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ABSTRACT

The interaction of a shock wave with a hot spot, a single vortex and a vortex street is studied within the framework of the two-dimensional compressible Euler equations. The numerical results obtained by the pseudospectral method and the finite difference MacCormack method are compared. In both the methods the shock wave is fitted as a boundary of the computational domain.

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Introduction

In their recent paper, Pao and Salas [1] presented a finite difference solution to the two-dimensional Euler equations governing the phenomena of shock wave interaction with an isolated vortex. Their study emphasized the acoustic aspects of the problem. Zang, Hussaini, and Bushnell [2] extended this numerical approach to the problem of turbulence amplification in shock wave interactions. In this work it was necessary to resolve rather complex fine-scale structure in order to draw meaningful conclusions about the transient processes that dominate turbulence amplification. The present study is a continuation of these efforts in two directions. First, to develop a highly accurate pseudo-spectral method capable of resolving the crucial small-scale structure on a relatively coarse grid. Second, to gain insight into the nonlinear dynamics of the transient processes involved in the passage of a shock wave over a single vortex, a vortex street and a hot spot.

Spectral methods have been demonstrated [3], [4] to be powerful alternatives to finite difference methods for the numerical solution of smooth flows. Recently, the work of Gottlieb, Lustman, and Orszag [5] and of Zang and Hussaini [6] have shown their applicability to simple one-dimensional compressible flows with shocks. The present paper discusses a Chebyshev pseudo-spectral method that has produced reliable, accurate and efficient solutions to complex, two-dimensional flows with a strong shock. Although the main emphasis is on the spectral technique, solutions to the governing equations obtained by the well known, second-order finite difference method originated by MacCormack are also given. This is the method that was used in [1] and [2]. The finite difference results for the present set of problems were calculated on a very fine grid and are used here for comparison with the solutions obtained with the spectral method.

Statement of the Problem

The physical problem that we model corresponds to an infinite, initially planar normal shock wave moving from left to right into a downstream region containing a flow field representative of one or more vortices, or a hot spot. In order to model the interaction of the shock wave with some given flow field ahead of it, it is only necessary to compute the flow field upstream of the shock. The physical domain, therefore, need consist only of the region between some left boundary $h(t)$, judiciously chosen such that it will be far from the interaction region, and the shock wave front itself $x_s(y,t)$. It is mapped onto the computational domain by the transformation,

$$X = \frac{x - h(t)}{x_s(y,t) - h(t)} ,$$

$$Y = \frac{\tanh(\alpha y) + 1}{2} ,$$

$$T = t.$$

The computational domain is thus $(X,Y) \in [0,1] \times [0,1]$. Note the stretching that has been used to handle the infinite extent of the lateral coordinate y. If the relative shock Mach number M_s is sufficiently high ($M_s > 2.08$), the flow upstream of the shock remains supersonic. In this case, the left boundary corresponds to a supersonic inflow, and all dependent variables can be prescribed on it. However, if the relative shock Mach number is low, then radiation-type boundary conditions are used at the left boundary. On the right, the computational region is bounded by the shock wave. Downstream of the shock the flow field is given analytically. The flow field immediately upstream of the shock, as well as the shape and velocity of the shock, are evaluated such that the Rankine-Hugoniot jump conditions and the compatibility condition reaching the shock wave from the upstream side are simultaneously satisfied.

The unsteady, two-dimensional, compressible, Euler equations in the computational plane are written in the form,

$$Q_T + \underline{A}Q_X + \underline{B}Q_Y = 0 \; ,$$

where $Q = [P,u,v,S]^T$ and

$$\underline{A} = \begin{bmatrix} U & \gamma X_x & \gamma X_y & 0 \\ a^2X_x/\gamma & U & 0 & 0 \\ a^2X_y/\gamma & 0 & U & 0 \\ 0 & 0 & 0 & U \end{bmatrix} \qquad \underline{B} = \begin{bmatrix} V & \gamma Y_x & \gamma Y_y & 0 \\ a^2Y_x/\gamma & V & 0 & 0 \\ a^2Y_y/\gamma & 0 & V & 0 \\ 0 & 0 & 0 & V \end{bmatrix} .$$

The natural logarithm of the pressure, the speed of sound, and the entropy are represented by P, a, and S, respectively, and γ is the ratio of specific heats. The velocity in the x and y directions are u and v, respectively. All variables are normalized with respect to reference conditions at downstream infinity, as in [1]. The contravariant velocity components are defined by

$$U = X_t + uX_x + vX_y$$

and

$$V = Y_t + uY_x + vY_y .$$

Subscripts denote partial derivatives with respect to the independent variables.

