STRENGTH THEORIES OF COMPOSITES:

STATUS AND ISSUES

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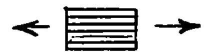
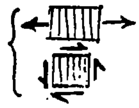
Central Issues

Under Uniaxial Stress

Fiber controlled? or Fiber dominated?



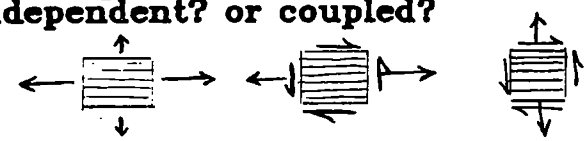
Matrix controlled? or Matrrix dominated?



Should tensile & compressive strength be related?

Under Combined Stress

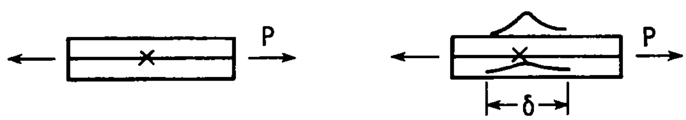
Are longitudinal & transverse strength independent? or coupled?



How to measure strength coupling?

ROLE OF MATRIX BINDER IN LONGITUDINAL STRENGTH (REFS. 1 AND 2)

MATRIX BINDER PROVIDES LOCAL REDUNDANCY

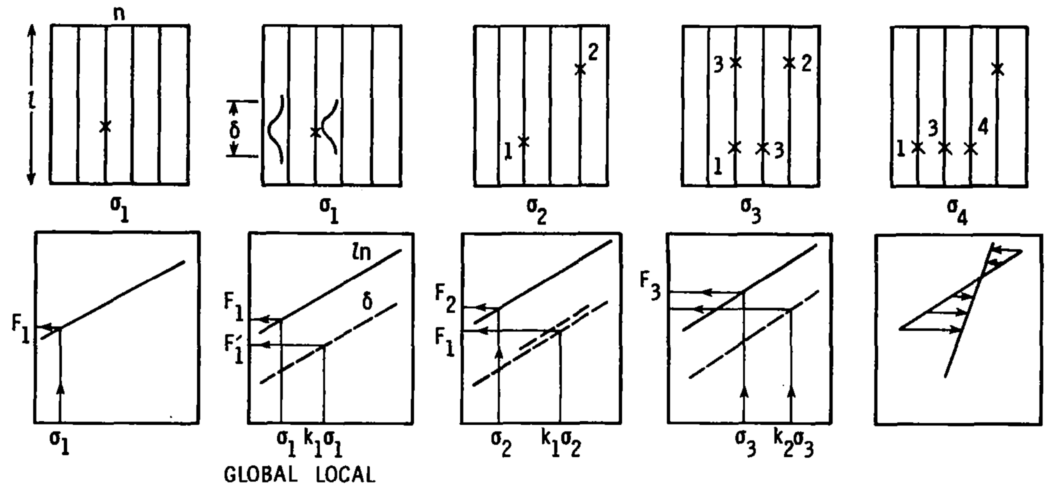


δ = INEFFECTIVE LENGTH ~ 10 d

FIBER BREAK	WITHOUT MATRIX NO. OF LOAD CARRYING FIBERS		WITH MATRIX NO. OF LOAD CARRYING FIBERS	
0	3	P/3	3	P/3
1	2	P/2	3-8	~ P/3-

LONGITUDINAL STRENGTH DEPENDS ON MATRIX EFFECTIVENESS PARAMETER $\delta\left(E_m/E_f,\ \sigma_{m_{ult}},\ \sigma_{INTERFACE}\right)$

LONGITUDINAL COMPOSITE FAILS SEQUENTIALLY (REF. 3)



LONG. COMPOSITE FAILURE IS SEQUENTIAL	SMALL δ MORE UNSTABLE BUT STRONGER	LARGE α MORE UNSTABLE WEAKER	MATRIX INCREASES α
$\frac{\sigma_1}{\sigma_{ult}} \sim .2$			

- LONG COMPOSITE FAILURE IS SEQUENTIAL, NO WELL-DEFINED PLANE OF FAILURE

LONGITUDINAL STRENGTH: FIBER-DOMINATED OR FIBER-CONTROLLED

HIGH LONGITUDINAL STRENGTH OF FIBER IS DUE TO:

DEFECTS ARE MINIMIZED BY SMALL FIBER DIAMETER

∴ $\sigma_{ult} \sim$ 600 ksi (STEEL ~ 200 ksi)

