DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA

(NASA-CR-173049) ON IDENTIFIABILITY OF FLEXIBLE STRUCTURE PARAMETERS Final Report, period ending 5 Jul. 1983 (Old Dominion Univ., Norfolk, Va.) 15 p HC A02/MF A01 CSCL 22B G3/18

N83-33932

Unclas
15039

ON IDENTIFIABILITY OF FLEXIBLE STRUCTURE PARAMETERS

By

S. M. Joshi, Co-Principal Investigator

and

G. L. Goglia, Principal Investigator

Final Report
For the period ending July 5, 1983

Prepared for the
National Aeronautics & Space Administration
Langley Research Center
Hampton, Virginia

Under
Research Grant NAG-1-102
Harold A. Hamer, Technical Monitor
Spacecraft Controls Branch (FDCD)

July 1983

DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
SCHOOL OF ENGINEERING
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA

ON IDENTIFIABILITY OF FLEXIBLE STRUCTURE PARAMETERS

By

S. M. Joshi, Co-Principal Investigator

and

G. L. Goglia, Principal Investigator

Final Report
For the period ending July 5, 1983

Prepared for the
National Aeronautics & Space Administration
Langley Research Center
Hampton, Virginia

Under
Research Grant NAG-1-102
Harold A. Hamer, Technical Monitor
Spacecraft Controls Branch, (FDCD)

Submitted by the
Old Dominion University Research Foundation
P. O. Box 6369
Norfolk, Virginia 23508

July 1983

ON IDENTIFIABILITY OF FLEXIBLE STRUCTURE PARAMETERS

By

S. M. Joshi[1] and G. L. Goglia[2]

ABSTRACT

This report investigates the identifiability of modal parameters of flexible structures. Expressions are derived for Cramér-Rao lower bounds for the modal parameters, that is, frequencies, damping ratios and mode shapes or slopes. The optimal initial state, which maximizes the trace of the Fisher information matrix in the absence of persistent input, is obtained. The concepts discussed are applied to a finite-element model of the 122 meter hoop/column antenna. The numerical results show that the identifiability of the structural frequencies is excellent, followed by that of the damping ratios and the mode-slopes.

INTRODUCTION

The design of a controller for large flexible space structures requires reasonable knowledge of the structural mode parameters. Since it is usually difficult to obtain accurate knowledge of the parameters by apriori modeling, it would be necessary to estimate the parameters in orbit. The identifiability of the parameters is investigated in this report using the Cramér-Rao lower bound. The expressions for Fisher information matrix and the optimal initial state for identification are obtained in the next section, followed by numerical results of the application of these concepts to the 122m hoop/column antenna.

[1] Research Associate Professor, Department of Mechanical Engineering and Mechanics, Old Dominion University, Norfolk, Virginia 23508

[2] Eminent Professor and Chairman, Department of Mechanical Engineering and Mechanics, Old Dominion University, Norfolk, Virginia 23508

IDENTIFIABILITY OF STRUCTURAL PARAMETERS

The objective of this report is to investigate the identifiability of the parameters, i.e., modal frequencies, damping ratios and mode shapes (or slopes) at the sensor locations. Since most established parameter estimation methods use discretized sensor outputs, consider the discrete-time, constant-coefficient system:

$$x(k+1) = Fx(k) + Gu(k) + v(k) \tag{1}$$

$$y(k) = Cx(k) + Du(k) \tag{2}$$

$$z(k) = y(k) + w(k) \tag{3}$$

where x, u, y represent $n\times1$, $m\times1$, $\ell\times1$ state, input and output vectors, v and w represent zero-mean, mutually uncorrelated white noise processes, and z is the observation (sensor output vector). F, G, C, D are appropriately dimensioned matrices. If $x(0)$, $v(k)$, $w(k)$ are Gaussian, the conditional density $f(Z_n/\theta)$ is Gaussian,

where

$$Z_N = \{z(0),\ldots,z(N)\} \tag{4}$$

and θ is the $p\times1$ vector of parameters to be estimated. Assuming $v = 0$,

$$f(Z_N/\theta) = C_1 \exp - \left[1/2 \sum_{k=0}^{N} \{z(k) - y(k)\ ^{\Gamma} W^{-1}\ z(k) - y(k)\}\right] \tag{5}$$

where C_1 is a constant and $W = \text{cov.}\{w(k)\}$. (If the apriori density of θ, $f(\theta)$, is Gaussian, the unconditional density $f(Z_N)$ will also be Gaussian).