Solution Techniques

Let k denote the time level and let Δt be the time step increment. The time discretization of eq. (12) is then as follows:

$$\tilde{Q} = [1 - \Delta t L^k]Q^k,$$

$$Q^{k+1} = \tfrac{1}{2}\,[Q^k + (1 - \Delta t\tilde{L})\tilde{Q}],$$

where the spatial operator L represents an approximation to $\underline{A}\partial_X + \underline{B}\partial_Y$. In the finite difference MacCormack method, the operators L^k and $\tilde{L}$ are evaluated as two points forward and two points backward differences in the predictor (left) and corrector (right) levels, respectively. In the pseudo-spectral method studied here, the solution Q is first expanded as a double Chebyshev series,

$$Q(X,Y,T) = \sum_{p=0}^{M} \sum_{q=0}^{N} Q_{pq}(T)\tau_p(\xi)\tau_q(\eta),$$

where

$$\xi = 2X - 1,$$

$$\eta = 2Y - 1,$$

and τ_p and τ_q are the Chebyshev polynomials of degrees p and q. The derivatives appearing in the spatial operators are then evaluated as

$$Q_X = 2 \sum_{p=0}^{M} \sum_{q=0}^{N} Q_{pq}^{(1,0)} \tau_p \tau_q,$$

where

$$Q_{pq}^{(1,0)} = \frac{2}{c_p} \sum_{\substack{m=p+1 \\ m+p \text{ odd}}}^{M} mQ_{mq},$$

and

$$c_0 = 2,$$

$$c_p = 1, \; p > 0.$$

The Q_Y derivative is evaluated in a similar fashion.

The evaluation of the shock wave shape and velocity followed the same procedure described in [1], except that in the spectral formulation, the derivatives that must be evaluated on the upstream side of the shock are expressed as Chebyshev expansions. At the left boundary, all variables were specified for supersonic inflow. For the case of subsonic inflow, the two velocity components and the entropy were specified, while the pressure was computed from a quasi-one-dimensional characteristic as described in [8].

The pseudo-spectral method has a tendency to develop slowly growing oscillations. Because of the global nature of this method they are spread over the entire flow field rather than being confined to the vicinity of sharp gradients. The underlying smooth solution can be recovered by a variety of filtering techniques. The results presented here were obtained by applying a von Hann window filter (see [6] for details) every 160 time steps. Another practical consideration is the explicit time-step restriction. The Chebyshev collocation points are clustered near the boundaries. Thus, smaller time-steps must be used in the pseudo-spectral calculations.

Results

Perhaps the simplest interaction to consider is that of a planar shock wave with a hot spot, as shown in Fig. 1. The flow field downstream of the shock wave situated at $x = 0$ at $t = 0$ is taken as a quiescent field whose temperature distribution σ, is given by

$$\sigma = \frac{\kappa}{2\pi} \exp\left(-((x-x_0)^2 + (y-y_0)^2)/2r^2\right),$$

where for this case $\kappa = 0.25$, $r = .125$, $x_0 = 0.5$, $y_0 = 0.0$, and $M_s = 3$ at $t = 0$. The vorticity distributions obtained by the finite difference method and by the spectral method after the shock wave has passed over the hot spot are shown in Fig. 3. The finite difference solution presented here, and in all cases that follow, was obtained with 75 mesh points in the X direction and 50 in Y. The spectral solutions were all obtained with 33 collocation points in the X direction and 17 in Y. There is very little difference between the two solutions. Both show the two counter-rotating vortices upstream of the shock, which is typical of this interaction. See [2] for more details on the physics.