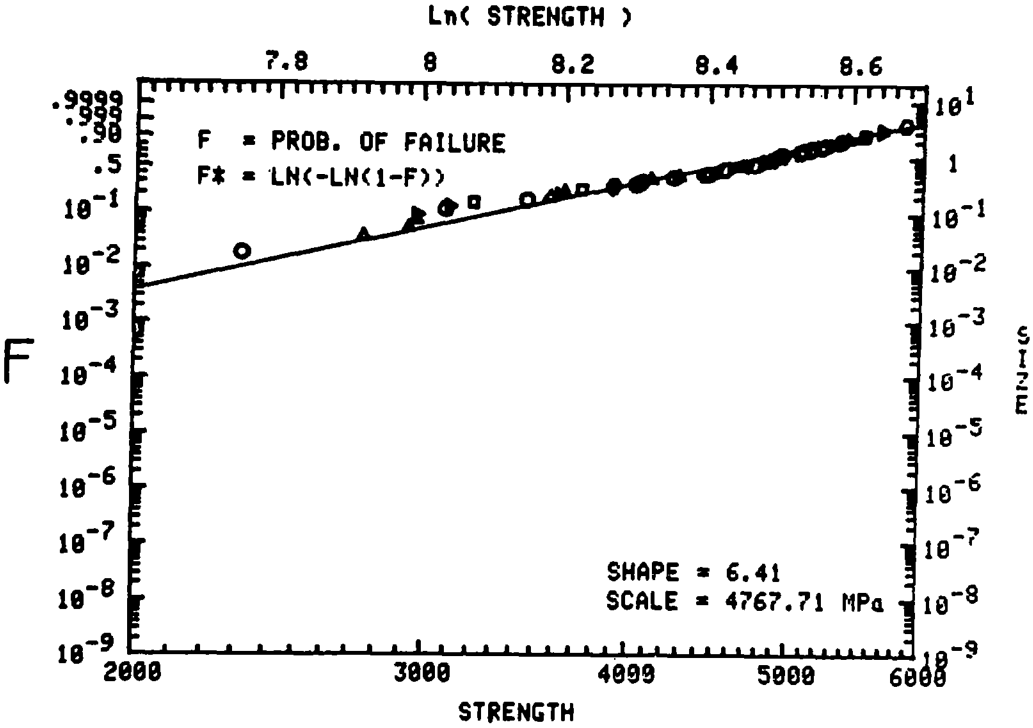
BUT EXISTENCE OF A SINGLE FLAW LEADS TO FAILURE
(WEAKEST LINK OF CHAIN)

∴ LARGE SCATTER

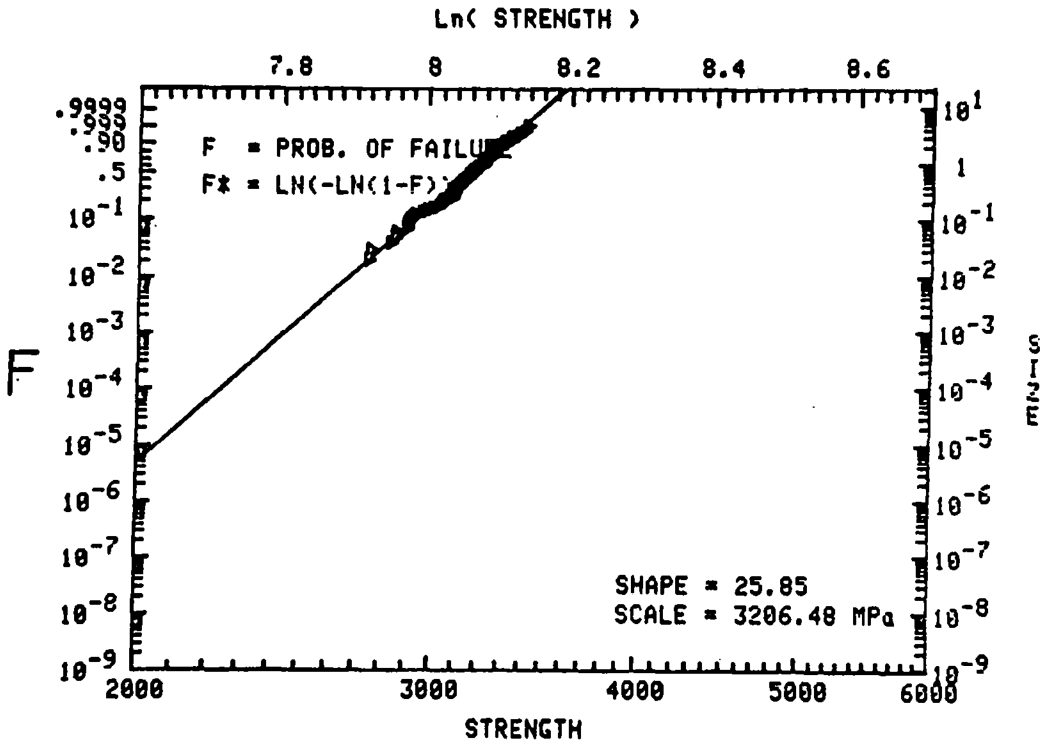
	COEFF. OF VAR.	SHAPE
FIBER	20%	6-8
STEEL	3-5%	25-50