If θ is any absolutely unbiased estimate of θ based on Z_N, it can be proved that (ref. 1)

$$E\left[(\theta - \theta)(\theta - \theta)^T/\theta\right] \geq J^{-1} \tag{6}$$

where

$$J = - E\left\{\frac{\partial^2}{\partial\theta^2} \ell n\ f\ (Z_N/\theta)/\theta\right\} \tag{7}$$

It can be proved from equations (5 and 7) that

$$J = \sum_{k=0}^{N} \frac{\partial y(k)}{\partial \theta}^{T} W^{-1} \frac{\partial y(k)}{\partial \theta} \qquad (8)$$

J is the Fisher information matrix, and J^{-1} is the Cramér-Rao lower bound on the covariance of the parameter estimation error. Thus ith diagonal element J^{-1} is the lower bound on the variance of error in estimating θ_i.

It is obvious from equation (8) that, J tends to equal a constant value as $N \rightarrow \infty$, in the absence of persistent inputs to the system (provided that the system is asymptotically stable). J is nonzero only if the initial state is nonzero. If persistent inputs are present, $J \rightarrow \infty$ as $N \rightarrow \infty$, since the information available keeps growing. If all the parameters are identifiable, the information matrix is nonsingular. An understanding of the degree of identifiability of different parameters can be obtained by comparing the Cramér-Rao standard deviations for those parameters, obtained using nonzero initial state and zero inputs.

Considerable work has been done on designing optimal inputs for identification. However, for investigating the relative degree of identifiability of various parameters, zero input can be assumed. If nonzero inputs are used during identification, accurate measurements of the inputs are required. In addition, to be of any consequence, the inputs will have to have reasonably large magnitudes. This could be particularly objectionable for a LSS, as large magnitude inputs may cause excessive elastic motion which would jeopardize the structural integrity of LSS. Also, nonzero input (e.g. torque in the case of the antenna) would require the estimation of the mode-slope at the location of the actuator, which may add to the number of parameters to be estimated. For these reasons, zero inputs are used in this study. The identification procedure could perhaps proceed as follows: after the deployment of the antenna, one or more torquers could be used to create nonzero modal amplitudes. The torquers would then be turned off and the sensor data would be collected for N samples. The data could be used (on-line or off-line) in an identification algorithm to obtain the estimates. Control system gains could subsequently be computed, and the control system turned on.

OPTIMAL INITIAL STATE FOR IDENTIFICATION

The sensitivity of $y(k)$ to θ_i, the ith parameter, is given by (assuming $u = 0$)

$$\frac{\partial y(k)}{\partial \theta_i} = \frac{\partial C}{\partial \theta_i} x(k) + C \frac{\partial x(k)}{\partial \theta_i} \tag{9}$$

The sensitivity state vector is given by

$$\frac{\partial x(k+1)}{\partial \theta_i} = \frac{\partial F}{\partial \theta_i} x(k) + F \frac{\partial x(k)}{\partial \theta_i} \tag{10}$$

The output sensitivity matrix is

$$\frac{\partial y(k)}{\partial \theta} = \left[\frac{\partial y(k)}{\partial \theta_1} \; \frac{\partial y(k)}{\partial \theta_2}, \ldots, \frac{\partial y(k)}{\partial \theta_p}\right]$$

$$= \left[E_1 \; x_a(k), \ldots, E_p x_a(k)\right]_{\ell x p}$$

where the $(p+1)$ n - dimensional, augmented state vector

$$x_a^T(k) = \left[x^T(k), \quad \frac{\partial x(k)}{\partial \theta_1}^T, \ldots, \quad \frac{\partial x(k)}{\partial \theta_p}^T\right] \tag{12}$$

and E_i are appropriate coefficient matrices. For zero input, the augmented state vector can be written as

$$x_a(k) = \Phi \; (k,0) \; x_a(0) \tag{13}$$

where $\Phi(k,0)$ is the transition matrix for the augmented state vector.

