA SP-463

THE A-TYPE STARS: PROBLEMS AND PERSPECTIVES

MONOGRAPH SERIES ON NONTHERMAL PHENOMENA IN STELLAR ATMOSPHERES

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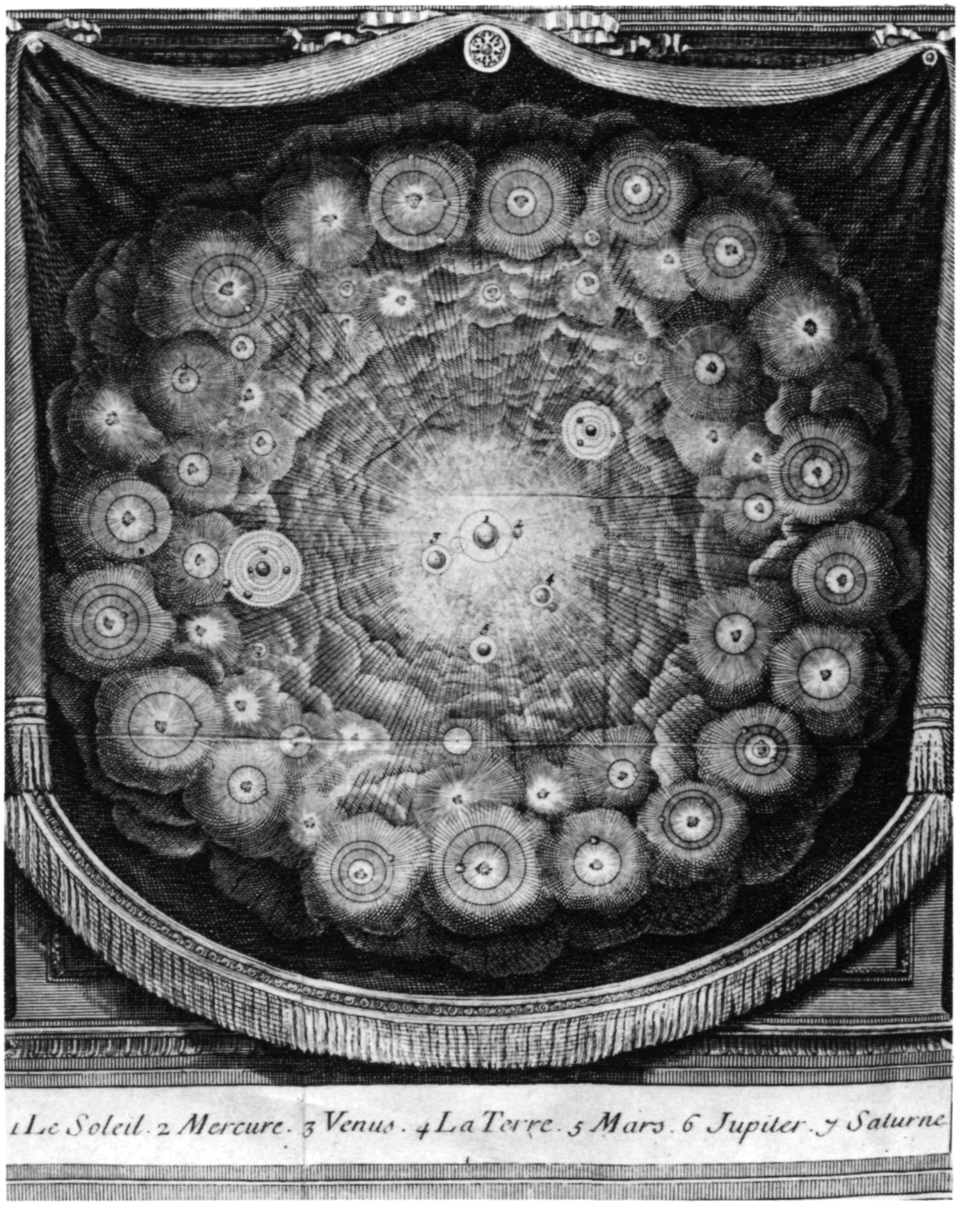

. . . Assurons-nous bien du fait, avant que de nous inquiéter de la cause. Il est vrai que cette méthode est bien lente pour la plupart des gens, qui courent naturellement à la cause, et passent par dessus la vérité du fait; mais enfin nous éviterons le ridicule d'avoir trouvé la cause de ce qui n'est point.

. . .

. . .

. . . De grands physiciens ont fort bien trouvé pourquoi les lieux souterrains sont chauds en hiver, et froids en été; de plus grands physiciens ont trouvé depuis peu que cela n'était pas.

–Fontenelle, Histoire des Oracles
Chapitre IV, pp. 20 et 23

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THE A-STARS:
PROBLEMS AND PERSPECTIVES

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MONOGRAPH SERIES ON NONTHERMAL PHENOMENA
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DEDICATION

in grateful appreciation
we dedicate this series and these volumes

to *Cecilia Payne-Gaposchkin*, who, with Sergei, set the spirit of empirical-theoretical atmospheric modeling by observing the following:

> "All true variable stars have variable atmospheres, but a variable atmosphere is probably the property of all stars, whether obviously variable in brightness or not [as witness the solar envelope]";

and who, by her intimate knowledge of particular stars, pioneered in the recognition of the fundamental importance of "individuality of stellar atmospheric characteristics."

to *Daniel Chalonge*, who sought, by ingenious meticulous observations, to make quantitative the features of qualitative classical taxonomy, thereby laying the foundations for showing the inadequacy of its two-dimensional, single-region atmospheric, character;

and who always opposed the spirit of a distinguished theoretical colleague's remark:

> "Don't show me those new observations of yours; they inhibit the range of my speculations."

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PREFACE

About one century has elapsed since the Henry Draper spectral classification scheme was introduced to try to identify physically alike kinds of stars from observational similarities in their visual spectra. The early one-letter classification has evolved into a very complex acronym with numerical subdivisions and symbols defining various kinds of peculiarity in stellar observations. During this time, our understanding of the variety of atmospheric regions which must exist to produce these peculiar spectral features has grown rapidly. The original classification was eventually interpreted in terms of an atmospheric model consisting of blackbody photosphere and a local-thermodynamic-equilibrium (LTE) reversing layer. Today, by analogy with the Sun, we recognize many layers above the photosphere which, although transparent to most visual wavelengths, have significant opacity in the far-UV, the X-ray, the infrared, and the radio spectral regimes. We recognize hot chromospheres, coronae and stellar winds, and even more extended regions such as cool emission envelopes, nebulae, and circumstellar dust clouds. Stellar classification and stellar atmospheric modeling are clearly in a stage of rapid continuing development. In particular, the farther the regions described here are found from the star, the more will their thermodynamic state be affected by nonthermal phenomena, and the less will they be controlled by the gravitational field of the star and by the local thermodynamic equilibrium associated with an extremely opaque medium.

More than two decades ago, the series *Stars and Stellar Systems* was generated under the general editorship of G. P. Kuiper. That series reviewed the status of astronomy and astrophysics at what we might call the beginning of a new era of extensive observations in the nonvisual wavelength regimes, many of them accessible only from space. We are now in the midst of this new era. These more recent observations continue to yield new insights into the outer atmospheric layers of the Sun and other stars. The new insights have forced us to reconsider the adequacy of the older HD system of classification and its classical successors, as well as of the assumptions underlying the classical theories for diagnosis and modeling of stellar atmospheres. All of these were reviewed in the Kuiper series' volume on *Stellar Atmospheres*. The present series will emphasize some of the current attempts to establish a new set of empirically based theoretical guidelines for treating stellar atmospheres. These new guidelines are intended as an elaboration of and also, where appropriate, as a revision of the classical guidelines, to permit a more comprehensive treatment which incorporates the recent new observations in a reasonable way.

To further put the current series, *Nonthermal Phenomena in Stellar Atmospheres*, in perspective in relation to the older series, *Stars and Stellar Systems*, the current series is far less comprehensive, being restricted to stellar atmospheres and, in some cases, subatmospheric boundary conditions. However, the approach is deliberately more critical because the new ideas required to interpret the new data are still in an early and frankly controversial stage of development, while the older theories on which the Kuiper series focused were relatively "standard" at that time. These new volumes are intended primarily as a stimulus to researchers to probe the unknown, starting with the new data, and not mainly as a compendium of what was known at the inception of the space era. The earlier series served that function well. For this reason the first priority in these volumes will be a review of the highest quality data, particularly the more recent data which exhibit nonthermal phenomena. Observation of the full electromagnetic spectrum exposes to view regions of the atmosphere that are transparent to visible light. The UV spectrum reveals the structures of chromospheres and coronae, the X-ray spectrum is emitted from regions which are very hot, the IR spectrum comes from all layers, hot and cold, which are opaque to IR photons, and the radio spectrum gives evidence of both thermal emission and of energetic nonthermal processes.

In the light of these new extensions of the wavelength regimes covered by stellar spectra, conventional taxonomy provides only a very provisional labeling, useful in classical statistical studies, but insufficient to reflect the intricate nature of physical phenomena and the variety of physical parameters that control the appearance of many stellar spectra. It is also true that theoreticians had long ago generally exhausted the possibilities for modeling stellar atmospheres with only effective temperature and gravity. They have, of course, continued to introduce physical improvements in their models, such as taking account of departures from LTE, or of the ionization in convection zones, etc. But these models, which are barely adequate for unambiguous fitting of the visible spectra, fail completely when confronted by the various new features observed in other spectral regimes. Because of this failure, it is not at all clear that stars which astronomers have called peculiar in the past are, in fact, fundamentally different from stars classified as normal. Until superior models emerge, we cannot be sure that the statistically defined abnormalities are anything more than spectral signatures of large-amplitude nonthermal processes which exist with smaller amplitudes in many stars classified as normal.

In the early days of these recent observational developments, many theorists thought of the new features largely as perturbations modifying the basic classical description. For example, they tried to perturb their models slightly by adding a superficial hot layer labeled "chromosphere," and by representing parametrically the emission features at the centers of such intense spectral lines as Hα and K(Ca II). But adding layers ad hoc without considering their possible interactions with the lower regions is physically inconsistent. Whether we talk about shells, winds, or magnetic features, they must be compatible, in the framework of physical laws, with the values of the other parameters characterizing the star. It may well be, for example, that a star of T_{eff} equal to, say, 10^4 K cannot have a dust shell of high opacity; or perhaps it can. However, we cannot blindly accept that such dust shells can occur without full investigation of the processes by which dust grains condense, grow, and are destroyed in a given stellar environment. Nor can we accept them without asking whether the IR and radio excesses they were introduced to produce may not come from chromospheric-coronal emission instead.

Historically, the analysis of stellar spectra may be thought of as proceeding in three sequential stages. The first is based on taxonomy and rough modeling and leads to very approximate estimates of a few basic parameters, such as T_{eff}, g, and chemical composition, while bypassing consideration of any anomalous features in the spectrum. The second is an attempt to explain each anomalous spectral feature in terms of some structural property of the stellar atmosphere, e.g., a circumstellar shell of some temperature and density at a certain distance from the star, a warm chromosphere, or a hot corona. This leads to a provisional, parametric description of the atmosphere of a star which is often physically contradictory: one group of spectral features may require a low density shell, another a high density shell. The red supergiant α Orionis (M2 Iab) offers a good example of the anomalies that are found in this second stage of analysis. For instance, the interferometrically measured diameter is found to decrease with increasing wavelength, which is explained by dust scattering in the circumstellar shell, but this model requires that the dust be located 1-3 stellar radii from the star, whereas observations of emission at 11μ put the dust no closer than 10 stellar radii. Another example is given by the variability of some B stars, such as γ Cas, which appears at different times in three different guises—as a normal B star, as a Be star, and as a B-type shell star. Still another case is that of Sirius, which has been regarded as a bona fide A0 main-sequence star for years, but which now appears to show some spectral anomalies, possibly linked with metallicity.

The third and ultimate stage of analysis, which is an order of magnitude more difficult than the preliminary ones, aims at models of the atmosphere that will be compatible with all known facts about the star and with the laws of physics. Such models will clearly be neither in LTE nor static, and, therefore, we shall be obliged to take account of all physical processes that may be operating in the star, including some not now recognized as important by astrophysicists.

It is the intent of this series of books to help set the stage for this last step. At the very least, these monographs will try to define some types of observations to be made and some types of models to be constructed before we can approach a full understanding of stellar structure. A good example is the solar case in which we can foresee what new observational and theoretical vistas might emerge from the Solar Maximum Mission and the Solar Polar Mission. Certainly the coming of radioastronomy, infrared astronomy, and space research have made it possible for us to handle the first two steps discussed above.

The principles of classical taxonomy are a necessary starting point for all parts of the HR diagram. In consequence, for each spectral type, the forthcoming volumes of this series will examine thoroughly those phenomena that are not included in the classical description. A typical problem that will be considered is the contrast between stars labeled B and Be, respectively. What parameters or physical processes have been overlooked that might provide a connecting link between these two subclasses of stars? It will be argued that there may be no such thing as a peculiar B star when observed over the whole spectral range. Had we begun the analysis with the far-UV region of the spectrum, our notion of what is normal would have been quite different, and any attempt at one- or two-dimensional classification would have led to a labeling system incompatible with the HD system or its successors.

We hope that readers of these books will sense the emergence of a new point of view in stellar diagnosis and in stellar astrophysics, a global approach which assigns to the whole spectrum and to all its features the same *a priori* weight as a basis for diagnosis, a physical approach which tries to attract the theoretical astrophysicist to the interpretation of the observed spectra, no matter how elaborate they may be, and an approach which considers each star as a physical object to be understood, by itself, in a coherent way, not simply statistically. Because so many of the data which exhibit nonthermal phenomena have come from observations made from space, we also hope that the delineation of the above trends will be useful in planning observing programs from space in the 1980's, particularly in the

UV, far-UV, and X-ray regimes, as well as in helping to coordinate these with ground-based observing programs in the visual, IR, and radio regimes. We trust these observations will also stimulate relevant laboratory investigations.

Classical schemes of stellar taxonomy are two-dimensional, with the two relevant parameters being, either implicitly or explicitly, temperature and pressure. The implication is that models of stellar atmospheres can also be described as a two-parameter family. The inadequacy of this approach has become apparent now that direct observations of chromospheric and coronal-like regions in other stars have been made. Indeed, two-dimensional classification schemes owe their success to the fact that they deal only with the photospheric spectrum, which in most classes of stars does not appear to be strongly perturbed by nonthermal processes.

The A-type stars offer a strong contrast to this general picture. Evidence for chromospheric activity remains elusive. Emission in the X-ray region of the spectrum is seen in some, but by no means all, A-type stars, and the explanation for the range in X-ray fluxes in stars of similar temperature and gravity is not known. There is, however, evidence that the *photospheric* spectra of A-type stars are *strongly* influenced by nonthermal phenomena. The visible spectra of many, perhaps even a majority, of A-type stars are peculiar in the sense that they cannot be explained in terms of a normal (solar) composition and classical thermal models. So prevalent are these peculiarities that it is probably true that there is no slowly rotating late A-type star that could reasonably be classified as normal.

The fundamental question, of course, is whether the line strength anomalies reflect true abundance anomalies or whether they can be explained in terms of unusual excitation and ionization conditions in an atmosphere of normal composition. The weight of the evidence presented in this monograph favors the first of these two possibilities and further suggests that the processes responsible for producing the abundance anomalies are intrinsic to the stars themselves. For an explanation of the observed spectra, we must consider mechanical energy fluxes, meridional circulation, radiative diffusion, structure of convective zones, magnetic fields, and probably other factors as well.

In this volume, the various classes of A-type stars are discussed in detail. Emphasis is placed on trying to determine empirically what physical characteristics, in addition to temperature and surface gravity, shape the emergent spectra of these stars. The efforts to incorporate relevant nonthermal processes into atmospheric models are also described. It is the purpose of this monograph to provide a clear picture of what is already known about these stars and to challenge observers and theoreticians alike to probe the unknown.

The A-type stars occupy a crucial transition region in the HR diagram between hot stars and cool ones. In their photospheres, hydrogen is neither totally ionized, as it is in hotter stars, or completely neutral, as it is in cooler stars. Electrons in an A-type atmosphere are produced by both hydrogen and metals. Elements such as C, Si, Mg, and Al, as well as H, He, and H^-, may contribute to the continuous opacity, and non-LTE effects may be important for all these elements. It is among the A-type stars that convection begins to become important, that observational diagnostics of chromospheric-like regions change from absorption to emission lines, and that, in supergiants, mass loss changes from high to low velocity flows. Because of these complexities, modeling of the A-type stars is difficult. On the other hand, because the spectral anomalies of the A-type stars are pronounced, this class of stars offers us a laboratory in which we can test our ability to model nonthermal phenomena. A better understanding of the A-type stars is likely to modify and improve models of stellar atmospheres throughout the HR diagram.

–Prepared by the Series' Senior Advisers
and the Organizers
Greenbelt, Paris, Tucson, January 1983

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ACKNOWLEDGMENTS

The A-type stars are an exceptionally complex class of objects. To do them justice, a monograph must deal with theory and observations, main-sequence stars and supergiants, peculiar stars as well as normal ones, stable stars and pulsating variables. Obviously no one person can be completely familiar with all of these areas of research.

In an effort to make sure that the present discussion of A-type stars was reasonably complete in those areas in which I have not carried out research myself–and balanced and unbiased in those areas in which I have–I submitted individual chapters to people who had made major contributions to the subjects discussed. Nearly everyone replied, always thoughtfully and often extensively. I wish, therefore, to express my sincerest thanks to H. A. Abt, S. J. Adelman, A. Baglin, W. K. Bonsack, M. Breger, G. W. Collins, P. S. Conti, C. R. Cowley, W. S. Fitch, J. N. Heasley, R. M. Humphreys, P. B. Kunasz, D. W. Kurtz, J. D. Landstreet, G. Michaud, N. D. Morrison, D. Moss, F. Praderie, D. Pyper Smith, M. A. Smith, A. B. Underhill, and W. Weiss. While these people are, of course, not responsible for any errors of fact or opinion that remain, the thought and time that they have so selflessly given to this project have produced substantial improvements in the manuscript. Leo Goldberg and Jean-Claude Pecker offered a number of very useful comments on the completed manuscript, as did Richard Thomas. Indeed, I am indebted to Dick Thomas not only for his provocative and valuable comments on the text, but also for persuading me to undertake this project.

The manuscript itself was written in part at the Observatoire de Paris, Sacramento Peak Observatory, and Kitt Peak National Observatory. I thank the directors of those observatories for their hospitality during my visits to each.

Reproducing a manuscript of this size is a substantial effort, and I am greatly indebted to the individuals involved. Linda W. Peterson was responsible for editing the text; besides ensuring accuracy and consistency of style, she measurably enhanced its clarity. Jeanne Koch was saddled with the task of deciphering my handwriting, a task she accomplished with remarkable speed and good will. The figures were photographed from the original texts by Bob Bishop as his final major project before retiring from the Mees Solar Observatory on Haleakala, Maui. I also wish to thank John Jefferies and Robert Milkey for stretching the administrative resources of the Institute for Astronomy nearly to the breaking point to complete the typing of this manuscript.

The penultimate sentence of *Huckleberry Finn* reads:

> . . . so there ain't nothing more to write about, and I am rotten glad of it, because if I'd 'a' knowed what a trouble it was to make a book I wouldn't 'a' tackled it, and ain't a-going to no more.

When I began writing this book on A-type stars, I expected that my own feelings would be similar. The generous support of all the people named here has made the project much less painful—and much more rewarding—than I could reasonably have hoped at the outset.

Sidney C. Wolff
Honolulu, January 1983

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RÉSUMÉ

Introduction

Les étoiles A de la séquence principale se trouvent dans une région du diagramme HR où l'on aurait prévu que les modèles atmosphériques, basés sur les hypothèses classiques de l'équilibre hydrostatique et radiatif, réusissent le mieux à décrire les spectres observés. Les zones de convection près de la surface ne sont pas étendues; l'hydrogène est la source principale d' opacité; la rotation est modérée; et les indices usuels de l'échec des modèles simples en équilibre thermodynamique (raies d'émission, variabilité irregulière à courte période. . .) sont absents. Néanmoins, deux sur dix des étoiles A montrent des anomalies spectrales assez frappantes pour être détectées sur des spectrogrammes à petite résolution, et beaucoup d'autres encore paraîssent anormales à plus grande résolution. On résume sur la Table R-1 (Bonsack, 1981) les caractéristiques des classes principales d' étoiles particulières de type A et B. Apparemment, des processus physiques qui sont masqués par les flux de masse dans les étoiles plus massives, ou par la convection dans les étoiles moins massives, jouent un rôle prédominant dans la formation du spectre émergent. La circulation méridienne, les zones de convection au-dessous de la surface, la pulsation, la séparation diffusive des éléments, et les champs magnétiques sont parmi les phénomènes que l'on considère importants. Comprendre les étoiles A de la séquence principale, c'est un problème redoutable, car nous ne pouvons actuellement construire des modèles de ces processus et de leurs interactions que d'une façon très limitée. Par contre, ces étoiles nous présentent aussi une occasion, en nous fournissant un laboratoire où nous pouvons éprouver les modèles atmosphériques qui tiennent compte des effets hydrodynamiques et hydromagnétiques.

Les supergéantes de type A posent une tout autre série de problèmes. Plusieurs d'entre elles subissent une perte évidente de masse, et la variabilité irrégulière ou semi-régulière est la norme. Ce qui est demandé ici est de définir, du point de vue observationnel, les caractéristiques des étoiles qui sont nettement hors de l'équilibre hydrostatique, et puis d'en construire des modèles.

Les Étoiles Normales de Type A

Dans la plupart des régions du diagramme HR, les étoiles "normales" sont la majorité. Mais parmi les étoiles A, les anomalies sont de règle plutôt qu'exceptionnelles. En effet, il n'y a probablement pas d'étoile de type A, à rotation lente, qui puisse raisonnablement être classée "normale." Parce que les anomalies qui intéressent les intensités des raies sont d'habitude très prononcées, la classification spectrale du système MK présente un moyen efficace de distinguer les étoiles normales des étoiles particulières. Mais la classification précise des étoiles A, en termes de température et de luminosité, est très difficile, à cause d'un manque de raies métalliques convenables à intensité intermédiaire. Les raies utilisables sont soit très fortes (Ca II λ3933 et les raies de Balmer de l'hydrogène), soit très faibles. De plus, les rapports des raies peuvent dépendre de propriétés atmosphériques telles que la microturbulence, aussi bien que de la température et la luminosité; et il faut être très circonspect en essayant d'extrapoler le système MK aux étoiles à composition anormale ou à structure atmosphérique anormale.

La classification photométrique représente une autre méthode de caractériser les étoiles de type A. Le système uvbyβ, qui mesure la température et la luminosité des étoiles A et fournit aussi une évaluation de l'effet de serre, est particulièrement utile. On a utilisé des modèles atmosphériques pour calibrer les indices uvbyβ. Les modèles et les observations sont cohérentes pour les étoiles A chaudes, mais pour les étoiles de type A avancé, les couleurs théoriques sont en désaccord avec les couleurs mesurées; ceci est probablement l'effet d'un traitement insuffisant de la convection. Les températures effectives déduites de la calibration des indices uvbyβ sont en bon accord avec celles déduites des mesures directes des distributions d' l'energie, en allant de l'ultraviolet lointain jusqu'aux régions visible et infrarouge du spectre.

La masse typique d'une étoile A0 est environ 2,5 $M_\odot$, celle d'une étoile de type A avancé est 1,5 $M_\odot$. Les valeurs typiques du rayon sont comprises entre 2,1 $R_\odot$ et 1,5 $R_\odot$; ces valeurs dépendent dans une certaine mesure de l'état évolutionnaire aussi bien que de la masse.

On a trouvé des étoiles normales de type A avancé dans des systèmes binaires spectroscopiques avec des périodes inférieures à 2,5 jours ou supérieures à 100 jours. Par contre, la plupart des étoiles Am sont membres de systèmes binaires dont les périodes sont comprises entre 2,5 et 100 jours. Il y a également une dichotomie dans la distribution des vitesses rotationnelles des étoiles Am et des étoiles normales de type A avancé. Il n'y a pas d'étoiles normales qui tournent plus lentement que ~40 km s^{-1}, et pas d'étoiles Am qui tournent plus vite que ~100 km s^{-1}. La synchronisation de marée a évidemment ralenti la rotation des étoiles de type A avancé qui font partie de systèmes binaires serrés.

On a cherché des preuves de la rotation différentielle dans les étoiles A, mais on n'en a pas trouvé. La recherche des champs magnétiques dans les étoiles normales de type A a également échoué; les champs effectifs longitudinaux plus élevés que 150 gauss doivent être très peu communs.

On a travaillé peu sur les abondances dans les étoiles normales de type A, en partie parce qu'il y a si peu d'étoiles normales à raies assez fines pour permettre de faire une analyse detaillée. Les résultats disponsibles sont assez gênants. Il y a, paraît-il, un groupe d'étoiles déficientes en metaux, dont le type spectral est voisin de A0; ces étoiles peuvent se trouver aussi fréquemment que les étoiles Ap. Il n'y a aucune explication satisfaisante à cette déficience.

Le Chauffage Non-Radiatif dans les Étoiles A

Dans le soleil, la chromosphère, la couronne, et la région de transition qui se trouve entre ces dernières, sont toutes les trois caractérisées par des températures considérablement au-delà des valeurs prédites par les modèles atmosphériques à équilibre radiatif. Dans le Chapitre 3, on aborde la question de l'identification de régions analogues dans les atmosphères extérieures des étoiles A. On a fait des recherches étendues des caractères des raies et du spectre continu dans les étoiles A qui seraient diagnostiques des régions à haute température. Parmi les caractères spécifiques qu'on a cherchés–sans les trouver–sont l'émission des raies H et K de Ca II, l'émission des raies h et k de Mg II, l'émission ou l'absorption de la raie $\lambda 10830$ de He I, et les raies d'émission de Si IV, C IV, et He II dans la région ultraviolette lointaine. Les preuves de la variabilité irrégulière des étoiles A, qui pourraient témoigner des processus hors équilibre, ne sont pas encore entièrement convaincantes.

Etant donné ces résultats négatifs, il est peut-être surprenant qu'on ait observé l'émission de rayons X par un certain nombre d'étoiles A. En certains cas au moins, l'émission est due surtout à un compagnon de type avancé. Mais en d'autres cas, on a vu l'émission dans des étoiles A qui ne sont pas actuellement connues pour membres de systèmes binaires. L'intensité de l'émission n'est pas liée à la rotation, comme c'est le cas pour les étoiles de type avancé. Par contre, la corrélation de la luminosité en rayons X avec la luminosité bolométrique, qu'on a trouvée pour les étoiles O et B chaudes ($L_x \sim 10^{-7}\ L_{bol}$), serait valable pour au moins quelques-unes des étoiles A. La découverte de l'émission de rayons X par des étoiles A est très récente, et l'on n'a fait que quelques tentatives de développer des modèles pour en rendre compte.

Les Étoiles Ap Magnétiques

Les étoiles Ap sont caractérisées par des raies fortes d'un ou de plusieurs des éléments suivants: Si, Cr, Sr, Eu, et d'autres terres rares. Les étoiles Ap typiques varient à des échelles de jours ou, par exception, de décennies. On observe des changements dans les intensités des raies spectrales, dans l'intensité du champ magnétique, et dans la luminosité et la couleur. Pour une étoile donnée, toutes ces quantités varient avec la même période. Le modèle qui explique le mieux ces variations est le modèle à rotateur incliné ou rigide. L'hypothèse fondamentale de ce modèle est que l'axe du champ magnétique est inclinée d'un certain angle par rapport à l'axe de rotation. Le champ est supposé localement constant–c'est à dire, lié à la surface de l'étoile–et en corotation. Un observateur lointain verra le champ magnétique varier en intensité au fur et à mesure que l'étoile tourne. On explique les variations spectrales en supposant que les éléments variables sont concentrés en taches sur la surface stellaire. Ce modèle peut expliquer la relation entre les phases des variations de la vitesse radiale et de celles de l'intensité des raies, qui sont typiques des étoiles Ap; le signe et l'amplitude des changements de la largeur des raies (l'effet de traversée–"crossover effect"); la relation entre le champ moyen superficiel et la moyenne de sa composante longitudinale, prise sur la surface stellaire; la variation du champ magnétique transversal, déduite des changements de la polarisation linéaire des raies métalliques individuelles, en fonction de la phase; et les changements des profils des raies fortes, comme Ca II λ3933, en fonction de la phase. Les variations photométriques sont, en grande partie, attribuables aux changements de l'effet de serre.

Les étoiles Ap "classiques" ont des températures semblables à celles des étoiles normales dont les types spectraux sont compris entre B avancé et A avancé. Mais on a récemment détecté des champs magnétiques dans quelques étoiles Bp à excès et à déficience d'hélium, et il est maintenant évident que le phénomène du rotateur oblique apparaît dans des étoiles aussi chaudes que celles de type B2.

Bien que les températures approximatives des étoiles Ap soient connues, il est difficile d'en faire une détermination précise. L'effet de serre, dû aux surabondances des métaux, baisse le flux dans la région ultraviolette du spectre, et ainsi modifie-t-il la distribution d'énergie aussi bien que la structure atmosphérique. Tout en admettant l'incertitude sur les modèles actuels de ces atmosphères complexes, on pense que les étoiles Ap sont des objets de la séquence principale; on a classé comme étoiles Ap environ 5 à 10% des étoiles de type B avancé et de type A chaud. La fréquence de systèmes binaires serrés parmi les étoiles Ap semble beaucoup moins élevée que parmi les étoiles A sans champ magnétique détectable. Les données disponibles concernant les binaires Ap indiquent des masses normales par rapport aux températures.

Les étoiles Ap tournent, en moyenne, beaucoup plus lentement que les étoiles normales de même température. On suppose que ces étoiles ont perdu du moment angulaire par quelque freinage magnétique. On a signalé des périodes de variation jusqu'à 75 ans. Si ce sont en effet des périodes rotationnelles, le processus de freinage a été très efficace.

Un résultat récent très intéressant est la découverte qu'au moins quelques-unes des étoiles Ap les plus froides varient en brillance à des échelles de minutes. Ces variations semblent liées aux pulsations nonradiales, et on les a expliquées en termes d'un "modèle à pulsateur oblique." L'hypothèse fondamentale de ce modèle est que la pulsation est symmétrique par rapport à l'axe magnétique, plutôt que par rapport à l'axe de rotation. Ceci étant, on peut déduire, à partir des amplitudes pulsationnelles observées, des quantités comme l'angle d'inclination de l'axe de rotation avec la ligne de visée, et celle de l'axe de rotation avec l'axe magnétique. Les géométries ainsi déduites sont compatibles avec les résultats obtenus plus tôt en utilisant le modèle à rotateur rigide pour interpréter les variations spectrales et magnétiques.

Tandis qu'on peut expliquer les variations des étoiles Ap de façon satisfaisante en invoquant le modèle à rotateur rigide, l'origine des anomalies des abondances reste quelque peu mystérieuse. On a proposé une variété de mécanismes qui relèvent des processus de synthèse nucléaire; mais étant donné l'état actuel de nos connaissances, aucun ne paraît satisfaisant. Une autre solution est la diffusion radiative, peut-être modifiée par des effets tels que l'accrétion et la turbulence. La difficulté principale de ce modèle est la stabilité atmosphérique qu'il exige, une stabilité largement au-delà de celle estimée probable par certains theoriciens. Par contre, ce modèle explique bien un certain nombre de propriétés des étoiles Ap. Des calculs de la diffusion en présence d'un champ magnétique intense viennent à l'instant de paraître. Puisque ces calculs prévoient des relations bien définies entre la géométrie magnétique et la distribution des éléments sur la surface stellaire, ils devraient permettre d'éprouver, de façon très rigoureuse, le modèle à diffusion.

Les Étoiles Am

La définition moderne des étoiles Am est la suivante: le phénomène Am est présent dans les étoiles qui ont une déficience apparente de calcium (et/ou de scandium) à la surface, et/ou un excès apparent du groupe de fer et des éléments plus lourds. Des étoiles qui satisfont à ces conditions, il n'y a qu'un sous-groupe qui satisfait aussi à la definition classique: une étoile Am est une étoile où le type spectral déduit des raies métalliques diffère de plus que cinq sous-types spectraux du type déduit de la raie K. Les étoiles Am se trouvent sur la séquence principale, à toutes températures effectives de 7400 K jusqu'à 10200 K. Hors de cette région, les étoiles Am apparaissent sans doute beaucoup moins fréquemment; mais les effets de sélection jouent probablement un rôle dans la détermination des limites apparentes de la température. Aux températures plus élévées, et plus basses, il est difficile de détecter la métallicité sauf à partir d'analyses détaillées des abondances, basées sur des spectrogrammes à grande résolution.

Les étoiles Am se distinguent des étoiles normales de type A par la fréquence de la binarité et par la distribution de vitesse rotationnelle, aussi bien que par les caractéristiques spectroscopiques. La plus grande partie—et peut-être la totalité—des étoiles Am de type spectral plus avancé que A4 sont membres de systèmes binaires spectroscopiques; et les deux-tiers des systèmes binaires connus ont des périodes orbitales inférieures à 100 jours. Par contre, peu d'étoiles normales de type A sont connues comme binaires, et celles-là tendent à des périodes orbitales longues ($P > 100$ jours). La plus grande vitesse rotationnelle observée jusqu'à présent pour une étoile Am est d'environ 100 km s^{-1}, et toutes les étoiles de type A avancé qui ont v sin $i < 40$ km s^{-1} sont probablement des étoiles à raies métalliques. La question fondamentale est la suivante: Laquelle de ces caractéristiques—la rotation ou la binarité—est liée à la cause de la métallicité—ou n'est-ce ni l'une, ni l'autre? Si toutes les étoiles Am sont, en effet, des binaires, on pourrait en déduire que les intéractions binaires produisent, d'une façon ou d'une autre, les anomalies des intensités des raies. Par contre, si quelques étoiles Am sont des étoiles simples, mais si toutes ces étoiles tournent lentement, la bonne solution serait que la rotation lente est nécessaire à la métallicité. En ce dernier cas, la prédominance de binaires parmi les étoiles Am serait due à une tendance au synchronisme, donc à la rotation lente, produite par les interactions de marée dans les systèmes serrés.

Les études des étoiles Am dans les amas limitent encore plus les modèles de l'origine de la métallicité. La découverte la plus importante est celle de plusieurs étoiles Am dans l'association d'Orion; ce résultat montre que la métallicité peut se produire en 1 à 3×10^6 ans. Il y a des preuves de variabilité dans les étoiles Am, mais la variabilité ne semble en aucun cas être due ni à la pulsation, comme pour les étoiles de type δ Sct, ni à la rotation d'une surface non-homogène, comme pour les étoiles Ap magnétiques. En ce qui concerne les anomalies des intensités des raies, les étoiles Am seraient membres d'un groupe beaucoup plus homogène que celui des étoiles Ap.

On a consacré des efforts considérables aux tentatives de décider si les anomalies observées dans les intensités des raies indiquent des anomalies des abondances. Jusqu'à présent, personne n'a construit de modèle atmosphérique qui puisse réproduire les intensités observées, à condition que les abondances soient solaires. Les contraintes observationnelles sont maintenant si rigoureuses qu'il n'est guère possible de changer la structure atmosphérique d'une étoile Am, en température ou en ionisation, des valeurs caractéristiques des atmosphères stellaires normales.

Il y a trois catégories générales d'explications des anomalies des abondances dans les étoiles Am: (1) les anomalies résultent de la synthèse nucléaire, ou à l'intérieur ou à la surface de l'étoile; (2) elles résultent de l'accrétion de grains ou de masse perdue par les étoiles evoluées; (3) elles sont dues à une séparation des éléments à l'intérieur de l'étoile, à cause des processus de diffusion radiative. Selon nos connaissances actuelles, seule la dernière de ces solutions serait compatible avec les observations.

Les Étoiles Variables de Type δ Sct

Le prolongement de la bande d'instabilité des Céphéides croise la séquence principale dans la région occupée par les étoiles de type A avancé et F chaud. Il est donc peu surprenant que la variabilité soit commune parmi les étoiles de type A avancé, et on expose au Chapitre 6 les propriétés des variables pulsantes qui se trouvent près de la séquence principale. Les variations de la luminosité de telles étoiles sont normalement de l'ordre de quelques centièmes de magnitude, bien que des amplitudes plus grandes que 0,3 mag ne soient pas inconnues. Les périodes sont inférieures à 0,3 jour. Pour plusieurs de ces étoiles, l'allure des courbes de lumière varie de façon marquée de cycle en cycle.

La plupart des étoiles qui présentent des variations ayant ces caractéristiques sont des étoiles de la population I, dans une phase évolutionnaire qui les place sur, ou peu au-dessus de, la séquence principale. Mais il y a aussi un groupe d'étoiles qui présentent des variations semblables, mais qui possèdent de grandes vitesses spatiales et de faibles abondances métalliques. Ce ne sont évidemment pas des membres de la population I. Elles pourraient être des étoiles naines de la population II, auquel cas elles seraient analogues aux "traînards bleus" que l'on voit dans plusieurs amas. Mais par contre, ces variables à déficience métallique pourraient être des objets en train d'évoluer vers le stade des naines blanches, ayant perdu une quantité considerable de masse pendant qu'elles étaient des géantes rouges. Au Chapitre 6, on désigne ces deux groupes d'étoiles, au nomd' étoiles de type δ Sct.

On définit comme étoiles de type δ Del, les géantes et supergéantes de type A avancé et F chaud dont le type spectral déterminé à partir de la raie K est en désaccord avec celui déterminé à partir des raies métalliques. Un grand nombre des étoiles de type δ Del sont aussi des variables photométriques, et pour cette raison on les discute au Chapitre 6, à côté des étoiles de type δ Sct. Mais la variabilité n'est point nécessaire pour qu'une étoile soit membre de la classe δ Del. Parce que leurs abondances ressemblent à celles des étoiles Am, on a proposé qu'au moins quelques-unes des étoiles de type δ Del sont des étoiles évoluées à raies métalliques.

Un tiers environ des étoiles situées sur ou près de la séquence principale, et dans la bande d'instabilité, varient de 0,01 mag ou plus; et la fréquence de variabilité paraît indépendante de l'âge stellaire. Les étoiles Am ne sont pas variables, et l'on a attribué cette absence de pulsation à la diffusion. Le mécanisme principal pour entraîner la pulsation est la zone d'ionisation de He II. Dans une atmosphère stable, l'hélium, n'étant pas soutenu par la pression radiative, tendra à se précipiter hors de l'atmosphère. Si la teneur en hélium devient suffisamment faible, l'étoile sera stable contre la pulsation. En même temps, d'autres éleménts qui sont fort soutenus par la pression radiative peuvent se concentrer dans les régions où se forment les raies, ainsi produisant les anomalies d'abondances caractéristiques des étoiles Am.

La plupart des étoiles de type δ Sct qu'on a bien observées, présentent une périodicité stable qui, en certains cas, subsiste pendant au moins plusieurs années. Dans un grand nombre d'étoiles, plusieurs modes radiaux sont excités à la fois. Dans la partie la plus froide de la bande d'instabilité, le mode fondamental tend à avoir une amplitude plus grande que celle des autres modes. La pulsation nonradiale existe partout dans la bande d'instabilité, mais on ne la voit pas dans chaque étoile. Des modes pulsationnels radiaux et nonradiaux peuvent coexister dans une même étoile. Les étoiles de type δ Sct qui font partie de la population I et qui ont des amplitudes plus grandes que 0,3 mag ne présentent pas d'habitude de multipériodicité complexe; le mode prédominant de leur pulsation est le mode radial fondamental. Au Chapitre 6, on décrit une série de techniques pour déterminer les modes de pulsation. En règle générale, on peut bien représenter les caractéristiques observées de la pulsation des étoiles de type δ Sct par les modèles théoriques disponibles.

Les Supergéantes

Dans les galaxies extérieures, comme dans la nôtre, les étoiles les plus brillantes aux longueurs d'onde visibles sont d'habitude de type B ou A. (La luminosité totale des supergéantes O est plus élevée, mais ce n'est que l'effet des corrections bolométriques, qui sont beaucoup plus grandes. On attribue le manque de supergéantes A à luminosités bolométriques semblables à celle des étoiles O à une perte de masse–peut-être catastrophique–aux stades antérieurs de leur évolution.) Les études spectroscopiques des supergéantes A peuvent donc offrir un des meilleurs moyens de déterminer si l'évolution des étoiles massives, et les processus physiques dans leurs atmosphères, sont les mêmes dans les autres galaxies que dans la nôtre. Mais jusqu'à présent on n'a fait que des études très limitées sur les supergéantes A. Se trouvant dans une région du diagramme HR où l'évolution est rapide, ces étoiles sont très peu nombreuses. Les observations faites à partir de satellites dans l'ultraviolet, où l'on observe des pertes de masse, et dans la région des rayons X, où il y a des preuves de chauffage non-radiatif, ont provoqué une grande partie de la recherche la plus récente sur les supergéantes des premiers types spectraux et de type avancé. Mais il n'y a guère de preuves de chromosphère ou de couronne dans les supergéantes A, et leurs flux relativement faibles dans les régions UV et X ne favorisent pas les observations à partir de satellites.

Les variations des supergéantes A sont extrêmement complexes. Les échelles de temps impliquées vont des heures jusqu' aux années. Le vent stellaire peut varier, aussi bien que les caractères photosphériques. Parmi les quantités variables sont la luminosité, la vitesse radiale, et les intensités et les profils des raies. Puisque les variations ne sont que semi-périodiques, et puisqu'on dispose rarement de mesures simultanées des différentes caractéristiques variables, on connaît assez peu des relations entre les caractéristiques variables, ou entre les changements photosphériques et les variations du vent stellaire.

Toutes les supergéantes A varient en vitesse radiale, avec des amplitudes typiques comprises entre 4 et 10 km s^{-1}. Une étude approfondie des variations de vitesse dans α Cyg a amené à l'identification de 16 modes de pulsation discrets, à périodes de 6,9 à 100,8 jours. Puisqu'un grand nombre de ces modes ont des périodes plus grandes que la période prévue pour le mode radial fondamental (14,3 jours), des pulsations non-radiales auraient lieu dans cette étoile. Les amplitudes et les phases des différents modes étaient stables sur un intervalle d'au moins 6 ans.

Des variations photométriques de 0,03 a 0,05 magnitudes sont typiques des supergéantes A. L'amplitude de la variation de lumière tend à croître en fonction de la luminosité, tandis que l'échelle de temps pour les variations semi-périodiques tend à augmenter en fonction de la luminosité croissante et de la température décroissante.

Une variété de techniques observationnelles permet de détecter la perte de masse et de déterminer le taux du flux massique. Dans les supergéantes A, les meilleures preuves de la perte de masse dérivent des analyses des absorptions larges dues aux éléments abondants une fois ionisés, qui se trouvent dans la région ultraviolette du spectre. On a vu des décalages en vitesse jusqu'à -240 km s^{-1}. De telles vitesses se comparent aux vitesses de libération photosphériques calculées, et elles fournissent donc des preuves assez claires de la perte de masse. On a signalé des variations de l'absorption ultraviolette, et des variations de l'émission à $H\alpha$ sont communes. Il paraît donc que le vent est variable. On a évalué les taux de perte de masse à partir de modèles des profils de raies, et à partir de mesures des excès infrarouges, qui proviennent de l'émission libre-libre dans le vent. Des valeurs typiques du taux de perte de masse se situent dans l'intervalle 10^{-7} à 10^{-9} $M_\odot$ par an; mais les différentes méthodes peuvent donner des valeurs très différentes pour une même étoile.

Il y a quatre catégories générales de théories de l'origine des vents stellaires: des vents froids entraînés par la pression radiative, des vents chauds coronaux produits par un gradient de pression gazeuse, des modèles héterogènes qui joignent les régions chaudes coronales aux vents froids entraînés par la pression radiative, et des modèles à flux "imparfait," qui attribuent l'origine des vents aux fluctuations ou instabilités du champ de vitesse photosphérique. Il est difficile de déterminer lequel de ces modèles s'applique aux supergéantes A, ou si aucun ne s'y applique, parce que les vents sont faibles et leurs caractéristiques sont mal définies. Puisque nos connaissances des processus physiques impliqués dans la perte de masse sont si limitées, le plus grand effort théorique s'est consacré à l'étude de la cinétique du transfert radiatif dans un milieu en mouvement. On spécifie le champ de vitesse et la structure de température comme données d'entrée, et on peut alors calculer les spectres émergents, y compris les profils des raies. Ce genre de modèle a fourni les meilleures estimations des taux de perte de masse dans les supergéantes A.

Les Étoiles Particulières de Type B

Du point de vue de la physique des atmosphères stellaires, la frontière à A0 est artificielle. On trouve des étoiles particulières parmi les premiers types spectraux, jusqu'au type B2. La limite du coté des températures élevées coïncide sans doute avec le début d'une perte de masse importante. Puisqu'on se sert d'un certain nombre de caractéristiques des étoiles Bp comme conditions aux limites pour les modèles des étoiles Ap aussi, on résume dans ce chapitre les caractéristiques des étoiles Bp.

Les plus chaudes des étoiles Bp sont les étoiles riches en hélium. Ces étoiles se trouvent dans une région très limitée du diagramme HR; les températures s'accordent (plus ou moins) avec le type spectral B2, et les gravités superficielles correspondent à la séquence principale. L'émission en Hα et l'absorption, décalée vers le violet, au doublet $\lambda\lambda$1548, 1550 de C IV témoignent de vents dans les étoiles riches en hélium. La plupart des étoiles riches en hélium qu'on a bien étudiées, possèdent des champs magnétiques intenses. On a interprété les variations de la luminosité, et de l'intensité des raies et du champ magnétique, dans le cadre du modèle à rotateur rigide.

La limite inférieure des températures des étoiles riches en hélium coïncide avec la limite supérieure pour les étoiles à déficience en hélium; en ce qui conceive la température, les domaines de ces deux groupes d'étoiles ne se recouvrent pas. Les étoiles pauvres en hélium ont des types spectraux compris entre B3 et B7. Les déficiences en hélium sont des facteurs qui varient de 2 à 15. Il existe des indices de trois sous-groupes distincts parmi les étoiles pauvres en hélium. Un des groupes, où les raies de P et de Ga sont renforcées, consiste probablement en un prolongement vers les températures élevées du groupe d'étoiles HgMn. Le deuxième groupe ressemble aux étoiles Si-λ4200 par la grande surabondance de Si. Et le troisième groupe se caractérise par des raies fortes de Ti et Sr. La variabilité photométrique et spectrale, et la présence d'un champ magnétique, caractérisent les deux derniers groupes. Les variations ressemblent beaucoup à celles qu'on observe dans les étoiles Ap magnétiques.

Les étoiles HgMn, ainsi dénommées à cause de l'intensité anormale des raies de Hg et Mn, sont les mieux étudiées de toutes les étoiles Bp. Ce groupe d'étoiles se limite au domaine de température compris entre 11000 et 16000 K (types spectraux de B6 à B9). Ces étoiles tournent lentement ($v \lesssim 100$ km s^{-1}); elles ne présentent pas de champ magnétique détectable, ni de variabilité spectrale. L'abondance de certains éléments, notamment la composition isotopique de Hg, varie de façon systématique en fonction de la température effective.

Les observations des étoiles Bp fournissent quelques contraintes importantes pour les modèles des étoiles particulières. Plusieurs étoiles de la population II présentent des abondances anormales qui ressemblent beaucoup à celles des étoiles Bp de la population I ayant une T_{eff} et un log g semblables. Il est donc évident que le mécanisme responsable des abondances anormales doit relever des caractéristiques des atmosphères stellaires, plutôt que de la masse ou de l'histoire évolutive. Depuis la découverte des étoiles riches en hélium, il est évident que les anomalies chimiques se trouvent partout dans cette région de la séquence principale, où les champs de vitesse dus soit à la perte de masse, soit à la convection, seraient sans importance. La stabilité atmosphérique et les abondances anormales paraîssent étroitement liées. La découverte des étoiles à excès et à déficience d'hélium dans de très jeunes amas fixe une limite supérieure d'environ 10^7 ans pour l'échelle de temps de l'évolution des abondances anormales. Ces contraintes sont compatibles avec les modèles à diffusion radiative. On ne dispose actuellement d'aucune autre explication globale des étoiles particulières du début de la séquence principale.

Les Modèles Atmosphériques

Le problème de la construction de modèles atmosphériques est complexe, surtout pour les étoiles A. Dans la méthode classique, on fait plusieurs hypothèses simplificatrices: l'atmosphère peut se représenter par des couches planes-parallèles et homogènes, elle est dans un état stationnaire, et elle est en équilibre hydrostatique et radiatif. Il est évident que ces hypothèses sont de mauvaises approximations des propriétés réelles d'au moins quelques groupes particuliers d'étoiles A. Des mouvements massiques—y compris la turbulence, la pulsation, et la circulation méridienne—peuvent être présents. La rotation peut modifier la forme de l'étoile, aussi bien que les taux de production d'énergie. Les champs magnétiques peuvent influencer la structure atmosphérique, tandis que le chauffage non-radiatif peut donner lieu à des chromosphères ou à des couronnes. Dans une atmosphère autrement stable, la diffusion peut donner lieu à une stratification des abondances. Les populations des niveaux atomiques peuvent s'écarter de l'équilibre thermodynamique local (ETL). Une perte de masse se produit dans les supergéantes. Contrairement aux étoiles plus chaudes et plus froides, l'hydrogène n'est que partiellement ionisé dans les étoiles A, et il y a, en général, deux zones de convection. Le Chapitre 9 résume les tentatives faites jusqu'à présent pour développer des modèles de ces étoiles complexes.

En principle, le calcul d'un modèle en ETL est direct. En réalité, il y a le problème pratique d'incorporer l'effet de serre, qui est important dans le spectre ultraviolet des étoiles normales de type A, et partout dans le spectre visible des étoiles particulières. Malgré ce problème, les modèles atmosphériques construits dans l'hypothèse de l'ETL et de l'effet de serre représentent assez bien le spectre de raies, aussi bien que le spectre continu, des étoiles normales de type A, aux longueurs d'onde supérieures à 1260 Å. Pour les longueurs d'onde inférieures à 1260 Å, les modèles à ETL surestiment l'opacité due à C I et Si I, et sousestiment le flux émergent. Des modèles en équilibre statistique fournissent une meilleure description du flux ultraviolet, notamment près de Ly α.

Mais tandis qu'une étoile presque normale comme Véga peut se représenter assez bien par des modèles en équilibre radiatif, si l'on tient compte de l'effet de serre et des équations d'équilibre statistique, il est évident que dans les étoiles particulières, y compris les étoiles magnétiques et Am, il faut avoir des modifications importantes de la structure atmosphérique ou de la composition chimique, ou des deux. Parmi les effets physiques pertinents sont la rotation, la diffusion, la convection, et les champs magnétiques.

Dans les étoiles de la séquence principale, la transition du transport d'énergie par rayonnement au transport par convection a lieu dans le domaine de température effective entre 6000 K et 8000 K; ce domaine comprend les étoiles de type A avancé. Les calculs démontrent que la convection peut transporter jusqu'à 6% du flux total dans les étoiles de type A avancé. La théorie classique à longueur de mélange montre qu'il y a deux zones convectives dans les étoiles A. Des traitements plus élaborés de la convection suggèrent que les éléments convectifs pénètrent la zone radiative voisine, et que la région entre les deux zones convectives est bien mélangée. Ce résultat montre que la séparation diffusive des éléments ne peut pas se produire dans la région entre les zones de convection d'hydrogène et d'hélium.

La rotation peut diminuer la gravité effective d'une étoile, modifier la forme superficielle, et baisser les taux de génération d'énergie. Les modèles d'étoiles en rotation uniforme ne prévoient pas de grands changements de la température et de la luminosité, sauf dans le cas des étoiles où la rotation est si rapide que la force centrifuge devient comparable à la force gravitationnelle. Les changements qui se produisent alors imitent les changements dûs à l'évolution stellaire, et il est difficile de séparer les deux effets. Il paraît que les modèles prédisent les bonnes tendances dans les relations entre les couleurs, les luminosités, et la rotation, mais que les effets prévus par les modèles d'étoiles en rotation uniforme sont plus petits que les effets observés. Malheureusement, les observations ne sont pas suffisantes pour déterminer si les étoiles tournent comme des corps solides, ou si elles subissent une rotation différentielle.

Il y a deux hypothèses de base pour l'origine des champs magnétiques stellaires. Ils peuvent être des champs fossiles, engendrés tôt dans le cycle évolutionnaire. Ils peuvent même être les restes du champ présent à l'origine dans le milieu interstellaire. Mais d'autre part, les champs stellaires peuvent être engendrés et soutenus par un mécanisme de dynamo actuellement en vigueur. A l'égard des étoiles Ap magnétiques, l'hypothèse du champ fossile a l'avantage que l'intensité du champ magnétique, aussi bien que l'orientation relative des axes magnétique et rotationnelle, sont essentiellement des paramètres libres, sans relation avec la vitesse angulaire actuelle de l'étoile. Il est donc plus facile de rendre compte des observations au moyen de cette théorie qu'à partir des modèles à dynamo. Il n'est pas facile de comprendre pourquoi on observe des champs magnétiques dans quelques étoiles mais pas dans les autres. Les conditions à l'époque de formation de l'étoile peuvent être le facteur décisif.

Un champ magnétique peut modifier la structure d'une atmosphère stellaire en introduisant un terme de force magnétique dans l'équation d'équilibre hydrostatique. On a calculé les effets à attendre, mais la comparaison directe avec les observations est difficile. La grandeur des modifications structurelles éventuelles est fonction de l'intensité du champ toroïdal, qui ne se mesure pas directement. Une des modifications prévues est une augmentation du rayon; mais il est difficile de distinguer les effets magnétiques des effets évolutifs.

Une méthode plus directe pour détecter les effets du champ magnétique sur les atmosphères stellaires est l'analyse de son influence sur la formation des raies, au moyen de l'effet Zeeman. Il existe d'amples calculs du transfert radiatif en présence d'un champ magnétique. Des observations récentes des caractéristiques de la polarisation dans les profils de raies individuelles ont été comparées en détail aux calculs. On en a tiré des conclusions très détaillées concernant les géometries des champs magnétiques stellaires.

De tous les processus physiques qui peuvent modifier la structure et la composition des étoiles A, celui qu'on a étudié le plus dans la décennie passée est la diffusion radiative. L'idée fondamentale de la théorie de diffusion, c'est que le profil de densité de la composante la plus abondante (l'hydrogène, bien entendu) dans une atmosphère stellaire est déterminé par l'équilibre des forces de gravitation et de pression, où le terme de pression comprend notamment la pression radiative. Pour toute autre espèce atomique (essentiellement une trace), la pression radiative ne sera pas nécessairement de la bonne grandeur pour créer un équilibre de forces. Les éléments qui subissent une pression excessive due aux transitions lié-lié ou lié-libre tendent à être poussés vers le haut, tandis que les éléments pour lesquels la force radiative est faible tendent à se précipiter vers le bas.

Une difficulté capitale des modèles à diffusion, c'est que les vitesses de dérive engendrées par la pression radiative sont extrêmement faibles (~ 1 cm s^{-1} à $\tau \sim 0{,}3$). Il n'est pas facile de savoir si les atmosphères stellaires sont assez stables pour que la diffusion radiative produise une stratification des abondances. On a fait des efforts considérables pour calculer les effets de la turbulence, la circulation méridienne, l'accrétion, les vents stellaires, les champs magnétiques, et des instabilités diverses sur le processus de diffusion.

Une autre méthode est de supposer que la diffusion est importante, et de déterminer dans quelle mesure ce processus peut rendre compte des propriétés des différents groupes d'étoiles particulières de type A. A cet égard, la diffusion rencontre un grand succès. Elle peut rendre compte du domaine de température et de vitesse rotationnelle où se trouvent les étoiles particulières, de l'échelle de temps pour l'évolution des anomalies, de la dichotomie de la pulsation et de la métallicité, et d'un certain nombre d'autres caractéristiques des étioles particulières.

Les Problèmes Actuels

Nos conclusions concernant les propriétés physiques des étoiles relèvent presque totalement de l'utilisation des modèles thermiques classiques pour interpréter les caractéristiques observables. De tels modèles supposent que l'atmosphère est chimiquement homogène, et qu'on peut en déterminer la structure à partir des équations d'équilibre hydrostatique et radiatif, convenablement modifiées au cas où une partie importante du flux est transportée par la convection. Ces modèles classiques présentent, sans doute, une première approximation raisonnable de la structure atmosphérique des étoiles réelles. Mais de tels modèles ne peuvent même pas représenter le spectre émergent total de Véga, le prototype d'une étoile "normale" de type A. Les complexités remarquables des étoiles particulières de type A ne peuvent pas se faire expliquer dans le cadre des modèles thermiques classiques.

De nouvelles observations, faites à partir de la terre et de l'espace, offrent des preuves critiques de l'importance pour la détermination de la structure atmosphérique de phénomènes tels que les champs de vitesse, les champs magnétiques, la rotation, la perte de masse, et le chauffage non-radiatif. Le Chapitre 10 présente quelques questions précises qui peuvent trouver une réponse grâce à des observations supplémentaires. Les étoiles de type A fournissent un laboratoire où l'on peut étudier une grande variété de processus non-thermiques—y compris l'accrétion, la perte de masse, la diffusion, la convection, les pulsations radiales et non-radiales, et l'interaction des plasmas avec les champs magnétiques. L'univers nous lance un grand défi—celui de caractériser les processus dynamiques qui déterminent la structure stellaire à chaque stade évolutionnaire, d'étudier les interactions des étoiles et de leur environnement, et de quantifier les façons dont les étoiles influencent l'évolution des structures plus grandes, comme les amas et les galaxies.

Table R-1
Les Propriétés Observées des Étoiles Particulières du Début de la Séquence Principale

Type: Am	Type: Ap, Sr-Cr-Eu
Température:	Température:
7000, 9000 K (10.000) (1)	8000, 12.000 K (1, 2, 6)
Abondances:	Abondances:
C, Ca, Mg, Sc: −0,5, −1,0 (2)	He: −1,0, −2,0 (2)
Groupe Fe: +0,5, +1,0	Mg: 0,0
Éléments Lourds: +0,5, +1,0	Si, Ca: 0,0, +1,0
H (eff):	V: 0,0, +0,5
Néant (11)	Cr: +3,0
Rotation:	Fe: +1,0, +1,5
Moyenne 40 km s^{-1} (3)	Sr: +2,0, +3,0
Max 125 km s^{-1} (3); 200(?) (4)	La: +1,0, +2,0
Binaires serrées:	Eu: +3,0, +6
Toutes (5)	*H*(eff):
	10^2, 10^4 gauss
	Rotation:
	Moyenne 30 km s^{-1} (8)
	Max 70 km s^{-1} (7)
	Binaires serrées:
	Rares (1)
Type: Ap, Si	Type: λ Boo (9, 19)
Température:	Température:
10.000, 16.000 K (1)	9.000 K
Abondances:	Abondances:
He: −1,0, −2,0 (2)	He: −1,0, −2,0
Si: +1,0, +2,0	Si, Ca, Mg: −1,0
Groupe Fe: +1,0, +2,0	Fe: −0,5
Terres rares: ≤ +6,0	O: 0,0
H(eff):	*H*(eff):
10^2, 10^4 gauss	Inconnu
Rotation:	Rotation:
Moyenne 46 km s^{-1} (8)	Normale (~100 km s^{-1})
Max 300 km s^{-1} (1, 7)	Binaires serrées:
Binaires serrées:	Inconnues
Rares (1)	

Source: D'après Bonsack (1980)
Note: S'il y a deux valeurs, elles dénotent les limites approximatives du domaine observé; on donne les abondances numériques en unités de $[X/H]$.

Table R-1 (suite)

Type: Ap, HgMn
Température:
  10.000, 15.000 K (1)
Abondances:
  He: 0,0, −1,0 (12)
  C: 0,0
  Cr: 0,0, +1,0
  Mn: +2,0
  Hg: 0,0, ou +3,0
H(eff):
  Néant (10, 11)
Rotation:
  Moyenne 30 km s^{-1} (8, 13)
  Max 100 km s^{-1}
Binaires serrées:
  Petit excés (13)

Type: Bp, He-pauvre
Température:
  14.000, 21.000 K (14)
Abondances:
  He^3 excès (18)
  He: −0,5, −1,0 (17)
  C: −1,0, −2,0
  N, O: 0,0, +1,0
  P, K: +2,0, +3,0
H(eff):
  Jusqu'à 10^4 gauss (16)
Rotation:
  Faible
Binaires serrées:
  Inconnues

Type: He-riche
Température:
  21.000, 30.000 K (14)
Abondances:
  He: ≅ H (14)
  C: 0,0
  O, N: +0,5, +1,0
  Mg, Si, Al: 0,0
H(eff):
  0, 3000 gauss (15)
  lie à la rotation
Rotation:
  0, 150 km s^{-1} (15)
Binaires serrées:
  Inconnues

Refs:
(1) Preston (1974)
(2) Bonsack & Wolff (1980)
(3) Abt (1975)
(4) Abt (1979)
(5) Abt (1961)
(6) Hack (1975)
(7) Wolff (1981)
(8) Abt et al. (1972)
(9) Sargent (1967)
(10) Conti (1970)
(11) Borra & Landstreet (1980)
(12) Heacox (1979)
(13) Wolff & Preston (1978)
(14) Osmer & Peterson (1974)
(15) Borra & Landstreet (1979)
(16) Wolff & Wolff (1976)
(17) Baschek & Sargent (1976)
(18) Hartoog & Cowley (1979)
(19) Baschek & Searle (1969)

SUMMARY

Introduction

The main sequence A-type stars occur in a region of the HR diagram where atmospheric models based on the classical assumptions of hydrostatic and radiative equilibrium might well be expected to achieve their greatest success in describing the observed spectra. Surface convection zones are not extensive; hydrogen is the primary source of opacity; rotation is moderate; and the usual signs that simple thermodynamic equilibrium models must fail—emission lines, irregular short-period variability, etc.—are missing. Yet two out of every ten A-type stars display spectral anomalies conspicuous enough to be detected on low-resolution spectrograms, and still more stars appear anomalous at higher resolution. The characteristics of the major types of peculiar A- and B-type stars are summarized in Table 1 (Bonsack, 1981a). Evidently, physical processes that are masked by mass flows in more massive stars, or by convection in less massive stars, play a dominant role in shaping the emergent spectra. Meridional circulation, subsurface convection zones, pulsation, diffusive separation of elements, and magnetic fields are some of the phenomena thought to be important. The problem of understanding the main sequence A-type stars is a formidable one, because our ability to model these processes and their interactions is currently quite limited. On the other hand, these stars offer us an opportunity as well by providing us a laboratory in which we can test model atmospheres that include hydrodynamic and hydromagnetic effects.

The A supergiants pose a different set of problems. Mass loss is clearly occurring in many of them, and irregular or semiregular variability is the norm. The challenge here is to define observationally, and then to model, the characteristics of stars that are clearly not in hydrostatic equilibrium.

The Normal A-Type Stars

In most parts of the HR diagram, "normal" stars are in the majority. Among the A-type stars, however, abnormalities are the rule, rather than the exception. Indeed, there are probably no slowly rotating late A-type stars that could reasonably be classified as normal. Because the line strength anomalies are usually quite pronounced, MK spectral classification offers an efficient method of distinguishing normal from peculiar stars. Accurate classification of

A-type stars in terms of temperature and luminosity is, however, quite difficult, since suitable metallic lines of intermediate strength are lacking. The usable lines are either very strong (Ca II λ3933 and the Balmer lines of hydrogen) or very weak. Furthermore, line ratios may depend on such atmospheric properties as microturbulence, as well as temperature and luminosity, and one must exercise considerable caution in attempting to extend the MK system to stars with abnormal composition or atmospheric structure.

Photometric classification offers an alternative method for characterizing the A-type stars. Particularly useful is the uvby β system, which measures the temperature and luminosity of A-type stars and provides an estimate of line blanketing as well. Line-blanketed model atmospheres have been used to calibrate the uvby β indices. The models and observations are consistent for early A-type stars, but the theoretical and measured colors disagree for late A-type stars, probably because of an inadequate treatment of convection. The effective temperatures inferred from the calibration of the uvby β indices are in good agreement with those derived from direct measurements of the energy distributions from the satellite ultraviolet through the visible and infrared regions of the spectrum.

The typical mass of an A0 star is $\sim 2.5\, M_\odot$ and of a late A-type star $1.5\, M_\odot$. Typical values of the radii are $2.1\, R_\odot$ to $1.5\, R_\odot$, with some dependence on evolutionary state as well as mass.

Normal late A-type stars have been found in spectroscopic binary systems with $P < 2.5$ days and $P > 100$ days. In sharp contrast to this result, the majority of Am stars are members of binaries with periods in the range 2.5 to 100 days. There is also a dichotomy in the rotational velocity distribution of Am and normal late A-type stars. There are no normal stars that rotate more slowly than ~ 40 km s^{-1} and no Am stars that rotate more rapidly than ~ 100 km s^{-1}. There is clear evidence that tidal synchronization has slowed the rotation of late A-type stars that are members of close binaries.

Evidence for differential rotation in A-type stars has been sought but not found. Searches for magnetic fields in normal A-type stars have also been unsuccessful; effective longitudinal fields greater than 150 g must be quite uncommon.

There has been little abundance work on normal A-type stars, in part because there are so few normal stars with lines sharp enough to permit detailed analysis. The results that are available are quite puzzling. There appears to be good evidence for a group of metal-deficient stars near A0, and the frequency of such stars may equal that of the Ap stars. The explanation for this deficiency is quite unclear.

Nonradiative Heating in A-Type Stars

In the Sun, the chromosphere, corona, and the transition region between them are all characterized by temperatures that exceed substantially the values predicted by model atmospheres in radiative equilibrium. Chapter 3 addresses the question of whether or not analogous regions can be identified in the outer atmospheres of A-type stars. Extensive searches have been made in A-type stars for line and continuum features that are diagnostic of high-temperature regions. Specific features that have been looked for include Ca II H and K emission, Mg II h and k emission, He I λ10830 in either emission or absorption, Lα emission, and Si IV, C IV, and He II emission lines in the satellite ultraviolet. The *only* A-type star in which chromospheric emission has so far been reported is α Aql (A7 IV-V, v sin i = 245 km s^{-1}).

There is evidence for emission in Mg II h and k and Ly α in this star. The evidence for irregular variability in A-type stars, which might be another indicator of nonequilibrium processes, is not yet completely convincing.

In view of these negative results, it is perhaps surprising that X-ray emission has been seen in a number of A-type stars. In at least some cases, the emission is primarily due to a late-type companion. In other cases, however, emission is seen in A-type stars not now known to be members of binaries. The strength of the emission is not correlated with rotation, as is the case for late-type stars. Rather, the correlation between X-ray and bolometric luminosities found for O- and early B-type stars ($L_x \sim 10^{-7}\ L_{\rm bol}$) seems to apply to at least some A-type stars. The discovery of X-ray emission from A-type stars is very recent, and only very tentative steps have been taken toward developing models to account for that emission.

The Magnetic Ap Stars

The Ap stars are characterized by unusually strong lines of one or more of the following elements: Si, Cr, Sr, Eu and other rare earths. Typical Ap stars vary on time scales of days to, in a few exceptional cases, decades. Changes are seen in the strengths of the spectral lines, in magnetic field strength, and in luminosity and color. In a given star the period of variation is the same for all these quantities. The model that best accounts for the variations is the oblique or rigid rotator model. The basic postulate of this model is that the axis of the magnetic field is inclined at some angle with respect to the rotation axis. The field is assumed to be locally constant, i.e., "frozen in" to the surface of the star, and to corotate with it. To a distant observer, the magnetic field will appear to vary in intensity as the star rotates. The spectrum variations are explained by assuming that the variable elements are concentrated in patches on the stellar surface. This model can explain the phase relationship between changes in radial velocity and line strength that typify Ap stars; the sign and amplitude of changes in line width (the "crossover" effect); the relationship between the mean surface field and its longitudinal component averaged over the stellar surface; the variation in the transverse magnetic field as inferred from the changes with phase of the linear polarization of individual metallic lines; and the changes with phase of the profiles of strong lines such as Ca II λ3933. The photometric variations are in large part due to changes in line blanketing.

The Ap stars as classically defined have temperatures like those of normal stars with spectral type late B to late A. Magnetic fields have recently been detected in helium-rich and helium-weak Bp stars, and it now seems clear that the oblique rotator phenomenon occurs in stars as early as spectral type B2.

While the approximate temperatures of the Ap stars are known, precise determinations are difficult. Line blanketing caused by the overabundances of metals depresses the flux in the ultraviolet region of the spectrum, thereby altering both the energy distribution and the atmospheric structure. Within the uncertainties of current models of these complex atmospheres, the Ap stars appear to be main sequence objects, and approximately 5 to 10 percent of the late B- and early A-type stars have been classified as Ap stars. The frequency of close binaries seems to be much lower among Ap stars than it is among A-type stars without detectable magnetic fields. The data that are available for Ap binaries indicate that their masses are normal for their temperatures.

The Ap stars rotate much more slowly on the average than do normal stars of the same temperature. Presumably these stars have lost angular momentum because of some form of magnetic braking. Periods of variation as long as 75 years have been reported. If these periods are indeed rotation periods, then the braking process has been extremely efficient.

A very interesting recent result is the discovery that at least some of the cooler Ap stars vary in brightness on time scales of minutes. These variations seem to be associated with non-radial pulsations and have been explained in terms of an "oblique pulsator model." The basic premise of this model is that the pulsation is symmetric about the magnetic rather than the rotation axis. With this assumption, one can derive from the observed pulsation amplitudes such quantities as the angles by which the rotation axis is inclined to the line of sight and to the magnetic axis. The inferred geometries are compatible with results obtained earlier by using the rigid rotator model to interpret spectrum and magnetic variations.

While the variations of the Ap stars seem to be explained adequately by the rigid rotator model, the origin of the abundance anomalies remains something of a mystery. A variety of mechanisms involving nucleosynthetic processes have been proposed, but given our present knowledge, all seem to be unsatisfactory. The alternative is radiative diffusion, possibly modified by such effects as accretion and turbulence. The primary problem with this model is that it requires an atmospheric stability that is well beyond what many theoreticians think likely. On the other hand, it does explain a number of the properties of Ap stars. Calculations of diffusion in the presence of a strong magnetic field are just now becoming available. Since these calculations predict definite relationships between the magnetic geometry and the distribution of elements over the stellar surface, some quite rigorous tests of the diffusion model should be possible.

Am Stars

The modern definition of Am stars is as follows: The Am phenomenon is present in stars that have an apparent surface underabundance of calcium (and/or scandium) and/or an apparent overabundance of the Fe group and heavier elements. Only a subset of the stars that meets these conditions also satisfies the classical definition that an Am star is one in which the metallic line and K line spectral types differ by more than five spectral subclasses. The Am stars are main sequence objects and are found throughout the temperature range 7400 K $< T_{\rm eff} <$ 10,200 K. While there can be no doubt that Am stars occur with very much lower frequency outside this temperature interval than within it, selection effects probably play a role in determining the apparent temperature boundaries. Metallicism is very difficult to detect at both higher and lower temperatures except through detailed abundance analyses based on high-resolution spectrograms.

The Am stars are distinguished from normal A-type stars not only by their spectroscopic characteristics but also by their binary frequency and rotational velocity distributions. The majority, and possibly all, of the Am stars with spectral types later than A4 are members of spectroscopic binary systems, and two-thirds of the known binaries are in systems with orbital periods less than 100 days. In contrast, few normal A-type stars are known to be binaries, and those few that are tend to have long ($P >$ 100 days) orbital periods. The largest rotational velocity observed for an Am star to date is ~ 100 km s^{-1}, and all of the late A-type stars with v sin $i < 40$ km s^{-1} are probably metallic line stars. The fundamental question is which, if either, of these characteristics—rotation or binary membership—is causally related to metallicism. If *all* Am stars are indeed binaries, then one might conclude that binary interactions somehow cause the line-strength anomalies. If, on the other hand, some Am stars are single, but all rotate slowly, then the correct conclusion might be that slow rotation is the necessary condition for metallicism. In the latter case, the preponderance of binaries among Am stars would be attributed to a tendency toward synchronism, and hence slow rotation, due to tidal interactions in close binaries.

Other constraints on models for the origin of metallicism come from studies of Am stars in clusters. Most significant is the discovery of several Am stars in the Orion Association, a result that demonstrates that metallicism can develop in 1 to 3×10^6 years. There is some evidence for variability in the Am stars, but in no case does the variability seem to be due to either pulsation, as in the δ Sct stars, or to rotation of an inhomogeneous surface, as in the magnetic Ap stars. In terms of their line-strength anomalies, the Am stars appear to be a much less heterogeneous group than the Ap stars.

Considerable effort has been devoted to attempts to determine whether the observed line-strength anomalies are indicative of abundance anomalies. To date, no one has devised a model atmosphere that can reproduce the observed line strengths if the abundances are required to be solar. Observational constraints are by now so severe that there is little latitude to alter the temperature and ionization structure of an Am star atmosphere from the values that characterize atmospheres of normal stars.

Explanations for abundance anomalies in Am stars fall into three broad general categories: (1) the anomalies are the consequence of nucleosynthesis, either in the stellar interior or on the surface of the star; (2) they are the result of accretion from grains or mass lost from evolved stars; (3) they are caused by a separation of elements within the star due to radiative diffusion processes. Given our present understanding, only the last of these possibilities appears to be compatible with the observations.

The δ Sct Stars

The extension of the Cepheid instability strip crosses the main sequence in the region occupied by late A-type and early F-type stars. It is therefore not surprising that variability is common among late A-type stars, and Chapter 6 discusses the properties of pulsating variables that lie near the main sequence. Luminosity variations of such stars are characteristically on the order of a few hundredths of a magnitude, although amplitudes larger than 0.3 mag. are not unknown; periods are less than 0.3 day; and in many stars the shapes of the light curves vary markedly from cycle to cycle.

Most of the stars that exhibit variations with these characteristics are Population I stars in a main sequence or early post-main sequence phase of evolution. However, there is also a group of stars that exhibit similar variations but that have high space velocities and low metal abundances. Obviously, these stars are not members of Population I. They may be main sequence Population II stars, in which case they are analogous to the blue stragglers seen in many clusters. Alternatively, the metal-poor variables may be objects that are evolving toward the white dwarf stage after having lost a substantial amount of mass while they were red giants. In Chapter 6, both types of stars are referred to as δ Sct stars.

The δ Del stars are defined spectroscopically to be those late A- and early F-type giants and subgiants with disparate K line and metal-line spectral types. Many δ Del stars are also photometric variables, and for that reason they are discussed in Chapter 6 along with the δ Sct stars. Variability is *not*, however, a requisite for membership in the δ Del class. Because their abundances resemble those of the Am stars, it has been suggested that at least some δ Del stars are evolved metallic-line stars.

Approximately one-third of the stars on or near the main sequence that lie within the instability strip vary by 0.01 mag. or more, and the incidence of variability seems to be independent of stellar age. The Am stars are not variable, and the absence of pulsation in them has been attributed to diffusion. The primary driving mechanism for pulsation is the He II ionization zone. In a stable atmosphere, helium, which is not supported by radiation pressure, will tend to settle out of the atmosphere. If the helium content becomes low enough, the star will be stable against pulsation. At the same time, other elements that are strongly supported by radiation pressure may be concentrated in the line-forming regions, thus producing abundance anomalies characteristic of Am stars.

The majority of well-studied δ Sct stars exhibit stable periodicity that, in some cases, persists for at least several years. In many stars, several radial modes are excited simultaneously. In the cooler portion of the instability strip, the fundamental mode tends to be of larger amplitude than the other modes. Nonradial pulsation occurs throughout the entire instability strip but is not seen in every star. Radial and nonradial pulsation modes may coexist in the same star. Population I δ Sct stars with amplitudes in excess of 0.3 mag. typically do not exhibit complex multiperiodicity; the dominant pulsation mode is usually the fundamental radial mode. A variety of techniques for determining pulsation modes are described in Chapter 6. In general, the observed pulsation characteristics of δ Sct stars can be well represented by existing theoretical models.

A Supergiants

In external galaxies, as in our own, the stars with the brightest visual magnitudes are usually of spectral type B or A. (The total luminosity of O supergiants is higher, but only because their bolometric corrections are much larger. The absence of A supergiants with bolometric luminosities comparable to those of O-type stars is attributed to—perhaps catastrophic—mass loss in earlier stages of evolution.) Spectroscopic studies of A supergiants therefore, potentially, offer one of the best ways to determine whether the evolution of massive stars and the physical processes in their atmospheres are the same in other galaxies as in our own. To date, however, the research on A supergiants has been quite limited. Because these stars occupy a region of the HR diagram where evolution is rapid, they are few in number. Much of the most recent research on supergiants of both earlier and later types has been stimulated by the availability of satellite ultraviolet observations of mass loss and X-ray observations that provide evidence of nonradiative heating. In the A supergiants, there is little evidence for chromospheres or coronae, and their comparatively low UV and X-ray fluxes limit satellite observations.

The variations of A supergiants are extremely complex. A variety of time scales is involved, ranging from hours to years. Variations can occur both in the stellar wind and in photospheric features. Variable quantities include luminosity, radial velocity, line strength, and line shape. Since the variations are only semiperiodic and simultaneous measurements of different variable characteristics are rarely available, rather little is known about the correlations of one variable characteristic with another or of photospheric changes with variations in the stellar wind.

All A supergiants vary in radial velocity with amplitudes typically in the range 4 to 10 km s^{-1}. A thorough study of the velocity variations in α Cyg has led to the identification of 16 discrete pulsational modes with periods ranging from 6.9 to 100.8 days. Since many of the modes have periods that exceed the expected period of the fundamental radial mode (14.3 days), nonradial pulsations must occur in this star. The amplitudes and phases of the various modes were stable over at least a 6-year interval.

Photometric variations of 0.03 to 0.05 mag. are typical of A supergiants. The amplitude of the brightness variation tends to increase with luminosity while the time scale for the semiperiodic variations tends to increase with increasing luminosity and decreasing effective temperature.

A variety of observational techniques allow the detection of mass loss and the derivation of mass loss rates. In the A supergiants, the best evidence for mass loss comes from analyses of the broad absorption features of abundant singly ionized elements in the ultraviolet region of the spectrum. Velocity shifts as high as -240 km s^{-1} have been seen. Such velocities are comparable to the (highly uncertain) estimates of the photospheric escape velocity, and so provide fairly clear evidence of mass loss. Variations in the ultraviolet absorption have been reported, and variations in Hα emission are common, so the wind appears to be variable. Mass loss rates have been estimated by modeling line profiles and from measurements of infrared excesses, which are produced by free-free emission in the wind. Typical mass loss rates are in the range 10^{-7} to 10^{-9} $M_\odot$ yr^{-1}, but the various methods can yield quite different mass loss rates for any given star.

Theories of the origin of stellar winds in A supergiants fall into four general categories: radiation pressure driven cool winds, hot coronal winds produced by a gradient in gas pressure, hybrid models that combine hot coronal regions with cool winds driven by radiation pressure, and "imperfect" flow models, which attribute the initiation of winds to fluctuations or instabilities in the photospheric velocity field. The determination of which, if any, of these models is applicable to A supergiants is difficult because the winds are weak and their characteristics poorly defined. Because our understanding of the physical processes involved in mass loss is so limited, most theoretical effort has been devoted to the study of the kinematics of radiative transfer in moving media. The velocity field and temperature structure are specified as input parameters for the models, and emergent spectra, including line profiles, can then be calculated. It is modeling of this kind that has provided the best estimates of mass loss rates in A supergiants.

Peculiar B-Type Stars

So far as the physics of stellar atmospheres is concerned, the boundary of A0 is an artificial one. Peculiar stars are found as early as spectral type B2, with the high temperature limit probably coincident with the onset of significant mass loss. Since a number of the characteristics of Bp stars are used as boundary conditions for models of both Bp and Ap stars, a brief review of the characteristics of Bp stars is presented in this chapter.

The hottest of the Bp stars are the He-rich stars. These stars occupy a very restricted region of the HR diagram; their temperatures correspond to (approximately) spectral type B2, and they have main sequence surface gravities. Emission at Hα and violet-shifted absorption in the C IV doublet λ1548, λ1550 both provide evidence that He-rich stars have winds. The majority of well-studied, He-rich stars have strong magnetic fields. Variations in luminosity, magnetic field strength, and line strengths are common and have been interpreted in terms of the rigid rotator model.

The low-temperature cutoff for the He-rich stars coincides with the high-temperature boundary of the He-weak stars, and there is apparently little or no overlap in the temperatures of these two classes of stars. The He-weak stars have spectral types in the range B3 to B7. The helium deficiencies range from factors of 2 to 15. There is evidence for three distinct subgroups of He-weak stars. One group, in which P and Ga lines are enhanced, probably form an extension to higher temperatures of the HgMn stars. The second group resembles the Si-λ4200 stars in the sense that Si is greatly overabundant; and the third group is characterized by strong lines of Ti and Sr. Magnetic fields and photometric and spectrum variability characterize the latter two groups, and these variations are quite similar to those seen in magnetic Ap stars.

The HgMn stars, so-called because lines of both Hg and Mn are anomalously strong, are the best studied of all the Bp stars. This class of objects is restricted to the temperature range 11,000 to 16,000 K (spectral types B6 to B9). These stars are slow rotators ($v < \sim 100$ km s^{-1}), do not have detectable magnetic fields, and are not spectrum variables. The abundances of certain elements, including most notably the isotopic composition of Hg, vary systematically with effective temperature.

Several important constraints on models of peculiar stars are provided by observations of the Bp stars. Because several Population II stars have abundance anomalies that closely resemble Population I Bp stars of similar T_{eff} and $\log g$, it seems clear that the mechanism for producing the abundance anomalies must depend on the characteristics of stellar atmospheres rather than on stellar mass or evolutionary history. With the discovery of the He-rich stars it is apparent that chemical peculiarities are found throughout the main sequence region where macroscopic velocity fields due either to mass loss or convection are likely to be of minimal importance. Atmospheric stability and abundance anomalies seem to be closely correlated. The discovery of He-rich and He-weak stars in quite young clusters sets an upper limit of $\sim 10^7$ years on the time scale for the development of abundance anomalies. These constraints are compatible with radiative diffusion models. No other comprehensive explanation for the peculiar stars of the upper main sequence is currently available.

Model Atmospheres

The problem of constructing model atmospheres is complex, and particularly so for A-type stars. Classically, several simplifying assumptions are made—that the atmosphere can be represented by plane-parallel homogeneous layers, that it is in a steady state, and in hydrostatic and radiative equilibrium. It is obvious that these assumptions are poor approximations to the actual properties of at least some specific classes of A-type stars. Mass motions, including turbulence, pulsation, and meridional circulation, may be present. Rotation may alter both the figure of the star and the rates of energy generation. Magnetic fields may affect atmospheric structure while nonradiative heating may induce chromospheres or coronae. In an otherwise stable atmosphere, diffusion will produce a stratification of abundances. Non-LTE level populations can occur. In supergiants, mass loss is taking place. In contrast to hotter and cooler stars, in A-type stars hydrogen is only partially ionized, and there are, in general, two convection zones. Chapter 9 presents a summary of attempts to date to develop models of these extremely complex stars.

In principle, the calculation of a classical model in LTE is straightforward. In practice, there are practical problems in incorporating the effects of line blanketing, which is substantial in the ultraviolet spectrum of normal A-type stars and throughout the visible in peculiar stars. Despite this problem, LTE line-blanketed models provide a fairly good representation of both the line and continuous spectra of normal A-type stars longward of $\lambda 1260$. The LTE models overestimate the opacity due to C I and Si I, and underestimate the emergent flux, for the region $\lambda < 1260$. Statistical equilibrium models provide an improved description of the ultraviolet fluxes, particularly near Ly α.

While a nearly normal star like Vega may be fairly adequately represented by radiative equilibrium models with proper inclusion of blanketing and the rate equations, it is clear that in magnetic, Am, and other peculiar stars there must be major changes in atmospheric structure, chemical composition, or both. Relevant physical effects include rotation, diffusion, convection, and magnetic fields.

In main sequence stars, the transition from radiative to convective energy transport occurs over the temperature range 6000 K $< T_{eff} <$ 8000 K; this range encompasses the late A-type stars. Calculations show that convection may carry up to 12 percent of the total flux in late A-type stars. Standard mixing length theory indicates that there are two convection zones in A-type stars. More sophisticated treatments of convection indicate that convective elements penetrate the adjacent radiative zone and that the region between the two convection zones is well mixed. This result shows that diffusive separation of elements cannot occur in the region between the hydrogen and helium convection zones.

Rotation can reduce the effective gravity of a star, alter its surface figure, and lower energy generation rates. Models of uniformly rotating stars predict that large changes in temperature and luminosity will occur only in stars in which rotation is so rapid that the centrifugal force becomes comparable to the force of gravity. The changes that do occur mimic the changes due to stellar evolution, and it is difficult to disentangle the two effects. It appears that the models predict the right trends in the relationships between colors, luminosities, and rotation, but that the effects predicted by models of uniformly rotating stars are smaller than those actually observed. Unfortunately, observations are not adequate to indicate whether stars rotate differentially or as solid bodies.

There are two basic hypotheses for the origin of stellar magnetic fields. They may be fossil fields generated early in the evolutionary cycle; they may even be remnants of the field originally present in the interstellar medium. Alternatively, the fields may be generated and maintained by a dynamo mechanism that is active at the present time. With respect to magnetic

Ap stars, the fossil field hypothesis has the advantage that both the magnetic field strength and the relative orientation of the magnetic and rotation axes are essentially free parameters that are unrelated to the present angular velocity of the star. It is therefore easier to account for the observations with this theory than with the dynamo models. It is unclear why magnetic fields are seen in some stars but not others; conditions at the time of star formation may be the critical factor.

A magnetic field can alter the structure of a stellar atmosphere through the introduction of a magnetic force term into the equation of hydrostatic equilibrium. Calculations of the effects to be expected have been made, but direct comparison with observations is difficult. The magnitude of any structural changes depends on the strength of the toroidal field, which cannot be measured directly. One predicted change is an increase in radius, but it is difficult to separate magnetic from evolutionary effects.

A more direct method for detecting effects of the magnetic field on stellar atmospheres is the analysis of its influence on line formation through the Zeeman effect. There are extensive calculations of radiative transfer in the presence of a magnetic field. Recent observations of polarization characteristics across individual line profiles have been compared in detail with the calculations; quite detailed inferences about the stellar magnetic geometries have been made.

Of all the physical processes that might alter the structure and composition of A-type stars, the one that has been studied most extensively in the past decade is radiative diffusion. The basic idea behind diffusion is that the density profile of the most abundant constituent (hydrogen, of course) in a stellar atmosphere is determined by the balance of gravitational and pressure forces, where the pressure term specifically includes radiation pressure. For any other (essentially trace) atomic species, radiation pressure may or may not be of precisely the right magnitude to lead to a balance of forces. Those elements experiencing an excess pressure due to either bound-bound or bound-free transitions will tend to be driven upward, while those elements for which the radiation force is weak will tend to settle downward.

A major problem with diffusion models is that the drift velocities induced by radiation pressure are extremely low (~ 1 cm s^{-1} at $\tau \sim 0.3$). It is unclear whether or not stellar atmospheres are stable enough for radiative diffusion to produce an abundance stratification. Extensive efforts have been made to calculate the effects of turbulence, meridional circulation, accretion, stellar winds, magnetic fields, and various instabilities on the diffusion process.

The alternate approach is simply to assume that diffusion is important and to determine how well this process accounts for the properties of the various types of peculiar A-type stars. Diffusion is extremely successful in this regard. It can account for the temperature and rotational velocity ranges in which peculiar stars are found, for the time scale in which anomalies develop, for the dichotomy between pulsation and metallicism, and a number of other characteristics of peculiar stars.

Outstanding Problems

Our deductions concerning the physical properties of stars depend almost entirely on the use of classical thermal models to interpret observable characteristics. Such models assume that the atmosphere is chemically homogeneous and that its structure can be determined from the equations of hydrostatic and radiative equilibrium, with appropriate modifications in those cases where a significant fraction of the flux is carried by convection. Such classical models offer, without doubt, a reasonable first approximation to the photospheric structure of real stars. Such models cannot, however, represent in its entirety the emergent spectrum even of Vega, the prototypical "normal" A-type star. The remarkable complexities of the peculiar A-type stars are not explained at all in the context of classical thermal models.

New observations, from the ground and from space, are providing critical evidence of the importance of such phenomena as velocity fields, magnetic fields, rotation, mass loss, and nonradiative heating in determining atmospheric structure. Specific questions concerning these various processes that might be answered through additional observations are presented in Chapter 10. The A-type stars provide a laboratory in which we can study a wide variety of nonthermal processes, including accretion, mass loss, diffusion, convection, radial and nonradial pulsations, and the interaction of plasmas with magnetic fields. The challenge that lies before us is great—namely, to characterize the dynamical processes that determine stellar structure at each evolutionary phase, to study the interactions of stars with their environment, and to quantify the ways in which stars shape the evolution of such larger structures as clusters and galaxies.

Table 1
Observed Properties of The Chemically Peculiar Stars of The Upper Main Sequence

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Type: Am
Temperature:
  7000, 9000 K (10,000) (1)
Abundance:
  C, Ca, Mg, Sc: −.5, −1. (2)
  Fe group: +.5, +1.
  Heavy elem: +.5, +1.
H(eff):
  None (11)
Rotation:
  Mean 40 km s^{-1} (3)
  Max 125 km s^{-1} (3); 200(?) (4)
Close Binaries:
  ~All (5)

Type: Ap, Sr-Cr-Eu
Temperature:
  8000, 12,000 K (1, 2, 6)
Abundance:
  He: −1., −2. (2)
  Mg: 0.
  Si, Ca: 0., +1.
  V: 0., +.5
  Cr: +3.
  Fe: +1.0, +1.5
  Sr: +2., +3.
  La: +1., +2.
  Eu: +3., +6.
H(eff):
  10^2, 10^4 g
Rotation:
  Mean 30 km s^{-1} (8)
  Max 70 km s^{-1} (7)
Close Binaries:
  Rare (1)

Type: Ap, Si
Temperature:
  10,000, 16,000 K (1)
Abundance:
  He: −1., −2. (2)
  Si: +1., +2.
  Fe group: +1., +2.
  Rare Earth: $\leq$ +6.
H(eff):
  10^2, 10^4 g
Rotation:
  Mean 46 km s^{-1} (8)
  Max 300 km s^{-1} (1, 7)
Close Binaries:
  Rare (1)

Type: λ Boo (9, 19)
Temperature:
  9000 K
Abundance:
  He: −1., −2.
  Si, Ca, Mg: −1.
  Fe: −.5
  O: 0.
H(eff):
  Not known
Rotation:
  Normal (~100 km s^{-1})
Close Binaries:
  Not known

Table 1 (Continued)

Type: Ap, HgMn	Type: Bp, He-Weak
Temperature:	Temperature:
10,000, 15,000 K (1)	14,000, 21,000 K (14)
Abundance:	Abundance:
He: 0., −1. (12)	He^3 excess (18)
C: 0.	He: −.5, −1. (17)
Cr: 0., +1.	C: −1., −2.
Mn: +2.	N, O: 0., +1.
Hg: 0. or +3.	P, K: +2., +3.
H(eff):	*H*(eff):
None (10, 11)	Up to 10^4 g (16)
Rotation:	Rotation:
Mean 30 km s^{-1} (8, 13)	Low
Max 100 km s^{-1}	Close Binaries:
Close Binaries:	Unknown
Slightly above normal (13)	
Type: He-Strong	References:
Temperature:	(1) Preston (1974)
21,000, 30,000 K (14)	(2) Bonsack and Wolff (1980)
Abundance:	(3) Abt (1975)
He: ≅ H (14)	(4) Abt (1979)
C: 0.	(5) Abt (1961)
O, N: +.5, +1.	(6) Hack (1975)
Mg, Si, Al: 0.	(7) Wolff (1981)
H(eff):	(8) Abt, Chaffee, and Suffolk (1972)
0, 3000 g, (15)	(9) Sargent (1967)
correlated with rotation	(10) Conti (1970a)
Rotation:	(11) Borra and Landstreet (1980)
0, 150 km s^{-1} (15)	(12) Heacox (1979)
Close Binaries:	(13) Wolff and Preston (1978a)
Unknown	(14) Osmer and Peterson (1974)
	(15) Borra and Landstreet (1979)
	(16) Wolff and Wolff (1976)
	(17) Baschek and Sargent (1976)
	(18) Hartoog and Cowley (1979)
	(19) Baschek and Searle (1969)

Source: From Bonsack (1981a).

Note: Two values indicate approximate limits of the observed range; numerical abundances are given in units of [*X*/*H*].