	WITH MATRIX	WITHOUT MATRIX
BUNDLE STRENGTH	550 ksi	350 ksi
BUNDLE SCATTER	4-5%	20-25%

- LONGITUDINAL COMPOSITE STRENGTH IS FIBER-DOMINATED WITH SUBSTANTIAL MATRIX INFLUENCE

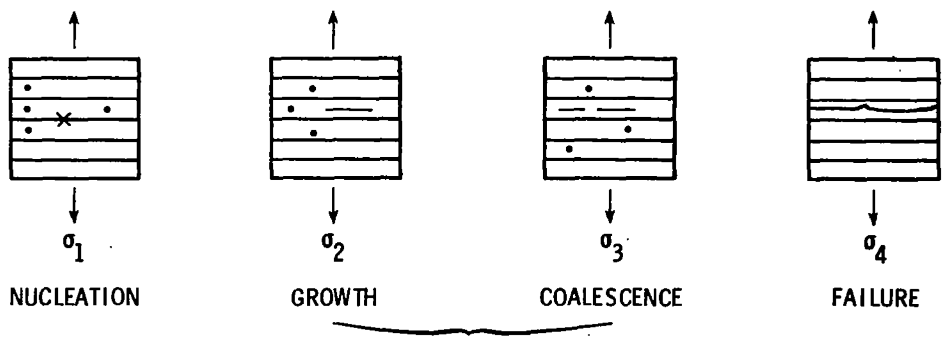


KEVLAR 49 FILAMENT (5CM)
INTRINSIC STRENGTH (.02CM/MIN,23C),N=54



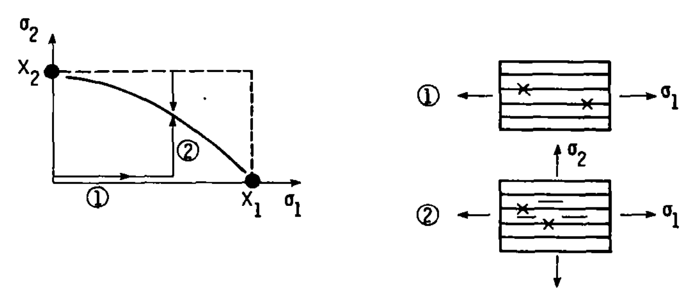
KELVAR 49-332/T403 STRAND
INTRINSIC STRENGTH (1CM/MIN),N=100

TRANSVERSE STRENGTH: MATRIX-DOMINATED OR MATRIX-CONTROLLED (REF. 4)