Figure 2 shows the velocity field for a single vortex about to interact with a shock wave of the same initial strength as in the previous case. The downstream conditions here are obtained by assuming a constant density field, calculating the velocity from the stream function,

$$\psi = \frac{\kappa}{2\pi} \log \sqrt{r^2 + (x-x_0)^2 + (y-y_0)^2} ,$$

the pressure from Bernoulli's relation, and the temperature from the equation of state. For the case shown in Fig. 2, the circulation $\kappa = 2$ and the softening scale r = 0.1. This model approaches an idealized incompressible point vortex at large distances but is much smoother near the center. Figure 4 shows the resulting pressure field after the shock wave has passed over the vortex. Overall, the results are qualitatively very similar, although the spectral method seems to be resolving the pressure field more accurately. See [1] for more details on the physics.

Finally, Figs. 5 and 6 show the results for the interaction with the Karman vortex street that simulates the conditions of the experiment reported in [7]. For this case, the stream function representing the vortex is given by the difference of ψ_+ and ψ_- where

$$\psi_{\pm} = \frac{\kappa}{2\pi} \log\left[\cosh\left(\frac{2\pi}{c} \sqrt{r^2 + (y \pm 1/2\, b)^2}\right) - \cos\left(\frac{2\pi}{c} (x \pm 1/2\, c)\right)\right].$$

To match the experiment, the circulation, core radius, shock Mach number and vortex separation parameters were determined as $\kappa = 0.186$, $r = 0.1$, $M_s = 1.3$, $c = .33$, and $b = .048$. For this calculation, the inflow Mach number was subsonic and radiation boundary conditions were applied at the left boundary. The results shown in Fig. 6 are in agreement with the experimentally observed [7] longitudinal compression and lateral elongation of the vortex field after passage through the shock. The finite difference results are noticeably smoother than the spectral ones. However, it is well known that the idealized Karman vortex street is unstable for all but one special ratio of the horizontal and vertical separations. The downstream flow has the stable ratio. Thus, it is likely that the upstream flow is physically unstable. It may be that the spectral results have captured this phenomena, but more computations are needed to settle the issue.

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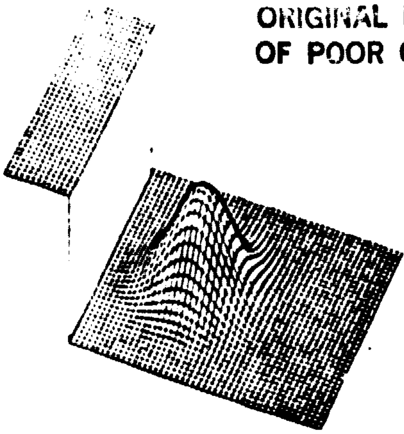

Fig. 1: Surface plot of entropy distribution for a hot spot about to interact with a Mach 3 shock wave.

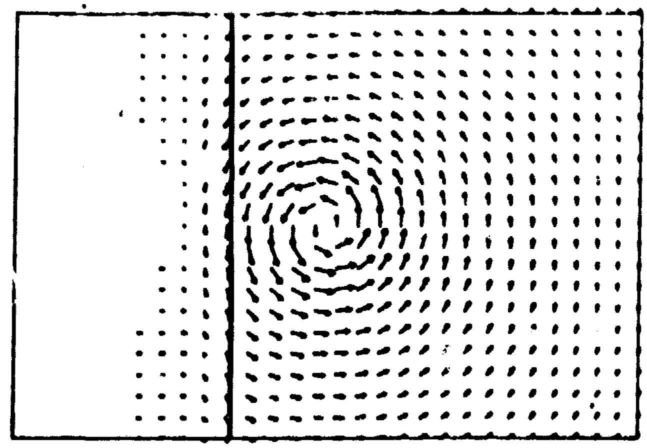

Fig. 2: Velocity field of a single vortex about to interact with a Mach 3 shock wave (solid curve). The velocity vectors represent perturbation from the mean values.