1

THE A-TYPE STARS

INTRODUCTION

The main sequence A-type stars pose very significant problems for those who would use spectra to derive the properties that characterize stellar atmospheres. In the absence of observational evidence, one might have expected the A-type stars to be the simplest of all the categories of stars. They possess no extensive surface convection zones; in the outer layers of the atmosphere the flux is carried entirely by radiation. Hydrogen is the primary source of opacity, rotation is moderate, and the usual signs that simple thermodynamic equilibrium models must fail—line or continuum emission that exceeds what would be predicted for model atmospheres in radiative equilibrium, turbulent line broadening, mass loss, irregular short-period variability, etc.—are either undetectable or much less evident than in hotter and cooler stars. Yet observations show that of all the spectral classes, the A-type stars possess the greatest variety. Two out of every ten display spectral anomalies conspicuous enough to be detected on low-resolution spectrograms (Cowley et al., 1969), and still more stars appear anomalous at higher resolution. Add to the high percentage of abnormal stars the fact that the spectral lines of the majority of normal stars are substantially broadened by rotation so that detailed analyses are difficult, and it becomes obvious why the study of main sequence A-type stars is, to a considerable degree, the study of peculiar objects.

The A supergiants present an entirely different set of problems. These stars are highly luminous; low-amplitude, irregular or semiregular variability in luminosity and radial velocity are common. The A supergiants are also losing mass, and the observational evidence indicates that the stellar winds are not constant with time. The challenge posed by the A supergiants is to develop a model for stars with extended atmospheres that are not in hydrostatic equilibrium. The first step must be an empirical one: to define in detail the nature of the variations in the photosphere and the wind, and to explore the coupling, if any, between them. Only then is it reasonable to try to build models for these very complex objects.

The purpose of this book is threefold: to describe the observations of the peculiar and normal A-type stars, both supergiants and dwarfs; to show how those observations serve to constrain theoretical models; and to evaluate the extent to which existing models simultaneously meet those constraints and reproduce the observed spectra.

The most unbiased approach to this subject would be to present first only the observations without interpretation in order to establish a baseline of strictly empirical facts about A-type stars. In practice, however, almost all of the "facts" about stars, apart from measurements of spectral types, colors, and line profiles, depend on models. The dependence may be weak, as in the derivation of effective temperatures from a combination of ground-based and spacecraft measurements; or it may be strong, as in the calculation of abundances. In either case, observations and modeling are inextricable, and, for that reason, I have chosen to assemble the information that we possess about

A-type stars by presenting, simultaneously, the observations and current interpretations of them. Throughout the discussion, I have tried to show explicitly how our deductions about the physical characteristics of stars depend on modeling and what uncertainties are introduced by the limitations of existing models. The various classes of A-type stars are treated separately, since they differ from one another in fundamental ways.

The discussion of the observations and their interpretation serves to underscore the diversity of physical phenomena—rotation, magnetic fields, turbulence, diffusion, mass loss—that affect the emergent spectra of A-type stars. Therefore, the penultimate chapter of this book is devoted to a critical evaluation of the progress already made in developing models that incorporate the relevant physical processes.

The emphasis of this manuscript is on the *current* status of our understanding of A-type stars. I have not adopted a historical approach, and much of the enormous effort that has been expended to achieve our present level of knowledge will not be apparent to the reader. For example, the wide acceptance of the rigid rotator model as the explanation of the variations of magnetic Ap stars is the result of literally hundreds of studies of individual members of this class of objects. Fewer then 10 percent of those papers are cited in this manuscript. Those papers that have been cited tend to be the papers that significantly advanced our picture of an entire class of objects, rather than of a single member of that class. Preference is also given to papers that provided the first or last pieces of a particular puzzle. While natural and probably unavoidable, this bias gives too little recognition to the contributions of those who painstakingly filled in the intermediate parts of the picture. As those who do puzzles—whether jigsaw or astronomical—know only too well, work on the middle portion usually requires the most skill and patience. To those scientists whose very important contributions to studies of the A-type stars have not been explicitly mentioned in the present monograph, I offer my apologies.

This volume includes no lists of the members of specific categories of A-type stars. Neither does it attempt to catalog all the observations of a particular object or class of objects. Computer technology has fortunately relieved authors of that burden. Data bases are maintained and constantly updated in several centers (Jaschek, 1977), and interested readers should take advantage of those facilities if they need bibliographical information on specific stars. I have, however, tried to provide enough references so that an individual who reads those more detailed discussions can discover all of the essential work in this field.

CLASSES OF MAIN SEQUENCE STARS

Because of the complexities and the enormous variety presented by the main sequence A-type stars, it is useful to begin with an overview of the various types of objects encompassed by this spectral range. The main subgroups of the A-type stars are described below.

- **The Magnetic Ap Stars.** These stars are distinguished spectroscopically by enhancements of the lines of one or more of the following elements: silicon, chromium, strontium, and europium. Lines of additional elements, including particularly other rare earths, may also be enhanced. The stars in this spectroscopic class for which accurate measurements of the Zeeman effect have been made all possess magnetic fields.
- **The Metallic Lined (Am) Stars.** Classically (Roman et al., 1948), the Am stars were defined to be those for which the spectral type derived from the K line of Ca II is at least five spectral subclasses earlier than the type that corresponds to the metallic line strength. A variety of broader definitions have been proposed, and their utility will be considered in detail in Chapter 5.
- **δ Del Stars.** These are stars with spectra like that of the prototype, namely a giant or subgiant with enhanced metal lines.
- **δ Sct Stars.** The stars in this group are pulsating variables that lie in the Cepheid instability strip or its extension toward the main sequence and that have periods of less than 1 day. These two restrictions effectively limit the group to Population I stars of types A and early F (Baglin et al., 1973). Unlike the preceding definitions, this one does not depend on spectroscopic criteria,

and therefore any of the other types of stars *may* also be δ Sct stars. The overlap, if any, of this and the other groups must be determined observationally.

The λ Boo stars constitute yet another group of peculiar stars. The criteria for membership in this group are as follows (Sargent, 1967):

1. Early A-type spectral class as determined from the hydrogen lines.
2. Weak metallic lines for their colors and hydrogen line spectral type and/or a luminosity, as derived from trigonometric parallaxes, that places them below the main sequence.
3. Low (Population I) radial velocities.

The λ Boo stars differ from the majority of peculiar A-type stars in two obvious ways: First, the λ Boo stars are characterized by metallic lines that are too weak rather than too strong. Second, their rotational velocities, which range from 75 to over 200 km s^{-1}, are quite high.

There are several arguments against the hypothesis that the λ Boo stars are highly evolved, Population II A-type stars (Sargent, 1965). Both their low radial velocities and high rotational velocities are in conflict with membership in an old stellar population. Many are members of binary systems, another property unusual among Population II stars, and λ Boo stars do not fall in the same part of the (M_v, B–V) plane as horizontal branch stars. One λ Boo star is probably a member of a visual binary system in which its companion appears to be a normal B5 V star. If this system is a physical one, then it demonstrates that young stars can have extraordinarily weak lines. Furthermore, the line weakening presumably does not reflect the composition of material from which the star formed, since its more luminous companion has a normal composition.

Only a handful of λ Boo stars have been identified, and Sargent (1965) estimates that they constitute 1 to 2 percent of the A-type dwarfs. Because of their small numbers, the characteristics of the λ Boo stars are the least well defined of all the classes of peculiar A-type stars. The stars that have been proposed so far for membership in this class are far from homogeneous in their composition and luminosity. Since so little is known about the λ Boo stars, they will not be discussed in any further detail in the present monograph. However, in evaluating models proposed to account for the composition of peculiar A-type stars, one should keep in mind the fact that there are some objects in which the abundances of the metals are unusually low rather than unusually high.

There are a few other special classes of A-type stars. Some have shell lines and are presumably cooler analogs of the Be stars. The Be phenomenon is discussed extensively in the book entitled *B Stars With and Without Emission Lines* (Underhill and Doazan, 1982), and that material will not be repeated here. The Herbig Ae stars also exhibit emission lines. These objects are exceedingly young, have apparently not yet reached the main sequence, and their emission is probably caused by the analog at higher mass of T Tauri-like activity. These stars are most appropriately treated in a discussion of pre-main sequence stars and are not included in the present monograph.

OVERVIEW

The main sequence A-type stars occur in a region of the HR diagram where mass motions in the atmosphere are expected to be relatively unimportant. Surface convection zones become extensive in stars of spectral types ~F0 and later, while mass loss in main sequence stars has been detected only at much higher temperatures. In the A-type stars, physical processes that are masked by mass flows in more massive stars, or by convection in less massive stars, probably play a dominant role in shaping the emergent spectrum. Meridional circulation, subsurface convection zones, pulsation, diffusive separation of elements, and magnetic fields are some of the mechanisms that are thought to be important. The observed spectrum of an A-type star reflects the interplay of all these processes, and a model of an A-type atmosphere must correctly balance their relative importance. Because macroscopic and microscopic velocity fields play such an important role in determining what we see, the emphasis in this book will be on the determination of physical parameters—temperature, surface gravity, rotation, binary characteristics, and line profiles—that may correlate with, or be diagnostic of, atmospheric motions. Considerably less emphasis will be placed on derivations of quantities, such as abundances, that are extremely model dependent. In my choice of material, I have stressed those areas of research that appear to me to be currently the most active or the most likely to prove productive. Whenever existing work appears to be controversial or to lead to unresolved contradictions with either observations or theory, I have explicitly so indicated.

This book is part of the series, *Nonthermal Phenomena in Stellar Atmospheres*, which is sponsored jointly by the National Aeronautics and Space Administration (NASA) and the Centre National de la Recherche Scientifique (CNRS) of France. It should be considered as a companion to the volume entitled *B Stars With and Without Emission Lines*, by Anne Underhill and Vera Doazan. The boundary at A0 that separates the subject matter of that book from this one is artificial in the sense that the physical processes that shape the emergent spectrum are quite similar for both B- and A-type stars. The volume by Underhill and Doazan provides an excellent treatment of the observation and interpretation of the spectra of normal stars, and quite rightly so, since normal (although a precise definition of this word would be difficult) stars are common among main sequence B-type stars. The discussions of spectral classification and photometry, particularly in the satellite ultraviolet, are both directly relevant to the work on A-type stars. The volume on B-type stars also provides an excellent introduction to the modeling of stellar atmospheres. The present monograph on A-type stars makes the assumption that the reader is already familiar with the kinds of material presented in *B Stars With and Without Emission Lines*, and the problems of interpreting the spectra of normal stars are summarized only very briefly here. The emphasis in the present monograph is on the interpretation of the remarkable spectral anomalies that are so common among A-type stars.

The challenge posed by the A-type stars is a formidable one. The appearance of the spectrum is apparently dominated by nonthermal processes, and currently our ability to model these processes and their interactions is limited. On the other hand, the A-type stars offer us an opportunity as well by providing a laboratory in which we can test models that include hydrodynamic and hydromagnetic effects. If the complexities of the A-type stars can ultimately be explained in detail, then nonthermal models will have passed a very stern test indeed.

2

THE NORMAL A-TYPE STARS

INTRODUCTION

In most parts of the HR diagram, the definition of a normal star is clear, and usually the title "normal" can be simply accorded to those stars that are in the majority. Such a procedure is not possible for the A-type stars because so many of them display strikingly anomalous line strengths; indeed, there are probably *no* slowly rotating late A-type stars that one could reasonably classify as normal (Abt and Moyd, 1973). A practical, but not entirely rigorous, definition is that a normal A-type star is one that at classification dispersions shows none of the anomalies characteristic of the magnetic Ap, Am, λ Boo, or other types of peculiar stars; that when subjected to detailed local thermodynamic equilibrium (LTE) analysis, appears to have a composition like the Sun's; and that exhibits no variability, either regular or irregular.

In practice, there are few high-resolution observations of normal A-type stars, and much of our knowledge of their temperatures, luminosities, and surface gravities depends on low-resolution spectroscopic or photometric measurements. The selection of normal stars for determination of binary frequency, rotational velocity distributions, etc., is also made on the basis of spectral and photometric classifications. It is therefore appropriate to begin with a discussion of the assumptions and limitations of classification systems. Readers interested in a more detailed review of the general principles of stellar taxonomy and of the various systems of spectral and photometric classification should refer to Fehrenbach (1958).

SPECTRAL CLASSIFICATION

There are two requirements for an effective system of spectral classification. First, the system must be defined in terms of observations of sufficient spectral resolution to permit the determination of types that accurately reflect both temperature and luminosity classes. Second, the observations required must be of as low a resolution as possible, so that classification can be extended to faint objects.

The system that has been most widely used is the MK system, which is defined by a set of standard stars whose locations on a two-dimensional plot of spectral type versus luminosity are known (Morgan and Keenan, 1973). Spectrograms suitable for classification on this system must have dispersions in the range 60 to 125 Å mm^{-1} and must be well widened. MK classes are derived by interpolation—that is, by comparison with the grid of standard stars—and depend on the totality of the appearance of the lines, blends, and bands in the ordinary photographic region of the spectrum. It is essential to understand that the MK system is an empirical one, based on real objects, and is independent of any assumptions about physical models. For limited regions of the HR diagram, self-consistent classification structures of higher accuracy and/or that reflect the importance of a third parameter can be devised. Examples of such systems, which may require observations at higher dispersion or of wavelength regions other than the photographic, are those developed by Walborn (1971) for OB stars, Osawa (1965) for Ap stars, and Barry (1970) for normal A- and F-type stars.

Accurate classification of A-type stars is difficult because the usable lines are either very strong (Ca II λ3933 and the Balmer lines of hydrogen) or very weak; suitable metallic lines of intermediate strength are lacking. The large numbers of anomalous stars with imprecisely defined characteristics (e.g., the Am stars) compound the problem. In their classification of the A-type stars listed in the original *Bright Star Catalog* (Schlesinger and Jenkins, 1940), Cowley et al. (1969) found that the most useful temperature indicator was the strength of the K line, while the ratios λ4178/λ4172 and λ4417/λ4481 were sensitive to luminosity. These four features are all blends, with λ4172 and λ4178 being dominated by neutral and singly ionized Fe and Cr, λ4417 by Ti II and Fe II, and λ4481 by Mg II.

In an attempt to relate the assigned spectral types to such atmospheric parameters as effective temperature (T_{eff}), surface gravity ($\log g$), microturbulent velocities, and abundances, Bolton (1971) calculated the strengths of these blends by convolving model atmosphere calculations of synthetic spectra with an instrumental profile appropriate for spectrograms of the resolution normally used for classification purposes. Although the correlation between models and observations is limited by the difficulty of evaluating precisely the response of the eye to a photographic plate, Bolton's calculations do show that while the line ratios are responsive to luminosity effects they also reflect chemical composition and microturbulent velocities. In fact, Bolton suggests that microturbulent velocities must correlate well with $\log g$ in order to account for the reliability of the ratio λ4417/λ4481 as a luminosity indicator. Unfortunately, observations of microturbulent velocities in giant A-type stars are too few to check this hypothesis. Bolton's calculations do demonstrate, however, that line ratios may depend on atmospheric properties other than temperature and luminosity, and that considerable caution must be exercised in any attempts to extend the MK system to stars with abnormal composition or atmospheric structure.

Another system that is very useful for the classification of O-, B-, A-, and F-type stars has been developed by Barbier, Chalonge, and Divan. The so-called BCD system was originally designed for use with low-dispersion photographic spectra obtained with objective prisms. This classification system is, however, a natural one to use in conjunction with new electronic detector systems, and it should be employed even more widely in the future than it has been.

The basic classification parameters are (Chalonge and Divan, 1952) D, a measure of the size of the Balmer discontinuity ($D = \log F_R - F_V$, where F_R and F_V are the fluxes longward and shortward of the Balmer discontinuity, respectively), and λ_1, which measures the effective wavelength of the Balmer jump according to a specified method. Since the shape of the continuum throughout the visible and near ultraviolet can also be measured, it is possible to derive color excesses as well. Chalonge and Divan (1973, 1977b) have defined additional parameters that can be derived from the same data set and that have the advantage of being sensitive to only spectral type and luminosity class. The quantities λ_1 and D that initially defined the BCD system are each dependent on both luminosity class and spectral type. The BCD parameters have been calibrated in terms of temperature and luminosity class for types O-F and all luminosity classes (Chalonge and Divan, 1973; 1977b). The measured parameters also depend on metallicity and are therefore useful in classifying peculiar stars as well (Chalonge and Divan, 1977a).

With the advent of spacecraft, spectral classification in the ultraviolet region of the spectrum has become feasible. General techniques and results have been reviewed extensively by Underhill and Doazan (1982). For the A-type stars, satellite observations have played a crucial role in the search for evidence of nonradiative heating (see Chapter 3) and in exploring the role of ultraviolet opacity in altering atmospheric temperature structure in stars of high metallicity and in causing brightness variations in magnetic stars with inhomogeneous surfaces (see Chapter 4).

PHOTOMETRIC CLASSIFICATION

As an alternative to the use of spectral line strengths for classification, a variety of photometric classification schemes, which depend on measurements of the continuum flux, have been devised. Two of the most widely used are the UBV and uvbyβ systems, and because both are widely applicable over the entire HR diagram, they provide the same kind of general framework for classifying stars as the spectroscopic MK system.

The UBV system makes use of three broad bandpasses (~1000 Å wide) centered approximately at λ3500, λ4300, and λ5500. The choice of these bandpasses was dictated (Johnson, 1963) by the need to limit the system to a small number of broad filters so that measurements of faint stars could be readily obtained, coupled with a desire to match approximately existing non-photoelectric systems based on the sensitivity of the eye (V) and photographic plates (B). The addition of a third ultraviolet filter allows a treatment of reddening and improves sensitivity of the system to temperatures of hot stars and to chemical composition (Becker, 1963). Although the UBV system does not provide a separation of luminosity classes, the colors do correlate well with spectral type for stars of similar composition. The relationship between these quantities for normal A-type stars has been derived by Cowley et al. (1970) and is given in Table 2-1.

While the UBV system is extremely useful, there are a number of problems in relating the observed colors to stellar characteristics. As noted, there is no luminosity discriminant, and

Table 2-1
Mean U, B, V Colors of Normal Stars

Luminosity:	V			IV			III		
Spectral Type	<B−V>	<U−B>	Number of Stars	<B−V>	<U−B>	Number of Stars	<B−V>	<U−B>	Number of Stars
B7							−0.10	−0.45	5
B8	−0.05	−0.31	20	−0.12	−0.44	2	−0.05	−0.37	19
B9	−0.04	−0.19	39	−0.00	−0.23	8	−0.04	−0.27	28
B9.5	−0.04	−0.14	40				−0.03	−0.14	8
A0	−0.01	−0.05	102	−0.02	−0.07	8	−0.01	−0.10	14
A1	+0.01	+0.00	104	+0.06	+0.03	7	+0.04	+0.07	5
A2	+0.05	+0.06	102	+0.06	+0.09	18			
A3	+0.09	+0.09	74	+0.09	+0.10	19	+0.10	+0.12	6
A4	+0.13	+0.11	21	+0.14	+0.12	7	+0.13	+0.14	6
A5	+0.16	+0.10	22	+0.14	+0.11	8	+0.16	+0.13	11
A6	+0.17	+0.10	4	+0.19	+0.12	12			
A7	+0.19	+0.09	19				+0.22	+0.11	10
A8	+0.25	+0.12	10	+0.24	+0.10	4	+0.22	+0.10	4
A9	+0.22	+0.08	2			0			
F0	+0.27	+0.09	19	+0.25	+0.09	10	+0.24	+0.12	7

Source: From Cowley et al. (1970).

the colors may be strongly affected by interstellar reddening. The system is not filter defined (the short wavelength cut-off is set by the atmosphere and the long wavelength cut-off by the sensitivity of the 1P21 photomultiplier used to define the system), so that measured colors depend on the equipment used and even on the altitude at which the observations are made. Transformations to the standard system may be nonlinear. Because the U filter includes the Balmer decrement, it is very difficult to relate observed colors to those calculated from models, particularly for A-type stars.

It was in part to remedy these deficiencies that Stromgren (1963a) devised a four-color, filter-defined system. The four filters are 180 Å to 300 Å wide, with the u filter centered at λ3500, the v filter at λ4110, the b filter at λ4670, and the y filter at λ5470. Very often four-color photometry is supplemented by narrow (~30 Å) and broad filters (~150 Å) centered on Hβ. The four colors plus Hβ allow discrimination of both temperature and luminosity classes even in reddened stars (Stromgren, 1966).

Specifically, in addition to the usual color indices (u-b) and (b-y), two other useful quantities are

$$c_1 = (u - v) - (v - b), \text{ and} \tag{2-1}$$

$$m_1 = (v - b) - (b - y). \tag{2-2}$$

The first is a measure of the Balmer discontinuity, and the second is sensitive to the amount of metal line blocking at λ4100 and provides a measure of composition.

For a normal reddening law, the color excesses E are related in the following way:

$$E(c_1) = 0.2\, E(b\text{–}y), \text{ and} \tag{2-3}$$

$$E(m_1) = -0.18\, E(b\text{–}y). \tag{2-4}$$

Therefore, the indices

$$[c_1] = c_1 - 0.2\,(b - y), \text{ and} \tag{2-5}$$

$$[m_1] = m_1 + 0.18\,(b - y), \tag{2-6}$$

are independent of reddening and are often useful.

A comparison with MK types or with theoretical models shows that, for main sequence stars later than ~A3, the hydrogen line index Hβ is an excellent measure of spectral type or temperature; the index c_1 depends primarily on surface gravity. Earlier than A0, the reverse is true. There is an intermediate region from A0 to A3 where neither c_1 nor Hβ can be used alone to yield accurate values of temperature and surface gravity. The (b–y) color is, of course, sensitive to temperature but cannot be used for reddened stars. However, linear combinations of the various indices can be constructed that will allow two-dimensional classification for normal stars (Stromgren, 1966) in the spectral interval A0 to A3.

Correlations between the four-color and MK systems have been derived by Oblak et al. (1976), and their results for main sequence stars are given in Table 2-2. This table should be compared with Table 2-3, which is taken from work by Crawford (1979).

Table 2-2
Relationship Between uvbyβ Indices and MK Types for Luminosity Classes V, IV-V, IV

Spectral Type	$\langle b-y \rangle$	$\langle m_1 \rangle$	$\langle c_1 \rangle$	$\langle \beta \rangle$
A0	0.002	0.148	1.033	2.860
A1	0.013	0.157	1.055	2.878
A2	0.030	0.169	1.082	2.880
A3	0.056	0.176	1.060	2.869
A4	0.072	0.184	1.039	2.862
A5	0.095	0.189	0.976	2.837
A6	0.104	0.191	0.970	2.827
A7	0.122	0.189	0.923	2.817
A8	0.138	0.187	0.888	2.793
A9	0.169	0.175	0.820	2.768
F0	0.172	0.182	0.799	2.766

Source: From Oblak et al. (1976).

Table 2-3
Average Photometric Indices for MK Spectral Types

	β (mag)	$(b-y)_0$ (mag)	m_0 (mag)	c_0 (mag)
A2 V	2.885	0.029	0.169	1.08
A3 V	2.871	0.055	0.172	1.08
A4 V	2.864	0.071	0.184	1.05
A5 V	2.841	0.090	0.195	0.96
A7 V	2.824	0.107	0.201	0.90
A8 V	2.789	0.132	0.194	0.87
F0 V	2.768	0.158	0.191	0.79
A3 III	2.869	0.048	0.174	1.14
A5 III	2.855	0.079	0.191	1.05
A7 III	2.823	0.097	0.204	1.00
A3 m	2.830	0.114	0.227	0.87

Source: From Crawford (1979).

Numerous other photometric systems, either general purpose or for a restricted part of the HR diagram, have been used by various groups (Golay, 1974, and references therein), but treatment of these systems falls outside the scope of the present discussion.

LUMINOSITY CALIBRATION

The ideal, but highly impractical, way to determine atmospheric parameters and abundances for stars would be to obtain high-resolution, photoelectric measurements of the entire spectrum. In practice, one must usually work with limited spectral coverage supplemented by broad- or intermediate-band photometry to define the continuum flux. The measured parameters must then be calibrated in terms of the physical parameters of the star—temperature, luminosity, and chemical composition. The uvby photometric system is particularly well suited for studying A-type stars, and the problem of calibration for that system will be discussed in detail. The general ideas are, of course, applicable to the calibration of any of the other photometric systems that are currently in use (Golay, 1974).

The calibration of photometric indices in terms of physical parameters proceeds in two steps. The first step is a strictly observational one, which establishes standard relationships between photometric indices, the position of the zero-age main sequence (ZAMS), and the absolute visual magnitude (M_v). The second step involves calibration of photometric indices with respect to T_{eff}, the effective temperature of the star, and M_{bol}, the bolometric luminosity. This second step can be achieved only through reference to model atmosphere calculations.

The empirical calibration of the uvbyβ system for A-type stars in terms of intrinsic color and absolute magnitude has been carried out by Crawford (1979). For calibration purposes, the A-type stars must be divided into two groups: those that are cooler than spectral type A2, where hydrogen absorption is at its maximum, and those that are hotter than A2. For the cooler A-type stars, both β, which measures the strength of absorption at Hβ, and b–y are sensitive to temperature, but the β index has the additional advantage of being insensitive to interstellar reddening. The c_1 index is the primary indicator of luminosity. Figure 2-1 shows the relationship between c_1 and β for bright A-type stars. This diagram is the analog in the uvbyβ system of the HR diagram, with luminosity increasing upward and temperature decreasing to the right. The lower envelope of the plotted points corresponds to the position of the ZAMS.

The calibration of β in terms of M_v, the absolute visual magnitude, is carried out through the observation of clusters of known distances (established by observations of F- and G-type stars in those same clusters, for which absolute magnitudes were determined from trigonometric parallaxes of field stars of the same type) in which the A-type stars are unevolved. The clusters used by Crawford (1979) for this purpose are the α Per open cluster, the Pleiades, and IC 4665.

A-type stars that have evolved significantly have higher luminosities than stars along the

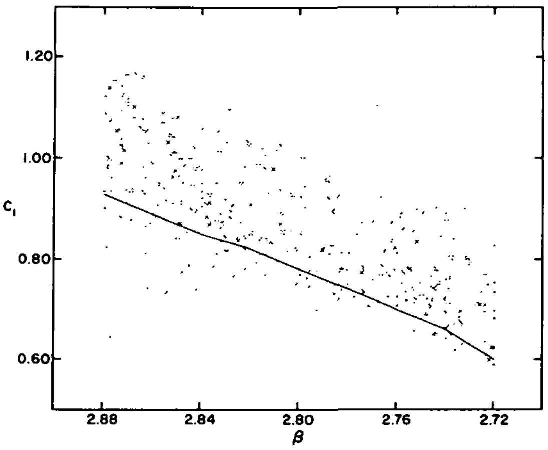

Figure 2-1. The relation between c_1 and β for the bright A-type stars. The line shows the standard relation given in Table 2-4. This relation has been chosen to approximate the lower envelope of the plotted points, allowing for observational error. The diagram is similar to an H-R diagram in that β is a temperature parameter, and δc_1 (the distance above the standard line) is a luminosity parameter (from Crawford, 1980).

ZAMS with the same temperature, or equivalently the same value of β. From observations of the Hyades, Coma, and UMa clusters, in which the A-type stars are evolved, Crawford (1979) finds that

$$M_V = M_V(\beta, \mathrm{ZAMS}) - 9\,\delta c_1, \qquad (2\text{-}7)$$

where δc_1 is equal to the observed value of c_1 minus the value of c_1 for the ZAMS measured at the same value of β.

Table 2-3 gives the standard relations between the various photometric indices and MK type; average photometric indices and luminosities for stars on the ZAMS are given in Table 2-4. Both tables are from the work by Crawford (1979). A graphical method for assigning MK spectral types to A- and F-type stars of all luminosity classes on the basis of uvby photometry has been described by Faber (1977). For stars of normal composition, the accuracy of the method is one luminosity class and 0.1 to 0.2 spectral classes and is independent of reddening.

Table 2-4
Standard Relations for A-Type Stars

β (mag)	b − y (mag)	m_1 (mag)	c_1 (mag)	M_V (ZA) (mag)
2.880	0.066	0.200	0.930	2.30
2.860	0.086	0.205	0.890	2.50
2.840	0.106	0.208	0.850	2.64
2.820	0.126	0.206	0.820	2.70
2.800	0.146	0.203	0.780	2.76
2.780	0.166	0.196	0.740	2.82
2.760	0.186	0.188	0.700	2.88
2.740	0.206	0.182	0.660	2.96
2.720	0.226	0.177	0.600	3.10

Source: From Crawford (1979).

It must be recognized explicitly that the standard relations are not valid for either the magnetic Ap or the Am stars. The continuous energy distributions of these two classes of stars are strongly modified by enhanced line absorption. The mean relationships are also, of course, invalid for double stars, and some of the scatter in the observed relationships may be caused by unrecognized close binaries. There is also evidence that the colors of A-type stars are modified by rotation in the sense that the c_1 index, which is sensitive to gravity, indicates a lower gravity for the more rapidly rotating stars. Rotational differences introduce a scatter of about ±0.2 mag into the calibration of M_V vs. β, and the effect is systematic in the sense that smaller distance moduli will be calculated for more rapidly rotating stars (Crawford, 1979).

Calibration of uvbyβ photometry of the A0 to A2 stars has not been completed, and the problems of doing so have been discussed by Stromgren (1966). As noted above, the spectral region A0 to A2 constitutes a transition region where β and c_1 are each sensitive to both luminosity and temperature. Through reference to model atmosphere calculations, Stromgren suggests that the temperature of an unreddened A0 to A2 star can be derived from a linear combination of u–b and b–y and M_v from a linear combination of β and c_1. For reddened stars the m_1 index, in combination with the index based on β and c_1 used to derive M_v, appears to be a useful temperature indicator.

ATMOSPHERIC PARAMETERS

The second step in the calibration procedure is to relate observed photometric indices to parameters, such as effective temperature, that characterize the stellar atmosphere. Recourse to model atmosphere calculations is mandatory, and the necessary assumption is that the parameters that characterize the model atmospheres also describe real stars. Indeed, determination of the extent to which this assumption is valid is the fundamental problem of stellar astronomy.

In order to illustrate the problems of comparing models with observations, the calibration of the uvby system will be analyzed in detail here, but the general ideas are applicable to any photometric system. There have been several attempts to calibrate the uvby indices in terms of T_{eff}, log g, and metal abundance (e.g., Matsushima, 1969; Olson, 1974). The most recent work is by Relyea and Kurucz (1978), and their discussion will be followed here.

There are three major sources of error in determining the relationship between photometric color indices and atmospheric parameters such as T_{eff}. First, the assumptions used to construct the models, and hence to calculate the emergent flux, may be incorrect or may provide an inadequate characterization of real stars. Second, the calculation of photometric indices from theoretical emergent fluxes may contain significant uncertainties. Third, measurements of the actual energy distributions of stars may be inaccurate.

In their calculations of the uvby indices, Relyea and Kurucz (1978) make use of a grid of model atmospheres computed by Kurucz (1979). These models include opacity due to 900,000 atomic lines in the form of distribution functions; most models cooler than 8500 K include convective flux with the ratio of mixing length to pressure scale height $\ell/H = 2.0$; and all models assume a microturbulent velocity of 2 km s^{-1}. Models were calculated for a range of metal abundances and surface gravities. Non-LTE effects were not included. (See Chapter 9 for an extended discussion of model atmospheres.)

In order to calculate colors that correspond to these models, it is necessary to construct a theoretical photometric system that accurately mimics the properties of the uvby system. Specifically, one must define the transmission of the atmosphere, the reflectivity of telescope mirrors, the transmission of the various filters, and the sensitivity of the photomultiplier. The calculated colors can then be transformed to the standard system through matching the detailed energy distributions of a few stars, for which uvby measurements have been made, with the energy distributions given by the models. *All* the relevant parameters are uncertain. Atmospheric transmission varies from place to place and night to night. The reflectivity of real (often dirty) mirrors may depart from the values quoted for fresh aluminum. The transmission of any given filter may change with time, and the characteristics of different filter sets may not be identical. A variety of photomultipliers with various sensitivities is commonly in use. The cumulative effect of these uncertainties is difficult to estimate. One can only hope it is not larger than the error (~0.015 mag) involved in transforming observed instrumental magnitudes to the standard photometric system (Crawford and Barnes, 1970).

Another source of uncertainty is in matching theoretical energy distributions with those actually observed for stars. Scanner observations are not continuous, but rather are discrete measurements made in wavelength intervals that contain no strong stellar or terrestrial lines. Furthermore, the accuracy of the photometric scans is not high. Intercomparison of independent scans of the same object by different observers indicates that differences as large as 6 percent are not uncommon (Breger, 1976a; 1976b). There may also be systematic errors in the calibration of scanner measurements. The most recent calibration of Vega, the star commonly used as a primary standard, still may contain errors of 3 percent in the ultraviolet and 2 percent in the visible (Hayes and Latham, 1975).

Despite these uncertainties, a number of general results, alluded to earlier, are clearly established by the calculations of uvby colors by Relyea and Kurucz (1978). First, the calculations confirm the conclusions, already drawn from observations alone, that b−y is sensitive to temperature throughout the A-type spectral region and that c_1 is sensitive to gravity. If the metal abundance is known, c_1 and b−y are therefore useful for a two-dimensional classification system for A-type stars. There is some ambiguity in interpreting c_1 in the temperature range $8500 \leq T_{eff} \leq 10{,}000$ K in that a given value may correspond to either high or low gravity, but this ambiguity can be resolved through the use of either m_1 or v−b indices. The m_1 index is sensitive to both composition and gravity for the A-type stars and is useful as an unambiguous indicator of metal abundance only for $T_{eff} \leq 7500$ K.

There are two ways to check the validity of the theoretical colors. The first is simply to compare the calculated and observed variations of specific colors as a function of one another (i.e., c_1 vs. b−y or m_1 vs. c_1); the second is to compare the calibration of the photometric indices in terms of effective temperature with the values obtained by integrating the observed energy distributions of individual stars.

The values of T_{eff} obtained by Relyea and Kurucz (1978) from their calibration of uvby photometry are compared in Table 2-5 with the values obtained by Code et al. (1976) from direct measurements of angular diameters (Hanbury Brown et al., 1974) and energy distributions longward of λ1100. The values of T_{eff} and $\log g$ in the columns labeled "derived" were obtained from calibrated grids of c_1 vs. b−y (method *c*by) or of c_1 vs. m_1 (method *cm*).

Plots of c_1 vs. b−y and m_1 vs. b−y for a large sample of stars (Hauck and Mermilliod, 1975), with no corrections applied for interstellar reddening, are shown in Figures 2-2 and 2-3. Loci derived from models with solar composition and the surface gravities expected for main sequence stars are superposed on the observed colors. While the agreement between theory and observation is good for the hotter stars (b−y < 0.05), discrepancies are significant throughout the A-type spectral range. Since molecular opacities are not included in the models, agreement between theory and observations cannot be expected for $T_{eff} \leq 6000$ K.

Table 2-5
Derivation of T_{eff} and Log g from uvby Photometry for Stars with Known Atmospheric Parameters

HR	Name	Sp	b − y	m_1	c_1	T_{eff} (Code)	T_{eff} (derived)	log *g* (Code)	log *g* (derived)	Method	Notes
2421.....	δ Gem	A0 IV	+0.007	+0.149	1.186	9270 ± 330	9180 ± 200	3.65 ± 0.17	3.6 ± 0.1	*c*by	
2491.....	α CMa	A1 V	−0.004	+0.158	0.982	9980 ± 160	9770 ± 200	4.31 ± 0.04	4.3 ± 0.1	*c*by	1
2943.....	α CMi	F5 IV	+0.272	+0.167	0.532	6520 ± 120	6610 ± 100	3.95 ± 0.04	3.8 ± 0.1	*c*by	2
3982.....	α Leo	B7 V	−0.041	+0.102	0.712	12220 ± 310	12190 ± 200	3.87 ± 0.22	3.6 ± 0.4	*cm*	
4534.....	β Leo	A3 V	+0.044	+0.210	0.975	8850 ± 350	8750 ± 200	4.16 ± 0.12	4.3 ± 0.1	*c*by	
5056.....	α Vir	B1 V	−0.114	+0.080	0.018	23930 ± 840	23830 ± 300	3.70 ± 0.09	3.8 ± 0.7	*cm*	
6556.....	α Oph	A5 III	+0.093	+0.168	1.039	8020 ± 340	8170 ± 100	3.77 ± 0.13	3.8 ± 0.1	*c*by	2
7001.....	α Lyr	A0 V	+0.004	+0.157	1.089	9660 ± 130	9400	3.94 ± 0.08	3.95	Defined	
7557.....	α Aql	A7 V	+0.137	+0.178	0.880	8020 ± 210	7870 ± 100	4.22 ± 0.08	3.9 ± 0.1	*c*by	2
8425.....	α Gru	B5 V	−0.061	+0.105	0.576	14060 ± 560	13400 ± 300	4.38 ± 0.23	3.8 ± 0.5	*cm*	
8728.....	α PsA	A3 V	+0.036	+0.206	0.991	8790 ± 310	8870 ± 200	4.31 ± 0.10	4.3 ± 0.1	*c*by	

Notes: (1) Sirius is a metallic-line star $[m/H] \approx +1.0$. (2) Models are in error at these temperatures.
Source: From Relyea and Kurucz (1978).

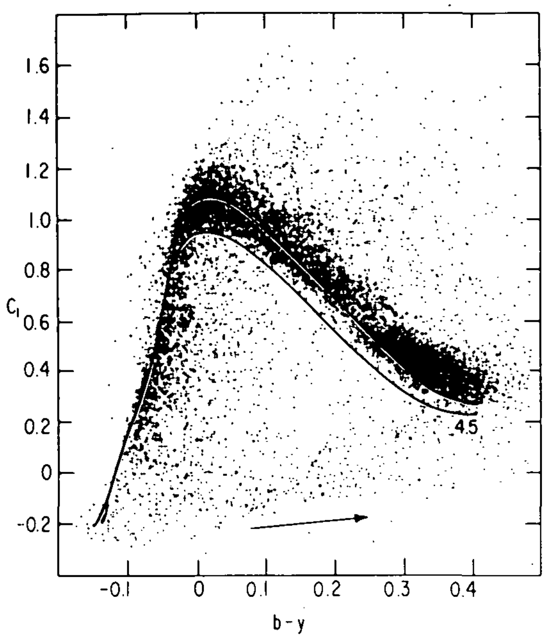

Figure 2-2. The (c_1) versus (b–y) diagram for models with log g = 4.0 and 4.5, [m/H] = 0.0. The theoretical relations are superposed on the plot of stars in the Hauck-Mermilliod Catalog with the direction of reddening indicated by an arrow (from Relyea and Kurucz, 1978).

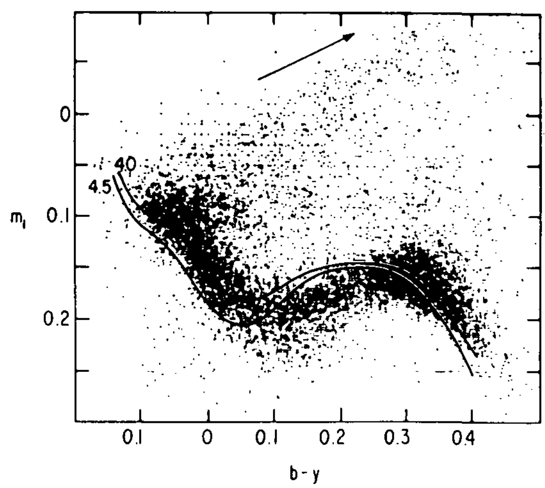

Figure 2-3. The (m_1) versus (b–y) diagram for models with log g = 4.0 and 4.5, [m/H] = 0.0. The theoretical relations are superposed on the plot of stars in the Hauck-Mermilliod Catalog with the direction of reddening indicated by an arrow (from Relyea and Kurucz, 1978).

The fact that the models represent the colors of early-type stars suggests that there are no major deficiencies in the absolute calibration of Vega, in the definition of the theoretical photometric system, or in the transformation of the theoretical colors to the uvby system. The failure of the models to reproduce the colors of the A-type stars, therefore, is probably indicative of some intrinsic problem with the calculations in the temperature range $6000 < T_{eff} < 8500$ K. Relyea and Kurucz (1978) consider a number of possibilities in turn, including improper representation of opacities, nonsolar abundances, an increase in microturbulent velocity, and rapid rotation. None of these effects appears significant enough to explain the observed discrepancies. A more likely explanation, Relyea and Kurucz conclude, is that the treatment of convection is inadequate. The A-type stars lie in a transition region between the hot stars ($T_{eff} > 9000$ K), where convection in the stellar envelope is negligible, and the cooler stars ($T_{eff} < 6000$ K) where convection is very strong. Conventional mixing length theory, which was adopted for these models, probably overestimates the efficiency of convection in stars in which the convective flux is low. In some cases, as Relyea and Kurucz point out, the width of the convective zone is smaller than the pressure scale height, and so the choice of $\ell/H = 2.0$ is "physically suspect." Since the locus of A-type stars in the two-color plot is bracketed by purely radiative models and the convective models with $\ell/H = 2.0$, it seems likely that much, and perhaps all, of the discrepancy between theoretical and observed colors is caused by improper treatment of partial convection.

The selection of the model atmosphere that best represents the continuous energy distribution of a star depends, directly or indirectly, on accurate measurements of stellar flux by means of narrow-band photometric scans. It might be natural to conclude, therefore, that such scans are necessarily superior to intermediate-band photometry for determining atmospheric parameters. During the past two decades, considerable effort has been devoted to measuring the energy distributions of A-type stars throughout the spectral region readily accessible from the ground (λλ3100 to 11000). The pioneering work of Oke (1964) has been followed by a long series of observations of normal A-type stars (e.g., Baschek and Oke, 1965; Wolff et al., 1968; Schild et al., 1971; Böhm-Vitense and Johnson, 1977; Adelman, 1978). A useful catalog of early scanner results has been published by Breger (1976b).

For ordinary (whatever that may mean!) main sequence A-type stars, however, uvby photometry yields essentially the same information as narrow-band scanner photometry, and uvby measurements can be made much faster and probably more accurately. The uvby system is designed to measure the size of the Balmer discontinuity and the slope of the Paschen continuum, and in practice these are the only two parameters derived from scanner photometry, even though the number of wavelengths at which measurements are made may be large. Furthermore, the errors in scanner photometry are typically larger than the errors in uvby photometry, primarily for two reasons. First, the network of spectrophotometric standards is not large, and the fluxes of the standards relative to one another are not as well determined as are the relative colors of uvby standards. Second, the random errors of scanner photometry appear to be larger, perhaps in part because extinction coefficients are usually not well determined. Because it is very time consuming to determine extinction at 20 or 30 different wavelengths, mean extinction curves are usually adopted—sometimes with disastrous results on marginally photometric nights that are becoming all too common at many older observing sites.

There are additional uncertainties in comparing observed with computed energy distributions that affect both narrow-band spectrophotometry and intermediate-band photometry. A major continuing problem is establishing the absolute calibration of one or more standard stars. A thorough discussion of the issues involved has been given by Hayes and Latham (1975). Hayes et al. (1975) summarize the methods used to establish the absolute calibration and describe as well their own and earlier work in this critical area. The primary standard selected has usually been Vega, and the procedure involves measurement of the standard star and of a standard blackbody source with precisely the same equipment. A primary source of uncertainty is the value of the horizontal extinction between the standard lamp and the telescope. After modeling atmospheric extinction and combining the results of absolute calibrations by various observers, Hayes and Latham (1975) suggested that the calibration listed in Table 2-6 be adopted. They also give monochromatic fluxes for Vega at λ5556 of

$$F_\lambda = 3.39 \times 10^{-9} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ Å}^{-1}, \quad (2\text{-}8)$$

$$F_\nu = 3.50 \times 10^{-20} \text{ ergs cm}^{-2} \text{ s}^{-1} \text{ Hz}^{-1}, \text{ and} \quad (2\text{-}9)$$

$$N_\lambda = 948 \text{ photons cm}^{-2} \text{ s}^{-1} \text{ Å}^{-1}. \quad (2\text{-}10)$$

Hayes and Latham estimate that the colors of Vega in the wavelength range λλ4036 to 8090 are accurate to ~2 percent and that the size of the Balmer discontinuity (λ3636 vs. λ4036) has been determined with an accuracy of 3 percent. Errors in the ultraviolet (λλ3300 to 3636) and in the near-infrared (λλ9700 to 10800) may exceed 2 percent because of the difficulty in modeling the extinction caused by ozone and water vapor. Of course, if Vega is variable (see Chapter 3), then errors in the calibration may be larger than 2 to 3 percent.

Until fairly recently, measurements of stellar energy distributions were restricted to that portion of the spectrum that can be observed from the ground. With the advent of rocket and satellite spectrographs, however, it has become possible to make direct observations of the fluxes of bright stars at wavelengths as short as λ1160. For a star with $T_{\text{eff}} \sim 10{,}000$ K, approximately 30 percent of the total flux is emitted at wavelengths less than λ3300, while less than 1 percent is emitted shortward of λ1100. For A-type stars, therefore, satellite and rocket observations, in combination with ground-based data, provide an essentially complete definition of the emitted spectrum. The bolometric correction is obtained directly, and if the angular diameter is known,

Table 2-6
Adopted Color Calibration for Vega

Wave-length (Å)	$1/\lambda$ (μ^{-1})	Adopted
3300.	3.030	+1.137
3350.	2.985	(+1.145)
3400.	2.941	+1.129
3450.	2.899	(+1.115)
3500.	2.857	+1.097
3571.	2.800	(+1.077)
3600.	2.778	...
3636.	2.750	(+1.059)
3680.	2.717	...
4036.	2.478	−0.299
4167.	2.400	−0.278
4255.	2.350	−0.263
4460.	2.242	...
4464.	2.240	−0.238
4566.	2.190	−0.199
4780.	2.092	...
4785.	2.090	−0.153
5000.	2.000	−0.103
5263.	1.900	−0.050
5556.	1.800	0.000
5840.	1.712	+0.062
6050.	1.653	...
6056.	1.651	+0.111
6370.	1.570	...
6436.	1.554	(+0.167)
6790.	1.473	+0.217
6800.	1.471	+0.219
7100.	1.408	+0.272
7550.	1.325	+0.361
7780.	1.285	(+0.398)
8080.	1.238	...
8090.	1.236	+0.429
8370.	1.195	...
8400.	1.190	...
8708.	1.148	(+0.450)
8804.	1.136	...
9700.	1.031	(+0.484)
9832.	1.017	...
9950.	1.005	...
10250.	0.976	(+0.571)
10256.	0.975	...
10400.	0.962	(+0.586)
10796.	0.926	...
10800.	0.926	+0.649

Source: Adapted from Hayes and Latham (1975).

then T_{eff} can be derived in a way that is nearly model independent. Specifically, the flux emergent at the stellar surface (F) is related to the flux outside the earth's atmosphere (f) by the expression

$$F = 4f/\theta_{\text{LD}}^2, \qquad (2\text{-}11)$$

where f is corrected for the effects of interstellar extinction and θ_{LD} is the angular diameter of the star. The effective temperature of the star is defined to be

$$T_{\text{eff}} = (F/\sigma)^{1/4}, \qquad (2\text{-}12)$$

where σ is the Stefan-Boltzmann constant.

A thorough discussion of the procedures involved in deriving T_{eff} from satellite data has been given by Code et al. (1976), while Beeckmans (1977) has intercompared the available ultraviolet measurements and estimated their errors. The fundamental calibration problem is the same for observations both from space and on the ground—a standard photometric source of some kind must be measured in precisely the same way as the stellar sources. In the case of satellite instrumentation, calibration is obviously possible only prior to launch. Rocket instruments can be calibrated both before and after flight, but substantial errors may still be introduced by uncertainties in the correction for atmospheric extinction that must be applied to measurements of the standard source made on the ground (Strongylis and Bohlin, 1979). The most extensive series of stellar measurements are those obtained with the S2/68 spectrophotometer aboard the TD-1 satellite and with OAO-2. Beeckmans finds that the absolute calibrations for the two experiments agree within 5 percent for $\lambda > 2000$ Å, but that the differences at shorter wavelengths are much larger and reach 35 percent near λ1500. The integrated fluxes for early-type stars that emit most of their flux below λ2000 are correspondingly quite uncertain. For A-type stars, however, the error in T_{eff} due to the uncertainty in the absolute calibration of the ultraviolet flux is only about 1 percent. On the other hand, uncertainty in the measured angular diameters may introduce errors as large as 3 to 6 percent in the determination of T_{eff} and is therefore a more important source of error than the flux calibration. Table 2-7 gives the effective temperatures and bolometric corrections derived by Beeckmans for six A-type stars with measured angular diameters. The standard deviations take into account all known sources of observational error. These values differ slightly from the ones derived by Code et al. (1976) from OAO-2 data alone (see Table 2-5).

MASSES AND RADII

The most reliable values of stellar masses and radii are derived from measurements of orbital parameters for spectroscopic binaries that also show eclipses. The best systems are those in which the stars are well separated so that tidal distortions are minimal and the velocity and light curves are not affected by gas streams. It is also important that the spectral lines of the two components be clearly resolved and that the two light minima be well defined.

Popper (1980) has recently reviewed critically the available data and listed the stars for which the errors in the estimates of the masses and radii are less than 15 percent. The basic parameters for the A-type stars that satisfy this criterion are listed in Table 2-8. The spectral types are inferred from photometry, because it is impossible to assign accurate spectral types, which by definition must be based on low dispersion spectra, to the components of double-lined binary systems. The values of T_{eff}, L, and M_v are based on calibrations that are also discussed by Popper.

There are alternative techniques for estimating stellar radii. While these techniques do not provide fundamental values of the radius in the same sense as orbital analyses, they do make it possible to derive radii for single stars. In these methods, the radius is derived from the angular diameter and parallax, and the observational problem is to obtain a reliable value for the angular diameter. Angular diameters for a few early-type stars have been measured directly with an intensity interferometer (Hanbury Brown et al., 1974).

Indirect methods (Gray, 1967; 1968) make use of the relationship

$$\frac{R}{D} = \sqrt{\frac{f_{E,\lambda}}{F_{s,\lambda}}} \, , \qquad (2\text{-}13)$$

where R is the radius of a star, D is its distance, $F_{s,\lambda}$ is the monochromatic flux emitted at the

Table 2-7
Effective Temperatures of A-Type Stars

Name	Spectral Type	B–V	Total Flux (10^{-6} ergs cm^{-2} s^{-1})	T_{eff}	Bolometric Correction
γ Gem	A0 IV	0.00	4.64 ± 0.30	9215 ± 450	−0.10 ± 0.09
α CMa	A1 V	0.00	109.70 ± 7.50	9870 ± 305	−0.15 ± 0.09
α Oph	A5 III	+0.15	3.59 ± 0.22	7980 ± 450	+0.03 ± 0.09
ϵ Sgr	A0 V	−0.03	5.33 ± 0.34	9370 ± 345	−0.18 ± 0.09
α Lyr	A0 V	0.00	28.90 ± 2.10	9540 ± 275	−0.19 ± 0.10
α Aql	A7 IV-V	+0.22	11.98 ± 0.74	7980 ± 310	+0.03 ± 0.09

Source: Adapted from Beeckmans (1977).

Table 2-8
Masses and Radii of A-Type Stars

Binary	Spectral Type	(B–V)	Mass ($M_\odot$)	R ($R_\odot$)	log T_{eff}
χ^2 Hya	B9	−0.09	3.61 ± 0.08	4.39 ± 0.04	4.063
$P = 2.27^d$	A0	−0.05	2.64 ± 0.05	2.16 ± 0.04	4.005
V 451 Oph	B9	−0.06	2.78 ± 0.06	2.65 ± 0.20	4.015
$P = 2.20^d$	A0	−0.02	2.36 ± 0.05	2.12 ± 0.20	3.985
RX Her	B9	−0.06	2.75 ± 0.06	2.44 ± 0.10	4.015
$P = 1.78^d$	A0	−0.02	2.33 ± 0.05	1.96 ± 0.10	3.985
β Aur	A1	+0.035	2.35 ± 0.03	2.49 ± 0.15	3.955
$P = 3.96^d$	A1	+0.035	2.27 ± 0.03	2.49 ± 0.15	3.955
SZ Cen	A7	+0.20	2.28 ± 0.02	3.62 ± 0.10	3.885
$P = 4.11^d$	A7	+0.22	2.32 ± 0.03	4.55 ± 0.10	3.878
EE Peg	A3	+0.10	2.08 ± 0.16	2.05 ± 0.10	3.927
$P = 2.63^d$	F5	+0.45	1.32 ± 0.04	1.29 ± 0.06	3.806
V 624 Her	Am	+0.19	2.1 ± 0.3	3.0 ± 0.3	3.890
$P = 3.90^d$	Am	+0.21	1.8 ± 0.2	2.2 ± 0.25	3.882
V 805 Aql	A5	+0.13	2.06 ± 0.07	2.10 ± 0.12	3.915
$P = 2.41^d$	F0	+0.28	1.60 ± 0.04	1.75 ± 0.12	3.855
RR Lyn	Am	+0.22	2.00 ± 0.05	2.50 ± 0.15	3.880
$P = 9.95^d$	F0	+0.29	1.55 ± 0.03	1.93 ± 0.15	3.850
WW Aur	Am	+0.14	1.98 ± 0.05	1.89 ± 0.04	3.910
$P = 2.52^d$	Am	+0.19	1.82 ± 0.05	1.89 ± 0.04	3.890
CM Lac	A2	+0.08	1.88 ± 0.09	1.59 ± 0.06	3.935
$P = 1.60^d$	F0	+0.28	1.47 ± 0.04	1.42 ± 0.06	3.855
RS Cha	A8	+0.20	1.86 ± 0.02	2.28 ± 0.15	3.885
$P = 1.67^d$	A8	+0.26	1.82 ± 0.02	2.28 ± 0.15	3.863
MY Cyg	Am	+0.30	1.81 ± 0.04	2.20 ± 0.04	3.848
$P = 4.00^d$	Am	+0.31	1.78 ± 0.04	2.20 ± 0.04	3.845
V 477 Cyg	A2	+0.07	1.78 ± 0.12	1.52 ± 0.06	3.940
HD190786	F2	+0.38	1.34 ± 0.07	1.20 ± 0.05	3.820
XY Cet	Am	+0.23	1.76 ± 0.02	1.88 ± 0.07	3.875
$P = 2.78^d$	Am	+0.27	1.63 ± 0.02	1.88 ± 0.07	3.860
TX Her	A8	+0.26	1.62 ± 0.04	1.58 ± 0.05	3.863
$P = 2.06^d$	F2	+0.36	1.45 ± 0.03	1.48 ± 0.05	3.827

Source: Adapted from Popper (1980).

stellar surface, and $f_{E,\lambda}$ is the monochromatic flux received at the earth. If $F_{s,\lambda}$ can be estimated either observationally or by comparison with model atmospheres, then the angular diameter $2R/D$ can be derived. If the distance is also known, then R can be easily calculated.

Two approaches to deriving $F_{s,\lambda}$ have recently been applied to extensive samples of early-type stars. The first method depends on the observational result that there is a very close relationship between visual surface brightness and the color index (V–R) (Barnes and Evans, 1976; Barnes et al., 1976).

The second method makes use of the fact that, although the stellar surface flux in the visible region of the spectrum depends strongly on effective temperature, the infrared flux is relatively insensitive to T_{eff}. Therefore, even a relatively poor first estimate of T_{eff} can lead to a relatively good estimate of the angular diameter, and successive iterations quickly converge (Blackwell and Shallis, 1977). Within ~10 percent, the two methods for obtaining angular diameters agree with each other and with direct determinations. The typical radius at A0 is ~2.1 $R_{\odot}$, and at A5 it is 1.9 $R_{\odot}$. Obviously, some scatter about these values will be introduced by evolutionary effects.

SPECTROSCOPIC BINARIES

The best data on the frequency of spectroscopic binaries among normal A-type stars comes from a study by Abt (1965). The sample included 55 stars with spectral types in the range A4 to F2 IV, V. Seventeen stars proved to be binaries, but none has a period shorter than 100 days. Subsequent studies have shown that there are some normal A-type stars in binaries with $P <$ 2.5 days (Abt and Bidelman, 1969). This result is in sharp contrast with earlier work which showed that all, or nearly all, Am stars are members of close binaries (Abt, 1961). This dichotomy in the binary properties of normal and Am stars probably reflects the importance of tidal interactions in close binaries. In late A-type stars, rotation and revolution are synchronized, or nearly so, for periods ~8 days or less (Levato, 1976). For periods in the range $2.5 < P < 8$ days, the synchronous rotational velocity is less than ~40 km s^{-1}, and all late A-type stars with such slow rotation show Am characteristics (Abt and Moyd, 1973; Wolff and Wolff, 1976).

The relationship between binary membership, rotation, and the Am phenomenon is discussed in detail in Chapter 5. The absence of normal stars in binary systems with orbital periods between 8 and 100 days is somewhat puzzling, but perhaps a survey of a larger sample of stars would lead to their discovery.

ROTATION

Rotation is not normally taken explicitly into account in modeling the emergent spectra of stars, because the direct effect of rotational distortion of the stellar surface on colors and line strengths is small except in the case where the rotational velocity approaches v_c, the velocity at which the centrifugal force counterbalances the gravitational force (Collins and Sonneborn, 1977). Among the A-type stars, however, rotational velocities and the appearance of the spectrum are closely correlated, in the sense that normal stars on the average rotate much more rapidly than the peculiar ones. Because this correlation may constitute an essential clue to the origin of the spectral anomalies, the distribution of rotational velocities will be discussed explicitly for each of the groups of A-type stars treated in this monograph.

All of the large surveys of rotational velocities published to date depend on a geometrical technique, suggested originally by Shajn and Struve (1929), that relates line profiles and specifically line widths to apparent rotational velocity v sin *i*. A critical discussion of the technique and a summary of the early studies of rotational broadening are given by Slettebak (1949). The essential assumption is that the unbroadened line contour is similar to that actually observed in narrow-lined stars of the same spectral class. The apparent stellar disk is then divided into strips, each producing that same narrow-lined contour with an equivalent width proportional to the area of the strip. The equatorial rotational velocity determines the radial velocity of each strip, and integration over the stellar disk yields the rotationally broadened contour. Although limb darkening is taken into account, a number of other potentially significant effects usually are not. The effects of gravity darkening, which is a consequence of the deformation of the star by rotation, are neglected. The unbroadened contour is assumed to be the same everywhere on the stellar disk, although variations in line strength are to be expected because of the dependence of the effective optical depth on distance from the center of the disk. Differential rotation is not considered, and the limb darkening is assumed to be a function of $\mu = \cos\theta$ only. Slettebak (1949) made some estimates of the possible effects of differential rotation and gravity darkening and found them to be small except for the case where the rotational velocity exceeded about $7/8\ v_c$.

Recently, models of rotating stars that take into account gravity darkening, rotational distortion, and variations in line strength over the stellar surface have been calculated by Collins (1974) and others. Collins' models make use of values of the polar radius and luminosity obtained from calculations of model interiors for rigidly rotating stars (Sackman and Anand, 1970) and depend on the assumptions (1) that the potential field is that of a rapidly rotating Roche model; (2) that the flux at any point on the surface is proportional to the local gravity; and (3) that the atmosphere at any point can be approximated by the plane parallel atmosphere (Kurucz, 1970) in LTE that corresponds to the local physical conditions. Differential rotation is not included.

In an extensive series of observations that include both photoelectric and photographic measurements of line profiles, Slettebak et al. (1975) have used the line profiles calculated by Collins to derive rotational velocities of a sample of stars in the spectral range O9 to F8. For the A- and F-type stars, their new values of v sin *i* are only 5 percent lower than the ones derived earlier by Slettebak through use of the geometrical technique of Shajn and Struve (1929).

Numerous observers (e.g., Slettebak, 1963; Kraft, 1965) have found that, particularly for v sin $i < 100$ km s^{-1}, rotational velocities can be estimated with good accuracy (± 10 percent) from visual inspection if a grid of standard stars is available. The primary purpose of the work of Slettebak et al. (1975) was to set up that grid, and future rotational velocity surveys will likely depend on intercomparisons with it. However, all published surveys of A-type stars antedate the work of Slettebak et al., and most have been reduced to the system established earlier by Slettebak, often by the simple expedient of using his original measurements as standards. Thus, these values of v sin *i* are systematically ~5 percent larger than would be obtained by using new modeling techniques, but such a small difference is immaterial for the present discussion, and no corrections will be applied to existing measurements.

Since the primary goal of assembling rotational velocity measurements is to allow an intercomparison of the velocity distributions of the various groups of A-type stars, the most useful surveys are those that meet the following two criteria. First, the survey must be large enough to allow statistical analysis, specifically calculation of the distribution of true rotational velocities. Second, the observations must be of a clearly defined sample so that selection effects and contamination by miscellaneous peculiar stars are both unimportant.

For the normal A-type stars, the survey that best satisfies the two criteria is that by Abt and Moyd (1973), who were primarily interested in distinguishing between the rotational velocity distributions of the Am and normal A-type stars. Their sample includes all the normal A-type stars in the catalog prepared by Cowley et al. (1969) with four-color photometry and with spectral types in the range A5 to A9, luminosity classes IV to V, where classical Am stars are also found. Altogether, 123 normal A-type stars were included, and the observed distribution of v sin *i* is shown in Figure 2-4.

On the assumption that the axes of rotation for the stars in this sample are randomly oriented, it is possible to derive the distribution of true

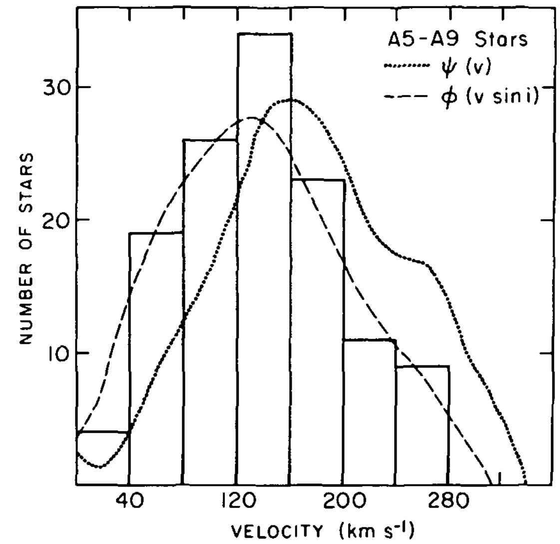

Figure 2-4. Distribution of observed rotational velocities (histogram) and of true rotational velocities ψ(v) (dotted line) for normal, main sequence A-type stars with spectral types A5 to A9. Dashed line shows distribution of apparent rotational velocities φ(v sin i) derived from ψ(v). Iteration to determine ψ(v) is continued until φ(v sin i) adequately fits the observations (from Wolff and Wolff, 1976).

rotational velocities ψ(v) from the observed distribution of apparent rotational velocities φ (v sin *i*). The technique devised by Lucy (1974) is particularly well suited for this purpose, because no assumptions need be made about the form of ψ(v). In applying this technique, one assumes a distribution ψ(v), calculates from that ψ(v) the corresponding φ(v sin *i*), and then obtains a new estimate for ψ(v) by comparing the calculated φ(v sin *i*) with the observed one. The iteration is repeated until the observed distribution is fit as well as the accuracy of the observations requires. Since ψ(v) is necessarily less regular than φ(v sin *i*), inasmuch as sin *i* acts as a smoothing function, the usual techniques of calculating ψ(v) directly from φ(v sin *i*) yield distributions that are more irregular than warranted by the statistical accuracy of the observations.

The distribution ψ(v) derived for the normal A-type stars is shown in Figure 2-4. The important feature of this diagram is that ψ(v) is essentially zero for $v < 40$ km s^{-1}. That is, there are no slowly rotating normal A-type stars in this sample. The same conclusion was reached earlier by Abt and Moyd (1973) for the same sample, although their limiting velocity occurred at $v \cong 100$ km s^{-1}. This difference is not surprising since Abt and Moyd used a different mathematical technique, and there are a variety of distributions ψ(v) that yield distributions φ (v sin *i*) that are compatible with the observations; the inversion process is not unique (Brown, 1950).

Whatever the mathematical uncertainties, it is clear that there can be few, if any, normal A-type stars with low rotational velocities. Only two of the stars observed by Abt and Moyd have v sin $i \leq 40$ km s^{-1}, and in those two cases the low value of v sin *i* can probably be attributed to a low value of the angle of inclination. Since the distinction between Am and normal A-type stars in this sample is based on classification spectrograms, which are fairly insensitive to rotation, the lack of slowly rotating normal stars is *not* a consequence of increasing visibility of subtle anomalies in slowly rotating stars.

Figure 2-5 shows the distribution of v sin *i* for main sequence stars as a function of spectral type. The sources of the data presented in this figure are given by Wolff et al. (1982). All stars, with the exception of magnetic ones, within the specified spectral ranges were used to construct the distributions of v sin *i*. Specifically, both Am and normal stars are included in the group of late A-type stars. For many years, the A spectral class was thought to harbor an unusually high number of slowly rotating stars. In fact, for all groups in the spectral range B0 to F0, the distribution of v sin *i* reaches its maximum value somewhere in the interval v sin $i < 100$ km s^{-1} and declines monotonically from that point with increasing v sin *i*. In the late A-type stars, spectroscopic binaries dominate the distribution at values of v sin *i* less than 50 km s^{-1}. This statement is not true for earlier spectral groups, with the possible exception of the middle B-type stars (see Wolff et al. [1982] for a discussion of this group). The large number of slowly rotating spectroscopic binaries among the late A-type stars probably

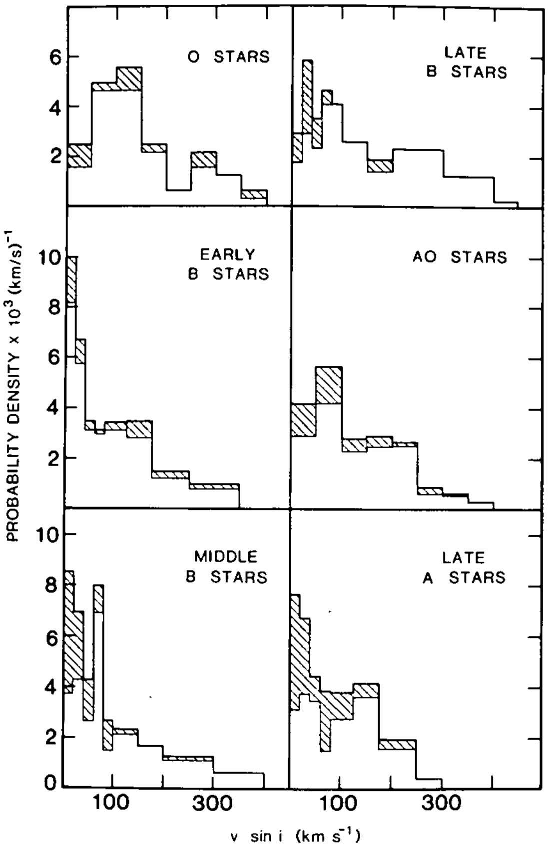

Figure 2-5. The observed distribution of v sin i for main sequence stars as a function of spectral type. Hatched areas show spectroscopic binaries discovered to date among the stars within each group (from Wolff et al., 1982).

reflects the impact of tidal synchronization of rotation and revolution. There are two reasons that this mechanism should be more effective in late A-type stars than in stars of earlier spectral classes. First, the envelopes of late A-type stars are partially convective, and tidal interactions are more efficient in convective than in radiative envelopes. Second, tidal forces seem to act throughout the main sequence lifetime of a star (Levato, 1976). By virtue of their lower mass, the late A-type stars spend longer on the main sequence than the other stars included in Figure 2-5, and are therefore most likely to have undergone a significant modification in rotation rates.

The line profiles of stars contain information not only about rotational velocities but about other velocity fields within the stellar photosphere as well. In principle, it is possible to distinguish such sources of line broadening as rotation, microturbulence, and macroturbulence from one another. Gray (1976) has discussed techniques for using Fourier transforms to search for profile differences that correspond to various types of line broadening. Because these methods have as yet been applied to only a small number of A-type stars, they will not be discussed in detail here. Only a few of the more interesting results will be presented.

Applications of Fourier techniques to the derivation of rotational velocities yield results that are generally in good accord with those derived with conventional methods (Gray, 1980b). In some specific cases, however, the values of v sin i derived from Fourier transforms of line profiles are remarkably discrepant. For example, Milliard et al. (1977) find that v sin i = 18 ± 2 km s^{-1} in Vega from a Fourier transform analysis of lines near λ1362. Gray (1980a), from a study of lines near λ4500, finds v sin i = 23.4 ± 0.4 km s^{-1}. A similar disagreement exists for Sirius. Smith (1976) finds an apparent rotational velocity of 17 ± 1 km s^{-1}, in good agreement with the value of 16 ± 1 km s^{-1} derived by Kurucz et al. (1977). From ultraviolet observations, Milliard et al. (1977) obtain v sin i = 11 ± 2 km s^{-1}. For both Sirius and Vega, the rotational broadening derived from Fourier techniques exceeds that derived by Slettebak et al. (1975) from direct analysis of the line profiles. Slettebak et al. estimate v sin i < 10 km s^{-1} for Vega, and v sin i = 10 km s^{-1} for Sirius. These discrepancies far exceed the estimates of observational error, and the explanation is not known.

Differential rotation is seen in the Sun and if present in other stars may cause marked changes in the Fourier transforms of line profiles. This effect has been looked for by Gray (1977). His calculations assume that differential rotation can be represented by an equation of the form

$$\omega = \omega_0 + \omega_2 \sin^2 \psi, \qquad (2\text{-}14)$$

where ω_0 and ω_2 are constant angular velocities and ψ is the colatitude measured on the star.

More marked differential rotation corresponds to larger values of ω_2, and so the dimensionless quantity

$$\alpha = \frac{\omega_2}{\omega_0 + \omega_2} \tag{2-15}$$

can be used to quantify the significance of differential rotation. Figure 2-6 shows a series of Fourier transforms for various values of α. Note that increasing differential rotation tends to suppress the first and third side lobes of the transform while leaving the second essentially unchanged.

Gray has obtained high-precision profiles of six A-type stars, and the side lobe pattern characteristic of differential rotation is not present in any of them. In some cases, however, the observed transforms are not at all what simple models

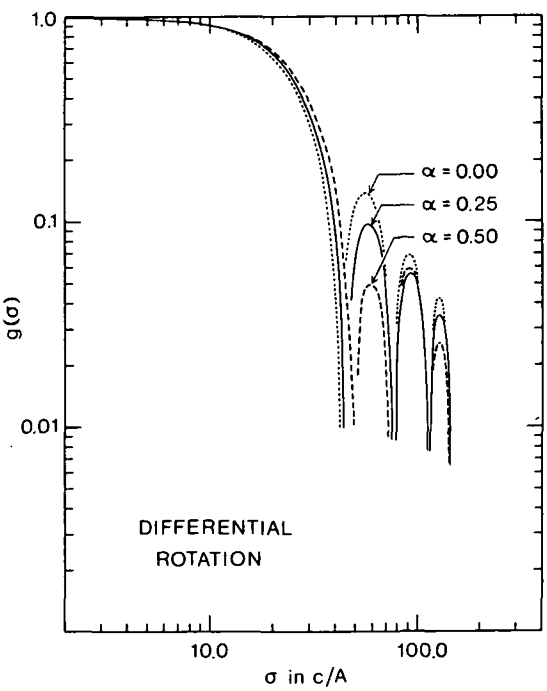

Figure 2-6. The Fourier transforms of line profiles showing the nonmonotonic behavior of peak side-lobe amplitude with increasing differential rotation as parameterized by the quantity α (from Gray, 1977).

predict. In one case, only the first side lobe is seen, and in another there are no side lobes at all. It seems likely that atmospheric models, which must explicitly include photospheric velocity fields, are simply inadequate to reproduce line profiles with an accuracy commensurate with the observations.

MICROTURBULENCE

Turbulence is probably present to some degree in all stellar atmospheres. A realistic model would include a spectrum of sizes and velocities for the turbulent elements (de Jager, 1980). In practice, only two limiting cases can be easily treated. If the size of a turbulent element is smaller than the photon mean-free path, then the velocity field is said to be microturbulent. The term macroturbulence refers to motions of turbulent elements that are large compared to the mean-free path of a photon. Macroturbulence produces a change in line width but not in line strength. In principle, macroturbulent and rotational broadening can be distinguished (Gray, 1976), but no observations of main sequence A-type stars have been obtained for this purpose. Because rotational broadening dominates macroturbulent broadening in virtually all low luminosity A-type stars, such observations are unlikely to prove worthwhile.

Microturbulence manifests itself through an increase in the Doppler parameter that characterizes the curve of growth. Determinations of this velocity parameter yield values of ~2 km s^{-1} in early A-type stars. The microturbulent velocity is higher in middle to late A-type stars, but the precise amount of the increase is uncertain (e.g., Baschek and Reimers, 1969; Chaffee, 1970). From an examination of several types of evidence, including line profiles, Smith (1973b) has concluded that the peak value of the microturbulence is ~4 km s^{-1}. It should be obvious from this discussion that, as classically used, macroturbulence and microturbulence are simply parameters that are adjusted in order to account for observed line widths and strengths. As Pecker and Thomas (1961) pointed out many years ago, any inferences about actual stellar velocity fields must depend on an analysis of the source function in the column of gas where the absorption lines are formed (see also Chapter 7).

MAGNETIC FIELDS

Strong magnetic fields have been detected in many of the peculiar stars of the upper main sequence. Magnetic fields of similar intensity are clearly not present in normal stars, and stringent upper limits to any such fields have been derived recently by Landstreet (1982). Measurements of the circular polarization in the wings of individual lines of hydrogen, helium, and iron were used to determine magnetic intensities (see Chapter 4), and Landstreet failed to detect the Zeeman effect in any of 31 normal O9.5 to F6 main sequence stars or in the five Am stars in his sample. Specifically, he was unable to confirm the presence of a magnetic field in either γ Vir N (F0 V), which had previously been reported to have a field that varied from −330 to +440 g (Boesgaard, 1974), or in α Leo (B7 V), in which Wolstencroft et al. (1981) had found evidence for circular polarization in Hβ. The median standard error of Landstreet's measurements is 65 g, and he concludes that effective longitudinal magnetic fields larger than 150 g must be quite uncommon among normal stars of the upper main sequence.

ABUNDANCES

In an analysis of the composition of a star, the abundance itself is often the only adjustable parameter. That is, one inserts into a model as much physics as seems appropriate or feasible. Any discrepancy between the calculated and observed equivalent widths is then attributed to an abundance difference. While objections can, and no doubt will, be made that this description greatly oversimplifies the process of determining abundances, there is sufficient truth in it to render, in this author's opinion, the composition of a star the least well determined of all its characteristics. For that reason, abundance studies will not be given much emphasis in the present monograph.

In the specific case of the A-type stars, most of the abundance studies to date have dealt with the peculiar stars. There are two reasons for this fact. First, the line strength anomalies are so spectacular that they virtually demand quantitative analysis. Second, there are very few normal stars with lines sharp enough to permit detailed study.

Nevertheless, abundance analyses have been carried out for a few early A-type stars, and the results are quite puzzling (Sadakane, 1981, and references therein). In the six stars studied, the scatter in the abundances of individual elements exceeds the factor of 2 that is normally adopted as a reasonable estimate of the uncertainty in the determination. In HR 7338 (A0 III), even after correction is made for contamination of the spectrum by its companion, the abundances of Al, Si, Sc, Ti, and Fe are lower than solar values by more than 0.6 dex. In α Lyr, Sadakane finds that the metals are deficient by about 0.4 dex, and a deficiency of Fe by about this same order of magnitude has been found by Dreiling and Bell (1980) as well. While a non-LTE analysis is required before we can accept with confidence the result that Vega is metal poor, the important point is that the metal abundances of A-type stars do not appear to be uniform.

Similar results have been obtained by Cowley et al. (1982), who used the method of wavelength coincidence statistics (Cowley and Aikman, 1980; see Chapter 4) to estimate the abundance of Fe in a sample of 34 late B- and early A-type stars with sharp lines. Basically, the technique depends on a search for wavelength coincidences between stellar and laboratory wavelengths. The larger the number of coincidences for weak lines, the greater the inferred abundance. Actual abundances are derived by comparing the wavelength coincidence statistics for stars in the sample for which detailed abundance analyses have already been carried out. Cowley et al. conclude from their sample that 10 to 20 percent of stars previously regarded as normal are deficient in iron, and presumably other metals as well, by factors of 2 or more.

The observations, therefore, seem to establish the reality of a group of metal-deficient stars near A0, and the frequency of such stars may equal that of the Ap stars. The explanation of the metal deficiency is quite unclear. Does the surface composition reflect the makeup of the star as a whole? If so, then abundance differences of A-type stars might reflect fluctuations in the composition of the interstellar material from which they formed, a point of view favored by Cowley et al. Or are the metal-deficient stars related to the λ Boo stars? The problem with this possibility is that, since one λ Boo star is apparently (Sargent, 1965) a member of a visual binary in which the primary is a B5 V star of normal composition, variations in primordial abundances probably are not the explanation for the metal deficiency of the λ Boo stars.

Data on the abundances of C, N, and O in A-type stars are remarkably sparse. The reason is simple (e.g., Lambert et al., 1982). The dominant ionization state of all three elements in A-type stars is the neutral one, and most of the detectable lines fall in the spectral region $\lambda6000$ to $\lambda10800$, where photographic observations are difficult. Silicon detectors with high quantum efficiency throughout this spectral region are now available at most large telescopes, and abundances of C, N, and O in Vega and Sirius have recently been derived by Lambert et al. (1982). In Vega, the abundances of all three elements are essentially solar. In Sirius, C is underabundant by a factor of 4, N overabundant by nearly a factor of 2, and O underabundant by a factor of 2. An overabundance of N, accompanied by underabundances of C and O, is suggestive of CNO processing. However, as Lambert et al. point out, CNO cycling preserves the sum of the CNO abundances. In Sirius, the sum is 8.76, while the solar value is 9.15 dex. The Sirius value would be compatible with a normal ratio of CNO/Fe only if $[Fe/H] \cong -0.8$. There is no evidence that Sirius is so metal poor.

An alternative possibility is that the CNO abundances in Sirius have been altered by diffusion processes. Sirius is classified as a "hot" metallic-line star, and calculations of radiative diffusion in such stars yield an abundance of N that is higher than that of either C or O (Michaud et al., 1976; Lambert et al., 1982).

This discussion of the limited data currently available on the composition of normal A-type stars raises more questions than it answers. An extensive survey of these stars is required to establish how uniform the abundances of early A-type stars really are and what mechanisms are responsible for the abundance differences that are seen in individual stars.

3

NONRADIATIVE HEATING IN A-TYPE STARS

There is strong evidence for nonradiative heating in stars of both very early and very late spectral types. Lines of anomalously high stages of ionization, including N V and O VI, are present in the winds of massive O- and B-type stars, while emission in Ca II H and K is a nearly ubiquitous indicator of chromospheric activity in late-type stars. It is only very recently, however, that new observational techniques have provided unambiguous evidence that nonradiative heating is present in A-type stars as well. This chapter summarizes the extensive data, negative as well as positive, that has been obtained in searches for chromospheres, coronae, and transition regions in A-type stars.

The existence of a stellar chromosphere can be inferred from the observation of line or continuum emission that exceeds what would be predicted for model atmospheres in radiative equilibrium, or from the identification of features caused by atoms in ionization or excitation stages that are significantly higher than would be expected for classic photospheric models. Particularly good diagnostics are emission features in the cores of lines that are collision dominated so that they reflect local conditions. The lines must also be strong enough so that they are formed in the outer portion of the atmosphere.

One of the most useful indicators of chromospheric activity in solar-type stars is emission in the Ca II H and K lines. There are, however, several practical difficulties in detecting similar emission in A-type stars (Dravins, 1981). The photospheric continuum fluxes near λ3933 in A-type stars exceed the solar flux at that same wavelength by a factor of 10 or more, and so chromospheric emission as intense as that seen in the Sun would have an equivalent width of ~ 0.3 mÅ or less. The rapid rotation characteristic of A-type stars will broaden any emission peaks in Ca II K. In late-type stars, the widths of H and K emission features increase with luminosity. If this effect extends up the main sequence (see discussion of Mg h and k below), then H and K reversals will be broader in A-type stars than they are in solar-type stars, and possible emission peaks may appear only as shallow undulations in the absorption line wings.

Despite the observational difficulties, searches for chromospheric indicators in main sequence A-type stars have been undertaken by several authors. Freire et al. (1977, 1978) and Freire (1979) report the results of a search for emission reversals in the cores of Ca II H and K in α CMa (A1 V), γ Gem (A0 IV), α Aql (A7 IV-V), and α Lyr (A0 V), and in the cores of the C II resonance multiplet (λ1335) and of the Si II lines (λλ1304, 1309). While Freire et al. find that some asymmetries, possibly due to photospheric velocity fields, are present in cores of H and K in Sirius and γ Gem, there is no evidence of reversal or emission in any of the lines observed. Specifically, the emission reported by Linsky et al. (1973) in α Lyr is not confirmed. The authors stress, however, that even modest rotational velocities (~10 km s^{-1}) are sufficient to render weak emission features undetectable.

Subsequent observations have failed to confirm even the asymmetry in the K line of Sirius. Both Griffin and Griffin (1979) and Czarny and Felenbok (1979) find that the K line is symmetric. In their original discussion of the asymmetry, Freire et al. (1977) suggest that the asymmetry might be caused by photospheric velocity fields induced by the companion to Sirius. The observation that shows the asymmetry was made at a slightly different phase in the binary orbit than the measurements that do not. The possibility of time variations in the photospheric velocity field cannot, therefore, be entirely ruled out.

In solar-type stars, chromospheric emission varies inversely with age. The average intensity of Ca II H and K emission in main sequence F- and G-type main sequence stars in young clusters, such as the Pleiades and Hyades, can exceed that of field stars by factors of 10 to 100. With this analogy in mind, Dravins (1981) obtained observations of eight A-type stars in the Hyades and IC 2391. No emission was detected.

In order to quantify the limits placed on chromospheres by the absence of observable emission, Freire et al. (1978) calculated a series of model atmospheres for α Lyr. Three types of models were considered: radiative equilibrium (RE) and local thermodynamic equilibrium (LTE), RE and non-LTE, and non-LTE with an artificially imposed temperature rise in the outer layers.

The observed lines span a range of optical depths. Central optical depth $\tau_L = 1$ occurs for the C II lines at $\tau_{5000} \cong 10^{-7}$, for the Si II lines at $\tau_{5000} \cong 4 \times 10^{-7}$, and for the Ca II lines at $\tau_{5000} \cong 3 \times 10^{-3}$. The model atmosphere calculations for all of these lines yield the same conclusion; that a temperature rise occurring at log $\tau_{5000} \leq -4$ would not produce detectable emission in the cores of any of these lines. A temperature rise at a deeper level in the atmosphere (log $\tau_{5000} \cong -3$) would produce a reversal, but that reversal would become undetectable for rotation as small as 10 km s^{-1}. However, a temperature rise at log $\tau \geq -4$ would also alter the Balmer continuum significantly and can be ruled out on the basis of the observed energy distributions.

High-resolution observations of the Ca II H and K lines have also been obtained for the supergiant α Cyg (A2 Ia). The K line is markedly asymmetric with considerable structure in the core (Linsky et al., 1973; McClintock and Henry, 1977). Observations by Marschall and Hobbs (1972) and Griffin and Griffin (1980) show, however, that much—and perhaps all—of the asymmetry can be accounted for by a number of sharp interstellar absorption features. Interstellar absorption is certainly to be expected in α Cyg, which is 0.5 kpc distant from the Sun and at a galactic latitude of 2°. Since the radial velocity in α Cyg is variable, a series of high-resolution measurements of the K line at various values of the stellar radial velocity would probably clarify the issue (McClintock and Henry, 1980). Many interstellar absorption features have, in fact, been observed in the ultraviolet spectrum of α Cyg (Praderie et al., 1980).

A specific search for chromospheric emission in the ultraviolet spectra of A supergiants has been carried out by Praderie et al. (1980), who observed HR 1040 (A0 Ia), α Cyg (A2 Ia), HR 2874 (A5 Ib), and α Car (F0 Ib) at a resolution of $\Delta\lambda \sim$ 0.1 Å with the International Ultraviolet Explorer (IUE). The primary chromospheric indicators (Ulmschneider, 1979)—namely, the resonance lines of Si IV $\lambda\lambda$1393.75, 1402.77, and CIV $\lambda\lambda$1548.19, 1550.77 and the line He II λ1640—are not seen in any of these stars. With the exception of α Car, all of these stars show evidence for mass loss; therefore the absence of chromospheric emission indicates that the extended envelopes around these objects must be cool. This result is in agreement with the conclusion by Böhm-Vitense and Dettman (1980) that highly luminous stars on the blue side of the instability strip show no evidence of chromospheric emission. The fact that the envelopes are cool does *not*, however, imply that nonequilibrium effects are absent. The mass flow velocities exceed the thermal velocities by a wide margin and the cause of the flow is unknown (see Chapter 7).

Two other lines that are known to be good indicators of chromospheric activity are the Mg II h and k doublet lines near λ2800. A search for chromospheric emission in these lines as well as in the far ultraviolet spectra of A-, F-, and G-type stars has been described by Böhm-Vitense and Dettman (1980). Their IUE observations, which include stars of all luminosity classes, generally span the wavelength regions λλ1900 to 3200 (0.2 Å resolution) and λλ1150 to 2000 (7 Å resolution). They find that there is a strong correlation between the presence of emission lines in the interval λλ1150 to 2000 and emission in the Mg II h and k lines, so that presumably observations of either can be used to detect stellar chromospheres. For high-luminosity stars, the Cepheid instability strip serves to separate the stars with chromospheric emission from those without, with only those stars on the red side of the strip showing emission. On the main sequence, chromospheric emission is seen only in stars later than spectral type F0. It must be noted, however, that the increase in the ultraviolet continuum radiation with increasing temperature makes it more difficult to detect chromospheric emission in the hotter stars. Böhm-Vitense and Dettman refer their measurements to the local continuum. The quantity that more properly measures chromospheric emission is the ratio of Ca II or Mg II emission to the total luminosity of the star. Use of this ratio, rather than equivalent widths, eliminates the apparent rate of decline of chromospheric activity in main sequence stars with increasing temperature (Dravins, 1976; Blanco et al., 1982). There is, in fact, direct evidence for chromospheric emission in α Aql, which is classified A7 IV–V (Kondo et al., 1977; Blanco et al., 1982). Both components of the Mg II h and k resonance doublet exhibit slight "bumps" on either side of the absorption cores. The line profiles are quite reminiscent of the profiles of Ca II H and K in the quiet solar chromosphere. The widths of the h and k features in α Aql are about 1.4 Å, and this width fits well with the linear relationship between absolute visual magnitude and line width established for stars of later type.

Another chromospheric indicator is Ly α emission. Emission in this line has been seen in α Aql (Blanco et al., 1980), thus confirming the presence of a chromosphere in this star. The flux per unit area at the stellar surface is F(Ly α) = 5.58×10^5 erg cm^{-2} sec^{-1}, and it exceeds the Ly α surface fluxes of the Sun, ϵ Eri, and Capella. Thus, the chromosphere of α Aql may, in fact, be a fairly powerful one. Praderie (1981) searched for, but failed to find, Ly α emission in Vega.

It seems quite clear from these data that chromospheric emission can occur in late A-type stars. The star α Aql rotates very rapidly (v sin i = 245 km s^{-1}), and perhaps chromospheric activity and rapid rotation will prove to be linked in A-type stars, as they are in stars of later types. Obviously, one would like additional measurements of stars with a range of temperature and rotational velocities, but because the emission features are so weak relative to the background radiation in these hot stars, such observations may be beyond the range of current techniques.

Another indicator of stellar activity is He I λ10830. At temperatures less than 20,000 K there should be very few atoms in the levels, which lie 20 volts above the ground state, that are responsible for producing either emission or absorption of this line. Emission at λ10830 can be produced in fairly dense regions and thus can serve as an excellent indicator of chromospheres in stars with effective temperatures that are significantly lower than 20,000 K. Absorption at λ10830 may be produced by back radiation from a corona (Zirin, 1975). The most extensive observations of λ10830 in stars are by Vaughan and Zirin (1968) and Zirin (1976). Only six A-type stars have been observed, including two normal and three peculiar main sequence stars and one supergiant. The line λ10830, in either emission or absorption, was not detected in any of them, and the upper limit on the strength of any feature at this wavelength is ~ 0.1 Å. Quantitative calculations of the corresponding limits on chromospheres or coronae in A-type stars have not been made.

Observations of the near-infrared spectrum (resolution equivalent to photographic observations at 6 to 12 Å mm^{-1}) of Vega have been obtained by Johnson and Wisniewski (1978) with a Michelson spectrophotometer. They report that the infrared lines of Ca II and O I have weak violet-shifted emission satellites and conclude that the emission is evidence for a thin, extended atmosphere, which may be expanding. Spectra of α CrB (A0 V), δ Cas (A5 V), α CMa (A2 V), and α Cyg (A2 Ia) do not show similar emission structures. Subsequent photographic (Griffin and Griffin, 1978) and Reticon (Barker et al., 1978) measurements fail to confirm the presence of emission in α Lyr. Two conclusions are possible: either α Lyr is variable, or the emission is instrumental in origin. While Johnson and Wisniewski offer some persuasive arguments against the latter possibility, it may prove equally difficult to devise a mechanism that could produce identical changes in two features as dissimilar as these particular lines of Ca II and O I.

Short-period variability of any kind in early A-type stars would constitute evidence of nonthermal processes. The instability strip crosses the main sequence in the region where late A- and early F-type stars occur and variability among these stars is common. However, there have been several reports of variability in early A-type stars that lie well to the blue of the instability strip. For example, variations in radial velocity, luminosity, and line profiles have been reported for Vega, which is *the* standard early A-type star. The history of these observations has been summarized by Wisniewski and Johnson (1979). The variations reported occurred in photometric brightness (amplitude ~0.04 mag) and in radial velocity (amplitude ~6 km s^{-1}), but most of the observations of variability were made before 1935. Nearly contemporaneous observations failed to agree on the period, if any, of the variations and the relative phasing between the light and radial velocity curves. In 1928 the International Astronomical Union (IAU) adopted Vega as a radial velocity standard, and modern observations indicate that it is suitable for that purpose (e.g., Abt and Snowden, 1973). A skeptic might conclude, inasmuch as the best measurements—those at the highest dispersion or made with the most modern instrumentation—are the ones that have failed to detect variability, that Vega is most likely constant. Wisniewski and Johnson point out, however, that Vega is both overluminous and too large in radius for its spectral type, presumably because it has evolved well away from the ZAMS-age main sequence (ZAMS). They further suggest that intermittent variability may be characteristic of the late stages of main sequence evolution. As additional support for the possibility that Vega may be a variable, they note that γ Gem and θ Vir have also been reported to show velocity variations.

The whole question of the variability of Vega has been reexamined by Fernie (1981), who describes quite frankly the natural scepticism of modern observers toward early reports of small amplitude variability. To quote Fernie:

> . . . How much credence is to be placed in these claims is a matter of personal judgment. The papers in which they appear are generally inadequate by today's standards: the procedures followed are often unclear; what comparison stars, if any, were used may go unstated; there are no quotations, let alone discussion, of probable errors; and some of the light curves drawn among observational points would outrage any modern referee. Add to this the fact that photometers of the period were relatively crude and the result is that claims of a few hundredths of a magnitude variability are now likely to be viewed with considerable skepticism.

Nevertheless, after very careful photometry Fernie finds evidence that on 3 of 14 nights Vega appeared brighter than its average value by 0.015 mag (2 nights) to 0.041 ± 0.005 mag (1 night). He also points out that the mass and radius of Vega match those of the δ Scuti variable AI Vel, and that low-amplitude light and velocity variations of Vega observed by Guthnick (1931) are almost identical in shape and time scale to those seen in AI Vel. Fernie, therefore, suggests that Vega occasionally exhibits δ Scuti-like variability. If this is true then a number of new questions arise concerning what triggers the intermittent pulsations and how frequently such variability occurs in stars that lie outside the pulsational instability strip as it is normally defined.

Short-period radial velocity variations in the sharp-lined A1 V star, θ Vir, have been reported by Beardsley and Zizka (1977). These investigators found that superposed on an orbital variation with a period of 17 years there is a periodic variation of 0.15 days with a semiamplitude of 5 km s^{-1}. The short-period variation, which was derived by consideration of deviations from smooth orbital motion, was present in all the observations made at the Allegheny Observatory during the time interval 1962 to 1974. The original dispersion of the observations was 40 Å mm^{-1}.