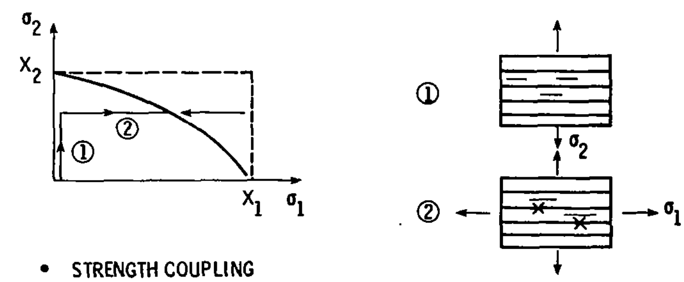


TRANSVERSE STRENGTH IS MATRIX-DOMINATED,
INFLUENCED BY FIBER AND PACKING, NOT WELL-DEFINED PLANE OF FAILURE

STRENGTH COUPLING UNDER COMBINED STRESS

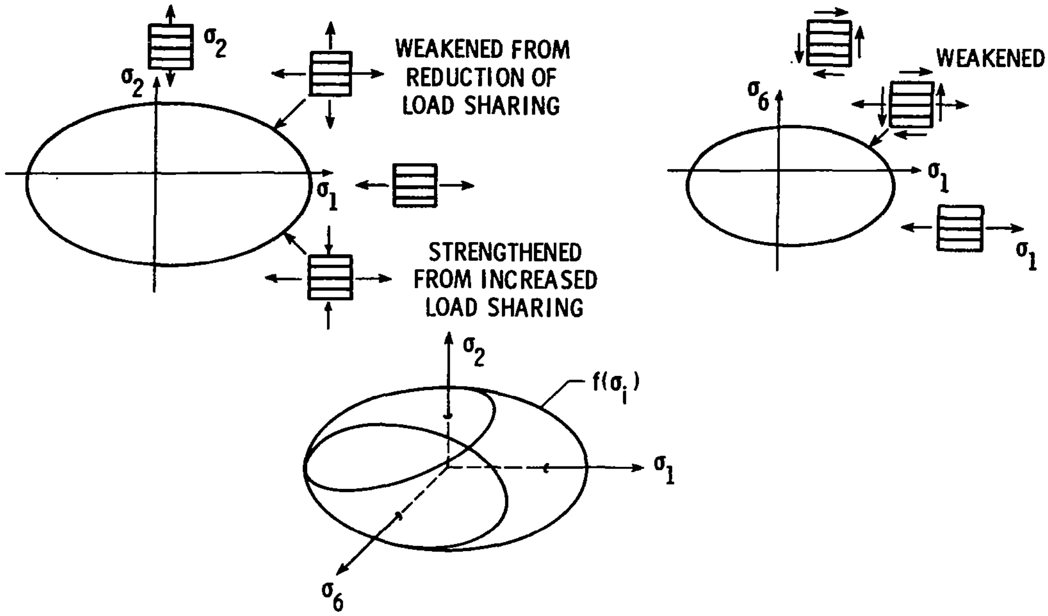


- BROKEN FIBER INITIATES TRANSVERSE CRACK, THEREFORE REDUCES TRANSVERSE STRENGTH



- STRENGTH COUPLING IS EXPECTED FROM PHYSICS OF FAILURE
- TRANSVERSE CRACK REDUCES LOAD TRANSFER, INCREASES δ, REDUCES LONGITUDINAL STRENGTH

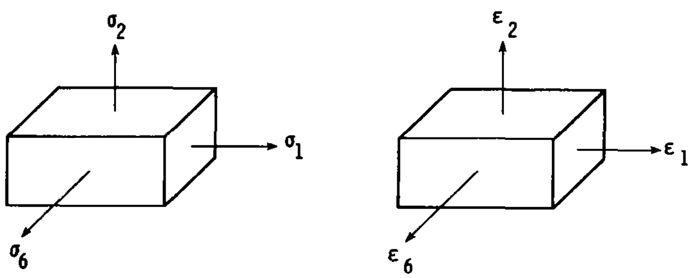
FAILURE SURFACE FOR STRENGTH COUPLING (REFS. 5 AND 6)



$$f(\sigma_i) = \sigma_i F_i + F_{ij} \sigma_i \sigma_j + F_{ijk} \sigma_i \sigma_j \sigma_k + \cdots \leq 1$$

TENSOR POLYNOMIAL EXPANSION OF SURFACE

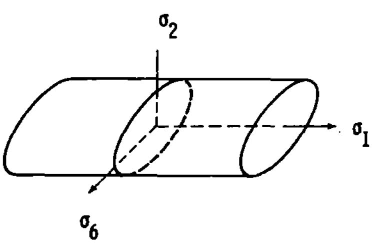
NONCOUPLING STRENGTH CRITERIA



MAX. STRESS (RANKIN)

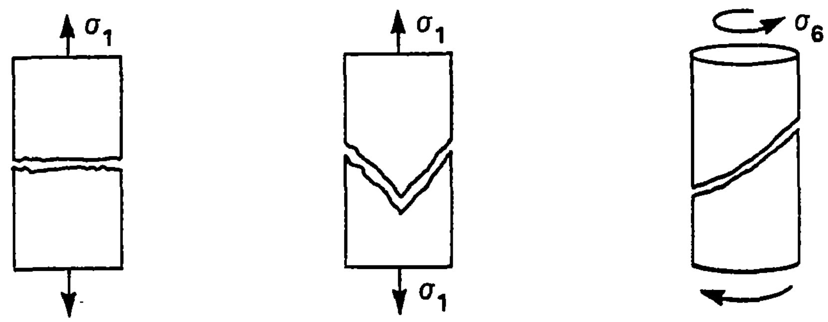
MAX. STRAIN (ST. VENANT)

BOTH CAN BE EXPRESSED AS TENSOR POLYNOMINALS (REF. 5)



TRANSVERSE/TRANSVERSE COUPLING
NO TRANSVERSE/LONGITUDINAL COUPLING
(REF. 7)

FAILURE TESTS REQUIRES WELL-DEFINED FAILURE MODES



if $\sigma_1^* < 2\sigma_6^*$	if $\sigma_1^* > 2\sigma_6^*$	$\sigma_1^* < \sigma_6^*$
Tensile failure mode	Combine failure mode	Tensile failure mode

Plane of Failure does not necessarily correspond to applied stress

Plane of Failure of composites are seldom well defined

Failure Criterion based on applied stress is NOT Mechanistic; it is Operational

CONSISTENCY IN STRENGTH THEORY FORMULATION

	Output		Mtl constant	Input
Deformation	ϵ_{ij}	=	S_{ijkl}	σ_{kl}
	ϵ_{ij}	=	α_{ij}	ΔT
Strength	σ_{ij}^*	=	( )	σ_{ij}

$\sigma_{ij} < \sigma_{ij}^*$ uniaxial

$\sigma_{ij} \leq f(\sigma_{ij}^*)$ combined

$$f(\sigma_{ij}) = F_{ij}\,\sigma_{ij} + F_{ijkl}\,\sigma_{ij}\,\sigma_{kl} + F_{ijklmn}\,\sigma_{ij}\,\sigma_{kl}\,\sigma_{mn} + \ldots$$