Since

$$x_a^T(0) = \left[x^T(0), \; 0^T\right] \tag{14}$$

Equation (13) can be written as

$$x_a(k) = \Psi(k)\ x(0) \tag{15}$$

where Ψ is the appropriate submatrix of Φ. Therefore,

$$\frac{\partial y(k)}{\partial \theta} = \left[\hat{E}_1(k)\ x(0),\ \hat{E}_2(k)\ x(0), \ldots,\ \hat{E}_p(k)\ x(0)\right]_{\ell x p} \tag{16}$$

where $\hat{E}_i(k) = E_i \Psi(k)$

Substituting equation (16) into equation (8), the (i,j) th element of J is given by:

$$J_{ij} = x^T(0) \left\{ \sum_{k=0}^{N} \hat{E}_i^T(k)\ W^{-1}\ \hat{E}_j(k) \right\} x(0) \tag{17}$$

One definition of optimal initial state is the $x(0)$ which maximizes the trace of J when constrained to be within a hyperellipsoid given by

$$x^T(0)\ S\ x(0) \leq d \tag{18}$$

where $S = S^T > 0$ and $d > 0$.

Theorem 1. - The $x(0)$ which maximizes $Tr(J)$ subject to (18) is the eigenvector of $S^{-1}H$ corresponding to its maximum eigenvalue, which lies on the surface of the hyperellipsoid in (18), where

$$H = \sum_{i=1}^{n} \sum_{K=0}^{N} \hat{E}_i^T(k)\ W^{-1}\ \hat{E}_i(k) \tag{19}$$

Proof. - The proof of this standard optimization problem is well known.

Optimal Initial State for LSS. - The structural equations of motion for a flexible LSS are uncoupled. The equation of motion (assuming zero input) for the ith mode $(i = 1, 2, .., n_q)$ is:

$$\ddot{q}_i + 2\rho_i \omega_i \dot{q}_i + \omega_i^2 q_i = 0 \tag{20}$$

ORIGINAL PAGE IS
OF POOR QUALITY

The $\ell \times 1$ output vector due to elastic motion is given by (assuming rate gyros are used)

$$y = \sum_{i=1}^{nq} \phi_i \dot{q}_i \tag{21}$$

The parameters to be estimated for the ith mode are:

$$\theta_i = \left[\rho_i, \omega_i, \phi_{1i}, \phi_{2i}, \ldots, \phi_{\ell i}\right]^T \tag{22}$$

Thus there are $(\ell+2)$ parameters per mode, or a total of $n_q(\ell+2)$ parameters. Since the parameters appear explicitly in the continuous equations of motion, it is more convenient to first write the continuous equations for the sensitivity states, and then to discretize them. The sensitivity state equations for the ith mode with respect to ρ_i is:

$$\frac{\partial \ddot{q}_i}{\partial \rho_i} + 2\rho_i \omega_i \left(\frac{\partial \dot{q}_i}{2\rho_i}\right) + \omega_i^2 \left(\frac{\partial q_i}{\partial \rho_i}\right) + 2\omega_i \dot{q}_i = 0 \tag{23}$$

(A similar equation can be written with respect to ω_i). In the absence of persistent input, the sensitivity with respect to the mode slopes is zero. Denoting $\theta_i = [\rho_i, \omega_i, \phi_{1i}, \ldots, \phi_{\ell i}]$, the output sensitivity matrix or the ith mode is

$$\frac{\partial y}{\partial \theta_i} = \left[\frac{\partial y}{\partial \rho_i} \quad \frac{\partial y}{\partial \omega_i} \quad \frac{\partial y}{\partial \phi_{1i}}, \ldots, \frac{\partial y}{\partial \phi_{\ell i}}\right]_{\ell \times (\ell+2)} \tag{24}$$