In an effort to confirm this variability, 19 radial velocity observations made on a single night and covering nine-tenths of one full cycle of the short-period variation reported by Beardsley and Zizka were obtained on March 8, 1976 with the coudé spectrograph of the 88-inch telescope on Mauna Kea (dispersion 13.6 Å mm^{-1}). The observations are shown in Figure 3-1. Obviously no variations of the kind reported by Beardsley and Zizka were present at the time these observations were made. The total range in velocity is less than half that found earlier and can be attributed entirely to observational error. After the fact, of course, it is impossible to confirm or refute the reality of

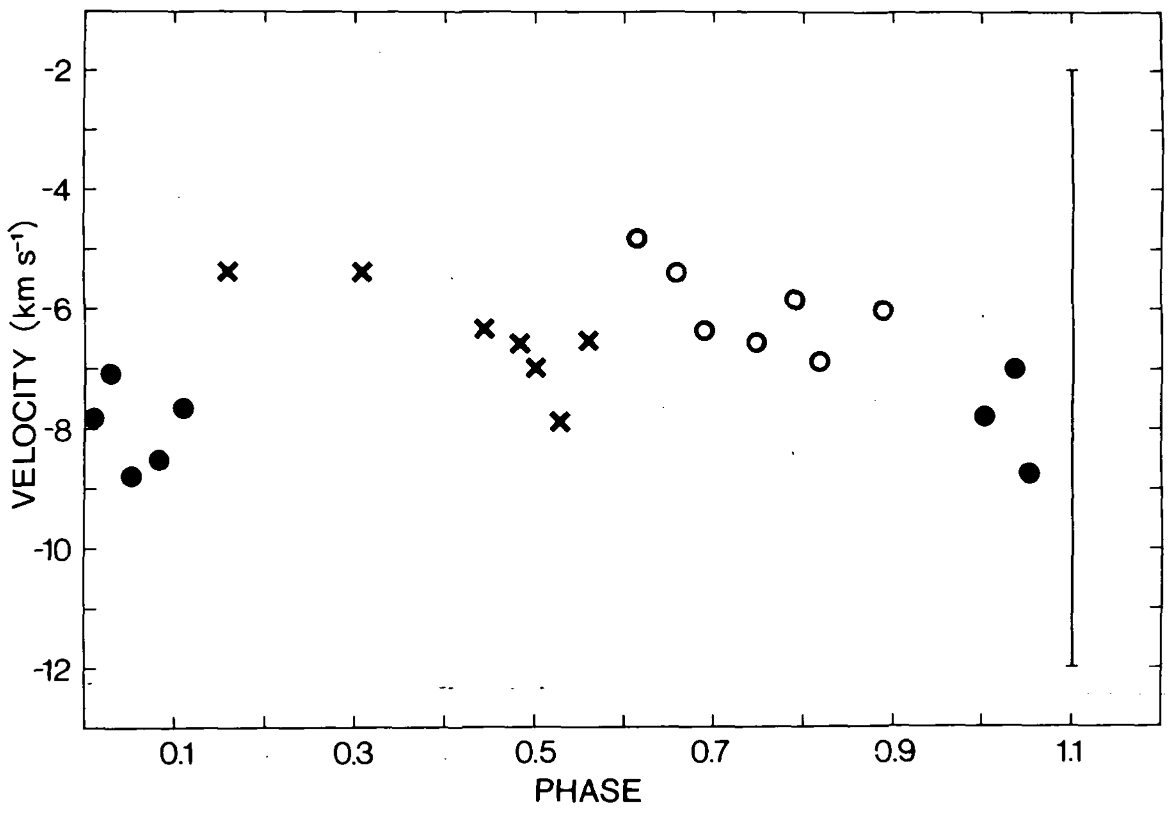

Figure 3-1. Radial velocity variations of θ Vir plotted on the assumption that $P = 0.152360$ day. Multiple exposures were obtained on each of three photographic plates, which are designated by filled circles, crosses, and open circles. Slight systematic velocity shifts between plates are apparent and indicative of uncertainties inherent in velocity measurements. The range of variation observed by Beardsley and Zizka is indicated by a vertical bar.

the short-period velocity variations observed at Allegheny, and it may be that the amplitude has decreased with time (cf., discussion by Beardsley and Zizka, 1977). It may also be that the original observations were overinterpreted. However, the work on θ Vir indicates the difficulty of establishing the reality, or lack thereof, of small amplitude variability of any kind. As an anonymous astronomer once remarked, "It takes only one observation to get a 'variable' star into the literature, but one thousand observations to get it out." Rarely does the presence or absence of short-period fluctuations appear to be so critical to our understanding of any specific object that astronomers are willing to devote sufficient telescope time to refute—or establish—beyond reasonable doubt reports of marginally detectable variability.

The detection of X-ray emission from stars constitutes, of course, a prima facie case for the existence of nonradiative sources of heating. It is only very recently, however, that X-ray devices have become sensitive enough to detect emission from A-type stars, and the interpretation of the observations is not clear.

A summary of the results of X-ray surveys to date has been given by Pallavicini et al. (1981), who also provide references to earlier work. X-ray emission has been observed in all types of stars. In late-type stars, the X-ray flux is correlated with rotational velocity, a result that is cited as evidence in favor of models that attribute coronal heating to dynamo-generated magnetic field structures. In early-type stars, the X-ray emission correlates well with luminosity ($L_x \sim 10^{-7} L_{\rm bol}$) and not with rotational velocity. The X-rays are presumably caused by some source of nonradiative heating in the extended envelopes that are produced by mass loss, which is ubiquitous in these stars (cf., Chapter 7).

Studies show that A-type stars can also be X-ray sources. On the average, the fraction of the total luminosity that is emitted in the form of X-rays is lower than for main sequence stars of either higher or lower temperature, a result that is probably related to the absence of strong surface convection zones or obvious mass loss in A-type stars. The range of X-ray emission in A-type stars is, however, quite broad. Fluxes range from 10^{27} ergs s^{-1} for stars such as Sirius and Vega to $\sim 5 \times 10^{30}$ ergs s^{-1} for the Algol system (Vaiana et al., 1981). Pallavicini et al. (1981) find no evidence for a correlation between X-ray flux and rotational velocities for A-type stars. However, the correlation between X-ray and bolometric luminosities found for O- and early B-type stars seems to be valid for at least some A-type stars (cf., Figure 3-2). An extrapolation of that correlation to A-type stars yields X-ray luminosities in the range of 10^{27} to 10^{28} ergs s^{-1}. It may simply be that A-type stars with higher X-ray fluxes are, like Algol, members of unresolved binary systems in which the X-ray flux is primarily due to a late-type companion.

A search for X-ray emission in specific classes of A-type stars has recently been completed by Cash and Snow (1982). Of nine Am stars that

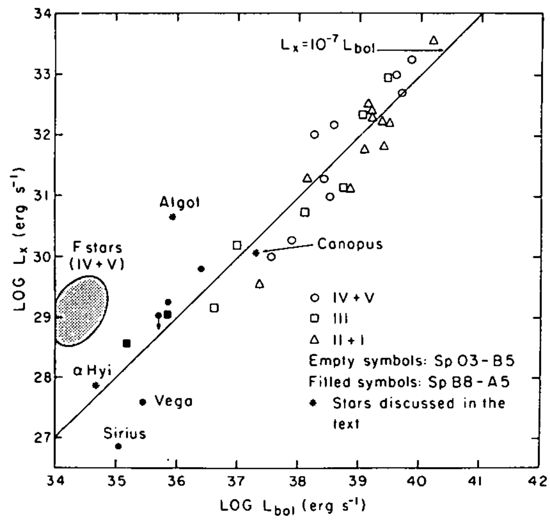

Figure 3-2. X-ray luminosity versus bolometric luminosity for stars of spectral types O3 to B5 (empty symbols) and of spectral types B8 to A5 (filled symbols). Different symbols indicate different luminosity classes. The position of main sequence and subgiant stars of spectral type F is also indicated schematically. Notice the good correlation between X-ray luminosity and bolometric luminosity for the entire group of stars of spectral type O3 to A5 as well as for the two subroups O3 to B5 and B7 to A5, separately. The straight line corresponds to the relationship $L_x = 10^{-7} L_{bol}$ (from Pallavicini et al., 1981).

were observed, four have been clearly detected as X-ray sources. With two exceptions, which can be easily explained, those Am stars that are members of close binaries were detected, while Am stars that are either single or members of wide binaries were not. Cash and Snow conclude from these data that Am stars as a class are not characterized by strong coronal emission.

Cash and Snow observed four HgMn stars and found evidence for X-ray emission from one (β Scl). Two magnetic Ap stars (ω Oph and 46 Dra) were detected as X-ray sources, and two others were not. Again the issue is clouded by a lack of information about the binary characteristics of these stars. The Ap star 46 Dra is known to be a member of a close binary, but data are too limited for the other stars to determine whether or not they are double. Despite this uncertainty, Cash and Snow believe that their X-ray observations support the contention that the atmospheres of some A-type stars are not static, inasmuch as X-ray emission has been detected in a significant fraction of the A-type stars surveyed so far, and such emission is taken as evidence for deposition of mechanical energy of some kind. There is evidence from IUE data for extended high-temperature atmospheres around a few Bp and Ap stars (Aydin and Hack, 1978; Stalio et al., 1978; Rakos et al., 1981), but many more observations are required before any conclusions can be drawn about the prevalence and strength of stellar winds in these stars. It is, of course, true that conventional theories of radiation pressure-driven winds do not predict that mass loss should occur in stars of such low luminosity.

It may seem surprising that most Ap stars with strong magnetic fields do not appear to be X-ray sources. Perhaps *both* turbulent motions *and* magnetic fields are required to provide the necessary heating. Another factor may be the difference in the character of the magnetic fields in solar-type and Ap stars. In the Sun, heating of the corona is associated with small-scale magnetic structures with closed field lines. The conventional picture of Ap stars involves large-scale dipolar structure with open field lines.

The feasibility of explaining the X-ray emission of A-type stars in terms of heating by acoustic flux has been explored by Fontaine and Villeneuve (1981). They argue that the acoustic flux depends very sensitively on the value adopted for the convective efficiency and that the range of likely values brackets the minimum acoustic flux needed to account for the X-ray emission of Sirius A. Acoustic heating cannot, however, explain the large ratio of X-ray to visible luminosity that is seen in many K and M dwarfs, and so a different mechanism must be involved in late-type stars (see also Renzini et al., 1977).

The discovery of X-ray emission from A-type stars is very recent, and only the first tentative steps have been taken toward developing models to account for that emission. Since the source of coronal heating in the Sun is not yet well understood, it is not surprising that an unambiguous identification of the relevant nonthermal processes in early-type stars still eludes us.

The detection of X-ray emission, which is the characteristic signature of a corona, in main sequence A-type stars prompted a careful search for spectral features that might be formed in the transition region between the hot corona and the cooler photosphere. Crivellari and Praderie (1982) have obtained both high (two stars) and low (seven stars) resolution data with IUE in the wavelength range $\lambda\lambda$1400 to 1800. Late A-type stars were selected for this study because their flux drops sharply shortward of λ1700 because of the presence of Si I photoionization discontinuities (λ1520 and λ1680), which reach their maximum strength near T_{eff} = 8000 K. The low-intensity continuum should facilitate the detection of weak emission caused by such characteristic transition region lines as Si IV λ1400, C IV λ1550, and He II λ1640. In fact, Crivellari and Praderie detected none of these lines. The upper limits on the emission measure for the transition region are lower than the value measured for α CMi (F6 IV to V). Crivellari and Praderie suggest two possible reasons for the absence of transition region emission lines in A-type stars. Either the transition region is at a higher temperature than is required to produce Si IV, C IV, and He II, or else the transition region is very thin and the temperature gradient steep so that the emission measure remains low. Obviously, the data available to date are limited, and a great deal of additional effort will be required to determine the structure of stellar transition regions and the changes of that structure with effective temperature.

It is evident from this discussion that the determination of the physical conditions in the outer layers of A-type atmospheres is an exceedingly difficult observational problem. These stars occupy a transition region between stars of type F and later, where emission lines are the hallmark of nonradiative heating, and stars of type B and earlier, where absorption lines of anomalously high ionization states are characteristic. On empirical grounds alone, the strengths of chromospheric, transition region, and coronal features must be weak. Nevertheless, X-ray emission has been detected in A-type stars of all temperatures, although not in all A-type stars of any given temperature. Chromospheric emission has been detected in late A-type stars. Transition regions must therefore also be present, but there is evidence that their structure differs in fundamental ways from analogous regions in solar-type stars. The presence, or even the absence, of specific features can tell us much about the structure of these regions. Since they occupy a crucial portion of the HR diagram where convection is diminishing in importance, studies of A-type stars may provide critical constraints on the roles of rotation and convection in determining the level of activity and perhaps provide some clues to the heating mechanism itself. However difficult, further study of the chromospheres and coronae of A-type stars is clearly warranted.

4

THE MAGNETIC Ap STARS

INTRODUCTION

The problem of the Ap stars, as Bidelman (1967) has stated, is that "stars of unusual spectrum are doing unusual things," and indeed the peculiarities of these objects have been recognized for about three quarters of a century. The prototypical Ap star is α^2 CVn, which is quite bright (V = 2.9) and can be easily observed. The spectrum of α^2 CVn was first classified as peculiar by Maury (1897), who commented on the weakness of the K line and the strength of the Si II doublet at $\lambda\lambda$4128,31. That this star was of particular interest became apparent when Ludendorff (1906) reported that several lines in the spectrum of α^2 CVn varied in intensity. It is curious, in view of the subsequent analyses of α^2 CVn, that the features reported by Ludendorff to be variable, including lines of Fe, Cr, and Mg, are among the lines that vary least in this star. He noted no variations in the Eu lines λ4129 or λ4205, but it is possible that these lines were outside the range of his instrument.

An extensive analysis of α^2 CVn was carried out by Belopolsky (1913), who showed that the line at λ4129 varied in a period of 5.5 days. This feature, and several other prominent lines in the spectrum of α^2 CVn, were attributed to Eu by Baxandall (1913). Belopolsky (1913) also derived the radial velocities of the Eu lines, and his discussion of the measurements is surprisingly close to the modern interpretation. After demonstrating that the radial velocity of the line at λ4129 varied in quadrature with the changes in intensity, Belopolsky commented (translation by Struve, 1942):

> . . . It is difficult to decide wherein to see the cause of this phenomenon. An obvious hypothesis suggests itself, namely that the central body is surrounded by a gaseous satellite or a gaseous ring having a condensation of matter at one point. This hypothesis is supported by the sign of the variable velocities (negative velocities preceding maximum of intensity of λ4129 and positive velocities following maximum), but the details of the observations still present difficulties which may perhaps be cleared up after more material has been accumulated.

The light curve of α^2 CVn was first measured photoelectrically by Guthnick and Prager (1914). A comparison of their results with a modern light curve (Wolff and Wolff, 1971) is shown in Figure 4-1. The two sets of observations have been phased together according to the period derived by Farnsworth (1932); a slightly better fit could be obtained for a shorter value of the period, but the observations of the Eu variations do not allow such a change.

As Figure 4-1 shows, Guthnick and Prager not only derived the correct amplitude for the variability but also discovered the asymmetry in the light curve, with the decline from maximum to

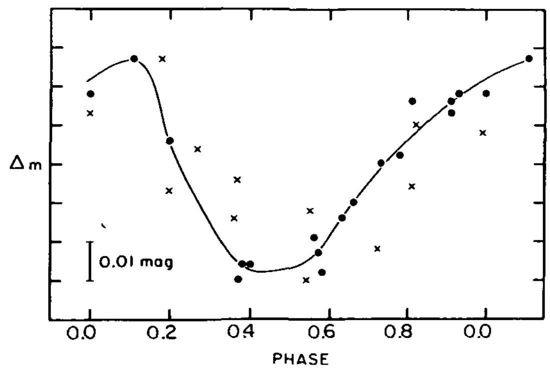

Figure 4-1. Photometric data for α^2 CVn. Crosses represent observations by Guthnick and Prager (1914); filled circles represent observations made with the b filter of the uvby system (Wolff and Wolff, 1971) (from Wolff, 1975).

minimum light occurring more rapidly than the rise from minimum to maximum. The accuracy of the 1914 observations is all the more remarkable because, due to the lack of sensitivity of their equipment, Guthnick and Prager were compelled to use as their comparison star δ UMa, which is more than 20° away from α^2 CVn.

Thus by 1914, it was established that α^2 CVn was a spectrum and photometric variable, that the extrema of the light curve coincided in phase with the extrema of the Eu line strength variations, and that the radial velocity and spectrum variations were in quadrature. Subsequent studies, particularly by Morgan (1933) and Deutsch (1947), showed that these properties are typical of all the Ap variables, although the details of the variations differ from star to star.

RIGID ROTATOR MODEL

No satisfactory model of the Ap stars was developed until after it was discovered that 78 Vir (Babcock, 1947) and most other sharp-lined Ap stars, including α^2 CVn, have variable magnetic fields. The Zeeman observations suggested that a star might possess an axis of symmetry other than the rotation axis, an assumption that is the essential step in formulating the rigid rotator model. This model was first proposed by Babcock (1949) himself:

> . . . It is true that I have suggested as a revised working hypothesis that intense magnetic activity may be correlated with rapid stellar rotation, but at this stage an equally good case can probably be made for the alternative hypothesis that the spectrum variables of type A are stars in which the magnetic axis is more or less highly inclined to the axis of rotation and that the period of magnetic and spectral variations is merely the period of rotation of the star.

Babcock (1960), of course, was never one of the major proponents of this second hypothesis, which is now referred to as the rigid rotator model. Instead, he proposed an empirical model based on the idea that stellar magnetic cycles were analogous to the solar cycle. Solar activity and rotation are thought to be intimately related, and so it seemed not altogether implausible that shorter periods might characterize stars with more rapid rotation and larger magnetic fields. This model was never developed to the point where it could account in any detail for the variety of phenomena associated with Ap stars.

An alternative possibility, the magnetic oscillator model, was suggested by Schwarzschild (1949; see also Ledoux and Renson, 1966). The model invokes nonradial gravity (g) modes to account for variations in magnetic field strength, colors, and radial velocities. This model, however, cannot easily explain the reversals in the polarity of the magnetic field or the wide range of observed periods (0.5 days to decades).

The only model that has been developed to the point where it can directly confront the observations is the rigid rotator or oblique-dipole rotator model. This model was developed by Stibbs (1950), who was unaware that Babcock had proposed and simultaneously dismissed it. The basic premise of the model is that, in analogy with the Sun and Earth, the axis of the stellar magnetic field, which is taken to be dipolar, is inclined at an angle β with respect to the rotation axis. The field is assumed to be locally constant, i.e., "frozen in" to the surface of the star and to corotate with it. To a distant observer, the magnetic field will, therefore, appear to vary in intensity as the star rotates (see Figure 4-2). The spectrum variations can be explained by assuming that the variable elements are concentrated in patches on the stellar surface. This hypothesis naturally accounts for the phase relationship between radial velocity and line intensity seen in α^2 CVn and other Ap stars. Stibbs worked out mathematically the consequences of the rigid rotator model and showed that it successfully explained the variations of HD 125248.

Over the years, the rigid rotator model has successfully met every observational challenge, including some that were quite unanticipated when

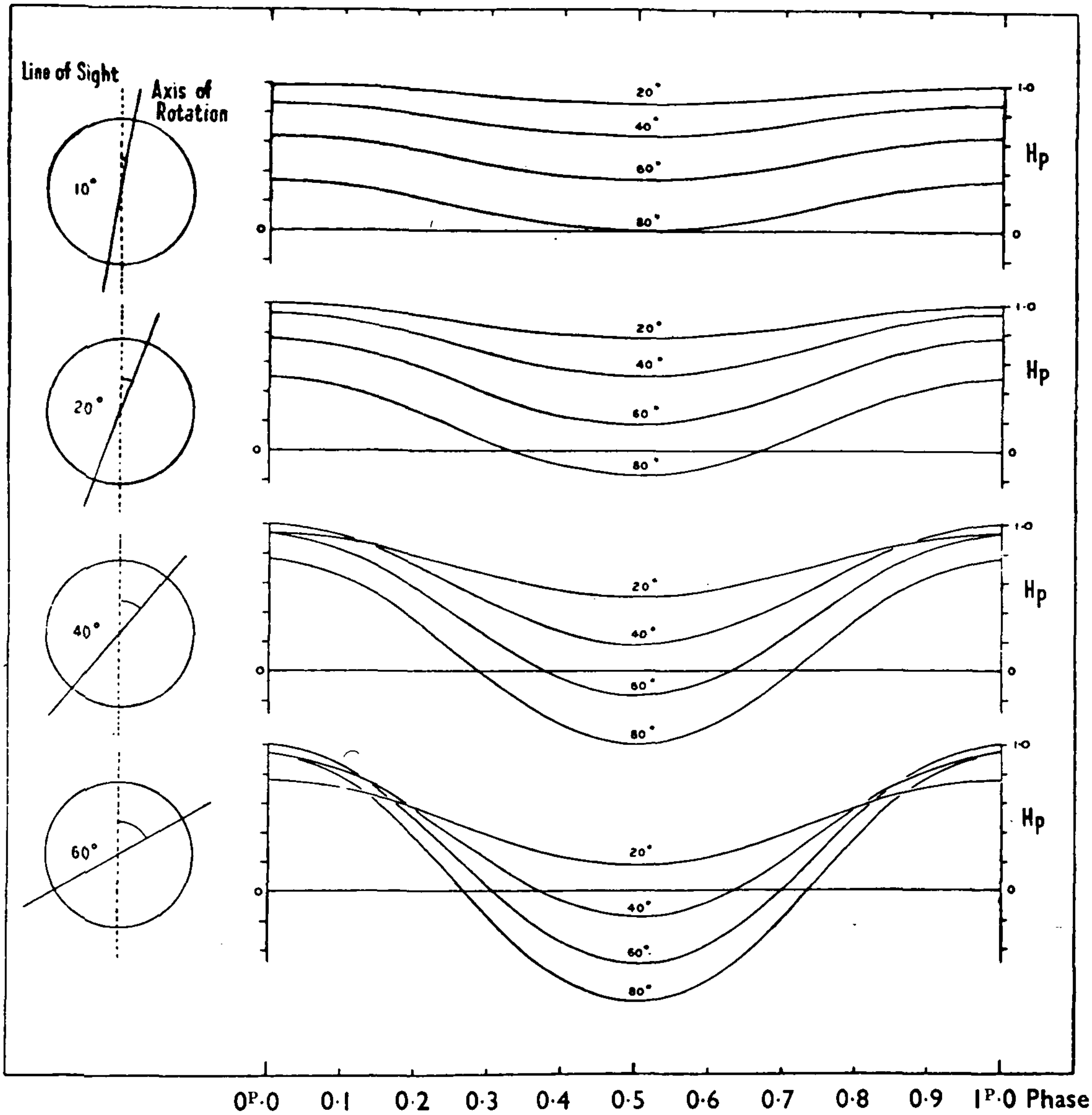

Figure 4-2. Apparent magnetic variations during the rotation of a star when viewed obliquely to the axis of rotation, the magnetic and rotational axes not being coincident. The four curves shown for each inclination to the line of sight are for colatitudes 20°, 40°, 60°, and 80° of the magnetic pole (from Stibbs, 1950).

it was originally proposed. Throughout this chapter, observational evidence in support of this model will be discussed. The model itself, however, is presented at the beginning of the chapter because, if it is kept in mind, a morass of otherwise confusing observational detail can be easily understood.

SPECTRAL CLASSIFICATION

In a pioneering study, Morgan (1933) showed that the Ap stars could be sorted into groups according to their predominant spectral peculiarity and that there was a relationship between the color of a star and its most conspicuous peculiar element. Morgan's work was extended by several other researchers, including, most notably, Jaschek and Jaschek (1958), who defined six groups of Ap stars—the λ4200-Si, Mn, Si, Si-Cr-Eu, Eu-Cr-Sr, and Sr groups. The λ4200-Si stars were so-called because for many years the high-excitation line of Si II at λ4200 remained unidentified (Bidelman, 1962). The Mn stars now are generally no longer included in a discussion of Ap stars, since subsequent research has shown that they differ in a number of fundamental ways from members of the other five categories defined by the Jascheks (Preston, 1971b). There are no confirmed variations in the brightness or spectrum of Mn stars. Magnetic fields have not been definitely detected in any Mn stars. (The report of a small magnetic field in κ Cnc [Preston et al., 1969] is almost surely incorrect.) The abundance anomalies of the Mn stars are quite different from those seen in the remaining five groups of Ap stars; overabundances of both Si and Mn are virtually never seen (Wolff and Wolff, 1974), and rare earth lines have not been detected in the Mn stars. Finally, the frequency of spectroscopic binaries among the Mn stars is quite similar to that of nonpeculiar stars (Wolff, 1978; Wolff and Preston, 1978a), while the binary frequency of the other five classes of Ap stars is extremely low (Abt and Snowden, 1973). Because of these differences, the Mn stars will be omitted from the following discussion of Ap stars, and in this book the nomenclature "Ap" will refer only to stars with enhanced lines of Si, Cr, Sr, and/or rare earths. (It is probable that the fundamental reason for the marked differences between Mn and Ap stars of similar temperature is the absence of a magnetic field in the former. Wolff and Wolff [1976] have presented a more extensive discussion of this issue.)

If the Mn stars are excluded from consideration, then the sequence of categories (λ4200-Si, Si, Si-Cr-Eu, Eu-Cr-Sr, and Sr) correlates in a statistical sense with color. The λ4200-Si stars are the bluest, and therefore presumably the hottest (Jaschek and Jaschek, 1967). The distribution of peculiarity types as a function of U–B is illustrated in Figure 4-3. The classifications are by Osawa (1965), and the Cr group also includes stars with enhanced Eu and/or Sr lines.

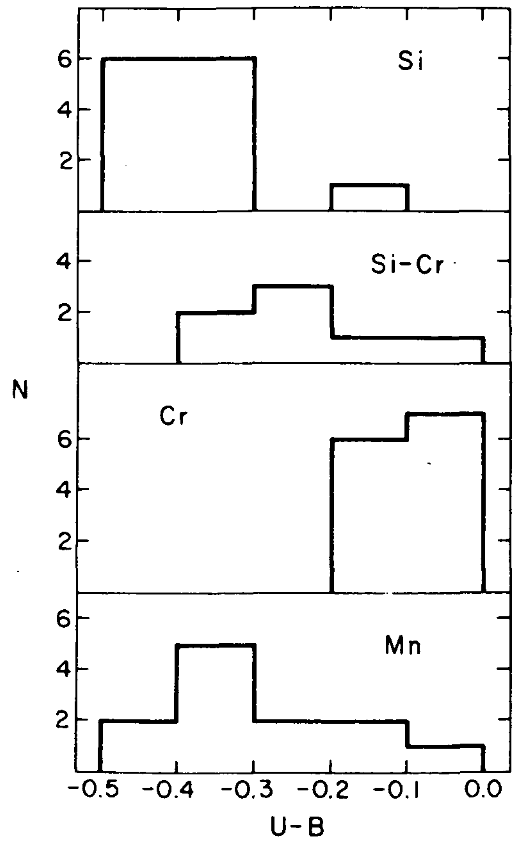

Figure 4-3. Number of Ap stars in various intervals of (U–B). All of the Ap stars for which (U–B) > 0 belong to the chromium group (from Wolff, 1968).

The correlation between color and spectral peculiarity, however, is only statistical. Stars with very different spectra may have identical colors. Indeed, the spectra of the Ap stars are bewildering in their diversity. With increasing spectral resolution, ever more categories seem to be required to provide an adequate classification system. In his survey at 60 Å mm^{-1}, Osawa (1965) divided 200 Ap stars into 16 different peculiarity classes. At still higher resolution (<10 Å mm^{-1}), it may be that no two Ap stars are exactly alike. Some indication of the range of line strengths observed can be obtained from the list of equivalent widths given by Adelman (1973b) for 21 cool Ap stars. Extensive line lists for individual stars also serve to emphasize the lack of detailed similarity among Ap stars (cf., Adelman, 1974a, 1974b, 1974c; Pyper, 1976; Rice, 1977; Adelman et al., 1979; and numerous others).

In an attempt to produce order from chaos, Cowley and coworkers have recently used statistical techniques to determine which elements are present in Ap stars and to search for correlations of the abundances of individual elements with one another. The basic technique used in the statistical approach to line identifications has been decribed by Hartoog et al. (1973). A list of measured stellar wavelengths is compared with laboratory wavelengths for a specific element. Agreement of a stellar and laboratory wavelength within a predetermined tolerance limit is taken to be a coincidence. The number of coincidences that would be expected simply on the basis of chance is estimated by generating sets of control or nonsense wavelengths and comparing those with the stellar line list. By making such comparisons a large number of times, it is possible to estimate the mean number of chance coincidences $\langle H \rangle$, and the standard deviation σ, of the distribution around $\langle H \rangle$. Hartoog et al. then define the significance S of any proposed element identification by the relation

$$S = \frac{H_0 - \langle H \rangle}{\sigma} , \qquad (4\text{-}1)$$

where H_0 is the number of coincidences with laboratory wavelengths.

An additional check on any proposed identification can be obtained by an examination of the distribution of wavelength residuals defined by

$$\Delta\lambda = \lambda_{\text{lab}} - \lambda_{\text{star}} . \qquad (4\text{-}2)$$

For a secure identification one would expect the distribution of $\Delta\lambda$ to peak at $\Delta\lambda = 0$, after correction for the stellar radial velocity.

One problem with this statistical approach is that it does not work well for atomic species with a few strong lines. It is also difficult to incorporate the information contained in the line intensities. On the other hand, it is impersonal, can be carried out automatically, and provides a quantitative figure of merit for each proposed identification. It may be the only practical method for dealing with a sample of Ap stars large enough to search for correlations of line strengths of various ionic species with one another. For a few stars, element identifications have been made both with conventional techniques and with the statistical technique developed by Hartoog et al. In general, there is good agreement between the two methods.

A summary of the early results obtained with the statistical method for a number of Ap, Am, Mn, and normal stars has been presented by Cowley (1976), who also discusses some of the practical problems associated with an analysis of this kind. The primary result by Cowley and coworkers is detailed confirmation of the dissimilarity of Ap star spectra. For example, in γ Equ, Nd is quite pronounced; in 10 Aql, a star which various spectroscopists have described as nearly identical to γ Equ, Nd is very much weaker. In contrast, Ce II is of comparable strength in the two stars. Sm II is strong in 10 Aql but weak in β Cr B. It is unusual to find Nd II and/or Sm II strong when Ce II is weak or absent, but this effect has been seen in 49 Cnc, in HD 25354 at one phase (but not at another), and in HD 51418. In β Cr B and HR 7575, Nd II and Sm II are weak, but Gd II is strong. Precisely the opposite is seen in HD 25354. All three elements are strong in γ Equ and in HR 465 at rare earth maximum. In HD 8441, Cowley and Henry (1979) find that Gd II is the only lanthanide that appears to be strong. The list of dissimilarities is endless.

The paper by Cowley and Henry also offers an interesting statistical approach to the problem of determining the degree to which Ap stars resemble one another. Specifically, they find that Ap stars that appear to be similar on the basis of their lanthanide spectra are not closely related in terms of the spectra of Fe peak elements. Obviously, any model for the origin of the line strength anomalies will have to account for their diversity from star to star.

Other results of the statistical element identification procedures are the absence of any transuranic actinide (Cowley et al., 1976); the possible presence of U II in a number of Ap stars (Cowley et al., 1977); and the low significance that must be attached to any possible identification of promethium (Hartoog et al., 1973, and references therein).

TEMPERATURES AND LUMINOSITIES

Determinations of the effective temperatures and absolute magnitudes of Ap stars are subject to substantial uncertainties. The anomalously strong absorption lines distort the appearance of the spectrum, so that the usual spectral classification criteria are invalid. Line absorption distorts the measured colors as well, so that the relation between color and T_{eff} that obtains for normal stars is not applicable.

It has been known for a long time that temperatures inferred spectroscopically disagree with those implied by such color indices as B–V. Most magnetic stars have been assigned spectral types of A0p or later. Indeed, it is on the basis of spectral classification that this group of stars has come to be referred to as Ap stars. The colors of many A0p stars, however, are bluer than the colors of normal stars of the same spectral type. Deutsch (1947), for example, found that A0p stars correspond in color and luminosity approximately to normal stars of type B8. He suggested that perhaps the helium lines were abnormally weak in Ap stars. Since the intensities of the helium lines are a major classification criterion, an apparent underabundance of helium would lead to the assignment of erroneous spectral types. The mechanism responsible for suppressing the helium lines and enhancing certain metallic lines characteristic of Ap stars was, of course, unknown at that time.

If Deutsch's hypothesis is correct, then the photometric colors should provide a better estimate of the temperatures of Ap stars than do spectral types. Provin (1953) and Abt and Golson (1962) showed that the Ap stars lie close to the locus in (U–B, B–V) plane defined by normal stars provided B–V $\leq$ +0.05 (Stepien and Muthsam, 1980), a result that lends support to the use of colors to derive temperatures for Ap stars. It is true that line blanketing may affect the colors, but only in a few of the coolest and most peculiar stars does line blanketing in the visible redden (B–V) by more than 0.05 (Baschek and Oke, 1965; Wolff, 1967). Visible spectrophotometry yields temperatures and surface gravities for the Ap stars that are like those of normal stars of spectral types B3 to A8.

This conventional viewpoint has been challenged by Leckrone (1973; see also Jamar et al., 1978; van Dijk et al., 1978). He finds that the close similarity in the visible region of the spectrum between the Ap and normal stars of similar UBV colors does not persist into the ultraviolet ($\lambda < 2500$ Å). Rather, the Ap stars are less luminous in the ultraviolet than normal stars of the same UBV colors, and the flux deficiency increases with decreasing wavelength from ~0.5 mag at λ 2460 to a magnitude or more at λ 1500 (see Figure 4-4). The ultraviolet flux deficiency is presumably caused by enhanced line and continuum sources of opacity, which are present throughout the region $\lambda < 3000$ K.

Two temperatures are relevant in discussing Ap stars, and the distinction must be kept in mind. The first is the temperature of the line-forming region, and this is the temperature that must be used in atmospheric modeling and abundance analyses. The second temperature is T_{eff}, which is a measure of the total luminosity of the star.

There is evidence that the line-forming regions are characterized by the temperatures that are derived from UBV colors (Preston, 1974, and references therein). The Si II/Si III ionization balance

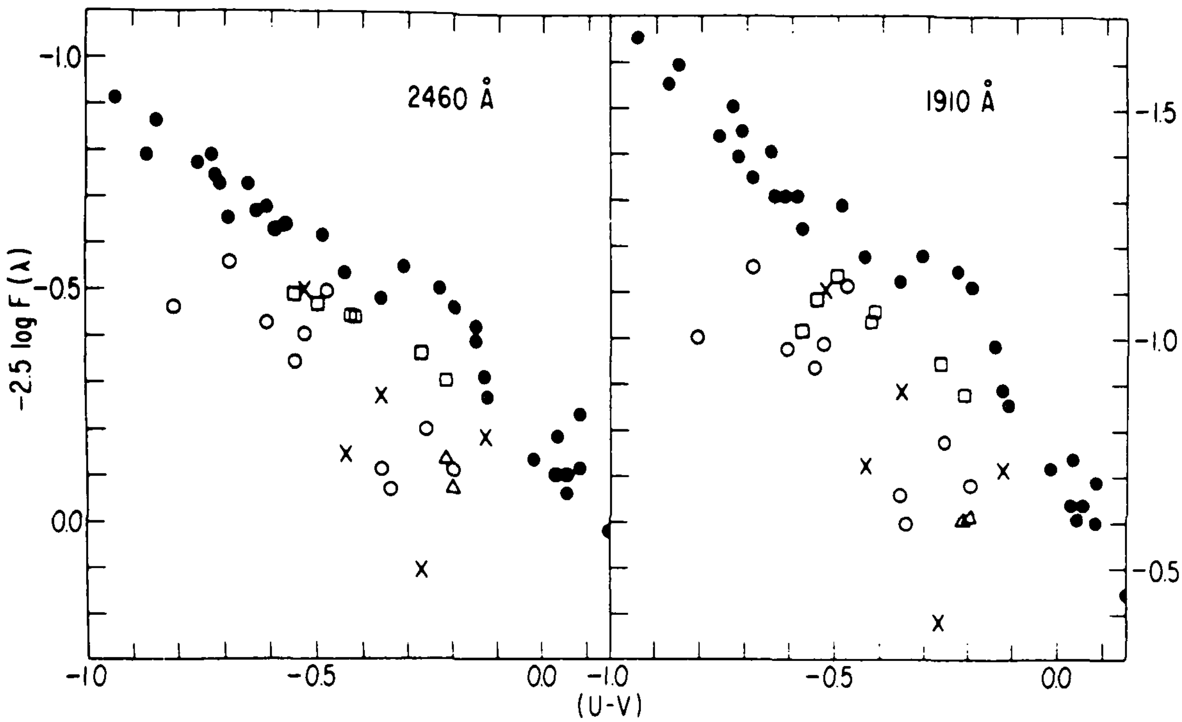

Figure 4-4. Normalized absolute fluxes in magnitude at 2460 Å and 1910 Å plotted versus U−V for various classes of Ap stars and for comparison standard dwarfs and giants. Darkened circles, normal stars. Open circles, Si 3955, 4200 stars. Crosses, other Si stars. Open squares, HgMn stars. Open triangles, Sr-Cr-Eu stars. Ap stars are deficient in −2.5 log F(λ) compared to normal stars of similar U−V. HgMn stars appear less flux deficient, for their U−V colors, than do Si or Sr-Cr-Eu stars (from Leckrone, 1973).

is in accord with the high temperatures implied by visible spectrophotometry. The variation in hydrogen line strengths with UBV colors is the same for normal and Ap stars. Similarity of temperatures inferred from photometry and spectroscopy in the visible region of the spectrum is not altogether surprising, since the depths of formation are not very different. Detailed model atmosphere analyses of the emergent spectrum must ultimately, however, take into account the fact that the distribution of flux with wavelength in an Ap star differs from that of a normal star characterized by the same temperature, as inferred from visible colors, hydrogen line profiles, and ionization balance in the line-forming regions. Spectrophotometric studies by Adelman (1981, and references therein) and collaborators show that standard models simply cannot reproduce the observed energy distributions in detail.

The effects of strong line blanketing on the emergent spectra of Ap stars and on their atmospheric structure have been explored in a series of papers by Muthsam (1979), Muthsam and Stepien (1980), and Stepien and Muthsam (1980). Models were calculated for plane-parallel atmospheres in hydrostatic and radiative equilibrium. Except for some limited test models, local thermodynamic equilibrium was assumed throughout. The key departure for these models was the inclusion of the opacity produced by 900,000 lines (Kurucz and Peytremann, 1975) and abundances representative of a "typical" Ap star. The line list includes primarily elements up to Z = 30; the laboratory data available for heavier elements, including the rare earths, is quite limited, and adequate treatment of these elements is not possible at the present time. The problems associated with a detailed comparison with specific Ap stars should already be apparent. There is no "standard" composition for Ap stars; abundances of specific elements may differ by orders of magnitude from one Ap star to the next. The composition of an individual Ap star is usually not well known. From the observations of spectrum variability, it is clear that the composition of a given star is not uniform over the stellar surface; composition may depend on optical depth as well, but there is little direct observational information on this issue.

Despite these limitations, Muthsam's calculations do clarify the role of blanketing and its effects on the atmospheric structure of Ap stars. In agreement with the observations by Leckrone (1973), Muthsam (1979) finds that an Ap star atmosphere mimics that of a normal star with a higher effective temperature. The $T(\tau)$ relationship is also found to be steeper in heavily line-blanketed Ap stars than it is in normal stars. The ultraviolet fluxes in the blanketed models are lower than the flux from normal stars, but the models do not reproduce the observations well, possibly because of inadequate atomic data or omission of relevant opacity sources, particularly shortward of $\lambda 1500$.

One critical point demonstrated by Leckrone's (1973) estimates of T_{eff} and by Muthsam's (1979) model atmospheres is that the total flux emitted by an Ap star is lower than that of a normal star with the same UBV colors. The Ap stars therefore are less luminous and have lower values of T_{eff} than their visible colors imply. An alternate method of determining effective temperatures has been described by Shallis and Blackwell (1979, and references therein). Let F_{E} be the total integrated flux from the star as measured at the earth and θ be the angular diameter of the star. Then

$$F_{\mathrm{E}} = \int_0^\infty F_{\mathrm{E},\lambda}\, d\lambda = \frac{\theta^2}{4}\, \sigma T_{\mathrm{eff}}^4 \,. \qquad (4\text{-}3)$$

At a specific wavelength λ_0,

$$F_{\mathrm{E},\lambda_0} = \frac{\theta^2}{4} F_{\mathrm{s},\lambda_0} = \frac{\theta^2}{4}\, \phi\, (T_{\mathrm{eff}}, g, \lambda_0) \ , \qquad (4\text{-}4)$$

where g is the surface gravity. If λ_0 is an infrared wavelength, then the function $\phi\ (T_{\mathrm{eff}}, g, \lambda_0)$ is fairly independent of the details of the specific model atmosphere used to evaluate it. In the case of Ap stars, both line blocking and backwarming, due to redistribution of ultraviolet flux, should be at a minimum in the infrared. Use of infrared magnitudes will, therefore, minimize the errors introduced by using models that do not incorporate these effects explicitly. Simultaneous solution of these two equations then yields values for θ and T_{eff}.

This technique has been applied by Shallis and Blackwell (1979) to three Ap stars, one Am star, and one HgMn star. These authors conclude that the values of T_{eff} are lower than their UBV colors imply, a result in agreement with the work of Leckrone. If accurate parallaxes are available, then linear radii can be estimated from the values of θ. Shallis and Blackwell find that the two Ap stars with good parallaxes have larger radii than do main sequence stars of comparable spectral type. Because properly blanketed models were not available, however, the significance of this result is unclear. Shore and Adelman (1979) have used the Barnes-Evans relationship between surface brightness and angular diameter (see Chapter 2) to obtain radii for Ap stars. While the radii derived from the Barnes-Evans relation agree *on the average* with values derived from the oblique rotator model, the two values for individual stars may differ by more than the observational error. Shore and Adelman suggest that these discrepancies may be caused by inadequate treatment of line-blanketing and flux redistribution (see also Muthsam and Weiss, 1979).

Radii of Ap stars can be estimated in two other ways: from their bolometric magnitudes and effective temperatures (Stift, 1974) or from the rigid rotator model. In the latter case, the following relationship holds

$$R \sin i = \frac{P\, v \sin i}{50.6} \ , \qquad (4\text{-}5)$$

where R is the radius in units of the solar radius,

P is the period in days, v is the equatorial rotational velocity in km s^{-1}, and i is the angle of inclination of the rotation axis to the line of sight. Since i is unknown, R can be derived in a statistical sense only (Preston, 1970b; Wolff, 1975a). The radii of Ap stars estimated in these two ways are larger than the radii of normal stars on the zero-age main sequence (ZAMS). In contrast to the radii derived by Shallis and Blackwell, however, they are probably not so large as to exceed the values expected for evolved stars that still lie within the main sequence band.

A fundamental property of the Ap stars that one would like to determine is their evolutionary status. Are they on the main sequence for the first time? Or are they highly evolved objects (e.g., Fowler et al., 1965)? An early argument in favor of the first hypothesis was that Ap stars in clusters appeared to lie on or close to the main sequence band defined by normal stars (e.g., Hyland, 1967). The precision of this statement, however, is limited because of the abnormal flux distributions of Ap stars, and recent studies have shown that Ap stars can deviate substantially from cluster main sequences. Hartoog (1976) has used models with enhanced metal abundances (Leckrone et al., 1974) to estimate the changes in B–V and M_v because of changes in composition. He finds that, relative to a star of normal composition, an Ap star may appear bluer by 0.1 mag and fainter in M_v by a magnitude or more. The fact that not all Ap stars in clusters appear subluminous is an indication that the effects of flux redistribution on the emergent spectrum are too complex to be accounted for entirely by the preliminary models of Leckrone et al.

There are other indications that the Ap stars are on the main sequence for the first time. Peculiar stars have been found in such young groups as the Orion and Scorpio-Centaurus associations, where there has been insufficient time for evolution of any but the most massive stars. Masses are known for two Ap stars and appear to be normal for their position on the main sequence (Abt et al., 1968; Bonsack, 1976). And, as noted above, the radii inferred for Ap stars from their rotational periods are compatible with models of stars on, or slightly evolved away from, the main sequence. The mean absolute magnitudes as determined from statistical parallaxes are also found to be the same for Ap and normal stars of the same UBV colors (Grenier et al., 1981).

FREQUENCY

The frequency of Ap stars among field stars has been estimated by several authors (e.g., Jaschek and Jaschek, 1967; Wolff, 1968; Hartoog, 1976; and Abt, 1979). There are uncertainties in these estimates, mainly because of the problem of establishing a sample of normal stars matched in luminosity and temperature. The question of whether or not the Ap stars are somewhat overluminous for their mass remains open (Bonsack, 1976). Because they are helium deficient, spectral types are a poor way of estimating temperature for hot Ap stars. Colors may be distorted by ultraviolet line blanketing in the hot Ap stars (Leckrone, 1973), by line blocking in the visible in cool Ap stars (Wolff, 1967), and by broad absorption features like the one at λ5200 in Ap stars of all temperatures. There may even be problems with the definition of a peculiar star. Durrant (1970) and Megessier (1971) have both claimed that the strengths of the Si lines form a continuum. There is apparently no sharp distinction between normal and Ap stars with respect to Si line strengths.

Despite these problems, the various estimates of the frequency of Ap stars are in generally good agreement, and results of one such survey are shown in Table 4-1, which was taken from Wolff (1968). The frequency is a maximum for colors that correspond (in normal stars) to spectral types B8 to A0 and drops sharply toward both earlier (hotter) and later (cooler) types. This table omits the He-weak Bp stars, which apparently form a continuation of the magnetic Ap stars to higher temperatures. Because the Bp stars are difficult to identify spectroscopically, their frequency is uncertain but is probably lower than that of Ap stars (see Chapter 8).

There have been several attempts to estimate the frequency of Ap stars in galactic clusters (e.g., Young and Martin, 1973; Hartoog, 1976; Abt, 1979), and work on this problem continues (Maitzen, 1981; North and Cramer, 1981). The primary

Table 4-1
Ratio of Ap Stars to Normal Main Sequence Stars

Color Index	$\frac{\text{Ap}}{\text{A (normal) + Ap}}$
$-0.19 \leq B-V \leq -0.10$	0.06
$-0.09 \leq B-V \leq 0.00$	0.13
$+0.01 \leq B-V \leq +0.10$	0.05
$+0.11 \leq B-V \leq +0.20$	0.01

uncertainties are twofold. First, despite extensive observational effort, the sample size remains quite small. Second, it is often difficult to determine whether or not an Ap star is actually a cluster member. Proper motion and radial velocity studies are rarely available, and because the Ap stars can be strongly affected by line blanketing, their positions in a color magnitude diagram are not a reliable indication of cluster membership. Despite these uncertainties, existing surveys agree that the frequency of Ap stars is lower in clusters and associations, perhaps by as much as a factor of 2, than among field stars.

A possible explanation of this effect has been offered by Abt (1979), who finds that the frequency of Si stars in clusters increases with age, and in the clusters older than 10^7 years reaches the same frequency as is observed among field stars. Abt finds no SrCr stars in young clusters, but their frequency in clusters older than 10^8 years is equal to that among field stars. Hartoog (1976) earlier found marginal evidence for an age dependence of the frequency of Si stars but concluded, in contrast to Abt's results, that the frequency of cool (SrCr) Ap stars was independent of age. Data by Maitzen (1981) support Hartoog's conclusions and, in addition, show that the strength of the peculiarities is uncorrelated with age. However, in all cases the sample sizes are so small that conclusions about the relationship, if any, between spectral peculiarities and age are scarcely secure.

Measurements of the $\lambda5200$ feature have been used to identify Ap stars among blue stragglers in open clusters (Maitzen et al., 1981). Approximately 8 percent of the blue stragglers are Ap stars, a frequency comparable to that in the field and in old open clusters (Abt, 1979).

SPECTRUM VARIABILITY

Spectrum variations in Ap stars were first noted three quarters of a century ago (Ludendorff, 1906), and studies of the changes in line intensities have continued to the present day (e.g., Floquet, 1979). Approximately half of the Ap stars show conspicuous spectrum variations (Bonsack, 1974), and an important early survey of their variable characteristics was carried out by Deutsch (1947). His study established many of the properties of the spectrum variations. A number of the stars were shown to vary periodically with periods of several days. (We now know that in any given star the periods of the spectrum, magnetic, and photometric variations are identical and may be as long as several years.) Deutsch also showed that not all elements vary in phase, although lines of a given ion always appear to vary together. The variable lines usually include those strong features that are characteristic of peculiar stars, including Sr, Cr, and Eu. The variation of Si II is, according to Deutsch, generally not pronounced. The phase relationships of the various elements are not the same from star to star. For example, in α^2 CVn, he found that Eu II and Ca II vary in phase, while in HD 125248 they vary out of phase. Deutsch cites many more examples of the diverse behavior of various stars, and observations during the past 35 years have only served to underscore this diversity.

Perhaps the most complete study of any Ap star is the analysis of α^2 CVn by Pyper (1969). In α^2 CVn, the equivalent widths of the rare earth lines vary periodically with $P = 5.46939$ days, and this same period applies to the variations in brightness and magnetic field. Figure 4-5 shows the phase relationship between the changes in line strength and radial velocity for the rare earths. Note that the extrema of the two curves do not coincide.

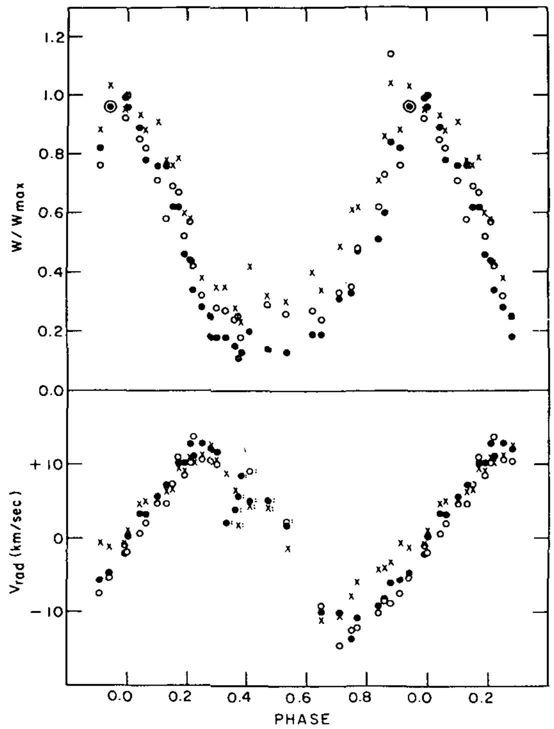

Figure 4-5. Equivalent widths and radial velocities for the following: filled circles, Eu II; crosses, Gd II; and open circles, Dy II. For a given ion at a given phase, the plotted point is the average value for all lines of that ion. For each line, the measured equivalent width has been divided by the maximum equivalent width in the cycle (see text) (from Pyper, 1969).

Line strength maximum occurs when the radial velocity is passing through its average value. This phase relationship is characteristic of Ap stars in general and is precisely what the oblique rotator model predicts. If the rare earth elements in α^2 CVn are concentrated in a single patch or spot on the stellar surface, the line strength maximum will occur when this patch is centered on the central meridian of the visible hemisphere. The net radial velocity as determined from the rare earth lines at this time will be zero. Prior to the phase of maximum line strength, the patch will have a net radial velocity toward the observer as it rotates over the approaching limb toward the central meridian. The opposite will occur as the line strengths decline. In α^2 CVn, and in the other Ap stars for which detailed measurements are available, the amplitude of the velocity variation is compatible with the period and radius of the star.

The behavior of the Fe peak elements (Ti, V, Cr, Mn, and Fe) in α^2 CVn is more complex. Figure 4-6 shows the changes in radial velocity, while Figure 4-7 illustrates the variations in line strength. The radial velocity curves are discontinuous. Four times during each cycle, the radial velocity increases from negative to positive values. During much of the cycle the lines are actually double. The variations in equivalent widths show similar discontinuities, and in each case, maximum line strength coincides in phase with mean radial velocity. On the oblique rotator model, these data imply that there are four distinct concentrations of Fe peak elements on the stellar surface. Multiple spots have been reported for a few other stars (e.g., Bonsack and Wallace, 1970; Megessier et al., 1979); but, in general, observations have been able to resolve only one or two abundance maxima.

Two techniques have been used to map the distribution of elements over the stellar surface. The first method was developed by Deutsch (1958a, 1970; see also Pyper, 1969). This approach assumes that the local equivalent width on the surface of the star can be represented by a development into spherical harmonics. The coefficients of this series expansion are then related to the coefficients of the Fourier series that describes the observed variations of equivalent width. Radial velocity and magnetic variations can be treated in a similar fashion. This technique has been applied by Deutsch (1958a) to HD 125248, by Pyper (1969) to α^2 CVn, and by Rice (1970) to HD 173650. Pyper's results for α^2 CVn are shown in Figures 4-8 to 4-10. The maximum abundance of the rare earths occurs near the negative magnetic pole, while the Fe peak elements occur in four patches distributed approximately along the magnetic equator. Some modifications to this approach have been suggested by Falk and Wehlau (1974) and Mihalas (1973).

An alternate method for determining the surface distribution of elements has been proposed by Khokhlova and coworkers (see Megessier et al., 1979, and references therein). The number of

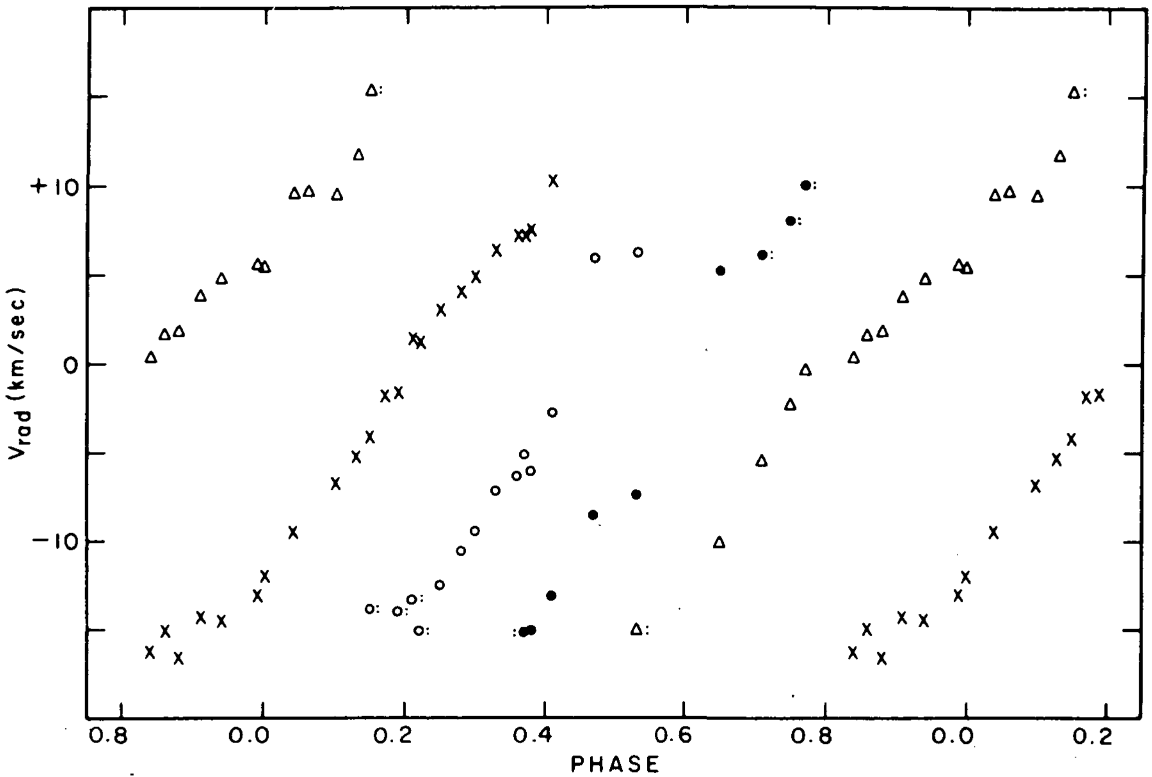

Figure 4-6. Variations in radial velocity of the iron-peak elements. Each plotted point is the mean value of v_{rad} *for Ti II, Cr II, and Fe II. Crosses, open circles, filled circles, and triangles represent data for individual concentrations or patches of material (from Pyper, 1969).*

spots on the star and their approximate longitudes are determined from the observations. One then assumes a model in which the sizes, abundances, and latitudes of the spots are also specified and calculates observed line profiles by integrating over the visible disk. The results of the calculations are then compared with observations, the model is modified and the calculations repeated until the observations are adequately represented.

Unfortunately, it is difficult to evaluate the accuracy, or even the uniqueness, of the derived maps of the distribution of elements over the stellar surface. The harmonic analyses depend on the shapes of the variation curves; and particularly for the magnetic field, these shapes are subject to substantial uncertainties caused by the inaccuracies of photographic photometry. While this problem may be solved with the availability of new photoelectric detectors, Megessier et al. (1979) have shown that the resulting maps will not be unique. The latitudes of the spots, their areas, and abundances are linked. A change in one quantity can be matched with compensating changes in the others in such a way that the observations can still be reproduced. Only the number of spots and their longitudes seem to be well determined.

This lack of uniqueness means that it is very difficult to relate the surface distribution of elements to the magnetic geometry. Specifically, many authors have assumed that coincidence of rare earth maximum with a magnetic extremum implies that the rare earths are located near the magnetic pole. In fact, without additional information, one is entitled only to conclude that the abundance maximum is located on the same rotational longitude as the magnetic pole. One result that is clear, however, from the observations is that the distribution of elements is usually not symmetric with respect to the magnetic equator.

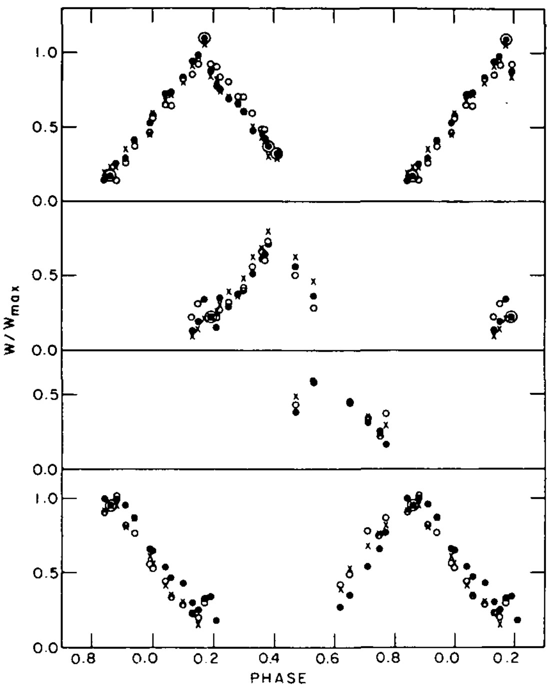

Figure 4-7. Variations in equivalent width of the iron-peak elements; filled circles, open circles, and crosses represent data for Ti II, Cr II, and Fe II, respectively. For each ion, measured equivalent widths are divided by the maximum value for the patch of material represented in the top panel of the figure (from Pyper, 1969).

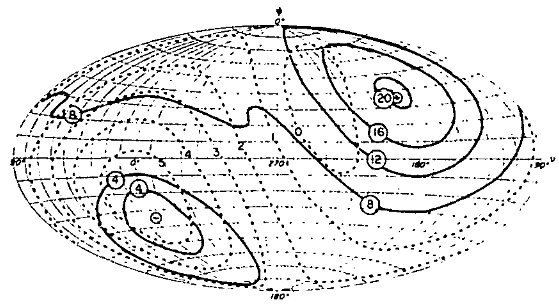

Figure 4-8. The oblique rotator model. Dashed curves, curves of constant local equivalent width for lines of the rare earths (Eu II, Gd II, Dy II). Solid curves, contours of |H|. Plus and minus signs indicate the axis of the magnetic dipole. Aitoff equal area projection (from Pyper, 1969).

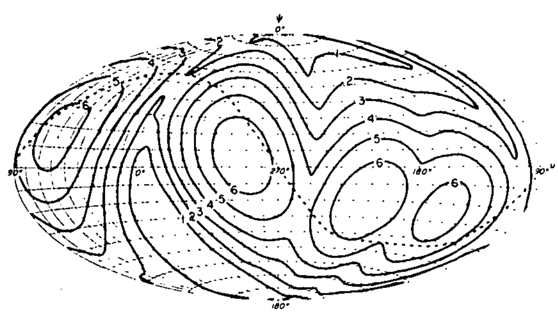

Figure 4-9. The oblique rotator model. Solid curves, curves of constant local equivalent width for the iron-peak elements (Ti II, Cr II, Fe II). Dashed curve, magnetic dipole equator (from Pyper, 1969).

Quite different abundance anomalies may be associated with the two magnetic hemispheres, and the distribution of rare earths on the surface of α^2 CVn is only one example of this common phenomenon.

In addition to variations in line intensity, changes in line profiles are seen in many Ap stars. In those stars in which Zeeman broadening is a significant component of the total line broadening, line widths vary in synchronism with the average surface magnetic field, integrated over the visible hemisphere of the star (e.g., Wolff and Wolff, 1970). Even more conspicuous is the crossover effect, so-called because it is observed when the magnetic field crosses through zero while changing from one polarity to another. The explanation of this effect is owed to Babcock (1956). The observational manifestation of the crossover effect is a difference in line widths in the two senses of circular polarization. The explanation for the difference in line widths is illustrated in Figure 4-11. Whatever the correct model for Ap stars, spectroscopic patches seem to be required in order to account for the crossover effect. One noteworthy success of the oblique rotator model is that it can account for both the order of magnitude and the sense (e.g., whether the right or left circularly polarized lines are broader) of the crossover effect.

Analyses of spectrum variations also provide the best evidence for the long-term stability of Ap stars. While magnetic data and, with a few exceptions, photometric measurements span only an

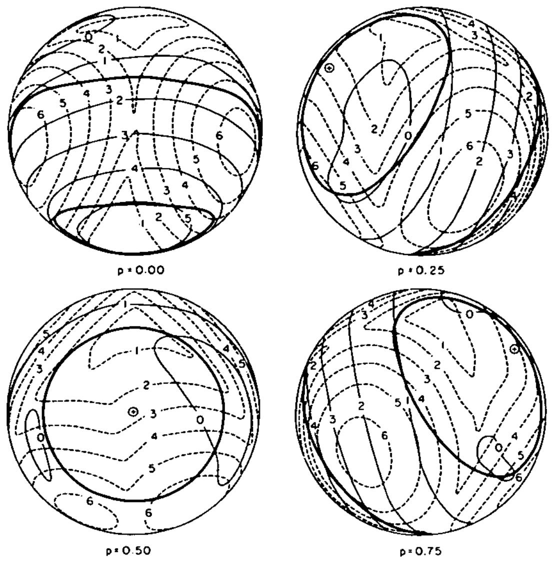

Figure 4-10. The visible hemisphere at four phases. Light solid and dashed curves are contours of the rare earth and the iron-peak elements, respectively. Heavy curve is locus where $H_z = 0$ (from Pyper, 1969).

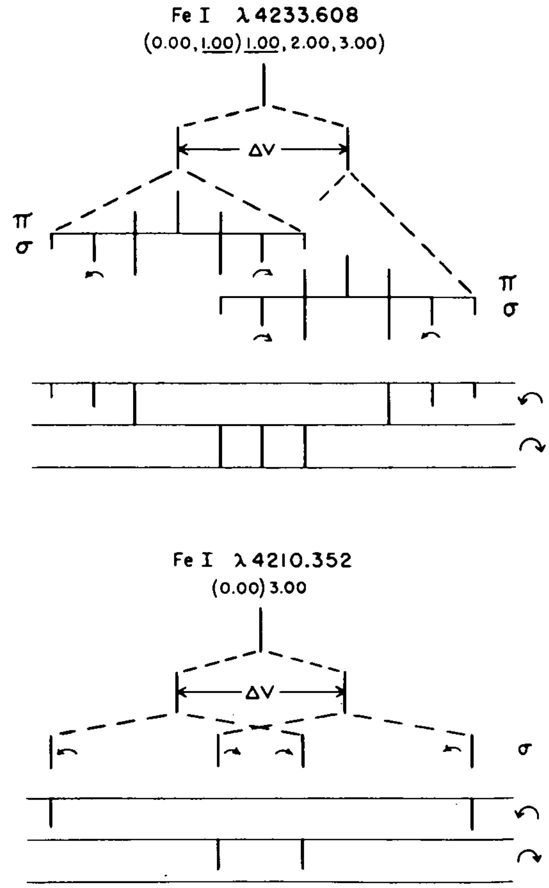

Figure 4-11. Diagrams illustrating the development of the crossover effect. The line is first split by the Doppler effect, corresponding to a differential velocity, Δv, between two areas on the star. If the two areas have longitudinal magnetic fields of opposite polarity, the lines are further split into their Zeeman patterns. The σ components are sorted out by the analyzer according to their sign of circular polarization, leading to a "broad" and a "sharp" profile in the two strips. If the field is not strictly longitudinal, the π components appear in both strips of the spectrum. In general, there occurs considerable overlapping of the profiles represented by the "lines" in the diagrams (from Babcock, 1956).

interval of 30 years, spectroscopic measurements for some Ap stars go back to the early part of this century. Stars like α^2 CVn (Pyper, 1969) and 56 Ari (Bonsack and Wallace, 1970) have been observed over time intervals during which thousands of cycles have elapsed. In no case is there evidence for any change in period or of phase shifts in the spectrum variations. This constancy is in sharp contrast to the Sun and other late-type stars with (presumably) dynamo-generated fields. For these objects, regions of activity tend to migrate due to differential rotation. The Ap stars, however, appear to be truly rigid rotators.

PHOTOMETRY

The anomalous line strengths of Ap stars distort their energy distributions, and this fact can be used to detect peculiar stars through photoelectric photometry. In addition, most Ap stars vary in brightness by a few percent and much of our knowledge of the periods of Ap stars has been derived from photometric observations.

The Ap and Am stars appear essentially normal in the UBV photometric systems. Measurements in these three broad bands do not, by themselves, contain sufficient information to distinguish normal from peculiar stars. However, intermediate- and narrow-band systems can be designed to make the distinction. Only a few of the most widely used systems will be discussed here. For a discussion of other photometric systems and their application to Ap stars, see Adelman (1981).

In 1967 Cameron showed that the m_1 index of the Stromgren uvby system was systematically larger for Ap stars than for normal stars with the same c_1 index. His results are shown in Figures 4-12 and 4-13. The m_1 index is defined by the relation $m_1 = (v-b) - (b-y)$, where the v band is centered at λ4110, b at λ4670, and y at λ5470. Direct measurements of line blocking (Wolff, 1967) show that absorption is very high in the v band. The amounts of line blocking in the b and y filters, however, are similar for a substantial range of abundances, so that the color b–y is fairly insensitive to composition. The sensitivity of m_1 to Ap characteristics is, therefore, probably caused by its response to line blocking near λ4100. The line blocking at λ4100 decreases with increasing temperature, and this fact probably accounts for overlap in the distributions of Ap and normal stars in the c_1–m_1 plane at the highest temperatures (see Figure 4-12).

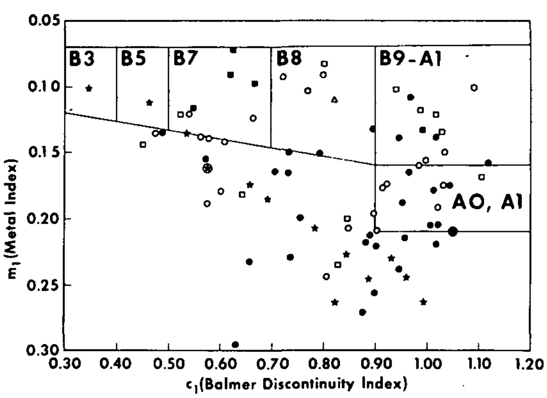

Figure 4-12. The distribution of the hot peculiar magnetic and broad-lined Babcock stars. Labeled boxes refer to regions occupied by nonpeculiar stars of luminosity classes III to V. Legend: star-shaped symbols, strong field magnetic stars; filled circles, other magnetic Ap stars; filled squares, other magnetic Bp stars; open circles, broad-lined Ap stars; open squares, broad-lined Bp stars; open triangle, sharp-lined nonmagnetic Ap stars (from Cameron, 1967).

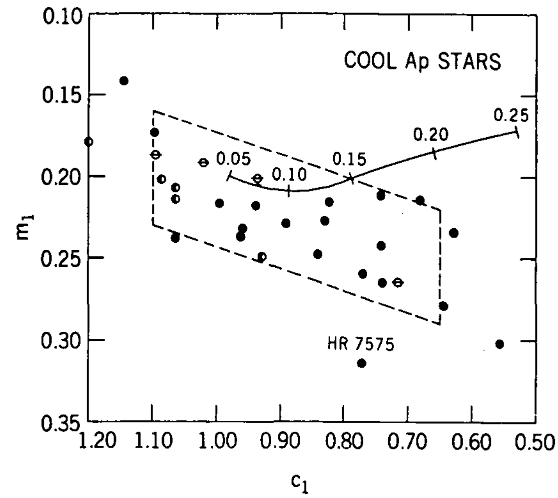

Figure 4-13. Distribution of the cool Ap Babcock stars in the (c_1, m_1)-diagram. The dashed parallelogram indicates the region occupied by Am stars. Nearly all of these stars are known (filled symbols) or suspected (half-filled symbols) to have magnetic fields (from Cameron, 1967).

If a standard filter system can be used to detect Ap stars with some degree of success, then one might expect that it would be possible to define an even more efficient filter system specifically for this purpose. One such system, which has been discussed by Maitzen (1976a, b) and by Maitzen and Seggewiss (1980), makes use of the standard y filter of the uvby system plus a g_1 filter centered at λ5015 and a g_2 filter at λ5240. The half-widths of the g_1 and g_2 filters are ~130 Å. The index $a = g_2 - (g_1 + y)/2$ is larger in Ap stars than in normal stars of the same (b–y). There is a broad depression in the flux of Ap stars in the λ5200 spectral region, and this depression becomes more pronounced with increasing temperature (Cowley, 1981), although there is substantial spread in values of the a index at all temperatures (Pyper and Adelman, 1982). In contrast to m_1, the a index is most effective in discovering hot (Si) Ap stars. Several attempts have been made to identify the cause of the depression in flux. The most recent discussion of this problem is by Maitzen and Muthsam (1980), who conclude that there are in fact two separate contributors to the λ5200 feature. At low temperatures (T_{eff} ~ 8000 K), a broad shallow feature is present that

Maitzen and Muthsam suggest can be explained entirely in terms of enhanced blocking by metallic lines. In possible contradiction to this conclusion, however, Pyper and Adelman (1982) find no correlation for the stars they have studied between line blanketing in the blue-violet and the strength of the broad absorption at λ5200. Direct measurements of metallic line blocking at λ5200 can be made from high-resolution spectrograms and ought, in principle, to be able to resolve this issue. Such measurements, however, can be made only relative to the local continuum and so cannot properly take into account any broad continuous features. At higher temperatures, a fairly sharp ($\Delta\lambda \sim 100$ Å) component is added to the broad feature, and it strengthens with increasing temperature. This sharp component remains unexplained. Its strength is well correlated with an absorption feature seen at λ1400 (Maitzen, 1980), which has been attributed to Si II autoionization lines (Jamar et al., 1978), but Buchholz and Maitzen (1979) have argued that Si II cannot be the only cause of the depression at λ5200 (see also Artru et al., 1981).

Spectrophotometry in the visible region of the spectrum provides the most complete characterization of the broad absorption features. Three conspicuous features have been noted (Kodaira, 1969). The λ5200 feature actually extends from λ4950 to λ5840 (Adelman, 1981). An absorption near λ4200 is also seen in most magnetic stars, but the third feature, which is near λ6300, is seen only infrequently. In many Ap stars, these broad features are variable, and correlations of their changes in strength with other spectrum and photometric variations may ultimately help to determine their origin.

Yet another photometric system that has been widely used in the study of Ap stars is the Geneva system (Hauck, 1976; Adelman, 1981, and references therein). This sytem includes an ultraviolet filter to measure flux shortward of the Balmer discontinuity; three intermediate-band filters cover the wavelength region λ3800 to λ4800; and three more cover the region λ5100 to λ6000. The particular advantage of this system is that it is very sensitive to the broad flux depressions near λ4200 and λ5200. The Geneva system is, therefore, quite useful for discovering and classifying Ap stars (e.g., North and Cramer, 1981).

The first study of photometric variations of the Ap stars was carried out by Guthnick and Prager in 1914. Their light curve for α^2 CVn is shown in Figure 4-1. With improvements in photoelectric instrumentation, fainter Ap stars became observable, and on the basis of an extensive survey, Abt and Golson (1962) concluded that all magnetic Ap stars vary in visible light. A pioneering survey of the variations of individual Ap stars was carried out by Rakos (1962, 1963). The analysis of the UBV variations of 16 magnetic stars by Stepien (1968) is representative of the results obtained with broadband photometry. Amplitudes are typically on the order of a few hundredths of a magnitude but range from the limit of detectability (~0.01 mag) to, in exceptional cases, one tenth of a magnitude or more. After comparing his measurements with earlier observations, Stepien concluded that there was no evidence for long-term changes in the periods, amplitudes, or shapes of the light curves. Like many other observers in this field, Stepien sought correlations of the light curves with other variable properties of the Ap stars; however, the light curves are quite diverse, and correlations are difficult to find. The magnetic and light curves may be either in phase or in antiphase. The amplitudes of the light curves are not correlated with the magnetic field strengths, periods of variation, spectral peculiarity, mean colors, or line widths. The light curves themselves can be quite complex, with double waves seen at some wavelengths and single waves at others.

There are significant advantages in using filters narrower than those of the UBV system to measure the brightness variations of Ap stars. First, amplitudes below the Balmer discontinuity and near λ4100 are often larger than those seen at other wavelengths. Variations may be more easily detected with a filter system that isolates these wavelengths. Second, narrowband observations often can be more easily interpreted in terms of a physical model than can broadband data. The uvby system is particularly well suited to photometry of

Ap stars, and extensive four-color photometry of Ap stars has been carried out by Wolff and coworkers (e.g., Wolff and Wolff, 1971). Light curves of HD 188041 are shown in Figure 4-14. The amplitude at λ4100 is much larger than at any other wavelength. Measurements of line blocking in the band pass of the v filter show that almost the entire variation is caused by changes in the strengths of lines that lie within this bandpass (Jones and Wolff, 1973). The amplitudes at the other wavelengths are small because the line opacity is much smaller and spectrum variations have, therefore, much less effect on emitted radiation. In other Ap stars, the explanation of the light variations is less straightforward, and the maximum amplitude may not occur at λ4100. Nevertheless, narrowband filters, such as those used in the uvby or Geneva systems, are much more likely than broadband filters to isolate wavelength regions where the photometric amplitude is large.

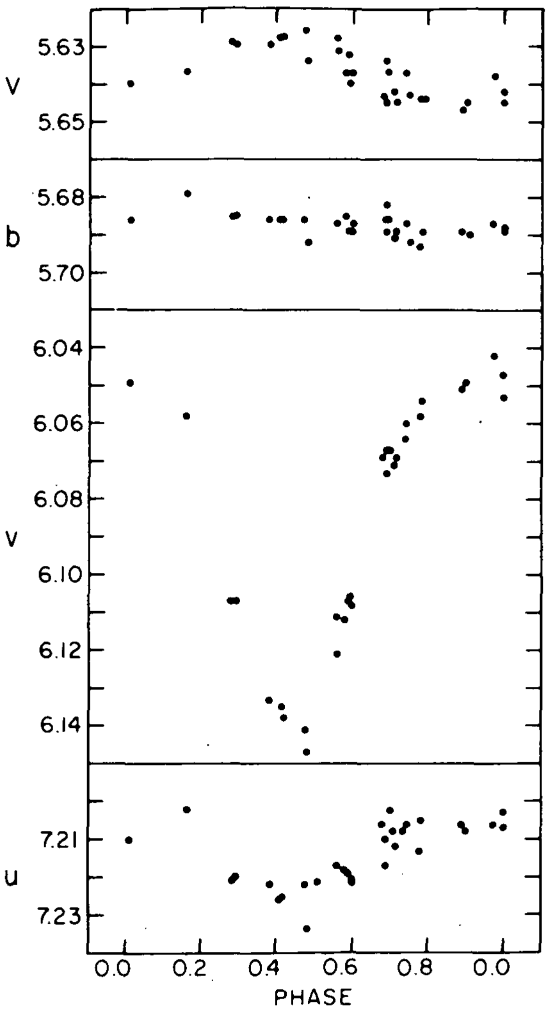

Figure 4-14. The light variations of HD 188041 plotted according to the ephemeris JD$_\odot$ (magnetic minimum) = 2432323 + 224.5E (from Jones and Wolff, 1973).

For stars in which the V, B–V, and U–B variations are all in phase, changes in temperature of a few hundred degrees can account for the change in brightness and color (Stepien, 1968). Temperature changes alone, however, cannot account for stars like α^2 CVn, which becomes redder as it brightens, or for stars that exhibit double waves at some wavelengths and not at others. A major insight into the origin of the complex light curves of Ap stars was achieved by Peterson (1970). He suggested that the light variations are a direct consequence of the spectrum variations. Specifically, he argued that changes in the continuous opacity, particularly that of Si, in the ultraviolet region of the spectrum, will cause a redistribution of flux in the visible and, hence, will produce changes in brightness. There are a number of arguments against the hypothesis that changes in the Si opacity are the primary cause of the light variations (Wolff and Wolff, 1971). However, the essential element of Peterson's hypothesis, namely that changes in ultraviolet opacity are a primary cause of luminosity variations in the visible region of the spectrum, was confirmed directly by Molnar (1973) when satellite observations with Copernicus became possible. Figure 4-15 shows the light curves of α^2 CVn as a function of wavelength. The star is essentially constant at λ2985. The variations shortward of this wavelength are in antiphase to the variations longward of λ2985 (see also Wolff and Wolff, 1971). Furthermore, the total flux integrated over all wavelengths is a constant independent of phase within the accuracy of the photometry. The invariance of the total flux and of the flux at λ2985 indicates that the photometric variations of α^2 CVn cannot be ascribed solely to changes in effective temperature. The Copernicus observations are, however, completely compatible with the hypothesis that strong blanketing in the ultraviolet redistributes flux longward of

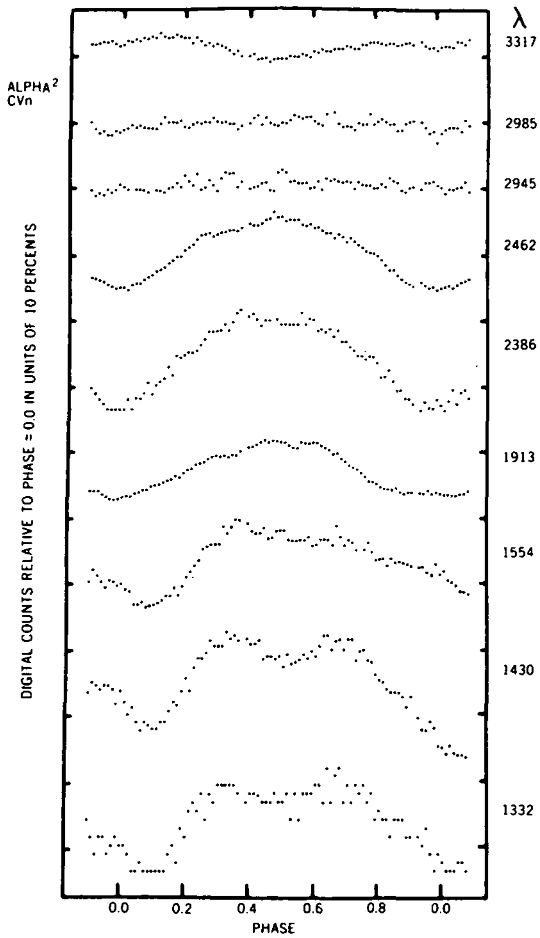

Figure 4-15. Far-ultraviolet light curves of α^2 CVn in digital counts relative to phase 0.0 in units of 10 percent. The filter wavelengths are to the right of the light curves (from Molnar, 1973).

λ2985, and that variations in blanketing cause changes in the visible flux. The precise elements responsible for the blanketing have not been identified unambiguously. Line blanketing caused by either Fe peak and/or rare earth elements seems the most likely source of the ultraviolet opacity (Wolff and Wolff, 1971; Molnar, 1973; Muthsam, 1979), with Si continuous opacity being a partial contributor in some stars (Molnar and Wu, 1978). Ultraviolet observations of several other Ap stars—including HD 215441 (Leckrone, 1974), ι Cas (Molnar et al., 1976), ε UMa (Mallama and Molnar, 1977), CU Vir and 56 Ari (Molnar and Wu, 1978)—show that the pattern of variations seen in α^2 CVn is typical of Ap stars as a class. A null wavelength for each star can be identified. On either side of this wavelength, brightness variations are in antiphase and compensate for each other in such a way that the effective temperature remains constant.

While spectrum variations are surely the dominant cause of the light variations, there are problems in accounting for the details of the light curves of some (pathological) Ap stars. At the very least, there are sources of continuous opacity that remain unidentified (Pilachowski and Bonsack, 1975; Bonsack, 1979).

The photometric variations discussed so far can be explained in terms of the rigid rotator model, in which the stellar surface is assumed to be inhomogeneous. There have been several attempts to measure variations that occur on a time scale significantly shorter than the rotation period. Such short periods would be evidence for some kind of pulsational instability. An extensive study of this kind was recently completed by Weiss (1978), who also provides references to earlier work. Because of the small scatter in existing light curves, it is clear that the photometric amplitudes produced by any pulsational instability are likely to be quite small—0.01 mag or less. Extremely careful measurements are required to detect such small amplitudes, and Weiss discusses the problems of such an observational program and the dangers of misinterpreting apparent features in light curves that are fragmentary or that were taken when the sky was less than perfectly photometric. Weiss concludes that the majority of Ap stars do not vary on time scales of minutes to hours. The one possible exception in his sample to this general rule is HD 24712, which is the coolest Ap star that he observed. Variations with a period of ~30 minutes have also been reported for 21 Com, a moderately cool Ap star (Percy, 1973, 1975; Weiss et al., 1980).

The possibility that the cool Ap stars might be pulsationally unstable has been confirmed by Kurtz (1982), who also shows that detailed analysis of the short-period variations can provide significant new constraints on the magnetic structure of these stars.

The star HD 101065 was the first peculiar star in which photometric variations with time scales on the order of minutes were unambiguously detected. Several different observers have now shown that HD 101065 varies with a principal period of 12.14 minutes and an amplitude of ~0.02 mag (Kurtz, 1978b; Kurtz and Wegner, 1979; Weiss and Kreidl, 1980). Sample light curves are shown in Figure 4-16. The star HD 101065 is an extraordinarily peculiar star. Its

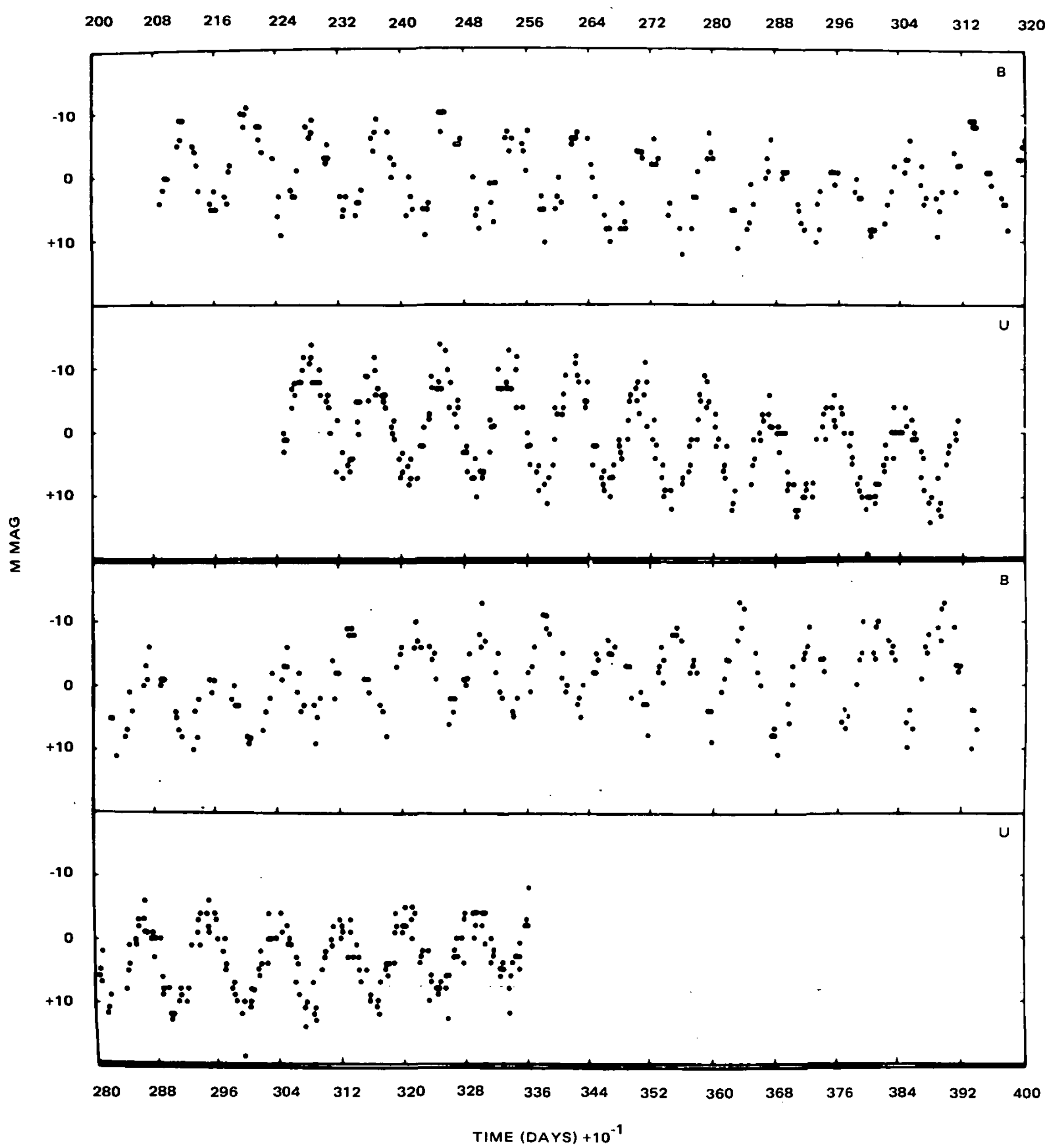

Figure 4-16. Simultaneous extinction corrected magnitudes in Johnson U and B for HD 101065 on JD$_\odot$ 2,443,644 normalized in the mean to zero. Drifts in the mean magnitude due to sky transparency and/or photometric sensitivity changes have been removed. The principal period is 12.141 ± 0.003 minutes (from Kurtz and Wegner, 1979).

spectroscopic characteristics were first described by Przybylski (1961), who showed that lines of the rare earth elements, particularly holmium and dysprosium, are extremely strong in this star. The Fe peak elements are present but are very weak (Cowley et al., 1977). Like the classical Ap stars, HD 101065 does have a strong magnetic field (Wolff and Hagen, 1976), but it is cooler than most (Wegner and Petford, 1974), and perhaps all (Przybylski, 1977) members of this class. The question of whether or not HD 101065 is a true Ap star remains open.

Nevertheless, encouraged by the results on HD 101065, Kurtz (1982) went on to observe several other cool Ap stars. Rapid oscillations were found for four Ap stars, in addition to HD 101065. Kurtz concluded that HD 24712 oscillates in two $\ell = 1$ modes with periods near 6.14 minutes; HD 83368 oscillates in both an $\ell = 1$ mode ($P = 11.67$ minutes) and an $\ell = 2$ mode with half the period of the $\ell = 1$ mode; HD 137949 (33 Lib) pulsates in a single mode ($P = 8.27$ minutes); and HD 128898 (α Cir) pulsates with periods near 6.8 minutes. (For a brief discussion of nonradial pulsation, see Chapter 6.)

After considering alternative models, Kurtz argues that the short-period variations of these stars can best be accounted for by an "oblique pulsator model," which postulates that the star pulsates about the magnetic rather than the rotational axis. This hypothesis is suggested by the fact that the period of the pulsational amplitude modulation in HD 24712 is equal to the rotational period with the maximum pulsation amplitude coinciding in phase with maximum magnetic field strength. Thus, maximum radial displacements occur at the magnetic poles where motion is along field lines. No radial motion occurs at the equator where the field lines are parallel to the stellar surface. The model therefore minimizes the problem of an oscillating plasma in the presence of a strong dipolar field. A frequency analysis shows that most of the pulsation amplitude in HD 24712 is carried in frequency triplets, where the spacing between the members of a given triplet is constant. The superposition of these frequencies then accounts for the changes in amplitude of the short-period pulsations. The amplitudes associated with the two dominant frequencies in HD 24712 are 0.0012 mag. That they can be identified at all is testimony to the precision of the photometry. Kurtz goes on to show that an oblique pulsator oscillating with a single frequency will give rise to a triplet of frequencies in its amplitude spectrum. These three frequencies are separated by the rotation frequency, and their amplitude ratios depend on i (the inclination of the rotation axis to the line of sight) and β (the angle between the magnetic and rotation axes). From the pulsation amplitudes one can, therefore, derive i, β, and $r = H_e(\mathrm{min})/H_e(\mathrm{max})$. For HD 24712, the values inferred for these three quantities are in reasonable agreement with those derived from Zeeman spectroscopy (Preston, 1972a).

Kurtz attributes the high frequencies of the two main oscillations in HD 24712 to p mode pulsations of very high radial overtones k. If the overtones are characterized by consecutive integers (k and k+1), then $k \sim 60$. Such oscillations are probably not incompatible with the diffusion hypothesis, inasmuch as they are confined to the outermost layers of the atmosphere.

The Ap stars in which short-period pulsations have been found to date lie within the instability strip. This fact suggests that these stars are analogs of the δ Sct stars, but that they are constrained for some reason to pulsate only in one or two very high overtones. An alternate possibility, which is less favored by Kurtz, is that perhaps all B, A, F, and G stars exhibit global p mode oscillations characterized by large values of k and low values of ℓ. The Sun is the one known example. In order to explain the fact that such pulsations are detected only in Ap stars, this hypothesis requires the additional postulate that in magnetic stars, but not in normal stars, all but one or two of the modes are suppressed. This possibility can be tested by searching for short-period oscillations in Ap stars that lie outside the instability strip.

The discovery of rapid oscillations in Ap stars offers an important new observational technique for deriving their basic characteristics. The possibility of determining their magnetic geometries has already been mentioned. Another important application is to the Ap stars with very long periods. If the frequencies associated with the rapid oscillations show the triplet splitting caused by rotation, then the periods of years that are associated with the spectrum and magnetic variations in such stars cannot be rotation periods (Kurtz, private communication).

MAGNETIC FIELDS

When an atom is placed in an external magnetic field, its energy levels are split into a number of sublevels. If LS coupling can be assumed, then each energy level characterized by the quantum numbers (nLJ) is split into $2J + 1$ sublevels characterized by a quantum number M. The sublevels are separated by an energy ΔE:

$$\Delta E = ge\hbar B/2\,mc, \tag{4-6}$$

where e is the charge of the electron, B is the magnetic field strength, and m is the mass of the electron. The Landé g factor can be evaluated from the expression

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}. \tag{4-7}$$

Emission lines that are otherwise single will thus be split into multiple components in the presence of a magnetic field. Transitions for which $\Delta M = \pm 1$ are called σ components, while π components are those for which $\Delta M = 0$. In any specific Zeeman pattern, the number of components, relative spacing, and intensity depend on the Landé g values of the two spectroscopic terms involved, while the scale of the pattern depends linearly on the strength of the magnetic field. The Zeeman patterns can be quite diverse (cf., Figure 4-17).

The measurement of longitudinal and transverse magnetic fields (Babcock, 1962) depends on the polarization properties of the σ and π components. If the magnetic field is transverse to the line of sight, then π components seen in emission are linearly polarized in the direction parallel to the field, while the σ components are linearly polarized perpendicular to the direction of the field. If the field is parallel to the line of sight, the σ components are circularly polarized in opposite senses while the π components vanish. (In a star we observe absorption lines, and the circular polarization is in the opposite sense to that seen in emission lines.) These effects are illustrated in Figure 4-18.