F_{ij} etc material constants dimensions $\left[\frac{1}{\text{stress}}\right], \left[\frac{1}{\text{stress}^2}\right] \ldots$

- Independent of mtls coordinates
- Allow mathematical operations

FAILURE STRENGTH MEASUREMENTS

Design loading conditions (direction of stress vector) which is capable of discriminating failure criteria

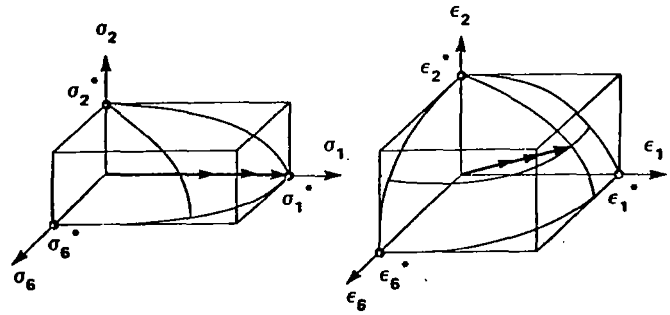
Stimuli

Failure strains		σ_1	σ_2	σ_6
ϵ_1^*	ϵ_1	S_{11}	S_{12}	S_{16}
ϵ_2^*	ϵ_2	S_{12}	S_{22}	S_{26}
ϵ_6^*	ϵ_6	S_{16}	S_{26}	S_{66}
		σ_1^*	σ_2^*	σ_6^*

Failure stresses

$$\underbrace{F_i\sigma_i + F_{ij}\sigma_i\sigma_j}_{} + \ldots = 1$$

Material response constants



One dimensional stress give rise to 3D strain

One dimensional strain give rise to 3D stress

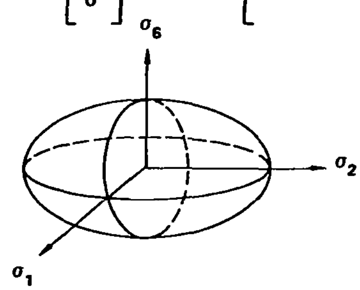
Consistant characterization desirable

THREE-DIMENSIONAL FAILURE CRITERION

Tensor-polynominal Failure Criterion:

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j + \ldots = 1 \qquad i,j = 1,2,3,4,5,6$$

$$F_i = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad F_{ij} = \begin{bmatrix} F_{11} & F_{12} & F_{13} & 0 & 0 & 0 \\ & F_{22} & F_{23} & 0 & 0 & 0 \\ & & F_{33} & 0 & 0 & 0 \\ & & & F_{44} & 0 & 0 \\ & & & & F_{55} & 0 \\ & & & & & F_{66} \end{bmatrix}$$



$\sigma_1, \sigma_2, \sigma_6$

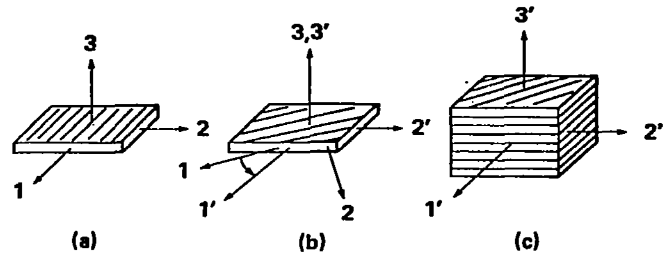
$\sigma_3, \sigma_4, \sigma_5$

2-D a failure surface **3-D a hyper-surface (6)**

- **How many independent tests? (12)**
- **What are the tests?**

NO. OF INDEPENDENT TESTS

Failure tensor F_i, F_{ij} follow tensor transformation rules



- 3-D failure criterion for lamina (a) only

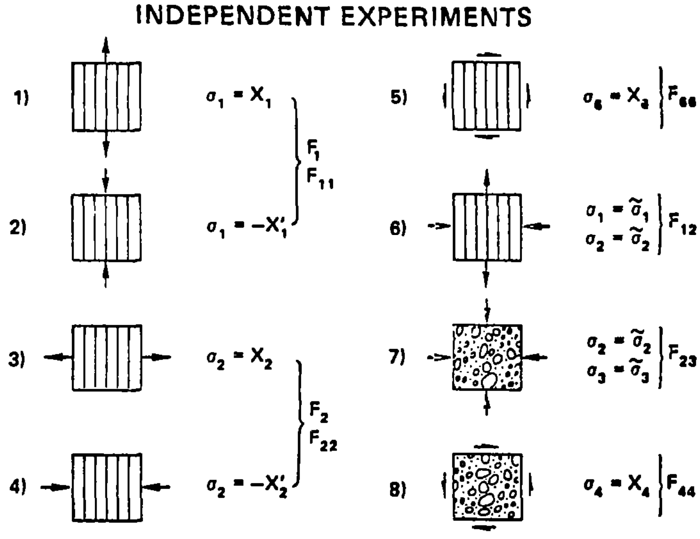
Symmetry condition of orthotropic lamina 2 = 3