The output sensitivity terms on the right-hand side of equation (24) can be easily obtained from equation (21) and equation (24) can be rewritten as:

$$\frac{\partial y}{\partial \theta_i} = \left[E_{i1} x_{a_i}, E_{i2} x_{a_i}, \ldots, E_{i,\ell+2} x_{a_i}\right] \tag{25}$$

where

$$x_{a_i} = \left[q_i, \dot{q}_i, \frac{\partial q_i}{\partial \rho_i}, \frac{\partial \dot{q}_i}{\partial \rho_i}, \frac{\partial q_i}{\partial \omega_i}, \frac{\partial \dot{q}_i}{\partial \omega_i}\right]^T \tag{26}$$

using equation (26) in equation (8) and simplifying, the trace of the (i, i)th submatrix of J (i=1,1,..,nq) can be written in the following form (since the initial conditions corresponding to the sensitivity state variables are zero):

$$Tr_i = x_i^T (0)\, H \;\; x_i\, (0) \tag{27}$$

where H_i depends upon the state transition matrix for equation (26) and W, and $x_i = (q_i \dot{q}_i)^T$. Therefore, the initial state $x_i(0)$ for the ith mode, which will maximize Tr_i subject to the constraint

$$x_i^T(0)\; S_i x_i(0) \leq d_i \tag{28}$$

(where $S_i = S_i^T \geq 0$, $d_i > 0$) is the eigenvector corresponding to the maximum eigenvalue of $S_i^{-1} \; H_i$, with a norm such that equality is satisfied in equation 28.

The computation of the optimal initial state requires the knowledge of the parameters. The apriori (approximate) information about the parameters could be used for this purpose. Also, it will be necessary to compute the inputs in order to achieve the optimal initial state. This could be approximately done using the controllability matrix based on apriori parameter information.

NUMERICAL RESULTS

In order to investigate the identifiability of various modal parameters, Cramér-Rao (C-R) bounds were computed for the hoop-column antenna. This was done by first creating nonzero initial state using the torquer at actuator location no. 1, and then propagating the sensitivity state vectors. Two, 3 axis rate gyros were assumed at locations 1 and 2 (fig. 2). Only the first 10 modes were considered (See Table I). The parameters to be estimated for the ith mode are: ρ_i, ω_i, ϕx_{1i}, ϕx_{2i}, ϕy_{1i}, ϕy_{2i},

$\phi_{z_{1 i}}, \phi_{z_{2 i}}$. Since the 10th mode has a natural frequency of 9.78 rad/sec ($\simeq$ 1.5 Hz), the sampling frequency was chosen to be 4 Hz (or sampling period = 0.25 sec.). Since the $(\rho\omega)^{-1}$ for the slowest (first) mode is approximately 130 sec., the data length for the computation was taken to be that value. The sensor noise in the rate gyros was assumed to have a standard deviation intensity (continuous-time) of 1 arc-sec/sec. (The C-R bounds for other noise levels can be obtained simply by multiplying by the appropriate factor).

An examination of the magnitudes of the mode-slopes for each mode corresponding to x, y, z axes revealed the contributions of each mode to th e gyro outputs. These contributions were divided into three groups: strong (corresponding to large magnitudes of mode-slopes), medium, and weak (small magnitudes of mode-slopes). Table II shows this classification, where asterisk (*), tic-marc (✓) and blank are used to denote strong, medium and weak contributions. It is sensible to try to identify only those modes using a particular axis gyro output, which make either strong or medium contribution along that gyro axis. Thus x-axis and z-axis gyro outputs are used to identify modes 1, 3, 4, 5, 6, 8, 10, while y-axis gyro outputs are used to identify modes 2, 7, and 9.