In principle, one should be able to detect either transverse or longitudinal magnetic fields in stars. In practice, it has proven easier to measure line displacements than line profiles, and so virtually all observations of magnetic fields made to date have been of the longitudinal component

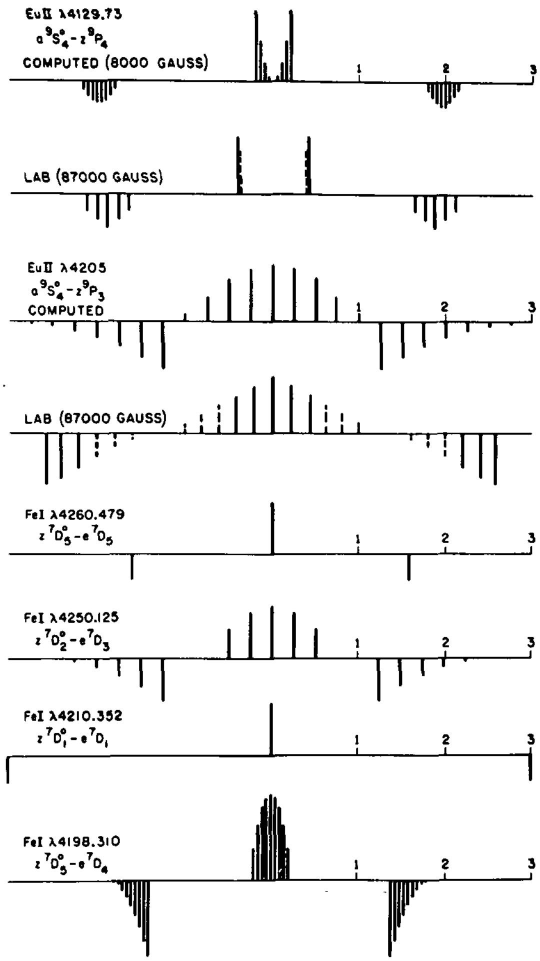

Figure 4-17. Zeeman patterns. The unit of displacement is the normal "Lorentz triplet." As is customary, the π components are shown above the axis, the σ components below. For the two lines of Eu II, computed (LS) patterns and observed laboratory patterns are shown (from Babcock, 1962).

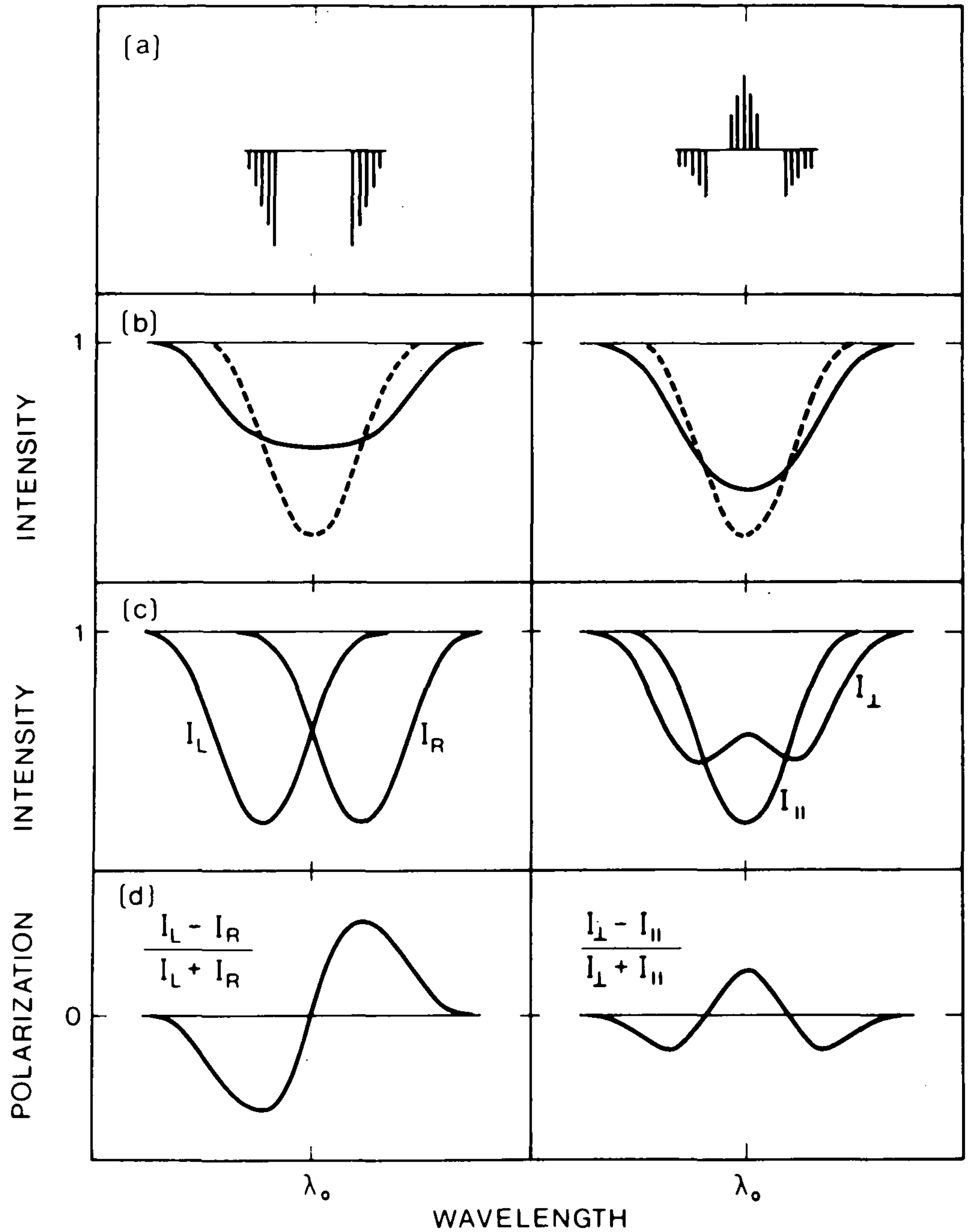

Figure 4-18. Wavelength dependence of emergent intensity and polarization across a spectral line split by a longitudinal (left-hand panels) and a transverse (right-hand panels) magnetic field. Panels (a) show the splitting of a spectral line for a $^3P_2 - {}^3D_3$ transition, π components above the line and σ components below, with the length of the bars indicating relative strengths. (In a longitudinal field, π components do not appear.) Panels (b) show the appearance of the stellar absorption line with a field present (solid lines) and absent (dashed lines). Panel (c) (left) shows the line profile as seen in right and left circularly polarized light, while panel (c) (right) shows the line profile seen in linearly polarized light parallel to and perpendicular to the field. Panels (d) show the net circular polarization (left) and linear polarization (right) across the lines. The profiles shown are schematic only (from Landstreet, 1980).

only. The average displacement in Angstroms of the σ components from λ_0, the position of the undisplaced line for $B = 0$, is

$$\Delta\lambda = \pm 4.67 \times 10^{-13} z B \lambda_0^2, \qquad (4\text{-}8)$$

where z is the mean displacement of the σ components, weighted by their intensities, in terms of the displacement for a normal Zeeman triplet. At $\lambda 5000$, $\Delta\lambda$ is approximately equal to 10^{-2} Å/kilg, and in stars the Zeeman splitting is normally smaller than the broadening due to other mechanisms (rotation, thermal and turbulent Doppler broadening, and collisional broadening).

Measurement of a stellar magnetic field is further complicated by the fact that the field is normally neither purely transverse nor longitudinal, but rather is inclined at some arbitrary angle to the line of sight. Babcock (1947) has shown that for weak features, the displacement of the centroid of a line, as seen in left or right circularly polarized light, is proportional to the longitudinal component of the field H_e, weighted by surface brightness and averaged over the visible hemisphere of the star. That is

$$H_e = \frac{\int H \cos\gamma \, I dA}{\int I dA}, \qquad (4\text{-}9)$$

where I is the surface brightness of area element dA and γ is the angle between the line of sight and the field vector at dA. For an oblique dipole rotator, H_e is given by the relationship (Stibbs, 1950)

$$H_e = \frac{1}{20} \, \frac{15 + u}{3 - u} \times H_p \, (\cos\beta \cos i + \sin\beta \sin i \cos 2\pi t/P), \qquad (4\text{-}10)$$

where β is the angle between the magnetic and rotation axes, i is the angle between the rotation axis and the line of sight, P is the period of rotation, H_p is the polar field strength, and u is the limb darkening coefficient. Landstreet [1982] has recently shown that the value of H_e derived from polarization data is not weighted by the limb darkened intensity. Therefore, $u \equiv 1$ in Equation 4-10.) If the star is also rotating, then Doppler shifts of regions with different magnetic intensities may yield quite complex line profiles. These effects have been discussed briefly in the section of this chapter on Spectrum Variability.

The first measurements of Zeeman splitting in stars were made by Babcock (1947). In order to separate left from right circularly polarized light, a Zeeman analyzer was mounted in front of the slit of the coudé spectrograph of the 100-inch telescope at Mt. Wilson. The analyzer consists of a quarter-wave plate plus a calcite crystal. The quarter-wave plate converts circularly polarized light to linear polarization, and the calcite crystal, which is doubly refracting, then separates the ordinary and extraordinary rays. The calcite is oriented in such a way that the two stellar images appear one above the other at the slit of the spectrograph. The thickness of the calcite is chosen to allow an appropriate separation at the plate of the two analyzed spectra, which are photographed simultaneously and appear side by side on the plate. Oblique reflections at the coudé flats produce a phase difference between vibrations parallel and perpendicular to the plane of incidence (thus converting circular to elliptical polarization). Compensation for these phase shifts can be achieved through the use of a Babinet-Soleil compensator, which is mounted in front of the quarter-wave plate and rotated during an exposure in such a way as to maintain a fixed relationship between the compensator axis and the configuration of the telescope mirrors. The observational techniques are discussed in more detail by Babcock (1962). Essentially similar devices were used for the extensive series of Zeeman measurements at Lick Observatory (Preston and Pyper, 1965) and at Mauna Kea (Wolff and Bonsack, 1972).

There are a number of limitations on the accuracy of the magnetic field strengths derived from photographic coudé spectroscopy. Errors inherent in the photographic process limit the accuracy to ±150 g (Preston, 1969c), and photographic detections of fields smaller than this limit or of intrinsic variability of this order of magnitude should be viewed with suspicion. Photographic measurements tend to emphasize the cores of lines. Neglect of the wings can lead to inaccurate estimates of the centroid of the line, and this error is probably one source of the anharmonic magnetic curves derived photographically for many Ap stars (Borra, 1974b). Errors of measurement increase markedly with line width, and reliable photographic measurements of H_e are possible only in stars with v sin $i < 40$ km s^{-1}. Finally, it is impossible, even in principle, to compensate entirely for the changes in polarization introduced by the oblique reflections at the coudé flats. Therefore, the spectrum that should be formed purely by light of one sense of polarization will be partially contaminated by light of the opposite sense of polarization. In the Lick system, the maximum contamination is about 20 percent (Preston and Pyper, 1965). Because of this contamination the measured value of H_e will be smaller than its true value.

In summary, photographic observations at a coudé spectrograph offer an efficient way of determining the sign of the magnetic field and the time scale of its variability. Other techniques are required for accurate estimates of its absolute value and the precise details of its changes with time.

With photoelectric techniques, H_e can be estimated from measurements of the polarization in the wing of a single line. Let us define the net polarization as

$$V = \frac{I_L - I_R}{I_L + I_R} , \qquad (4\text{-}11)$$

where I_L and I_R are the intensities at a given wavelength as viewed in left and right circularly polarized light, respectively. Then (see Figure 4-19)

$$V(\lambda) \cong \frac{I(\lambda) + (dI/d\lambda)\, 2\Delta\lambda - I(\lambda)}{I(\lambda) + I(\lambda) + (dI/d\lambda)\, 2\Delta\lambda}$$

$$\cong \Delta\lambda\, (dI/d\lambda)/I(\lambda) \; ; \qquad (4\text{-}12)$$

here $\Delta\lambda$ is given by Equation (4-8). If the line under consideration is a Balmer line, then $dI/d\lambda$ is small, and the line width is large compared with

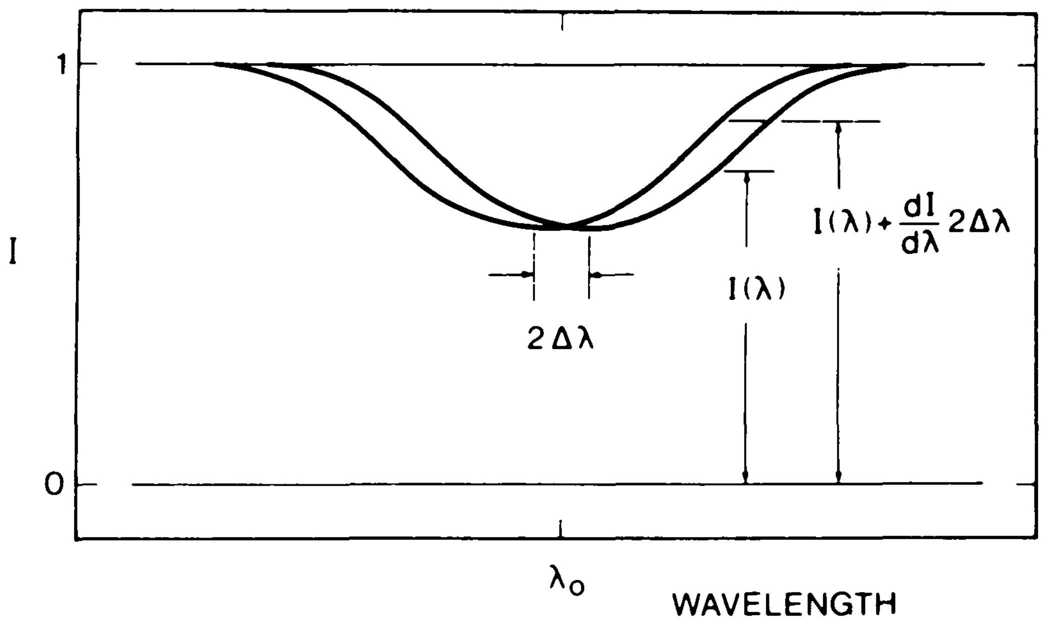

Figure 4-19. Change in line depth of an absorption line when viewed in right and left circularly polarized light in a narrow wavelength band. The difference in the intensity seen in right and left circularly polarized light at a specific wavelength is indicated. The field is assumed to be longitudinal (from Landstreet, 1980).

the sources of Doppler broadening. Then the measured polarization, which is always small (~1 percent for $H_e \sim 10^4$ g), can be easily interpreted in terms of the magnetic field strength (Angel and Landstreet, 1970). This technique is virtually insensitive to rotational broadening and, hence, has provided most of the available information on the magnetic properties of rapidly rotating stars.

The same technique can be applied to metallic lines, although Doppler broadening or very large Zeeman splitting may distort the line profile in such a way that Equation (4-12) is no longer applicable. However, with new panoramic detectors, simultaneous measurements of the polarization of hundreds of metallic lines can be made, and fields of a few tens of gauss should be readily detectable. Accuracies of this order have already been achieved through use of a radial velocity template for simultaneous measurement of the Zeeman shifts of many lines (Brown and Landstreet, 1981; Borra et al., 1981).

All three of these techniques for deriving Zeeman splitting have been applied to Ap stars. Since Babcock's (1947) detection of a field in the Ap star 78 Vir, fairly complete magnetic data have been obtained for perhaps 50 stars. Detections have been made in perhaps twice as many more stars, and nondetections have been reported for still more objects. Major summaries of magnetic data have been published by Babcock (1958) and Borra and Landstreet (1980). The latter paper also provides extensive references to the literature on stellar magnetism.

The primary result of the studies of stellar magnetism is that fields can be detected in most stars with enhanced lines of Si, Sr, Cr, and/or Eu; these are the Ap stars as defined here. Magnetic fields are also seen in some He-rich and He-weak stars, which are probably hotter analogs of the Ap stars (Borra and Landstreet, 1979; Landstreet et al., 1979). Strong magnetic fields have not been detected in other types of stars. In particular, Babcock's (1958) report that magnetic fields are present in at least some Am and HgMn stars has not been confirmed. Photoelectric polarimetry in the wings of Hβ by Borra and Landstreet (1980) sets upper limits of 100 to 300 g for a number of stars in these two categories (standard deviations range from 44 to 200 g).

In order to be definitely measurable photographically, the longitudinal magnetic field, H_e, must exceed ~200 g. In the majority of Ap stars, field strengths range from a few hundred to 1000 to 2000 g. The star with the largest longitudinal field reported to date is HD 215441 in which H_e varies from 12 to 20 kilg. Early work by Babcock (1960) suggested that in some Ap stars the magnetic variations were not periodic. However, more recent work has established periods for most of the stars that Babcock classified as irregular variables, and the present point of view is that any Ap star, observed sufficiently well, can be shown to be periodic (e.g., Preston, 1971b).

An interesting question is whether or not there are intrinsic fluctuations in magnetic intensity superposed on an otherwise regular periodicity. The scatter seen in some photographically derived magnetic curves is distressingly large. In the case of 78 Vir, however, Preston (1969c) feels that the scatter can be accounted for entirely in terms of the random and systematic errors inherent in the measurement of photographic plates. Bonsack (1977) argues that the observations of the Si-type star HD 133029 cannot be dismissed so easily. Photographic observations show a nonreversing field with an average longitudinal component of +2300 g. There are occasional excursions of 1 kilg in either direction, with downward excursions occurring more frequently. The magnetic variations are not synchronous with either the photometric or radial velocity variations, both of which have a period of 2.88706 days. Borra and Landstreet (1980) have observed HD 133029 photoelectrically at Hβ, and they do find that H_e varies periodically with an amplitude of 1 kilg and with the same period as the luminosity. Irregular variations in this or any other Ap star observed by Borra and Landstreet do not exceed 200 g.

It is difficult to see how to reconcile the photoelectric and photographic observations. One possibility is that for some inexplicable reason, the errors in the photographic observations of HD 133029 are larger than they are for the other Ap stars observed by Bonsack at Mauna Kea. The other possibility is that the photographic and photoelectric techniques measure different things. The Hβ data sample the whole disk and measure H_e integrated over the stellar surface. If the field is highly inhomogeneous, with very large values of H_e occurring only over a small portion of the visible hemisphere, then the photographic measurements, which emphasize the line cores, will oversample those portions of the disk where the field is weak (Borra, 1974a, 1974b). This problem is compounded in those cases where there are abundance patches, and measurements of metallic lines do not sample the disk uniformly.

Many of the photographic magnetic curves are strongly anharmonic, and inferences concerning the field geometry have been drawn from the shapes of these curves (Böhm-Vitense, 1967; Landstreet, 1970). Photoelectric polarimetry of Hβ yields essentially sinusoidal magnetic curves (see Figure 4-20) for all but a few stars (HD 32633, HD 37017, and HD 37776), and this result has been interpreted as evidence for the suggestion by Borra (1974b) that the anharmonic curves are an artifact of the process of measuring photographic plates. An alternate possibility has been discussed by Hensberge et al. (1979), who point out that magnetic curves derived from photographic measurements of the Zeeman splitting of metallic lines and from Hβ polarimetry are not directly comparable since different element distributions are involved. The anharmonic magnetic curves

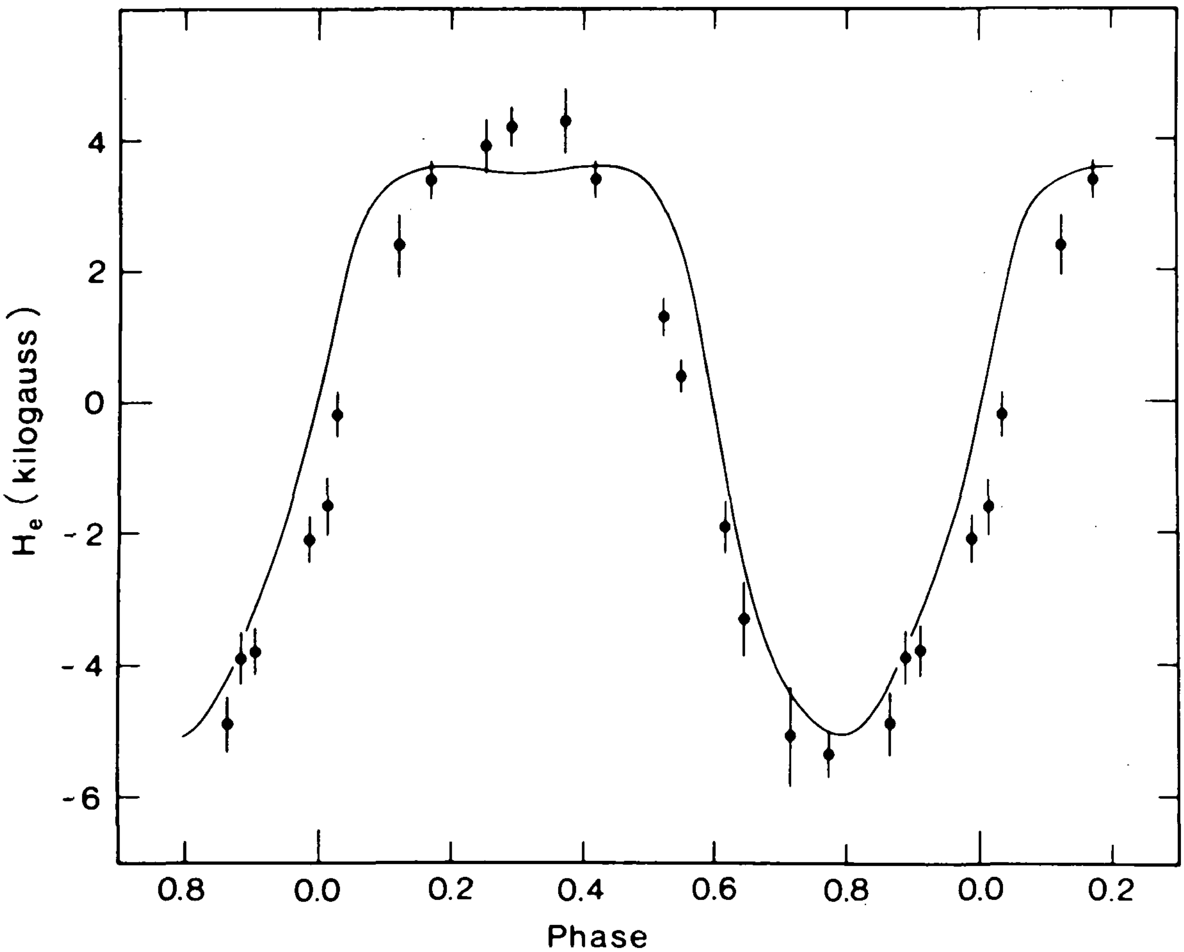

Figure 4-20. Hβ magnetograph observations (filled circles) of the magnetic field of 53 Cam plotted against phase. Error bars indicate ±1 standard error. The smooth curve is a freehand average of Babcock's photographic magnetic curve (from Borra and Landstreet, 1977).

derived from metallic lines may, Hensberge et al. suggest, be due to the nonuniform distribution of elements over the stellar surface. In either case, it is clear that magnetic geometries cannot be inferred from photographic data alone.

In a few stars the magnetic field is large enough to produce resolved Zeeman patterns. Such patterns can occur only if the Zeeman splitting $\Delta\lambda$ exceeds the Doppler broadening (Preston, 1971b). With H in kilogauss, λ in Angstroms, and v, the Doppler width, in km s^{-1}, this condition is

$$\frac{1.4 \times 10^{-4}\ \lambda\ Hz}{v} > 1 \ , \qquad (4\text{-}13)$$

where z is the ratio of the σ splitting to that of a normal Zeeman triplet. At λ4300, this condition becomes

$$H > \frac{1.7\ v}{z} \ . \qquad (4\text{-}14)$$

The smallest plausible value for v is ~3 km s^{-1}, a value set by the resolution of a typical coudé spectrograph and normally exceeded by stellar rotational broadening. With $z = 1$, H must exceed ~5 kilg for the σ components to be resolved.

Since the width of a Zeeman pattern depends only on the strength of the magnetic field, and not on its direction, one can define a scalar magnetic field, averaged over the visible hemisphere of the star without regard to sign, according to the relation

$$H_s = \frac{\int HIdA}{\int IdA} \ . \qquad (4\text{-}15)$$

The value of H_s depends linearly on the splitting of the σ components. For a dipole field, H_s is equal to 0.80 H_p, where H_p is the polar field strength, when the star is viewed pole-on. When the star is viewed equator-on, H_s = 0.64 H_p. (Under these same circumstances, H_e varies from 0.3 H_p to 0.0.)

For many years, HD 215441 was the only star known to have a magnetic field large enough to produce resolved Zeeman patterns (Babcock, 1960). In this star, H_s varies from 32000 to 34000 g in a period of 9.488 days (Preston, 1969a). However, certain lines, specifically those with anomalous Zeeman patterns in which the π group splits into two components with displacements that are comparable to the displacements of the σ groups, can be resolved even when most lines cannot be. With the recognition of this fact (Preston, 1969b, 1969d) it became possible to determine H_s for many additional Ap stars.

Measurements of the complete cycle of variation of H_s are available for HD 215441 (Preston, 1969a), β CrB (Wolff and Wolff, 1970), HD 126515 (Preston, 1970a), and 53 Cam (Huchra, 1972). The results for β CrB are shown in Figure 4-21. The mean surface field H_s varies approximately in antiphase with the longitudinal component of the field H_e. The total amplitude of the

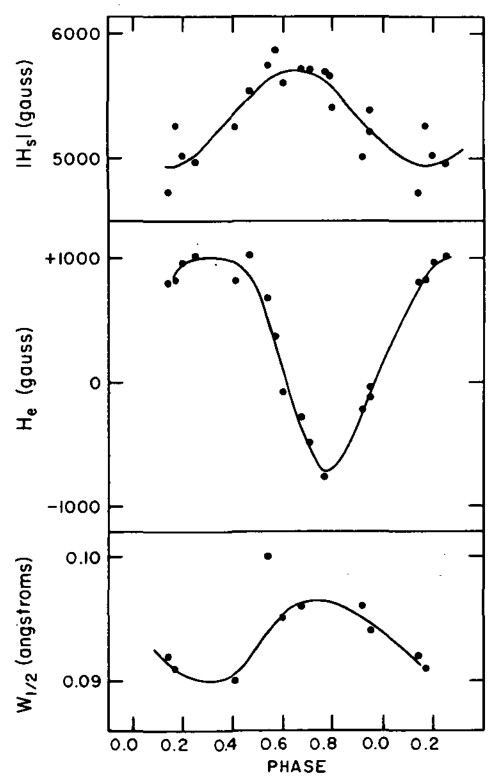

Figure 4-21. Variations in total surface field $|H_s|$, longitudinal field H_e, and line width $W_{1/2}$ in β CrB (from Wolff and Wolff, 1970).

variation in H_s is about 800 g, and it reaches a maximum of 5700 g. Note, too, that the line widths in β CrB vary in phase with H_s. Zeeman broadening is a significant factor in determining the line widths in this star.

The variations of H_s in HD 215441, HD 126515, and 53 Cam are like those of β CrB in the sense that H_s varies approximately sinusoidally and with the same period as H_e.

The observations provide strong constraints on the magnetic geometries of these four stars. From Babcock's early measurements, it was obvious that magnetic fields in Ap stars must be primarily dipolar (e.g., Preston, 1971b). Toroidal fields cannot produce strong longitudinal Zeeman splitting in the integrated light of an entire stellar hemisphere. Multipole fields of order higher than a dipole also produce small effects in integrated light. For a magnetic quadrupole, H_s must be on the order of 20 to 40 kilg in order to produce values of H_e in the range 1 to 2 kilg. Such a large value of H_s would produce much larger Zeeman splitting than is actually seen in Ap stars. The sharpness of the lines and the purity of the Zeeman effect rule out the possibility that Zeeman splitting seen in integrated light is produced in small areas of the stellar disk with very intense fields.

If stellar magnetic fields were strictly dipolar, then one would expect H_s to exhibit a double wave variation for each complete cycle of H_e, provided H_e reverses sign during the cycle. That is, H_s should reach a maximum each time we view one of the poles, positive or negative, of the dipole. Only a single maximum of H_s, coinciding approximately in phase with one extremum of H_e, is observed, and this fact requires that the field be hemispherically asymmetric. To first order, this asymmetry can be represented by a dipole with its center of symmetry displaced along the axis of the dipole from the center of the star by a significant fraction of the stellar radius. Values of the displacement found for the stars studied to date range from 0.1 to 0.36 R_*.

A fit to the observations of H_s and H_e in HD 126515 is shown in Figure 4-22.

This offset dipole model fails in two respects. It cannot account for deviations from a sinusoid

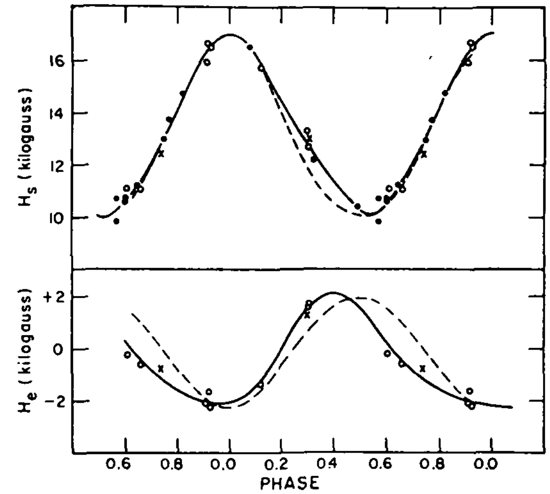

Figure 4-22. Variation of the mean surface field H_s and the effective field H_e with phase in the 130-day cycle of HD 126515. Crosses, open circles, and filled circles, observations made in 1957, 1960, and 1969, respectively. Solid curves, freehand representations of the variations. Dashed curves, variations for a decentered-dipole model (from Preston, 1970a).

in the H_e curve, but as noted earlier the anharmonic H_e variations are probably an artifact of the measuring process (Borra and Landstreet, 1977; Hensberge et al., 1979). This model also cannot account for the phase shift of 0.1 cycles between the extrema of H_e and H_s. This phase shift is seen in β CrB as well as in HD 126515 and is probably real. A decentered dipole in which the decentering direction is not constrained to be parallel to the axis of the dipole can account for the phase shift (Stift, 1975). A dipole plus quadrupole field geometry yields values of H_s and H_e that are indistinguishable observationally from those calculated for an offset dipole (e.g., Wolff and Wolff, 1970).

Observations of the polarization characteristics of individual lines contain considerably more information about the magnetic geometry. Such measurements are now available for β CrB (Borra and Vaughan, 1977), α^2 CVn (Borra and Vaughan, 1978), and 78 Vir (Borra, 1980a). For 78 Vir, Borra (1980b) has compared the observed line profiles and polarization characteristics at several different phases with the predictions of various dipole and quadrupole magnetic field distributions (see Chapter 9, Figures 9-8 and 9-9). He finds that the Zeeman patterns can be successfully reproduced by an oblique rotator (a significant test of the validity of this model) that includes both a dipole plus a modest quadrupole magnetic field. There are many more stars that could be analyzed in the same way, and studies of this kind offer the best method of determining the magnetic geometries of the Ap stars.

An alternative magnetic geometry, the so-called "symmetric rotator," has been proposed by Krause (1971), Krause and Oetken (1976), and Oetken (1977). These authors argue that if the dynamo theory correctly explains the origin of the magnetism in Ap stars—and it is not at all clear that this hypothesis is correct—then the magnetic field distribution should be either symmetric or antisymmetric under reflection through the rotational equator. Various combinations of dipole plus quadrupole fields can meet the condition of symmetry. One case that has been considered in detail is a model in which the dipole and quadrupole are parallel to one another and lie in the plane of the rotational equator. In a second model, the dipole lies in the equatorial plane while the quadrupole is orthogonal to it and parallel to the axis of rotation. For stars with symmetric field reversals, the magnetic variations can be well represented by a dipole in the rotational equatorial plane, and there is little to distinguish the symmetric rotator from the oblique rotator model. However, if the magnetic reversal is not symmetric, then there are major differences between the two models. The oblique rotator model achieves nonsymmetric reversals by changing β, the angle between the magnetic and rotation axes. The symmetric rotator, however, can account for such magnetic variations only by adding a large quadrupole component. A quadrupole field can produce a net longitudinal field that is no larger than 5 percent of the field strength at the magnetic pole. Therefore, if a quadrupole field makes a large contribution to H_e, the average surface field must also be very large. Direct measurements of H_s seem to rule out the symmetric rotator model (Borra and Landstreet, 1978; Deridder et al., 1979), although this conclusion has recently been challenged by Oetken and Krause (1981), who argue that the effects of a nonuniform distribution of elements over the stellar surface have not been properly taken into account.

Estimates of H_s at a single phase are available for 26 additional Sr-Cr-Eu Ap stars (Preston, 1971a). (In general, the rotational broadening in Si Ap stars is so high that the Zeeman patterns cannot be resolved.) The values of H_s were estimated from resolved Zeeman patterns ($H_s > 6$ kilg) or from line broadening ($H_s < 6$ kilg). Values of H_s that are less than 2 to 3 kilg are quite uncertain, because for such small fields the Zeeman broadening is comparable in magnitude to broadening due to other causes. The frequency distribution of H_s is shown in Figure 4-23. The distribution peaks at 2 to 3 kilg and decreases smoothly with increasing H_s. The maximum values of H_s exceed 10 kilg. Preston finds no correlation between H_s and UBV colors, Eu II line strengths, or rotational velocities. It should be noted, however, that for all stars in the sample, v sin i is less than 10 km s^{-1}.

Borra and Landstreet (1980) have also looked for a correlation between magnetic field strength and rotation. For rapid rotators one can measure only H_e, not H_s, but Landstreet et al. (1975) had suggested that H_e tended to be smaller in rapidly rotating Ap stars. Based on a larger sample, Borra and Landstreet conclude that there is *marginal*

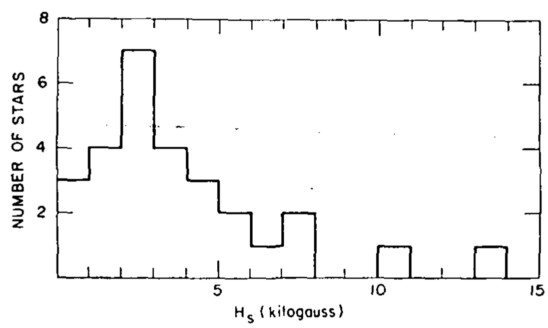

Figure 4-23. Frequency distribution of H_s for 28 Sr-Cr-Eu stars with v sin $i \leq 10$ km s^{-1} (from Preston, 1971a).

evidence (their italics) that rapid rotators have smaller fields than slower rotators. However, large fields can occur in rapidly rotating stars. In HD 124224, for example, H_e varies from −400 to +800 g. They infer that H_p in this star must be at least 7 kilg. The equatorial rotational velocity of HD 124224 is 300 km s^{-1}.

A final question about magnetic geometries concerns the relative orientations of the magnetic and rotation axes. The angle of inclination β, between the magnetic and rotation axes, can be inferred from the magnetic curves. Let r be the ratio of the minimum to maximum values of H_e (with the sign of the field taken explicitly into account). Then (e.g., Preston, 1967a)

$$r = \frac{H_e\ (\text{min})}{H_e\ (\text{max})} \tag{4-16}$$

$$= \frac{\cos\beta\cos i - \sin\beta\sin i}{\cos\beta\cos i + \sin\beta\sin i}\ .$$

If the axes of rotation of Ap stars are oriented at random, then for a given value of β, it is possible to calculate the distribution of r. Sample distributions are shown in Figure 4-24. The predominance of values of r near −1 was interpreted by Preston (1967a) as evidence that regions of high magnetic intensity are located preferentially near the rotational equator. This conclusion is valid whether or not the field is dipolar (Landstreet, 1970). Subsequent data have tended to fill in the distribution of r. Specifically, there is now a second maximum near r = +0.5. Preston (1971b) interpreted this secondary maximum as evidence for a bimodal distribution of β. However, the sample size is small, and statistical tests show that one cannot reject the hypothesis that the observed distribution is the consequence of a single, medium high value for the obliquity (β = 60°). The observed distribution is also compatible with a 50/50 admixture of high obliquity (80°) and low obliquity (20°), and a variety of other possibilities including a random mixture of β (Hensberge et al., 1979; Borra and Landstreet, 1980). The distribution of r is incompatible with the hypothesis that β is

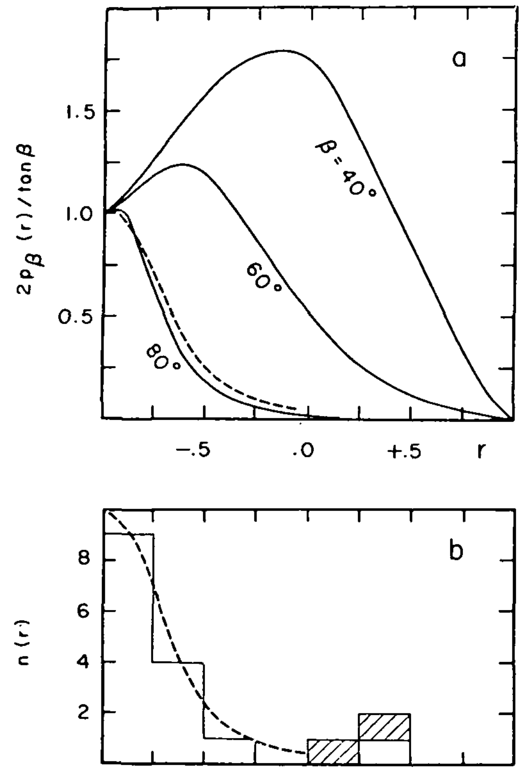

Figure 4-24. (a) Probability distributions of the observed ratio $r = H_e(min)/H_e(max)$ given by Equation (4-16) in the text for three values of β. (b) Histogram of the observed r values for the known periodic magnetic variables. The dashed curves in (a) and (b) are smoothed freehand representations of the histogram (from Preston, 1967a).

always large (80°), but additional constraints on β can be obtained only by a substantial increase in sample size.

Obviously, considerably more information concerning magnetic geometries is contained within the line profiles. A discussion of more theoretical analyses of Zeeman measurements is given in Chapter 9.

BINARY FREQUENCY

A by-product of Babcock's (1958) magnetic survey of Ap stars was an indication that the frequency of spectroscopic binaries is unusually low. A systematic search for radial velocity variations among Ap stars by Abt and Snowden (1973) has confirmed this result. Only 20 percent of the Si and Sr-Cr-Eu stars included in the survey were found to be members of spectroscopic binaries. The frequency of spectroscopic binaries among the normal A-type stars is 43 percent. Abt and Snowden find that the frequency of visual binaries among the Ap stars is normal.

The analysis of binary orbits can provide crucial information on stellar masses. Unfortunately, no magnetic Ap stars are known to be members of eclipsing systems, and only two have been found to be components of double-lined binaries. One of these is HD 98088, which has a period of 5.905 days. While the system does not eclipse, the orbit is inclined at a large angle with respect to the plane of the sky, and the minimum mass of the Ap star is found to be 1.7 $M_\odot$. The hydrogen line spectral type of the Ap primary is A3 V and the secondary is an A8 V star. If the mass of the secondary is normal for its type, then the mass of the Ap star is 2.2 $M_\odot$. This value is the same as would be expected for a normal star at that location on the main sequence (Abt et al., 1968). The analysis of the orbit of the Sr-Cr-Eu star HR 3724, which is a member of a visual binary, also yields a mass that is, within the observational uncertainties, normal for its spectral type (van Dessel, 1972).

A considerably more puzzling situation is presented by the orbital analysis by Bonsack (1976) of HR 2727, which is a Sr-Cr-Eu star. In this case, the mass of the Ap star is found to be 0.75 times the mass of its companion. If both stars obey the normal mass-luminosity law, then the companion star should be three times more luminous than the Ap star. In fact, Bonsack finds that in the region λλ4000 to 4500 the two stars are of comparable luminosity. In other words, the Ap star appears to be overluminous for its mass, and this conclusion remains valid even if redistribution of flux from the ultraviolet is taken into account. If this result is confirmed, then the implication is that the internal structure of magnetic stars differs in some significant way from that of normal stars.

Observations of HD 98088 may ultimately provide some additional constraints on the internal structure of magnetic stars. The present lower bound on the period of apsidal motion in this system is 700 years. If the internal structure of HD 98088 were like that of a normal star, the apsidal period should be ~500 to 700 years and certainly should not exceed 1000 years. Therefore, observations over the next 50 years should clearly establish whether or not the present lower bound on the apsidal period is close to its actual value, as it must be if the internal structure of HD 98088 is normal (Abt et al., 1968; Wolff, 1974b).

In close binaries, tidal forces can lead to synchronization of revolution and rotation. In normal late B-type and early A-type stars, there is probably complete synchronization to periods of ~4 days and a tendency toward synchronism for periods as long as 15 days (Plavec, 1970; Levato, 1976; Wolff, 1978).

A systematic discussion of synchronism in Ap stars cannot be made since only three are known to have periods less than 8 days. For two of these, 41 Tau (Wolff, 1973) and HD 98088 (Abt et al., 1968), rotation and revolution are synchronized. The third, HR 710, has an orbital period of 2.9978 days. There is marginal evidence for magnetic variations with a period of 15.9 days, but whether these variations are due to intrinsic changes in field strength or to rotation with a period of 15.9 days is unclear. Whichever hypothesis is adopted there are severe problems in the interpretation of HR 710 (Bonsack, 1981). Several other stars with much longer periods, including HR 2727 with P(orb) = 46.3 days, HD 8441 with P(orb) = 106 days, HR 465 with P(orb) = 273 days, and HD 216533 with P(orb) = 16.03 days, clearly do not show synchronism of rotation and revolution (Preston and Wolff, 1970; Wolff and Morrison, 1973; Bonsack, 1976; Floquet, 1979). The available data therefore suggest, but do not prove, that Ap stars are similar to normal ones with respect to the period range in which synchronism of rotation and revolution occur.

ROTATIONAL VELOCITIES

The rotational velocities of the magnetic Ap stars are, on the average, much lower than those of normal stars of the same temperature. Indeed, it is the unusual sharpness of the lines in these stars that makes possible photographic measurements of their magnetic field strengths. Surveys of the rotational velocities have been carried out by Abt et al. (1972) and by Preston (unpublished). Figure 4-25, which was constructed from the data supplied by Preston, shows the distribution of rotational velocities for the magnetic Ap stars. The stars have been grouped according to color into three temperature classes; these classes correspond approximately to the Si, Si-Cr, and Sr-Cr-Eu spectral classifications. The data are also summarized in Table 4-2, which is taken from Wolff (1981). The masses for each of the three groups have been estimated from a comparison of their colors with theoretical evolutionary tracks.

Several important points are illustrated in Figure 4-25. First, there are a few Ap stars, particularly among the Si group, that rotate quite rapidly. For example, the equatorial rotational velocity of 56 Ari is ~175 km s^{-1} (Bonsack and Wallace, 1970) and that of HD 124224 is ~300 km s^{-1} (Borra and Landstreet, 1980). Therefore, slow rotation is not a necessary condition for the appearance of spectral peculiarities, at least in the Si Ap stars. Statistical studies of Ap stars in clusters show that slow rotation is not a sufficient condition either (Abt, 1979). There are many slowly rotating normal stars (see also Wolff and Preston, 1978a). While slow rotation is one of the more obvious characteristics of the Ap stars as a group, the detailed relationship between rotation and the presence of abundance anomalies is complex.

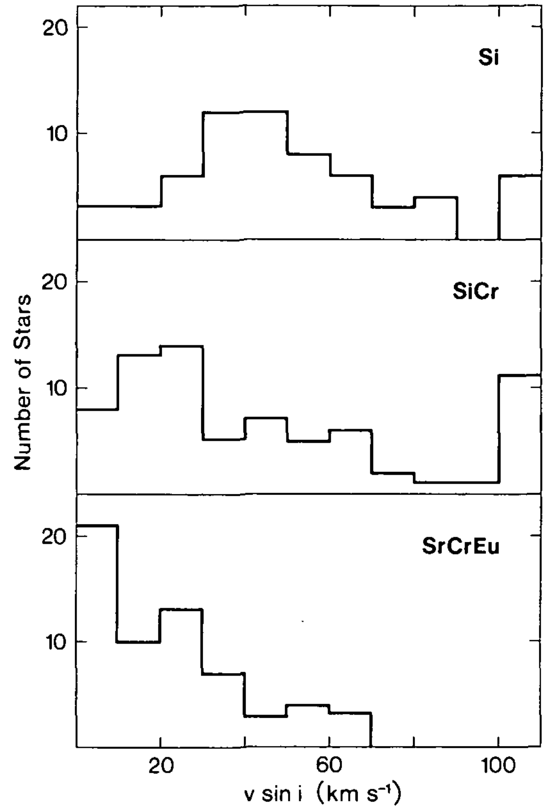

Figure 4-25. Distribution of apparent rotational velocities of the magnetic Ap stars.

The second important property of the rotational velocities of Ap stars is the overall decrease in <v sin *i*> with decreasing mass. As Figure 4-25 shows, this decrease is caused by two factors: (1)

Table 4-2
<v sin *i*> For Field Stars

Group	<Mass>	<Age> (years)	No. of Stars	<v sin *i*> (km s^{-1})
1	4 $M_\odot$	5×10^7	63	54
2	3 $M_\odot$	10^8	64	36
3	2 $M_\odot$	3×10^8	61	24

Source: From Wolff (1981).

there are no cool Ap stars with extremely high rotational velocities, and (2) while many cool Ap stars rotate very slowly (v sin $i < 10$ km s^{-1}), virtually none of the hot Ap stars do so.

The slow rotation of the Ap stars is attributed to magnetic braking of some kind. The natural explanation for the decrease in v sin i with decreasing temperature is that the Ap stars with lower mass evolve more slowly and that the braking mechanism has a longer time in which to act. Possible braking mechanisms are discussed in more detail in the section of this chapter on Evolutionary Changes.

PERIODS

If the rigid rotator model is correct, then in each Ap star the period of variation must be equal to the period of rotation. This in turn implies that there must be a relationship between period and line width, in the sense that stars with broader lines (more rapid rotation) will have shorter periods. Specifically, for an oblique rotator

$$v = \frac{50.6 R}{P} , \tag{4-17}$$

where v is the equatorial rotational velocity in km s^{-1}, R is the stellar radius in units of the solar radius, and P is the period in days. Of course, one can measure only v sin i, the projected value of the rotational velocity, and not v itself. Therefore, in a plot of v sin i against P, all Ap stars should fall below the relationship defined by Equation (4-17), provided a suitable mean value of the radius is selected. Such a plot is shown in Figure 4-26. The discovery that the line widths of Ap stars vary inversely with their periods was an early source of support for the oblique rotator model (Deutsch, 1956).

The initial studies of Ap stars typically yielded periods of a few days, which could plausibly be interpreted as rotation periods. It has gradually become clear, however, that a few stars have much longer periods. In 1969, Wolff (1969b) showed that the period of HD 188041 was 224.5 days, and that no shorter periods could represent the

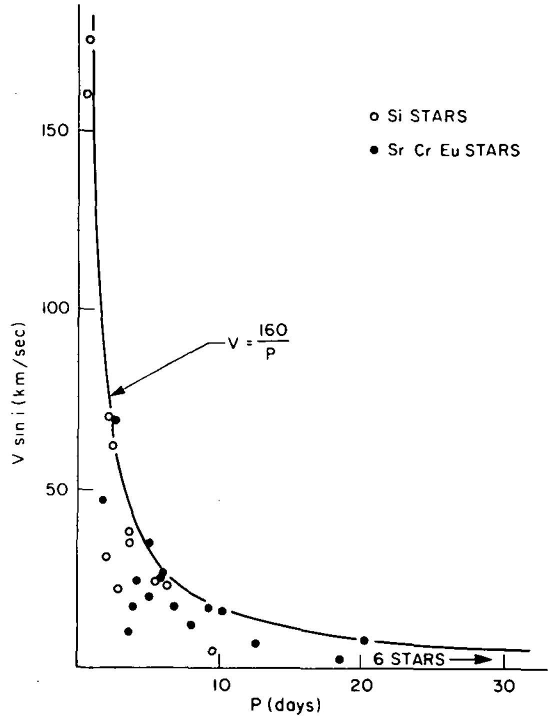

Figure 4-26. A plot of projected rotation v sin i versus period P for periodic Ap stars. The curve represents the relation between equatorial rotational velocity and period for stars with radius R = 3.2 $R_{\odot}$, as discussed in the text. The curve is a reasonable upper envelope for the plotted points (from Preston, 1971b).

magnetic data. Such a long period had been proposed by Babcock (1954), but because coudé observations are typically obtained only near full moon, periods close to one month could not be ruled out on the basis of Babcock's data alone (Preston, 1967b).

Much more surprising was the discovery that the Ap star HR 465 has a period of 22 to 24 years (Preston and Wolff, 1970). The rare earth lines in this star during September 1960 were unusually strong even for an Ap star (Bidelman, 1967). Observations of HR 465 from 1966 through 1967 by Preston and Wolff indicated no changes in line strength over a period of 15 months, and their spectra bore no resemblance to that obtained 7 years earlier by Bidelman. Plate files at the Hale Observatories were searched for additional spectrograms of HR 465, and observations were found that went back to 1936. From these data, Preston and Wolff concluded that there was no evidence for variations on time scales of a day, a week, a month, or a year. However, the spectroscopic data, as well as fragmentary UBV photometry and magnetic field measurements, could be well represented by a period of 22 to 24 years. Subsequent magnetic data (Wolff, 1974a) support this period. The next rare earth maximum was predicted to occur in approximately 1984, and there is evidence that the rare earth lines are strengthening on schedule (Cowley and Rice, 1981).

Another star for which there is good evidence for a period of decades is γ Equ. Bonsack and Pilachowski (1974) argued that the observational evidence available up to that time favored the hypothesis that the magnetic field had been declining monotonically from a value of +500 g in 1948 to −150 g in 1973. If the magnetic variations are symmetrical about $H_e = 0$, then the implied period may be as long as 72 years. Subsequent observations by Borra and Landstreet (1980) yield $H_e \cong -400$ g; this value is in agreement with the period suggested by Bonsack and Pilachowski (see also Scholz, 1979).

Photometric searches for other Ap stars with periods in excess of ~15 days have been summarized by Wolff (1975b). Only the cooler (non-Si type) Ap stars have such long periods, and the period-frequency distribution derived by Wolff for the cool Ap stars is shown in Figure 4-27. More than half of these Ap stars have periods less than 3 days, and the number of Ap stars decreases rapidly with increasing period.

Several authors have noted the similarity between the period of HR 465 and the length of the solar cycle (e.g., Krause and Scholz, 1981). They then suggest that the long-period variations are not to be associated with stellar rotation but with some other (usually unspecified) mechanism. On the basis of the data in Figure 4-27, Wolff argues the contrary point of view. The frequency distribution is smooth as a function of period. There is no discontinuity between $P < 30$ days, where the rigid rotator model is certainly correct, and $P >$ 30 days. Rather, Ap stars are found at all values of P from days to years. It is also true that the variable properties—amplitudes, phase relation-

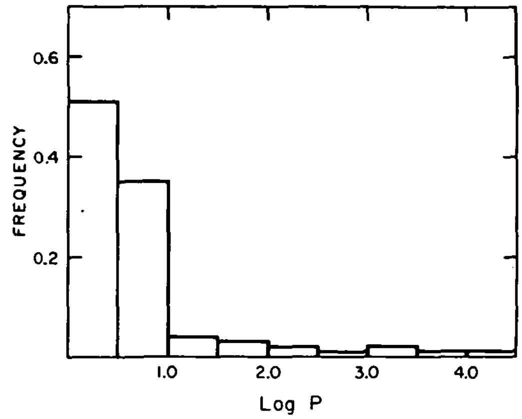

Figure 4-27. Distribution of periods for non-Si Ap stars (from Wolff, 1975).

ships, etc.—of the long-period Ap stars are in every way similar to those of the short-period Ap stars. These arguments of similarity and continuity, according to Wolff, are strong support for the hypothesis that in all Ap stars, the period of variation should be identified with the period of rotation. Such long rotation periods imply, of course, that some kind of powerful deceleration mechanism must have operated to brake the rotational velocities of Ap stars.

TESTS OF THE RIGID ROTATOR MODEL OF THE AP STARS

Observations that support the validity of the rigid rotator model have been mentioned in several of the preceding sections of this chapter. The correlation between period and line widths was one of the earliest, and remains one of the strongest, pieces of evidence in favor of this model. Another notable success of the rigid rotator model is its ability to explain the sign and amplitude of the crossover effect (Babcock, 1956). The line splitting in α^2 CVn provides very strong evidence that the distribution over the surface of this star is nonuniform (Pyper, 1969). The oblique pulsator model proposed by Kurtz (1982) to explain photometric variations that occur on time scales of minutes assumes that the rigid rotator model is correct and seems to substantiate this assumption in a number of ways.

Measurements of H_s provide powerful support for the rigid rotator model, and for the hypothesis that the magnetic field is predominantly dipolar as well. In β Cr B, for example, spectroscopic measurements of v sin i, coupled with the value of the period, imply that $i < 30°$. The nearly symmetric reversal of the magnetic field then implies that $\beta > 86°$, where β is the angle between the magnetic and rotation axes. The polar field, which is given by the relation (Stibbs, 1950)

$$H_p = \frac{3.3\ H_e\ (\max)}{\cos\ (\beta - i)}, \qquad (4\text{-}18)$$

should exceed 6 kilg. The surface field H_s varies from (0.80 to 0.64)H_p, and so H_s in β Cr B should exceed ~4 kilg. The measured values range from 4800 to 5800 g (Wolff and Wolff, 1970).

If the rigid rotator model is correct, then the mean transverse field, and therefore the electric vector of the linearly polarized light, should describe a 360° rotation during the course of each complete magnetic cycle. Borra and Vaughan (1976) have observed the polarization of Fe II λ4520.2 at eight different phases in the cycle of β Cr B and find that the linear polarization of this line behaves in precisely the manner predicted by the rigid rotator model.

Variations in the profile of the Ca II K line in Ap stars can best be explained in terms of abundance variations over the stellar surface and, therefore, provide additional support for the rigid rotator model. Figure 4-28 shows profiles of the K line measured at two different phases of the cycle of 52 Her (Wolff and Preston, 1978b). In this, and in several other Ap stars, the variation in the equivalent width of the K line is primarily due to changes in the line wings (e.g., Preston, 1969c; Preston et al., 1969; Sadakane, 1974). An increase in abundance will lead to variations in the wings, but not the core of a line on the damping portion of the curve of growth. Another point illustrated by Figure 4-28 is that the central intensity of the K line is quite high even though the changes in the line wings suggest that the line is on the damping portion of the curve of growth. The line center is probably filled in by continuous radiation from portions of the star where the Ca abundance is low.

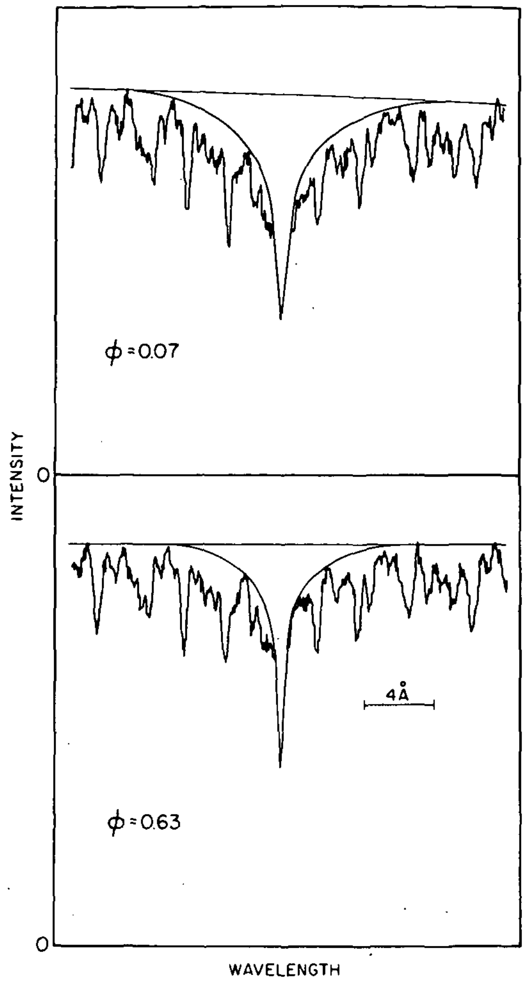

Figure 4-28. Intensity profiles of Ca II λ3933 in 52 Her measured near the phases of maximum (top panel) and minimum (bottom panel) line strength (from Wolff and Preston, 1978b).

An alternate explanation for the K line variations has been proposed by Snijders (1975). In stars with a constant Ca abundance, but with other abundances varying over the stellar surface, the radiation field, and hence the Ca II lines, will vary. This mechanism, however, probably cannot account for variations of a factor of 2, and such large variations are common among the cooler Ap stars.

Further evidence for surface inhomogeneities has been provided by detailed study of the magnetic variations of individual elements. In β Cr B (Preston, 1967b) and HD 188041 (Wolff, 1969a), the magnetic variations of the individual atomic species differ from one another. An example is shown in Figure 4-29. However, in those cases where two stages of ionization of a given element can be observed, the amplitudes of the magnetic variations are the same for both. It is very difficult to understand these results (particularly for such pairs of elements as Gd II and Ce II or Mn I and Fe I, which have similar excitation and ionization potentials and yet markedly different magnetic variations), unless there is some kind of spatial separation of the various atomic species. Two distinct magnetic fields cannot be produced by a single volume of gas.

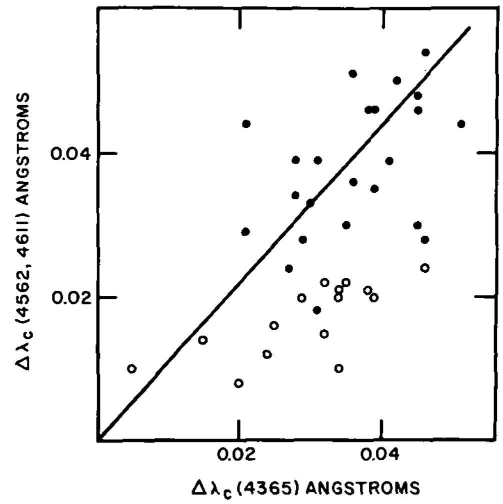

Figure 4-29. Zeeman displacements near magnetic maximum (abscissae) versus Zeeman displacements approximately 0.35 cycles later (ordinates) for Gd II (filled circles) and Ce II (open circles). The straight line is the best least squares fit to the Gd II data (from Wolff, 1969a).

EVOLUTIONARY CHANGES

A demonstration that any of the properties of the Ap stars–strength of spectral lines, rotational velocities, magnetic field intensities–change with time would provide very strong constraints for models of the origin of these stars.

Abt (1979) has reported that the frequency of Ap stars in clusters and associations increases with the age of the group. The sample size, however, is small, and this very interesting result needs to be strengthened by observations of additional clusters.

A quantity that might well be expected to change with time is rotational velocity. Two different models–selective accretion from the interstellar medium (Havnes and Conti, 1971) and diffusion plus mass loss (Strittmatter and Norris, 1971)–have been proposed to account for the abundance anomalies of Ap stars. Both models also predict significant loss of angular momentum on time scales of 10^7 to 10^9 years. These time scales are of the same order of magnitude as the main sequence lifetimes of magnetic Ap stars.

The first observational evidence for a correlation between rotation and age was offered by Wolff (1975a). For cool Ap stars, she showed that there appears to be a correlation between period and radius, in the sense that stars with larger radii tend to have longer periods. Since stellar radii tend to increase with evolution away from the ZAMS, the stars with larger radii should be older. A period-radius relation is therefore equivalent to a period-age relation. The scatter in the relationship, however, is large and some correlation is to be expected simply because of conservation of angular momentum as a star evolves.

The best way to search for a correlation between rotational velocity and age is to observe Ap stars in clusters with known ages. Despite unavoidable problems–small sample size, uncertain membership status, possible cluster-to-cluster variations in $\langle v \sin i \rangle$–such observations have been made by several authors. The data currently available are shown in Figure 4-30. There is clear evidence for a correlation between age and rotation for the hot Ap stars, including He-rich, He-weak, and Si-type objects, but no correlation for cool Ap stars (Hartoog, 1977; Abt, 1979). Figure 4-30 includes only three stars from the Orion Association. Recently, Joncas and Borra (1981) have used photometric techniques to identify many new Ap and He-weak stars in Orion, and preliminary measurements show that most of these stars have detectable magnetic fields (Borra, 1981). In those cases where a time scale for the magnetic variations has been estimated, the periods are ~1 to 2 days. The addition of these stars greatly strengthens the correlation between age and v sin i for the hot Ap stars.

In a discussion of the data in Figure 4-30, Wolff (1981) considers possible braking mechanisms. Mass loss at a rate of 10^{13} to 10^{14} g s^{-1} in the presence of a magnetic field can account for loss of angular momentum on the time scale of 6 X 10^7 to 6 X 10^8 years. This is the time scale required to explain the correlation for hot Ap stars in Figure 4-30. Wolff argues, however, that mass loss is unlikely to be the braking mechanism. First, there is no reason to expect radiation-driven, thermally driven, or centrifugally driven winds in Ap stars. Second, the required mass loss rate is so high that the entire atmosphere down to optical depth $\tau \sim 5$ would be lost in 10 to 100 years. This time scale is much shorter than the diffusion time scale (Michaud, 1970) and may pose problems for any model in which the abundance anomalies are confined to the outer layers of the star only. Third, because the magnetic fields of Ap stars are presumed to be independent of Ω, the angular velocity decreases exponentially. Unless the time constant for this decrease happens to be critically matched to the stellar lifetime, mass loss will result in virtually no braking or an excessive amount of it. The fact that Ap stars are rotating, but only very slowly, would then depend on a coincidence for which there is no apparent physical explanation (Mestel, 1975).

The alternative process, namely accretion from the interstellar medium, suffers from none of these problems (Mestel, 1975). If material is accreted by a rotating star with a strong magnetic field, then accretion will be halted at that distance from the stellar surface where the magnetic energy density and the kinetic energy density of the infalling material are comparable. The interstellar material will be held up at this boundary by the magnetic field, but the region within the boundary is essentially devoid of gas. This situation is unstable (Rayleigh-Taylor instability) and the gas will eventually enter the magnetosphere. Here it will be forced into corotation, and if the centrifugal force exceeds the gravitational force, the gas will move along field lines toward the equator. Ultimately, it will break loose from the magnetosphere and will carry angular momentum away with it. Typical interstellar densities (1 cm^{-3}) yield an e-folding time compatible with the value inferred from Figure 4-30. Braking will terminate when the centrifugal force at the boundary of the magnetosphere no longer exceeds gravity. The corresponding rotational period is on the order of weeks, so this process leads naturally to periods of the length that are typically seen in Ap stars.

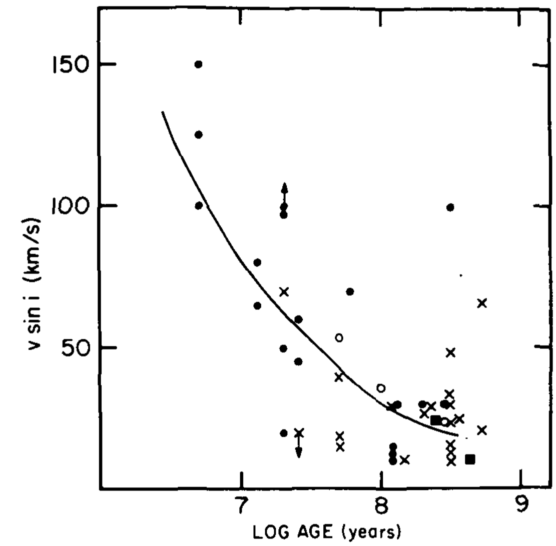

Figure 4-30. Plot of v sin i as a function of cluster age. Filled circles and crosses represent hot and cool Ap stars, respectively. Open circles represent data for field stars in Table 4-2. Solid line is fit to measurements of 20 Si stars derived by Abt (1979). All 20 stars are included in Figure 4-30, although in many cases new values of v sin i have been obtained (from Wolff, 1981).

It must be stressed, however, that even if (temporary) accretion from the interstellar medium is the correct explanation for the slow rotation of Ap stars, it cannot account for their abundance anomalies. The time required to accrete sufficient material to produce the large overabundances that are present even in very young Ap stars seems to be prohibitively long (Havnes, 1976).

Another mechanism must be sought to explain the much longer rotational periods (~years) that are seen in some Ap stars. Wolff suggests that pre-main sequence braking due to mass loss in the presence of a large magnetic field is the most likely explanation for such long periods. In this case, the fact that the angular velocity decreases exponentially is an advantage. Extensive mass loss can yield arbitrarily low values of the rotational velocity.

Combining these arguments, Wolff then concludes that pre-main sequence braking is a major factor in determining the rotational velocities of cool Ap stars. Braking continues after these stars reach the zero-age main sequence, but v sin i for these stars is already so low that it is difficult to detect additional decreases in line width. A strong correlation between (main sequence) age and v sin i for these stars is not expected and is not seen (Hartoog, 1977). The hotter Ap stars, however, evolve so quickly during the pre-main sequence phase of their evolution that implausibly high mass loss rates would be required to produce significant loss of angular momentum. For these stars, most of the magnetic braking occurs during the main sequence phase of evolution, and observational evidence for this gradual loss of angular momentum seems quite strong (Abt, 1979).

Evidence that magnetic field strengths also vary systematically with stellar age has been presented by Borra (1981). From Hβ polarimetry of 13 Ap and He-weak stars in the Orion Association, he concludes that the magnetic fields of the Orion stars are stronger by about a factor of 3 than the fields of the older Ap stars near the Sun. Borra interprets his observations as favoring the hypothesis that the fields in Ap stars are fossil fields. The e-folding time implied by the observations for decay of these fields is $\sim 10^8$ years, a value significantly below Cowling's (1945) estimate of 10^{10} years for the ohmic decay time for a plasma at rest. Borra suggests that mass motions, possibly circulation currents driven by rotation, are responsible for the shorter decay time. For more detailed discussion of this possibility, see Mestel and Moss (1977), Moss (1977), and Mestel et al. (1981). Very recently, Moss (1982) has calculated evolutionary models for a star with a mass of 2.25 $M_\odot$ and a dipolar magnetic field. The magnetic and rotation axes were taken to be parallel; rigid body rotation and conservation of angular momentum were adopted. With these assumptions, Moss finds that the surface field declines monotonically with an e-folding time of $\sim 3 \times 10^8$ years, a value comparable to such a star's main sequence lifetime.

However satisfying this agreement between theory and observation may seem, a caveat is in order. Most of the young stars with large magnetic fields are actually He-weak Bp stars, which are more massive than the stars classically defined as Ap stars. Therefore, existing data can be interpreted equally plausibly as a correlation between mass and field strength. More data are required to determine whether $\langle H_e \rangle$ varies with mass at a given age or with age at a given mass.

If both rotation and magnetic field strength decline as an Ap star ages, do the abundance anomalies become progressively more pronounced at the same time? A number of authors have suggested that they do (e.g., Maitzen, 1976; Gerbaldi and Morguleff, 1981). Perhaps the best evidence for this effect comes from studies by Joncas and Borra (1981, 1982), who use the a index to measure the depression at λ5200 in Orion OB1 and in the Upper Scorpius Association. They find that the depression is smaller in these very young stars than in older Ap stars of similar temperature in the solar neighborhood and conclude from this result that metal enrichment must take place on time scales of a few times 10^7 years.

The question of whether or not there are systematic changes in composition with time merits much more study. The extreme diversity of the spectrum anomalies of Ap stars makes it difficult to define peculiarity in terms quantitative enough to allow a search for correlations between peculiarity and other observable parameters. The a index does provide a quantitative measure of peculiarity but the cause of the broad depression at λ5200 is not known, and anomalies in the a index cannot yet be related directly to elemental abundances.

ABUNDANCES

The classification of Ap stars depends only on the appearance of the spectrum. Obviously, one would like to know if the changes in appearance along the Ap sequence from λ4200 Si stars to Sr stars reflect changing ionization conditions or if there are real abundance differences among the various classes of Ap stars.

An abundance analysis of the Ap stars is subject to considerable uncertainty. Many of the stars are spectrum variable, and so the abundance is not uniform over the stellar surface. Magnetic fields can have a significant effect on radiative transfer in lines and hence on the line intensities. The excess blanketing in Ap stars, particularly in the far ultraviolet, has a major effect on the temperature structure of the atmosphere. No abundance analysis to date has attempted to take all of these effects into account. Rather, the hope has been that a conventional LTE analysis of a sample of Ap stars will provide fairly accurate estimates of differential abundances within the sample. As additional support for the approximate validity of LTE models, researchers often cite the facts that (1) the continuous energy distributions of normal and Ap stars in the region λλ3200 to 10800 are indistinguishable after suitable corrections are made for line blanketing; (2) that the profiles of the Balmer lines of normal and Ap stars are identical; and (3) that the Si III/Si II ionization balance is the same in normal and Ap stars of similar color (Adelman, 1973a, and references therein).

Even if one accepts the basic premise that an LTE analysis can provide a reasonable estimate of the composition of an Ap star, it is often difficult to intercompare the results of different authors. The derived abundances depend on the temperature scale, the atmospheric model, and the choice of atomic parameters, including *gf* values. In attempting to sort out the systematics of abundances in Ap stars, the most useful studies, therefore, are ones in which a significant sample of objects has been treated in an identical manner. In the discussion that follows, we will focus on studies of this kind. References to abundance analyses of individual stars can be found in Adelman (1973a) and Bonsack and Wolff (1980).

Early surveys of Ap stars to establish abundance behavior of specific elements were carried out by Sargent, Searle, and coworkers. Reviews of this work have been presented by Sargent (1964) and Sargent and Searle (1967b). Subsequently, a search for Ne in Ap stars was reported by Sargent et al. (1969). Megessier (1971) determined the abundance of Si in the λ4200 stars, and Adelman (1973a) completed an abundance analysis of essentially all the observable elements in 21 cool Ap stars.

The primary conclusions that can be drawn from these surveys are best illustrated by examining the results obtained by Adelman (1973a) for his sample of 21 Sr-Cr-Eu stars. Figure 4-31 shows the logarithmic abundances with respect to hydrogen derived for these stars as well as for two normal stars. Overabundances for the Fe-peak elements are typically one to two orders of magnitude, with Cr showing the largest enhancement. Overabundances of the rare earths are even larger; the abundance of Eu is estimated to be typically 10^5 times larger than the solar value. The abundances of the heavy elements in cool Ap stars are, however, quite uncertain. Elements like uranium occur only in stars with the most complex spectra in which line blending can be a severe problem. For example, strong absorption features used to derive abundances of uranium and neodymium are at least in part due to lines of chromium (Cowley and Arnold, 1978; Cowley, 1981). Future studies may substantially revise the abundances plotted in Figure 4-31 for elements with $z \geq 57$.

The abundances of the Ap stars differ in several important respects from those derived for Am stars (see Chapter 5), which have similar temperatures. In general, while the abundances of the Ap and Am stars overlap for some elements, the abundances of Cr, Sm, Gd, Mn, Nd, and Eu are much higher in the Ap stars than in the Am stars. For Y, and probably Ni, the reverse is true. For the Am stars, the abundances of all the elements correlate well with one another. For the Ap stars, the Fe-peak abundances are well correlated with each other

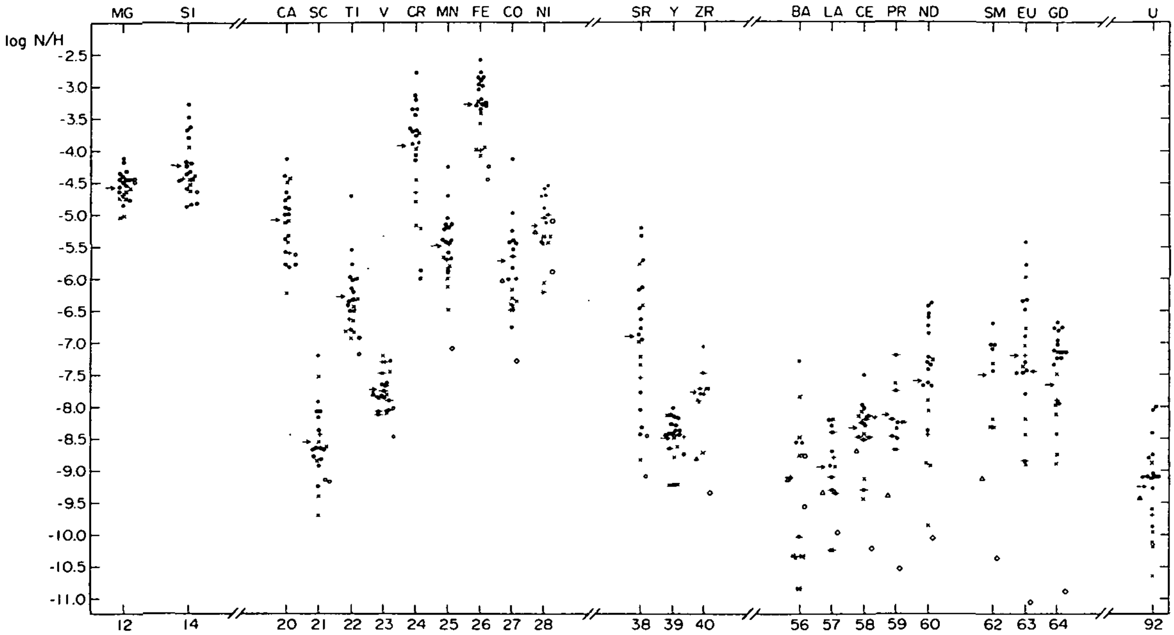

Figure 4-31. Log N/H values, the absolute abundances, as a function of atomic number. Closed circles, cool Ap stars with $T_{eff} > 9450$ K; +, HD 81009; crosses, the other cool Ap stars; open circles, normal stars. Uncertain values are indicated by bracketing horizontal lines except for HD 81009, for which a plus sign with a diagonal is the symbol. Arrows indicate the mean elemental abundances for those cool Ap stars with deduced abundances. In case all the cool Ap program stars did not have abundances deduced for an element, a second average was made by assuming that those cool Ap stars without lines measurable for equivalent widths had solar abundance values. These averages are indicated by open triangles. The atomic number for each element with deduced abundances is given below the diagram, while the corresponding chemical symbol is above it. Those elements for which abundances were not determined in the normal stars have their solar abundances indicated by open diamonds; the solar abundance of U is log U/H = −12.30 (from Adelman, 1973a).

but not with the rare earths. This last result is in agreement with the results derived by Cowley and Henry (1979) from statistical classification techniques. The scatter in the abundances of the Ap stars is larger than is typically seen in Am stars, particularly for the rare earths, and substantial differences in abundance occur for stars of approximately the same effective temperature. For most elements, there is a tendency for the overabundance to increase with increasing temperature (Adelman, 1973a; Bonsack and Wolff, 1980). The observed abundances do not seem to correlate with either rotation or magnetic field strength (for specific examples, see Bonsack and Wolff, 1980). In the absence of a similar survey of Si stars, it is impossible to determine in detail how similar the abundances of hot and cool Ap stars really are. Overall, the abundances for most elements seem fairly similar, but a few clear differences do exist. For most of the cool Ap stars, the abundance ratio Si/Mg is normal, while in the Si stars, Si/Mg exceeds the solar value by a factor of 25 or more (Searle and Sargent, 1964). The reverse is probably true of Cr. The ratio Cr/Fe appears to be larger in the cool Ap stars than in the Si stars. Other possible systematic trends in composition are more subtle, and differences in the techniques of analysis may mask—or introduce—discrepancies between the abundances inferred for the Si and the Sr-Cr-Eu stars. A full-scale study of the Si stars seems warranted. Such a study should set upper limits on the abundances of elements like the rare earths, which are conspicuous in cool Ap stars but not in hot ones.

Whether the absence of rare earth lines in the hottest Si stars is solely due to ionization might be answered by careful upper limits on the equivalent widths of singly ionized rare earths in the visible part of the spectrum, and by direct measurements of doubly ionized rare earths (Aikman et al., 1979). Another question that such a survey might answer is whether or not there is a clearcut separation between normal and Ap stars. Both Durrant (1970) and Megessier (1971) find that there is a smooth transition in the abundance of Si from normal stars to the λ4200 Si stars.

Another advantage of studying the hotter (Si) Ap stars is that abundances can be derived for certain lighter elements that are normally unobservable in the cool Ap stars. Preliminary studies indicate that He, C, O, and Ne are all underabundant in Si stars (Sargent and Searle, 1967b; Sargent et al., 1969).

A final question in relation to abundance studies of Ap stars is whether the line strength anomalies are due to real abundance effects or rather to abnormal excitation and ionization conditions within the stellar atmosphere. It is universally believed (discussed in the next section) that the apparent abundance anomalies are confined to the outer layers of the star only and are not characteristic of the stellar interior. In this sense, the line strength anomalies are an atmospheric phenomenon. It seems impossible, however, to reproduce the observed line intensities by a combination of a normal (solar) atmospheric composition and departures from LTE. The arguments have been set out in detail for HD 34452 by Tomley et al. (1970). In HD 34452, He is found to be deficient by a factor of 10. Calculations of the effects of departures from LTE in B-type stars show that the equivalent widths of the blue He lines are virtually unaffected (Auer and Mihalas, 1973). The fact that the He I lines in HD 34452 are weak eliminates the possibility that they are formed at such small optical depths that the model atmospheres are grossly inadequate.

Weak lines of He I might result if helium were excessively ionized. Even if this were the case, however, the excited lines of neutral helium would be substantially weakened only if He II is predominantly in excited states (which implies an appreciable abundance of He III). Since He II λ4686 is not present, this hypothesis is untenable. The He I lines could also be weakened by either an underpopulation of the lower levels from which they arise or an overpopulation of their upper levels. However, the excessive strength of the lines of Si III, which have similar excitation potentials, argues against this possibility.

Similar arguments can be made for chlorine, which is overabundant by a factor of 1000 in HD 34452. In the atmosphere of HD 34452, chlorine is mostly singly ionized. Since the second ionization potential of chlorine is nearly the same as the first ionization potential of helium, any effect of the radiation field that would tend to enhance the population of Cl II should have the same effect on He I, but He I is observed to be underabundant. Any excess radiation that might yield an overpopulation of the levels from which the Cl II lines arise should also yield an overpopulation of S II levels of comparable excitation potentials, but the corresponding S II lines are not seen.

Analogous arguments could be made for other Ap stars but in general have not been, because the assumption that the apparent abundance anomalies are real is almost universally accepted. At the end of their discussion of this issue Tomley et al. (1970) comment:

> . . . We feel that our case for the reality of the abundance anomalies is sufficiently strong to place the burden of proof upon those who disagree with the results. It is incumbent upon such critics to specify an atmospheric structure such that its radiation field explains the measured line strengths, hydrogen line profiles, and colors with a normal composition. We have been unable to do so.

To date, no one has met this challenge for any Ap star.

ORIGIN OF ABUNDANCE ANOMALIES

While the variations of the Ap stars seem to be explained adequately by the rigid rotator model, the origin of the abundance anomalies remains something of a mystery. All of the theories proposed to date, with one exception, appear to be fundamentally flawed. The one remaining possibility, radiative diffusion, requires an atmospheric stability that is well beyond what many theoreticians think likely.

All models for the origin of the abundance anomalies postulate that the abnormal composition is characteristic only of the outer layers of the star. The line strengths cannot be indicative of the composition of the star as a whole (e.g., Adelman, 1973b). If the atmospheric abundances of Eu were characteristic of the interior as well, virtually the entire galactic abundance of this and other rare earths would be locked into the Ap stars. It is also true that there are no late-type giants with compositions that are closely similar to those derived for Ap stars. If the anomalies were present throughout the peculiar stars, then it should be possible to identify their descendants among stars that have evolved away from the main sequence.

A number of Ap stars are found in galactic clusters or visual binaries, the other members of which have apparently normal composition. This observation argues against the possibility that Ap stars formed from material with an anomalous composition. The Ap stars are unlikely to be highly evolved objects (see the section on Temperatures and Luminosities). Their presence in a number of very young clusters and associations is taken as evidence that the time scale for the production of the abundance anomalies can be no longer than 10^7 years.

Beyond these rather simple properties, there is little agreement about which are the characteristics of Ap stars that any acceptable theory must explain. Indeed, the characteristics deemed most significant are likely to depend on the theory being espoused. With respect to abundance patterns, for example, proponents of the radiative diffusion model are likely to emphasize the general deficiencies of the normally abundant light elements, including helium, and the large overabundances of normally rare elements like mercury, platinum, and the rare earths. Proponents of theories that involve nuclear processing are likely to emphasize the facts that the odd-even pattern of abundance ratios is maintained in most cases in Ap stars, and that *r*-process elements such as uranium and thorium have been identified in Ap stars.

In a review of abundance patterns in Ap stars, Cowley (1980a) has stressed the importance of examining the data dispassionately, with no specific theory in mind, in order to determine what kinds (note Cowley's use of plural) of physical processes are likely to be involved. The specific question that Cowley addresses is, in fact, the fundamental one. Are the abundance anomalies that we now see the result of local (i.e., within or near the Ap star) nucleosynthesis? Or are they the consequence of some process, a kind of fractionation, that concentrated elements selectively in the photospheres of Ap stars? In principle, an observational distinction between these two hypotheses should be possible. Cowley points out the kinds of correlations one might look for. If nuclear processing is a prime cause, then one would expect to see the odd-even alternation of abundances, low abundances near Sc, high abundances at the Fe-peak, and perhaps some evidence of the *s* and *r* process abundance peaks. In addition, there should be a general correlation of abundances with atomic number, since elements in the same row of the periodic table tend to be built up by a similar nucleosynthetic process. If, on the other hand, the abundance anomalies are due to some kind of sorting process that takes place within the stellar atmosphere, then the composition should correlate with atomic structure rather than with atomic number.

Unfortunately, the observations do not allow a clearcut distinction between these possibilities. Often, not enough adjacent elements can be measured in order to search for violations of the odd-even effect. Transition probabilities, as well as ionization and excitation balances, may be sufficiently uncertain to render absolute values of the calculated abundances suspect. If radiative diffusion is effective, and elements are driven upward from fairly deep layers within the stellar envelope, then elements with adjacent atomic numbers will pass through the same isoelectronic sequence, and abundances may show some correlation with atomic number even in the absence of local nuclear processing.

Cowley (1980a) has reviewed the evidence concerning the possible role of nuclear mechanisms in producing the abundances seen in Ap stars. Two kinds of observations are relevant. One can search for violations of patterns typical of nuclear processing, or one can look for patterns that match the expected results of specific nuclear processes, such as the *r* process. He concludes that in the Ap stars, even though departures from solar abundances are large, there are no definite cases in which the abundance of an element with an odd atomic number *Z* exceeds that of its neighbors in the periodic table with even *Z*. The lanthanides follow two basic patterns, although there are many star-to-star variations on these themes. The most common pattern is one in which Ce is strong, while the abundances of the lanthanides of intermediate and greater mass decrease progressively. In other stars, Eu is dominant, the intermediate and heavier lanthanides are clearly present, and Ce is weak. In HR 465, the lanthanides that are missing–La, Pr, Tm, and Lu–all have odd *Z*. These patterns could be encompassed within nucleosynthesis schemes that include varying contributions from the *r* and *s* processes. In HR 465, uranium and thorium are also present. Both elements are heavier than bismuth, the last alpha-stable element, and can only be made in the *r* process.

Unfortunately, as Cowley (1980a) also notes in his review, there are at present no nucleosynthesis schemes that can account in any detail for a majority of the properties of the Ap stars. The specific proposals that have been made have been discussed critically by Adelman (1973a) and Catalano (1976). Three different sites have been proposed for nuclear processes–the surface of the Ap star, its interior, and a nearby companion star.

In 1965, Fowler et al. suggested that the Ap stars are highly evolved objects that have already passed through the red giant phase and returned to the main sequence where they are now observed. The anomalous abundances are attributed to helium and silicon burning and to the *r* and *s* process in the stellar interior. Products of these nuclear reactions are then brought to the surface through mixing. One primary deficiency of this model is that helium is typically underabundant in Ap stars–not overabundant as one would expect of highly processed material. Interior nuclear reactions cannot account in detail for the abundance patterns seen among light elements (C, O, and Ca) or among the rare earths. In common with all nuclear theories, this model does not attempt to explain the correlation between temperature and abundance that is seen for many elements, for the inhomogeneous distribution of the abundance anomalies over the stellar surface, or for the association between magnetic fields and chemical peculiarities. The hypothesis that the Ap stars are highly evolved also seems in conflict with their membership in very young clusters.

As an alternative, Brancazio and Cameron (1967) proposed that magnetic fields might accelerate protons and induce spallation reactions at the stellar surface. The net effect is to break down heavier elements into H and He. Acceleration of α particles may also induce nuclear reactions, but in this case the net result is constructive, and heavy elements may be built up. The problems with this model are numerous. If both protons and α particles are accelerated, nuclear reactions with protons will dominate, and heavy elements will be broken down and not built up. Therefore, a selective acceleration mechanism is required. Large overabundances of Fe and Si cannot be produced simultaneously, since much of the Fe excess is due to capture of α particles by Si and adjacent elements. This process will also tend to smooth out the odd-even alternation of abundances hear the Fe-peak, and this prediction is contrary to observations.

Another class of models proposes that the nuclear reactions occur in binary companions to the Ap stars (e.g., van den Heuvel, 1967; Guthrie, 1967; Guthrie, 1971, and references therein). The specific scenario is as follows. The star that is to become an Ap star is originally the less massive of the two components of a close binary system. The more massive component evolves more rapidly and reaches a stage where it explodes as a supernova. The light elements (H, He, C, N, and O) are the first to reach the secondary. Since the velocity ($\sim$5000 km s^{-1}) of the exploding material greatly exceeds the escape velocity of the secondary, this material sweeps past the secondary and very little is deposited on it. Heavier elements expelled by the primary include products of explosive carbon burning, iron peak elements formed during the quasi-equilibrium burning of silicon, *s* process elements formed when the primary was a red giant, and *r* process elements formed at the center of the explosion. If these heavy elements are deposited on the secondary, then this scenario can account schematically for the abundance distribution of many of the elements in Ap stars. There are, however, some specific problems noted by Guthrie (1971). For example, the overabundances of the rare earths are much too high relative to abundances measured for Sr, Y, and Zr. Other difficulties have been discussed by Adelman (1973a) and Catalano (1976). The binary hypothesis cannot account for the relationship between composition and temperature. If the heavy elements were produced in a supernova explosion, their relative abundances ought to resemble solar values, and they do not. The kinematics of such systems might be expected to be different from the kinematics of normal A-type stars and they are not. Present theories of supernovae in binaries suggest that the explosion is unlikely to disrupt the system. If Ap stars were produced in the manner described by Guthrie, they should still have (collapsed) companions, yet most Ap stars are single. The supernova hypothesis also has difficulty explaining systems with one Ap star and one normal star, and such systems, while rare, do exist.

Two nonnuclear processes for explaining the abundance anomalies of Ap stars have been discussed extensively in the literature. The first, magnetic accretion (Havnes and Conti, 1971), depends on the interaction of the stellar magnetic field with the interstellar medium. At the boundary of the magnetosphere ($\sim$15 AU for a typical Ap star), most atoms in the interstellar medium are singly ionized. Light elements from the interstellar medium will be deflected when they encounter the magnetosphere, but heavier elements, because of their larger mass-to-charge ratio, may penetrate it. If they become ionized again, their radius of gyration is reduced and they may be captured by the star. Once captured, the particles describe periodic orbits as they spiral along magnetic field lines. Accretion onto the stellar surface occurs preferentially at the poles. This model has the dual advantages of predicting an inhomogeneous distribution of surface elements and of explaining the slow rotation of the Ap stars (see the section on Evolutionary Changes). It can also account in an approximate way for the kinds of abundance anomalies that are seen (but see Guthrie, 1971).

There are, however, two primary problems. Some clusters contain a wide variety of peculiar stars (Abt, 1969), and it seems unlikely that the interstellar medium is sufficiently inhomogeneous that accretion of such diverse kinds of material would occur over such small volumes. Even more troublesome is the problem of time scale. Some very young Ap stars have quite pronounced abundance excesses. It appears to be impossible to accrete the amount of material required in the time available (Havnes, 1976). The problem is only partially alleviated by postulating that the accreted material is not mixed into the star but remains in the line-forming region ($\tau < 0.1$). Even in this case, the accretion time scale is longer than the diffusion time scale; and Michaud (1976) has, therefore, argued that if both processes are operative, diffusion must dominate. Accretion will, at most, only slightly perturb the effects of diffusion.

The remaining model attributes the abundance anomalies to radiative diffusion (Michaud, 1970). Because this process has been offered as an explanation for all the peculiar stars of the upper main sequence, it is discussed extensively in Chapter 9. The basic premise is that, in a stable atmosphere, those elements that experience an excess force caused by radiation pressure, transferred either through bound-bound or bound-free transitions, will be driven upward in the atmosphere and concentrated in the line-forming regions. Those elements that have few transitions in the wavelength region where the stellar flux is at its maximum will tend to sink. The numerous successes of this theory are summarized in Chapter 9. The primary deficiency of the model is that it is impossible to demonstrate, with our present knowledge of hydrodynamics, that the extreme atmospheric stability required for this process to have any net effect actually obtains. Nevertheless, the diffusion model is the only one extant that provides a plausible explanation of most of the Ap star anomalies. Even Cowley (1981), who has maintained a healthy skepticism toward all models of the origin of the Ap stars, has said:

> . . . Without doubt, the diffusion hypothesis is in best accord with the majority of "facts" about [chemically peculiar] stars as we think we know them today. The facts are explained in terms of plausible physics with a minimum appeal to unknown mechanisms. If there could be only one theory allowed to us, it would, on the basis of everything we think we know now, surely have to be diffusion.

The specific problem of diffusion in a magnetic field has been analyzed by Michaud et al. (1981). Magnetic fields can influence the diffusive separation of elements by suppressing macroscopic motions and by influencing the motions of ionized particles. If densities are low and collisions infrequent, then ions will tend to spiral along magnetic field lines. Michaud et al. estimate that the critical density is $N_p \sim 10^{13}$ cm^{-3} which corresponds to $\tau_{5000} \sim 10^{-2}$. An ionized element can diffuse freely, uninfluenced by the magnetic field, until it reaches this density. As the ion continues to diffuse upward into regions of lower density, it will follow the field lines more and more closely. Diffusion will stop where the field lines are horizontal. The distribution of elements over the stellar surface will then reflect the magnetic geometry. Figure 4-32 shows a variety of possible magnetic field structures for various ratios of H_q/H_d, where H_q and H_d are the strengths of the field at the poles of a magnetic quadrupole and dipole, respectively. The quadrupole and dipole are assumed to be aligned and to have the same sign. Diffusion in a magnetic field may cause the elements to be distributed in patches or rings, and the magnetic latitudes of these features depend on H_q/H_d.

If this hypothesis is correct, magnetic and spectrum variations are linked in a predictable way, and Michaud et al. calculate the variations in line profiles, line strengths, H_e, and H_s that are to be expected for specific Ap stars on the basis of their models. Unfortunately, a complete set of observational data is not yet available for any one star. Presumably such data could be obtained and might finally provide a critical test of diffusion. If a radiative diffusion model can actually predict the surface distribution of elements in Ap stars—a feat attempted by no other model—then the remaining skeptics, and they are many, might finally capitulate.

The converse, unfortunately, is probably not true. There are a number of parameters in the model that can be adjusted to yield different surface distributions. Zeeman spectroscopy cannot distinguish between an offset dipole and a centered dipole plus quadrupole, yet the patterns of field lines are quite different in these two cases. In decentered dipoles, field lines are always vertical at the poles, and they are horizontal only in one region of the star, even if the decentering parameter is large. As Figure 4-32 shows, a dipole plus quadrupole field can yield much more complex patterns of field lines.

A second relevant parameter is age. For example, the ions Si I to IV can be supported by the radiation field but Si V cannot (Alecian and Vauclair, 1981b). The reservoir of Si is therefore finite and after ~10^6 years it will have all diffused to the surface. A horizontal magnetic field is required to support Si, and the overabundance will therefore initially occur in regions where the field lines are horizontal (see Chapter 9). As time passes, and more Si arrives from below, the Si will diffuse horizontally to regions where the field lines are vertical. Since Si is not as well supported here, some will settle and the apparent abundance will decrease. On this hypothesis, both the abundance of Si and its distribution over the stellar surface become complex functions of age, a quantity that is almost never known with precision.