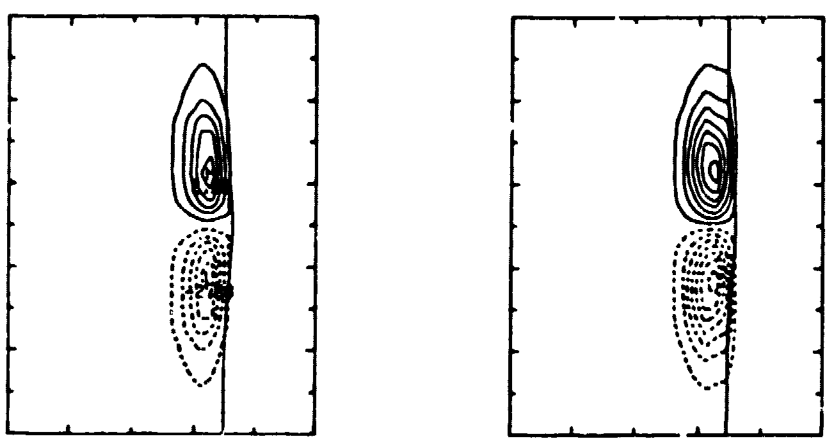

Fig. 3: Vorticity fields at t = 0.20 computed by pseudo-spectral (left) and finite difference (right) methods for a hot spot after interaction with a Mach 3 shock wave (solid curves).

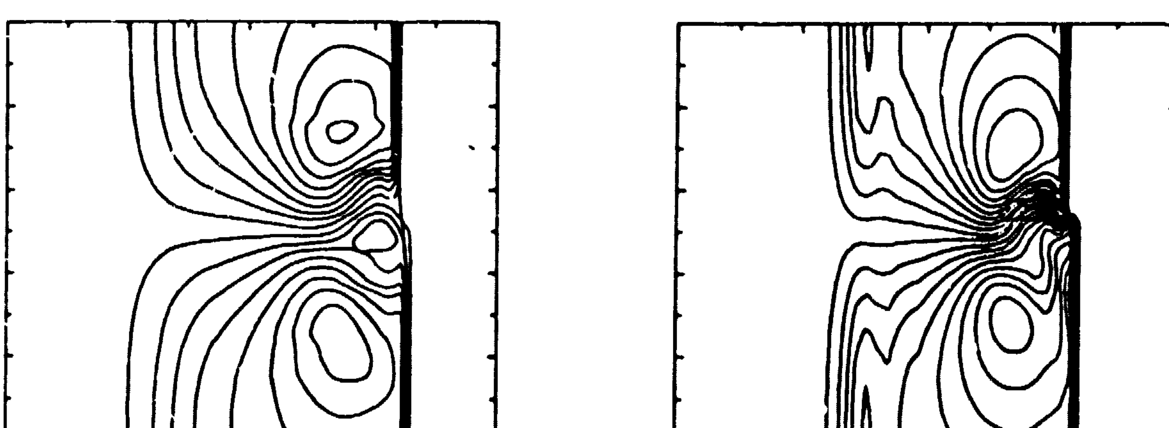

Fig. 4: Isobars at t = 0.20 computed by pseudo-spectral (left) and finite difference (right) methods for a single vortex after interaction with a Mach 3 shock wave.

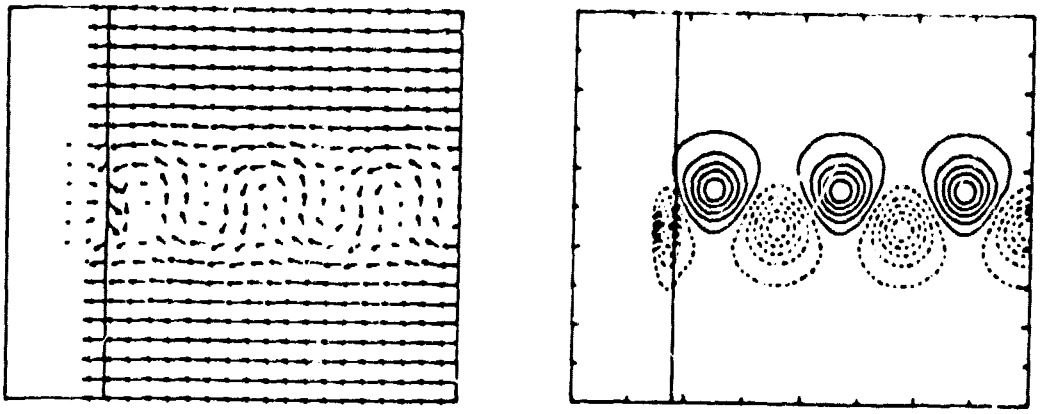

Fig. 5: Velocity field (left) and vorticity contours (right) for a Karman vortex street about to interact with a Mach 1.3 shock wave (solid curves). The velocity vectors represent perturbations from mean values. Negative contour levels are are indicated by dashed lines.