Nonzero initial states were attained using torquers (along appropriate axis) at actuator location no. 1. The torquers were assumed to generate rectangular waves with the same frequency as the lowest-frequency mode to be estimated (for that axis), with nominal amplitude of 10 ft-lbs. (optimal initial conditions were not used). The torquer was turned off after two periods (i.e., 4 pulses), and the state at that time was used as the initial state in order to compute the C-R bounds. One computer run was made for each (x, y and z) axis. The results revealed that the C-R bounds for modes with medium contribution to a particular axis gyro output (e.g., mode 1 for x-axis in Table IV) are very large. For example, the C-R standard deviation at 130 sec. for ρ_1 using x-axis gyro data is 0.4 (ρ_1 is 0.01). Therefore, only those modes which make strong contributions to a particular axis should be identified using gyro outputs along that axis. Figure (4) shows typical C-R standard deviations as they evolve with time. Since the higher-frequency modes decay much faster than the low-frequency modes, the C-R standard deviations for the higher-frequency modes attain their steady-state quickly. From the data obtained, it can be concluded that the natural frequencies generally have the best identifiabililty (i.e., the smallest C-R standard deviations), followed by damping ratios, and then mode-slopes. The excellent identifiability of the damping ratios is especially welcome, since it is usually the worst-known (apriori) of the parameters. Unlike the mode-slopes, the identifiability of the damping ratios (ρ_i) improves as ρ_i's decrease.

The data generated is useful for comparing identifiability of different parameters rather than for obtaining the absolute identifiability. Trial application of algorithms such as the maximum likelihood method will reveal the actual accuracy with which the parameters can be identified. As mentioned earlier, the C-R bounds for modes with weak or medium contributions to the sensor outputs are large. That is, those modes have poor identifiability with respect to sensor outputs along particular axes. Therefore, if one can compute the estimated accuracy of identification while performing identification, one can conclude (assuming all modes are excited) that the mode-slopes for the poorly identifiable modes are small, and can be ignored (i.e., assumed to be zero) while designing the control systems. The C-R bounds also indicate which sensor outputs should be used to identify which modes. That is, the structure of the model used in identification can exclude modes which make weak or medium contributions to the sensor outputs, and have poor identifiability.

CONCLUDING REMARKS

The identifiability of the modal parameters of large flexible structures was investigated. The expression for the optimal initial state which maximizes the trace of the Fisher information matrix (in the absence of persistent inputs) was obtained. Cramér-Rao (C-R) lower bounds were computed for the modal parameters of the 122 meter hoop/column antenna. It was found that modal frequencies generally have the lowest C-R bounds, followed by damping ratios and mode-slopes (or shapes).