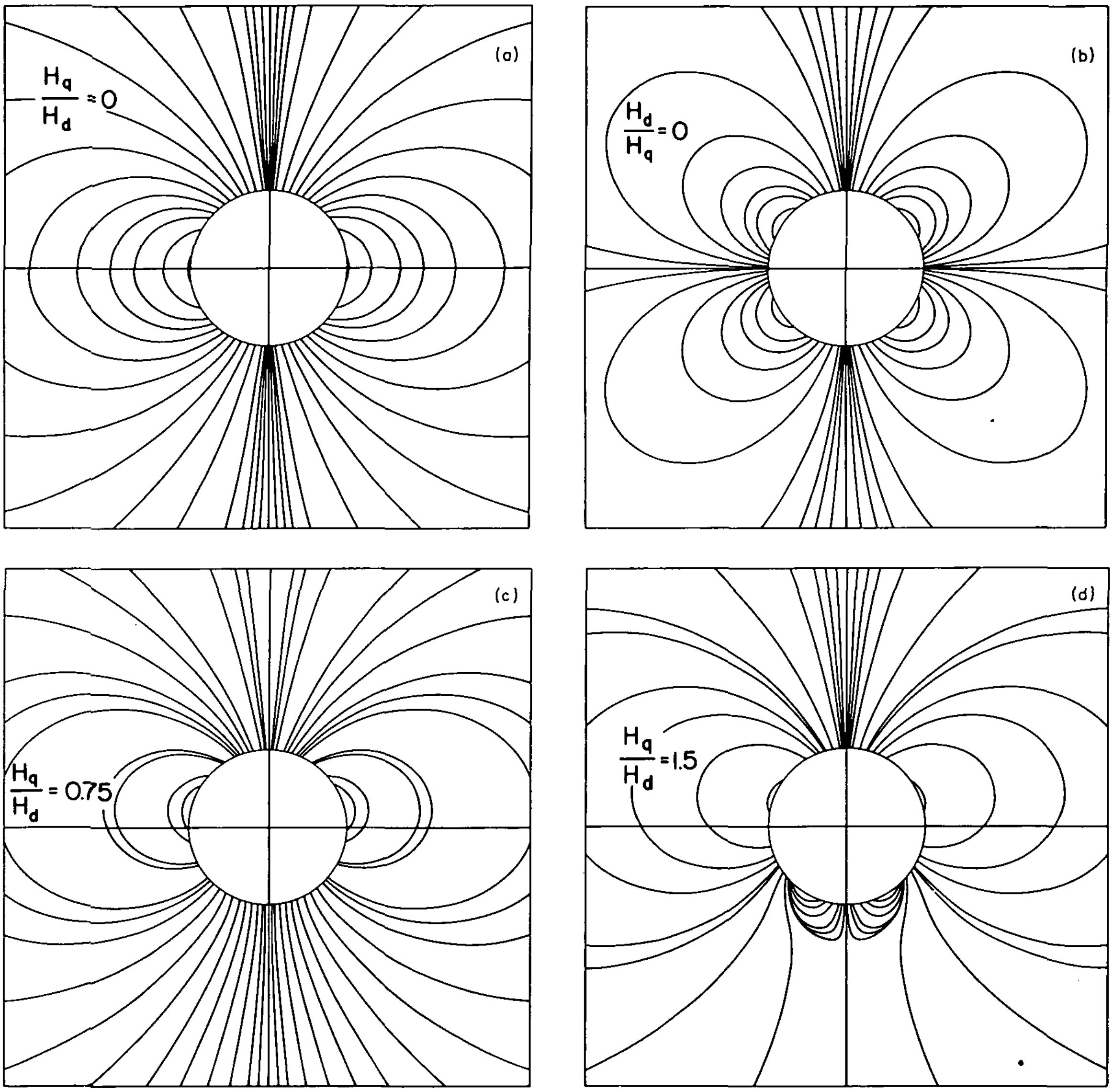

Figure 4-32. Magnetic field lines for a dipole (part a), a quadrupole (part b), and aligned dipoles plus quadrupoles (parts c and d). The line density is not representative of the field strength. The stellar surface is represented by a circle. The three-dimensional configuration is obtained by rotating around the vertical axis. For diffusion calculations the regions on the star where the magnetic field is horizontal and where it is vertical are most important. This varies considerably as the quadrupole strength varies from 0.75 to 1.5 that of the dipole, as allowed by the magnetic field observations. Note that magnetic field observations do not require, but do not rule out, the presence of octopoles (from Michaud et al., 1981).

A third significant parameter is turbulence. Michaud et al. (1981) argue that if turbulence is present, it will be more effectively suppressed in regions where the magnetic field lines are vertical. Diffusion can therefore compete more effectively with mass motions in such regions, and abundance anomalies should occur only where the magnetic field is vertical. Note that this prediction is the *converse* of the prediction for a star with no turbulence.

This discussion serves to underscore a basic problem with diffusion (see Chapter 9). It is such a fragile process that a whole host of other processes can drastically modify its effects. It is therefore very difficult to devise critical observational tests. On the other hand, the Ap stars are extremely complex. A theory with only a few free parameters probably cannot account adequately for their diversity.

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5

THE Am STARS

DEFINING CHARACTERISTICS

According to the classical definition, Am or metallic lined stars are objects in which the K line is considerably weaker than would be expected for the average metallic line type (Titus and Morgan, 1940; Roman et al., 1948). An additional condition is that there must be no possibility of explaining the observed spectrum in terms of two normal stars. That is, the spectral type inferred from metallic lines near the K line must be identical to that derived from lines at longer wavelengths. Often a more restrictive definition is used—an Am star is one in which the metallic line and K line types differ by more than five spectral subclasses. Some magnetic stars have weak K lines, but in general these stars also show strong enhancements of a small number of elements (Eu II, Cr II, Sr II, in particular) and can be distinguished from the Am stars on that basis.

In terms of identifying all the members of a class of stars with spectral anomalies produced by the same physical mechanism, this definition has a number of limitations. The spectral criteria used are most sensitive to line strength variations in late A-type stars and are relatively insensitive among early A-type stars. The definition does not allow for Am stars with marginal peculiarities and does not take into account the information that might be obtained from the study of weak lines.

Based on a survey of A-type stars at high resolution, Conti (1970) has proposed an alternate definition of the Am class:

> . . . The Am phenomenon is present in stars that have an apparent surface underabundance of Ca (and/or Sc) and/or an apparent overabundance of the Fe group and heavier elements.

As Conti emphasized, this definition admits to the Am class three different kinds of stars: those with weak Ca (Sc) and strong metallic lines (e.g., 63 Tau); those with only weak Ca (Sc) lines (α Gem B); and those with only strong metallic lines (17 Hya). The common characteristic is a difference between the Ca (Sc) lines and the other metallic lines. The reality of the second subgroup, however, has been challenged by Smith's (1974) abundance analysis of the prototype α Gem B, which yielded an overabundance of a factor of 5 for iron. The definition as phrased also presupposes an identity between line-strength anomalies and abundance anomalies. The evidence favors this identity, but it is a point that must be proved, not simply assumed.

With this broader definition, a variety of photometric techniques has been developed to search for Am stars. The m_1 index can be used to isolate Am stars with enhanced blanketing by metallic lines (Stromgren, 1963a; Cameron, 1967; Milton and Conti, 1968), while narrowband photometry at the K line is effective in identifying stars that are deficient in calcium (Henry, 1969). The photometric measurements suggest that there is a smooth transition between normal and metallic line stars (Hesser and Henry, 1971). Metallicism is not a discrete phenomenon that results in clearly isolated anomalies in line strength.

A more important consequence of the broadened definition of Am stars is that it allows a search for members of this class at spectral types earlier than A5. Among the early A-type stars, the hydrogen lines pass through a broad maximum in strength and are fairly insensitive to temperature. Spectroscopic identification of stars with metallic lines that are too strong for the temperature of the star, which is inferred from the Balmer lines, becomes difficult. Furthermore, the K line is on the flat part of the curve of growth, and its strength is relatively insensitive to abundance changes. However, detailed abundance analyses at high dispersion of Sirius (A1 V) and 68 Tau (A2 IV) show that both have abundance anomalies similar to those found in classical Am stars (Conti et al., 1965; Strom et al., 1966). A specific anomaly that is common to 68 Tau, Sirius, and the Am stars is an apparent deficiency of scandium, which manifests itself through an anomalous ratio of Sc/Sr line strengths. A further survey at high dispersion (10 Å mm^{-1}) shows that about half of the early A-type stars with sharp lines exhibit anomalous ratios of Sc/Sr (Conti, 1965a). Conti suggests that these stars are hotter analogs of the Am stars and that metallicism, therefore, extends into the early A-type stars. Indeed, it is this observation that suggests the inclusion of a scandium deficiency as one of the defining characteristics of Am stars.

LUMINOSITY AND TEMPERATURE

The early studies that attempted to define the characteristics of the Am stars and their position in the HR diagram have been reviewed by Smith (1971). In many respects the Am stars resemble F-type stars with low surface gravity. Like the F-type stars, Am stars rotate slowly as a group and their colors are fairly compatible with the metallic line spectral types. Certain characteristic spectral anomalies, including a weak Ca II K line and enhanced Sr II λ4077, suggest that, according to MK criteria, they lie above the main sequence.

Subsequent studies, however, demonstrate conclusively that the Am stars are not high luminosity F-type stars, and the evidence has again been reviewed by Smith (1971). Observations of both hydrogen line profiles (Böhm-Vitense, 1960; van't Veer-Menneret, 1963; Conti, 1965b; Baschek and Oke, 1965) and the visibility of the higher members of the Balmer series (Jaschek and Jaschek, 1959; Komarov, 1965) indicate that the hydrogen lines are broad and that the pressures in the atmospheres of Am stars are like those in dwarf stars. Broadband colors (e.g., UBV indices) are strongly affected by enhanced metallic line blanketing and cannot be used to infer temperatures (Baschek and Oke, 1965; Ferrer et al., 1970). In general, the effect of line blocking on the colors of Am stars increases with decreasing wavelength (Mendoza et al., 1978), and quite strong effects are seen in the ultraviolet (λλ1500 to 2700) region of the spectrum (van't Veer-Menneret et al., 1980, and references therein). Indices, such as the (b–y) color of the Stromgren system, that are relatively insensitive to blocking by metallic lines yield temperatures that are compatible with the hydrogen line spectral types, and hence with an A-type spectral class.

In order to determine the position of the Am stars in an HR diagram, it is necessary to correct the observed colors or energy distributions for line blanketing. Approximate techniques for doing so have been described by Baschek and Oke (1965) and Strittmatter and Sargent (1966). Both studies concur in finding that after deblanketing corrections have been made, the Am stars lie to the *left* of the main sequence defined by normal A-type stars; the particular clusters studied were Coma, Praesepe, and the Hyades. Strittmatter and Sargent interpret this result in terms of theoretical models, which indicate that at a given color, rapidly rotating stars appear more luminous than slowly rotating ones. Since the Am stars rotate on the average very slowly, they therefore appear to be somewhat subluminous. In view of the approximate nature of the deblanketing procedure, it is not clear how precisely determined the positions of the Am stars in the HR diagram actually are, and it would be interesting to reexamine the conclusions of Strittmatter and Sargent with modern model atmosphere calculations that incorporate blanketing explicitly and with observations that use photometric indices that are insensitive to line blocking. However, for the purposes of the present discussion, the important result of existing studies is that the Am stars appear to be main sequence objects. Further supporting evidence is presented in the next section.

In order to characterize a star, it is important to know its temperature as well as its luminosity. Smith (1973a, 1974) has examined early A-type and F-type stars in detail at high spectral resolution in order to determine the temperature domain for metallicism. Fm ($0.20 < b-y < 0.25$) stars are quite rare. The overall incidence of Am stars seems to peak at ~50 percent at $b-y = 0.15$ and drops to essentially zero at $b-y$ ~0.25 (cf., Figure 5-1). The lowest temperature found by Smith (1973a) for any Am star is 7400 K. Selection effects probably affect both this determination of the lower temperature boundary and the apparent rate of decline of the frequency of metallicism with decreasing temperature. It is difficult to detect F-type metallic line stars at low resolution, and at least one model for the origin of metallicism (diffusion) predicts that the magnitude of the abundance anomalies should decrease with decreasing temperature. Smith (1974) has also carried out abundance analyses of the early A-type stars α Gem A and B, as well as θ Leo, which was previously regarded as a spectral standard for the class A2 V. Relative to α Lyr, all three show enhanced lines of Fe and other heavy elements and in this respect resemble classical Am stars. In sharp contrast to cooler Am stars, Sc and Ca are enhanced in the hot Am stars. However, Sr is enhanced even more, so that the ratio Sc/Sr remains anomalous in accord with Conti's (1970) definition of metallicism. Helium appears to be deficient by factors of

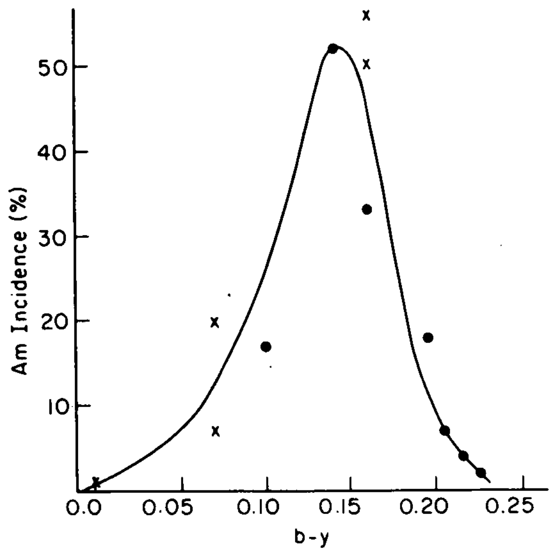

Figure 5-1. Percentage incidence of Am and Fm stars along the main sequence. Dots and crosses represent results of two different surveys (from Smith, 1973a).

3 to 5 in the well-studied Am stars that are hot enough to exhibit lines of He I (Lane, 1981). The hottest stars in which Am characteristics have been identified are α Gem A (T_{eff} = 10,200 K) and Sirius (10,150 K; Bell and Dreiling, 1981).

The low-temperature boundary for the HgMn stars is ~11,000 K (Wolff and Wolff, 1974), and a natural question is whether the Am and HgMn groups form a continuous sequence. One difficulty in establishing a link between these two groups has been the absence of transition objects. For example, Hg II λ3984, one of the defining features in HgMn stars, is probably absent in the two hottest Am stars (α Gem A and α CMa), although this conclusion is complicated by the presence of Fe I λ3983.96. A deficiency of helium does seem to be one abundance anomaly characteristic of both groups.

The question of the relationship between HgMn and Am stars has been examined in detail by Lane (1981). She finds that many elements show definite trends of abundance with T_{eff} that are continuous between the two classes of stars.

Furthermore, most of the curves relating observed abundances to T_{eff} pass through the value that corresponds to normal (solar) composition in the temperature interval 10,000 K to 11,000 K. Only Mn and Zr are likely to be significantly overabundant; therefore, Lane suggests that the absence of transition stars may be caused by the difficulty of recognizing such objects at the classification dispersions used for large-scale surveys. More work, both observational and theoretical, is needed to clarify this issue.

STAGE OF EVOLUTION

Insight into the evolutionary development of peculiar stars can be obtained from studies of cluster members for which the age is well determined. Several authors, including Jaschek and Jaschek (1967), Conti and van den Heuvel (1970), Smith (1972), Hartoog (1976), van Rensbergen et al. (1978), and Abt (1979) have searched for Am stars in clusters and associations.

The important results of these various surveys, which are in excellent agreement with one another, have been summarized by Abt (1979):

1. Even if the four purported Am stars with v sin $i > 200$ km s^{-1} are excluded from consideration, there are several Am stars in the Orion Association. Metallicism can, therefore, develop in 1 to 3×10^6 years.
2. The mean frequency of Am stars in clusters and in the Ori Ic Association is the same as it is among field stars.
3. The frequency of Am stars does not depend on the age of the cluster in which they are found (but see Hartoog, 1976), provided the cluster is young enough to have main sequence A-type stars. The anomalies do disappear as Am stars evolve away from the main sequence, so there is an upper limit on the ages of Am stars.
4. There is only marginal evidence for statistically significant differences in the frequency of Am stars from cluster to cluster.

The fact that metallicism occurs with the same frequency among field stars and in clusters of widely disparate ages argues that Am stars are not highly evolved objects, but rather are in the main sequence phase of evolution. This conclusion is supported by observations of spectroscopic binaries (Conti, 1967). The frequency of short-period spectroscopic binaries is nearly constant along the main sequence from B to G if, and only if, the Am stars are included in the sample. If they are excluded, then the binary frequency among late A-type stars is abnormally low. Conti argues that this result indicates that the Am stars are in the same state of evolution as other close binaries on the main sequence and therefore are unevolved.

It should also be noted that the masses of Am stars are by definition normal. Five of the sixteen spectroscopic binary systems used by Popper (1980) to determine the relationship between spectral type and mass for A-type stars contain at least one Am star.

VARIABILITY

Variability, either due to pulsation (δ Sct stars) or to rotation of an inhomogeneous surface (magnetic stars), is common among A-type stars. However, variability is absent, or nearly so, in the Am stars and the lack of variability may be an important clue to the cause of metallicism.

Variability in A-type stars may manifest itself in three ways: changes in magnetic field, changes in the position or strength of spectral lines, or changes in luminosity. Babcock (1958) measured magnetic fields of the order of a few hundred gauss in 7 of 14 Am stars included in his survey. However, conventional photographic Zeeman spectroscopy has failed to confirm Babcock's results (Conti, 1969a), while more precise photoelectric measurements of the circular polarization in Hβ by Borra and Landstreet (1980) set upper limits of 100 to 200 g on the longitudinal field present in the eight Am stars observed. There is no confirmed evidence of spectrum variability in Am stars apart from the radial velocity variations associated with binary motion. The variations seen by Abt (1961) in 32

Vir may be related to the fact that one component of this double-lined binary is a δ Sct variable (see Chapter 6).

Several extensive searches for photometric variability in the Am stars have yielded, with few exceptions, negative results. Breger (1970) found no variability on time scales of 1 to 6 hours in several Am stars in the Hyades and Praesepe. Winzer (1974) found at least six Am stars to be variable with amplitudes of ~0.01 mag and periods of 1.8 to 5.5 days. Of the six, HR 976 had the largest amplitude, and subsequent observations by Abt and Levy (1976) show that the phase relationship between the velocity and light curves, and the amplitude of the light curve, are both typical of what is seen in ellipsoidal variables. On the basis of their results for HR 976, Abt and Levy suggest that it is highly likely that the other photometric Am variables in Winzer's sample are also ellipsoidal variables.

Böhm-Vitense and Johnson (1978) have presented evidence for both color and luminosity variability in Am stars on time scales of years. Comparisons of their spectrophotometry of three Am stars and several normal stars with measurements made about 10 years earlier (Baschek and Oke, 1965) show that the differences in the energy distributions for the Am stars are four times larger than for the standard stars. Böhm-Vitense and Johnson also find that the values of V and B−V measured by various observers (Blanco et al., 1968) for Am stars vary more than the values for normal late A-type stars. From these data, Böhm-Vitense and Johnson conclude that all Am stars are probably variable on a long time scale.

This conclusion has been challenged by Lane and Lester (1980), who obtained spectrophotometric scans of several Am stars. They find evidence that 81 Tau may have varied by a small amount in the wavelength region $\lambda < 3636$ during the course of their five night observing run; the other nine Am stars that they observed were constant. More importantly, Lane and Lester compare their observations both with the spectrophotometry of earlier observers, including Baschek and Oke (1965) and Böhm-Vitense and Johnson (1978), and with uvby colors. This latter comparison is possible because uvby colors can be inferred quite accurately from the measured energy distributions. They find evidence for variability only in τ UMa and possibly 15 Vul. For both stars, the only observations that deviate significantly from the mean colors obtained by averaging all the available data are those by Baschek and Oke. In the case of τ UMa, nearly contemporaneous four-color data by Stromgren and Perry (1965) do not confirm the discrepant spectrophotometric measurements. Lane and Lester therefore conclude that either the peculiar energy distribution measured on that one occasion is due to some transient event, or, alternatively, that the observation by Baschek and Oke is not correct. Whatever the case for τ UMa, the constancy of the other Am stars does not support the suggestion by Böhm-Vitense and Johnson that all metallic line stars vary on a long time scale.

Further observations, with careful attention to possible systematic errors, are required to determine the nature of the variability, if any, of the Am stars. For the immediate purpose of modeling these stars, however, the most important result of the search for variability is the absence of changes in brightness on time scales of minutes to days. There is thus no evidence for either pulsation or surface inhomogeneities.

BINARY FREQUENCY

Fundamental work on the binary characteristics of Am stars has been carried out by Abt and described in two important papers (Abt, 1961, 1965). In the first, which reports the results of a radial velocity survey of Am stars, Abt concludes that 22 of 25 stars in the sample are spectroscopic binaries. Since some orbits must be viewed at small angles of inclination, Abt suggests that all Am stars are probably spectroscopic binaries. In the second paper, Abt determines that only 17 of 55 normal stars in the same range of spectral types are members of spectroscopic binary systems. Furthermore, none of the normal binaries has a period less than 100 days, while approximately two-thirds of the Am stars are members of such close binary systems. The frequency of spectroscopic binaries with periods longer than 100 days is similar for Am and normal A-type stars.

It is clear from the observations that the binary characteristics of the Am stars differ markedly from those of nonmetallic lined stars of similar spectral class. The question is whether this difference is a cause of metallicism or is only an indirect consequence of it. If *all* Am stars are indeed binaries, then one might conclude that binary interactions somehow cause the line strength anomalies. If, on the other hand, some Am stars are single, but all rotate slowly, then the correct conclusion might be that slow rotation is the necessary condition for metallicism. The preponderance of binaries among Am stars would be attributed to the tendency toward synchronism, and hence slow rotation, caused by tidal interactions in close doubles.

Some of the problems of trying to establish whether *all* Am stars, and *none* of the normal A-type stars, are members of close binary systems have been discussed by Batten (1967a). Of the original sample of 25 Am stars, Batten considers that radial velocity variations have been clearly established for only 17. The amplitudes found by Abt (1961) are very small for five stars, and Batten feels that additional observations are required to confirm the apparent variability; the velocities of the remaining three stars were constant. Batten also points out that in the Sixth Catalog of Spectroscopic Binaries (Batten, 1967b) there are 73 binaries with periods less than 50 days whose primaries have spectral types in the range A4 to F2 and that are not classified as Am stars. Batten suggests that the spectral classifications be confirmed, but inasmuch as line strengths vary continuously from normal to marginal to pronounced Am stars, the definition of metallicism can become a critical issue. In fact, only 16 of 25 stars in Abt's original sample fit the strong (a difference of five subtypes between the K line and metallic line types) definition of Am stars (Abt, private communication).

Some clarification of the relationship between Am and binary characteristics has been achieved through more recent studies. Abt and Bidelman (1969) have undertaken the project suggested by Batten—the accurate spectral classification of A-type primaries in binary systems. *Outside* the period interval $2.5 < P < 100$ days, Am and normal stars do coexist. Within this period range, however, all main sequence primaries with spectral types in the range A4 to F1 have metallic line or peculiar spectra.

In an effort to determine whether there are single Am stars, Conti and Barker (1973) have examined in detail five stars in the Coma Cluster. High-resolution abundance analyses show that all have Am characteristics (Smith, 1971), and only two proved to be variable in radial velocity. This sample is obviously too small for any statistical analysis. It is not possible to conclude that any specific star is single, since an unfavorable angle of inclination may conceal orbital motions. Nevertheless, Conti and Barker argue that so many Am stars exhibit constant velocity that at least some are likely to be single. The fact that some Am stars are members of binaries with quite long periods is another argument against scenarios for the origin of these stars that require an interaction between binary components.

DISTRIBUTION OF ROTATIONAL VELOCITIES

If at least some Am stars are not binaries, then it is logical to ask whether some other physical characteristic is uniquely correlated with metallicism. An obvious possibility is rotation. With one possible exception, which will be discussed in detail below, the maximum rotational velocity observed for Am stars is $\sim$100 km s^{-1} (Abt and Moyd, 1973). Since the techniques used to detect a weak K line through spectral classification, or to detect an enhancement of the m_1 index, are virtually insensitive to rotation, the absence of rapidly rotating Am stars is real.

There have been several attempts to determine whether slow rotation is both a necessary and sufficient condition for metallicism. The basic data were obtained by Abt and Moyd (1973), who derived values of the apparent rotational velocity, v sin i, for 66 Am and 123 normal A5 to A9 IV and V stars. On the assumption that the axes of rotation are randomly oriented, it is possible to calculate the distribution of equatorial rotational velocities $\psi(v)$ that will yield $\phi(v \sin i)$, the observed distribution of apparent rotational velocity.

Abt and Moyd show that one possible solution for $\psi(v)$ indicates that there is a negligible overlap in the rotational velocity distributions of the Am and normal A-type stars. They therefore conclude that slow rotation is both a necessary and sufficient condition for metallicism. However, there are many functions $\psi(v)$ that yield the same $\phi(v \sin i)$ (Brown, 1950). Application of the technique suggested by Lucy (1974) yields the maximum likelihood solution; the results are shown in Figures 5-2 and 5-3 (Wolff and Wolff, 1976). This solution shows essentially no overlap in the rotations of Am and normal A-type stars for $v < 40$ km s^{-1}, but considerable overlap in the velocity range $40 < v < 100$ km s^{-1}. (The magnetic Ap stars have been excluded from this discussion. Obviously, however, the rotational velocities of the Ap and Am stars do overlap.)

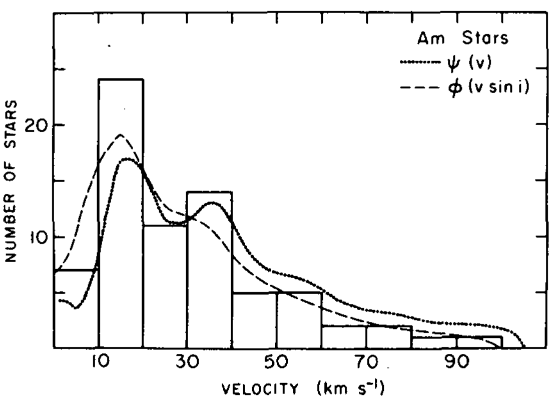

Figure 5-2. Distribution of observed rotational velocities (histogram) and of true rotational velocities $\psi(v)$ (dotted line) for the Am stars. Dashed line shows distribution of apparent rotational velocities $\phi(v \sin i)$ derived from $\psi(v)$. Iteration to determine $\psi(v)$ is continued until $\phi(v \sin i)$ adequately fits the observations (from Wolff and Wolff, 1976).

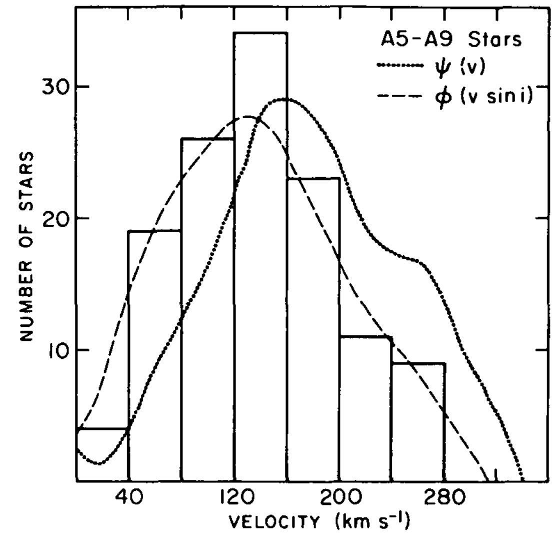

Figure 5-3. Same as Figure 5-2 for normal stars with spectral types A5 to A9 (from Wolff and Wolff, 1976).

Because $\psi(v)$ cannot be uniquely determined, it is impossible from these data alone to determine whether all slowly rotating late A-type stars exhibit metallic line characteristics. However, there are other observations that suggest slow rotation and the degree of metallicism, if any, are at best poorly correlated. In a study of rotational velocities of stars in binaries with periods less than 5 days, Abt and Hudson (1971) find that most, but not all, late A-type stars rotate synchronously and that there is an overlap in equatorial rotational velocities of normal and Am stars. Specifically, Abt and Hudson find that the rotational velocities of the Am stars fall in the range $12 < v < 101$ km s^{-1}, while the rotational velocities of normal stars lie in the interval $40 < v < 149$ km s^{-1}. The absence of normal stars with $v < 40$ km s^{-1}, and the overlap in the rotations of Am and normal stars for $v > 40$ km s^{-1}, is in excellent agreement with the results inferred from the maximum likelihood solution for $\psi(v)$. In a more recent paper, Abt (1975) has derived projected rotational velocities for the 44 marginal metallic line (Am:) stars listed in the catalog by Cowley et al. (1969). Abt finds that there is a strong overlap in the rotational velocity distributions of the Am and Am: stars, although on the average the Am: stars rotate slightly faster. Kodaira (1976) has reexamined this question by using the m_1 index to infer the degree of metallicism. He finds a correlation between m_1 and rotation in the sense that the most slowly rotating stars show the most anomalous line strengths, but the scatter in the relation is very large. From a somewhat larger sample, Burkhart (1979) has concluded that while the most peculiar stars all have v sin $i < 55$ km s^{-1}, there is no smooth correlation between rotation and metallicity. It is clear that factors other than rotation are important in determining the appearance of the line spectrum. The other factors do not appear to be age, the angle at which the star is viewed, or the previous history of mass exchange between binary components (Abt and Moyd, 1973).

Recently, the whole question of the relation between rotation and metallicism has been confused by Abt's (1979) report that there are four Am stars in the Orion Association with v sin i > 200 km s^{-1}. Abt also shows that even in clusters as old as $10^{8.3}$ years, there are some slowly rotating stars that do not have abnormal spectra. Abt therefore suggests slow rotation is not a necessary condition for metallicism, but that some stars, including ones with fairly high rotation, become peculiar (for whatever reason) and subsequently lose angular momentum as they age through tidal or magnetic braking. Abt suggests that the necessary condition for metallicism is membership in a closely spaced binary.

If there are indeed Am stars with v sin $i >$ 200 km s^{-1}, then existing models for the origin of Am characteristics (see below) are surely incorrect. Some uncertainties in the classification of the Orion stars have been discussed by Bonsack and Wolff (1980). The identification of the four stars as Am stars was based on low-resolution spectrograms and simply means that the Ca II K line type is earlier than the metallic line type, with the hydrogen line type intermediate to the two. Photometry (Warren and Hesser, 1977), however, does not support the Am classification. The m_1 index is not abnormally high in any of the stars, as it is in classical Am stars. The colors of one of the stars (HD 36670) correspond to a spectral type of A0 or earlier; Abt (1979), however, has assigned types of A3/A5/F1 based on the Ca II K line, hydrogen lines, and metallic lines, respectively, so that the photometric and spectral types are in clear conflict. Two of the four stars—Brun 405, classified A0.5/A2/A2, and BD-5°1331, classified B8/B9.5/B9.5—are of an earlier spectral type than classical Am stars, and in fact lie in a temperature region where spectral classification criteria are fairly insensitive to Am characteristics. Michaud (1980) has also pointed out that some of the peculiar Orion stars are more massive than typical Am stars. He suggests that perhaps the Orion stars have strong magnetic fields and that, as angular momentum is lost, they will come to resemble Sr-Cr or Si stars rather than Am stars. Rapidly rotating Si stars are quite common. Existing observations are inadequate to determine whether or not the Orion stars have magnetic fields, or whether as Abt suggests, they are close binaries.

APPARENT ABUNDANCES

Since the pioneering work by Greenstein (1948, 1949), there have been many abundance analyses of the Am stars. Most useful for understanding the systematics of this class of objects are studies of a large number of stars in which uniform data are treated in a uniform manner. A particularly good example is the study of microturbulence and abundances in 16 Am stars by Smith (1971). The analysis was carried out through the use of standard atmospheric models that assumed hydrostatic and radiative equilibrium and local thermodynamic equilibrium (LTE) level populations. No allowance was made for the alteration of atmospheric structure caused by enhanced blanketing by metallic lines. More recent explorations of this particular problem suggest that proper inclusion of blanketing is unlikely to alter the abundance patterns to a significant degree (Smith, 1973a; Lane, 1981). While there may be some systematic errors in an analysis of this kind, the hope is that the Am stars are sufficiently similar to one another that differential analyses have some meaning.

The derived abundances relative to iron are shown in Figure 5-4. Iron itself is overabundant relative to standard stars by several percent to as much as a factor of 5, with some tendency for the enhancement in Fe to increase with increasing temperature.

The main characteristics of Figure 5-4 are the marked deficiencies of specific light elements (C, Mg, Ca, and Sc) and general overabundances of the heavy elements. The abundance anomalies rarely exceed a factor of 10. The scatter in the relative abundances of the light elements is probably intrinsic to the stars. For iron group elements, the scatter in the relative abundances derived for Am stars is no larger than it is for the standard stars. The star-to-star variations in the absolute abundances for Fe, and therefore for the other elements, exceed by a wide margin Smith's estimate of the observational error. Other important features of Figure 5-4 are the absence of any clear demarcation between the Am and standard stars and of any distinct subclasses of Am stars. These results have been confirmed and extended by IUE observations of the lighter elements by Lane (1981). On the whole, the Am stars appear to be a much less heterogeneous group than magnetic Ap stars (cf., Cowley and Henry, 1979).

ABUNDANCE ANOMALIES OR LINE STRENGTH ANOMALIES?

The fundamental question concerning the Am stars is whether the line strengths reflect real abundance anomalies or simply an anomalous population of specific excitation and ionization states due to an unusual or peculiar atmospheric structure. The latter possibility has been explored extensively in the literature. Greenstein (1949) showed that the metallic line characteristics are not caused by the superposition of spectra from two stars. A comparison of the strengths of lines

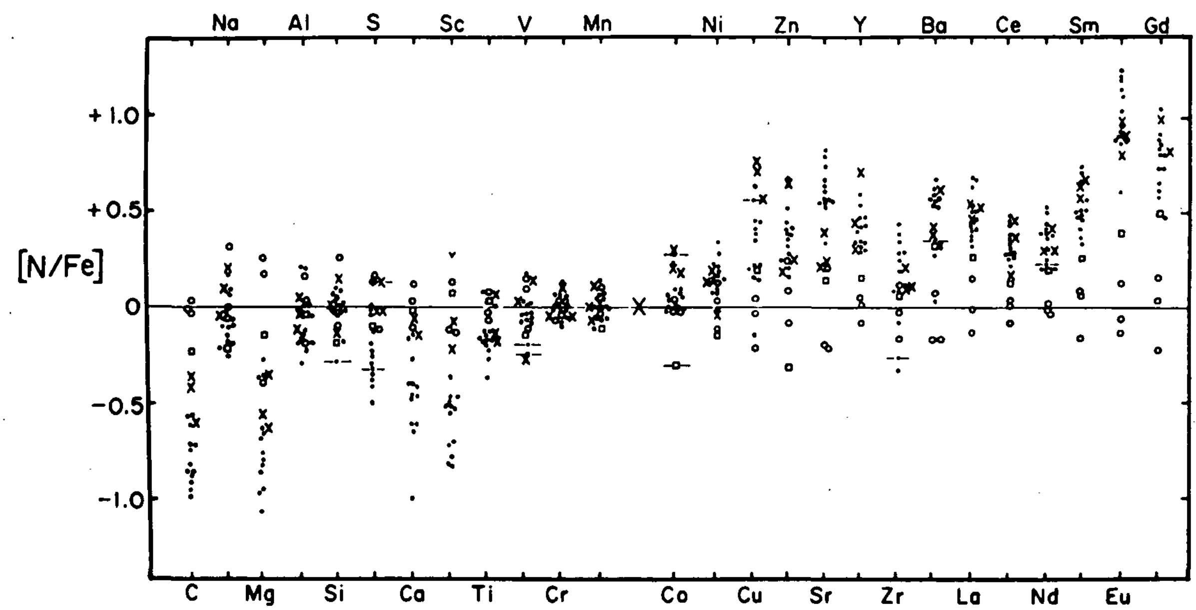

Figure 5-4. The relative abundances of the elements. The ordinate represents logarithmic abundance ratios with respect to iron and relative to the means of the standard stars. Dots represent Am stars; crosses, "marginal" Am stars; circles, standards; squares, HR 906 (a "standard" star which is apparently enhanced in iron). Bracketing horizontal lines denote more uncertain values (from Smith, 1971).

arising from normal and excited metastable levels of Fe I showed that dilution effects are not present, and that the metallic lines do not therefore originate in an extended envelope or circumstellar shell. The elements with abnormally low abundances have ionization potentials in the range 12 to 16 volts, and excess second ionization caused by radiation near the Lyman lines could account for the apparent deficiencies. Such excess emission should also reduce the strengths of the Balmer lines and perhaps even result in Balmer emission. The strengths of the Balmer lines, however, are normal.

Excessive ionization will also result if the electron pressure is abnormally low, and Böhm-Vitense (1960) has suggested that the apparent underabundance of Ca might be explained in this way. In order to test this possibility, Conti (1965b) constructed a series of models in which the gravity was artificially reduced in the layers of the atmosphere where the metallic lines are formed ($\tau < 1.0$). The gravity was assumed to be normal in deeper layers, in order to reproduce the continuum and hydrogen line profiles, both of which are consistent with gravities typically derived for dwarf stars. Models with surface gravities lowered somewhat from those seen in normal dwarf stars fit the observations, a result in accord with measurements of the Cr I to Cr II and Fe I to Fe II ionization balance. However, the decreased gravity is insufficient to account for the apparent Ca deficiency and does not significantly alter the ratio Ca/Fe. Conti also showed that models with arbitrarily higher boundary temperatures and arbitrarily higher He abundances are unable to account for the observed ratios of Ca/Fe.

The advent of space observations has made possible a more direct test of the importance of a lowered electron pressure on the ionization in the atmospheres of Am stars. The abundance of Mg is higher than that of Ca, and the Mg II h and k lines are formed higher in the atmosphere than the Ca II K line. The fact that the Mg II h and k lines in Am and normal A-type stars are indistinguishable demonstrates unambiguously that a lower density in the upper layers of the atmosphere cannot be the explanation for the deficiency of Ca (Böhm-Vitense, 1980).

An alternative source of overionization has been suggested by Böhm-Vitense (1976). She postulated that Am stars have an active hydrogen convection zone while rapidly rotating stars do not, that the convection powers a chromosphere and corona, and that these regions cause in some way the overionization. However, recent measurements of Mg h and k yield no evidence of chromospheric emission (Böhm-Vitense, 1980), while X-rays are detectable in only a few Am stars and may be directly related to their binary properties (Cash and Snow, 1982).

Cowley (1980b) cites evidence that the line depths in Am stars are greater than in either magnetic or normal dwarf stars. Both this observation and Conti's measurement of a slightly lowered electron pressure argue that the atmospheric structure of the Am stars is abnormal in some way. However, semiempirical studies like those of Conti indicate that such differences, whatever they may be, probably cannot produce the observed line strengths if the atmospheric abundances are constrained to be solar. Furthermore, as Conti (1970) has emphasized, there is no direct evidence for extreme departures from LTE. Emission lines and peculiar line profiles are absent. Lines of a given element that arise from different excitation and ionization states, as well as from metastable and permitted states, yield consistent abundances. Elements like Sr and Sc show very different abundance anomalies, even though their ionization and excitation potentials are similar. In the absence of a physically consistent non-LTE theory, one cannot be *certain* that the apparent abundance anomalies of the Am stars are real. However, observational evidence favors this hypothesis, and the next section examines its consequences.

ORIGIN OF ABUNDANCE ANOMALIES

Explanations for abundance anomalies in Am stars fall into three broad general categories. The anomalies are the consequence of nucleosynthesis, either in the stellar interior or on the surface of the star; they are the result of accretion from grains or mass lost from evolved stars; they are caused by a separation of elements within the star due to diffusion processes.

The arguments against the first two hypotheses are strong. The observed abundances do not follow the patterns expected for nucleosynthesis. Specifically, the sizes of the anomalies do not correlate well with cosmic abundances, with position in the periodic table, or with the nucleosynthetic groups to which the elements belong. In any case, nuclear reactions in stellar interiors appear an unlikely explanation for metallicism because Am stars have been found in groups like Ori Ic that are only 1 to 3 $\times$ 10^6 years old. In such groups, stars with masses of $\sim$2 $M_\odot$ have not had time to evolve to the point where nuclear burning of heavy elements can take place. Since magnetic fields have not been detected in Am stars, there is no obvious mechanism for producing energetic particles that could lead to nuclear reactions at their surfaces. The possibility that material enriched in heavy elements has been transferred to the Am star from an evolved companion seems improbable because, when they have been observed, the secondaries in Am systems prove to be either normal or Am stars with normal masses.

Another argument against nucleosynthetic processes is the observation that Sr, Y, and Zr, the magic number *s* process elements do not all show similar overabundances. Furthermore, there may not be conservation of heavy particles. Smith (1971) has pointed out that the pronounced deficiency of C is not compensated by overabundances of neighboring elements.

Accretion can produce anomalous abundances only if the process is selective or if the material accreted has an abnormal composition. Strong surface magnetic fields are an obvious mechanism for producing preferential accretion of specific elements (Havnes and Conti, 1971), but there is no evidence that magnetic fields are present in Am stars. Accretion of material enriched in heavy elements via a supernova explosion (van den Heuvel, 1968) appears unlikely because of the high frequency of Am stars. Approximately 50 percent of the stars of spectral type $\sim$A7, and all of the slowly rotating stars of this type, have Am characteristics. It is also difficult to see how accretion can account for different compositions in two components of a close binary, and such differences have been observed (Conti, 1969b).

The third mechanism that has been proposed to account for metallicism is diffusion (Michaud, 1970; Watson, 1970; and Smith, 1971). The basic hypothesis, which is explored in more detail in Chapter 9, is that the deficiencies of C, Ca, Sc, and other elements are the result of gravitational settling, while the overabundances of the heavy elements are caused by selective radiation pressure transferred through line absorption processes. The excess pressure causes specific elements to be driven upward in the atmosphere and to be concentrated in the photosphere. The expected drift velocities are very slow ($\sim 10^{-4}$ to 10^{-5} cm s^{-1}), and so diffusion will be ineffective in any region where there are substantial mass motions caused by turbulence, convection, meridional circulation, or other mechanisms. Model atmosphere calculations of late A-type stars indicate that a surface hydrogen convection zone, as well as a subsurface helium convection zone, should be present (e.g., Praderie, 1967a). Diffusive separation of elements cannot, therefore, occur in the surface layers of Am stars. The original models proposed by Watson and Smith postulated that the diffusion occurred in the radiative zone that separates the two convective regions. Subsequent theoretical calculations (Toomre et al., 1976) indicate that there will be strong convective overshooting into the adjacent radiative zones, and models that involve diffusive separation in the intermediate radiative region are untenable. The viewpoint adopted more recently by proponents of the diffusion model is that gravitational settling leads to a depletion of helium in the region where convection would normally occur because of the second ionization of helium. Diffusion can then take place below the He I and H I convection zone, and the entire convective region near the stellar surface reflects the impact of diffusive separation of elements (Baglin, 1972).

If diffusion does actually occur, then it offers a natural explanation for many of the observed characteristics of Am stars. The absence of metallicism in rapid rotators is attributed to meridional circulation. The high frequency of metallicism is simply due to the fact that most slowly rotating A-type stars have atmospheres stable enough for diffusion to occur. The time scale for diffusion is compatible with the presence of Am stars in very young clusters and associations. There is evidence that abundance anomalies decrease with increasing rotation and microturbulence (Lane, 1981), both of which should reduce the efficiency of diffusion. The apparent dichotomy between pulsation and metallicism is attributed to atmospheric disturbances produced by pulsation that disrupt the diffusion process. The questions that remain are whether in practice diffusion actually does occur in Am stars and whether it can account in detail for the specific abundance anomalies that are observed (cf., Chapter 9 for a more complete discussion).

CONCLUSIONS

In his review of Am stars written more than a decade ago, Conti (1970) formulated the following ten conclusions about Am stars:

1. Am star atmospheres are characterized by either deficient Ca (Sc) or overabundant heavier elements, or both.
2. These anomalies are probably indicative of real abundance differences and cannot be explained by known physical atmospheric effects.
3. Although they have a high microturbulence, this parameter is not peculiar to Am stars; other stars in the same spectral region share this characteristic.
4. The Am star atmospheres are anomalous *only in their composition.*
5. As a group, Am stars are slowly rotating, due in many cases to their being members of close binaries.
6. Am stars are mostly unevolved, and their anomalies could well be acquired early in their youth.
7. The Am phenomenon is found in only a limited region of the HR diagram, near the main sequence and between F0 and A0.
8. The different behavior of Ca (Sc) and other metals suggests that two different, but possibly related, processes are involved in the Am phenomenon.
9. Nuclear processes in stellar interiors or on the surfaces of Am stars do not seem to be involved in the Am problem.
10. Physical separation processes may prove to be the key to our understanding of these curious stars.

Nearly half the references cited in the present review of Am stars are to papers published since 1970. The major advances during that time have been in the elaboration of diffusion calculations and in the development of better models of convection. However, the conclusions reached by Conti remain a valid summary of the basic characteristics of Am Stars.

6

THE δ SCT STARS

DEFINITION OF CLASS

The extension of the Cepheid instability strip corsses the main sequence in the region occupied by late A-type stars. One might, therefore, expect pulsational instability to be a common property of A-type stars near the main sequence, and observations show this to be true. The detailed nature of the variability, however, is complex, and a thorough understanding of it can greatly enhance our knowledge of the atmospheric structure of A-type stars.

Eggen (1956a) was the first to suggest that there exists a class of variables, distinct from the RR Lyr stars, near the main sequence at spectral types A and F. Furthermore, his early light curves of ρ Pup (Eggen, 1956a) and δ Del (Eggen, 1956b) established several important characteristics of this group of variables. First, the amplitudes typically are on the order of a few hundredths of a magnitude. Second, while some stars, such as ρ Pup, have very stable light curves, other stars, including δ Del, show marked changes in the light curve from cycle to cycle. Third, the periods are short—less than 0.3 days. Eggen's results for ρ Pup and δ Del are shown in Figures 6-1 and 6-2.

The measurement of the radial velocity variations in the δ Sct and related variables is difficult,

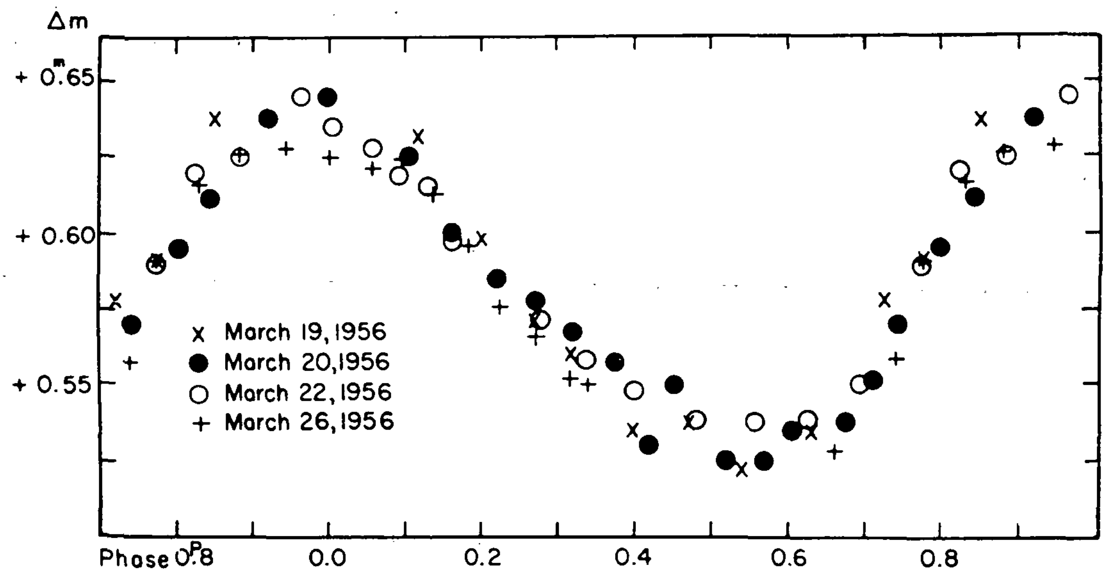

Figure 6-1. Light curve of ρ Pup from photoelectric observations on 4 nights (from Eggen, 1956a).

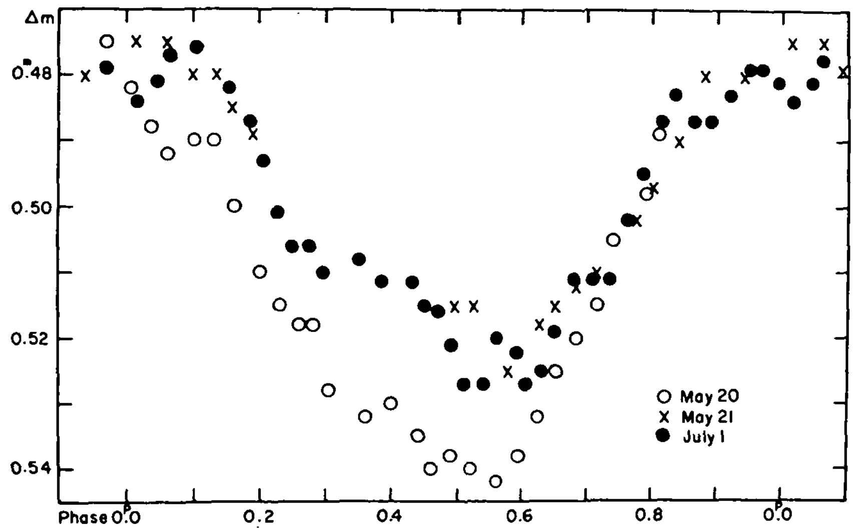

Figure 6-2. Light curves of δ Del observed on 3 nights (from Eggen, 1956b).

and relatively little work has been carried out in this area. The problem is that the radial velocity variations are small; the observed amplitudes generally fall in the range from 2 to 30 km s^{-1} and, for the majority of stars, are less than 10 km s^{-1}. High-dispersion spectra are required in order to achieve this level of accuracy. There is an additional requirement that exposure times should not exceed 10 percent of the pulsation period; for a typical period of ~2 hours, the exposure time should therefore be on the order of 10 minutes. The simultaneous requirements for high spectral resolution and short exposure times mean that, to date, only the brightest δ Sct-type stars have been studied spectroscopically. Figure 6-3 illustrates the photometric and radial velocity variations of ρ Pup derived from simultaneous spectral and spectrophotometric observations (Danziger and Kuhi, 1966). The relative phasing of the luminosity and radial velocity variations appears to be typical. Danziger (1967) suggests that, if one considers only those stars for which simultaneous measurements of brightness and velocity are available, maximum light precedes minimum radial velocity by 0.00 to 0.25 cycles. More recent data suggest that the phase shift between the inverse radial velocity and light curves is constant for all δ Sct stars with predominantly radial pulsation and is equal to $\Delta\theta = 0.09 \pm 0.015$ (Breger et al., 1976).

While the general characteristics of the A- and F-type variables near the main sequence have been known for some time, extensive effort has been devoted during the past decade toward

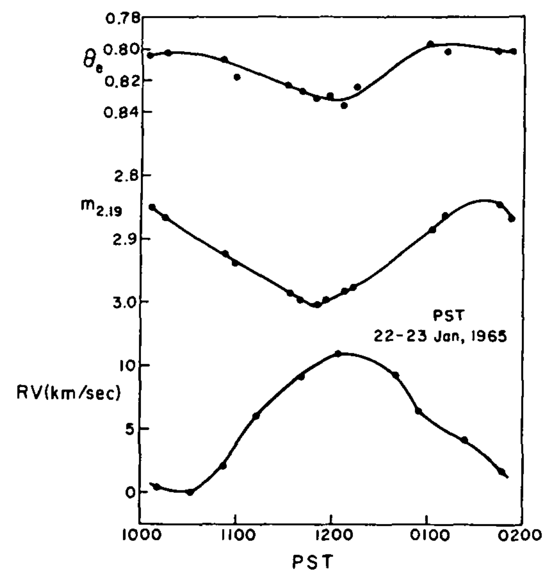

Figure 6-3. Radial velocity, light, and temperature of ρ Pup plotted as a function of Pacific Standard Time. The light curve was measured in a 50 Å bandpass centered at $1/\lambda(\mu m) = 2.19$. The temperature is given in terms of $\theta_e = 5040/T_{eff}$. Radial velocity measurements are relative, not absolute (from Danziger and Kuhi, 1966).

determining their evolutionary status, the stability of the variations, and the relation between pulsation, rotation, and composition. The remainder of this chapter will deal with the available observations in detail and draw heavily on the excellent review papers by Baglin et al. (1973) and Breger (1979).

Before proceeding with the discussion, however, it is worthwhile to anticipate some of the major results of the work to date and deal with the question of how many classes of A- and F-type variable stars exist. Historically, two classes have been hypothesized and distinguished according to the following definitions (Kukarkin et al., 1969):

> RRs: RR Lyrae-type variables with the period not exceeding $0^{d}.21$ (dwarf Cepheids). Belong to the population of the disk, are absent in clusters. Their luminosity is $2^{m} - 3^{m}$ fainter than the luminosity of RRab and RRc stars. A typical representative—SX Phe.
>
> δ Sct: δ Scuti-type stars. Pulsating variables of spectral class A (late subclasses) and F, the amplitudes of light variation do not exceed, as a rule, $0^{m}.1$ (rarely up to $0^{m}.3$). The form of light curve strongly varies usually. According to many characteristics resemble dwarf Cepheids, but differ from them by the small amplitudes. Are [found] in the Hyades-type clusters. Similar to the RRs-type stars, their periods do not exceed $0^{d}.2$. A typical representative—δ Sct.

However, the distinction based on amplitude has been challenged by a number of observers (Baglin et al., 1973, and references therein; Breger, 1979). The dwarf Cepheids as defined above are not distinguished from the δ Sct stars by period, color, mass, composition, space motion, or other fundamental characteristics. There does seem to be a group of metal-poor variables with shorter periods and probably lower mass, lower luminosity, and higher space velocities than the δ Sct stars of solar composition, but the two groups of stars cannot be distinguished on the basis of amplitude. In this book all A- and F-type stars with periods less than 0.3 days will be called δ Sct stars, a terminology suggested by Breger (1979). However, it should be recognized that this class encompasses two groups of stars with different evolutionary histories.

Lists of known δ Sct stars have been published by Breger (1979) and by Eggen (1979), who prefers the terminology "ultrashort period Cepheids" for this class of objects. Both lists are quite useful but are not complete. A thorough literature search or reference to computerized data bases is required to determine whether any specific star has been reported to be variable.

INCIDENCE OF PHOTOMETRIC VARIABILITY

There have been a number of searches for photometric variability among the A- and F-type stars (e.g., Danziger and Dickens, 1967; Breger, 1969; Jorgensen et al., 1971). Figure 6-4 shows the location of known δ Sct stars in the HR diagram. The quantity M_v was derived from a calibration of the uvby system for nonvariable Population I stars. The dashed lines indicate the observed boundaries of the instability strip. While a few variables may lie outside these limits, pulsation is very much more probable within them. From a calibration of the uvby colors based on model calculations (Breger, 1977), the boundaries of the instability strip are estimated to be as follows (Breger, 1979):

Blue Edge:	8800 K on the ZAMS
	8400 K at $M_v = 0.65$
Red Edge:	7500 K on the ZAMS
	6950 K at $M_v = 1.7$

The frequency of variability within these boundaries derived from any specific survey depends strongly on the precision of the observations. Figure 6-5 shows the distribution of amplitudes for the δ Sct stars; the frequency increases

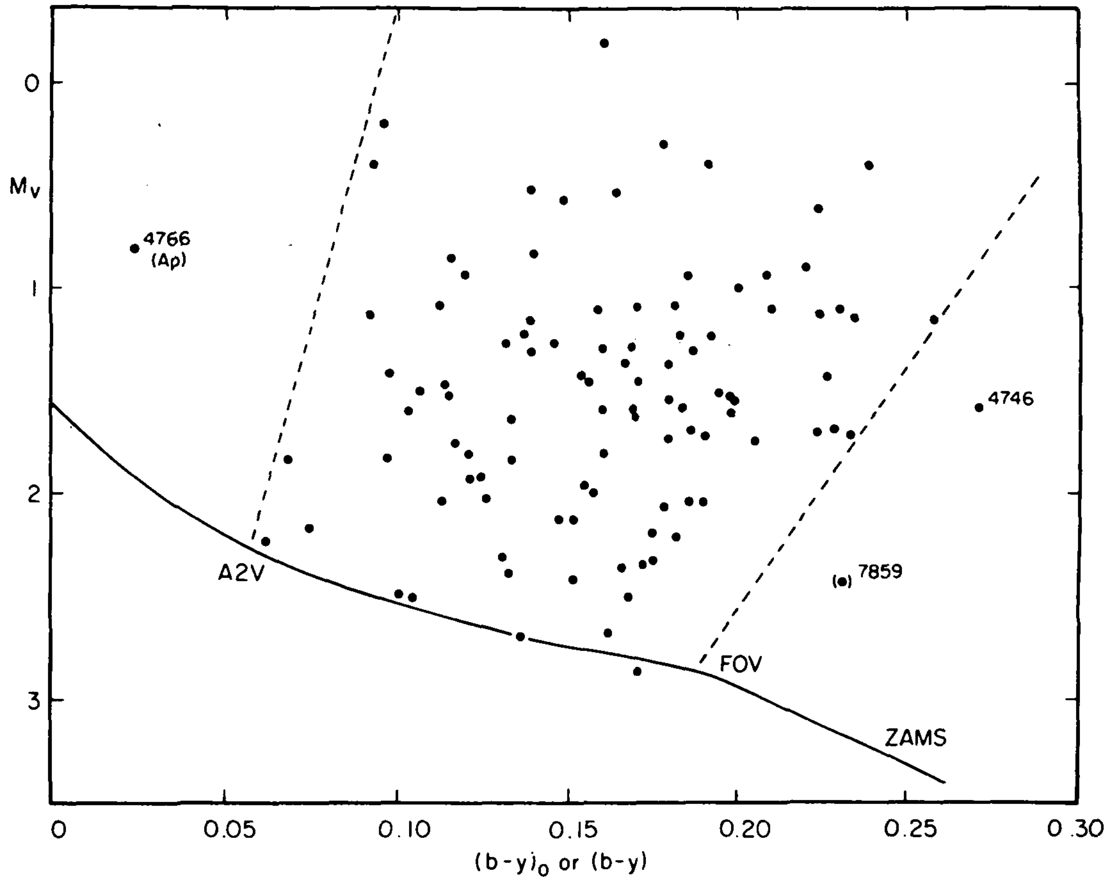

Figure 6-4. Position of δ Sct stars in the HR diagram. Some metal-poor stars with uncertain luminosities have been excluded. Dashed lines refer to the instability strip borders (from Breger, 1979). Recent data indicate that HR 4746 is probably constant.

nearly exponentially with decreasing amplitude. Approximately one-third to one-quarter of the stars within the instability strip vary by 0.01 mag or more.

The results of searches for δ Sct stars in galactic clusters have been summarized by Breger (1975a) and Slovak (1978). In general, the incidence of variability is similar to that found for field stars, if careful allowance is made for the limiting accuracy of the various surveys. The frequency of variability appears to be independent of age in clusters young enough for the instability strip to be well populated. Specifically, two possible members of NGC 2264 and one member of NGC 7789 have been found to be variable; these results suggest that stars as young as 10^6 years and as old as 10^9 years can exhibit δ Sct characteristics. The coexistence within the instability strip of variable and nonvariable stars in the same cluster rules out initial composition as a major factor in determining whether or not a star pulsates.

Only about one-third of the stars in the lower instability strip are variables, and determination of which physical factors—rotational velocity, atmospheric structure, age, binary properties—inhibit or encourage pulsation would undoubtedly provide important insight into the atmospheric structure of A-type stars. Studies of this problem indicate that rotation and metallicism are the characteristics most closely correlated with pulsation.

Figure 6-6 shows the distribution of apparent rotational velocity, v sin *i* for dwarf (within one

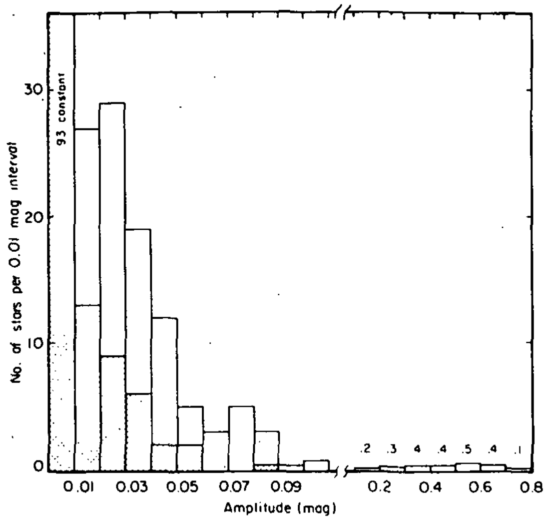

Figure 6-5. Histogram of amplitudes observed in δ Sct stars. Known constant stars have also been included and counted as having amplitudes between 0.00 mag. and 0.01 mag. (from Breger, 1979).

magnitude of the main sequence) and giant δ Sct stars. These distributions should be compared with those for Am and normal A-type stars (cf., Figures 5-2 and 5-3). As Figure 6-6 demonstrates, the rotational velocities of δ Sct stars span a wide range, and the rotational velocity distributions of variable and nonvariable stars clearly overlap. The distribution of v sin *i* for the giants is quite similar to that for nonvariable A-type stars, with the exception that none of the variables rotates at a velocity greater than 200 km s^{-1}. Very rapid rotation may inhibit pulsation. The distribution of v sin *i* for dwarf δ Sct stars shows that there are very few pulsating variables near the main sequence that are also slow rotators.

Figure 6-7 illustrates the incidence of variability among A-type stars with normal spectra. There is no satisfactory explanation for the fact that variability is common but not ubiquitous within the instability strip. However, the variability in these stars is small and the stars are very close to stability. It may be that small differences in atmospheric structure determine the amplitude of variability and that many of the

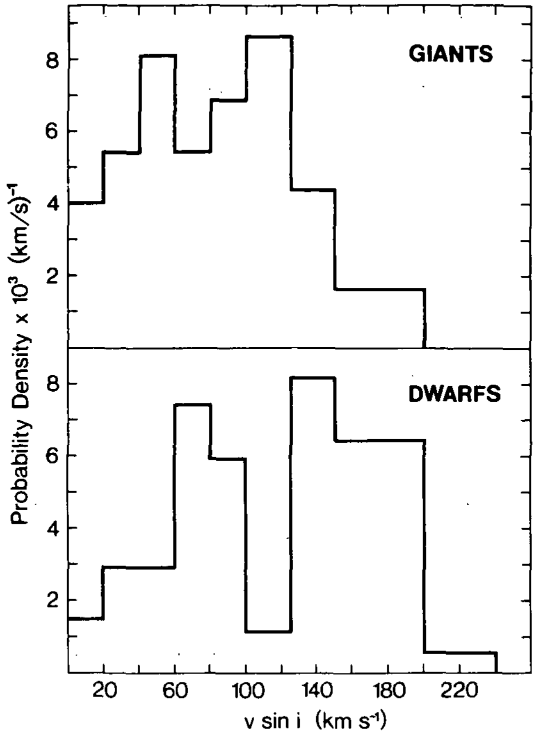

Figure 6-6. Distribution of v sin i for δ Sct variables. Rotational velocities are from Uesugi (1976).

apparently constant stars are variable at levels below the threshold (~0.01 mag) of detectability. It is still impossible to predict the amplitude variation for a δ Sct star from its other characteristics.

The incidence of variability in A-type stars with anomalous spectra is summarized in Figure 6-8. Near the main sequence there is complete exclusion between pulsation and pronounced metallicism. There are, however, two marginal Am stars (marginal in the sense that the difference between the K line and metallic line types is less than five subclasses) near the main sequence that exhibit low-amplitude light variability (Kurtz, 1978a). In giants and subgiants, the coexistence of pulsation and metallicism is quite common.

The A-type stars above the main sequence with abnormal spectra are named after the prototype of the class—δ Del. The δ Del stars are defined *spectroscopically* to be those late A- and early F-type giants and subgiants with disparate

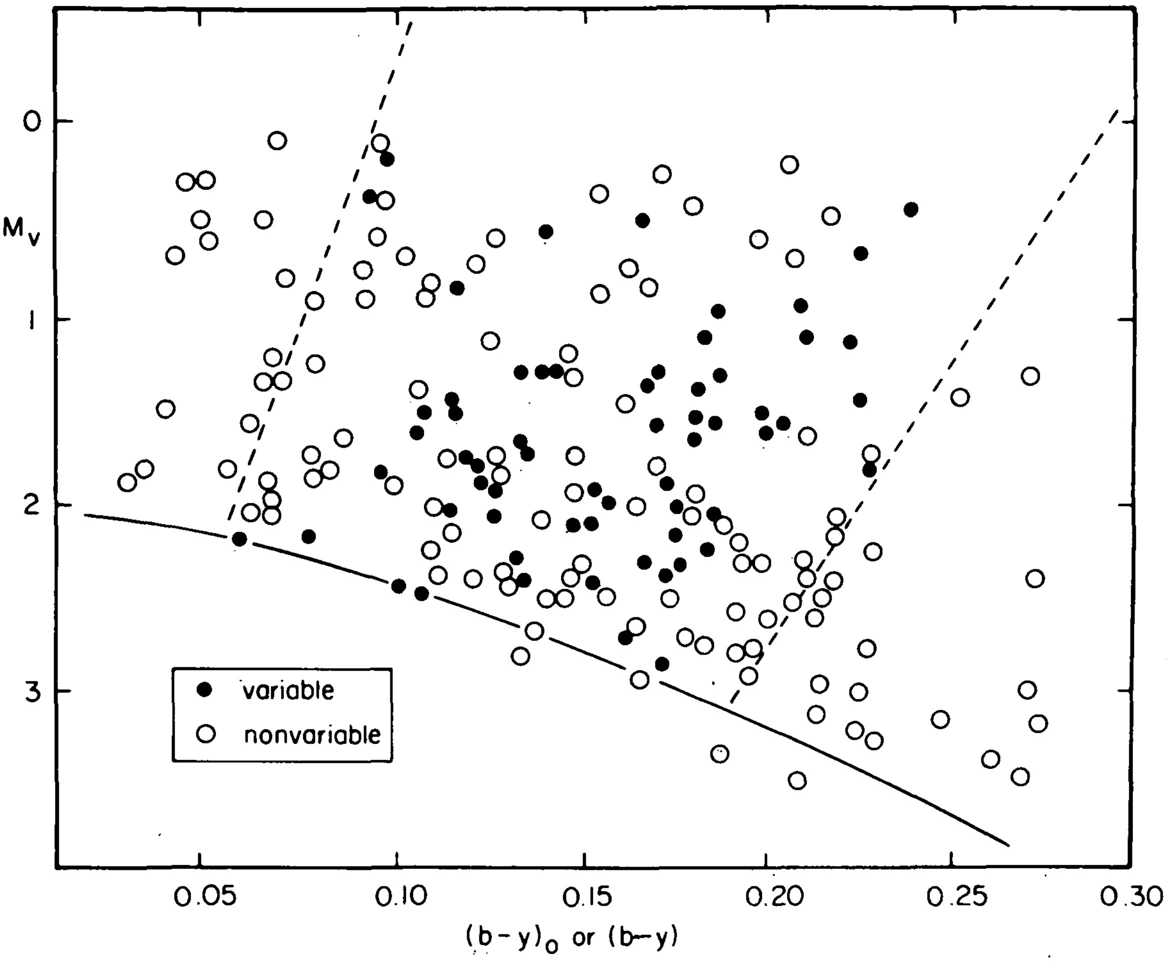

Figure 6-7. Incidence of variability of stars with known normal spectra. Detectable pulsation is common but occurs in less than 50 percent of the stars (from Breger, 1979).

K line and metal line spectral types. Photometric studies show that many δ Del stars are also photometric variables, although variability is *not* a requisite for membership in the class. In the δ Del stars, therefore, pulsation and metallicity, and even very pronounced metallicity, can coexist (Kurtz, 1980a, 1980b). On the basis of their abundances, position in the HR diagram, and rotational velocities, Kurtz (1976) has concluded that at least some of the δ Del stars are evolved metallic line stars. A picture of Am stars as unevolved objects and δ Del stars as evolved Am stars is, however, probably too simplistic and will no doubt be modified as we define more precisely the characteristics of each group. There are, for example, Am stars that seem to lie above the main sequence (e.g., 22 Boo; Burkhart et al., 1980), and some stars with line strengths like those of the δ Del stars are dwarfs (Baglin, private communication).

The absence of slowly rotating δ Sct stars near the main sequence that is illustrated in Figure 6-6 may be entirely equivalent to the observation that classical Am stars do not pulsate (Breger, 1970), since data for nearby field stars suggest that all late A-type stars with v sin $i <$ 40 km s^{-1} are Am stars. Exceptions to this general dichotomy between pulsation and metallicism in dwarfs have been reported from time to time but have not been confirmed by independent observations. An interesting example is 32 Vir, which is an A8m star that was reported to be a

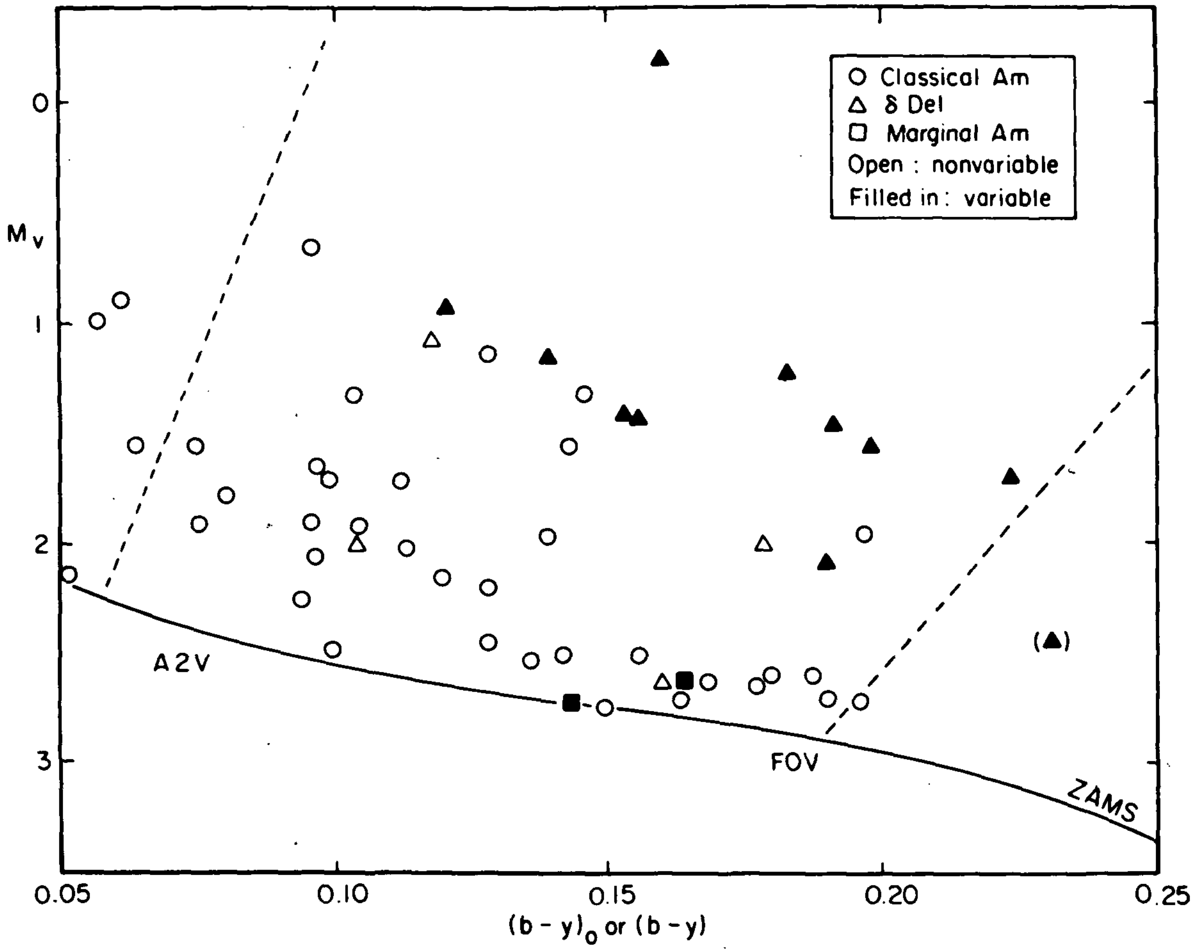

Figure 6-8. Incidence of variability for stars with abnormal spectra. The diagram shows that classical Am stars do not pulsate but that pulsation is common among δ Del and marginal Am stars (from Breger, 1979).

δ Sct variable. A detailed analysis, however, shows that 32 Vir is a double-lined binary. Two models for this system have been proposed that would maintain the dichotomy between metallic line characteristics and pulsation in dwarf stars. Kurtz et al. (1976) have pointed out that it is the primary component in the 32 Vir system that has a composition typical of Am stars. However, it may, in fact, be the secondary that is variable. As an alternative model, Mitton and Stickland (1979) have presented evidence that the secondary star is less massive but hotter than the primary. It appears that the primary may be an evolved Am star, with the luminosity of a subgiant, and should be classified as a δ Del star. Therefore, 32 Vir cannot be considered as a good counterexample to the hypothesis that metallicism and pulsation in stars near the main sequence are mutually exclusive.

The absence of pulsation in classical Am stars is usually attributed to diffusion (e.g., Baglin, 1972; Breger, 1972). In a stable atmosphere, helium will tend to sink due to gravity (Baglin, 1972), which is not balanced by radiation pressure. The helium content in the He II ionization zone, which is the primary driving mechanism for the pulsation, may be reduced to the point that the star becomes stable against pulsation. At the same time, other elements that are strongly supported by radiation pressure may be concentrated in the outer portion of the atmosphere, thus producing the abundance anomalies characteristic of

Am stars. Cox et al. (1979) and Saez et al. (1981) have argued that even the coexistence of pulsation and metallicism can be explained within the framework of the diffusion model. If gravitational settling reduces helium content in the stellar atmosphere to $Y < 0.18$, then the He II convection zone will disappear and metallic line characteristics can quickly be produced. The residual helium, however, is sufficient to drive pulsation. (A critical assessment of radiation diffusion and its application to A-type stars is given in Chapter 9.)

PERIODICITIES

The time scales for variations of δ Sct stars range from 30 minutes to 5 hours (Breger, 1979). In many stars, both the amplitudes and lengths of the cycles vary. There are two possible explanations of this kind of variability: the variation may be quasi-periodic, with both period and amplitude unstable; or, alternatively, variations may be strictly periodic, with the cycle-to-cycle changes caused by the interaction of two or more modes of pulsation with definite periods. If the latter explanation is correct, then measurement of ratios of the periods for the various modes can be used to determine the mode of pulsation (fundamental, first overtone, etc.), type of pulsation (radial or nonradial), and possibly even the mass and, therefore, evolutionary state of the star.

The issue of whether the variations are strictly periodic or only quasi-periodic remains unresolved. Based on their analysis of HR 8006 and HR 9039 and consideration of other data in the literature, LeContel et al. (1974) argue that the irregularities in the light curve are real and that many δ Sct stars do not oscillate in a periodic mode or in a mixture of periodic modes. LeContel et al. attribute the complexity of light curves to nonlinear effects in the atmosphere. Specifically, an interaction between pulsation and convection might be an example of such an effect. Similar conclusions have been reached for 14 Aur (Morguleff et al., 1976a), HR 242 (Smyth et al., 1975), and 44 Tau (Morguleff et al., 1976b). Other data are in conflict with this hypothesis. In their analysis of 21 Mon, Stobie et al. (1977) positively identified six frequencies that were present throughout a 58-day time span. Stobie et al. argue that the persistence of some frequencies for more than 8 weeks is contrary to the suggestion by LeContel et al. (1974) that nonlinear atmospheric effects are a dominant factor in shaping the light curves.

This problem has also been studied by Fitch (1976). He and coworkers have observed seven δ Sct stars intensively and have found stable periods in all but one; that one is 4 CVn, and since all the observations were made near full moon the aliasing is severe. Fitch suggests that the difference between his results and those of other observers is that his more extensive observations are better suited for analyzing the complex variations of δ Sct stars. Additional support for the stability of the periods of δ Sct stars comes from the work of Warman and Pena (1978), who find that multiple periods can be derived that adequately represent the observations of HR 432, HR 515, HR 812, and HR 8006 throughout two entire observing seasons separated by 5 years. Kurtz (1980c) has recently rediscussed the photometric variations of those δ Sct stars that appear to show unstable light curves. He concludes that the evidence for frequencies changing on time scales of 24 hours is weak; that δ Sct stars in general have stable frequencies that can be associated with discrete modes of pulsation; and that the analysis of the light curves is very difficult because both radial and nonradial modes may be excited simultaneously, and the frequencies of the various modes may be very close together.

In summary, the available data establish that the majority of well-studied δ Sct stars exhibit stable periodicity. Whether or not the amplitudes associated with those frequencies are stable remains an open question (Breger, 1980b). An enormous amount of observing time—Fitch (1976) suggests at least 10 closely spaced nights and Breger (1979) as many as 30 nights *per star*—would be required to establish or disprove the statement that all δ Sct stars are strictly periodic.

PULSATION CHARACTERISTICS

The theoretical treatment of pulsation in the lower portion of the Cepheid instability strip has been reviewed by Petersen (1975), while a summary of the present status of the models has been given by Stellingwerf (1979). Linear stability analyses of models of δ Sct stars (cf., Petersen, 1975, and references therein) show that models within the instability strip with a normal helium abundance ($Y = 0.30$) are unstable in the fundamental mode and in the first several overtones. The pulsation modes are excited primarily in the helium ionization zone, as is true of the classical Cepheids, which occupy the upper portion of the instability strip.

A schematic, but highly simplified, explanation of the complex physical process that is involved is as follows. During the phase of maximum compression, the stellar atmosphere is heated, but in the region of the He II ionization zone much of the energy released by the compression is absorbed by the process of ionization of He II to He III. The flow of radiation to the exterior of the star is impeded by two mechanisms. First, luminosity depends on temperature, which is lower than in the absence of ionization. Second, the opacity is large and causes a damming up of radiation in the ionization region. The excess pressure eventually becomes sufficient to lift the overlying layers of the star and to force an overall expansion of the atmosphere. While He II is highly opaque to ultraviolet radiation ($\lambda < 229$ Å), which is present in large quantity at temperatures of ~50,000 K that are characteristic of the He II ionization region, He III is effectively transparent. Once He II becomes mostly ionized, ultraviolet radiation can flow freely through this layer of the star, the temperature decreases, the pressure drops, the weight of the overlying layers compresses the star, He III recombines with an electron to form He II, and the cycle is repeated. Of course, all stars possess ionization zones, and a stability analysis is required to show which pulsate and which do not.

Again in oversimplified terms, the red edge of the instability strip marks the locus of stars in which the He II ionization zone is so deep that the increased pressure cannot lift the overlying layers. The blue edge bounds the region of the HR diagram in which the weight of the overlying layers is insufficient to force recompression.

A detailed linear survey of stellar models intended to represent the δ Sct stars has been carried out by Stellingwerf (1979). Growth rates, periods, and other parameters for the first six radial modes were calculated for models with hydrogen and metal abundances, respectively, of $X = 0.7$ and $Z = 0.005$, and masses in the range 0.2 to 2.0 $M_\odot$, as well as for $X = 0.7, Z = 0.02$, and $M = 2\,M_\odot$. The models span the temperature range $7000\text{ K} < T_{\mathrm{eff}} < 8300\text{ K}$. The first group of models was intended to represent objects that have classically been referred to as dwarf Cepheids, while the remaining models were for stars with normal Population I masses and abundances. Stellingwerf finds that both sets of models yield periods and period ratios of the magnitudes observed. However, by a judicious choice of Z, it is possible to obtain correct period ratios for any reasonable mass (cf., Petersen, 1978). Therefore, multiperiod analyses of light curves, in the absence of other information, are not adequate to determine either the masses or the evolutionary status of δ Sct variables.

Figure 6-9 shows the results of Stellingwerf's calculations for Population I models. Several important characteristics of radial pulsations in this temperature, luminosity, and mass range are illustrated in the diagram. First, the models are pulsationally unstable, and the region of instability in the HR diagram coincides well with the area occupied by δ Sct stars. Pulsation occurs in modes at least as high as the fifth overtone; the fourth and fifth overtones, in most cases, show the highest growth rates and exhibit both red and blue edges that lie fairly close to the boundaries of the observed instability strip. The "blue edge," which marks the high-temperature boundary of the region of instability, occurs at progressively higher temperatures for successively higher modes of oscillation.

A number of model properties can be compared directly with observations. First, the model periods are equal to those (0.02 to 0.25 days) actually observed. For a given mass and mode,

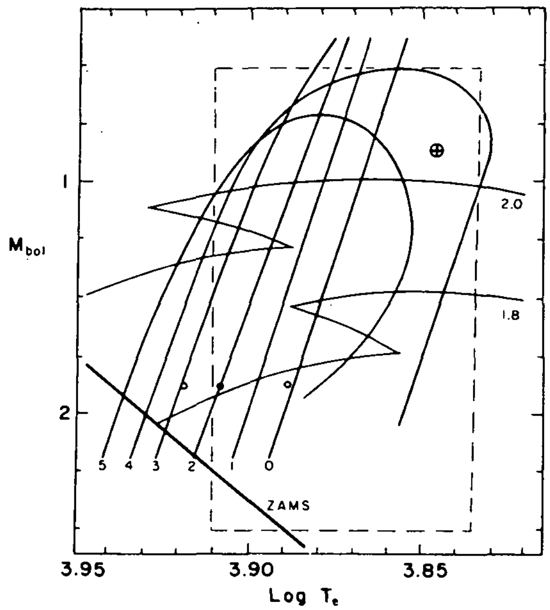

Figure 6-9. Models for stars with $M = 2\ M_\odot$, $Z = 0.02$. Blue edges are indicated by diagonal lines and labeled by mode number. Also shown are evolutionary tracks for $M = 2.0\ M_\odot$ and $1.8\ M_\odot$. Box encloses region occupied by δ Sct stars. Crossed circle denotes a model with $P_0 = 0.1874$ days and $P_2 = 0.1176$ days, values very close to those observed in δ Sct itself (from Stellingwerf, 1979).

the period of radial pulsation increases with increasing radius, and hence the models yield a *P–L–C* (period, luminosity, color) relation of the form

$$\log P = -0.29\,M_{\mathrm{bol}} - 3.23 \log T_{\mathrm{eff}} + C, \tag{6-1}$$

where P is the period in days and C is equal to 11.96, 11.85, and 11.76 for the first three modes, respectively. An observational *P–L–C* relation has been derived by Breger and Bregman (1975) who assume that the average period, which is derived by simply counting cycles, provides a useful measure of stellar variability. Full multiperiod analyses are, of course, available only for a very limited number of δ Sct stars. The relationship derived by Breger and Bregman is

$$M_{\mathrm{v}}\ (\pm 0.24\ \mathrm{mag}) = -2.64 \log P + 7.0\,(\mathrm{b{-}y}) - 2.48, \tag{6-2}$$

or

$$\log P = -0.35\,M_{\mathrm{bol}} - 2.7 \cdot \log T_{\mathrm{eff}} + 9.86. \tag{6-3}$$

In the first equation, no attempt has been made to take account of the specific mode of oscillation. The second equation was derived by assuming that every δ Sct star cooler than $T_{\mathrm{eff}} = 7800$ K pulsates in the fundamental mode and every star hotter than this value in the first overtone. The agreement between the coefficients derived observationally and theoretically is quite satisfactory.

An updated observational *P–L–C* relation has been derived by Breger (1979):

$$M_{\mathrm{v}} = -3.052 \log P + 8.456\,(\mathrm{b{-}y}) - 3.121\ (\pm 0.31). \tag{6-4}$$

The difference in coefficients between this equation and the earlier one provides a measure of uncertainty in the relation. The individual data points are shown in Figure 6-10, and only three stars deviate markedly from the overall trend. It is clear that the periods, despite the uncertainty in their precise values, do provide a measure of the intrinsic characteristics of stellar pulsation, and that the models do represent the observations.

As noted earlier, Breger and Bregman (1975) found that the stars hotter than $T_{\mathrm{eff}} = 7800$ K pulsated in the first overtone and those cooler than this temperature in the fundamental mode. This conclusion depends on the fact that the inferred Q values are smaller for the hotter stars, and this result implies radial overtones. Many stars also have nonradial modes, and in such cases, a definite determination of the pulsation

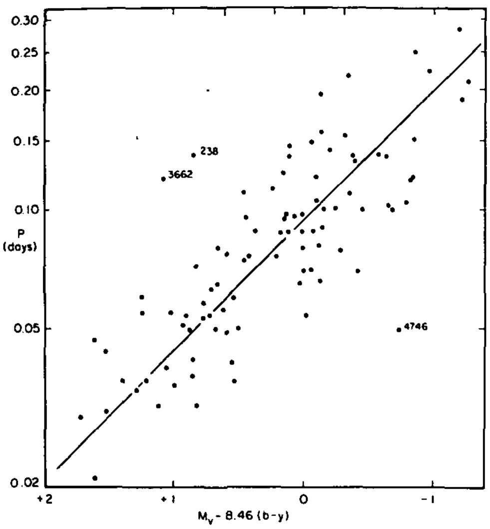

Figure 6-10. Comparison of periods (or time scales of variations) with absolute magnitude after a color correction. This diagram shows that despite uncertainties in the period determinations, the derived periods do have meaning in a statistical sense (from Breger, 1979).

mode from Q alone is not possible. Nevertheless, examination of Figure 6-9 shows that T_{eff} = 7800 K coincides very closely with the fundamental blue edge for radial pulsation. Stellingwerf (1979) argues that low order radial modes tend to dominate the pulsation and that observations and theory are therefore again in excellent agreement. Other properties that can be calculated theoretically, such as the phase lag between minimum radius and maximum luminosity and the quantity $(\Delta R/R)/\Delta M_{bol}$, are also in reasonable agreement with the observations.

The δ Sct stars pulsate in a variety of ways, and the models for δ Sct stars provide guidelines for determining the mode of the pulsation. The possibilities have been described very succinctly by Unno et al. (1979):

> . . . Stars are like musical instruments which have various modes of oscillation and tones . . . . The normal modes in a spherically symmetric star are characterized by the eigenfunctions that are proportional to the spherical harmonics: $Y_\ell^m(\theta, \phi)$ (ℓ = 0, 1, 2, . . .; m = 0, ±1, . . ., ±ℓ). In particular, the radial modes are the special cases of ℓ = 0. The other harmonics are called the dipole (ℓ = 1), the quadrupole (ℓ = 2), the octapole (ℓ = 3) oscillations, etc. The eigenfrequencies depend on ℓ but are degenerate by (2ℓ + 1)–folds in m. The normal modes belonging to the harmonics ℓ are further distinguished by the number of nodes, k, in the radial component of displacement from the center to the surface of a star. The k values are 0 for the fundamental mode, 1 for the first overtone mode, 2 for the second overtone mode, etc. The normal modes are classified by the radial quantum number k and the angular quantum number ℓ. When "the Zeeman splitting" is introduced by rotation or by magnetic field, the azimuthal quantum number m has to be added.

Pure radial or spherically symmetric pulsation, which is simply an alternate expansion or contraction of the star as a whole, corresponds to ℓ = 0. Nonradial pulsations, on the other hand, cause the shape of a star to deviate from spherical symmetry. Since adjacent regions of the star are out of phase in terms of their radial velocity and photometric variations, nonradial oscillations are more difficult to detect observationally. The 5-minute solar oscillation is an example of nonradial pulsation, and radial and nonradial pulsations coexist in many β Cep stars. Radial pulsation can be characterized by the constant Q, which relates the period and mean density of a variable star:

$$P(\bar{\rho}/\rho_\odot)^{1/2} = Q. \qquad (6\text{-}5)$$

Heuristic derivations of this relation have been given by Cox (1980). Values of Q derived by Stellingwerf (1979) for a typical δ Sct star $M = 2\,M_\odot$, T_{eff} = 7700 K, M_{bol} = 1.207) are given in Table 6-1.

Table 6-1
Characteristics of Pulsation

Pulsation Mode	Period (days)	Q (days)	P_i/P_{i-1}
Fundamental	0.1121	0.03259	
1st Overtone	0.0865	0.02514	0.772
2nd Overtone	0.0705	0.02050	0.815
3rd Overtone	0.0591	0.01719	0.838
4th Overtone	0.0506	0.01470	0.856
5th Overtone	0.0441	0.01283	0.872

Source: Adapted from Stellingwerf (1979).

The relationship for Q can be expressed in observational terms (Breger and Bregman, 1975):

$$\log Q = -6.454 + \log P + 0.5 \cdot \log g + 0.1\,M_{bol} + \log T_{eff}. \tag{6-6}$$

In principle, comparison of observed and calculated values of Q can determine the mode of radial pulsation. Breger and Bregman find that the observed values of Q are compatible with pulsation in the first three radial modes. There is no correlation between Q and luminosity, but there is a dichotomy between the Q values for stars hotter and cooler than T_{eff} = 7800 K. Table 6-2 summarizes the results obtained by Breger and Bregman. The Q values of the stars cooler than 7800 K correspond to pulsation in the fundamental mode, while the hotter stars pulsate in the first and second overtones.

Mode typing can also be carried out for multiperiod δ Sct stars on the basis of the ratios of the periods of individual modes. Period ratios near 0.77 are seen in many δ Sct stars and are obviously compatible with excitation of the fundamental and first overtone.

The evidence for nonradial pulsations in δ Sct stars rests mainly on the observation of unusual period ratios in multiperiodic stars. An excellent example is the star 1 Mon, for which multiperiod analyses have been carried out by Shobbrook and Stobie (1974) and by Balona and Stobie (1980). Table 6-3 lists the frequencies, with their corresponding amplitudes, that are required to fit the observations of 1 Mon, and Figure 6-11 shows the resulting representation of the light curves. The existence of close frequencies rules out the possibility that the oscillation is entirely caused by low order radial modes. Shobbrook and Stobie show that the frequency spectrum can be accounted for by the nonlinear superposition of three nonsinusoidal

Table 6-2
Temperature Dependence of Mean Q-Values

Sample	No. of Stars	Mean Q-Values (days)	
		Hot Group	Cool Group
All stars	59	0.022 ± 0.001	0.030 ± 0.001
Hotter than 8070 K	12	0.023 ± 0.002	. . .
7810 to 8070 K	12	0.021 ± 0.002	. . .
7450 to 7800 K	17	. . .	0.030 ± 0.002
Cooler than 7450 K	18	. . .	0.031 ± 0.002
Petersen and Jorgensen (1972)	38	0.023 ± 0.002	0.028 ± 0.002

Source: From Breger and Bregman 1975.

Table 6-3
Parameters of Sinusoidal Components of the Light Variations in 1 Monocerotis, From Observations of Shobbrook and Stobie

	Frequency Description	Frequency (c/d)	Amplitude[a] (pc)	Hel. JD of Max. Light 2441681+
(1)	f_1	7.34620 ± 0.00008	8.73 ± 0.08	0.7233 ± 0.0002
(2)	f_2 or $f_1 + f_L$	7.47533 ± 0.00012	6.24 ± 0.08	0.7548 ± 0.0003
(3)	f_3 or $f_1 - f_L$	7.2172 ± 0.0003	2.24 ± 0.08	0.8063 ± 0.0008
(4)	$f_1 + f_2$ or $2f_1 + f_L$	14.8223 ± 0.0004	2.01 ± 0.08	0.7324 ± 0.0004
(5)	$2f_1$	14.6934 ± 0.0004	1.59 ± 0.08	0.7140 ± 0.0005
(6)	$2f_2$ or $2f_1 + 2f_L$	14.9494 ± 0.0008	0.92 ± 0.08	0.7493 ± 0.0009
(7)	$2f_3$ or $2f_1 - 2f_L$	14.565 ± 0.002:	0.50 ± 0.08:	0.757 ± 0.002:
(8)	$3f_1$	22.035 ± 0.002:	0.42 ± 0.08:	0.755 ± 0.002:
(9)	$2f_1 + f_2$ or $3f_1 + f_L$	22.164 ± 0.002:	0.38 ± 0.08:	0.768 ± 0.003:
(10)	$f_1 + 2f_2$ or $3f_1 + 2f_L$	22.296 ± 0.003:	0.25 ± 0.08:	0.735 ± 0.005:
(11)	— —	1.000:	0.35:	0.61:[b]
(12)	f_L	0.1291:	0.50:	2.12:
(13)	f_4[c]	12.105:	0.30:	0.767:

[a]To convert these amplitudes to intensities on the Johnson V system, they should be divided by 1.04.

[b]This period of 1 day probably derives from the reductions rather than the star.

[c]The reality of this period is questionable.

Source: From Shobbrook and Stobie (1974).

waves with frequencies f_1, f_2, and f_3. Balona and Stobie find the same values of f_1, f_2, and f_3, and they therefore conclude that 1 Mon has maintained stable frequencies for at least 7 years. They argue that the strongest oscillation is due to an overtone radial pulsation ($\ell = 0$), while the two other oscillations can be identified with nonradial dipole modes ($\ell = 1$) split by stellar rotation. Furthermore, the inferred rotational velocity is compatible with the observed value of v sin i. The alternative explanation for the multiple frequencies that are observed is that the variations are modulated by an external force; specifically, the variations in phase and amplitude might be caused by tides raised in the primary, which is the δ Sct star, by a companion star. However, the inferred orbital velocity variation substantially exceeds the limit set by observations, and so the data strongly support the hypothesis that 1 Mon exhibits nonradial pulsations.

Another example of a thorough analysis of a star with nonradial pulsations is the work on 14 Aur by Fitch and Wisniewski (1979). The star 14 Aur is a single-line spectroscopic binary with a period of 3.788568 days, and the analysis is complicated by the fact that the equilibrium figure of the star is nonspherical.