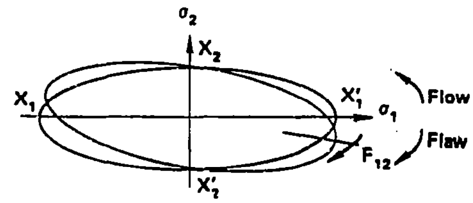
Tensor Notation	Contracted Notation	
$F_{22} = F_{33}$	$F_2 = F_3$	
$F_{1122} = F_{1133}$	$F_{12} = F_{13}$	12 – 4 = 8
$F_{2222} = F_{3333}$	$F_{22} = F_{33}$	
$F_{1313} = F_{1212}$	$F_{55} = F_{66}$	

Component not associate with planer properties

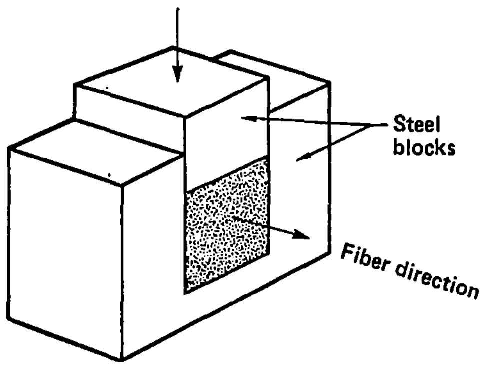
F_{2233} (F_{23}) Transverse strength coupling
F_{2323} (F_4) Shear

INDEPENDENT EXPERIMENTS

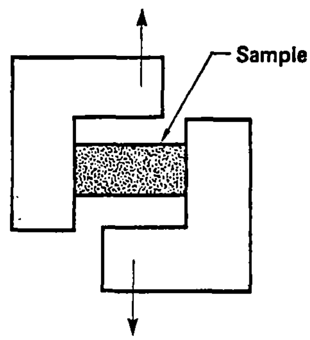




EXPERIMENTS ASSOCIATE WITH THICKNESS DIRECTION



Experiment No. 7
Biaxial compression



Experiment No. 8
Shear test

SHOULD TENSILE AND COMPRESSIVE STRENGTH BE RELATED?

FOR UNIAXIAL TENSION: $\sigma_1 \neq 0, \quad \sigma_i = 0, \quad i \neq 1$

$$F_{11}\,\sigma_1^2 + F_1\,\sigma_1 = 1 \quad \Rightarrow F_1 = \frac{1}{X_1} - \frac{1}{X_1'}, \; F_{11} = \frac{1}{X_1 X_1'}$$

$\sigma_1 (F_{11}, F_1) = \sigma_1 (X_1, X_1')$ — X_1 TENSILE STRENGTH; X_1' COMPRESSIVE STRENGTH

$$F_1 = \frac{1}{X_1} -$$

ALTERNATE EQUIVALENT FORM:

$$\tilde{F}_{11}\,(\sigma_1 - \tilde{\sigma}_1)^2 = 1$$

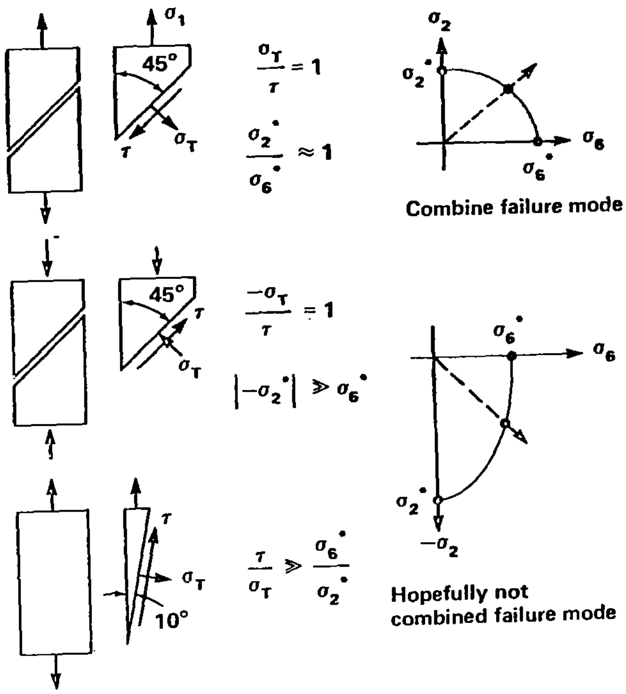
$$\tilde{F}_{11}\,\sigma_1^2 - (2\tilde{\sigma}_1 \tilde{F}_{11})\,\sigma_1 = 1 - \tilde{\sigma}_1 F_{11}$$