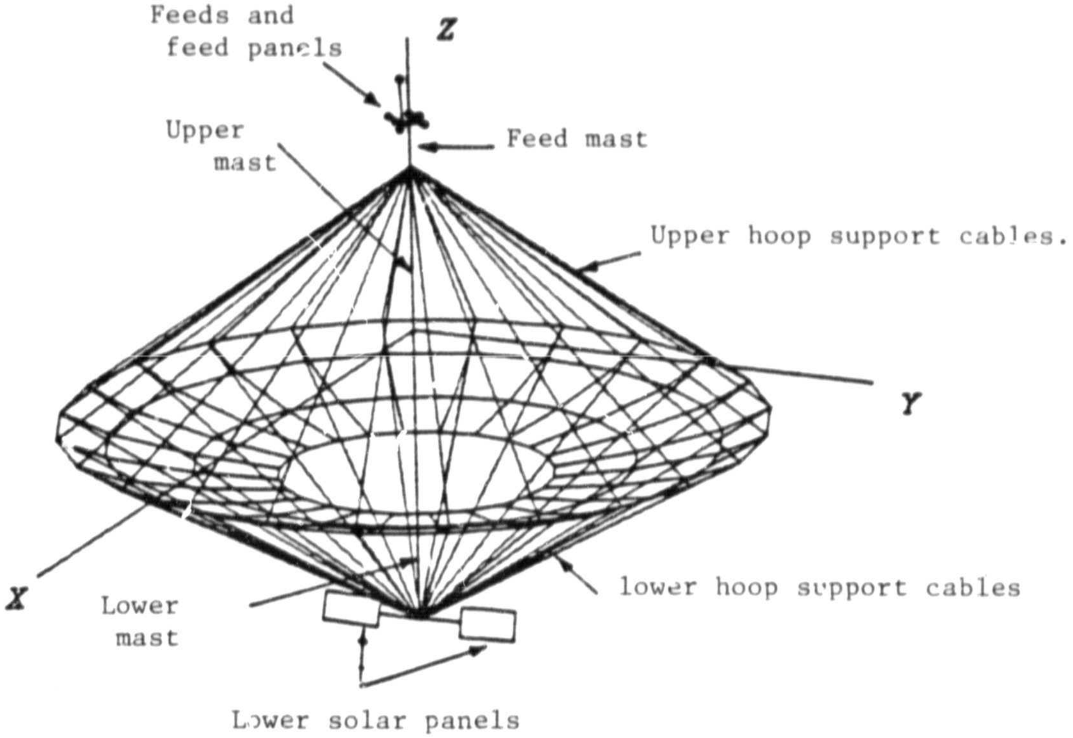


Figure 1. Hoop/Column antenna concept

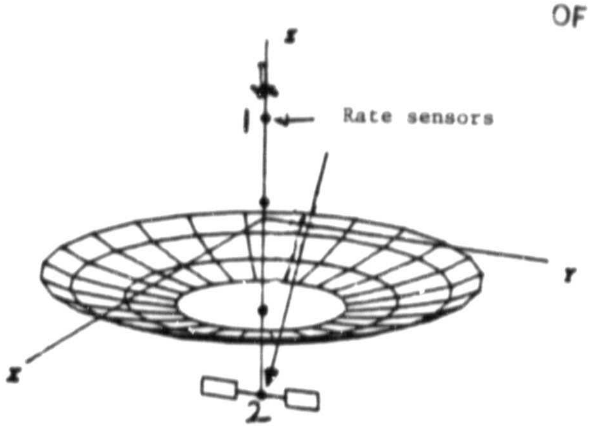


Figure 2. Assumed sensor locations

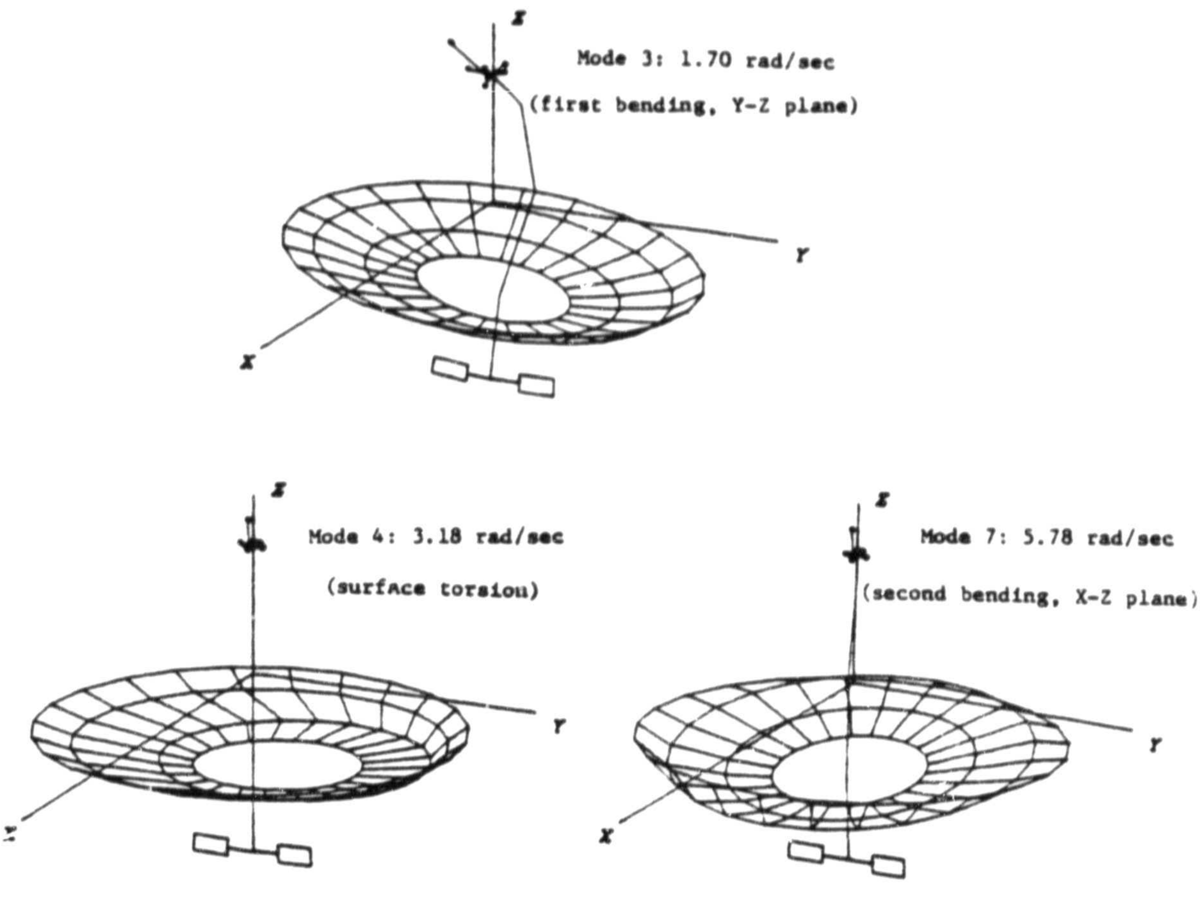
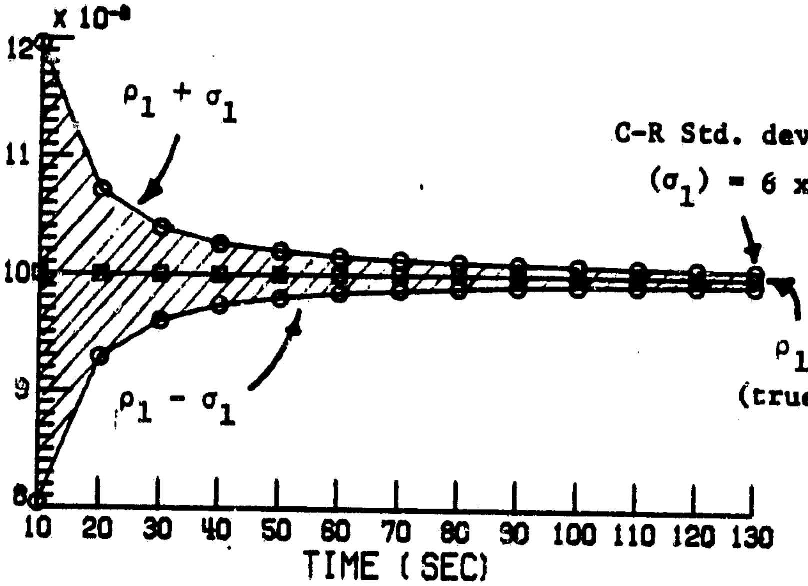


Figure 3. Typical antenna mode shapes

Evolution of C-R bounds (typical)



Examples of C-R bounds

Mode	ω	$\sigma\%$	ρ	$\sigma\%$	$\lvert\phi_1\rvert$	$\sigma\%$	$\lvert\phi_2\rvert$	$\sigma\%$
1	0.75	.04	.01	0.65	0.5E-2	0.28	0.16E-4	52
3	1.7	.006	.01	0.87	0.37E-2	0.57	0.37E-3	3.7
10	8.8	.08	.01	12	0.24E-2	15	0.49E-2	8.8

($\sigma\%$ = C-R Standard deviation expressed as a percentage of the parameter)

Figure 4. Cramér-Rao bounds

TABLE I. STRUCTURAL MODE FREQUENCIES (RAD/SEC)

0.75	1.35	1.70	3.18	4.53	5.59	5.78	6.84	7.4	8.78
10.85	11.24	15.05	15.4	15.75	15.85	16.04	18.84	18.84	18.99

TABLE II. MODAL CONTRIBUTIONS TO OUTPUTS

*: Strong ✓: Medium (blank): Weak

Mode / Axis	1	2	3	4	5	6	7	8	9	10
x	✓		*	✓	*	✓		*		*
y		*					*		*	
z	*		✓	*	✓	*		✓		✓