Unfortunately, determination of the mode of oscillation—radial vs. nonradial—cannot always be made from an analysis of the frequencies present in the light curve. Since the periods of radial

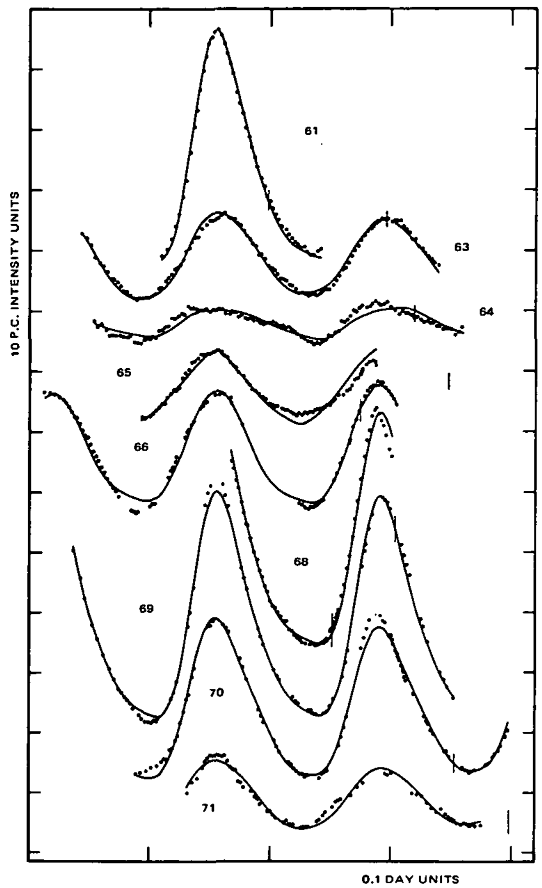

Figure 6-11. Light curves for 1 Mon obtained over an 11-day observing period. The continuous curves are computer fitted from the parameters listed in Table 6-3. The ordinate and abscissa zero points are arbitrary, but the zero of heliocentric JD is marked by the vertical lines and the JD is 2441600 plus the number against each graph. The 7.7-day variation in the amplitudes is clearly seen (Shobbrook and Stobie, 1974).

and nonradial oscillations in δ Sct stars are comparable in length, the mode cannot be classified from the cycle length alone in variables that exhibit only a single mode. Even in multiperiod stars, extensive observations are required for mode typing. For example, in HD 31908, Kurtz (1980a) finds three equally spaced frequencies which he attributes to nonradial oscillation. However, the ratio P_2/P_1 of the two most prominent frequencies is very close to that predicted for radial pulsation in the fundamental and first overtone. If the third frequency were not measured—and it would have been missed if either its amplitude were a factor of 2 smaller or the noise in the data a factor of 2 larger—HD 31908 would probably have been interpreted as a radial oscillator.

There are two other methods for distinguishing radial from nonradial pulsation in δ Sct stars. Balona and Stobie (1979) have shown that the phase difference between the B–V color curve and the V luminosity curve depends on the mode of oscillation. Table 6-4 gives the phase differences calculated for various values of the phase lag ψ between the flux and radius variation and for a range of values of f, where f is the ratio of flux to radius variation (Balona and Stobie, 1980). As the table shows, the phase differences are not very sensitive to ψ or f. The data in Table 6-4 indicate that not only is it possible to distinguish radial ($\ell = 0$) from nonradial ($\ell \neq 0$) oscillations, but that it is also possible to distinguish the nonradial modes $\ell = 2$, $\ell > 4$, and ℓ = odd from one another. An application of this technique to HD 188136 has been discussed by Kurtz (1980b), who also gives references to similar studies of other δ Sct stars. An analysis of phase shifts led to the identification of modes in 1 Mon discussed earlier in this section.

Variations in line profiles and radial velocity can also be used to determine the mode of oscillation. A schematic illustration of the effects to be expected has been presented by Smith and McCall (1978). Figure 6-12 shows a velocity vector diagram for a rotating star in nonradial pulsation. The oscillation illustrated is the quadrupole mode, $\ell = 2$. The value of m determines the nature of the wave, with $m = 0$ corresponding to a standing wave. Values of $m \neq 0$ correspond to traveling waves, with $m > 0$ indicating a retrograde mode. Figure 6-13 shows the changes to be expected in line profiles for the oscillation shown in Figure 6-12. Nonradial pulsations result in very small changes in radial velocity but in large changes in line widths and asymmetry. Radial pulsation in a rotating star may also produce asymmetric line profiles (Duval and Karp, 1978), but these asymmetries must be accompanied by large changes in radial velocity.

Table 6-4
The Phase Shift $\Delta\phi = \phi_V - \phi_{B-V}$ Between the Light and Color Variations Calculated for Even Spherical Harmonic Orders, ℓ, of Nonradial Oscillation

		f					
ℓ	$\psi°$	5	10	15	20	25	30
0	90	−0.061	−0.031	−0.021	−0.016	−0.013	−0.011
	100	−0.064	−0.032	−0.021	−0.016	−0.013	−0.011
	120	−0.065	−0.030	−0.020	−0.015	−0.012	−0.010
	140	−0.057	−0.024	−0.015	−0.011	−0.009	−0.007
2	90	0.078	0.046	0.032	0.024	0.019	0.016
	100	0.069	0.042	0.030	0.023	0.019	0.016
	120	0.050	0.033	0.024	0.019	0.015	0.013
	140	0.033	0.022	0.017	0.013	0.011	0.009
4	90	0.414	0.339	0.279	0.234	0.199	0.172
	100	0.415	0.340	0.280	0.233	0.198	0.171
	120	0.424	0.349	0.284	0.231	0.192	0.162
	140	0.441	0.373	0.295	0.225	0.174	0.140
6	90	0.460	0.422	0.386	0.352	0.322	0.295
	100	0.461	0.423	0.387	0.353	0.323	0.296
	120	0.466	0.431	0.397	0.363	0.331	0.302
	140	0.474	0.447	0.419	0.387	0.353	0.318
8	90	0.477	0.455	0.433	0.411	0.391	0.371
	100	0.478	0.455	0.434	0.412	0.392	0.372
	120	0.480	0.461	0.441	0.421	0.402	0.382
	140	0.485	0.470	0.455	0.439	0.423	0.405

Note: A range of f, the ratio of flux to radius amplitudes, and ψ, the phase lag between flux and radius variation is tabulated. For all odd values of ℓ, $\Delta\phi$ is identically zero.
Source: From Balona and Stobie (1980).

Campos and Smith (1980) have applied these basic concepts to mode typing of ρ Pup and δ Sct. For ρ Pup they find that the line profiles do not change with phase, but that the radial velocity varies quasi-sinusoidally with an amplitude of $2K = 11.5$ km s^{-1}. They conclude, therefore, that ρ Pup is a radial oscillator and that the lack of variable asymmetry in the line profiles is caused by that star's low rotational velocity. The assumption that δ Sct pulsates in a purely radial mode, however, fails to account for the observed variation of 30 percent in line width. Figure 6-14 shows the improvement in fit if radial and nonradial oscillations are assumed to be simultaneously excited. The differences due to nonradial oscillations are subtle, and extremely high signal-to-noise data are clearly required for mode typing on the basis of line profiles. An independent analysis of the photometric variations, however, does support the conclusion that both radial and nonradial oscillations are present in δ Sct (Fitch, 1976). Further confirmation comes from an analysis of the phase differences between the light and color variations. From their study of BVI photometry, Balona et al. (1981) conclude that the principal variation of δ Sct is a radial pulsation, while the secondary variation can be identified with a quadrupole oscillation.

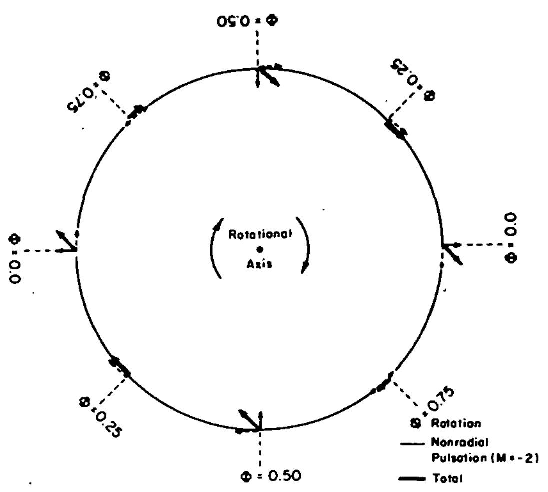

Figure 6-12. Velocity vector diagram for a rotating nonradially pulsating star viewed pole-on. An observer at each location in the equatorial plane sees the result of the vector addition of pulsation and rotation integrated across the disk. Using the diagram, one can predict the sense of asymmetry and the approximate width of the line for a given mode ($\ell = -m = 2$, in this case) (from Smith and McCall, 1978).

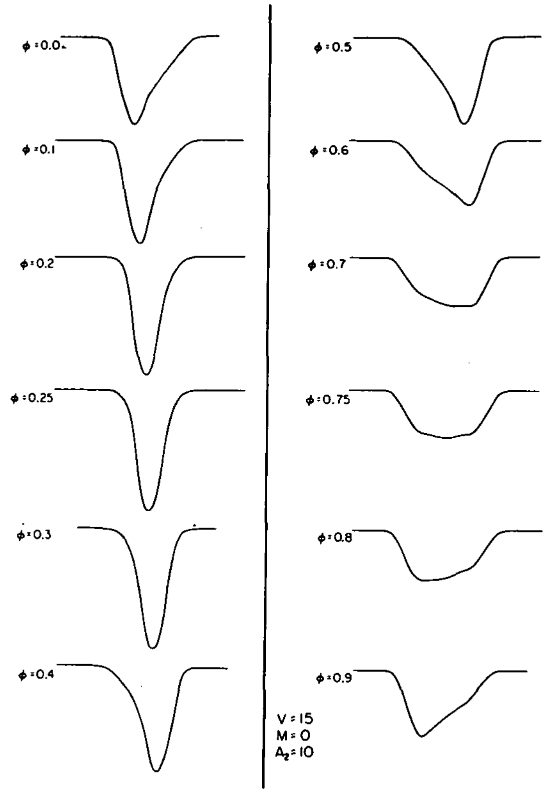

Figure 6-13. Evolution of a line profile in the presence of a traveling wave nonradial mode. The illustration depicts the sectoral quadrupole mode ($\ell = 2$, $m = -2$). The profile evolution runs backward in time with a retrograde mode ($m = +2$) (Smith and McCall, 1978).

A more extensive study of mode typing through observations of line profile changes has recently been completed by Smith (1982). Three of the nine stars studied were found to be pure radial oscillators, three more showed at least two nonradial modes and one radial mode, and the remaining three are apparently pure nonradial oscillators with at least two excited modes. In three of the stars, there is evidence for rotational splitting of the nonradial modes, while in 14 Aur A the splitting of 3 nonradial modes appears to be caused by tidal perturbation by a close companion. In general, Smith's results, based on modeling of line profiles, are in agreement with earlier conclusions derived from multifrequency analyses of photometric data.

The conclusions that can be reached on the basis of existing data concerning modes of pulsation have been summarized recently by Breger (1980b; see also Fitch, 1980). In many stars, several radial modes are excited simultaneously. In the cooler portion of the δ Sct instability strip, the fundamental mode tends to be of larger amplitude than the other modes. Nonradial pulsation occurs throughout the entire instability strip but is *not* seen in every δ Sct star. Radial and nonradial pulsation modes may coexist and may be coupled through resonances or rotation. Population I δ Sct variables with amplitudes in excess of 0.3 mag typically do not exhibit complex multiperiodicity; the dominant pulsation mode is usually the fundamental radial mode. Pulsation constants for the ℓ = 0, 1, 2, and 3 modes have been calculated by Fitch (1981) for evolutionary sequences with masses of 1.5 to 2.5 $M_\odot$ and standard Population I composition ($X = 0.70, Z = 0.02$).

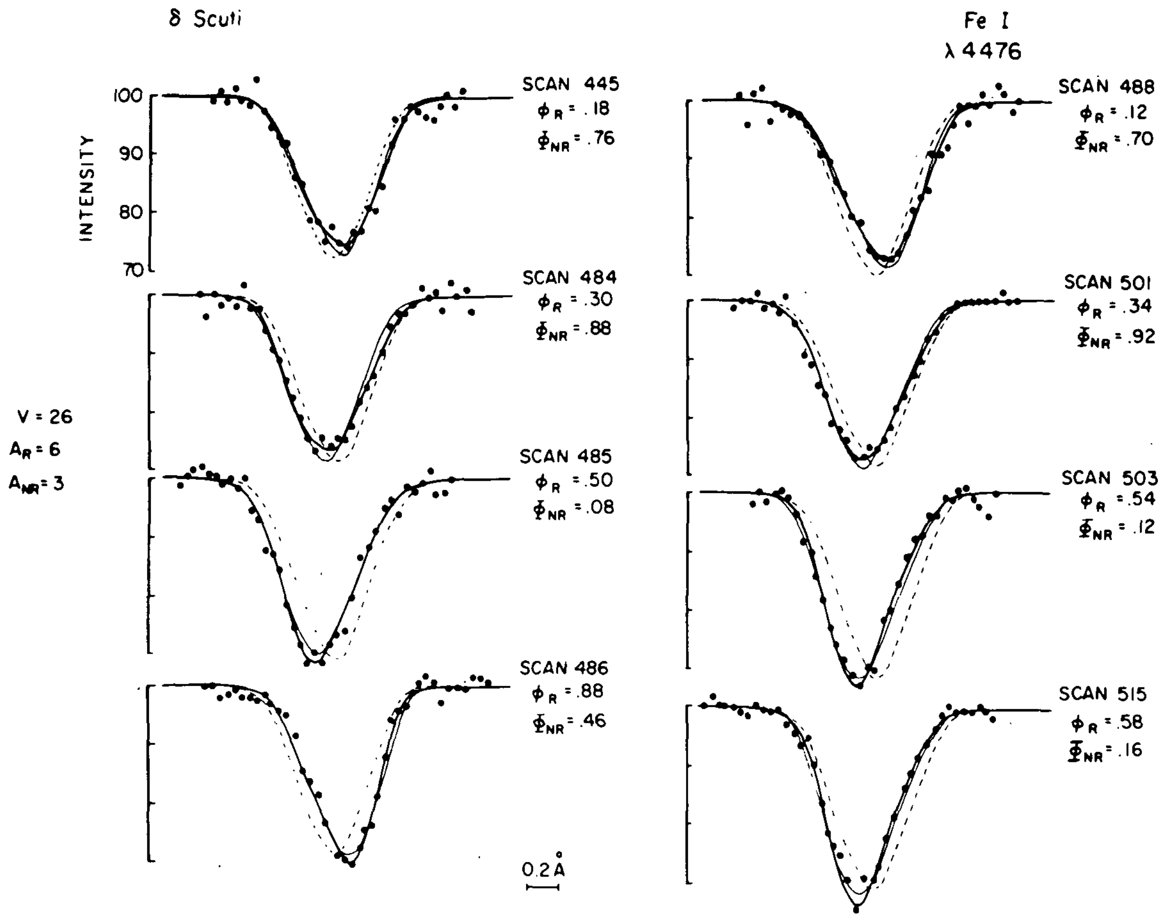

Figure 6-14. Reticon observations of the λ4476 line profile in δ Sct modeled with radial pulsation alone (light solid line) and with the adopted radial plus nonradial pulsation combination (thick solid line) (from Campos and Smith, 1980).

DWARF CEPHEIDS

The dwarf Cepheids, or RRS or AI Velorum stars, were first defined as a class by Smith (1955). The members of this class are pulsating variables with periods less than one-quarter of a day, photometric visual amplitudes greater than 0.3 mag, and spectral types in the range A to F. The critical question is whether the dwarf Cepheids form a distinct class of objects or are simply those δ Sct stars—that is, those stars with Population I masses and composition—that happen to exhibit, for whatever reason, light variations with unusually large amplitudes.

The idea that a classification based on photometric amplitude could be used to define a group of stars with other distinctive characteristics was reinforced by observations of SX Phe, which is often taken as *the* prototypical dwarf Cepheid. This star has a high space velocity, a low luminosity according to its trigonometric parallax, and a low abundance of metals. Furthermore, the ratio of the periods of the first overtone to the fundamental, P_1/P_0 = 0.778, is larger than the value predicted by early models of pulsating variables with Population I masses and composition. Bessell (1969) therefore suggested that the dwarf Cepheids were post-helium flash stars with a mass of about 0.5 $M_\odot$.

Recent surveys by McNamara and Feltz (1978), Eggen (1979), and Breger (1980a), however, challenge the hypothesis that all dwarf

Cepheids share the characteristics of SX Phe. Specifically, the majority of dwarf Cepheids cannot be distinguished from the classical δ Sct stars on the basis of any of the following characteristics:

- **Distribution of Periods.** If allowance is made for the fact that rapid rotation appears to exclude large amplitude photometric variability and for the period-luminosity relation shown by δ Sct stars, then the distributions of periods for the dwarf Cepheid and δ Sct stars are identical (Breger, 1980a).

- **Composition.** The abundances of the dwarf Cepheids have been inferred from the photometric index $\delta m_1 = m_1$ (Hyades) $- m_1$ (star) and from ΔS, which measures the difference between the K line and hydrogen line spectral types. The calibration of these indices is uncertain inasmuch as the masses of the dwarf Cepheids are, a priori, unknown, and either index may be luminosity sensitive. Furthermore, the calibration of δm_1 is based on stars that do not pulsate. Nevertheless, the observations show that δm_1 and ΔS for the majority of dwarf Cepheids match the values typical of classical δ Sct stars. Only four of the dwarf Cepheids (GD 428, SX Phe, CY Aqr, and DY Peg) in Breger's survey exhibit abnormally weak metallic lines.

- **Space Motions.** The space velocities derived for the dwarf Cepheids depend on the adopted luminosity. Fortunately, however, the assumption that they are classical δ Sct stars corresponds to a higher luminosity, and hence a greater distance and greater space velocity, than the assumption that they are post-helium flash stars with masses of 0.5 $M_\odot$. The hypothesis that the dwarf Cepheids equal the δ Sct stars in luminosity yields low (Population I) space velocities except for the four stars (GD 428, SX Phe, CY Aqr, and DY Peg) that also have very weak metallic lines (McNamara and Feltz, 1978; Breger, 1980a).

- **Masses.** One of the original arguments for the hypothesis that the dwarf Cepheids are low mass Population II objects was that the period ratios derived for the multiperiodic variables could not be fit by Population I models. An additional complication was the lack at that time of multiperiod analyses for classical δ Sct stars; because of their larger amplitude, dwarf Cepheids are easier to study. Recently, however, Stellingwerf (1979) and Cox et al. (1979), who developed deep envelope, linear, nonadiabatic models that incorporate improved opacity tables, have shown that the observed period ratios can be accounted for by Population I models. The one exception is SX Phe, which can be represented only by models with a reduced helium content ($Y \sim 0.1$). The new models also show that neither the absolute values of the period nor the period ratios of δ Sct and dwarf Cepheids depend strongly on mass (Petersen, 1978). The periods are, however, very sensitive to composition, and by varying Z (Stellingwerf, 1979), and Y (Cox et al., 1979), one can account for the observed periods by any reasonable mass.

Masses for δ Sct and dwarf Cepheid stars can be derived if information other than simply the periods is available. The absolute magnitude M_{bol}, combined with T_{eff}, defines the mass very accurately (Stellingwerf, 1979), but M_{bol} is usually not known. If the calibration of uvby photometry used to derive $\log g$ for static stars is applicable to dwarf Cepheids and δ Sct stars, then the value of $\log g$ plus the period can be used to derive the mass. This technique has been applied by Breger (1980a) and by McNamara and Feltz (1978), who have found that the masses of dwarf Cepheids lie in the range from 1 to 2.5 $M_\odot$.

The most persuasive evidence of Population I masses for dwarf Cepheids comes from the application of the Wesselink method. By comparing the change in radius of a pulsating variable inferred from photometric measurements with the radius change derived from integration of the radial velocity curve, one can derive the radius of the star. Application of this method to dwarf Cepheids yields radii of ~3 $R_\odot$ for AD CMi (Breger, 1975b), RS Gru (McNamara and Feltz, 1976; Balona and Martin, 1978), and DY Her (McNamara, 1978). In combination with the values of log g derived photometrically, the derived radii imply masses in the range 1.9 to 2.8 $M_\odot$. Interestingly, an attempt to apply the Wesselink method to two metal-deficient dwarf Cepheids failed (Van Citters, 1976). Equal temperatures measured at different phases implied greatly different radii, and the explanation for the discrepancy is unclear.

The three hypotheses that have been proposed to account for the dwarf Cepheids have been summarized by Breger (1980a). The first is that the dwarf Cepheids are normal Population I stars in a main sequence or post-main sequence phase of evolution. In this hypothesis, the dwarf Cepheids are simply those δ Sct stars with amplitudes that exceed the (arbitrary) limit of 0.3 mag. The observational determinations of the masses, composition, and space velocities of the majority of dwarf Cepheids are compatible with this hypothesis.

At least four dwarf Cepheids, however, exhibit high space velocities and low metal abundances and do not appear to be properly classified as main sequence or near-main sequence Population I stars. There are two possible explanations for this group of stars. They may be main sequence Population II stars; pulsation periods, surface gravities, and the trigonometric parallax for SX Phe all rule out membership on the horizontal branch. If these stars are on the main sequence, their ages are only $\sim 2 \times 10^9$ years, and either metal-poor stars have been formed very recently in our galaxy or else the evolution of some metal-poor stars has been delayed (McNamara and Feltz, 1978). On this hypothesis, the metal-poor dwarf Cepheids are analogous to the blue stragglers seen in many clusters. The alternative explanation is that the metal-poor dwarf Cepheids are highly evolved objects that have lost a great deal of mass during the red giant phase of evolution and are now evolving toward the white dwarf stage with luminosities much lower than expected for normal horizontal branch stars. In principle, a choice can be made between these two hypotheses by measuring the masses of the metal-poor dwarf Cepheids, but in practice there is sufficient uncertainty in the calibration of log g that the mass determinations are not accurate enough to exclude either hypothesis.

In summary, observational surveys have shown that there is a continuity in the properties—masses, space velocities, composition, and periods—of the classical δ Sct stars and most dwarf Cepheids, that a distinction between the two groups based on photometric amplitude is meaningless, and that both groups of stars are in main sequence or early post-main sequence phases of evolution. In addition, however, there is a small group of large amplitude variables in this same part of the HR diagram that, on the basis of their composition and space velocities, appear to be Population II stars. Whether they are main sequence blue stragglers or highly evolved post-helium flash stars remains to be determined.

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7

A SUPERGIANTS

INTRODUCTION

During the past decade increasing effort has been devoted to the study of the most luminous stars, and a very useful monograph on this subject has been recently published (de Jager, 1980). The reasons for the renewed interest are twofold. First, new observational techniques in the X-ray, far-ultraviolet, infrared, and radio regions of the spectrum allow observations not only of stellar photospheres but also of the mass outflows and extended envelopes that are ubiquitous in the most luminous stars. Second, theoretical methods are now available that allow the solution of coupled equations of dynamics and radiative transfer. Recent modeling of supergiants has re-emphasized the inappropriateness of treating these stars as isolated thermodynamic systems. The high-velocity winds from early-type supergiants can strongly modify the surrounding interstellar medium and may play a role in triggering—or limiting—star formation in dense clouds. And, of course, massive stars are thought to be a major factor in the chemical enrichment of galaxies. Heavy elements, synthesized in the interiors of these stars, may be returned to the interstellar medium either gradually through stellar winds or violently through supernova explosions to be incorporated into new generations of stars.

To date, only a limited amount of work has been devoted to the A-type supergiants. Because they occupy a region of the HR diagram where evolution is rapid, they are few in number. Furthermore, the emergent spectrum is modified by mass loss to a much smaller degree than is the case for O, B, and M supergiants. There is little evidence for chromospheres or coronae, and the comparatively low UV and X-ray fluxes limit satellite observations. Nevertheless, because the A-type supergiants are fairly evolved objects, in which substantial mass loss may have already occurred, an understanding of their present structure, past evolution, and mass loss rates can provide critical information on the importance of presupernova mass loss in shaping the chemical evolution of galaxies. Moreover, ultraviolet line profiles, which indicate that mass loss is occurring in some of the brightest A supergiants, are morphologically quite distinct from the profiles seen in O and early B supergiants. The structure of the stellar wind in A supergiants is evidently unique in some way that is not yet understood.

TEMPERATURE AND LUMINOSITY

Effective temperatures can be determined directly for stars for which both absolute fluxes and apparent radii are known. A two-telescope interferometer has been used to measure directly the angular diameter of α Cyg (A2 Ia) (Bonneau et al., 1981). The inferred radius is $145 \pm 45\, R_{\odot}$, where most of the uncertainty is due to the uncertainty in the distance of α Cyg. The value of T_{eff} is in the range 8150 to 8250 K, depending on the choice of the bolometric correction.

Direct measurements of the radius are not available for other A supergiants. As an alternative approach to estimating T_{eff}, Johnson (1966) has used color indices to interpolate between stars of earlier and later spectral type with measured angular diameters. Flower (1977) has recently rederived temperature scales and bolometric corrections for stars of all luminosity classes by taking into account new ultraviolet and infrared observations. Again, temperatures for the A-type supergiants must be established by interpolation, and Table 7-1 compares the temperature scales derived by Flower and Johnson. The temperatures suggested by Flower and Johnson for A2 supergiants are 700 to 1000 K higher than the value derived by Bonneau et al. for α Cyg.

A-type supergiants are absent in the solar neighborhood, and reliable parallaxes are available for none of them. The best estimates of absolute magnitude are for stars in clusters or associations, for which distances can be obtained through fitting of zero-age main sequences (ZAMS's). A number of secondary calibration techniques based on such criteria as the appearance of O I λ7774 (Parsons, 1964; Osmer, 1972) have been discussed by Bouw and Parsons (1971). Figure 7-1, which is taken from their paper, shows that spectral classification provides only an approximate indication of luminosity. Some additional quantitative methods are also available. For example, Rosendhal (1974) has shown that measurement of the strengths of the Si II lines λ6347 and λ6371 can yield absolute magnitudes for B9 to A2 supergiants that are accurate to within ±0.5 mag. The strengths of these lines are expected to be insensitive to both temperature and gravity in this spectral range. The observed strengthening of these saturated lines may therefore be because of a positive correlation between turbulence and luminosity. There is also a good correlation between the net strength (absorption minus emission) of Hα and luminosity, but the usefulness of this criterion as a luminosity indicator is limited by intrinsic stellar variability and by the fact that the slope of the correlation depends fairly strongly on temperature (Rosendhal, 1973b).

Figure 7-2 shows an HR diagram for supergiants in our own galaxy. With rare exceptions,

Table 7-1
Effective Temperatures and Bolometric Corrections for A Supergiants

Spectral Type	T_{eff}[a]	B.C.[a]	T_{eff}[b]	B.C.[b]
A0	9400	−0.38	–	–
A1	9100	−0.30	–	–
A2	8900	−0.21	9120	−0.17
A4	–	–	8790	−0.10
A5	8300	+0.01	8510	0.00
A8	–	–	8205	+0.09
F0	7500	+0.14	7800	+0.14

[a]From Johnson (1966).
[b]From Flower (1977).

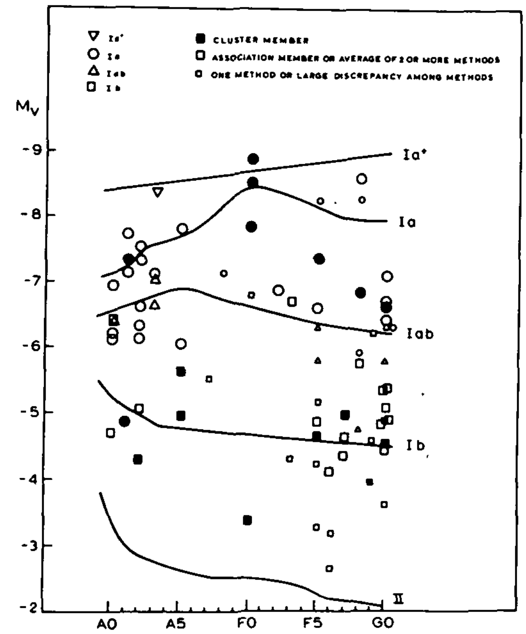

Figure 7-1. Absolute visual magnitudes of A and F supergiants—a calibration of the Yerkes spectral classification system. Note the large dispersion of the points. The Ia+ class in this diagram is defined by one point only (from de Jager, 1980; based on data from Bouw and Parsons, 1971).

the late B and early A supergiants are the brightest stars in the visual region of the spectrum (see also Underhill and Doazan, 1982). The limiting magnitude is approximately $M_v = -8.5$. The same general characteristics are seen in the Large Magellanic Cloud (LMC), although the relative number of A supergiants is somewhat higher (Humphreys, 1979). Figure 7-3 shows the same stars, but now the ordinate is the bolometric rather than the absolute visual magnitude (Humphreys and Davidson, 1979). In this diagram, the O-type stars are more luminous than the A supergiants because of the very large bolometric corrections that must be applied to the former. Evolutionary tracks with mass loss (Chiosi et al., 1978) are superimposed. The A supergiants clearly lie beyond the region of core hydrogen burning and may be in the phase of core helium burning. However, the physics of these late stages of evolution is not well understood. An example of the failure of the models is the fact that the HR diagram is well populated in the region of the early B-type stars, while calculations suggest that these stars should be rare. According to the models, these stars should be in the hydrogen shell-burning phase and should be evolving rapidly to the right. The absence of A supergiants with bolometric luminosities comparable to those of O-type stars is usually attributed to, perhaps catastrophic, mass loss in earlier stages of evolution (e.g., Humphreys and Davidson, 1979).

THE MOST LUMINOUS A SUPERGIANTS

In external galaxies, as in our own galaxy, the stars with the brightest visual magnitudes usually are of spectral type B or A. The importance of these stars is twofold. First, if an accurate luminosity criterion can be found, then the "superluminous" A-type stars may provide a distance scale for nearby dwarf galaxies where other techniques are often inapplicable. Second, spectroscopic studies of these superluminous objects can be used to determine whether the evolution of massive stars and the physical processes in their atmospheres are the same in other galaxies as in our own. The superluminous B supergiants are discussed by Underhill and Doazan (1982) in *B Stars With and Without Emission Lines.*

A prototypical example of a superluminous A-type star in our own galaxy is HD 160529 (A2 Ia^+), which has been analyzed in detail by Wolf et al. (1974). One would expect extremely luminous stars to be near the limit of stability, and indeed HD 160529 shows large variations in radial velocity, color, and magnitude. The photometric amplitude is 0.27 in y and 0.18 in (u–v). Major

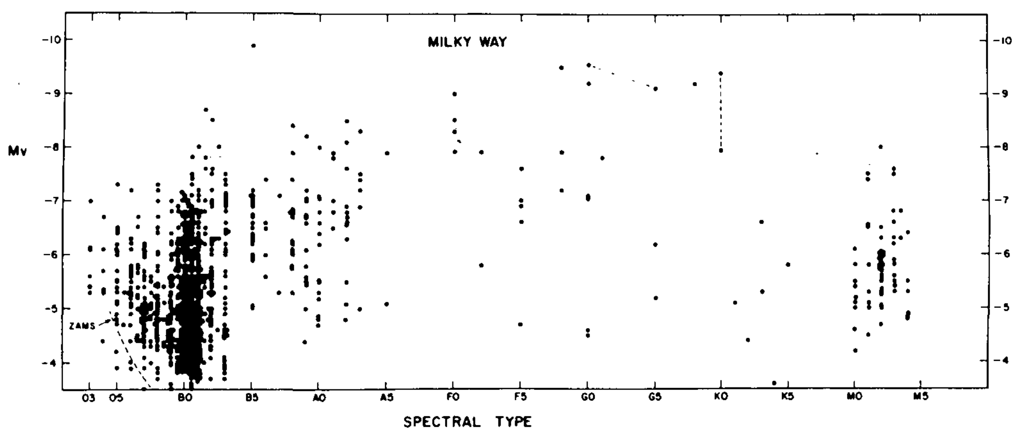

Figure 7-2. The H–R diagram, M_v versus spectral type, for the supergiants and O stars in the Milky Way (from Humphreys, 1978b).

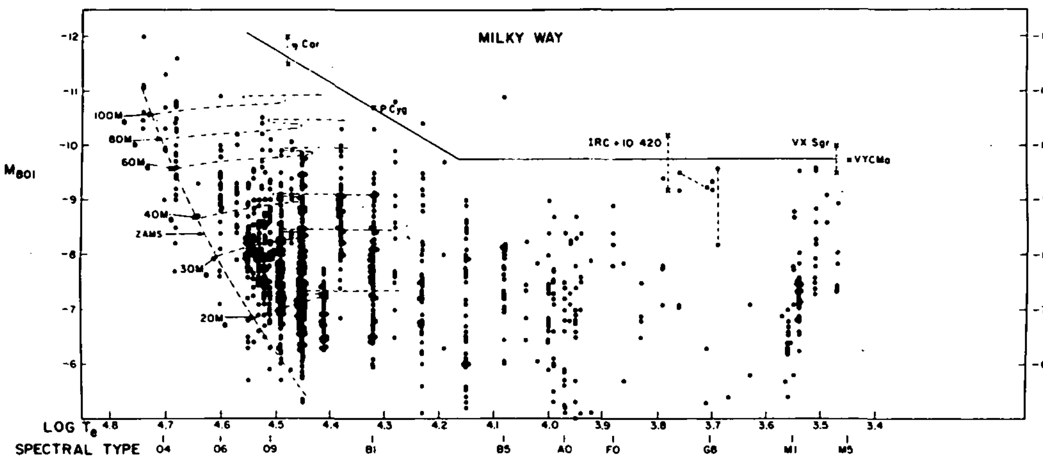

Figure 7-3. The "theoretical" HR diagram, M_{bol} versus log T_e, for the galactic supergiants. The position of the ZAMS and evolutionary tracks with mass loss are shown. The solid line defines the approximate upper boundary of the supergiant luminosities. The positions of two peculiar stars, η Car and P Cyg, and three supergiant infrared sources are also indicated (from Humphreys and Davidson, 1979).

variations typically occur on time scales of days; the observations to date are not well suited to detecting fluctuations with shorter periods. The radial velocity varies with an amplitude of 40 km s^{-1}, and line splittings with velocity differences as high as 42 km s^{-1} are seen in Fe II, Ti II, and Cr II. There is evidence for a progression—albeit a highly variable one—in the velocities of the Balmer lines, in the sense that lower members of the series usually have more negative velocities. This result may be caused by a velocity gradient in the atmosphere, or may reflect a relatively greater contamination by emission in the lower members of the Balmer series. Emission at Hα is strong and is seen in Hβ as well. The velocities of the metallic lines depend on excitation potential, and hence on depth in the atmosphere. Lines of higher excitation potential, which are presumably formed deeper in the atmosphere, have lower velocity amplitude.

Obviously, local thermodynamic equilibrium (LTE) curve-of-growth analyses are of limited validity in a star like HD 160529. However, the excitation temperature is found to be 7800 K and the microturbulent velocity, which is variable, falls in the range of 7 to 14 km s^{-1}. By assuming that the excitation temperature applies to optical depth $\tau = 0.1$, Wolf et al. (1974) find that the effective temperature is $T_{eff} = 8800$ K, and log $g = 1$. These values are estimated from LTE models in strict hydrostatic equilibrium.

Wolf et al. also argue that the velocity fields can account for many of the characteristics of HD 160529. Since the star is fainter when the line splitting is evident, much of the variation in brightness may be caused by variable line blanketing. A velocity field will alter the equation of mechanical equilibrium, and hence the gas pressure in the stellar atmosphere. Since the electron pressure depends on gas pressure and temperature, the ionization state will also be altered by a velocity field; such changes may account for the observed variations in the Balmer discontinuity (Groth, 1972). Photospheric velocity fields may supply the source of nonradiative heating that is required by the fact that He I λ5876 is seen in absorption in HD 160529. Wolf et al. also note a correlation between line splitting seen in the photospheric absorption lines of Ti II, Fe II, and Cr II and the intensity of the Fe II emission lines at λ5991, λ6432, and λ6516. This correlation may reflect the increased dissipation of energy due to enhanced velocity fields manifesting themselves through line doubling, and it also suggests that the emission occurs in the vicinity of the photosphere (essentially a chromosphere) rather than in an extended shell. Of course the physics involved—the mechanism for converting kinetic to excitation energies, the efficiency of this mechanism, the temperature rise produced by it—remains unexplored.

An initial survey of the superluminous stars in the Magellanic Clouds was carried out by Feast et al. (1960); there are at least a dozen A supergiants with $M_v < -8.0$ (Humphreys and Davidson, 1979) in the Large Magellanic Cloud. In fact, the star with the brightest absolute visual magnitude known outside our own galaxy is HD 33579 (A3 Ia^+) in the Large Magellanic Cloud. For this star, $M_v = -9.8$ (Humphreys, 1979). The brightest star in the Small Magellanic Cloud (SMC) is HD 7583, with $M_v = -9.3$. Both stars have been analyzed spectroscopically (Wolf, 1972; 1973, and references therein), but in less detail than HD 160529 because of their faintness. Spectroscopically derived values of T_{eff} and $\log g$ yield masses of about 25 $M_\odot$ for both stars. The uncertainty in this number is difficult to estimate but is surely substantial since it depends on atmospheric parameters derived from plane-parallel, unblanketed, LTE models in hydrostatic equilibrium.

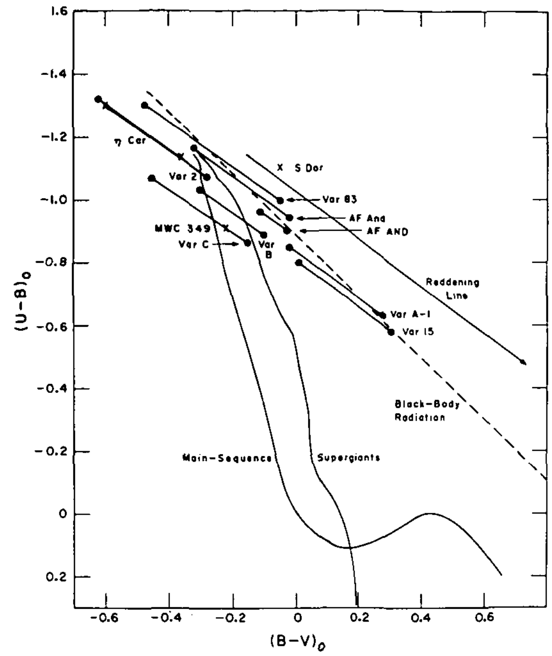

Figure 7-4. The $(U-B)_0$ versus $(B-V)_0$ diagram for eight blue variables plus η Car, MWC 349, and S Dor. The M 31 and M 33 stars are shown as lines representing the range of intrinsic colors they would have for the upper and lower values for their interstellar reddening. Eta Car is also shown as a line for two different determinations of the intrinsic colors. All of these stars have ultraviolet excesses and lie near the blackbody radiation line (from Humphreys, 1978a).

Apart from novae and supernovae, visually the brightest objects in galaxes are a group of irregular variables, which were first described in detail by Hubble and Sandage (1953), and are usually referred to as Hubble-Sandage stars. These stars have a strong ultraviolet continuum, with no Balmer discontinuity. The slopes of their Paschen continua are similar to those of A supergiants (cf., Figure 7-4). Emission lines of H, He I, Fe II, and [Fe II] are present, and at maximum light the luminosities of these stars are near $M_v = -10$ (Humphreys, 1975; 1978a). The photometric variations of several members of this class have been summarized by Sharov (1975), and sample light curves are shown in Figure 7-5. The character of the variability varies greatly from star to star; in some cases only a single outburst has been observed, while in others strong oscillations are omnipresent.

In discussing the evolutionary status of these stars, Humphreys (1979) and Humphreys and Davidson (1979) have emphasized the spectroscopic similarities between the Hubble-Sandage variables and S Doradus in the Large Magellanic Cloud and η Car. All are apparently massive, highly luminous stars with extended atmospheres. There is evidence for mass loss in all of these objects. Humphreys and Davidson suggest that massive stars ($M > 60\ M_\odot$) in late (post-core H-burning) stages of evolution may go through brief periods of semicatastrophic mass loss ($\sim 10^{-3}$

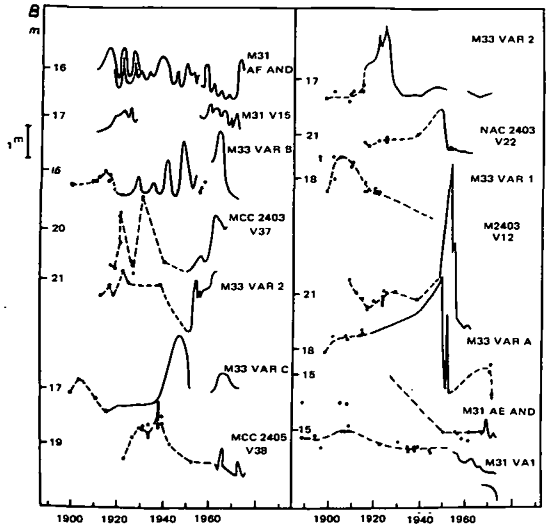

Figure 7-5. Light curves of bright irregular variables in M 31, M33, and NGC 2403 (from Sharov, 1975).

$M_{\odot}\ yr^{-1}$ for η Car), perhaps driven by pulsational instabilities with multiple outbursts (circa 1890, 1840, and earlier for η Car).

MASS LOSS

Mass loss is ubiquitous among highly luminous OBA stars. This fact was firmly established from observations by Morton (1967), who found that the ultraviolet resonance lines of highly luminous stars were displaced from their rest positions by one thousand kilometers per second or more. Since the observed velocities exceed the photospheric escape velocities by a wide margin, material must be lost from the star. Subsequent satellite data (e.g., Snow and Morton, 1976) have confirmed and extended the rocket observations by Morton. An excellent summary of the most recent results on mass loss in B supergiants can be found in Underhill and Doazan (1982).

A variety of observational techniques are available to detect mass loss, either directly or by inference from measurements of expansion or extension of photospheric or near-photospheric layers. The following describes two particularly significant mass loss indicators in the visible (Hutchings, 1970).

1. **Emission, particularly in Hα, Hβ, He I 5876.** Broad emission can generally be taken as evidence that the atmosphere is extended and is either rotating or expanding. The line profile depends on such factors as the extent, density, and velocity field of the stellar envelope.

2. **A dependence of radial velocity on excitation potential and/or a regular progression of radial velocities along the Balmer series.** Near the stellar surface, if LTE obtains at least approximately, one would expect temperature, and hence excitation, to decrease with increasing height above the stellar photosphere. A velocity gradient may manifest itself, therefore, in a systematic correlation between radial velocity and excitation potential. In the case of the Balmer series, the stronger the line, the lower the density at which it saturates, and so the lowest members of the Balmer series tend to sample the velocity fields in the outer portions of the atmosphere.

Of course, caution must be used in interpreting either of these indicators in terms of mass loss. In Be stars, Hα is very strong, but $\dot{M}$, so far as we can tell, is smaller than in B supergiants. Also, velocity gradients may conceivably be present in stars that are not losing mass. Nevertheless, these two indicators, taken in conjunction with such other stellar parameters as T_{eff}, v sin i, and M_{bol}, offer the most useful diagnostics of mass loss in the visible region of the spectrum. Strongly displaced UV resonance lines remain the best evidence for mass loss, but observations of these features can only be made from beyond the Earth's atmosphere. Indeed, it was the advent of routine satellite observations (i.e., Copernicus and IUE) that spurred extensive work on mass loss during the past decade.

A high-dispersion (17.8 Å mm^{-1}) photographic survey of Hα emission in luminous O9 to A5 stars was undertaken by Rosendhal (1973b), and representative profiles for A-type supergiants are shown in Figure 7-6. With increasing luminosity, the absorption of Hα first weakens, then becomes narrower with a kink in

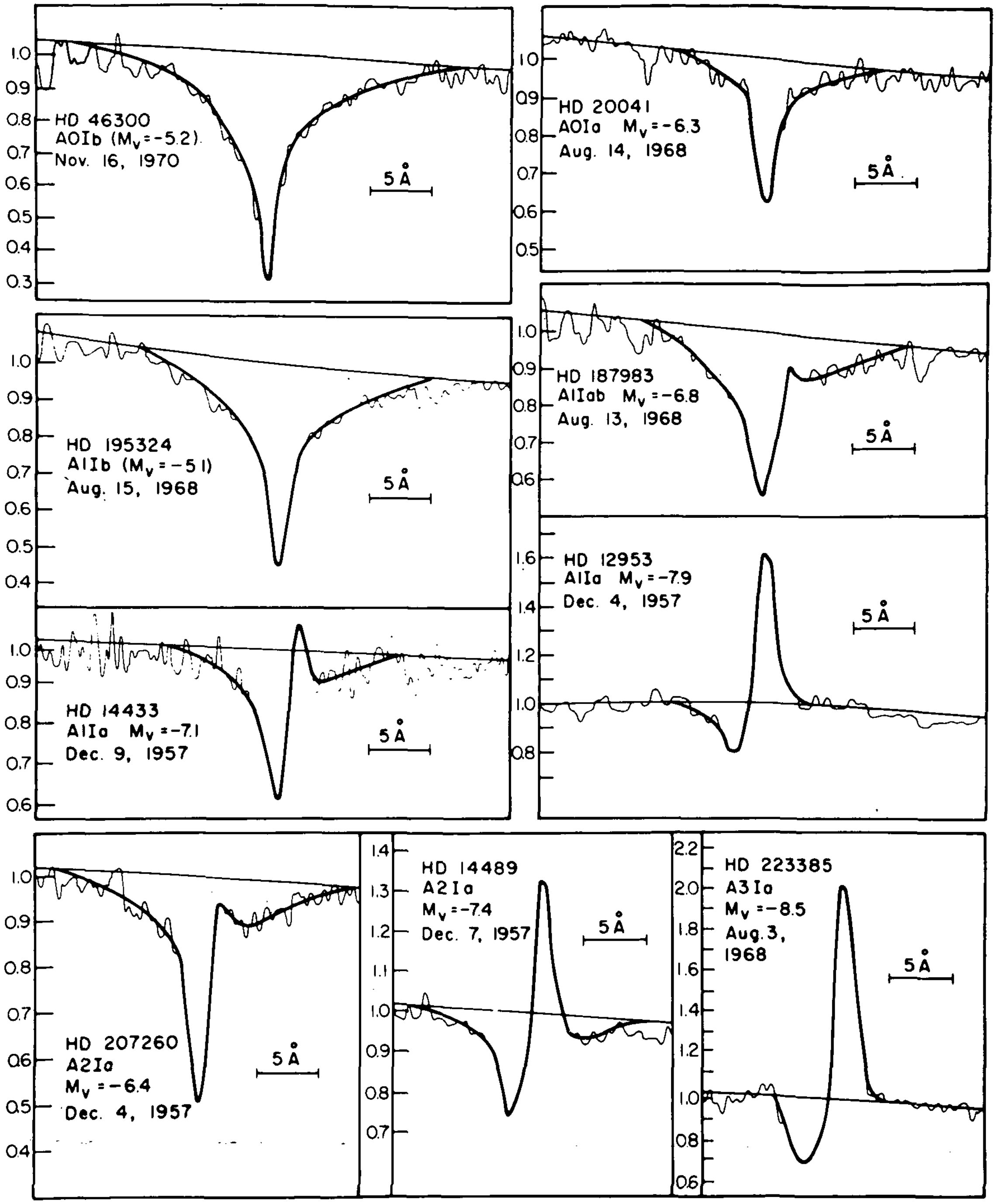

Figure 7-6. Representative Hα profiles in A supergiants (from Rosendhal, 1973b).

the red wing. At still higher luminosities, emission above the level of the continuum is seen in the red wing, while at the highest luminosities emission dominates over absorption. A similar pattern is seen in B supergiants as well, although the slope of the relationship between the net strength of the line (absorption minus emission) and absolute magnitude is temperature dependent. Among B8 to A3 stars, emission at Hα disappears for stars fainter than absolute visual magnitude −6.8 to −7.0, or absolute bolometric magnitude −7.0 to −7.6. The limiting *bolometric* magnitude for Hα emission is approximately the same for early B supergiants. (These generalizations do not apply to the Herbig Ae and Be stars, which are generally thought to be in the pre-main sequence phase of evolution.)

Rosendhal (1973b) has also searched for evidence of velocity gradients in A supergiants, and he finds that the velocity differences Fe II-Si II, C II-Si II, C II-He I, and He I-Si II are well within the 4 to 7 km s^{-1} uncertainty that is attributable to errors in measurement and effective wavelengths. There are large differences between the velocities of Hα and the other lines in the red part of the spectrum in the sense that the Hα velocities are more negative by about 10 km s^{-1} at $M_{bol} = -7.0$ and 50 km s^{-1}, or more, at $M_{bol} = -8.5$. Rosendhal offers persuasive arguments that these velocity differences are caused by flows in the atmosphere, and not simply by a progressive filling in of Hα by red-displaced emission, which becomes increasingly important in stars with higher luminosities.

The ultraviolet spectra of four A supergiants have been examined for evidence of mass loss (see also Underhill and Doazan, 1982). The brightest, α Cyg (A2 Ia), has been studied extensively Kondo et al., 1975; Lamers, 1975; Snow and Morton, 1976; Barbier et al., 1978; Lamers et al., 1978). A new study by Praderie et al. (1980) compares α Cyg with HR 1040 (A0 Ia), HR 2874 (A5 Ib), and η Leo (A0 Ib). In contrast to stars of earlier type that are losing mass, the ultraviolet lines of A supergiants do not exhibit P Cyg profiles. The absence of emission may be partly caused by blending with metallic absorption lines (Hensberge et al., 1982). The resonance lines of abundant singly ionized elements (e.g., C II, Si II, Fe II, Mg II) are blueshifted by up to 240 km s^{-1} in α Cyg and ~150 km s^{-1} in HR 1040 and HR 2874. The surface escape velocities for all these stars probably lie in the range of 230 to 320 km s^{-1}, although these values are quite uncertain because of uncertainties in the mass estimates for these stars. Thus, the velocity shifts constitute fairly clear evidence for mass loss. No mass loss is seen in η Leo.

Excess emission in the infrared and radio regions of the spectrum is seen in many early-type stars and is attributed to free-free transitions in a plasma at large distances from the surface of the star. This emission is evidence for the presence of an extended body of gas around these stars and is usually taken as a strong indicator that mass loss is occurring. Excess emission may also be present in the Balmer continuum (Underhill, 1981).

Wright and Barlow (1975; see also Panagia and Felli, 1975) have shown that the rate of mass loss from a homogeneous sphere of hydrogen gas expanding at velocity v is given by the relation

$$\dot{M} = 0.095\, \mu Z^{-1} (\gamma g_\nu)^{-1/2} \mathrm{v}\, S_\nu^{3/4} \cdot D^{3/2}\, \nu^{-1/2}\, M_\odot \mathrm{yr}^{-1}, \qquad (7\text{-}1)$$

where v is in km s^{-1}, S_ν is the flux at frequency ν in janskys due to free-free emission, D is the distance of the star in kiloparsecs, μ is the mean atomic weight of the gas, Z is the average charge on each ion, γ is the average number of electrons per ion, and g_ν is the value of the Gaunt factor at frequency ν.

Observational results, particularly for luminous O- and B-type stars, can be fairly well represented by models in which the mass flow begins at low velocities near the stellar photosphere and accelerates outward until a maximum or terminal limiting velocity v_∞ is reached. The value of v_∞ can be estimated from the very sharp blue edges of the displaced ultraviolet absorption lines. Radio free-free emission originates far out in the stellar envelope where the flow has reached its terminal velocity; therefore, mass loss rates inferred from radio data are fairly independent of the assumptions made in constructing the model and are generally assumed to be quite reliable. Measurements of α Cyg by Abbott et al. (1980), show that the upper limit of 0.4 mJy for the flux at λ = 6 cm implies a mass loss rate that is less than $2.5 \times 10^{-7} M_\odot \mathrm{yr}^{-1}$. More recent data have reduced this upper limit to $1.5 \times 10^{-7} M_\odot \mathrm{yr}^{-1}$ (Abbott, private communication). The result is valid, however, only if the estimate of the stellar radius is correct, if hydrogen is mostly ionized, and if there is no deceleration in the outer layers of the wind. Models of the stellar wind in α Cyg (Kunasz and Praderie, 1981; Kunasz and Morrison, 1982) suggest that both of the last two conditions may be violated.

In principle, the same method can be used to derive mass loss rates from infrared measurements of free-free emission. However, the infrared emission arises from a region fairly close to the stellar surface where the flow is still accelerating. Therefore, the mass loss rate can be derived only if the variation of velocity with distance from the photosphere $v(r)$ is known. Infrared excesses are found to be small relative to the uncertainties in measuring them and in allowing properly for interstellar reddening. The accuracy of mass loss rates derived from infrared data is, therefore, limited. For α Cyg, Barlow and Cohen (1977) derive $\dot{M} = 6.9 \times 10^{-7}\ M_\odot\ yr^{-1}$. This value is higher than the upper limit obtained from radio measurements, and there are two possible explanations for the discrepancy. First, the infrared observations may be in error since Morrison and Hackwell (private communication) failed to find an infrared excess in α Cyg at 20 μ. Second, the function $v(r)$ used by Barlow and Cohen to interpret their data was that derived from data for P Cyg and that velocity law is almost certainly not appropriate for α Cyg.

Modeling of emission at Hα and of the ultraviolet absorption lines can also yield estimates for $\dot{M}$. Models of this kind have been calculated by Kunasz and Praderie (1981) and Kunasz and Morrison (1982) for α Cyg. The models themselves will be discussed in more detail later in this chapter, but the estimates for $\dot{M}$ fall in the range 3.1×10^{-9} to $4.0 \times 10^{-7}\ M_\odot yr^{-1}$.

An alternative method for deriving the mass loss rate of α Cyg has been described by Hensberge et al. (1982). These authors use fits to the profiles of ultraviolet resonance lines of Fe II to determine the parameters that characterize the wind of α Cyg. The calculations, which make use of the Sobolev approximation with the underlying photospheric spectrum taken to be the lower boundary condition, favor a stellar wind model in which the outflow velocity increases rapidly to its limiting value of $v_\infty = 240\ km\ s^{-1}$. Since Fe II lines are used in deriving the mass loss rate, an estimate of the fraction of atoms in this ionization state is required. Hensberge et al. show that the contribution of the wind to the absorption feature due to the resonance line of Al III at λ1854.7 is very small, and this fact implies that the wind is fairly cool. Unless a source of nonthermal heating is present that simultaneously enhances Fe++ without destroying Al+, at least 11 percent of the iron in the wind is singly ionized. The corresponding mass loss rate is $\dot{M} = 1$ to $5 \times 10^{-9} M_\odot yr^{-1}$, a value that is compatible with the lower bound on $\dot{M}$ obtained by Kunasz and Praderie (1981) from their analysis of Mg II λ2802, but is much smaller that the value of $\dot{M} = 1.7$ to 4×10^{-7} $M_\odot$ derived by Kunasz and Morrison (1982) from their analysis of Hα. The latter authors argue that a mass loss rate as low as $5 \times 10^{-9}\ M_\odot\ yr^{-1}$ yields no filling in at all of Hα. The explanation for the discrepancy in the two analyses is not known.

Table 7-2 lists the mass loss rates derived for a number of A supergiants. The wide range of values derived for a given star through the use of different methods reflects our lack of understanding of the detailed structure of the stellar wind. Typically, the mass loss rates are at least an order of magnitude smaller than values derived for early B supergiants (Underhill and Doazan, 1982).

In this discussion of the winds in A supergiants, no mention has been made of the mechanism responsible for the mass outflow. In fact, the mechanism is not known. Radiation pressure almost certainly plays a role in accelerating the flow, but nonradial pulsations and other mass motions within the star are likely to be important in initiating the wind (Lucy, 1976). More extensive discussion of this issue will be found in the section on Variations in the Stellar Wind.

PHOTOSPHERIC VARIABILITY

The variations in A supergiants are extremely complex. A variety of time scales are involved,

Table 7-2
Mass Loss Rates

Star	Spectral Type	Observational Data	$\dot{M}$ ($10^{-6}\ M_\odot\ yr^{-1}$)	Source
HR 1040	A0Ia	Infrared excess	0.42	Barlow and Cohen (1977)
		UV lines	0.01	Praderie et al. (1980)
η Leo	A0Ib	Infrared excess	0.047	Barlow and Cohen (1977)
		UV lines	<0.001	Praderie et al. (1980)
			0.0003	Kondo et al. (1976)
HD 12953	A1Ia	Infrared excess	0.80	Barlow and Cohen (1977)
9 Per	A2Ia	Infrared excess	0.52	Barlow and Cohen (1977)
α Cyg	A2Ia	Infrared excess	0.69	Barlow and Cohen (1977)
			0.011	Lamers et al. (1978)
		UV lines	0.01–0.07	Praderie et al. (1980)
		Radio emission	<0.25	Abbott et al. (1980)
		UV lines	0.003–0.15	Kunasz and Praderie (1981)
		UV lines	0.001–0.005	Hensberge et al. (1982)
		Hα	0.17–0.40	Kunasz and Morrison (1982)
HR 2874	A5Ib		0.003	Praderie et al. (1980)

ranging from hours to years. Variations can occur in either the photosphere or the extended envelope that is associated with mass loss. Variable quantities include luminosity, radial velocity, line strength, and line shape. Since the variations are only semiperiodic, extensive observations are required to determine the characteristics of the variations, and simultaneous measurements are required to determine the correlations, if any, of one variable characteristic with another.

Variations on an intermediate time scale have received the most complete study. Particularly important is the early work on radial velocity variations by Abt (1957), who obtained observations of nine A- and F-type supergiants nearly every night for 30 consecutive nights. All of the stars proved to be variable with characteristic periods in the range 4 to 30 days and amplitudes typically in the range 4 to 10 km s^{-1}. The derived values of the pulsation constant Q, which is given by the relationship

$$Q = P(\rho/\rho_\odot)^{1/2}, \tag{7-2}$$

where P is the period and $\rho_\odot$ is the mean density of the Sun, are comparable to values calculated theoretically for pulsation in the fundamental mode. Because of the limited time interval covered by his observations, Abt was unable to determine whether the semiregular variations that he observed were actually caused by the superposition of several pulsational modes.

An extensive set of radial velocity data is available for α Cyg. During the years 1927 to 1933, Paddock (1935) obtained radial velocities on 399 nights from 794 plates taken with the Mills spectrograph at Lick Observatory, and these data have been analyzed by Lucy (1976). On 6 occasions, 18 or more spectrograms were obtained during the course of a single night. While the amplitude spectrum shows a peak at 5.6 hours, a careful statistical analysis shows that the peak is not significant. Therefore, there is no evidence for short period (~ hours) variations in radial velocity in α Cyg. This result is in agreement with theoretical expectations. For an isothermal atmosphere, the critical period below which standing atmospheric oscillations are impossible can be estimated from the relation (Lamb, 1945)

$$P_L = 4\pi c_s/\gamma g, \quad (7\text{-}3)$$

where c_s = the speed of sound, γ is the ratio of specific heats, and g is the acceleration due to gravity. For α Cyg, Lucy finds P_L = 8.4 days. Therefore, even if an oscillation with P = 5.6 hours were present, it would appear as a traveling wave with a wavelength that is short compared to the pressure scale height in the atmosphere. It would, therefore, manifest itself as a form of turbulence and would not affect the overall radial velocity of the star. Lucy argues that this theoretical result strengthens the conclusion that the apparent short-period oscillations are due to observational error.

After averaging over the semiregular variations ($P \sim 10$ days), Lucy finds some evidence for long-period (P = 846.8 or 776.4 days) changes in radial velocity, possibly due to orbital motion. Unfortunately, additional data from Mt. Wilson and Victoria do not confirm the long-period variations, probably because fluctuating systematic errors in the additional measurements are nearly half the observed amplitude of 6 km s^{-1}.

After allowance for the long-period variations, Lucy finds evidence for semiregular velocity changes in α Cyg with an amplitude of 10.3 km s^{-1} and a time scale of about 10 days. On the basis of a harmonic analysis, Lucy shows that these variations can be accounted for by the simultaneous excitation of at least 16 discrete pulsational modes with periods ranging from 6.9 to 100.8 days. The amplitudes and phases are stable over at least the 6-year interval spanned by Paddock's data. For a reasonable choice of mass, luminosity, and radius, the expected period of the fundamental radial mode in α Cyg is P_0 = 14.3 days. The six terms with the largest amplitudes in the harmonic analysis have periods comparable to this value. However, several other terms have even longer periods—an impossible situation for radial oscillations. Lucy, therefore, argues that nonradial pulsations must occur. Furthermore, certain periods occur in pairs, a result that suggests rotational splitting of degenerate nonradial modes. The inferred rotational velocity of 10.4 km s^{-1} is compatible with observed line widths. Lucy goes on to suggest that the observed macroturbulence is to be associated with surface motions generated by the superposition of nonradial oscillations. Microturbulence, on the other hand, can be associated with oscillations with periods less than the value (~8.4 days) below which atmospheric waves become progressive.

It is, of course, to be expected that a star of the extent of α Cyg would not exhibit simple radial pulsation, a point that has been emphasized by de Jager (1980). The radius of α Cyg is approximately 10^8 km. A disturbance propagating at the speed of sound requires nearly half a year to traverse this distance. It is therefore clear that nonradial pulsation must be the characteristic mode of oscillation.

Photometric variations are also commonly seen in supergiants, and the correlations of period and amplitude with stellar characteristics have been summarized by Burki et al. (1978) and Maeder (1980) in papers that also give extensive references to earlier literature on this subject. The amplitudes of the brightness variations in A supergiants are typically 0.03 to 0.05 mag and tend to increase with luminosity. For the most luminous (Ia) stars, the amplitudes of the A and F supergiants are somewhat smaller than the amplitudes observed in stars of type G0 and later. Typical periods can readily be derived for the semiperiodic variations, and for B–G supergiants these semiperiods tend to increase with both increasing luminosity and decreasing effective temperature. For A supergiants, periods range from 10 to 100 days. Empirically derived pulsation constants are uncertain, but for at least some A supergiants Q seems to exceed the theoretical values for pulsation in the fundamental mode. This result may be taken as further evidence for nonradial pulsation. The pulsation of massive stars is discussed in more detail in Chapter 7 of the book *B Stars With and Without Emission Lines* (Underhill and Doazan, 1982).

In principle, additional information on the pulsation mechanism can be obtained from an analysis of the phase relationship between color and luminosity changes. Unfortunately, these relationships have been studied in only a few supergiants. In HD 14489 (A2 Ia) and possibly HD 12953 (A1 Ia), maximum light seems to correspond to the bluest color, but precisely the opposite phase relationship is seen at both earlier and later spectral types (Burki et al., 1978). In general, only a single semiperiod has been observed, and it is not known if these same relationships repeat from cycle to cycle.

Very little is known about the correlation of one kind of variability with another. Simultaneous spectroscopic and photometric observations are generally not available. Some correlations of various spectroscopic properties have been searched for and found. For example, Rosendhal and Wegner (1970) have reported a strong correlation between changes in radial velocity and the microturbulence inferred from curves of growth in two A supergiants, and they argue that large-scale and small-scale mass motions must be coupled. Radial velocity variations of different metallic lines and of the hydrogen lines in α Cyg have been intercompared by Inoue (1979). The measured velocities differ from one element to another and are presumably dependent on the depth of line formation. Such measurements can be used to probe the changes in pulsational characteristics at various layers in the atmosphere.

There is also rather limited information on variability with time scales much longer or shorter than days or weeks. Some highly luminous A supergiants (HD 160529, HD 37836, and S Doradus) are known to vary on a time scale of hours (Appenzeller, 1974; Sterken, 1976), while the long-term variations of several extreme A supergiants are illustrated in Figure 7-5. A considerably expanded data base is required in order to determine whether or not similar long- and short-term variability is characteristic of A supergiants with more modest luminosities.

VARIATIONS IN THE STELLAR WIND

The study of variations in the winds of supergiants is crucial in providing constraints for models of mass loss. Since the wind traverses the region of line formation in about a day, changes that occur on time scales longer than 1 day may reflect variations in the properties of the wind as a whole. Changes on shorter time scales would offer a measure of small-scale inhomogeneities in the wind (e.g., Carlberg, 1980). If correlations could be established between photospheric variations and changes in the density or velocity structure in the wind, then it might be possible to identify specific hydrodynamical mechanisms as drivers of the flow.

A primary diagnostic of the low density gaseous regions beyond the photosphere is Hα, and variations in the strength and profile of this line in A supergiants are common. For example, on the basis of only two or three random spectrograms of each of seven A supergiants, Rosendhal (1973a) found that four showed variations in the intensity of Hα that exceeded 20 percent. The nonvariable A supergiants tended to be later in spectral type or lower in luminosity than the variable ones.

The winds of supergiant stars produce conspicuous spectral lines in the ultraviolet region of the spectrum. Most ultraviolet observations have concentrated on the spectral region $\lambda < 2000$, and the emphasis to date has been on studies of O and B supergiants. Line identifications are available, however, for α Cyg (A2 Ia) in the spectral region $\lambda\lambda$1800 to 3017 (Underhill, 1977; Barbier et al., 1978). While no P Cyg emission profiles are observed, the resonance lines of Mg II and Fe II in α Cyg and other A supergiants that are losing mass are shifted to shorter wavelengths by up to 250 km s^{-1} (Lamers et al., 1978; Praderie et al., 1980). Sample spectra are shown in Figure 7-7.

Data on the variability in the ultraviolet of A supergiant winds are extremely limited because only scattered observations, usually widely separated in time, are available. For example, Morrison (private communication) and Praderie et al.

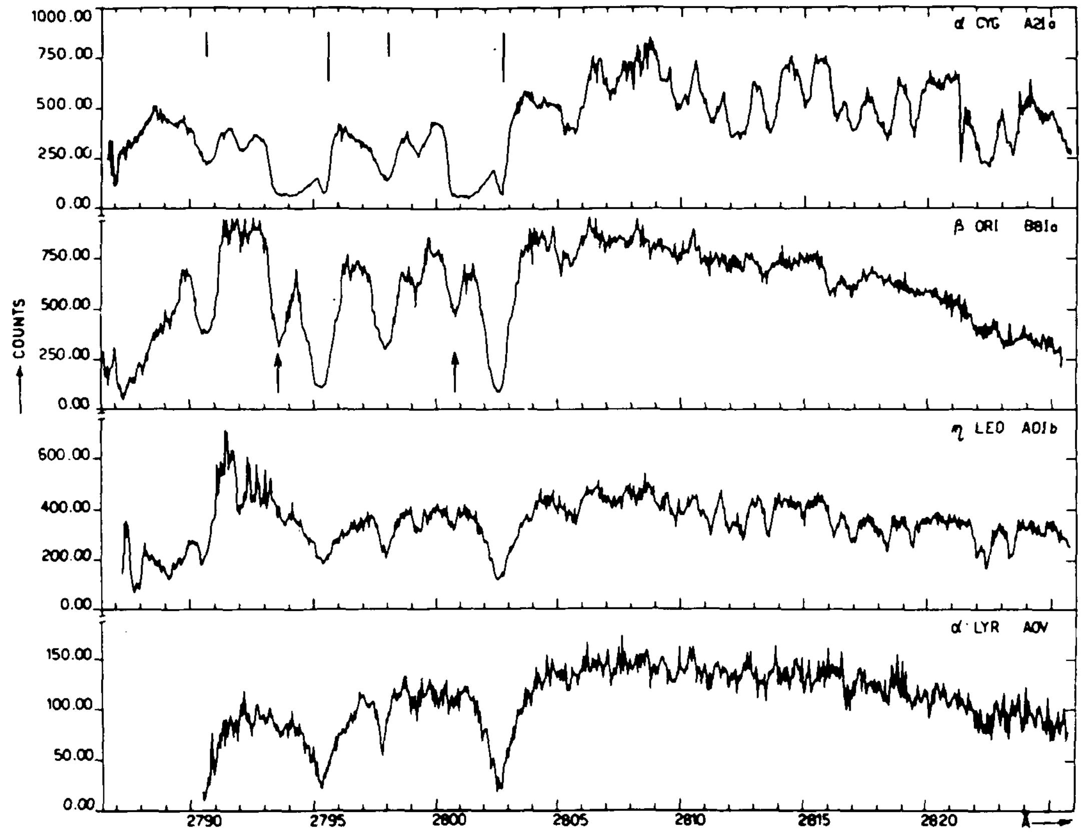

Figure 7-7. BUSS spectra of α Cyg (No. 20), β Ori (No. 30), η Leo (No. 7), and α Lyr (No. 9) in the region 2785 to 2825 Å. The ordinate gives the flux in an arbitrary linear scale. The laboratory wavelength of (long vertical lines) the Mg II resonance lines and (short vertical lines) the subordinate lines is indicated. In α Cyg, the Mg II lines consist of a sharp feature at the rest wavelength, which is caused by an interstellar component, and a broad component. The broad absorption extends to −267 km s^{-1}, and the flat bottom of this feature suggests it is saturated. In β Ori, components of Mg II are shifted by −190 km s^{-1} from the rest wavelength. These features suggest a shell may be present. In η Leo and α Lyr, the Mg II lines are symmetric and show no evidence of mass loss (from Lamers et al., 1978).

(1980) observed nearly identical Mg II line profiles, and their measurements span nearly an entire year. Because the Mg II lines are saturated, they are not very sensitive to structural changes in the wind, but at least the data show no evidence for changes in the terminal velocity. Lamers et al. (1978) found evidence for variability in the Fe II resonance lines of α Cyg. In May 1976, absorption components were seen at −125 and −195 km s^{-1}, relative to the center of mass velocity of the star. In September 1976, the component at −125 km s^{-1} was absent.

Lamers et al. also find some evidence for variability in η Leo (A0 Ib). While they see no mass loss in their own data, asymmetries were observed in the Mg II resonance lines a year earlier (Kondo et al., 1975). Variations are also clearly seen in β Ori (B8 Ia), which is the only other star observed by Lamers et al.; the ultraviolet measurements therefore strengthen the conclusion that variations are a common characteristic of the winds in A supergiants. In their analysis, Lamers et al. argue that violet-shifted components are evidence for high concentrations of Fe II at specific regions in the stellar envelope, and that variations in these components indicate that mass loss is not stationary but that matter must be emitted in "puffs." The crucial question is how these density enhancements are related to variations in Hα and to the nonradial oscillations that are observed in α Cyg. Such oscillations may reflect the kind of atmospheric instability that Cannon and Thomas (1977) postulate as the primary mechanism for initiating a stellar wind.

A specific search for coupling between photospheric velocity fields and mass loss has been carried out by Rosendhal (1972) in the visible region of the spectrum. No correlation was found between the velocities, either radial or microturbulent, derived from metallic lines and the strength and velocity of the emission components of Hα (see also Inoue, 1979). Rosendhal concludes that there is no coupling between either macro- or microturbulence and mass flow from the star, and, furthermore, that microturbulence in α Cyg cannot be attributed to a velocity gradient in the flow leading to mass loss. Rosendhal adds two important caveats to this result. First, his observations span only 16 days and do not include the full range of variation that has been seen in α Cyg. Second, changes in Hα emission strength can be caused not only by changes in mass loss rate, but also by structural changes in the extent of the envelope or in the flow velocity. This second conclusion is supported by Kunasz and Morrison (1982), who have shown that the widths of both emission and absorption Hα features decrease as the assumed velocity profile softens.

In addition to density variations in the winds of A supergiants, there may be variations in the level of ionization. Underhill (1980a) has reported strong, narrow emission lines at λ1550 due to C IV on one of four spectrograms of HR 1040 (A0 Ia). She finds evidence for weak C IV emission on three other spectrograms, while subsequent observations by Praderie et al. (1980) show no emission at all. Sharp, undisplaced C IV lines are seen in many late-type stars and are attributed to coronal emission. It would be worthwhile to try to confirm Underhill's observation in order to determine if hot plasma does sporadically occur in the wind of this or other A supergiants.

ROTATION AND OTHER SOURCES OF LINE BROADENING

The velocity fields in the atmospheres of supergiants are complex and may contribute to the broadening of spectral lines in a variety of ways. Rotation is one obvious source of line broadening. Since mass loss is ubiquitous, portions of the atmosphere are expanding, and a depth-dependent expansion velocity can contribute to the line widths and may yield either asymmetric or shifted profiles. Turbulence can also alter line shapes. Two limiting cases for turbulence can be easily treated. If the size of a turbulent element is smaller than the photon mean-free path, then the velocity field is said to be microturbulent. Macroturbulence is used to refer to motions of turbulent elements that are large compared to the mean-free path of a photon. Microturbulence may manifest itself through an increase in the Doppler parameter that characterizes the curve of growth. Macroturbulence produces a change in line width but not equivalent width. Obviously, discussing turbulence in terms of these two limiting cases is an extreme oversimplification. In fact, there must be a "spectrum of turbulence" in any real stellar atmosphere (de Jager, 1980). Unfortunately, observations are seldom of sufficient precision to provide a complete characterization of stellar velocity fields.

Rotational and macroturbulent line broadening in A supergiants has been discussed by Rosendhal (1970a). The range in line widths observed by him in 18 A supergiants is small— 27 to 49 km s^{-1} if the line broadening is expressed as an apparent rotational velocity. Of course, if rotation were the only source of Doppler broadening, then at least some stars should be viewed nearly rotational pole-on and should have sharp lines. The fact that none does indicates that mass motions in the atmosphere are a significant broadening agent. Rosendhal assumes that a star with the narrowest lines is seen pole-on, so that its line widths are entirely due to macroturbulence, and that this same macroturbulence characterizes all other A supergiants. Typical values of the rotational velocity are then ~13 km s^{-1}. Rosendhal thus concludes that in A supergiants turbulence is the dominant source of line broadening, but that rotation is still clearly present. Furthermore, the estimated rotations are compatible with conservation of angular momentum during evolution, provided that stars evolve with no radial exchange of angular momentum. This result is somewhat surprising if mass loss from these stars has been as extensive as is generally believed.

Microturbulent velocities are usually estimated by comparing observed and calculated curves of growth for metallic ions. Analyses of several A supergiants have been carried out by Rosendhal (1970b) and Aydin (1972). Both studies find values for the microturbulence in the range 5 to 15 km s^{-1}, with variations from element to element, star to star, and at different times in the same star. Figure 7-8 shows curves of growth for Fe II lines in 6 Cas derived from spectrograms taken about 15 days apart. While the

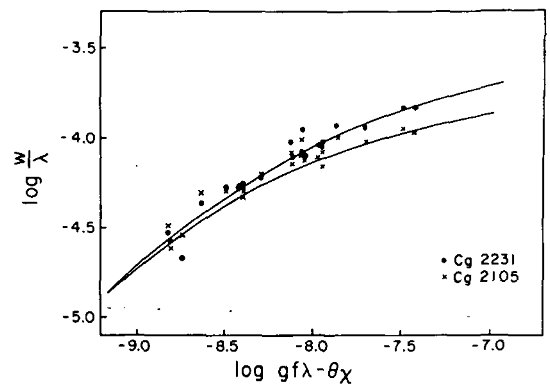

Figure 7-8. Observed curves of growth for Fe II lines obtained from measurements of two different spectrograms of 6 Cas. The two observations were separated by about 15 days. Solid lines are possible fits of theoretical curves of growth to the observed points (from Rosendhal and Wegner, 1970).

linear portions of the two curves are quite similar, the strongest lines differ in equivalent width by a factor of 1.6. As Rosendhal comments, the changes in the deduced microturbulent velocity are not subtle. He also finds clear evidence for a correlation between microturbulent and radial velocities in two of the three stars studied, while Aydin sees evidence for a similar correlation in one of the six stars in his sample. Rosendhal argues that this correlation indicates that the various scales of motion are strongly coupled. Radial velocities presumably reflect large-scale motions, while microturbulence depends on motion on a much smaller scale. In HD 92207, the equivalent width of Hγ has been shown to vary in phase with the microturbulent velocity (Buscombe, 1973). Aydin finds evidence that microturbulence increases with increasing luminosity.

It should be apparent from this discussion that macroturbulent and microturbulent velocities are simply parameters that are used to characterize line strengths and widths. Whether or not these quantities provide meaningful diagnostics of the actual photospheric velocity fields is clearly questionable. With the advent of modern electronic detectors, which allow precise measurements of line shapes, attempts have been made to derive the characteristics of the velocity field through Fourier analysis of observed profiles. Work in this area has been reported by Gray (1976), Smith and Gray (1976), Ebbets (1978), Karp and Fillmore (1980), and others.

The validity of Fourier analysis techniques can be tested through theoretical calculations of line profiles for model atmospheres in which the velocity fields are completely specified, and an analysis of this kind has been carried out by Mihalas (1979). (Mihalas also provides a useful review of earlier work on this complex subject.) In Mihalas' study, line profiles were calculated by solving for the source function in the comoving fluid frame and computing emergent intensities by means of a formal solution in the observer's frame. Equivalent widths, line depths, line widths, and line asymmetry parameters were derived for a range of rotational, expansion, and microturbulent velocities. The calculations show that atmospheric expansion and microturbulence, either singly or in combination, produce curves of growth that are similar in shape. From curves of growth alone, one can infer that a velocity field is probably present, but one can determine very little about the relative amounts of microturbulence and expansion.

Line profiles provide considerably more useful diagnostics than do equivalent widths. In an expanding atmosphere, the position of the absorption minimum shifts monotonically to shorter wavelengths with increasing line strength. Stronger lines, which saturate farther out in the flow, sample larger flow velocities. Velocity gradients distort line shapes, and, in particular, weak lines are skewed toward higher frequencies. In contrast, strong lines absorb effectively over a large range of velocities and tend toward symmetry. Mihalas (1979) shows that measurements of line shifts, widths, and asymmetries for absorption features with a range of intensities may allow determination of whether or not expansion is occurring in stellar photospheres and, in favorable cases, may even yield estimates of the expansion rate. The results, however, will be highly model dependent, and the solutions may well prove not to be defined uniquely by observations of the accuracy that is realistically obtainable (see also Karp, 1978). The problem is compounded by the fact that in real stars there is likely to be a spectrum of turbulent velocities and a depth dependence in all the relevant parameters. Available observations are inadequate to determine if line shifts and asymmetries, with the dependence on line strength derived by Mihalas, are actually observed.

In summary, there is clear evidence for nonthermal motions in the atmospheres of A supergiants. The parameters that characterize some manifestations of these motions, such as the shape of the curve of growth or the line widths, have been labeled with terms having obvious physical connotations—rotation, macroturbulence, and microturbulence being specific examples. Present diagnostic techniques, however, simply do not allow us to assert that these connotations are closely correlated with actual photospheric velocity fields of the type implied by the nomenclature.

THE ORIGIN OF THE STELLAR WIND

The A supergiants pose a special problem for the theoretician in that a complete model must account not only for the photospheric spectrum but for the spectrum produced in the stellar wind. Below, we examine first some general ideas concerning the origin of stellar winds and the treatment of line formation within winds. With that background, we can then look at the problem of modeling the mass loss of A supergiants. Models of stellar winds are also discussed by Underhill and Doazan (1982) and will be treated in depth by Underhill and Conti (to be published in 1984).

Theories of the origin of stellar winds fall into four general categories: (1) cool radiation pressure models, (2) hot coronal models, (3) hybrid models that combine hot coronal regions with cool winds driven by radiation pressure, and (4) "imperfect" flow models (cf., Cassinelli, 1979).

In order to account for the P Cyg profiles observed by Morton (1967) in the far-ultraviolet of early-type stars, Lucy and Solomon (1970) argued that mass loss could be produced by radiation pressure. Absorption of radiation in resonance lines, especially by abundant elements with atomic transitions near the stellar flux maximum, transfers momentum from the photons to selected ions in the stellar envelope. Typically about 100 strong lines are required to yield the mass loss rates that have been derived for O and B supergiants. This model has been explored in considerable detail by Castor et al. (1975), who made use of the Sobolev approximation. Weber (1981) and Leroy and Lafon (1982a; 1982b) have treated the problem of winds from hot stars in a more general way that is valid even for low velocities, which occur at the base of the flow. Because of the Doppler effect, absorption lines shift progressively away from line center into adjacent unabsorbed regions of the photospheric spectrum, and rapid acceleration to high velocities can occur. The models that have been calculated to date all assume radiative equilibrium, an assumption that will surely have to be altered as our understanding of stellar winds advances (Leroy and Lafon, 1982b).

The second class of models attributes mass loss to a hot corona (e.g., Parker, 1958). On this model, the expansion of the outer layers of the stellar atmosphere to form a wind is caused by a gradient in the gas pressure. The origin of the coronal heating is unclear. In stars with convection zones, dissipation of mechanical energy may provide a source of heating. For early-type supergiants, Hearn (1975b) suggests that the same waves that result in macroturbulent line broadening may carry sufficient energy to account for the necessary coronal heating, but this suggestion has been challenged by Berthomieu et al. (1976).

Neither the coronal model nor the cool radiation pressure-driven wind model can account for the lines of C IV, N V, Si IV, and other ions seen in the winds of OB stars. Therefore, hybrid models that combine a corona plus cool wind have been proposed by various authors. For example, in his model of ζ Ori (O9.5 Ib), Hearn (1975a) finds that the temperature of the corona must be at least 2.6×10^6 K in order to account for the observed mass loss rate. Because of the high density of OB star winds, radiative losses are high; Hearn argues that the corona must be fairly narrow and that Hα and the lines of C IV, N V, Si IV, and other ions must be formed in the outer, cooler regions of the wind. It is in this cool region that acceleration of the flow by radiation pressure takes place.

A modified version of the corona plus cool wind model has been proposed by Cassinelli et al. (1978), and Cassinelli and Olson (1979). In this model, the O VI and N V lines in O-type stars and the C IV lines in B supergiants are attributed to Auger ionization, whereby two electrons are moved from C+, N++, and O+++ following the K shell absorption of X-rays. In the original models, the X-rays were assumed to arise from a thin (<0.1 R_*) hot ($T \sim 5 \times 10^6$ K) corona at the base of the flow. Subsequent direct observations of X-ray spectra of early-type stars (Cassinelli et al., 1981) show that the X-rays are not as strongly attenuated as one would expect for a thin corona at the base of a spherically symmetric wind. Either the hot material must be distributed throughout the wind (Natta et al., 1975; Lucy and White, 1980), or else the attenuation is less than expected because of the temperature structure or geometry of the overlying material (Rosner and Vaiana 1979; Waldron, 1980). Snow et al. (1981) favor a two-component model for the X-ray emitting region. The hard X-rays can best be explained by a high temperature (10^7 K) region at the base of the wind, while the soft X-rays seem to come from a region (2 to 3×10^6 K) above most of the wind. The corona plays no role in driving the stellar wind, but serves only to account for the observed level of ionization.

Radiation-driven winds are unstable (Carlberg, 1980, and references therein), and Lucy and White (1980) have argued that the nonradiative heating required to produce X-ray emission may be caused by this instability. Essentially, Lucy and White propose that the winds can be approximated by a two-component structure with radiatively driven blobs ploughing through, and confined by the ram pressure of, an ambient gas that because of shadowing is not itself radiatively driven. The hypersonic, chaotic motions produce heat through shocks. Alternate coronal models simply postulate high-temperature regions without attempting to account for the source of nonradiative heating.

The fourth class of models attributes the initiation of winds to fluctuations or instabilities in the photosphere and, specifically, the photospheric velocity field. Thomas (1973) argues that photospheric velocity fields, chromospheres-coronae, and stellar winds are inextricably intertwined and that mass loss depends on storage within the star of nonthermal kinetic energy, with material pushed out of the star from below the photosphere (Cannon and Thomas, 1977; Thomas, 1979).

The determination of which, if any, of these models is applicable to A supergiants is difficult because the winds are weak and the observational constraints are few. The physical state of the winds in supergiants changes progressively from a high level of ionization and rapid outflow to relatively low ionization and a moderate rate of outflow as one goes from spectral type B0 to A0 (Underhill, 1980b). Excess ionization and X-ray emission, which are the signatures of chromospheres and coronae, have not been seen in α Cyg or other A supergiants. The imperfect flow model predicts that variations should be seen in the wind. While Hα does vary in α Cyg, the time scale is not clearly established. The Mg II resonance lines in α Cyg did not vary during the 11 months for which observations are available (Praderie et al., 1980; Morrison, private communication), but since these lines are saturated, they are sensitive only to changes in the terminal velocity and are relatively insensitive to other structural changes in the wind. It seems likely that radiation pressure is the dominant force in accelerating the winds of A supergiants to their terminal velocities. However, the mechanism for initiating the flow and for, perhaps, determining the mass loss rate remains unknown.

MODELS OF THE STELLAR WIND

Because our understanding of the underlying physical processes involved in mass loss is so limited, modeling of stellar winds has focused almost exclusively on what Mihalas (1978) describes as the kinematics of radiative transfer in moving media. The velocity field v(r) and temperature structure of the stellar wind are specified as input parameters for the models, and the density structure $\rho(r)$ is determined by the equation of continuity

$$\dot{M} = 4\pi r^2 \rho(r) \mathrm{v}(r). \tag{7-4}$$

A variety of mathematical methods are then available that allow the calculation of emergent spectra, including line profiles. Excellent reviews of these techniques have been given by Mihalas (1978) and Hummer (1976), both of whom give extensive references to the significant literature in the field. Modeling of stellar winds is also treated in the volume entitled *O, Of, and Wolf-Rayet Stars* by Underhill and Conti (1984). Because thorough reviews are already available, and because there have been only a few applications to A supergiants, the present discussion will be restricted to a summary of the basic assumptions and limitations of the models.

If the material in a stellar atmosphere is moving with velocity v(r), where r is the radial distance from the center of the star, then frequencies measured by an external observer at rest will be Doppler shifted relative to the frequencies in the rest frame of the atoms in the atmosphere. The relationship between these frequencies is given by

$$\nu' = \nu - \nu_0 (\boldsymbol{n} \cdot \mathrm{v}/c), \tag{7-5}$$

where ν is the frequency in the observer's frame, and ν' is the frequency in the atom's frame at which a photon traveling in direction $\boldsymbol{n}$ was emitted or absorbed. The opacity and emissivity, as seen by an external observer, therefore depend on the angle, and the radiative transfer equations must specifically incorporate this directional dependence. Solutions have been obtained for this problem in both the low velocity regime (velocities that do not exceed a few thermal Doppler widths) and the high-velocity limit (Sobolev's approximation). Only recently have methods, which depend on transforming to the moving frame of the material rather than working in the observer's frame, been developed for handling intermediate velocities (cf., Mihalas, 1978, and references therein).

The case of low velocity flows has been treated by a number of authors (e.g., Hummer and Rybicki, 1968; Kalkofen, 1970; and other references given by Hummer, 1976). The primary effect of a velocity field is to distort the monochromatic optical depth scale. For example, an expansion decelerating outward changes the opacity in such a way that the long wavelength side of a spectral line is more transparent than the short wavelength side. The changes in line profiles, relative to the static case, can be pronounced. The form of the velocity law can also play an important role in determining the line source function (Simonneau, 1973; Mihalas et al., 1975). Only for low velocities are the changes in the source function relatively small. In this case, the redistribution of opacity from the line core to the shortward side of the line is not great enough to allow substantial flux to be carried in the core, while the reduction of radiation in the shortward wing is approximately compensated for by an increased transparency in the longward wing.

High-speed flows ($\geq$ 10 thermal or random Doppler units), including large-velocity gradients, are frequently encountered in stellar winds. The effects of such a wind on line profiles can be easily seen in the case where the wind is taken to be spherically symmetric and the emission in the line is optically thin. Figure 7-9 shows a schematic diagram of an expanding stellar envelope. The material directly between the star and the observer may contribute to an emission component or alternatively may absorb incident photospheric radiation and scatter it out of the line of sight. In either case, the resulting feature will be Doppler shifted shortward of its undisplaced position. From the parts of the stellar atmosphere projected against the sky (emission lobes), we may observe either thermal emission or emission due to scattering of radiation from both the photosphere and the envelope. The radial velocities of material in the emission lobes range symmetrically from positive to negative values, and if $\tau \ll 1$, the resulting emission feature from these lobes alone will be positioned symmetrically about the undisplaced position of the line. The region behind the stellar disk will be occulted, and hence the maximum flow velocity can be observed directly only on the shortward side of the line. The resulting line profile will typically have P Cyg characteristics: violet-shifted absorption accompanied by redshifted emission.

In realistic cases, of course, the stellar envelope is not transparent to radiation, and it is necessary to solve the coupled equations of statistical equilibrium and radiative transfer in a moving medium. If both the flow speed and its gradient are large, then an important simplification, first recognized by Sobolev (1947), becomes possible.

According to Equation (7-5), the relationship between the frequency ν in the observer's frame

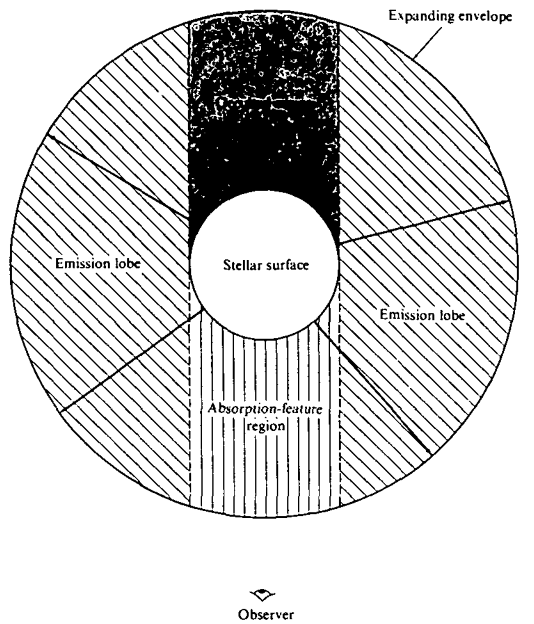

Figure 7-9. Schematic diagram of expanding envelope surrounding a stellar surface. The material in the occulted region is blocked from view by the stellar disk and cannot be seen by an external observer (from Mihalas, 1978).

and ν' in the frame moving with the outflowing material is

$$\nu' = \nu - \nu_0 (\boldsymbol{n} \cdot \mathrm{v}/c). \tag{7-5}$$

The mean thermal speed of an atom of mass M is

$$\mathrm{v}_{\mathrm{th}} = \sqrt{2kT/M}, \tag{7-6}$$

where T is the temperature. It is customary to measure velocity in terms of v_{th}:

$$u = \mathrm{v}/\mathrm{v}_{\mathrm{th}}. \tag{7-7}$$

Frequency displacements from line center can also be measured in Doppler units by defining

$$\Delta\nu_D = \nu_0 \mathrm{v}_{\mathrm{th}}/c, \tag{7-8}$$

which yields

$$x' = x - \mu u, \tag{7-9}$$

where $x \equiv (\nu - \nu_0)/\Delta\nu_D$, and x' is defined similarly. In this expression, $\mu = \cos\theta$, where θ is the angle between the direction of flow and the direction of the emitted photon. Now suppose that all radiation in a rapidly expanding flow is emitted at line center ($x' = 0$). Then all the radiation observed in the stationary frame at x will be emitted from the surfaces at which

$$x = \mu u. \tag{7-10}$$

Surfaces of constant line-of-sight velocities can be constructed for various velocity laws $u(r)$. Figure 7-10 shows two examples, one for an accelerating flow, the other for a decelerating flow. This figure illustrates one of the major difficulties in using line profiles as diagnostics of the structure of expanding atmospheres. Because the constant velocity surfaces extend over a large portion of the extended stellar atmosphere, radiation observed at frequency x in the stationary frame is actually formed over a range of values of r. Therefore, there is no unique relationship between frequency and depth in the envelope. It is impossible, even in principle, to derive the depth dependence of such quantities as density and temperature from line profiles with a spatial resolution greater than the range of r from which radiation at a given observed frequency arises. The problem is even worse for decelerating flows than for accelerating ones. In the case of decelerating flows, the line of sight may intercept a constant velocity surface more than once, and the physical conditions at the intercepts may differ markedly. These intercepts may also be radiatively coupled with one another, and in that case the Sobolev approximation, which is described below, is inadequate.