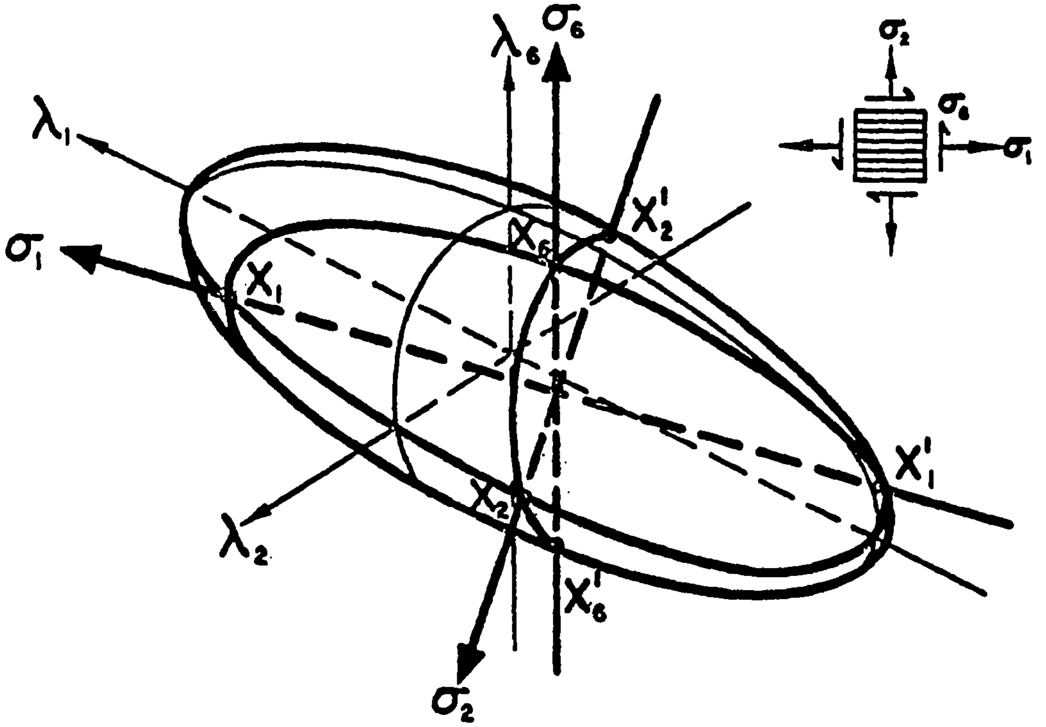
NOW, INTERPRET $\tilde{\sigma}_1$ AS INTERNAL STRESS (A MATERIAL CONSTANT).

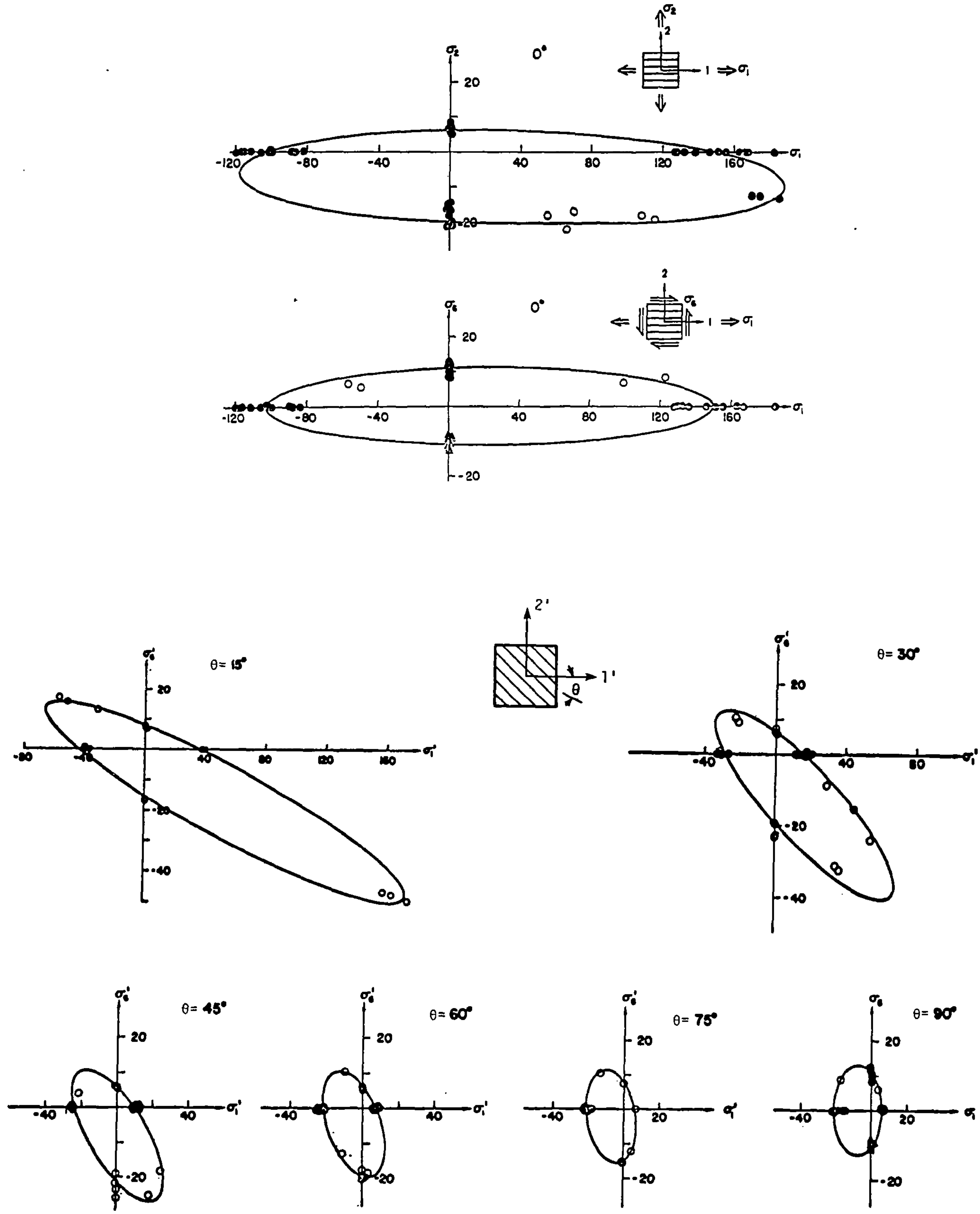
STRESS ANALYSIS MUST OPERATE ON $(\sigma_1 - \tilde{\sigma}_1)$,