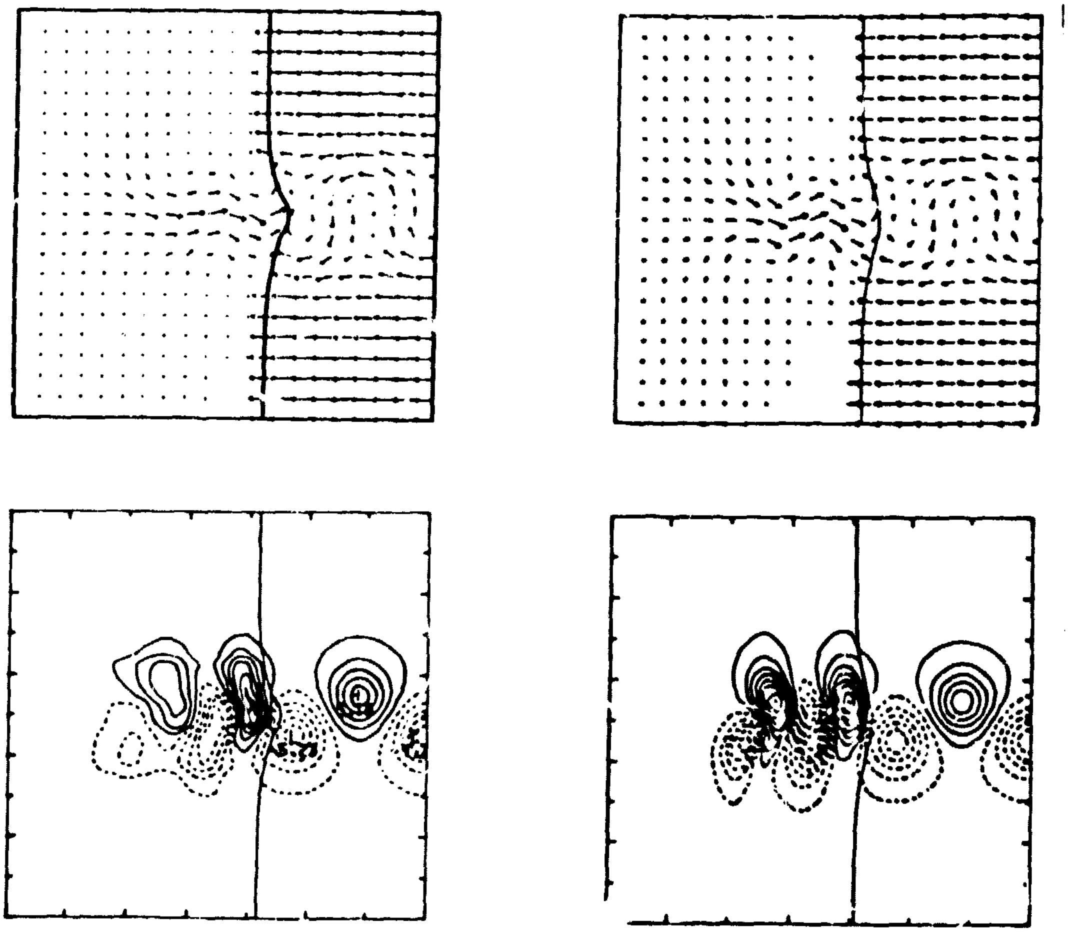

Fig. 6: Velocity fields (top) and vorticity contours (bottom) at $t = 0.36$ computed by pseudo-spectral (left) and finite difference (right) methods for a Karman vortex street after interaction with a Mach 1.3 shock wave (solid curves). The velocity vectors represent perturbations from mean values. Negative contour levels are indicated by dashed lines.