Let us examine now the formation of a spectral line in an optically thick, expanding stellar envelope. Let us also assume that the effects of a background continuum can be ignored. Consider a given line of sight that intercepts a surface of constant velocity that produces radiation at a specific frequency x. Because of thermal motions and the intrinsic width of the line profiles, the

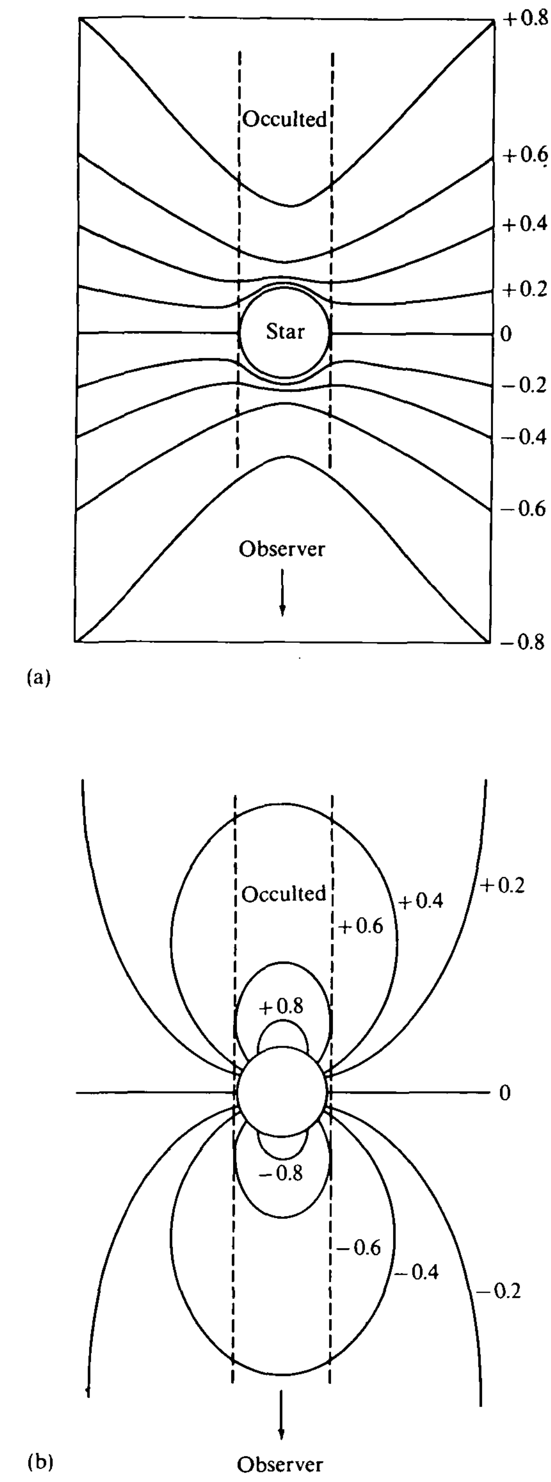

Figure 7-10. Surfaces of constant radial velocity, v_z = constant. (a) $v(r) = v_\infty(1 - 1/r)^{1/2}$. Curves are labeled with v_z/v_∞. (b) $v(r) = r^{-1/2}$ (from Mihalas, 1978).

geometric thickness of the constant velocity surface is not zero, and the opacity and emissivity at frequency x are produced in a shell whose width depends on the ratio of the Doppler and atomic broadening to the macroscopic velocity gradient. Because of global expansion of the envelope (and global recession of each point from every other point) other shells in the envelope cannot interact with radiation emitted at frequency x. The velocity gradient makes it possible for photons to escape from the stellar envelope. Although the optical depth of the shell giving rise to radiation at frequency x may be large, it is finite, and the radiative transfer problem is confined to that particular shell. If the probability that a photon emitted from that shell will escape freely to infinity is large, then the so-called escape probability method of Sobolev (1947; see also Castor, 1970) is applicable. This method has been used extensively in recent years in the analysis of lines formed in the winds of OB supergiants (cf., Hummer, 1976; and Mihalas, 1978 for reviews and references). An atlas of theoretical P Cyg profiles for a variety of velocity laws has been calculated by Castor and Lamers (1979; see also Surdej, 1979; Olson, 1978, 1981). Because the Sobolev approximation is valid only for velocities that substantially exceed the thermal and microturbulent velocities, profiles calculated in this way are accurate only in the line wings, but not in line cores.

As noted earlier, one of the problems with basing the equations of radiative transfer for a moving medium in the rest frame of the observer is that angle and frequency are coupled in a way that renders calculation, particularly of scattering terms, difficult. The advantages of working in a frame that is comoving with the material have been recognized for a long time (McCrea and Mitra, 1936), and recently techniques have been developed that allow for solution of the line formation problem in spherical atmospheres expanding with any velocity (Mihalas et al., 1975), including specifically the case of small flow velocities. Transformation to the comoving frame allows full treatment of variations with depth of the flow velocity and the variation with depth of other physical parameters can be taken fully into account. Solutions can be obtained for both the high- and low-velocity limits as well as for intermediate velocities.

A detailed comparison of theoretical P Cyg profiles calculated by a comoving frame formalism and by the Sobolev approximation has been discussed recently by Hamann (1981). He finds that the deviations from the Sobolev approximation increase with increasing values of v_D/v_∞, the ratio of Doppler to wind velocity. Even if v_D/v_∞ is as small as 0.1, the deviations can be substantial and are not restricted to the line core. In the case of α Cyg, the ratio v_D/v_∞ falls precisely in the relevant range. The wind velocity is approximately 300 km s^{-1} and the Doppler broadening is 30 to 50 km s^{-1}. The error in the Sobolev approximation also increases with the optical depth of the line under consideration.

In a recent study, Kunasz and Praderie (1981) have applied comoving frame techniques to line formation in the wind of α Cyg, with specific reference to the Mg II resonance lines. A special problem posed by α Cyg and the other A supergiants observed to date is the absence of P Cyg structure in the lines formed in the wind. Instead, A supergiant lines are characterized by a very sharp, violet absorption edge, saturation extending most of the way from the violet edge to line center, and no significant features, either absorption or emission, longward of line center. Recently, Hensberge et al. (1982) have suggested that blending by metallic absorption lines may be a major factor in explaining the absence of emission. None of the usual indicators of a high temperature (chromospheric) region have been seen in early A supergiants, and Cassinelli et al. (1981) did not detect X-ray emission from α Cyg.

In calculating models for α Cyg, Kunasz and Praderie use the method described by Mihalas et al. (1975). The photospheric radius and temperature are assumed to be known, and the flow is postulated to be smooth and spherically symmetric. The variation of velocity, temperature, and microturbulent velocity with distance from the photosphere are input parameters for the calculation.

Kunasz and Praderie find two classes of solutions that may account for the line profiles seen in the wind of α Cyg. In very dense winds ($\dot{M} > 10^{-5}\ M_{\odot}\ \mathrm{yr}^{-1}$) with the acceleration occurring near the photosphere, thermalization allows very little radiation to escape the zone where acceleration occurs, and no emission can be observed. Absorption to redward of line center, however, is expected unless the photospheric line is extremely narrow or the abundance of singly ionized Mg much lower than expected for the adopted photospheric temperature. In addition to problems with the photospheric spectrum, this model can be ruled out because the required mass loss rates exceed by nearly two orders of magnitude the upper limit of $\dot{M} < 2.5 \times 10^{-7}\ M_{\odot}$ derived from radio observations of α Cyg (Abbott et al., 1980).

In the second class of models, the P Cyg emission components cancel against photospheric absorption on the longward side of line center. This model requires that most of the outflowing material lie below the zone of rapid acceleration and that $\rho(r)$ be small in the accelerating layers. Since the amount of Mg^+ in the wind depends both on the rate of mass loss and on the ionization balance in the wind, a family of solutions is possible. A plausible lower limit, corresponding to domination of the ionization balance by Mg^+, is $\dot{M} = 3 \times 10^{-9}\ M_{\odot}\ \mathrm{yr}^{-1}$, while a value of $\dot{M}$ larger than the upper limit set by the radio data is allowed. The velocity field in the wind is uncertain, but turbulent velocities of 50 to 60 km s^{-1} are required in order to produce a saturated absorption core. Deceleration in the outer layers of the flow, may be required. The physical interpretation of these supersonic turbulent velocities is unclear.

While models of this kind are clearly useful for indicating what types of hydrodynamic and thermal structures may be present in the wind of α Cyg, they are incomplete. Changes in Hα have been seen and indicate that the wind is variable. The photospheric layers of α Cyg are characterized by a complex pattern of nonradial oscillations, and these modes may propagate outward into the wind—they may even play a crucial role in initiating the flow. If so, then the assumptions of steady-state, spherically symmetric flow that are made in current models are unlikely to be correct. The study of mass loss in A supergiants is still a very new field, and the rather simple models discussed here are likely to undergo substantial modification during the coming decade.

8

PECULIAR B-TYPE STARS

OVERVIEW

So far as the physics of stellar atmospheres is concerned, the boundary at A0 is an artificial one. The magnetic stars extend, with no break in continuity, to temperatures at least equivalent to that of middle B-type stars. Classical Am stars are restricted to spectral types of A4 and later, but this limit is imposed by the original definition of the class. There are early A-type stars that share the anomalies of the late Am stars—weak lines of calcium and/or scandium accompanied by an enhancement of heavy metals—and these stars are usually referred to as hot Am stars. It has been suggested that the same physical phenomenon (radiative diffusion) is responsible for the spectral anomalies of the B-type HgMn, He-weak, and He-rich stars as well. Observations and models of the Bp stars may, therefore, provide critical insight into the origin of the anomalies of the A-type stars. A brief summary of the properties of the peculiar B-type stars can be found in the book *B Stars With and Without Emission Lines* by Underhill and Doazan (1982). However, because of the close relationships between the peculiar A- and B-type stars, it seems worthwhile to examine their characteristics in the present monograph as well. The discussion here is, by no means, a complete catalog of the properties of the Bp stars. Rather, emphasis is placed on describing those features of the Bp stars that seem, in light of our present knowledge, to set the most important constraints on models that attempt to account for the origin of their spectral anomalies.

It has been apparent for a long time that there are two distinct groups of abnormal A-type stars—the magnetic Ap stars and the Am stars. These stars can be distinguished spectroscopically, since the absorption lines that are characteristically strong or weak in the two groups are quite different. Detailed studies show that the Ap and Am stars differ in many other ways as well (cf., Chapters 4 and 5). The Ap stars have strong magnetic fields, but no fields have been detected in the Am stars. The Ap stars exhibit spectrum, photometric, and magnetic variations; the Am stars appear to be, with few exceptions, nonvariable. The binary frequency of the Ap stars is unusually low, and of the Am stars unusually high.

Wolff and Wolff (1976) have summarized the arguments in favor of the hypothesis that there are, in fact, two continuous sequences of peculiar stars that parallel the main sequence and extend not only throughout the A spectral class but into late and middle B-types as well. The Wolffs suggest that the distinction between the two groups is the presence or absence of a magnetic field, and that such other properties as abundances or variability are causally related to the magnetic field. The Wolffs' suggestion that the magnetic field is the primary factor that determines the nature of peculiar stars may or may not be correct. What is clearly true is that there are two types of peculiar stars in the temperature range 10,000 K $< T_{\text{eff}} <$ 16,000 K, and that the differences between these two groups closely parallel the differences between the Ap and Am stars with temperatures less than 10,000 K.

Among the late B-type stars, the nonmagnetic sequence is identified with the HgMn stars, so-called because the most prominent spectral anomalies are strong lines of Hg and/or Mn. The HgMn stars have no magnetic fields and are distinguished from their magnetic counterparts of similar temperature (the Si stars) in that they are nonvariable and have a much higher frequency of spectroscopic binaries. One potential problem with identifying the HgMn stars as hotter analogs of the Am stars is an apparent break in the sequence of nonmagnetic stars in the temperature interval (approximately) $10{,}000 < T_{eff} < 11{,}000$ K. The HgMn stars lie on the high-temperature side of this range, while the hot Am and classical Am stars lie on the cool side. No transition objects, such as an Am star with strong Hg lines, have been identified. Whether or not the absence of transition objects, and for that matter the absence of any peculiar nonmagnetic stars with temperatures between 10,000 K and 11,000 K, is caused by selection effects, peculiarities in atmospheric structure or the specifics of atomic structure and atomic processes is an issue that remains unresolved.

At temperatures higher than ~16,000 K, the distinction between magnetic and nonmagnetic stars, if any, becomes considerably less clear. The confusion is probably caused by inadequate observations of the hotter stars. The primary defining spectroscopic characteristic is the abundance of helium, and that particular characteristic appears to be unrelated to the presence or absence of a magnetic field. (This lack of correlation between helium abundance and magnetic intensity obtains for cooler Ap stars as well. Helium is deficient in both Si and HgMn stars.) Helium is characteristically deficient up to temperatures equivalent to spectral type B2. Peculiar B-type stars with higher temperatures are He-rich. Some, but not all, of both the He-rich and He-weak stars have strong magnetic fields and exhibit spectrum and photometric variability of the kind typically seen in magnetic Ap stars. Very little is known about the frequency of binaries among the He-rich and He-weak stars (Jaschek and Jaschek, 1981). The following sections present brief summaries of the observed properties of the He-rich, He-weak, and HgMn stars.

He-RICH STARS

Two quite distinct categories of stars have been referred to as He-rich (Hunger, 1975). The first consists of low mass, presumably highly evolved objects in which the He abundance is abnormally high throughout the entire star. The second includes stars with main sequence temperatures and gravities. The atmospheric abundance of helium in this latter group is abnormally high ($n_H \sim n_{He}$), but the abundance of helium in the stellar interior is apparently normal. Only this second category of He-rich stars will be discussed here.

The first He-rich star to be studied in any detail was σ Ori E, which was found, by Berger (1956), to exhibit both unusually strong He lines and a helium discontinuity. As its name implies, σ Ori E is a member of a multiple star system. Accurate distances can be determined for other (nonpeculiar) members of the system, and the color and luminosity of σ Ori E place it on the main sequence as defined by stars with a normal hydrogen-to-helium abundance ratio (Greenstein and Wallerstein, 1958).

Only with the completion of extensive southern hemisphere surveys (Hiltner et al., 1969; MacConnell et al., 1970) were enough He-rich stars identified to allow a systematic discussion of their properties. Eight He-rich stars were studied by Osmer and Peterson (1974), who found that these objects form a well-defined group in the (g, T_{eff}) plane (see Figure 8-1). The low-temperature cutoff for the He-rich stars coincides with the high-temperature boundary of the He-weak stars, and there is apparently little or no overlap in the temperatures of these two classes of stars. The high-temperature boundary of the He-rich stars is not well defined but apparently does not extend into the O-type stars. On theoretical grounds, the high-temperature boundary is generally thought to coincide with the onset of substantial mass loss. Since the helium abundance anomalies cannot be characteristic of the star as a whole, extensive mass

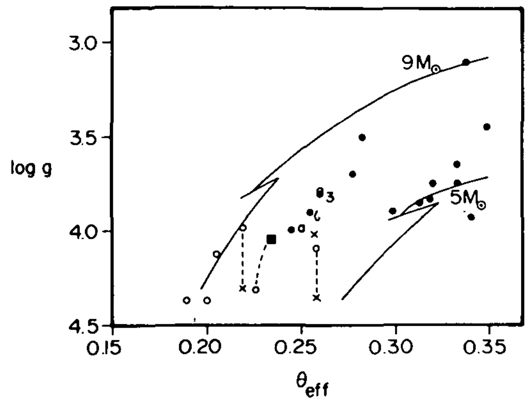

Figure 8-1. The helium-rich (o and x) and helium-weak (●) stars as a function of surface gravity and θ_{eff}. Moderate-dispersion (o) and high-dispersion (x) results are shown as well as the values obtained for σ Ori E (■). Evolutionary tracks for stars of 9 $M_\odot$ and 5 $M_\odot$ are also shown (from Osmer and Peterson, 1974).

flows can presumably compete with, and ultimately overwhelm, whatever process is responsible for concentrating helium in the outer layers of the star.

In marked contrast to peculiar stars of later spectral types, He-rich stars may have quite high rotational velocities. Five of the ten He-rich stars analyzed by Borra and Landstreet (1979) rotate at velocities in excess of 150 km s^{-1}.

Possible explanations for the origin of the abundance anomalies have been discussed by Osmer and Peterson (1974). They argue that the He-rich stars must be young. Several are found in the Orion Association, and one (W66) is a member of NGC 6530, which is less than 10^6 years old (Odell, 1981). Masses, radii, and luminosities can be estimated for those He-rich stars that belong to clusters, and in all cases the derived values are consistent with main sequence B-type stars with masses of $\sim 9 M_\odot$. The He-rich stars in the field are located near the galactic plane ($|b^{II}| < 18°$), as one would expect for massive Population I stars (MacConnell et al., 1970).

Since some He-rich stars occur in clusters in which the majority of stars have a normal He abundance, the He enrichment must have occurred after the stars formed. Osmer and Peterson argue that the He overabundance is unlikely to be caused by nuclear processing followed by either mass transfer in a binary system or by mixing during an advanced stage of stellar evolution. The observed CNO abundances are not in accord with the predicted results of nuclear reactions. Furthermore, a fully mixed star with the composition of the He-rich objects is likely to have a mass of only $\sim 2 M_\odot$ (Liebert et al., 1971). How such a remnant could be produced in the Orion Association, where only stars with masses in excess of $\sim 20 M_\odot$ have evolved beyond the main sequence, is altogether unclear.

The observations of the He-rich stars are, Osmer and Peterson argue, compatible with radiative diffusion models. Radiation pressure, of course, cannot support a normal abundance of helium. Because of saturation effects, the radiative force can exceed gravity only for extremely low helium abundances, and one would normally expect to observe a helium *deficiency* in B-type stars (Vauclair et al., 1974). If there is, however, a net outflow of mass, as is true of early B-type stars with stellar winds, and if at some place in the atmosphere the flow and diffusion velocities are comparable, then a concentration of helium may occur (Vauclair, 1975).

This picture is modified if a magnetic field is present (Shore, 1978). Flow of ionized material across magnetic field lines is inhibited, and in stars in which the stellar wind is weak, mass loss may occur only at the magnetic poles. Observations show that the strength of the wind in early B-type stars increases with increasing luminosity. At the magnetic poles, where the wind is strongest, outflow will begin to dominate diffusion in the hotter B-type stars, and there can be no He overabundance in the polar caps. Only near the magnetic equator will the flow be sufficiently inhibited so that a balance between downward diffusion of helium and mass loss can be achieved.

Satellite observations in the ultraviolet region of the spectrum allow direct tests of these ideas. The stellar winds of the He-rich stars are strong enough to be detected in the CIV doublet at λ1548, λ1550, and preliminary measurements of the variation of the wind with time and as a function of helium line strength have been reported by Shore and Adelman (1981). The diffusion model predicts that the region of the stellar surface in which the helium enhancement occurs should move from the magnetic poles to the magnetic equator with increasing temperature. In those stars in which the helium is overabundant only in a magnetic polar cap, a region of enhanced stellar wind should coincide with one or both of the magnetic poles and should co-rotate with the stellar photosphere. The diffusion model requires that the magnetic field suppress the wind near the magnetic equator, and it will presumably exert some control over the dynamics of the flow near the magnetic pole. The wind, therefore, cannot be spherically symmetric and may take the form of plumes and jets. Observations of the stellar wind at four phases of the cycle of the He variable HD 37017 are compatible with this picture, although because there are ambiguities in mapping the surface distribution of elements (see Chapter 4), the interpretation of the results is not unique.

Observations of the C IV profiles in several other He-rich stars have been described by Barker et al. (1982a). A feature common in most of the He-rich stars observed to date is violet-absorption wings, which may extend as far as 600 km s^{-1} from line center and are indicative of a stellar wind. Emission of C IV is present in some, but not all, He-rich stars. Barker et al. present evidence that Hα emission is strongest in the most rapid rotators, but C IV emission is strongest in sharp-lined, He-rich stars. Whether members of the latter group of stars have sharp lines because they are viewed pole-on or because they are intrinsically slow rotators is unclear at this time. Barker et al. also find evidence that the wind in the He-rich star HD 184927, like that in HD 37017, varies with time. It is evident from these preliminary studies that ultraviolet observations offer a powerful new technique for studying the interplay of mass loss, abundance anomalies, and magnetic field geometries.

As this discussion indicates, if the diffusion model for the He-rich stars is correct, then one would expect these stars to be variable with periods equal to the rotation period, to show evidence of mass loss, and to have strong magnetic fields. In general, the observations are in accord with this expectation.

The He-rich star in which the variability has been most thoroughly documented is σ Ori E. Work on this star was stimulated by Walborn's (1974) discovery of strong, variable Hα emission. The emission can be as broad as 1800 km s^{-1} and varies on a time scale of hours. Subsequent observations have established that a number of other properties of σ Ori E are also variable and that the variations have a period of 1.19 days. The strength of the photospheric helium lines changes by nearly 50 percent (Hunger, 1974; Thomsen, 1974). There are two eclipse-like drops of nearly 0.2 mag in brightness during each cycle (Hesser et al., 1976). Variable shell lines appear in the Balmer spectrum at the time of these eclipses (Groote and Hunger, 1976, 1977). The radial velocity variations of the photospheric lines are less than 4 km s^{-1} (Bolton, 1974), but the shell lines vary by as much as ±60 km s^{-1}.

The helium spectrum variations suggest that σ Ori E may be analogous to magnetic Ap stars and should be interpreted in terms of the rigid rotator model. The eclipses and Hα emission, on the other hand, suggest the star may be a binary with an accretion disk. The discovery of a magnetic field in σ Ori E by Landstreet and Borra (1978) argues strongly that the first of these two hypotheses is the correct one.

The relative phasing of the variations of σ Ori E is shown in Figure 8-2. Note that the magnetic field must be very strong. A lower limit for Bp, the field strength at the magnetic pole, is 10,000 g. In their oblique rotator model of σ Ori E, Landstreet and Borra assume that helium is enhanced in a ring around the magnetic equator. This placement is dictated by the coincidence in phase of helium maximum and the zero crossing of the magnetic field strength. The eclipses and shell lines also appear when the line of sight is nearly in the plane of the magnetic equator. Landstreet and Borra argue that this phase relationship is the consequence of the interaction of a stellar wind with the magnetic field. In the vicinity of the magnetic equator, the outflow of gas is either halted or greatly impeded, thereby leading to an increase in

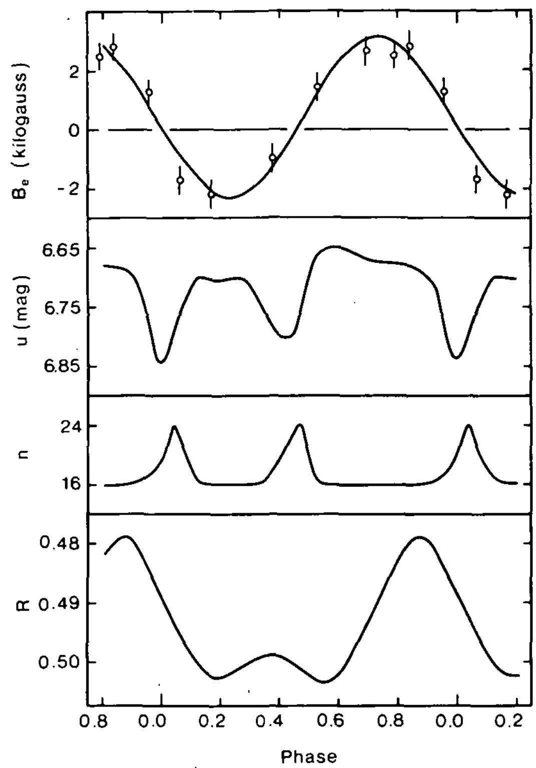

Figure 8-2. Observed variation of σ Ori E as a function of phase. Top to bottom: longitudinal magnetic field strength B_e (error bars are ± σB, and the smooth curve is the best-fit sine wave); Stromgren u magnitude; number n of the last visible Balmer line; and variation in the photoelectric line strength index R which measures the strength of He I λ4026 (small R corresponds to large equivalent width). The lowest three smooth curves are hand-drawn fits to the data (from Landstreet and Borra, 1978).

density above the photosphere and to shell absorption features. Absorption or scattering in the shell may account for the eclipses. The magnetic field is strong enough to dominate the flow to the necessary distance from the stellar surface. Note that the relationships of the magnetic field structure, helium surface distribution, and wind geometries are precisely those required by the diffusion model. The high velocities seen in the Hα emission are not directly explained, but a model of the stellar wind has not yet been developed.

One troubling problem for both the binary and oblique rotator models of σ Ori E is the absence of photospheric velocity variations. Landstreet and Borra suggest that the saturated cores of the He lines may be constant with phase and that changes are restricted to the weaker, unsaturated line wings. Radial velocity measurements with conventional techniques tend to emphasize the line cores. With new, high signal-to-noise photoelectric detectors, it would be possible to search for the variations in line wings that are predicted by the oblique rotator model.

Spectrum variations, including variations in Hα emission, have been observed in a number of other He-rich stars (e.g., Pedersen and Thomsen, 1977; Pedersen, 1979). Searches for magnetic fields in 10 He-rich stars resulted in positive detections in 7 (Landstreet and Borra, 1978; Borra and Landstreet, 1979; Barker et al., 1982b), and 6 have fields that exceed 1 kilg. It appears that the magnetic fields of the He-rich stars are larger than those that are typical of cooler peculiar stars. It may be that the dichotomy between magnetic and nonmagnetic stars extends to the He-rich objects. Variations in helium line strength, Hα emission, luminosity, and magnetic intensity seem to be well correlated, and variations have not yet been detected in the nonmagnetic He-rich stars (Pedersen, 1979; Borra and Landstreet, 1979). The similarity with cooler peculiar stars, where variations are seen only in magnetic stars, is obvious. However, the number of well-studied, He-rich stars is very small, and a larger sample of objects must be observed before firm conclusions can be drawn.

The intense emission at Hα and the extended violet wings of the C IV lines indicate that the stellar winds in He-rich stars are strong. In O-type stars that are losing mass, free-free emission from the circumstellar shell produces excess emission in the infrared. A search for similar infrared excesses in He-rich stars has yielded positive results. Groote et al. (1980) reported that 11 of 15 helium variables exhibited infrared excesses. The excesses were most prominent in the M band (4.9 μm), and the measured fluxes were in some cases 1.5 times larger than expected from the photosphere alone. In the case of σ Ori E, the infrared flux was found to be greatest at the phases when the shell lines are most prominent.

Mechanisms for producing an infrared excess have been discussed by Havnes (1981). He concludes that both thermal emission from dust and synchrotron radiation from precipitating electrons are unlikely to be the cause of the infrared excesses of the helium stars. As in the case of O-type stars, free-free emission from circumstellar gas offers a more plausible explanation. The free-free absorption coefficient is proportional to λ^2. For an optically thick envelope, the radiation at a specific wavelength is approximately equal to the thermal emission from the surface where the optical depth at that wavelength is given by the relation $\tau(\lambda) = 1/3$ (Cassinelli and Hartmann, 1977). These two results combine to imply that the effective radius of the star, and hence the emitted flux, increase with wavelength. Unless the mass loss rate is very high, it is only at wavelengths of 5 μm and beyond that the flux excess becomes large enough relative to the background photospheric flux to be readily measured.

While an infrared excess in the helium variable stars is, therefore, perhaps not surprising and might be related to mass loss, an excess in cooler peculiar stars which are noted for their quiescence, is quite unexpected. Recent measurements by Groote and Kaufmann (1981) seem to show that excesses are present in peculiar stars of all types. According to Groote and Kaufmann, excesses of 50 percent are common at spectral type A3, and excesses of a factor of 2 are not at all unusual. These results must be viewed with caution. The excesses are generally seen only at 5 μm, a region where, because of low atmospheric transmission, accurate photometry is very difficult. Additional observational difficulties are presented by the fact that, although color indices of early A-type stars are defined to be zero, the expected fluxes of normal stars at 5 μm are less than at the V band by more than three orders of magnitude.

In an attempt to confirm the reported infrared excesses, Bonsack and Dyck (1982) have recently obtained JHKLM photometry for a sample of 22 Ap and Bp stars and 18 normal stars. The infrared colors of the peculiar stars proved to be similar within the measurement errors to those of normal stars and to agree with the predictions of normal model atmospheres to within 20 percent at worst. Even the excess previously reported for σ Ori E was not confirmed. Bonsack and Dyck also show that in the earlier measurements there is a clear tendency for the apparently faint stars to show larger excesses at 5 μm than do the brighter stars. It appears, therefore, that infrared excesses are *not* common in Bp and Ap stars, although further careful measurements of the He-rich stars with Hα emission may be warranted.

He-WEAK STARS

The He-weak stars were first identified by Garrison (1967), who called attention to a group of stars in the upper Scorpius Complex that have helium lines that are much weaker than one would expect from their hydrogen line spectral types or photometric colors. In HD 191980, for example, the Balmer line type is B5, but the ratio He I λ4471/Mg II λ4481 corresponds to type B8 (Keenan et al., 1969). The colors are compatible with the earlier type. Extensive references to the literature on He-weak stars can be found in Landstreet and Borra (1982).

The He-weak stars have spectral types in the range B3 to B7, but the frequency of He-weak stars is not well determined. In general, these stars cannot be identified spectroscopically at classification dispersions, but rather must be identified through a discrepancy between the photometric colors and spectral type. The primary spectral classification criterion is the strength of the helium lines, while the colors are nearly independent of helium content. A low helium abundance, therefore, results in a later spectral type than is normal for the measured colors. Large-scale surveys of B-type stars that include both photometric and spectroscopic measurements have not generally been undertaken. Perhaps the best data come from the work on He-line strengths by Nissen (1974, 1976), who finds that in Sco-Cen, Lac OB I, and Ori OB I, approximately 25 percent of the B3 to B7 stars are deficient in helium by a factor of 2 or more, relative to other cluster members. The existence of He-weak stars in such young associations indicates that the time scale of formation of these objects must be $\sim 10^7$ years or less.

Satellite observations show that the flux distribution in the ultraviolet of the majority of He-weak stars is normal for their UBV colors, although a few He-weak stars are fainter in the ultraviolet than normal stars with the same UBV colors. Possibly the flux deficiency is caused by line blocking by many faint spectral features (Bernacca and Molnar, 1972; Ciatti et al., 1978).

Abundance analyses of 12 He-weak stars by Norris (1971a) show that these stars do not constitute a homogeneous group. Instead, the abundances of the heavier elements seem to define three distinct subgroups of He-weak stars (Landstreet and Borra, 1982). One resembles the Si-λ4200 stars in the sense that Si is greatly overabundant. In the second group, P and Ga lines are enhanced, as they are in HgMn stars. The third group is characterized by strong lines of Ti and Sr. The helium deficiencies range from factors of 2 to 15. The He-weak stars occupy a very restricted region of the HR diagram, bounded on the high-temperature side by the He-rich stars and on the low-temperature side by HgMn and classical Si-λ4200 stars.

An extensive survey of the magnetic properties of the He-weak stars has recently been completed by Landstreet and Borra (1982). They report that their observations are consistent with the view that the P-Ga stars are indeed analogs of the HgMn stars, while the Si and SrTi stars are related to the magnetic Ap and He-rich stars. Like the HgMn stars, the P-Ga stars are not variable, and none of the members of this class measured to date has a detectable magnetic field. In contrast, spectrum variability and strong magnetic fields are present in at least some Si and SrTi He-weak stars. Furthermore, at least some members of these latter two classes exhibit precisely the same kind of variability that is characteristic of cooler magnetic Ap stars. The two best-studied He-weak variables are a Cen (Norris, 1971b, and references therein) and HR 7129 (Balona and Martin, 1974; Wolff and Wolff, 1976).

The variations of HR 7129, which are illustrated in Figures 8-3 through 8-6, can be accounted for by the oblique rotator model. The period of the star is 3.670 days. The magnetic field is the second largest one measured to date in a main sequence star. The field varies from +7000 to −5000 g, and the decline of 12000 g from maximum to minimum occurs in only 1.5 days. The He I and Si II spectrum variations are in antiphase, as is often true in Bp variables, and the Si II radial velocities vary in quadrature with the Si II line intensities. A marked crossover effect is also seen.

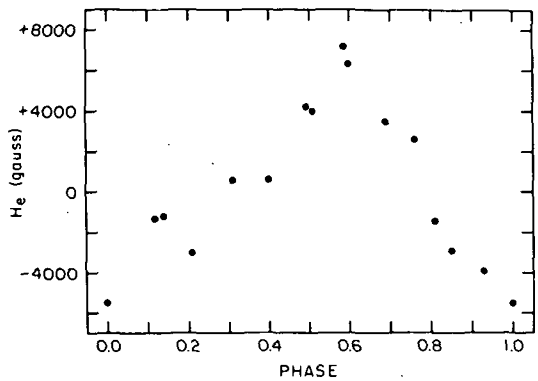

Figure 8-3. Longitudinal magnetic field variation of HR 7129 plotted according to the ephemeris HJD (magnetic minimum) = 2,442,248.89 + 3.670E (from Wolff and Wolff, 1976).

An oblique rotator model (Hensler, 1979) that fits these observations has caps of helium- and silicon-enriched material at the opposite magnetic poles. The axis of the magnetic field is inclined to the rotation axis at an angle of 87°, and the field strength at the poles is H_p = 33000 g.

An analogous model for the spectrum variations of a Cen has been derived by Mihalas (1973). In earlier models of a Cen (Norris and Baschek, 1972), the assumption was made that the surface gravity was constant over the stellar surface. Mihalas points out that this condition implies a strong violation of horizontal pressure equilibrium, since the pressure scale height in the He-poor region is considerably larger than in the He-rich zone. Mihalas argues that it is physically more plausible that horizontal pressure equilibrium obtains.

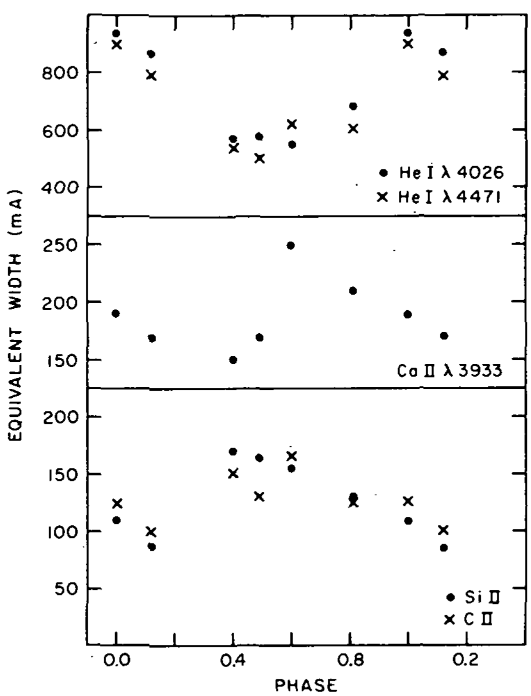

Figure 8-4. Equivalent width variations in HR 7129 (from Wolff and Wolff, 1976).

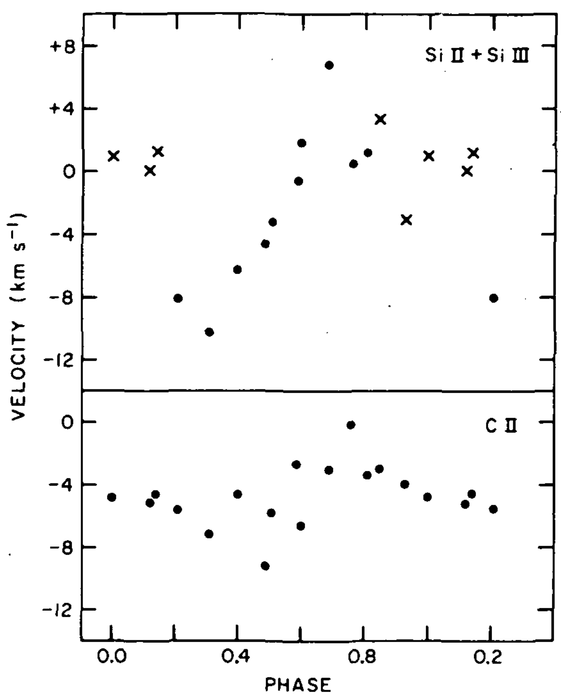

Figure 8-5. Radial velocity variations in HR 7129. In the upper panel, the filled circles denote measurements of lines of both Si II and Si III, while the crosses represent only lines of Si II, those of Si III being too weak to measure (from Wolff and Wolff, 1976).

The hydrostatic equation is

$$\Delta p = \rho g + F = \rho g_{\text{eff}}, \qquad (8\text{-}1)$$

where $p = \rho kT/\mu m_{\text{H}}$, and F stands for the nongravitational forces. Since the mean molecular weight μ is lower in He-poor zones than in He-rich ones, horizontal pressure equilibrium can be maintained only if g_{eff} increases. This increase can be achieved if magnetic forces are directed inward in the direction of gravity in the He-poor zones.

Mihalas' analysis points out an outstanding deficiency of virtually all models for the spectrum variations in Ap and Bp stars. Specifically, existing models tend to be nonphysical in the sense that they show only the distribution of elements over the stellar surface. The implications of that distribution for the structure of the atmosphere are usually not addressed.

A detailed study of line profiles should, in principle, provide useful diagnostics of atmospheric structure. The neutral helium lines in the He-weak star 3 Sco have been analyzed by Norris and Strittmatter (1975) for this purpose. They find that the members of the diffuse series of helium are too broad and shallow relative to the metallic lines. They show that this discrepancy can be explained if the helium is distributed nonuniformly either with depth in the atmosphere or in patches over the stellar surface. Available profiles are inadequate to distinguish between these two possibilities.

It is obvious from the terminology used—He-rich, He-weak, HgMn—that the most prominent spectral anomalies of the Bp stars are not all the same. More detailed studies show that the anomalies vary systematically with temperature, the He-rich stars being the hottest members of the Bp

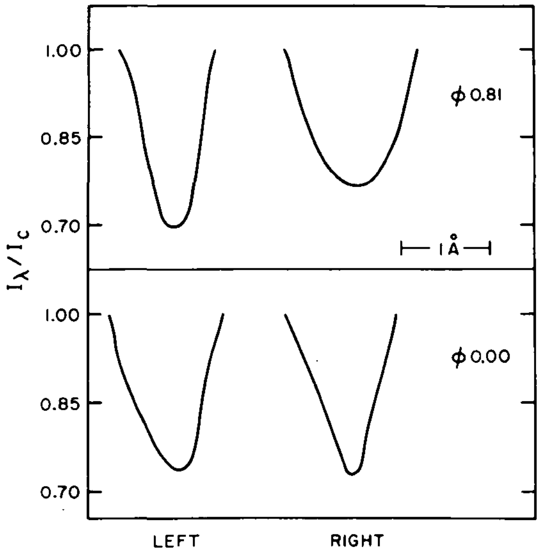

Figure 8-6. Line profiles of Ca II λ3933 in left and right circularly polarized light at phases 0.81 (upper panel) and 0.00 (lower panel). I_λ/I_c represents the line intensity normalized to the intensity of the continuum (from Wolff and Wolff, 1976).

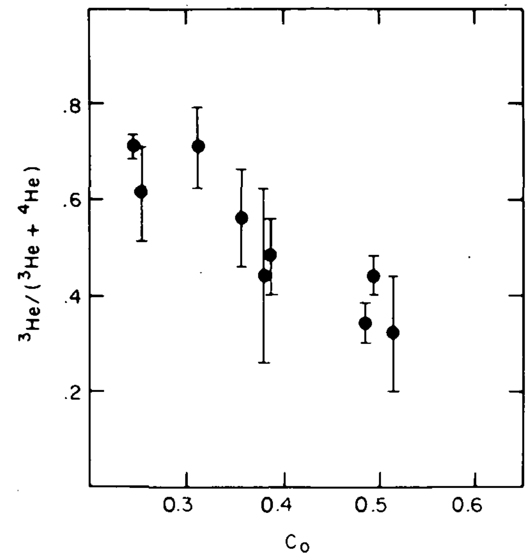

Figure 8-7. The fractional ^{3}He content as a function of c_0 for the definite and probable ^{3}He stars. The error bars indicate 1 σ uncertainty in the measured fractional ^{3}He content (from Hartoog and Cowley, 1979).

category and the HgMn stars, the coolest. Line strengths of other elements also correlate with temperature, and one of the most striking examples of this phenomenon occurs within the He-weak group.

It has been known for a long time that ^{3}He is overabundant in some Bp stars. In 3 Cen A, Sargent and Jugaku (1961) found that $^3He/^4He \sim 5$, and ^{3}He is clearly present in ι Ori B (Dworetsky, 1973). A systematic search for ^{3}He in the spectra of Bp stars has been carried out by Hartoog and Cowley (1979), and their results are illustrated in Figure 8-7. Among the ^{3}He stars, the fractional abundance of this isotope increases with increasing temperature. The ^{3}He stars occupy a very restricted region of the HR diagram bounded on the cool side by He-weak stars that show no evidence of ^{3}He, and on the hot side by the He-rich stars. The total abundances of helium in the ^{3}He stars are 5 to 20 times lower than in normal B-type stars.

These results are particularly important for models of the Bp stars, since it is very difficult to produce an overabundance of ^{3}He through nuclear processing. Such a close correspondence between T_{eff} and isotopic composition only compounds the problem. The only explanation offered for the ^{3}He anomaly involves a combination of radiative diffusion plus mass loss. Schematically, the argument goes as follows. In the He-rich stars, the overabundance of He has been attributed to a balance between stellar wind and diffusion velocities. If this hypothesis is correct, then at slightly lower temperatures this balance may be achieved for ^{3}He only. At still lower temperatures, both ^{3}He and ^{4}He will tend to sink, thus leaving an atmosphere that is deficient in helium but of normal isotopic composition. Detailed models of this process have been developed by Michaud et al. (1979).

The abundances of elements heavier than helium in the He-weak stars can be bizarre. In 3 Cen A, which is the best studied member of this class, phosphorus is overabundant by a factor of 100, krypton by a factor of 1300, and gallium by perhaps as much as a factor of 8000 (Jugaku et al., 1961). The different members of the He-weak class, however, are far from uniform in chemical composition. Whether or not variable and nonvariable He-weak stars have distinctively different compositions, as existing observations suggest, is a question that merits further study.

HgMn STARS

The HgMn stars are the most thoroughly studied of all the groups of Bp stars. The class was recognized by Morgan (1933) in his spectral classification study of peculiar A-type stars. We now know that the HgMn stars are restricted to the temperature range 11,000 to 16,000 K, which corresponds approximately to spectral types B6 to B9 (Wolff and Wolff, 1974). The systematically later types assigned by Morgan and others reflect the abnormal weakness of the helium lines. The strength of the helium spectrum is a primary classification criterion for middle and late B-type stars.

Discovery of HgMn stars is difficult because many known examples do not appear abnormal on spectrograms of the type customarily used for spectral classification (Osawa, 1965). The line blanketing in HgMn stars is small (Wolff, 1967) and does not distort the colors sufficiently to allow detection by intermediate or broadband photometric techniques (Cameron, 1966).

An efficient technique for finding HgMn stars was devised by Nariai (1967). The strongest lines of Mn II in the optical region of the spectrum lie in the interval λλ3440 to 3500. These lines can be easily detected in HgMn stars at 50 Å mm^{-1} but are not seen in normal stars at that same dispersion. Ultraviolet surveys at moderate resolution (Wolff and Wolff, 1974) and high-dispersion spectroscopic surveys in the region λλ3700 to 4700 (Preston, 1972b) have provided extensive lists of HgMn stars. Detailed analyses of stars identified by these surveys have established the characteristics of the HgMn group.

The frequency of HgMn stars is quite high and varies with both temperature and rotation (Wolff and Preston, 1978). At the low-temperature end (~11,000 K) of the range in which they occur, HgMn stars constitute about 8 percent of all B-type stars. The frequency of HgMn stars increases

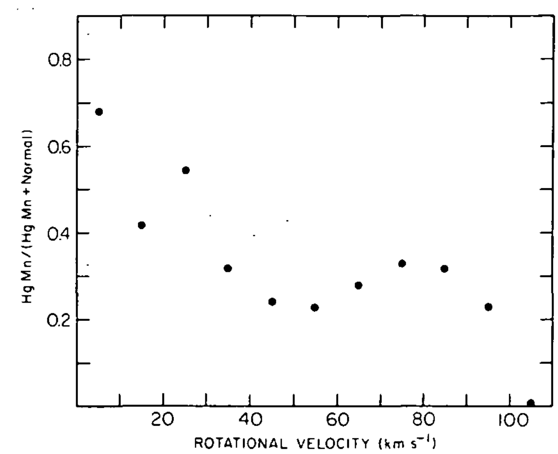

Figure 8-8. Fraction of stars showing HgMn characteristics as function of true rotational velocity (from Wolff and Preston, 1978).

with increasing temperature and reaches a value of ~25 percent at a temperature of 15,000 K. No HgMn star rotates in excess of 100 km s^{-1}, and their frequency increases with decreasing rotation for velocities less than 100 km s^{-1} (see Figure 8-8). Slow rotation is a necessary but not a sufficient condition for the appearance of HgMn characteristics.

Several surveys have been undertaken to determine the frequency of spectroscopic binaries among the HgMn stars (Abt and Snowden, 1973; Aikman, 1976; Wolff and Preston, 1978). These surveys concur in showing that the binary frequency among the HgMn stars is the same as among normal stars. The frequency of double-line HgMn binaries may be somewhat higher. Synchronism of rotation and revolution is common in close binaries where the components are of equal mass, and a majority of slowly rotating late B-type stars have HgMn characteristics.

Repeated attempts to detect magnetic fields in HgMn stars have been unsuccessful (Preston, 1971; Borra and Landstreet, 1980). The HgMn stars do not vary in luminosity or radial velocity, apart from orbital motions, and the visible spectrum does not change with time. There have been occasional reports of spectrum variations in the satellite ultraviolet (Rakos and Kamperman, 1977; see also Rakos et al., 1981; Renson and

Manfroid, 1981), but these reports need confirmation.

Perhaps most extraordinary and puzzling of all the features of the HgMn stars is their composition. As the name of the class implies, lines of both Mn and Hg are abnormally strong. In addition, lines of elements not seen at all in normal stars may be quite prominent. Numbered among these elements are platinum, gallium, phosphorus, xenon, and krypton. The abundances of some elements, including notably Mn itself, correlate with effective temperature (see Figure 8-9). Most extraordinary is the correlation of Hg isotopic abundances with T_{eff}. Figure 8-10 shows profiles of Hg II λ3984 in several stars, and Figure 8-11 shows the systematic variation of the centroid wavelength of this line with color index (and hence temperature). The isotopic composition of Hg varies from a nearly solar mixture in the hotter stars to nearly pure ^{204}Hg in the coolest ones (White et al., 1976). A diffusion model, which depends on a balance between gravitational and radiation forces so critical that the small mass differences of the mercury isotopes become important, has been developed to explain the observations (Michaud et al., 1974). No alternative explanations are available. Certainly a correlation of isotopic composition with T_{eff} is very difficult to explain by nuclear processes.

The abundances of other heavy elements in HgMn stars pose a different kind of problem. The

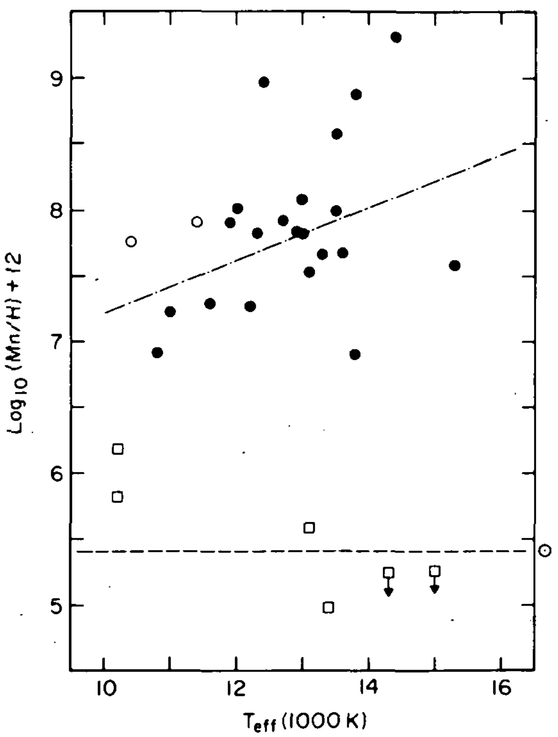

Figure 8-9. Manganese abundances plotted against stellar effective temperatures. Filled circles, HgMn stars; open circles, SiCr stars; open squares, normal stars; dashed line, solar abundances; dot-dashed line, least-squares fit to the data (from Heacox, 1979).

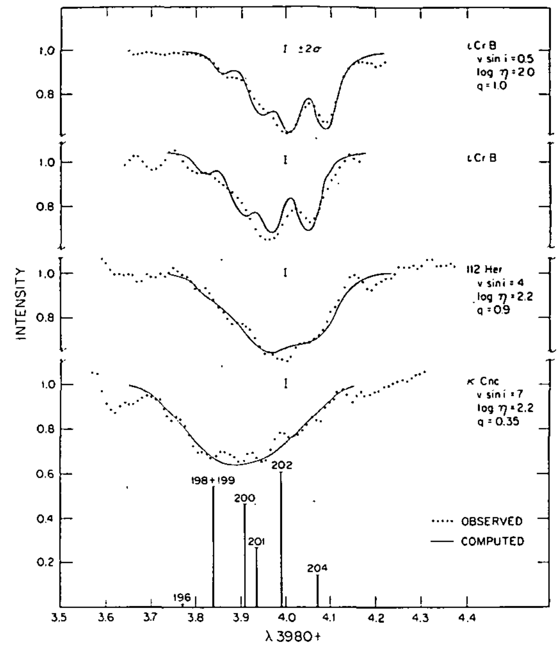

Figure 8-10. Dotted curves represent profiles of Hg II λ3984 for three stars observed with a Fabry-Perot interferometer; solid curves represent theoretical profiles computed for isotopic mixtures defined by the relationship log α = q(A−202)log$_{10}$e, in which q is a dimensionless mix parameter, A is the atomic mass, and α = [N(A)/N(202)]$_*$/[N(A)/N(202)]$_\odot$. Two independent observations are shown for ι CrB. Vertical bars at bottom indicate wavelengths and intensities of isotopic components for the terrestrial mixture (q = 0) (from White et al., 1976).

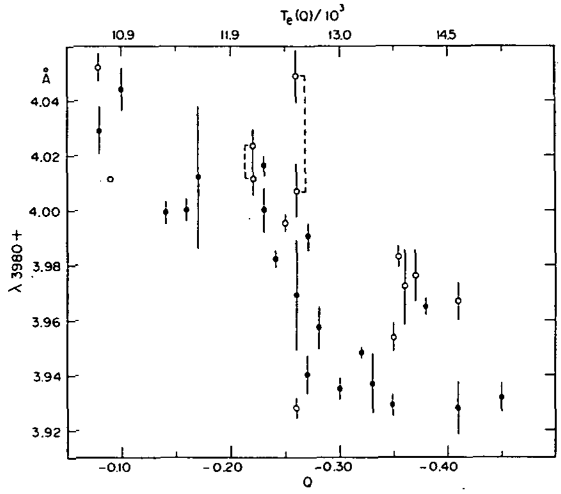

Figure 8-11. A plot of measured centroid wavelength versus Q = (U–B) – 0.72 (B–V) for 33 Hg stars. Vertical bars denote mean errors. Filled and open circles denote single stars and components of spectroscopic binaries, respectively. Dotted lines connect components of the double-lined binaries 46 Dra and 41 Eri which are plotted at the positions of their observed Q values. The Q for 41 Eri is uncertain because of a close visual companion (from White et al., 1976).

most comprehensive study is by Heacox (1979). He finds that there is no correlation between rotational velocity and the abundance of any element. This result is particularly puzzling, since the likelihood that a star will show HgMn characteristics *is* correlated with rotational velocity (see Figure 8-8). Heacox also finds a great diversity in the abundance patterns of HgMn stars of similar temperature, surface gravity, and v sin *i*. An example is shown in Figure 8-12. Note, for example, that Be is much more abundant in υ Her than in ϕ Her, which in turn has more Sc, Y, and Zr. Heacox notes further that the elements that exhibit the greatest abundance anomalies (Be, Ga, Xe, Hg, and Pt) do not do so in all stars. A particularly noteworthy example is 53 Tau, which no one doubts is a member of this class, but in which Hg is not seen. The extreme diversity of the HgMn stars has been confirmed by a number of other abundance studies (e.g., Jacobs and Dworetsky, 1981).

A distinction between nuclear and nonnuclear explanations for the origin of the abundance anomalies of Bp and Ap stars might be achieved if clear violations of nuclear abundance patterns were observed. Elements with even atomic numbers are typically more stable than their immediate neighbors in the periodic table and, if nuclear processes are the dominant mechanism in producing the element distribution, are expected to be more abundant. Several attempts have been made to determine whether the odd-even effect is violated in HgMn stars (e.g., Allen and Cowley, 1977; Allen, 1977; Heacox, 1979; Maguzzu et al., 1981).

There is good evidence that the ratio Mn/Fe is greater than unity in some HgMn stars. In addition, the P/S ratio appears to be greater than one in several stars, and there seems to be no way to produce so high a ratio by means of nucleosynthesis (Cowley and Aikman, 1975). An odd-*Z* anomaly at yttrium is also fairly well established for some HgMn stars. The possibility always exists, however, that these conclusions might be modified by improvements in *gf* values or in the calculations of atmospheric models.

The alternative to nuclear processing as a cause of the abundance anomalies is, once again, radiative diffusion. Quite detailed models have been

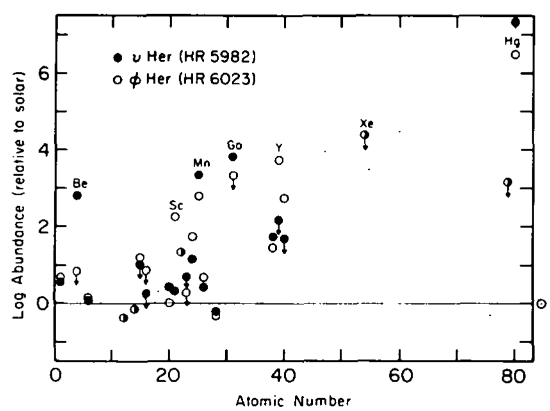

Figure 8-12. Abundances in υ Her and ϕ Her expressed in logarithmic units relative to solar abundances and plotted against atomic number. Filled circles, υ Her; open circles, ϕ Her; arrows indicate upper limits (from Heacox, 1979).

calculated for specific elements, including boron (Borsenberger et al., 1979), calcium and strontium (Borsenberger et al., 1981), and manganese (Alecian and Michaud, 1981). The agreement in some cases is remarkably good (see Chapter 9). However, a major problem is presented by the great diversity of the HgMn stars. Does the diffusion theory contain enough free parameters to accommodate this diversity? Or are there multiple causes for the observed anomalies?

POPULATION II Bp STARS

A number of Population II horizontal branch stars are known to possess composition anomalies similar to those of Population I Bp stars. It has been known for quite some time that the horizontal branch stars in globular clusters are He weak (Greenstein and Munch, 1966; Greenstein and Sargent, 1974). The similarities between Population II and Population I Bp stars, however, can go far beyond the similarity in helium abundances. The high-latitude star Feige 86 has strong lines of phosphorus, a property that is characteristic of HgMn and He-weak stars of Population I (Sargent and Searle, 1967a). In fact, the only substantial difference in the metallic abundances of Feige 86 and 3 Cen A is a greater deficiency of carbon in the former.

In a very recent study, Hartoog (1979) has demonstrated that Feige 86 shares with 3 Cen A a very high abundance of ^{3}He. Figure 8-13 shows the profiles of λ6678 in Feige 86, HR 7467 (a star known to have a high ratio of ^{3}He/^{4}He), and a normal B-type star. Figure 8-14 compares the position of Feige 86 in the (log g, log T_{eff}) plane with the location of ^{3}He Population I stars. The similarities between Feige 86 and Population I Bp stars argue strongly that the mechanism responsible for producing the observed abundance anomalies depends on T_{eff} and log g and not on the past evolutionary histories of the peculiar stars. Another implication of these results, of course, is that the apparent composition of the horizontal branch stars cannot be considered indicative of their primordial abundances.

Observations of Feige 86 with IUE show that gallium and mercury are present, and the line He

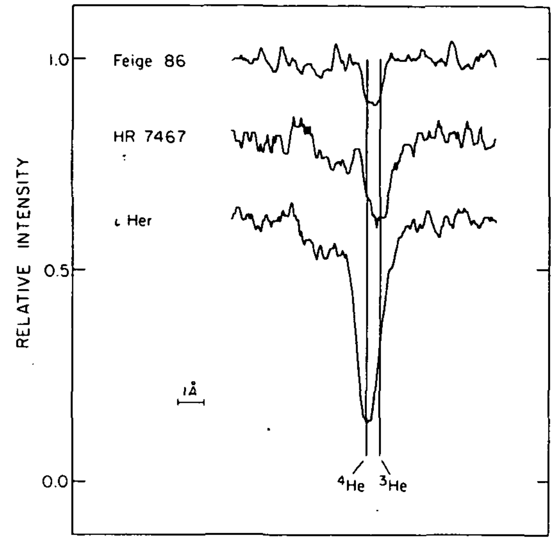

Figure 8-13. The average λ6678 profile in Feige 86 is compared with the profile in the known ^{3}He star HR 7467 and the normal star ι Her. The ^{3}He and ^{4}He positions are marked (from Hartoog, 1979).

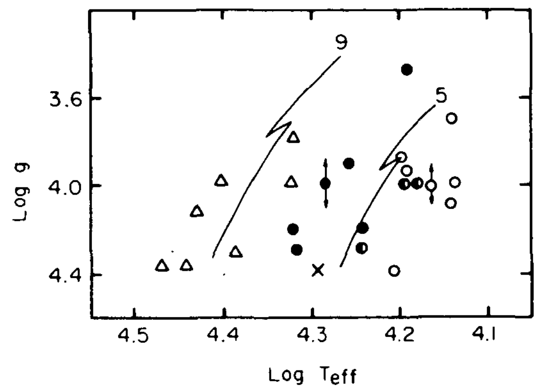

Figure 8-14. Triangles, the (log T_{eff}, log g) diagram for the He-rich stars; solid circles, ^{3}He stars; partially filled circles, marginal ^{3}He stars; open circles, He-weak (no ^{3}He) stars; cross, Feige 86 (from Hartoog, 1979).

II λ1640 is primarily due to ^{3}He. The IUE data also provide quite unambiguous evidence for non-radiative heating in this star (Hack, 1980). Lines of N V, C IV, and Si IV are seen, and each has three components. One component is blueshifted by about 1.5 Å and the other is redshifted by 0.4 Å. The ionization of these features is much too large to be explained by photospheric heating, since $T_{eff} \cong$ 17,000 K to 18,000 K for Feige 86. The presence of multiple components suggests that large mass motions are also present. Such effects may be difficult to reconcile with the radiative diffusion model for the origin of the abundance anomalies.

A few other examples of Population II Bp stars are known. Lines of phosphorus are present in Feige 92 (Sargent and Searle, 1967a). The field horizontal branch star HD 97859 resembles the Si stars and is also a spectrum variable (Searle and Sargent, 1972; Stalio, 1974). The old disk horizontal branch star 38 Dra is similar to the HgMn stars (Adelman and Sargent, 1972). The line Hg II λ3984 is definitely present in 38 Dra. The primary differences with respect to HgMn stars of Population I are somewhat higher abundances of both Mn and Zr in 38 Dra.

The primary arguments for identifying these stars as members of Population II are kinematical and are presented in the papers cited above.

SUMMARY

A number of the characteristics of Bp stars are referred to, either explicitly or implicitly, in developing models for the origin of the abundance anomalies of both Bp and Ap stars. Some of the most important constraints on model building are as follows:

1. **Correlation of Abundances with Surface Properties.** The abundances of specific elements, including He and Mn, correlate well with temperature as do the isotopic compositions of He and Hg. Population II stars have abundance anomalies that closely resemble those of Population I stars of similar $\log g$ and T_{eff}. These observations indicate that the mechanism responsible for the abundance anomalies depends on the characteristics of stellar atmospheres rather than of their interiors, masses, or past evolution.

2. **Temperature Range of Peculiar Stars.** With the discovery of He-rich stars, it is now apparent that peculiar stars are found throughout the temperature range in which neither mass loss nor convection are thought to be important. Peculiar stars are not found in regions where macroscopic velocity fields are strong enough to be directly observable. This result suggests that atmospheric stability correlates closely with abundance anomalies. However, it is clearly true that many stars in this temperature range, including slow rotators, have normal compositions.

3. **Diversity.** Abundance analyses, particularly of HgMn stars, have established that the peculiar stars are extremely diverse. Whatever model is proposed to explain the abundance anomalies must include either multiple physical processes, or one process with many free parameters, to accommodate this diversity.

4. **Time Scales.** A fundamental constraint on models is the time scale for the observed characteristics to develop. The best evidence for changes with time of rotational velocities and magnetic fields comes from studies of He-rich and He-weak stars in young clusters and associations (Wolff, 1981; Borra, 1981). An upper limit for the allowed time scale for the development of peculiarities also depends primarily on observations of these stars. Few peculiar stars of later type (and lower luminosity) have been identified in extremely young stellar groups.

9

MODEL ATMOSPHERES

MODEL ATMOSPHERES–THE GENERAL PROBLEM

The goal of calculations of model atmospheres is to provide a characterization of the dependence of physical variables–temperature, density, pressure, etc.–on depth in the atmosphere. With that characterization, it then becomes possible to describe the emergent spectrum and, through comparison of theory with observation, to determine the structure and composition of stars. The problem of constructing model atmospheres is, however, too complex to be solved in the absence of simplifying assumptions. In the case of the Sun, and presumably other stars as well, the atmosphere is not homogeneous but exhibits complex, small-scale structure. Mass motions, including convection, turbulence, pulsation, and meridional circulation induced by rotation, may be present. Rotation itself may alter the figure of the star and even the rates of energy generation and may thereby alter the emergent spectrum. Magnetic fields, if present, will affect atmospheric structure while sources of nonradiative heating can induce chromospheres or coronae. In an otherwise stable atmosphere, diffusion may change the chemical composition in various layers in the atmosphere. Non-local thermodynamic equilibrium (LTE) level populations occur. In supergiants, the atmosphere is highly extended, and mass loss is certainly taking place.

Obviously, incorporating all of these physical processes into the calculation of a model stellar atmosphere is well beyond current capabilities. In order to make any progress at all, it is necessary to make a number of simplifications, and the assumptions classically adopted are as follows (e.g., Mihalas, 1970, 1978):

1. The atmosphere can be described in terms of plane-parallel, homogeneous layers. The first of these two conditions is applicable to stars in which the depth of the atmosphere is small relative to the radius of the star. The plane-parallel approximation is therefore quite good for main sequence stars and wholly inappropriate for supergiants. Very little is known about the homogeneity–or lack thereof–of stars other than the Sun. In the absence of observational constraints, it is necessary to assume that model atmospheres are a reasonable representation of the mean or average conditions in the star.

2. The atmosphere is in a steady state. The equation of radiative transfer and the populations of the various atomic levels are assumed to be invariant with time. Transient phenomena, such as pulsation, shocks, and variable heating by a binary companion, are assumed to be absent.

3. The atmosphere is in hydrostatic equilibrium. All velocity fields–either large or small scale, random or systematic–are neglected, and it is assumed that pressure and gravitational forces are in balance.

4. The atmosphere is in radiative equilibrium. Transport of energy by such hydrodynamical processes as convection is neglected, and it is assumed that there are no energy sources in the atmosphere.

In the case of the A-type stars, it is clear that these assumptions are poor approximations to the actual properties of at least some specific classes of stars. The usual approach in modeling A-type stars has therefore been to relax one or more of these assumptions and to analyze the resulting changes in atmospheric structure.

The sections that follow will first examine modern solutions to the classical atmospheres problem. These solutions depend, of course, on the validity of the four assumptions stated above. Then various models that incorporate additional physical processes will be discussed. A basic review of the model stellar atmospheres of early-type stars can also be found in the volume on B-type stars by Underhill and Doazan (1982), which is part of the present series of monographs.

LTE MODELS

In principle, the calculation of a model photosphere in which the assumption of LTE is a good approximation is straightforward (e.g., Mihalas, 1970). In this case, the problem is reduced essentially to the determination of $T(\tau)$, the variation of temperature with optical depth in the atmosphere. The occupation numbers of the atomic levels and the ionization balance will depend on both the electron density N_e and the temperature. Opacities, which are determined by the level populations and ionization balance, also depend on T and N_e, as does the pressure. The pressure and density distribution, however, can be derived by integration of the equation of hydrostatic equilibrium. Once $\rho(\tau)$ and $N_e(\tau)$ and the opacities are derived and the source function estimated, it is possible to solve the equation of radiative transfer and then check if the condition of radiative equilibrium is satisfied. In general, it will not be satisfied, because the initial assumption for $T(\tau)$ may well be in error, but various iterative techniques are available for improving the adopted values for $T(\tau)$.

A primary practical problem in applying this approach is that line blanketing is substantial in the ultraviolet region of the spectrum of normal A-type stars and in the visible as well for Ap and Am stars. This blanketing must be taken into account if the calculated spectrum is to be compared to that observed for a real star. The effects of increases in opacity on the spectrum and atmospheric structure have been described by Kurucz (1979). The alteration in the $T(\tau)$ relationship due to enhanced ultraviolet opacity is illustrated in Figure 9-1. In layers that are optically thick, the increased opacity impedes the flow of radiation, while in the optically thin layers the flow is enhanced. In order to maintain a constant flow of energy, the temperature must increase in the deeper layers and decrease near the outer boundary of the atmosphere. The impact of blanketing on the observed energy

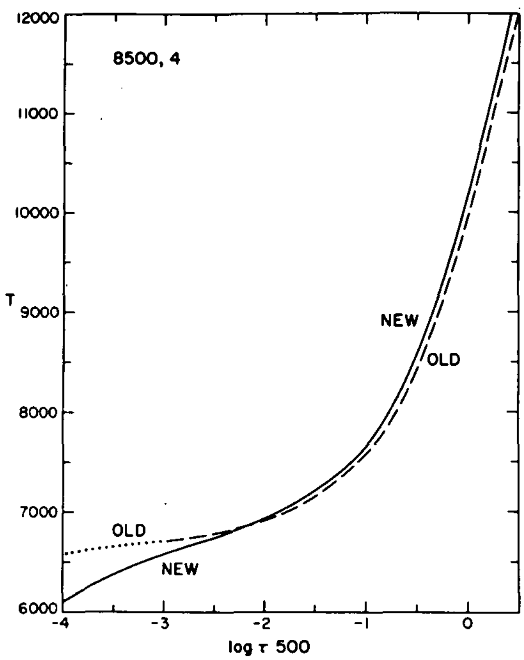

Figure 9-1. The effect on the T–τ relation of increasing the line opacity for a model with T_{eff} = 8500 K and log g = 4. The model labeled "new" has more opacity (from Kurucz, 1979).

distribution of a late A-type star is shown in Figure 9-2. The decrease in flux in the ultraviolet due to enhanced opacity is compensated for by an increase in visible flux. Furthermore, the slope of the energy distribution in the visible is sensitive to changes in ultraviolet opacity; thus, effective temperatures estimated from the visible flux alone will be in error if the opacity in the model is incorrect.

The change in temperature structure resulting from an enhancement of the ultraviolet opacity also produces changes in line profiles. Line cores are typically deeper because of the lower surface temperature, while the line wings and equivalent widths may be either weaker or stronger depending on the depths of formation. Obviously, abundances, surface gravities, and other quantities inferred from line profiles may be in error if the opacity used in the model atmosphere calculation is incorrect.

The most extensive set of models that attempts to treat realistically the opacity caused by metallic lines is the one calculated by Kurucz (1979). One million lines have been included statistically in the calculations through the use of distribution functions to characterize the opacity. The models themselves assume plane-parallel geometry, hydrostatic equilibrium, local thermodynamic equilibrium, no molecules in the equation of state, and radiative plus convective energy transport, with the mixing length to pressure scale height taken to be 2.0. Abundances were assumed to be equal to the solar values. The models range in temperature from 5500 to 50,000 K and in the log of surface gravity from 4.5 down to the radiation pressure limit. In the temperature range 5500 to 10,000 K, models were also calculated for abundances of 1/10 and 1/100 the solar values. Kurucz also makes extensive references to, and comparisons with, earlier work on model stellar atmospheres.

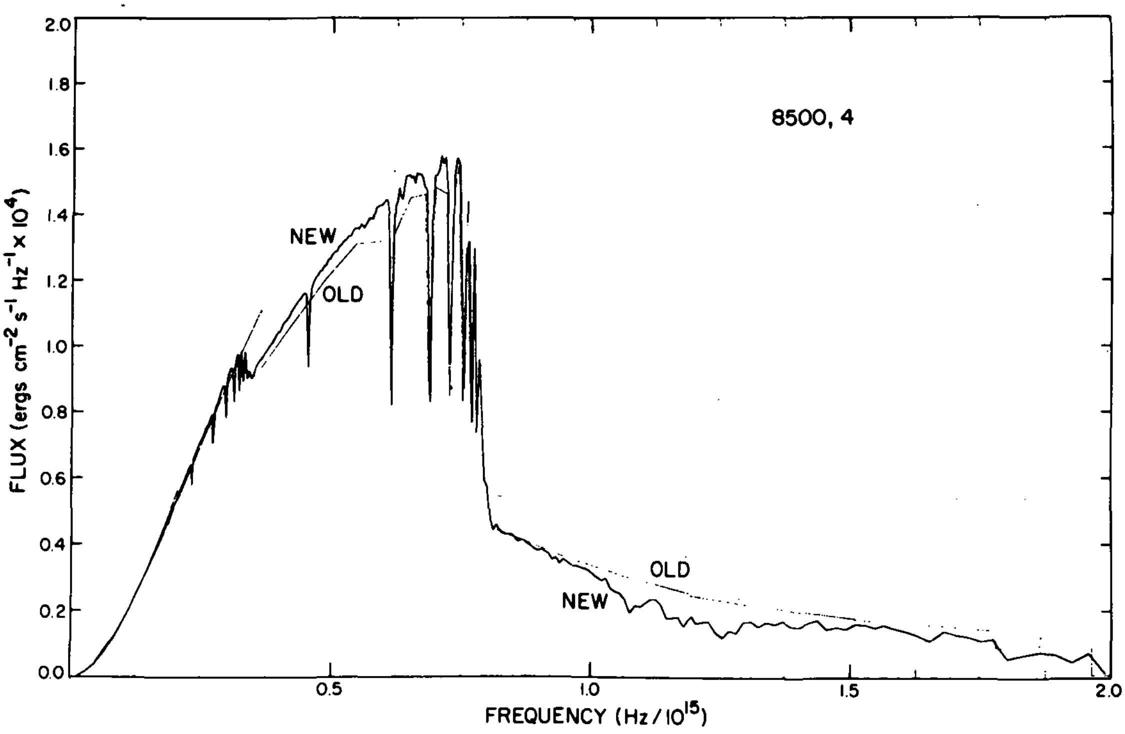

Figure 9-2. The effect on the flux F_ν of increasing the line opacity for a model with T_{eff} = 8500 K and log g = 4 (from Kurucz, 1979).

Two basic kinds of comparisons with observations are possible. First, one can compute various observable parameters, such as color indices, determine the correlations of these quantities with one another (e.g., two-color plots), and then see whether or not the same correlations are seen in observational data. Comparisons between theoretical and observed uvby colors have already been described (see Chapter 2) and used to estimate the effective temperatures of A-type stars. As noted in the earlier discussion, the computed and measured colors are in good agreement except for the late A-type stars, where the discrepancy is likely to be caused by an inadequate treatment of convection.

An even more rigorous requirement for the blanketed LTE models is that they reproduce the observed spectrum—lines as well as continuum—of a specific star. Comparisons of theory and observations have been made for Vega by Kurucz (1979) and, in more detail, by Dreiling and Bell (1980). This latter paper also gives extensive references to earlier work on both observations and models of Vega. (For a similar analysis of Sirius, see Bell and Dreiling, 1981.)

In their study, Dreiling and Bell consider the following six separate characteristics, all of which must be accounted for by an adequate model:

1. The absolute fluxes at $\lambda5556$ and $\lambda10400$.
2. The relative fluxes over the interval $\lambda\lambda3300$ to 10800.
3. The hydrogen line profiles.
4. The curves of growth for various metallic ions, including specifically Fe I, Fe II, and Ti II.
5. The absolute fluxes and line blocking in the ultraviolet (below the atmospheric cutoff) region of the spectrum.
6. The infrared fluxes.

Dreiling and Bell find that measurements of the first two of these quantities indicate that the effective temperature of Vega is T_{eff} = 9650 K, in excellent agreement with the value of 9660 ± 140 K obtained by Code et al. (1976) from the angular diameter of Vega and from OAO-2 and ground-based observations of the energy distribution. The Balmer line profiles yield log g = 3.9 ± 0.2, a value consistent with the size of the Balmer discontinuity and with the surface gravity inferred from the radius and estimated mass of Vega. The abundances derived from the Ti II and Fe II curves of growth are about a factor of 2 less than solar. The ultraviolet fluxes and line blocking are consistent with the model calculations, but are too imprecise to constitute a rigorous test. Indeed, Dreiling and Bell suggest that the model of Vega is sufficiently well understood that the calculated fluxes can be used to provide a calibration of the ultraviolet energy distribution and of infrared flux measurements between 1 and 10 μm as well. Their models do not, however, reproduce either Ly α or the UV spectrum for $\lambda < 1240$.

STATISTICAL EQUILIBRIUM MODELS

While the overall agreement between the calculated and observed emergent spectrum of Vega is fairly satisfactory, some discrepancies are apparent (Dreiling and Bell, 1980). Particularly pronounced is the problem with the violet wing of Ly α. At $\lambda1135$, for example, the observed flux is more than five times greater than that predicted by LTE models. There is also evidence for a discrepancy between the abundances of metallic elements derived from visible and satellite ultraviolet lines (Castelli and Faraggiana, 1979), although this result has been challenged by Sadakane and Nishimura (1981).

The basic assumption of all LTE models is that the occupation numbers of atomic states depend only on T and N_e and hence reflect only local conditions. In fact, of course, excitation and ionization are strongly influenced by the radiation field, which is, in turn, dependent on the state of the gas throughout the atmosphere (e.g., Mihalas, 1970). The equation of radiative transfer and the rate equations which describe the processes that populate and depopulate the various atomic states must be solved simultaneously. Model atmospheres obtained through such an approach are properly termed statistical equilibrium models. They are also often described, incorrectly, as non-LTE models. Obviously, the latter term is inappropriate inasmuch as a whole range of potentially important nonequilibrium effects are not included in the calculations.

Statistical equilibrium models have been calculated for dwarf (Frandsen, 1974) and supergiant (Kudritzki, 1973) A0 stars. In the A0 dwarf models (T_{eff} = 10,000 K), the statistical equilibrium calculations for pure hydrogen atmospheres indicate that there is a temperature rise in the outer layers of the atmosphere from a minimum of about 7300 K to a maximum between 8500 and 9000 K. The n = 1 level of hydrogen is overpopulated, whereas the levels with $n > 3$ are underpopulated relative to their LTE values. The same features are present in statistical equilibrium models of B-type stars (Auer and Mihalas, 1970). For $\log g > 3$, differences between the LTE and statistical equilibrium calculations for A0 stars become apparent only in the cores of the hydrogen lines. The supergiant models, which were calculated for T_{eff} = 10,000 K, $\log g = 1$, and a range of helium abundances show large changes in the radiation field relative to LTE models, not only in the lines but in the continuum as well. Furthermore, the continuum radiation field, including the hydrogen discontinuities, depends on the helium abundance. Unless the He/H ratio is somehow known in advance, the size of the Balmer discontinuity cannot be used to infer temperature or surface gravity. Calculations of this kind serve to underscore the problems of interpreting the spectra of supergiants.

Hubeny (1981) has recently taken a major step forward by calculating models for Vega that take account of departures from LTE of H, H^-, C I, Si I, S I, N I, and Mg II, as well as Ly α blanketing in a fully consistent way. He finds that the statistical equilibrium models are essentially indistinguishable from LTE models throughout the visible continuum. The largest change occurs in the magnitude of the Balmer discontinuity, the value of which is increased by 0.02 mag in the statistical equilibrium models. Departures from LTE produce larger effects in the ultraviolet with the most significant changes occurring in the violet wing of Ly α. Specifically, LTE calculations overestimate the opacity caused by C I and Si I and therefore underestimate the emergent flux, with the greatest errors occurring in the interval $\lambda\lambda$1100 to 1260. The flux observed in the violet wing of Ly α can be explained in terms of a model in radiative equilibrium (see also Snijders, 1977), and there is no need to postulate the existence of a chromosphere (Praderie et al., 1975). The statistical equilibrium model that best agrees with the observations is characterized by T_{eff} = 9660 K and $\log g = 4$ (Hubeny, 1981).

This discussion of various types of theoretical calculations raises some interesting philosophical points concerning the construction of models (see also Hubeny, 1981). Calculation of a model incorporating all of the physical processes that may be important in determining the emergent spectrum of a star is beyond the present state of the art. Even solution of the rate equations for a fully blanketed model is not now possible, and, furthermore, many of the essential physical constants (photoionization cross sections, collision cross sections, etc.) are quite uncertain. In model building, therefore, the approach frequently adopted is to make alterations to a basic LTE model in order to incorporate one or more additional processes that are thought to be of major significance. Often these alterations are made without checking whether they affect materially the temperatures, densities, and level populations that characterized the LTE model. Unfortunately, completely consistent calculations usually are feasible only for a set of assumptions that are so restricted they cannot possibly be an adequate description of reality. For example, Snijders (1977) used an LTE model atmosphere to evaluate the rate equations for C I and Si I. This approach is not consistent, because departures from LTE in C I and Si I may change the opacity and hence the temperature structure of the atmosphere. However, this approximate calculation does provide an improved representation of the Ly α wings. In contrast, Kurucz's line-blanketed models are fully consistent in the sense that they are based on a consistent set of assumptions. Because of neglect of departures from LTE, they do not adequately represent the Ly α wings but provide a better representation of the visible spectrum than do unblanketed, statistical equilibrium models. One justification for calculating models that are known to be incomplete is that a differential comparison of models may provide insight into the relative importance of various assumptions used in the computations.

In the case of the A-type stars, it is obvious that objects of similar T_{eff} and log g can have very different emergent spectra in the visible or the ultraviolet or both. While a nearly normal star like Vega may be adequately described in terms of radiative equilibrium models with proper inclusion of blanketing and the rate equations, it is clear that in magnetic, Am, and other peculiar stars there must be major changes in atmospheric structure, chemical composition, or both. In the following sections, we will discuss the influence of such effects as diffusion, rotation, convection, and magnetic fields on the observable spectrum.

CONVECTION

In a stellar atmosphere, energy can be transported by either radiation or convection. In the A-type stars, convection is relatively unimportant in the sense that it transports only a small fraction of the energy, and it does so only in the coolest members of this spectral group. Nevertheless, convection—or its absence—may play a crucial role in determining the appearance of the line spectrum of A-type stars. Diffusive separation of elements, which has been postulated as an explanation for many of the observed abundance anomalies, can occur only in regions that are stable against convection. A thorough review of convection has been given by Zahn (1980), who also provides extensive references to the literature on this subject. The discussion of convection here will focus on its observable effects in A-type stars.

A discussion of the basic physics of convection can be found in Mihalas (1970). An atmosphere in radiative equilibrium will be unstable against convection if an element of gas, displaced either up or down, experiences forces that tend to move that element further in the direction of its displacement. In stellar atmospheres, this condition occurs when the temperature gradient is very steep, so that the element, whose motion is assumed to be adiabatic, cannot accommodate itself to its surroundings. In A-type atmospheres, steep temperature gradients are associated with regions where hydrogen and helium are being strongly ionized. In such regions, the opacity increases sharply and steep temperature gradients are required to maintain the flow of energy through them. A model envelope for an A-type star, which illustrates these interconnections, is shown in Figure 9-3. As the figure shows, there are two convection zones in A-type stars. One is due to the ionization of H and He; the other is due to the ionization of He^+. The structure of the convective regions is therefore fundamentally different from that of both hotter and cooler stars.

Thermal convection is described by equations for conservation of mass, momentum, and entropy (e.g., Zahn, 1980), and this set of equations is highly nonlinear. The flows are turbulent with motions on a range of interrelated scales. Solution of the general equations is impossible at the present time and approximations are required.

The most commonly applied approximation is the mixing length theory. The physical picture underlying the mixing length theory is that energy transport is by turbulent elements that move up and down. Both upward moving elements, which have an excess of thermal energy over the ambient environment, and downward moving elements, which have a deficiency, contribute to the heat flow. After traversing some typical distance, which is referred to as the mixing length, the element loses its identity by dissolving and merging with its surroundings, depositing its thermal energy, positive or negative, in the process. The convective flux is proportional to $[\rho(z) \bullet v(z) \bullet \alpha \bullet (\nabla - \nabla')]$, where $\rho(z)$ is the mass density of convecting elements, $v(z)$ is their velocity, α is the ratio of the mixing length ℓ to the pressure scale height H, $\nabla(z)$ is the temperature gradient of the ambient medium, and $\nabla'(z)$ is the temperature gradient experienced by the

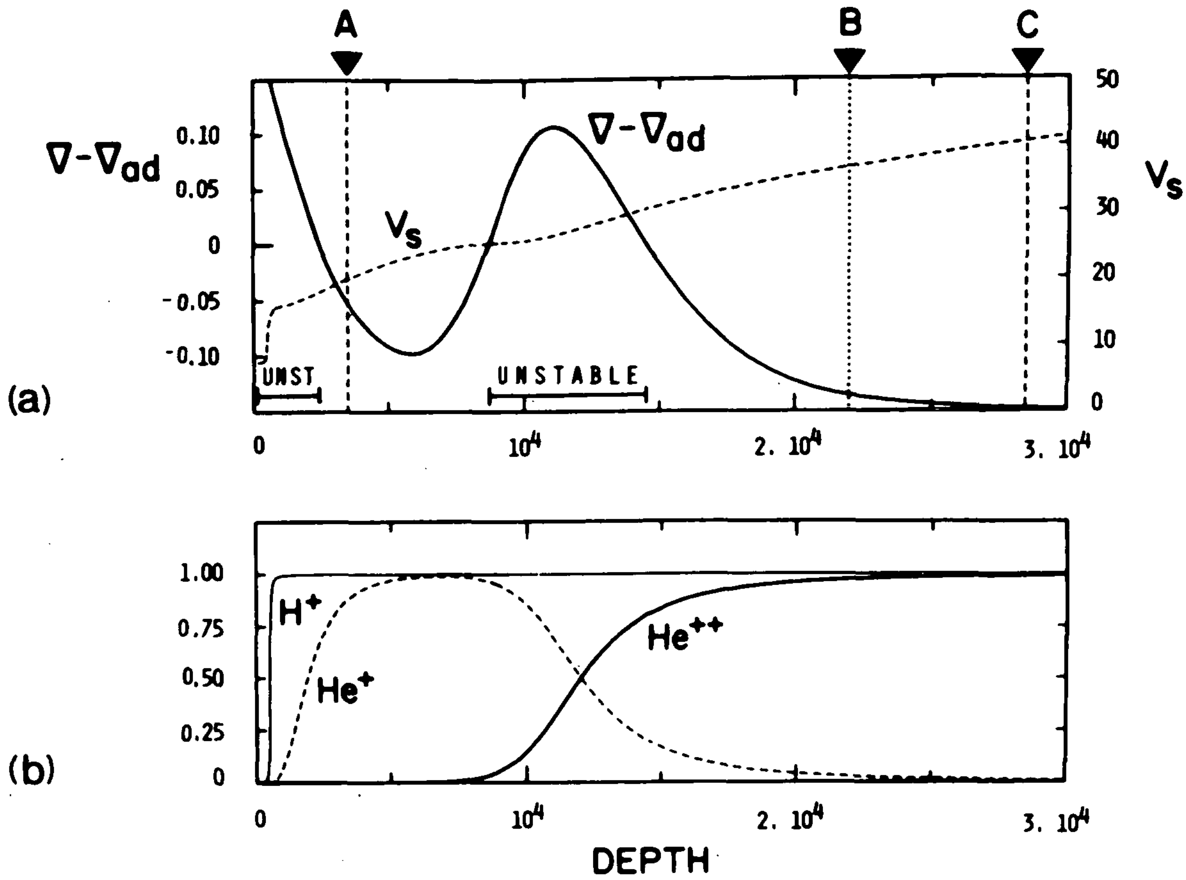

Figure 9-3. A mixing length model for the envelope of an A-type star has two convectively unstable zones. This A star has a mass of 1.8 $M_\odot$, an effective temperature of T_{eff} = 8000 K, a surface gravity of 1.15 cm s^{-2}, and its chemical composition is X = 0.602, Y = 0.354, Z = 0.044. The mixing length was chosen equal to the pressure scale height. Figure 3a presents the superadiabatic gradient $\nabla - \nabla_{ad}$ and the sound velocity V_s (in km s^{-1}) as functions of the depth (in kilometers) below the surface; regions where $\nabla - \nabla_{ad}$ is positive identify zones of convective activity in the mixing length treatment. Figure 3b shows how the proportion of the ions H^+, He^+, and He^{++} vary with depth (from Toomre et al., 1976).

convective elements as they pass through the geometrical depth z in the atmosphere.

One of the difficulties in the mixing length approximation is the choice of an appropriate value for ℓ/H, the ratio of the typical distance traversed by a turbulent element to the local pressure scale height. In practice, this ratio is essentially a free parameter that can be selected to force theory into accord with observations. In the case of the Sun, the radius inferred from models of the convective region is very sensitive to ℓ/H. Forcing agreement between observed and calculated values of the radius indicates that $\ell/H \sim 1.5$.

In main sequence stars the transition from radiative to convective energy transport occurs over the temperature range 6000 K < T_{eff} < 8500 K; this range encompasses the late A-type stars. The depth of the convection zone decreases with increasing temperature over this temperature interval, and for most of the late A-type stars, its depth is smaller than the pressure scale height. For these stars, the physical meaning of a value for the mixing length that is comparable to, or even exceeds the pressure scale height is unclear, but ℓ/H is generally chosen to be in the range of 1.0 to 2.0.

Standard mixing length theory (Böhm-Vitense, 1958) predicts that the onset of convection will be abrupt. That is, there exists a temperature T_1 such that all stars with $T_{eff} > T_1$ will be in radiative equilibrium, while in all stars with $T_{eff} < T_1$ at least some of the energy transport will be by convection. For fixed T_{eff} and ℓ/H, the efficiency of convective energy transport decreases with $\rho(z)$ and hence with decreasing gravity. The temperature T_1, therefore, will decrease with increasing luminosity.

There are three characteristics of stars that might be expected to change abruptly at T_1 and might therefore be used to define its value. First, it is generally believed that chromospheric activity is closely linked to an efficient hydrogen convection zone (although recent observations of X-ray emission in A-type stars show that non-radiative heating is certainly not limited to such stars). Measurements of emission in Ca II H and K and Mg II h and k have therefore been used to define the temperature at which convection becomes efficient enough to power chromospheric activity. Convection can also be expected to damp pulsation. Thus, the red boundary of the Cepheid instability strip may be used to define the onset of convection. Böhm-Vitense and Nelson (1976) show that the red boundary of the Cepheid instability strip stretches from $T_{eff} \cong 6400$ K at $\log L/L_\odot = 2$ to $T_{eff} \cong 5500$ K at $\log L/L_\odot = 4$, and that within the errors this line coincides with the high-temperature boundary for chromospheric activity. The requirement that this boundary also coincide with the onset of convection according to the mixing length theory then yields $\ell \sim H$, if the additional assumption is made that ℓ/H is the same for all stars.

Convection will also redden the observed colors of a star, and if the onset of convection is indeed abrupt, then this effect may be observable. The following argument has been presented by Böhm-Vitense and Canterna (1974). Suppose that all stars with $T_{eff} > T_1$ are in radiative equilibrium, while a portion of the energy is transported by convection in stars with $T_{eff} < T_1$. If a radiative star with $T_{eff} = T_1$ has a color B–V = C, then a star with $T_{eff} = T_1 - \epsilon$, where ϵ is small, will have B–V = $C + \Delta$. The star will appear redder, because once convection becomes important, the temperatures in the layers that contribute to the fluxes in the U and B bands decrease (Böhm-Vitense, 1970). The magnitude of the color change Δ depends on the amount of flux carried by convection, but is likely to be on the order of 0.05 to 0.10 mag. Model calculations by Relyea and Kurucz (1978) are shown in Figure 9-4. If the onset of convection is indeed abrupt, then there should be few stars with colors between C and $C + \Delta$. From an examination of field stars and members of galactic clusters, Böhm-Vitense and Canterna (1974) find evidence for a deficiency of stars in approximately the color interval expected. A similar "gap" in the distribution

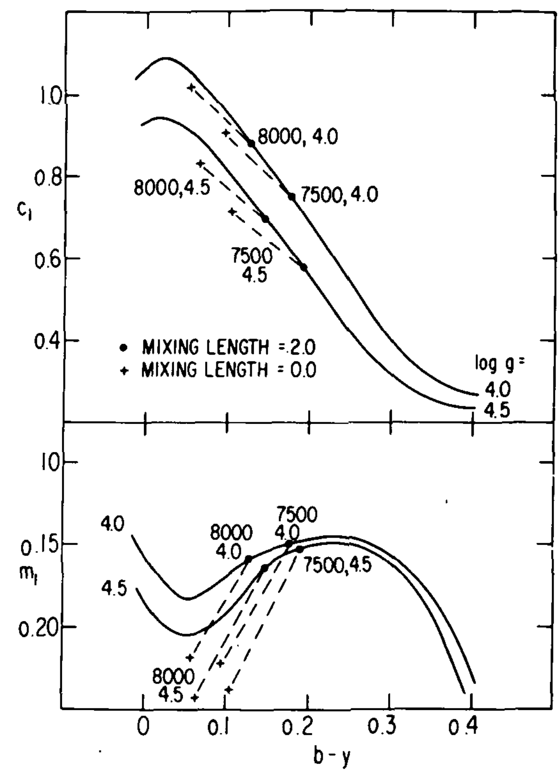

Figure 9-4. The (c_1) versus (b–y) and (m_1) versus (b–y) diagrams for models including convection with ratio of mixing length to pressure scale height of 2.0 compared to model colors for no convection (from Relyea and Kurucz, 1978).

of stars in color-color plots is seen in the much larger sample of data in the catalog prepared by Hauck and Mermilliod (1975; see diagrams in Relyea and Kurucz, 1978). Böhm-Vitense and Canterna also suggest that the location of the gap varies from cluster to cluster and that studies of its position may provide clues to the interaction of rotation and convection (see also Böhm-Vitense, 1978). The value of T_1 is typically about 7500 K.

The mixing length theory is, of course, entirely unsuited to studies of such dynamical questions as the coupling between convection and rotation. A second question of importance in A-type stars is that of the extent of the convective regions. As we have seen, the mixing length theory indicates that there should be two distinct convective layers separated by a region of stability. Early models of Am stars suggested that observed abundance anomalies were caused by diffusive separation of elements which occurred in this zone (Praderie, 1967a; Watson, 1970; Smith, 1971). Because diffusion can be effective only in extremely quiescent regions, it is essential to determine if the two unstable regions are linked by convective overshooting into the intervening radiative zone.

The structure of the He^+ convection zone in A-type stars has been analyzed by Toomre et al. (1976; see also Latour et al., 1981), who make use of the anelastic approximation (Latour et al., 1976). This approximation, which has been applied extensively in meteorological studies, consists mainly of filtering out high-frequency acoustic waves, which probably transport only a small fraction of the energy, and analyzing only the lower frequency convective modes. Toomre et al. (1976) find that the convective heat flux is two orders of magnitude greater than that predicted by the standard mixing length theory. Convection may carry up to 12 percent of the total flux of a late A-type star. The anelastic modal calculations also suggest strong penetration of convective elements into the adjacent radiative zone. This result rules out any diffusion model in which the separation of elements takes place in a stable region between the H and He^+ convection zones. Calculations by Nelson (1980) confirm the conclusion that the region between the two convection zones is well mixed. An alternative diffusion model, proposed by Vauclair et al. (1974), postulates that helium will settle out through the bottom of the second convection zone, which will then disappear, thus establishing the stability required for diffusion to occur.

ROTATION

Rotation can affect the emergent spectrum of a star in a variety of ways. In a rotating star, centrifugal forces serve to reduce the effective gravity and may, therefore, alter energy generation rates, temperature, and density in the central regions of the star. Rotation may also alter the shape of a star, with uniform rotation reducing the polar radius and increasing the equatorial radius. The corresponding distribution of temperature and luminosity over the stellar surface is nonuniform and the emergent spectrum may differ from that of a nonrotating star. Rotation may also lead to mixing within a star caused by a meridional circulation, and particularly in A-type stars such mixing—or the lack of it—may play a crucial role in determining the appearance of the observed spectrum.

A review of stellar rotation, with special emphasis on the theoretical treatment of its impact on the internal structure of stars, has been published by Moss and Smith (1982). An excellent and extremely comprehensive monograph on stellar rotation by Tassoul (1978) is also available. In this section, therefore, we will restrict our examination of stellar rotation to A-type stars, with special emphasis on the role of rotation in modifying the structure of the atmosphere and in determining the appearance of the emergent spectrum.

The conditions at the center of a rotating star are determined by the weight of the overlying layers. Since centrifugal forces act to reduce the effective gravity, the central conditions in a rotating massive star will mimic those of a nonrotating star of lower mass—lower luminosity, lower temperature, and higher pressure (the central pressure increases with decreasing mass for $M > 1.5\ M_\odot$).

For A-type stars the change in luminosity for a star rotating uniformly at the critical velocity (i.e., the velocity at which centrifugal and gravitational forces balance) is 5 to 8 percent, with the conditions at the center of the star matching those of a nonrotating star with about 2 percent less mass (Sackmann, 1970). For differentially rotating models, the situation is more complex (cf., Tassoul, 1978), but at least for massive stars the central conditions depend on the total angular momentum of a star and not on the way it is distributed through the star (Bodenheimer, 1971).

The distortions in shape and the lower luminosity that are both the consequences of rapid rotation may cause significant changes in the emergent spectrum. The dependence of surface flux on aspect can be estimated according to a result by von Zeipel (1924). In a rigidly rotating star that is in hydrostatic and radiative equilibrium, the radiative flux over an equipotential surface is proportional to the local gravity. Of course, von Zeipel also showed that stars in radiative and hydrostatic equilibrium cannot rotate rigidly (e.g., Collins, 1970), but the assumption usually made is that his result holds to a good degree of approximation, even for stars that rotate differentially. Other factors can also modify the relationship between flux and local surface gravity. For example, convective envelopes will redistribute the flux horizontally and reduce the variation of flux with g; in fact, this variation will also depend on the depth and extent of the convection zone (Lucy, 1967; Osaki, 1970; Anderson and Shu, 1977).

Models of rotating stars have been calculated by a variety of authors (Collins, 1970; Moss and Smith, 1982, and references therein). In order to calculate the emergent spectrum, it is necessary to adopt the polar radius and luminosity derived from calculations of the interior structure of a rapidly rotating star; to assume a potential field, which is usually taken to be that of a rapidly rotating Roche model; to adopt a relationship between g and flux, and von Zeipel's result is usually taken as a good approximation. The local atmospheric structure is taken to be identical to that of a nonrotating star of the same temperature and surface gravity. Integration over the stellar surface then yields line profiles, continuum fluxes, and other observable quantities.

An extensive set of model calculations, which yield monochromatic magnitudes and Stromgren colors for rotating stars of types B0–F8, has been carried out by Collins and Sonneborn (1977). Figure 9-5 shows the effect of rotation on the theoretical color magnitude diagram. *The effects of rotation are large only if rotation approaches the critical velocity.* Indeed, the absolute values of the changes in observable quantities usually depend on the square of the fractional angular velocity (i.e., the ratio of the actual to the critical angular velocity).

In general, a rapidly rotating star will have a spectrum appropriate to a star of lower temperature and surface gravity. For example, LTE calculations show that Fe I λ4476 increases in strength with increasing rotation (see Figure 9-6). The marked sensitivity of this line to rotation is caused by the strong temperature dependence of

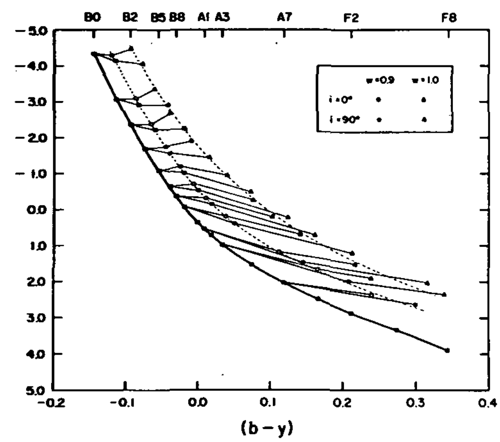

Figure 9-5. The effect of rotation on the theoretical color-magnitude diagram. Solid line, observed main sequence; squares, nonrotating models. Models of constant w, the ratio of centrifugal to gravitational forces, lie at approximately a constant distance above the main sequence. Note that the majority of rotation effects occur for values of w between 0.9 and 1.0 (from Collins and Sonnenborn, 1977).

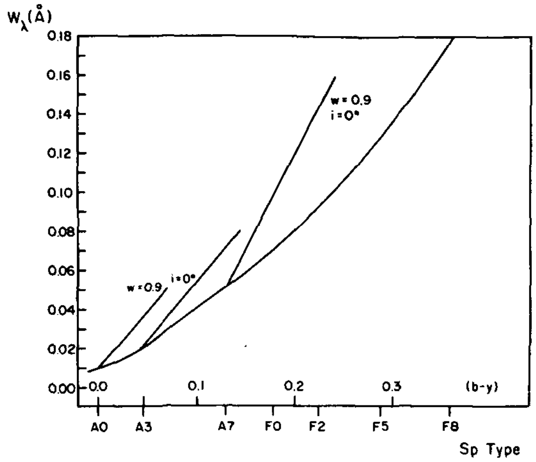

Figure 9-6. Variation of the equivalent width of Fe I λ4476 with spectral type and rotation. Only the rotational effect of pole-on models is displayed because of the weakness of the line in the rapid rotators seen equator-on with spectral type A7 and earlier (from Collins, 1974).

both the excitation and ionization of neutral iron (Collins, 1974). Because all quantities (line strengths, colors, etc.) of a rotating star change in such a way as to mimic a nonrotating star of a lower temperature, only differential effects, which generally depend on a high power of the fractional angular velocity, will lead to anomalies in the emergent spectrum. That is, to first–or even second–order one would not expect to find Fe I λ4476 to be too strong for the observed continuum colors. None of the spectral features analyzed to date yield detectable differential effects apart from the obvious Doppler broadening of the lines (Collins, 1974).

Examination of Figure 9-5 will show that the effect of rapid rotation is to mimic the changes caused by stellar evolution in the sense that the spread of the main sequence is increased and the region above the zero-age main sequence (ZAMS) becomes populated. For field stars, whose ages are generally unknown, it is therefore difficult to disentangle the effects of rotation and evolution in order to see whether the changes in luminosity and temperature predicted by models of rotating stars actually occur. In clusters, however, where all of the stars have nearly the same age and are of uniform chemical composition, such a test is possible, and a number of authors have searched for a dependence of colors and magnitudes on rotational velocities in nearby clusters. For example, in the Hyades (Baschek and Oke, 1965), Coma, and Praesepe (Strittmatter and Sargent, 1966), the Am stars, with appropriate corrections applied for line blanketing, tend to lie to the left of the main sequences defined by other cluster members. Since the Am stars are known to be abnormally slow rotators, this result is in the sense predicted by the model calculations illustrated in Figure 9-5. In the Hyades (cf., Figure 9-7) and Praesepe there is a correlation between ultraviolet color and rotation in the sense that rapidly rotating late A- and early F-type stars show an ultraviolet excess. In this temperature range the size of the Balmer discontinuity is sensitive to luminosity, and the correlation is in the sense that rapidly rotating dwarf stars have colors that look a little more like giant stars (Kraft and Wrubel, 1965; Dickens et al., 1968).

It should also be noted explicitly that because displacements in an HR diagram due to rotation

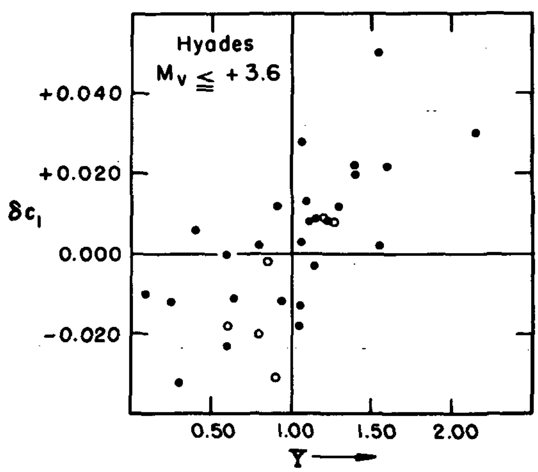

Figure 9-7. Departures δc_1 from the mean Hyades main sequence as a function of $Y = v \sin i / \langle v \sin i \rangle$. Stars of large projected rotation have $Y > 1$ and stars of small projected rotation have $Y < 1$ (from Kraft, 1970).

and evolution are similar, rotational velocity effects may make clusters appear to be older than they are.

While the models of rotating stars clearly predict the right trends in the relationships between colors, luminosities, and rotation, there remain some questions concerning the detailed comparison between theory and observations. Color magnitude diagrams are difficult to use for testing the models because the effects of rotation depend on the absolute rotational velocity rather than on the directly observable quantity $v \sin i$. Furthermore, unrecognized spectroscopic binaries will also be displaced above the cluster main sequences (Maeder and Peytremann, 1972). Differential reddening and the attendant uncertainties in determining intrinsic colors may mask correlations between rotation and colors (Dickens et al., 1968). After taking these problems into account, Maeder and Peytremann (1972) conclude that the rotational velocity effects predicted by models of uniformly rotating stars later than spectral type A7 are smaller than those actually observed. The models by Collins and Sonneborn (1977) predict larger effects for rotational velocities that exceed 90 percent of the critical velocity, but few stars seem to rotate this rapidly.

It is obvious from the above discussion that it would be very useful to know whether real stars characteristically rotate differentially or as solid bodies. Unfortunately, there are very few observational constraints. The spread in the main sequences of Praesepe and Hyades may be large enough to rule out solid body rotation for late A-type stars, but the uncertainties are large enough that the observations cannot distinguish between various laws of nonuniform rotation (Smith, 1971). For early A-type stars the observed spread is compatible with either uniform *or* strong differential rotation (Smith and Worley, 1974).

In addition to changes induced in the emergent spectra by distortions of the surface figure, rotation apparently plays another role in determining the observed characteristics of A-type stars. Abundance anomalies typical of magnetic or metallic lined stars are seen in a majority of the slowly rotating late A-type stars but not in the rapidly rotating ones. The explanation usually offered is that the abundance anomalies are confined to the outer layers of the atmosphere and that in rapid rotators mixing due to meridional circulation replenishes the atmosphere with material of normal composition, thereby diluting and masking any abundance anomalies (Baglin, 1972; Michaud, 1982).

MAGNETIC FIELDS

The fundamental questions that must be resolved by theoretical treatments of magnetism in A-type stars are two: What is the origin of the observed field? Why are fields observed in some, but not all, members of this class?

Two hypotheses have been offered to account for the presence of magnetic fields in stars of the upper main sequence. The first of these, the "fossil field" theory (Cowling, 1945), postulates that the magnetic fields are remnants of the field originally present in the interstellar medium, compressed and magnified by the star formation process (for reviews and references, see Moss and Smith, 1982; Mestel, 1976). Some magnetic flux must be dissipated in the process, since conservation of flux during the protostellar collapse would yield a configuration in which the magnetic energy would exceed the gravitational energy, and clearly under those circumstances, collapse could not occur. The Ohmic decay time for A-type stars with dipolar fields is $\sim 10^9$ to 10^{10} years, and so a fossil field can, in principle, survive long enough to be observed in main sequence objects.

One potential problem with the fossil field hypothesis is that stars with masses comparable to or less than the masses of A-type stars are expected to pass through a phase during their contraction to the main sequence in which they are fully convective. Various studies suggest that the magnetic flux will be expelled from convective regions, a kind of magnetic "brainwashing" (Mestel, 1976) that will destroy the primeval field. Some recent numerical experiments, however, suggest that magnetic flux may become concentrated into tubes until it becomes strong enough to influence and resist convection locally (Galloway and Weiss, 1981). In this way, some primeval flux may survive in a localized form, and after evolution through the fully convective phase is complete, diffuse to a more uniform configuration.

A modification of the fossil field theory is that, while the primeval field may be destroyed during the convective (Hayashi) phase of pre-main sequence evolution, fluid and perhaps highly turbulent motions may generate large-scale magnetic fields through a dynamo process. If these fields survive after the main sequence is reached, the magnetism now observed could still be considered of fossil origin in the sense that it was generated early in the evolutionary cycle. The key point is that a fossil field is one that is no longer being maintained against spontaneous decay.

In contrast to this hypothesis, there are models in which the fields are generated and maintained by a dynamo mechanism which is active at the present time (again, see Mestel, 1976; Moss and Smith, 1982, for reviews). The dynamo is in the core of the star and must be strictly steady. If this suggestion is indeed correct, then as Mestel pointed out, one might expect to see a correlation between magnetic field strength and the present value of the rotation rate. While a detailed theory is not available, a plausibility argument can be made along the following lines. Consider the case where turbulent motions and, more particularly, nonuniform rotation in the convective core generate a steadily growing magnetic field. As the field increases, it will interact with the motions generating it in the sense of limiting them until a balance is achieved and the growth of the field is stopped. The field strength should therefore be a function of the angular velocity of the star. Observations, however, suggest that the correlation, if any, is apparently in the wrong sense. Borra and Landstreet (1980) find marginally significant evidence for weaker magnetic fields among the more rapidly rotating Ap stars, although some rapid rotators, particularly the He-rich Bp stars, do have large fields. Normal A-type stars, on the average, rotate much more rapidly than the Ap stars and apparently do not have observable fields.

In summary, with respect to the Ap stars, the fossil field theory has the advantage that both the magnetic field strength and the relative orientation of the magnetic and rotation axes are essentially free parameters that are unrelated to the present angular velocity of the star. This condition may be required to account for the observed characteristics of magnetic A-type stars. While dynamo theories cannot be ruled out, a requirement of a successful dynamo model is that it be able to account for the observation that the magnetic flux is not an increasing function of angular velocity. For an extensive discussion of dynamo versus fossil field models, see Weiss et al. (1976).

The second question—why magnetic fields are observed in only some A-type stars—remains unanswered. A part of the explanation probably lies in the competition between meridional circulation and magnetic fields. If meridional circulation is dominant, then the circulation currents will tend to pull the magnetic lines of force beneath the surface of the star and the magnetic field will be unobservable (Strittmatter and Norris, 1971). An interesting problem arises for dynamo theories. If the dynamo mechanism is to be effective, rotation must be rapid enough to generate a field but slow enough that meridional circulation does not conceal it.

There are, of course, many slowly rotating nonmagnetic stars. The absence of a detectable field may reflect a condition at the time of star formation. One can imagine that stars form with a range of values of angular momentum and magnetic fluxes, and only those with the right balance of magnetic field and rotational velocity become magnetic stars. Alternatively, one might consider the possibility that all stars possess some primeval field, but suffer loss of flux caused by convection, instabilities, or whatever. The magnetic stars are then those objects that have managed to retain some of their primeval field. This hypothesis requires that the magnetic field not be strongly concentrated to the stellar interior

(Mestel et al., 1981). Since the factors that shape these processes are unknown, any speculation on such issues must remain no more than that—speculation.

A magnetic field can affect the atmosphere of a star in three ways. It can alter the structure of the atmosphere through introduction of a magnetic force term into the equation of hydrostatic equilibrium. It can affect the transfer of radiation in spectral lines, and it may influence or be an essential component of the mechanism that is responsible for generating the abundance anomalies in Ap stars. We will examine each of these effects in turn.

Schematically, the magnetic field enters the equations of atmospheric structure through the pressure:

$$P = P_g + P_B + P_R, \tag{9-1}$$

where $P_B = B^2/8\pi$ is the magnetic pressure component, P_g is the gas pressure, and P_R the radiation pressure. The equation of hydrostatic equilibrium then is of the form

$$\frac{d}{dr}(P_g) + \frac{d}{dr}(P_R) + \frac{d}{dr}\left[\frac{B^2}{8\pi}\right] = -\rho g(r), \tag{9-2}$$

where ρ is the density and g the gravity. Hubbard and Dearborn (1982) have calculated the effects of a large global magnetic field on the envelopes of 2 to 5 $M_\odot$ stars. They consider the case of toroidal flux tubes that are carried from the interior of a star to its surface either by diffusion or by buoyancy. They find that for a 1000 g toroidal field in the stellar photosphere, the radius of a 2 $M_\odot$ star, which is a typical mass for an A-type star, is 20 percent larger than in the nonmagnetic case (see also Peterson and Theys, 1981). Hubbard and Dearborn also find no discernible changes in the structure of the core or of the envelope deeper than the outer 10^{-4} of the star's mass. The energy generation and evolution of the star is, therefore, unaffected. The star will appear displaced in the HR diagram somewhat redward of the position occupied by a nonmagnetic star of the same age and mass.

A primary uncertainty in comparing these results with observations is that the strength of the toroidal field can be measured directly only through its effect on line profiles, and only very large fields can be detected in this way (see Chapter 4). In most stars, only the poloidal component of the field is measurable, and the correlation between poloidal and toroidal field strengths is unknown. There is, of course, considerable evidence that the radii of magnetic Ap stars exceed the ZAMS values, but there is controversy about whether or not the radii are too large to be attributed entirely to evolutionary effects (Preston, 1970b; Wolff, 1975; Shallis and Blackwell, 1979). It is also true that many Ap stars lie redward of the main sequence (Wolff, 1967), but the significance of this fact is unclear because the atmospheric structure, and hence the emergent spectrum, are strongly altered by the extreme line blanketing in these stars. Bonsack (1976) has found evidence that at least one Ap star (HR 2727) is overluminous for its mass. This result suggests an abnormal internal structure for Ap stars, but a simple increase in radius cannot account for the unusual properties of HR 2727.

The effect of a dipolar field, slightly distorted by additional toroidal electric currents, has been calculated by Stepien (1978). He finds that if the vertical component of the magnetic force is directed inward, the atmospheric structure differs negligibly from the nonmagnetic case. If the vertical component of the magnetic force is directed outward, the effective gravity is close to its nonmagnetic value in the photosphere but is reduced by 70 to 80 percent higher in the atmosphere. For realistic magnetic fields, the surface figure of the star departs from a spherical shape by less than 3 percent. Temperature variations over the stellar surface of 2 to 3 percent, with the stronger magnetic pole being the hotter one, will also occur due to the nonspherical shape.

Since changes in the density structure of the atmosphere of a magnetic star occur only above

$\tau = 1$, the continuum in the visible region of the spectrum is unaffected. Lines, particularly the cores of the hydrogen lines, are altered. Unfortunately, however, models of Ap stars are not yet adequate, particularly because of the difficulty of treating blanketing, to allow the use of hydrogen line profiles as a diagnostic of the expected effects. The temperature variations are likely to be masked by the much larger changes in blanketing because of a nonuniform distribution of elements over the stellar surface.

A more direct method for detecting effects of the magnetic field on atmospheric structure is the analysis of its influence on line formation through the Zeeman effect. A general review of this subject as it relates to Ap stars has been given by Hardorp et al. (1976). In principle, measurement of the polarization characteristics of individual spectral lines as a function of distance from line center can provide strong constraints on the field geometry. In practice, it is only recently that new electronic detectors, with their precise photometric characteristics, have provided observations of an accuracy sufficient to exploit this technique.

The limitations of photographic data are illustrated in the detailed study of the magnetic Ap star β CrB by Freedman (1978). Radiative transfer in a magnetic field was calculated by use of the method described by Rachkovsky (1969). The magnetic geometry and other atmospheric parameters were taken from the best existing observational results. Line profiles calculated for left and right circularly polarized light are found to be generally compatible with observations, although line blending in β CrB is so severe that it hampers detailed comparison. Partly for this reason and partly because of the limited photometric accuracy of the photographic plates used in the study, the comparison between theory and observation provides more of a consistency check on the magnetic geometry inferred previously from direct measurements of Zeeman splitting rather than an independent determination of it. The profile fitting does confirm the basic dipole nature of the field, although modest departures from a true dipole cannot be ruled out. There is no evidence that the magnetic field has drastically altered the atmospheric structure. The gravity inferred spectroscopically is in satisfying accord with that derived from the mass and radius. The microturbulent velocity is less than 1.5 km s^{-1}, and this low value may be a consequence of the stabilizing effect of the magnetic field. Magnetically split lines appear stronger than single lines, but neglect of this magnetic intensification in abundance analyses, in general, does not lead to errors in excess of a factor of 3.

Considerably more information is contained in the polarization properties of individual lines. An excellent example of the kind of detailed analysis that is now possible has been published by Borra (1980). In order to calculate line profiles, Borra divided the visible disk of the star into about 600 equal area elements dA. For a specific magnetic geometry, the local magnetic field strength and orientation were calculated for each area element dA. The transfer equations for a specific spectral line were then solved through the use of Unno's (1956) analytical solutions. Integration of the resulting Stokes parameters over the visible hemisphere of the star, with allowance for rotation, limb darkening, and line weakening, yields the polarization characteristics that should be observed through the spectral line.

This technique has been applied in detail to 78 Vir (Borra, 1980). The best fit is found for a combination of dipole plus quadrupole fields. Although the fit is excellent, the model itself is almost certainly not unique. In particular, a dipole plus quadrupole geometry and a decentered dipole geometry yield quite comparable results and cannot be distinguished. However, the observations do rule out a number of magnetic geometries, including a pure dipole.

The inferred distribution of the magnetic field over the stellar surface of 78 Vir is illustrated in Figure 9-8 and the fits to the line profiles are shown in Figure 9-9.

One disadvantage in applying this kind of analysis to 78 Vir is that the Zeeman broadening is small relative to the Doppler broadening, and the accuracy with which the magnetic geometry can be derived is correspondingly limited. Over

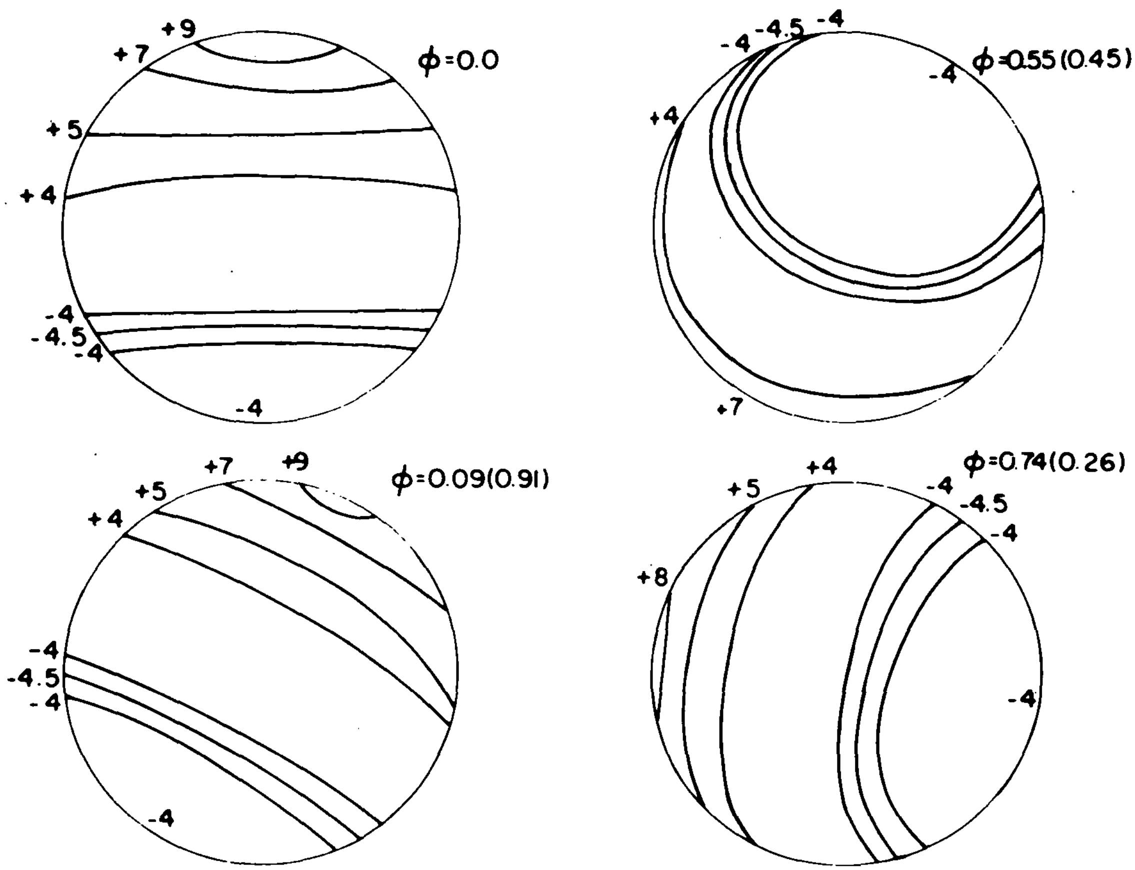

Figure 9-8. The visible disk of 78 Vir, as represented by a model that includes both magnetic dipole and quadrupole components, is shown in selected phases. The lines (labeled in units of $B_p/10$) join points of equal field strength. The plus sign indicates north polarity; the minus sign, south polarity. Therefore, the field does not go to zero between the lines labeled +4 and −4, it simply becomes transverse and stays at about $4B_p/10$ at that point (from Borra, 1980b).

the next decade, however, similar studies for additional stars should provide a much clearer picture of the large-scale characteristics of magnetic fields in Ap stars.

The evidence to date is compatible with the hypothesis that spectrum variability, which is attributed to rotation of an inhomogeneous (spotted) stellar surface, is seen only in stars with surface magnetic fields. It is reasonable to suppose that the magnetic fields are responsible for confining elements to spots and for limiting differential rotation and other large-scale motions, which would tend to smooth out any inhomogeneities. However, detailed models of the interaction of magnetic and velocity fields in a stellar atmosphere do not yet exist.

It is also possible that magnetic fields play a crucial role in whatever mechanisms are responsible for generating the observed abundance anomalies and the inhomogeneous distribution of those anomalies over the stellar surface. In the case of the accretion model, a magnetic field is essential in order to ensure selective accretion of certain elements. The magnetic lines of force also control the patterns of flow, with ionized elements spiraling down the lines of force and accumulating in the regions of the magnetic poles, thus producing a nonuniform surface distribution (Havnes and Conti, 1971).

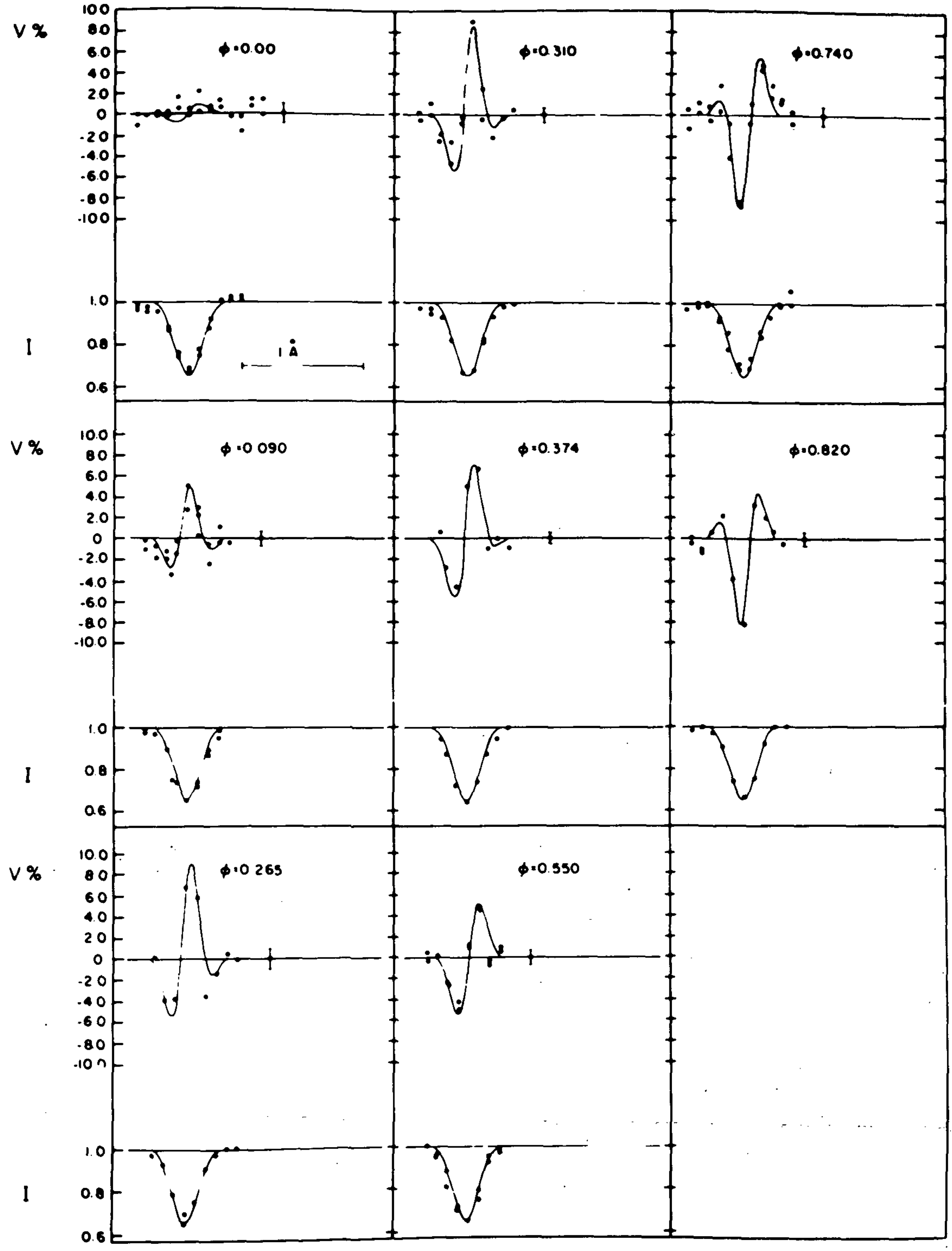

Figure 9-9. The circular polarization and line profiles observed in Fe II λ4520.2 (dots) are compared to those generated by the model illustrated in Figure 9-8 (continuous line). The observed points are spaced by 0.086 Å, and wavelength increases left to right. The error bars (± σ) show two standard deviations. Each scan is identified by the phase taken at the midpoint of observation (from Borra, 1980b).

Magnetic fields will also have an impact on radiative diffusion, and possible consequences are discussed in the next section of this chapter.

DIFFUSION

The possibility that diffusion processes might lead to a dependence of composition on depth within a star has been discussed extensively. As early as 1926, Eddington showed that diffusion cannot be important within convection zones because of the high velocities associated with convective motions. However, there are extensive regions in many types of stars where convection is thought to be unimportant and where diffusion might take place. Gravitational settling of the heavier elements in the Sun was discussed by Aller and Chapman (1960), while Greenstein et al. (1967) suggested that gravitational settling might account for the deficiency of He that is a nearly universal characteristic of B-type stars in the galactic halo. However, it is to Michaud (1970) that one must give primary credit for investigation of the role that diffusion might play in accounting for the peculiarities of main sequence A- and B-type stars.

The basic idea behind diffusion is that the density profile of the most abundant constituent (hydrogen, of course) in a stellar atmosphere is determined by the balance of gravitational and pressure forces, where the pressure term specifically includes radiation pressure. For any other (essentially trace) atomic species, radiation pressure may or may not be of precisely the right magnitude to lead to a balance of forces. Those elements experiencing an excess pressure due to either bound-bound or bound-free transitions will tend to be driven upward, while those elements for which the radiation force is weak will tend to settle downward.

The physics involved in diffusion has been discussed extensively in the literature (Michaud, 1970, and references throughout the remainder of this chapter) and will not be repeated here. However, a simplified approach, which illustrates the important ideas, has been presented by Michaud (1976). The equilibrium abundance of gas i with pressure p_i is given by the equation

$$\frac{\partial \ln p_{i,\mathrm{eq}}}{\partial r} = \frac{F_i}{kT} = \frac{-g\mathrm{Am}_\mathrm{p} + Ag_{\mathrm{rad},i}\, m_\mathrm{p}}{kT}, \tag{9-3}$$

where F is the total force on element i, m_p is the mass of a proton, g_rad is the acceleration due to radiation pressure, and A is the atomic mass number. (For the purposes of this discussion, the effects of both thermal diffusion and the electric field have been neglected. The first simplification is justified in the outer layers of a stellar atmosphere and the second is valid for heavy elements.) If T and g_rad are constant, then

$$\frac{p_{i,\mathrm{eq}}}{(p_{i,\mathrm{eq}})_0} = \exp\left[\frac{\mathrm{Am}_\mathrm{p}}{kT}\,(g_{\mathrm{rad},i} - g)\,\Delta r\right]. \tag{9-4}$$

Similar equations apply to protons and neutral hydrogen, except that g_rad is negligible. Let us define the concentration by the relation

$$c_i = \frac{p_i}{p_\mathrm{p} + p_i}. \tag{9-5}$$

In a stellar atmosphere, $p_i \ll p_\mathrm{p}$, where the subscript p refers to the protons. Since diffusion is a slow process that only gradually changes the actual concentration gradient toward its equilibrium value, it is plausible to assume that the diffusion velocity of species i relative to the protons is, to first order, proportional to the difference between the actual and equilibrium gradients, i.e.,

$$\begin{aligned} w_{ip} &= D_{ip}\left[\frac{\partial \ln c_{i,\mathrm{eq}}}{\partial r} - \frac{\partial \ln c_i}{\partial r}\right] \\ &= D_{ip}\left[-\frac{\partial \ln c_i}{\partial r} - \frac{\partial \ln p_{p,\mathrm{eq}}}{\partial r} + \frac{\mathrm{Am}_\mathrm{p}}{kT}\,(g_{\mathrm{rad},i} - g)\right]. \end{aligned} \tag{9-6}$$

The diffusion coefficient, D_{ip}, depends on temperature, T, the number density of protons, N, and the degree of ionization of the diffusing element, Z, according to the relation, $D_{ip} = 1.5 \times 10^8\ T^{5/2}/(NZ^2)$. The quantity, $g_{rad,i}$, includes terms specific to the transition considered. When $A >> 1$, then the term

$$\frac{\partial \ln p_{p,eq}}{\partial r} \tag{9-7}$$

is negligible, and

$$w_{ip} = D_{ip} \left[- \frac{\partial \ln c_i}{\partial r} + \frac{Am_p}{kT} \cdot (g_{rad,i} - g) \right] \tag{9-8}$$

When $\partial \ln c_i/\partial r = 0$ and $g_{rad} >> g$, then

$$w_{ip} = 3 \times 10^{14}/N; \tag{9-9}$$

here N is the number density of protons. At an optical depth of $\tau \sim 0.3$ in an A-type star, $w_{ip} \sim 1$ cm s^{-1}. (This estimate is valid [cf., Michaud 1976] for elements with low abundances and properly placed lines.)

From this estimate, we can see immediately one of the major problems with diffusion models. The velocities induced by radiation pressure are extremely slow. Since there are a number of other processes, such as turbulence or meridional circulation, which might result in considerably higher velocities, there is some (quite natural) skepticism about the effectiveness of diffusion in real stellar atmospheres. Unfortunately, the determination from first principles of whether or not diffusion actually occurs is beyond our present knowledge of hydrodynamics. The point of view generally adopted during the past decade has been that even in the absence of proof, it is worthwhile to explore the consequences of the hypothesis that diffusion does lead to a stratification of elements in the atmospheres of A- and B-type stars. As the following discussion will show, one can scarcely help but be impressed by the diversity of phenomena that can be explained by this assumption.

The first major result of diffusion calculations is a prediction of whether an element will appear to be overabundant or underabundant. Momentum can be transferred to an atom through either continuous or bound-bound (line) absorption processes. Michaud (1970) has estimated that the mass fraction that can be supported by absorption in a single line is $\sim 10^{-7}$ to 10^{-6}, with the limit set by the fact that the line ultimately becomes saturated. Therefore, bound-bound transitions are able to support only those elements of low cosmic abundance. For example, the rare earths, for which normal abundances correspond to mass fractions $\sim 10^{-8}$ to 10^{-9}, cannot only be supported by line transitions but can also be driven upward in a stellar atmosphere and, depending on the boundary conditions, may be concentrated in the observable layers. This concentration will result in apparent overabundances. Lines cannot support overabundances of elements with high cosmic abundances. Momentum, however, can be transferred through either photoionization or autoionization, and these processes may yield enhanced abundances of already abundant species.

One specific prediction of the diffusion model is, therefore, that only those elements with lines or ionization continua that fall in the region where stellar flux is high will appear to be overabundant. Elements with unsuitably placed lines or with ionization potentials which are too low or too high will appear to be underabundant. In A-type stars, for example, very little flux is emitted in the Lyman continuum. Therefore, those neutral elements that are naturally very abundant and whose ionization potentials substantially exceed 13.6 eV should show marked deficiencies. The elements He and Ne fall in this category, and both are typically underabundant in peculiar B-type stars. (These elements cannot be observed in A-type stars.) On the other hand, elements like Sc, Sr, Y, Zr, and the rare earths, which have numerous lines and ionization potentials in the range 10.5 to 13.6 eV, will tend to be pushed upward in the atmosphere; in fact, overabundances of such elements are observed.

In a qualitative sense, diffusion can account for, or is compatible with, a number of the other characteristics of peculiar stars along the upper main sequence (for a review, see Bonsack and Wolff, 1980).

Temperature Range. Peculiar stars in the main sequence band are restricted to spectral types B2 to F0 (Abt and Moyd 1973; Osmer and Peterson, 1974). Later than F0, surface convection is extensive, while mass loss is probably ubiquitous in stars of spectral types earlier than B2. Either convection or mass loss would be expected to mask the effects of diffusion.

Ages. Peculiar stars are observed in many clusters and associations, including the Orion and Scorpio-Centaurus groups (e.g., Abt, 1979). This fact requires that the time scale for developing apparent abundance anomalies be $\sim 10^7$ years or less. A typical time scale for developing a chemical composition gradient through diffusion is $\sim 10^4$ years (Michaud, 1970).

Rotational Velocities. In a rotating star, matter is expected to circulate from pole to equator, with the loop being closed by internal circulation in meridional planes. If the flow velocities associated with this circulation pattern become too high, then diffusion will be unable to establish or maintain an abundance gradient. One might, therefore, expect abundance anomalies to appear only in slowly rotating stars. In the nonmagnetic peculiar (Am and HgMn) stars, the maximum observed rotational velocity is $\sim$90 km s^{-1}, while in the magnetic stars, rotational velocities can be as high as $\sim$200 km s^{-1}. Presumably, the magnetic field acts as an additional stabilizing force, thereby allowing diffusion to proceed at higher rotational velocities than are possible in the absence of a field.

Evolutionary Status. Several Population II horizontal branch stars are known to exhibit line strength anomalies similar to those of Population I peculiar stars (Sargent and Searle, 1967a; Baschek and Sargent, 1976; Hartoog, 1979), including a high abundance of ^{3}He. This fact argues strongly against the hypothesis that the anomalies are related to the evolutionary history of the stars in which they are seen. Rather, it seems more likely that the anomalies depend only on the present conditions—temperature, pressure, rotation, etc.—in the stellar atmosphere. Radiative diffusion is clearly an example of a process that satisfies this condition.

Correlation of Abundance with Temperature. The line strength anomalies vary strongly with stellar effective temperature. In the middle to late B-type magnetic stars, the defining characteristic is an enhancement of lines of Si II. At slightly lower temperatures, Cr lines are often enhanced; and in the middle and late A-type magnetic stars, lines of Sr, Cr, and the rare earths are prominent. Even after allowance is made for changing excitation and ionization effects, the absolute abundances of at least some elements correlate well with temperature. For example, Bonsack and Wolff (1980) find that the abundances of the Fe-peak elements increase with increasing temperature. In principle, diffusion can explain such effects, since radiation pressure depends on temperature. In practice, few detailed calculations have been attempted.

The one element for which temperature-dependent effects have been explored in some detail is helium. Lines of He are prominent in the spectra of B-type stars and are found to be abnormally weak in peculiar stars of spectral type mid to late B. It has been known for some time that the ratio ^{3}He/^{4}He is anomalously high in some peculiar stars with early to middle B-type spectra (Sargent and Jugaku, 1961; Dworetsky, 1973). In a recent survey of He-weak stars, Hartoog and Cowley (1979) have shown that the ratio ^{3}He/(^{3}He + ^{4}He) actually correlates very closely with the stellar effective temperature in the sense that the quantity ^{3}He/(^{3}He + ^{4}He) increases from 0.3 at T_{eff} = 15,000 K to 0.7 at T_{eff} = 21,000 K. Some of the hottest peculiar stars (near spectral type B2) actually appear to exhibit overabundances of He (Osmer and Peterson, 1974).

Radiative diffusion is, of course, ineffective for He. Because of saturation effects, the radiative force can exceed gravity only for extremely low helium abundances (Vauclair G. et al., 1974); therefore, one would normally expect to observe a helium deficiency. Indeed, detailed calculations (Michaud et al., 1979) of diffusion without mass loss show that, if the stellar envelope is stable, diffusion always leads to underabundances of ^{4}He and, if enough time is available, to the replacement of ^{4}He by ^{3}He. Diffusion cannot explain the overabundances of ^{4}He seen in the He-rich stars unless stellar winds or magnetic fields are also present (see also Vauclair, 1975). If these calculations are correct, He-rich stars should be found only among the hottest of the peculiar stars and in a region in the HR diagram where significant mass loss is thought to occur. The observations are entirely compatible with this prediction. More detailed calculations of the change in helium isotopic composition with time are required to determine whether diffusion can also explain the correlation between effective temperature and the abundance ratio ^{3}He/^{4}He.

Shore (1978) has carried out preliminary estimates of the effects of a stellar wind in the presence of a strong magnetic field. He suggests that the He-weak stars of middle B-type are those in which a magnetic field suppresses mass outflow to such a degree that no concentration of helium can build up, except possibly near the magnetic pole where material can flow outward freely along field lines. In the hotter He-rich stars, the stellar wind is of course stronger, and Hα emission is actually observed in several of them. For these stars, Shore suggests that only near the magnetic equator will outflow be sufficiently inhibited so that a balance between downward diffusion of helium and mass loss can be achieved. Specifically, Shore predicts that He-weak stars should exhibit a concentration of helium near the magnetic poles, if indeed there is any enhancement at all; in the He-rich stars, an enhancement of helium should occur around the magnetic equator. Because of the question of the uniqueness of the models of element distribution over the stellar surface, Shore's predictions for the He-weak stars have not been rigorously tested. However, Landstreet and Borra (1978) do find evidence that in the helium-rich variable σ Ori E, helium is most strongly enhanced either in a ring that coincides approximately with the magnetic equator or in the two regions where the magnetic and rotational equators cross. Such configurations are in accord with the model developed by Shore.

Dichotomy Between Pulsation and Metallicism. The δ Sct and Am stars occupy the same region of the HR diagram, yet these two sets of stars appear to be completely disjoint. Classical Am stars do not pulsate, and pulsating main sequence stars do not exhibit the abundance anomalies characteristic of metallicism. This dichotomy can be explained in terms of element separation by means of radiative diffusion. In late A-type main sequence stars, there are initially two convection zones, and δ Sct-type pulsation is driven by the He$^+$ ionization zone (Chevalier, 1971). However, in the absence of mixing, helium diffuses downward out of the atmosphere, depleting the He$^+$ ionization zone until convection in this region induced by the ionization of He$^+$ is no longer possible. The disappearance of the He$^+$ convection zone both suppresses pulsation and allows elements to diffuse to the surface from deep in the atmosphere, thereby building up abundance anomalies (Baglin, 1972; G. Vauclair et al., 1974).

There are two groups of peculiar stars that do pulsate: the "marginal" Am stars, whose abundance anomalies are less pronounced than those of classical Am stars; and the δ Del stars, which are evolved stars with abundance anomalies that resemble those of the Am stars (Kurtz, 1976). Cox et al. (1979) claim that these two groups of stars can be accounted for by diffusion models that also take into account the role of turbulence (G. Vauclair et al., 1978). Their calculations show that if helium is underabundant by only a factor of 3 or so, the He$^+$ ionization zone can still drive pulsation but convection will be suppressed. Since the upward diffusion of metals occurs more rapidly than the downward diffusion of helium (Michaud et al., 1976; G. Vauclair et al., 1978), mild metallicism and pulsation may coexist.

If helium is indeed present, then microscopic diffusion is not completely efficient and some sort of partial mixing must occur. If it does then it may be possible to explain a second important difference between the δ Del and classical Am stars. In δ Del stars the abundance of Ca is normal (Kurtz, 1976), while a Ca deficiency is one of the defining characteristics of the Am stars. Calculations show that the radiative force on Ca passes through a local minimum just below the convective zone, and only a small amount of mixing with deeper layers would be adequate to restore a normal abundance of this element in the photosphere (G. Vauclair et al., 1978; Saez et al., 1981).

Precisely what factors determine whether or not a star is stable enough for diffusion to occur remain unclear. Rotation is presumably important since all peculiar stars rotate fairly slowly. However, rotation cannot be the only relevant factor. The rotational velocity distributions of normal, Am, and δ Sct stars show a considerable degree of overlap.

To date most comparisons between radiative diffusion models and observations have been of the kind described here. That is, one simply calculates the radiative and gravitational accelerations on particular ionic species and thereby determines whether the abundance of that element will increase or decrease over its normal value in the observable regions of the stellar atmosphere. Estimation of the actual abundance of any particular element is very difficult because of the large number of other physical processes that may modify diffusion (for review, see Vauclair, 1981). These processes include the following effects.

1. Turbulence. Because of overshooting, turbulence is expected in A-type stars both above and below the convection zones and may be induced by meridional circulation as well. The impact of turbulence on diffusion has been studied by G. Vauclair et al. (1978; see also Vauclair, 1981).

The total diffusion velocity is taken to include two components:

$$V_D = D_{12} \left[-\left(1 + \frac{D_T}{D_{12}}\right) \frac{1}{c} \frac{\partial c}{\partial r} + \frac{Am_p}{kT} g_{eff} \right] , \qquad (9\text{-}10)$$

where D_{12} is the molecular or microscopic diffusion coefficient of particles 2 through 1, c is the number concentration of particles 2, and D_T, the turbulent diffusion coefficient is defined by

$$V_M = -D_T \frac{1}{c} \frac{\partial c}{\partial r} , \qquad (9\text{-}11)$$

with V_M being the resultant turbulent diffusion velocity. The quantity g_{eff} is defined by the relation

$$g_{eff} = g_R - g_{GT} , \qquad (9\text{-}12)$$

where g_R is the upward acceleration due to the radiation force and g_{GT} is the downward acceleration due to gravitational and thermal forces (Michaud et al., 1976). The effect of turbulence is to reduce the equilibrium concentration gradient by $(1 + D_T/D_{12})$. If $D_T = 0$, there is no turbulence; if $D_T \to \infty$, no concentration gradient can develop.

In order to estimate quantitatively the effects of turbulence on the concentration gradient, it is necessary to know the value of D_T/D_{12}. Unfortunately, our knowledge of hydrodynamics is too limited to permit the calculation of D_T. In their calculations of turbulence plus diffusion, G. Vauclair et al. (1978) treat D_T as a free parameter and infer its value by requiring that calculated and observed abundances agree. They find from their analysis that the underabundance of Sc seen in Am stars implies that He must also be underabundant, and that the value of D_T implied by these results is compatible with a large fraction of the other observed anomalies. However, the calculated overabundances of the heavier elements are still much higher than those actually observed. Vauclair et al. suggest that either the radiation forces have been overestimated or else a stellar wind is present (see also Michaud et al., 1976).

2. Boundary Conditions. The suggestion that a (possibly selective) stellar wind might account for the discrepancy between the observed and calculated abundances of heavy elements in Am stars brings up another problem in calculating the effects of diffusion–proper treatment of the boundary conditions. In particular, a specific element will be driven upward if it experiences an excess of radiation pressure in the envelope of the star below the layers where observable lines are formed. If g_R becomes less than g_{GT} in the line-forming region, then the element can become trapped there, its concentration can build up, and an apparent overabundance will be observed. This situation will occur, for example, for elements that reach a stage of ionization in the stellar atmosphere with no lines from the ground state at wavelengths longer than λ911 (that is longward of the Lyman limit). Both Hg III and Mn III are examples (Michaud et al., 1974; Alecian and Michaud, 1981). Elements for which the radiation force remains high throughout the line-forming regions will be driven out of the star and will not exhibit anomalous abundances. The determination of which elements will be ejected from the star, and which ones will be retained by it, requires a detailed non-LTE calculation of the line formation process.

A general (nonselective) stellar wind can, if high enough, entirely prevent radiative diffusion in stellar atmospheres. Estimates indicate that abundance gradients will be completely destroyed if the wind velocity exceeds the diffusion velocity by more than a factor of 100. If the two velocities are comparable, then the abundance anomalies will differ from those that develop in the absence of a stellar wind (Vauclair, 1975; Cox et al., 1978). For stars in the mass range 1.5 $M_\odot$ to 4 $M_\odot$, mass loss at a rate of 5.0×10^{-13} to 6×10^{-14} $M_\odot$ yr^{-1} is sufficient to eliminate any change in atmospheric abundances due to diffusion (Vauclair, 1981). Such mass loss rates are well below the present threshold for direct detection of stellar winds.

A second boundary condition that may have to be taken into account is the rate of accretion from the interstellar medium. Indeed, selective accretion was postulated by Havnes and Conti (1971) as the major source of the abundance anomalies in magnetic stars. Michaud (1976), however, has argued that accretion at most can do no more than perturb slightly the effects of diffusion. If the accretion model were correct, then, given the density of the interstellar medium, the very large overabundances seen in magnetic stars could be produced within a main sequence lifetime only if the accreted material remained above optical depth $\tau \sim 0.1$ If, however, the atmosphere is sufficiently stable that the accreted matter is not mixed throughout the stellar envelope, then diffusion can also occur. Michaud argues that typical diffusion time scales are much shorter than accretion time scales and that diffusion must therefore dominate (see also Michaud, 1980).

3. Magnetic Fields. Magnetic fields can affect diffusion in a variety of ways. Ionized elements will tend to diffuse along field lines, while a horizontal field will inhibit diffusion in a vertical direction. The possible relevance of diffusion in a magnetic field as an explanation for the inhomogeneities or "spots" that are characteristic of magnetic Ap stars is obvious (e.g., Shore and Adelman, 1974; Michaud et al., 1981). Magnetic fields may also serve to trap elements that would be driven entirely out of the atmosphere of a nonmagnetic star. For example, rare earth elements are overabundant in magnetic peculiar stars, but are not seen in nonmagnetic ones. This observation may be due to the fact that, even though radiation pressure exceeds gravity for the rare earths everywhere in an A star envelope, rare earth ions can be trapped by horizontal magnetic field lines.

A second effect of magnetic fields is to split a single line into its Zeeman components. Because these lines are less saturated, radiative acceleration is increased, and the abundance that can be supported in the stellar atmosphere is increased.

The specific case of diffusion of Si has been calculated in some detail (Vauclair et al., 1979; Alecian and Vauclair, 1981; Michaud et al., 1981). The basic argument is as follows. In a nonmagnetic star, the radiation force is able to support an abundance of Si that is within a factor of 2 of the normal abundance. If a horizontal magnetic field is present, then diffusion of ionized Si is hindered by the difficulty of crossing the magnetic lines of force. Only neutral Si can move freely in the vertical direction, and because it is low in abundance (most of Si is in the form of Si II and Si III at the temperatures of 12,000 to 14,000 K considered in these calculations), its lines are not saturated, and the excess of the radiation force over gravity is large. If the Si ionizes after being driven upward, then the magnetic field lines prevent its sinking. The specific prediction is that Si should be initially overabundant only in the (magnetic) equatorial regions of magnetic stars. The reservoir of Si is, however, finite; the ions Si I–IV can be supported by radiation pressure but Si V cannot. After $\sim 10^6$ years, all of the available Si will have diffused to the surface. As time passes, the Si in the line-forming region will also diffuse horizontally to regions where the field lines are vertical. Since Si is not as well supported here, some will settle out of the atmosphere, and the apparent abundance will decrease. If these arguments are correct, then the abundance of Si and its distribution over the stellar surface will depend in a complex way both on magnetic geometry and stellar age (Michaud et al., 1981).

While this specific calculation is relevant only to the hotter A-type stars, the arguments do illustrate the interplay that must exist between diffusion, if it does occur, and magnetic fields. Detailed calculations of diffusion in the presence of a magnetic field have recently been carried out by Michaud et al. (1981), who make some quite specific, and probably testable, predictions about the relationship between the magnetic field geometry and the distribution of elements over the stellar surface (see Chapter 4).

4. **Meridional Circulation.** A rotating star is ellipsoidal in shape, with the consequence that its temperature at the poles exceeds that at the equator. Simple models show that in such a situation meridional circulation carries material from the poles toward the equator, with the loop being closed by flows within the star. The effect of laminar flows of this kind on diffusion has been estimated in an approximate way by S. Vauclair et al. (1978). They find that if the macroscopic velocity associated with meridional circulation V_M is less than the diffusion velocity V_D, then diffusive separation of elements is unimpeded. For $V_D < V_M < 100\ V_D$, the abundance anomalies are reduced; for $V_M > 100\ V_D$, no diffusive separation can develop or be maintained.

Meridional circulation, at least in the absence of a magnetic field, is necessarily unstable. If angular momentum is conserved along flow lines, then an increasingly pronounced gradient of angular velocity will develop within the star. The shear instability will ultimately lead to turbulence and mixing, which will destroy any previously established variation of composition with depth in the atmosphere. These effects have been discussed by Vauclair (1976, 1977), who suggests that Am stars can exist only when the meridional circulation patterns are stable and that Am stars are converted to δ Sct stars by mixing induced by the shear flow instability. Vauclair's estimate of the duration of the stable phase is much shorter than that required by observations of the frequency of Am stars. However, given our limited understanding of the flows within rotating stars, and of the changes in flow patterns with time, this discrepancy is not too surprising.

It is clear from this discussion that an understanding of the hydrodynamics of rotating gaseous ellipsoids is yet another prerequisite for a complete model of radiative diffusion and its effects on the emergent spectra of stars.

An important theoretical discussion of this problem has recently been published by Tassoul and Tassoul (1982), who have treated the hydrodynamical problem of meridional circulation in the radiative zone of stars without magnetic fields. Earlier calculations had indicated that the circulation velocity should vary as ρ^{-1}, and velocities of tens to hundreds of meters per second would occur in the outer layers of stars. Indeed, a major argument against diffusive separation of elements has been the apparent inability of this process to compete with the mixing induced by meridional circulation. After properly taking into account the outer boundary conditions, Tassoul and Tassoul find that the circulation velocities do not show the ρ^{-1} dependence, but remain uniformly small. The rotational motion is turbulent rather than laminar. Using this new model of meridional circulation, Michaud (1982) finds that the He II convection zone can disappear, and chemical separation of elements can occur, only in stars in which the equatorial rotational velocity is less than 90 km s^{-1}. As Michaud comments, this limiting velocity is embarrassingly close to the limiting velocity actually observed for HgMn and Am stars.

5. **Instabilities.** An instability that may affect diffusion has been identified by Lin (1979) through solution of the equations governing a coupled system of radiation plus fluid motions. In Am stars, which have extensive surface convection zones, diffusion takes place below the convection zone and the instability proposed by Lin does not come into play. In HgMn stars, however, there is no surface convection zone and static models indicate that those elements that are driven upward by radiation pressure are concentrated in thin layers at shallow optical depths. Lin's calculations show that such thin layers are unstable. Schematically, what happens is that photons continue to exert pressure on the layer and seek their way through by pushing aside the overabundant element, which he refers to as an impurity. The impurity is concentrated into drops, which must fall because their optical thickness exceeds that of the original layer, and they cannot be supported by radiation pressure. As the drops fall, the impurity will diffuse out of drops and be swept upward by the radiation between the drops; thus, the entire process is a dynamic one. Radiation pressure, which is responsible for concentrating the impurity in a thin layer, ultimately breaks up that layer into blobs. The blobs then sink and evaporate, with the evaporated impurity being driven upward, perhaps to form a new layer. Additional calculations are required to show whether the layer forms intermittently or whether some kind of stationary circulation pattern is established.

6. **Other Effects.** Calculation of the apparent abundance excesses or deficiencies caused by radiative diffusion requires a detailed treatment of both atmospheric structure and line formation processes, with non-LTE effects explicitly included. Whether or not an element that is forced upward by radiation pressure in the stellar envelope will remain bound to the star depends on the ionization processes, and hence on temperature and density, in regions where the optical depth in the continuum is as small as $\log \tau_{5000} \sim -6$. Even in deeper layers of the atmosphere, radiative forces may be affected by departures from LTE of the level populations and by departure of the source function from the Planck function. Overabundances of metals in the line-forming regions increase the opacity due to lines and suppress the ultraviolet flux, thereby altering both the temperature structure of the atmosphere and the radiative forces (Jamar et al., 1978).

The obvious impossibility of incorporating all of these effects in a realistic way into stellar models would seem to render futile any attempt to compare calculations of the effects of radiative diffusion with observed abundance anomalies. Nevertheless, such comparisons have been carried out for several specific elements, and the model calculations agree remarkably well with observed abundances. Most notable are the discussions of boron (Borsenberger et al., 1979), calcium and strontium (Borsenberger et al., 1981), and Mn (Alecian and Michaud, 1981).

Calculations of the abundance anomalies produced by radiative diffusion require a determination of the radiative force throughout the stellar envelope and must include specifically the outer portions of the atmosphere where the density is so low and collisions so infrequent that thermodynamic equilibrium cannot be maintained. Non-LTE effects on both the atmospheric structure and line formation processes must be taken into consideration. In the models to date, magnetic fields and various sources of turbulence that might reduce the effects of microscopic diffusion have been neglected, although Borsenberger et al. (1979) have concluded that an overabundance of B can occur only if a magnetic field is present.

A comparison of predicted and observed abundances of Mn is shown in Figure 9-10. The maximum abundance of Mn that can be supported by radiation pressure follows closely the upper envelope of observed Mn abundances. Any form of partial mixing would, of course, serve to reduce the overabundance of Mn below that calculated for these simple models, which include no mixing. The agreement between theory and observation is particularly impressive inasmuch as there are no adjustable free parameters in the theory.

Most of the observational attempts to refute the significance of diffusion in producing abundance anomalies in A-type stars have involved the search, either directly or by inference, for mass motions in the atmospheres of peculiar stars. Large-scale turbulence would, of course, lead to mixing and prevent the diffusive separation of elements.

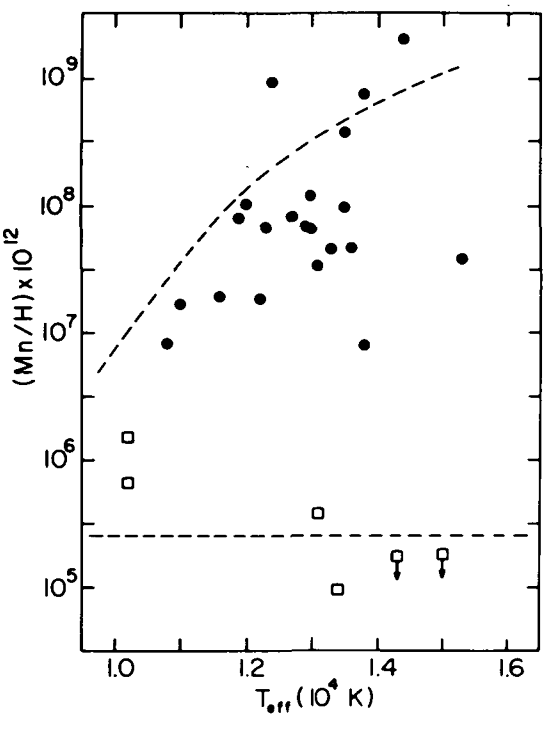

Figure 9-10. Abundances of Mn in 21 HgMn stars. The dashed line represents the maximum amount that can be supported in a Mn cloud. The filled circles represent HgMn stars while open squares represent normal stars. Arrows denote upper limits. The radiative acceleration can support the largest overabundances of manganese observed and can account for their variation with effective temperature (from Alecian and Michaud, 1981).

For example, from a study of high-resolution lines profiles of three HgMn stars, Smith and Parsons (1976) found evidence that the line profiles in ι CrB, and possibly HR 4072 and χ Lup as well, were asymmetric in the sense that there seemed to be weak absorption features displaced approximately 8.6 km s^{-1} shortward of the main lines. Smith and Parsons argued that these asymmetries were evidence for material flowing outward at approximately the speed of sound and that it would be difficult to reconcile such mass motions with the diffusion hypothesis. They did point out a number of difficulties with their interpretation of the data. Apart from the asymmetries, Fourier analysis of the line profiles indicated that macroturbulence was negligible; no evidence of downward flow was seen; and there is no known mechanism for driving the flow. More recently, Dworetsky (1980) has shown quite convincingly that ι CrB is a close spectroscopic binary and that the blue displaced absorption features seen by Smith and Parsons in that star are actually due to the companion.

Ironically, theoretical calculations completed before the discovery of the binary nature of ι CrB show that line asymmetries should actually occur in the lines of some elements if diffusion is the cause of the abundance anomalies (Michaud, 1978). Specifically, diffusion models predict that some elements should lie in thin layers above the photosphere ($\tau \sim 10^{-5}$). The velocity distribution for such elements is nonisotropic and non–Maxwellian, and the resulting line asymmetries will mimic those caused by a stellar wind. Asymmetries do not necessarily therefore imply that the atmosphere is unstable. No line asymmetries are predicted for elements like helium and oxygen that are not supported by radiation pressure. Unfortunately, there are no observations suitable for testing these predictions about line shapes.

Rapid rotation may be present in some Am stars. Abt (1979) has reported that there are four

Am stars in the Orion Association with v $\geq$ 200 km s^{-1}. If these stars are indeed Am stars, then diffusion cannot account for their abundance anomalies. Meridional circulation would surely dominate diffusion in such rapidly rotating stars. Abt, therefore, argues that slow rotation is *not* a necessary condition for the appearance of Am stars. Rather, he suggests that the Am phenomenon is present as a consequence of some other stellar property–probably membership in a close binary system–and that the low rotational velocity which is characteristic of most Am stars develops because of tidal interactions during the main sequence phase of evolution. These observations by Abt are in conflict with the conventional wisdom that slow rotation and metallic line characteristics are closely related.

In any stars with v sin $i \geq 200$ km s^{-1}, precise characterization of the abundance anomalies is very difficult. Bonsack and Wolff (1980) have pointed out that two of the four stars discussed by Abt are much earlier in spectral type than classical Am stars and, in fact, lie in a temperature region where spectral classification criteria are fairly insensitive to Am characteristics. Photometric data, specifically m_1 indices (Warren and Hesser, 1977), do not support the Am classification. Michaud (1980) has suggested that perhaps these four stars possess strong magnetic fields that serve to stabilize the atmosphere against meridional circulation, hence accounting for the spectral anomalies seen by Abt. If this hypothesis is true, then as rotation slows down due to magnetic braking, the abundance anomalies should come to resemble those of magnetic peculiar stars rather than metallic line stars. Because of the importance of these stars to our overall understanding of the relationship between spectral peculiarities and rotation, and of the possible role of diffusion in A-type stars, it is essential that additional observations be made at high resolution to characterize their spectra and to search for magnetic fields and radial velocity variations.

Many stars of spectral type A are X-ray sources with fluxes typically in the range 10^{27} to 10^{30} ergs s^{-1}. Some stars with spectral anomalies are apparently included among those A-type stars that emit X-rays, although most peculiar stars, magnetic stars included, are definitely not X-ray sources at the present levels of detectability. In the case of the Am stars, there is some evidence that the X-ray emission is associated with the secondary and does not come from the Am star itself (Cash and Snow, 1982). Cash et al. (1979) have argued that X-ray emission and diffusion are incompatible, and the fact that at least some peculiar stars are X-ray sources rules out diffusion as the source of the abundance anomalies. The suggestion by Cash et al. is that the X-rays are caused by either a hot corona or (less likely) flare activity and that either is most likely due to heating as a consequence of mass motions involving a substantial amount of energy. Extensive mass motions, of course, would mask completely the effects of diffusion. The mechanism of X-ray production in A-type stars is, however, not at all clear. The X-ray luminosity in A-type stars correlates well with the bolometric luminosity and not with the rotational velocity, while the reverse is true in late-type stars where dynamo-generated fields in stars with extensive convective zones are thought to play a dominant role in coronal heating (Pallavicini et al., 1981). Until the heating mechanism for early-type stars is better understood, any inference about the link between X-ray emission and photospheric and subphotospheric velocity fields in A-type stars is premature.

Michaud (1980) has examined the observational data from precisely the opposite point of view and finds evidence that separation of elements must occur. In the case of the solar wind and solar cosmic rays, the abundances of the isotopes of He and heavy elements ($Z \geq 6$) do not match the photospheric abundances and are, furthermore, not constant with time. Michaud argues that it is more plausible to attribute these abundance anomalies to chemical differentiation, either due to the acceleration mechanism or diffusion or both, rather than to nuclear processing. He stresses that this chemical differentiation occurs despite large turbulence in the solar photosphere and despite the existence of a corona, which implies a substantial source of nonradiative heating.

Michaud offers a second argument that relates more directly to the A-type stars. In magnetic Ap stars, the variation of line strengths with time is attributed to the rotation of a star with an inhomogeneous surface. That is, there are concentrations or spots of particular elements, and their changing visibility as the star rotates produces the observed spectral variations. Michaud shows that the requirement that these spots survive during an appreciable fraction of the stellar lifetime, and not dissipate due to horizontal (turbulent) diffusion, implies that the star must also be stable enough so that microscopic diffusion can occur in the vertical direction.

From this discussion, it is evident that the role of radiative diffusion remains a controversial issue. Many researchers remain quite skeptical that such a fragile process can actually be important in stellar atmospheres. It is also true that in other parts of the HR diagram, mixing seems to be a *more* efficient process than theoretical models suggest. Nevertheless, diffusion is the only mechanism identified so far that can provide a coherent explanation of the various types of peculiar stars that are found along the upper main sequence. Definitive tests of the diffusion model may shortly be possible, since quite definite predictions are now being made about the precise abundance anomalies that should be seen and about the relationship between magnetic geometries and the surface distribution of elements (see Chapter 4). Whatever the outcome of these tests, it is quite clear that the detailed modeling of hydrodynamical effects, which must be taken explicitly into account in order to calculate the consequences of diffusion, have greatly advanced our understanding of dynamics of stellar atmospheres. From both an observational and theoretical standpoint, the exploration of the diffusion model and its consequences has proven to be an extremely fruitful line of research.

SUMMARY

As this chapter demonstrates, the A-type stars pose unique problems for anyone who attempts to construct model atmospheres for them. In their photospheres, hydrogen is neither completely ionized, as it is in the hot stars, nor completely neutral, as it is in the cool stars. There are, in general, two convection zones for stars in the temperature range 7500 K $< T_{eff} <$ 12,000 K. Electrons in the atmosphere are produced by both hydrogen and the metals. Continuous opacity sources include not only hydrogen and helium, as in the hot stars, but also C, Si, Mg, Al, and H^-, as in the cool stars. Modeling therefore requires specification of the abundances of these elements, as well as a treatment of non-LTE effects for each. Autoionization features may contribute to the opacity. The hydrogen lines and the Balmer discontinuity are at their maximum strength. The A-type stars are often casually treated as if they resemble either hotter or cooler stars. It is precisely because the A-type stars are unique that such an approach is both misleading and uninformative. As Francoise Praderie has commented, the A-type stars "have something special, they are not banal."

The usual schemes for classifying stars are two-dimensional, with the two relevant parameters being, either implicitly or explicitly, T_{eff} and log g. The implication is that appropriate atmospheric models can also be described as a two-parameter family. Pecker (1973) has commented that

> "This general idea has been, for years, of a great help; nowadays it looks more like a severe and artificial limitation, which has been often acting as a brake against any progress in the interpretation of stellar spectra."

Foreshadowing subsequent work by himself, Thomas, and others, Pecker goes on to argue that a stellar atmosphere should be regarded as a transition region between the interior of a star and the interstellar medium. The stellar interior is a blackbody in thermal equilibrium; temperature determines every other property of the stellar interior except the density. In contrast, the interstellar medium is characterized by a great many degrees of freedom. Parameters other than the temperature and the radiation field are required to describe the state of the medium.

The transition between these two regimes is gradual. In what Pecker terms the lower photosphere radiation escapes, so the Planck law is no longer valid. So long as collisions dominate, however, the populations of various atomic states are determined by temperature and density. At higher levels in the atmosphere, the Boltzmann and Saha equations begin to break down. At still higher levels, even the assumption of radiative equilibrium is no longer valid, and various sources of nonradiative heating begin to play a role. With this picture, a two-dimensional classification is no longer adequate. Mechanical energy fluxes, magnetic fields, and a host of other factors will influence atmospheric structure. Indeed, every star may be unique! Pecker suggests that the two-dimensional classification schemes have been successful only because abnormalities are, for the most part, restricted to the outer nonphotospheric layers of stellar atmospheres, and the visible spectrum is not affected. An example of this phenomenon is the range of X-ray emission seen in A-type stars with similar values of T_{eff} and $\log g$.

For the A-type stars, however, two-dimensional classification schemes fail to describe adequately even the photospheric spectrum. The models described in this chapter attempt to evaluate the importance of a variety of other factors, including magnetic fields, rotation, diffusion, etc., in shaping the emergent spectra. These models are primarily useful for delineating the magnitude of the effects that are likely to be seen. The significance of the A-type stars is that, because the photospheric spectra depart so strongly from the predictions of LTE models, we have an opportunity to determine empirically the importance of various hydrodynamical and hydromagnetic processes.

After reading an early draft of this chapter, Dick Thomas (private communication, slightly paraphrased) summarized it as follows:

1. The only "mathematically-computable" models are LTE-blanketed ones like those of Kurucz. We know that they are logically and thermodynamically incomplete, but they are all we have and they reproduce some spectral features.

2. The "standard" extension of such models is to add chromospheres and coronae, which are [known to be] now universal across the HR diagram. What varies from star to star is the nonradiative heating mechanism. Because there is no solid evidence for what is in the chromospheres of A-type stars (vague things on Mg II in a few stars); and because the evidence of mass-fluxes is mainly in supergiants; and because the coronal evidence comes from [still incomplete] X-ray data, this [manuscript] prefers not to discuss what chromospheres-coronae are like in A-type stars.

3. The nonthermal modifications of standard Kurucz-type photospheres are diffusion and magnetic effects. The former is *one* possible way to get the apparent difference in abundances. Therefore, although there is really no physical reason to think such diffusion can exist, we adopt it. We have yet no models that treat the effects of magnetic fields in perturbing Kurucz-type atmospheres or in producing nonradiative fluxes to heat a chromosphere.

This summary is obviously very pessimistic—but perhaps realistic—about our level of understanding of the atmospheres of A-type stars. Readers can judge for themselves the validity of this point of view. It is certainly true that the study of A-type atmospheres is in its infancy. Because the A-type stars occupy a crucial transition region in the HR diagram between hot stars and cool ones—a region where convection begins to become important, where the observational diagnostics of chromospheric-like regions change from absorption to emission lines, where X-ray emission is a sometime thing, and where, in supergiants, mass loss changes from high to low velocity flows—a better understanding of these stars is likely to modify models of stellar atmospheres throughout the HR diagram.

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10

OUTSTANDING PROBLEMS

OVERVIEW

The study of stellar atmospheres has been a major focus of astrophysics throughout this century. As a consequence, we think we can now determine the basic properties of a star—effective temperature, luminosity, and composition—with a considerable degree of confidence. The present evolutionary status, past history, and future development of most stars are also thought to be well understood. To a greater degree than is perhaps desirable, this certainty about the correctness of our present views has permeated the earlier chapters of this book.

A word of caution is therefore in order. We must never lose sight of the fact that our deductions concerning the physical properties of stars depend almost entirely on the use of classical thermal models to interpret observable characteristics. These models assume that the atmosphere is chemically homogeneous and that its structure can be determined from the equations of hydrostatic and radiative equilibrium, with appropriate modifications in those cases where a significant fraction of the flux is carried by convection. But is this approach adequate? Or do departures from such thermal models, small as they may be, play a key role in shaping atmospheric structure?

The apparent success of the classical models is due to several factors. They offer, without doubt, a reasonable first approximation to the photospheric structure of real stars. The temperatures, luminosities, and even the abundances inferred by means of model atmospheres are, to first order, almost certainly correct. Until very recently, however, our view of the universe has been exceedingly myopic. For most stars, the wavelength regions that can be observed from the ground simply do not provide the clearest evidence for, or the most useful diagnostics of, nonthermal phenomena. New observations, particularly those made from spacecraft, now demand that we move beyond the first order approximations of classical thermal models.

Variations of any kind constitute prima facie evidence of the inadequacy of classical thermal models, which assume that atmospheres are homogeneous and in a steady state. Another factor in the long-standing failure to develop nonthermal modeling techniques is the difficulty of establishing the nature, or even the reality, of low-amplitude variability. Photoelectric techniques have been used to measure luminosity changes with a precision of 1 to 3 percent for much of this century. In the specific case of the A-type stars, however, brightness variations are typically of a similar order of magnitude. Until recently, therefore, photometric data have been adequate only for establishing the fact of variability but have been inadequate to characterize that variability in detail.

The study of spectrum variability is even more difficult. So long as observations of line profiles had to be made photographically, their precision was necessarily limited. Astronomers were—and remain—justifiably skeptical of many reports of changes in line shapes and intensities. Even in cases where variations were clearly established, a satisfactory explanation of them in terms of existing models usually proved impossible. Therefore, many researchers have given low priority to studies of variability.

New observational techniques are beginning to alter profoundly our restricted view of stellar atmospheres. It has become quite clear that in no case can the emergent spectrum of a star be represented in its entirety by classical thermal models. New techniques must, and are, being developed to try to incorporate such phenomena as velocity fields, magnetic fields, rotation, mass loss, and nonradiative heating in calculations of the structure of stellar atmospheres.

Compelling evidence of the significance of such factors has come from satellite observations in the ultraviolet ($\lambda < 3000$) and X-ray regions of the spectrum. Ultraviolet lines provide excellent diagnostics of chromospheric activity and mass loss, while X-ray emission is an indicator that regions with coronal-like temperatures must be present.

New ground-based data are also leading to an increased interest in nonthermal phenomena. Advances in detector systems have revolutionized our ability to make spectrophotometric measurements. Photon counting devices, such as the Reticon (Vogt et al., 1978) and the intensified Reticon systems (Shectman and Hiltner, 1976) and the multianode microchannel array detector (Timothy et al., 1982), offer simultaneously greater sensitivity and higher precision. Accurate measurements of line profiles can be used to test model atmospheres with increased rigor. Quite subtle changes in line strengths and shapes can also be measured with confidence. Even small variations may have important implications for the atmosphere as a whole. In particular, they may provide the only direct observational evidence for changes in velocity fields, which may be a controlling factor in determining mass flows and rates of nonradiative heating.

The A-type stars occupy a region of the HR diagram where the hallmarks most diagnostic of nonthermal phenomena in both hotter and cooler stars are, for the most part, absent. Emission lines indicative of high-temperature, chromospheric-like regions have not been detected in either supergiants or main sequence stars. There is evidence for mass loss only in supergiant stars, but not in A-type dwarfs. However, nonthermal phenomena may manifest themselves in a variety of other ways, and the inadequacies of classical thermal models become apparent in a detailed analysis even of Vega, the prototypical "normal" A-type star. The metallic lines in this star are rotationally broadened ($v \sin i = \sim 20$ km s^{-1}; Milliard et al., 1977; Gray, 1980a); the Doppler broadening velocity of 2.5 km s^{-1} that is required to fit the curves of growth for Fe I, Fe II, and Ti II exceeds the value (< 2 km s^{-1}) expected for thermal broadening alone and implies that some form of microturbulence is present (Dreiling and Bell, 1980); an infrared excess may have been detected (Morrison and Simon, 1973), and there is evidence for variable X-ray emission (Vaiana et al., 1981). There are also some inconsistencies between the line-blanketed LTE models and the observations. The abundances inferred from lines of Fe I and Fe II are in disagreement, although it is unclear whether the problem is caused by inadequacies in the models or in the *gf* values. The abundances derived from Fe II and Ti II are lower than solar values by about a factor of 2, and it would be interesting to know whether the inferred abundances are correct or if they somehow reflect the limitations of the models. An obvious next step is to see how well the models represent line shapes, as well as line strengths.

Despite these problems, we can, in fact, feel considerable satisfaction at how well the emergent spectrum of Vega can be represented by line-blanketed LTE models combined with some ad hoc assumptions about photospheric mass motions. The features that are not well represented, such as velocity fields and X-ray emission, are often regarded as relatively minor perturbations of an atmosphere that is well understood. Whatever satisfaction we do feel, however, must surely be tempered when we examine the peculiar A-type stars. Many—and perhaps most—A-type stars exhibit spectra that are remarkable in their abnormalities. In the context of classical thermal modeling, the observed line strengths correspond to extraordinary abundance anomalies that have no counterpart among stars later than spectral-type

F0 or earlier than B2. It is argued in earlier chapters of this book that the apparent abundance anomalies are real, and that they are characteristic only of the stellar photosphere and not of the star as a whole. Furthermore, the line strength anomalies are correlated, at least in a statistical sense, with rotation, magnetic field, and pulsation. The observations, therefore, present clear evidence that nonthermal processes are a dominant factor in determining the emergent spectra of peculiar A-type stars.

The variety of phenomena that apparently have a significant effect on what we see, and that should therefore be included in nonthermal models, is overwhelming. Model building is likely to prove successful only if we first focus on establishing observationally what factors are most important and how they interact with one another. Much of the present book has been concerned with precisely these issues. It is clear, however, that new instruments for both ground-based and space astronomy, which will become available during the next decade, will allow a much more aggressive attack on specific problems. It therefore seems worthwhile to review some areas where additional work is necessary to resolve existing conflicts between theory and observations, or where new data are likely to lead to marked improvements in current models.

UNSOLVED PROBLEMS–SUPERGIANTS

The study of A supergiants received new impetus with the discovery that mass loss is occurring in at least the brightest members of this class. However, despite the substantial recent progress described in Chapter 7, there remains a number of quite significant unanswered questions concerning the evolutionary status and atmospheric structure of these stars.

In particular, there was little discussion in Chapter 7 of the evolution of A supergiants, and no attempt was made to compare their positions in the HR diagram with theoretical evolutionary tracks. This omission was intentional, because very little can be stated with certainty about either. Evolutionary tracks for massive stars, including mass loss, indicate that core hydrogen burning terminates at log $T_{\text{eff}} = 4.4$ or higher. Since this temperature exceeds that of A supergiants by a wide margin, it has usually been assumed that these stars are well past the main sequence phase of their evolution and are in the phase of core helium burning (e.g., Brunish and Truran, 1982). This conventional picture has been challenged on observational grounds by Meylan and Maeder (1982). From a comparison of theoretical evolutionary tracks with observations of supergiants in young galactic clusters, they find that 40 percent of the supergiants fall outside the calculated main sequence band. On the basis of model lifetimes, the expected number is 8 percent. Meylan and Maeder argue that the main sequence phase may extend as far as spectral type A.

This conclusion conforms to earlier suggestions made by several other authors that the theoretical main sequence band is too narrow (see Chapter 7). Mass loss alone is insufficient to broaden the calculated main sequence by the necessary amount. Meylan and Maeder suggest that additional mixing due to shear flow instabilities, convective overshoot, or meridional circulation may increase the width of the main sequence band. Alternatively, strong winds may increase the stellar radius, lower T_{eff}, and contribute to an apparent broadening of the main sequence. Therefore, the first unanswered question concerning A supergiants is a fundamental one. What phase of evolution are they in?

All supergiants probably vary in luminosity and radial velocity. The amplitudes decrease with decreasing luminosity and in many supergiants are of the same order as the observational error. For this reason, and because the variations seem to be only semiregular at best, the nature of the variability is not well defined. The questions that might be answered by extensive study are many. What is the cause of variability? What can variations tell us about atmospheric velocity fields? Is there evidence for stable frequencies of variation that can be associated with nonradial pulsation? If so, what can we infer about the atmospheric structure from the pulsations? What portion of the observed line broadening is caused by nonradial pulsations? Is there any evidence for coupling between changes in the photospheric velocity fields and mass flows as measured from Hα emission, ultraviolet absorption features, and infrared excesses?

The study of α Cyg by Lucy (1976) stands as a model of what can be achieved through observation and analysis of the variations of supergiants. The penultimate paragraph of his paper is worth repeating:

> . . . If α Cyg is typical of the early- and intermediate-type supergiants, then each of these highly luminous stars is such that the diligent application of a 50-year-old technology is capable of yielding a set of accurate pulsation periods whose number exceeds the number of parameters governing its structure. Such sets of periods would therefore provide an extraordinarily stringent test of stellar evolution theory. Moreover, since models of these stars are not homologous under variations of such parameters as mass, luminosity, and mean molecular weight, a fit to an observed set of periods will require not only that stellar evolution theory be correct but also that these parameters have been correctly chosen. There is, therefore, the possibility of using such periods for the theoretical determination of the distances, masses, and compositions of these supergiants.

Of course, a program of the kind required is impossible given modern-day pressures on telescope time. Nevertheless, it is becoming clear that many astrophysical problems can be solved only through long-term synoptic observations. A way should be found to support at least a few of the more important programs of this kind.

A number of diagnostics are available for estimating mass flows and mass loss rates in A supergiants. Unfortunately, detailed measurements of Hα and ultraviolet line profiles, radio and infrared excesses are available only for the brightest A supergiant—α Cyg. Those measurements are not contemporaneous, and the magnitude of the infrared excess is in dispute. With the advent of Space Telescope, it will be possible to extend measurements of stellar winds to a much larger sample of A supergiants, to map out the dependence of the winds on temperature and luminosity, and to search for temporal variations in the flow. Sophisticated mathematical techniques are available for modeling the winds of A supergiants, which have much lower flow speeds than winds in stars of earlier spectral type. For α Cyg itself, models of different spectral features yield incompatible mass loss rates. Future work should attempt to develop a consistent model that can represent all the mass loss indicators. Such work should obviously be extended to include a representative sample of A supergiants.

UNSOLVED PROBLEMS—MAIN SEQUENCE STARS

The basic challenge posed by the main sequence A-type stars is to determine why some exhibit extraordinary line-strength anomalies while others appear blandly normal. There are a variety of quite feasible observations that should help to clarify this issue.

Of crucial importance is a clearer definition of the temperature and luminosity range in which chemically peculiar stars can be found. For many years, the emphasis was on the study of peculiar A-type stars. It is now apparent, however, that peculiar B-type stars are also common (see Chapter 8). The line strength anomalies are more conspicuous in the A-type stars, but this fact may be due to the difference in temperatures. The higher ionization that characterizes B-type atmospheres would render the lines of singly ionized Fe-peak and rare earth elements, which dominate the visible spectrum of A-type stars, relatively inconspicuous. This statement would be true even if the compositions of B-type peculiar stars were identical to those of the cooler peculiar A-type stars. A prerequisite to a better understanding of chemically peculiar stars in general is a clearer definition of the properties of peculiar B-type stars. What are the characteristic line-strength anomalies that can be used to identify them? How do the anomalies change as a function of effective temperature? In the A-type stars, the characteristic anomalies differ for stars with and without magnetic fields. Is the same true for B-type peculiar stars? How does the frequency of peculiar stars vary with effective temperature? With rotation? With age?

Schematically, at least, stars with conspicuous peculiarities seem to be found only in regions of the HR diagram where large-scale mass motions have not been directly measured (and are not expected on theoretical grounds). Mass loss seems to terminate the sequence of peculiar stars at (approximately) spectral type B2. The cool boundary of the sequence seems to be determined by the onset of convection. Studies of stars in the boundary zones may provide some additional constraints on models for the origin of spectral anomalies. For example, if radiative diffusion is indeed effective in peculiar B- and A-type stars, then it should also produce abundance anomalies in stars cooler than the low temperature boundary as it is now defined for classical Am stars. The anomalies for the metals in cooler stars are likely to be small, but might be detected through detailed spectroscopic analyses. Diffusion may, however, be a major factor in determining the abundances of lithium, beryllium, and boron. Because these three light elements are destroyed by nuclear reactions fairly close to the stellar photosphere, determinations of their relative abundances can provide crucial constraints for models of macroscopic transport as a function of depth within stars (Vauclair et al., 1978).

In addition to providing a laboratory in which to study hydrodynamical processes, the chemically peculiar stars are one of only a few kinds of astronomical objects in which we can study directly the interaction of plasmas with strong magnetic fields. Interactions of this kind undoubtedly play a fundamental role in a variety of astrophysical phenomena, ranging from stellar activity cycles to the ejection of material from nuclei of active galaxies. In the chemically peculiar stars, however, we can measure both the strength of the field and its impact on mass flows. Diffusion in a magnetic field may produce surface inhomogeneities of the kind that typify Ap stars. Measurements of line profiles and magnetic field strengths as a function of phase in Ap and Bp stars would afford a rigorous test of diffusion models. The model predictions also depend on the role of turbulence in the atmosphere, and observations offer a way of studying its significance (Michaud et al., 1981).

Potentially even more interesting may be the study of mass loss in magnetic stars. In early Bp stars, both Hα and ultraviolet line profiles provide clear evidence for mass loss. The strength of the magnetic field is also directly measurable in these stars, and the strength of the wind and the magnetic intensity vary in phase. The wind itself is, therefore, obviously not spherically symmetric (see Chapter 8). Studies of the variations in such stars can be used to test our models of mass outflow in the presence of a strong magnetic field.

Systematic changes in the characteristics of peculiar stars as they age would, if they occur, provide important constraints on models for the origin of the spectral anomalies, for loss of angular momentum, and for the decay of stellar magnetic fields. There is evidence that significant correlations between age and the other properties of peculiar stars do indeed exist, and this evidence has been discussed in several earlier chapters of this book. The results are intriguing but ambiguous. In particular, there are few known examples of very young peculiar stars with spectral types A0 or later. The youngest peculiar stars now known are also systematically the most massive ones. Therefore, it is unclear whether the reported correlations show a dependence of spectral characteristics on age, or only on mass. We must enlarge the sample of peculiar stars with known ages, a difficult task since it requires observations of faint stars in distant clusters and associations.

The study of peculiar A-type stars may provide important tests of our ability to model specific physical processes. For example, accretion from the interstellar medium should play a significant role in determining the composition of the atmospheres of white dwarfs, but observations indicate that accretion rates may be significantly lower than simple models predict. On the basis of approximations similar to those used to derive accretion rates for white dwarfs, Strittmatter and Norris (1971) estimate that a peculiar A-type star can accrete a new atmosphere (defined as the amount of material down to optical depth 5) in only 600 years. This time scale is much shorter than the main sequence lifetime of A-type stars and somewhat shorter even than the diffusion time for helium. Strittmatter and Norris postulate that weak magnetic fields must be present, even in such apparently nonmagnetic objects as the HgMn stars, in order to inhibit accretion at rates that would obliterate the abundance anomalies. New techniques have reduced the upper limits on the magnetic fields in HgMn and certain other Bp stars to quite low levels. Is it possible, therefore, to turn the problem around? Can we use the existence of peculiar Ap and Bp stars to reevaluate models of accretion from the interstellar medium?

An understanding of the process of tidal synchronization and its efficiency is important in modeling the evolution of close binaries, including X-ray binaries. In the A-type stars, there is apparently a marked increase in the efficiency of synchronization with decreasing mass (see Chapter 2). Existing observations, which should be confirmed and extended to a larger sample, show that a majority of the late A-type stars in close binaries rotate slowly, while among early A-type binaries the distribution of v sin *i* is more uniform. Presumably the difference is due to the fact that tidal interactions are more effective in convective than in radiative envelopes. Again questions arise. Can we go beyond simply explaining the observations in a schematic way? Can we instead use the observations to improve our models of dissipative processes in stellar atmospheres?

As noted, the efficiency of tidal synchronization depends on the nature of the convection zone. The same thing is true of diffusion and a number of other physical processes in stellar atmospheres. The transition from radiative to convective envelopes occurs in the A-type stars, and so we can test our models of convection, including particularly models of partial convection, by studying its effects on the colors and other observables in A-type stars.

Convection is thought to be intimately associated with chromospheric and coronal activity. Support for this apparent association is offered by the A-type stars, in which convection zones are weak and chromospheric emission has not been detected. However, X-ray emission definitely is present in some, but not all, A-type stars. A number of problems are posed by the available data. What determines whether or not an A-type star will be a source of X-rays? What is the role of binary companions in producing the observed emission? What is the relationship, if any, between spectral peculiarities and X-ray emission? And what is the source of nonradiative heating in stars with (relatively) quiescent atmospheres?

In the magnetic stars, the variations discussed were associated either with the rotation of an inhomogeneous surface or with, in the cooler Ap stars, nonradial pulsation. From time to time, however, observers have reported intrinsic, nonperiodic fluctuations in magnetic field strengths and line profiles. For the most part, these reports have not been confirmed. Are the fluctuations real? If so, what do they tell us about the relationship between magnetic fields and hydrodynamic processes? It is this author's impression that intrinsic variability has been reported more frequently for Ap stars with the shortest rotation periods. Is this impression correct? If so, is it because of the difficulty of measuring the properties of stars with broad lines? Or are we seeing evidence of a fragile balance between magnetic forces and circulation currents in rapidly rotating stars?

Throughout the text, the absence of variability in the nonmagnetic stars has been emphasized. In a comparison of magnetic and nonmagnetic stars, this emphasis is surely appropriate. Nevertheless, variations have been reported from time to time in both Am and HgMn stars. In all cases, the amplitudes of variability are comparable in magnitude to the observational error. But is it correct to dismiss the reported effects as either not proved or unimportant? Or are the atmospheres of some peculiar stars not really as stable as we think they are? Obviously very careful observations, with the best instrumentation and meticulous attention to experimental error, will be required to establish the point one way or the other.

Simultaneous excitation of two, or even three radial modes is fairly common in stars that populate the lower portion of the instability strip. Yet models with simultaneously excited radial modes have not been successfully constructed. What factors determine the mode(s) of oscillation?

Some of the most interesting recent work on both magnetic Ap and δ Sct stars involves the study of nonradial pulsations. Potentially, the determination of modes of oscillation can tell us much about atmospheric structure. A variety of techniques for identifying modes is available, and the results of the different approaches are generally consistent. Yet again, however, the question arises. What factors determine whether or not excitation of nonradial modes will occur? Fitch (1980) has offered the somewhat radical point of view that nonradial modes in stars above the main sequence usually occur only if either rapid rotation or a close companion causes departures from spherical symmetry. Is this hypothesis correct? If so, what are the implications for constructing models of these stars?

Finally, it is worthwhile to recall once again one of the most remarkable properties of the A- and B-type stars, namely, the extraordinary differences in the line spectra of stars with similar values of T_{eff}, log g, and magnetic field strength. Do many "normal" A-type stars have metal abundances of a factor of 2 below that of the Sun? If so, why? What is the explanation for the diversity of the peculiar stars? Is it due to the interaction of such properties as convection, turbulence, and magnetic geometry with a single process such as diffusion? Does it reflect the results of multiple processes–diffusion, accretion, and possibly others–on the composition of the atmospheres? Or does it indicate differences in the initial composition of individual B- and A-type stars? Nature has, through whatever mechanism, concentrated in the outer atmospheres of the peculiar stars many of the elements that are thought to be produced in advanced stages of nucleosynthesis. These elements are unobservable in most other classes of stars. If we could somehow separate intrinsic effects, such as diffusion, from extrinsic ones, then the chemically peculiar stars of the upper main sequence might be used to address one of the most fundamental problems of all–the chemical evolution of the Galaxy.

THE FUTURE

A point of view all too prevalent among the astronomical community is that stellar atmospheres are basically well understood and that few surprises are likely to result from further studies of them. The emphasis during the past 2 decades has been on the discovery of new astronomical phenomena, and indeed this emphasis has been appropriate because the number of discoveries is unparalleled in the history of astronomy. Quasars, X-ray stars and galaxies, the microwave background, pulsars, and γ-ray bursts were all discovered during the past quarter century, and virtually all were identified through either the application of new techniques to astronomy or the opening up of new wavelength regions. A provocative discussion of discoveries in astronomy has recently been published by Harwit (1981).

As Harwit points out in his book, discoveries follow almost immediately the introduction of new observational techniques. However, there is no clear coupling between discovery and analytical understanding of new phenomena. This point is illustrated very nicely by work on Ap stars. The detection of variations in these stars was one of the more remarkable, and inexplicable, discoveries of early stellar spectroscopy. Yet many elements of our current understanding of these stars are a product of very recent work. The explanation of the light variations in Ap stars, the discovery of Ap stars with periods of several years, the recognition of the extension of the Ap phenomenon to spectral types as early as B2, the mapping of magnetic geometries, the study of the interaction of stellar winds with magnetic fields and their relationship to abundance anomalies, the measurement of pulsations in Ap stars, and the development of radiative diffusion models to account for the abundance anomalies are all achievements of the past 15 years. The list of unsolved questions is extensive enough to suggest that major advances are still to come.

In fact, it seems quite clear that the recognition of the importance of nonthermal phenomena in determining atmospheric structure is leading to a profound change in our approach to stellar astrophysics. A decade ago, the emphasis in stellar research was on the increasing elaboration of equilibrium models—inclusion of line blanketing, relaxation of LTE equations governing atomic level populations, treatment of radiative transfer in spherical rather than plane-parallel geometry—and on the use of those models to catalog atmospheric temperatures, pressures, and compositions. Stars are not, however, the isolated objects in perfect equilibrium that are envisioned by classical models. The emphasis now in stellar research is on the measurement and interpretation of such dynamical processes as mass loss, variability, activity cycles, and atmospheric heating. At the present time, a full understanding of these processes may seem as remote as the possibility of determining stellar compositions seemed in the nineteenth century. Indeed, it is certainly one of the major achievements of the human intellect that we now know of what the stars are made. The challenge that lies before us is no less great—to achieve an understanding of the evolution of stars from formation to death, to characterize the dynamical processes that determine stellar structure at each phase of evolution, to study the interactions of stars with their environment, and to quantify the ways in which stars shape the evolution of such larger structures as clusters and galaxies.

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