WHICH IS AN OPERATIONAL INCONVENIENCE

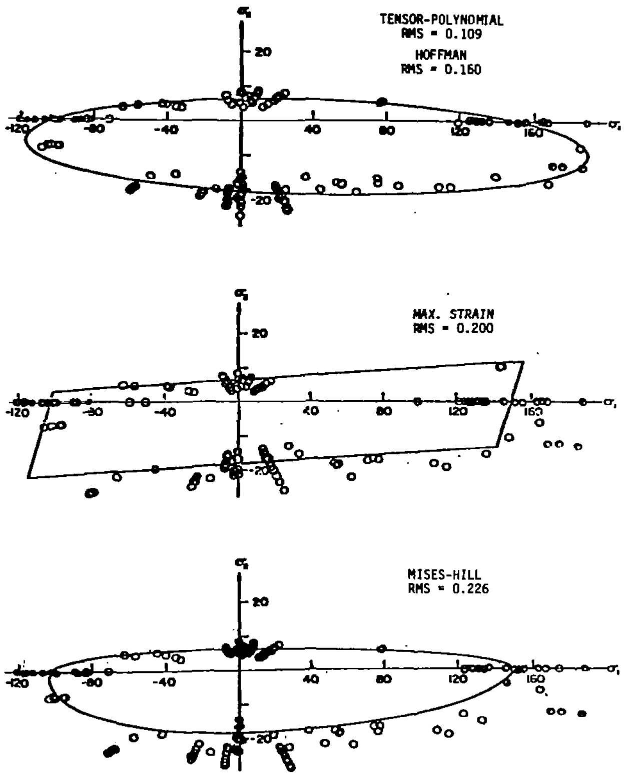
Off-axis sample to measure shear strength



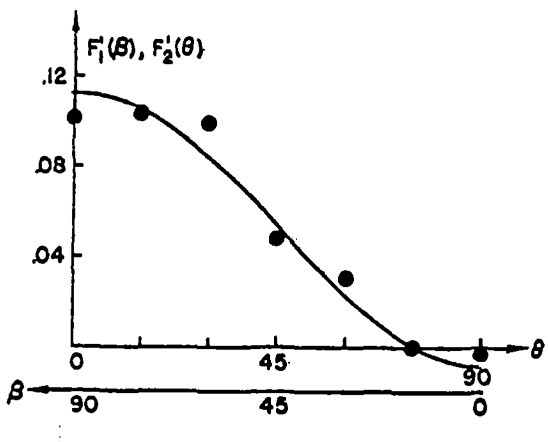


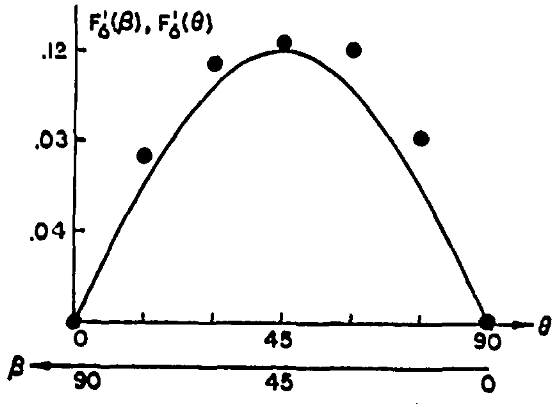


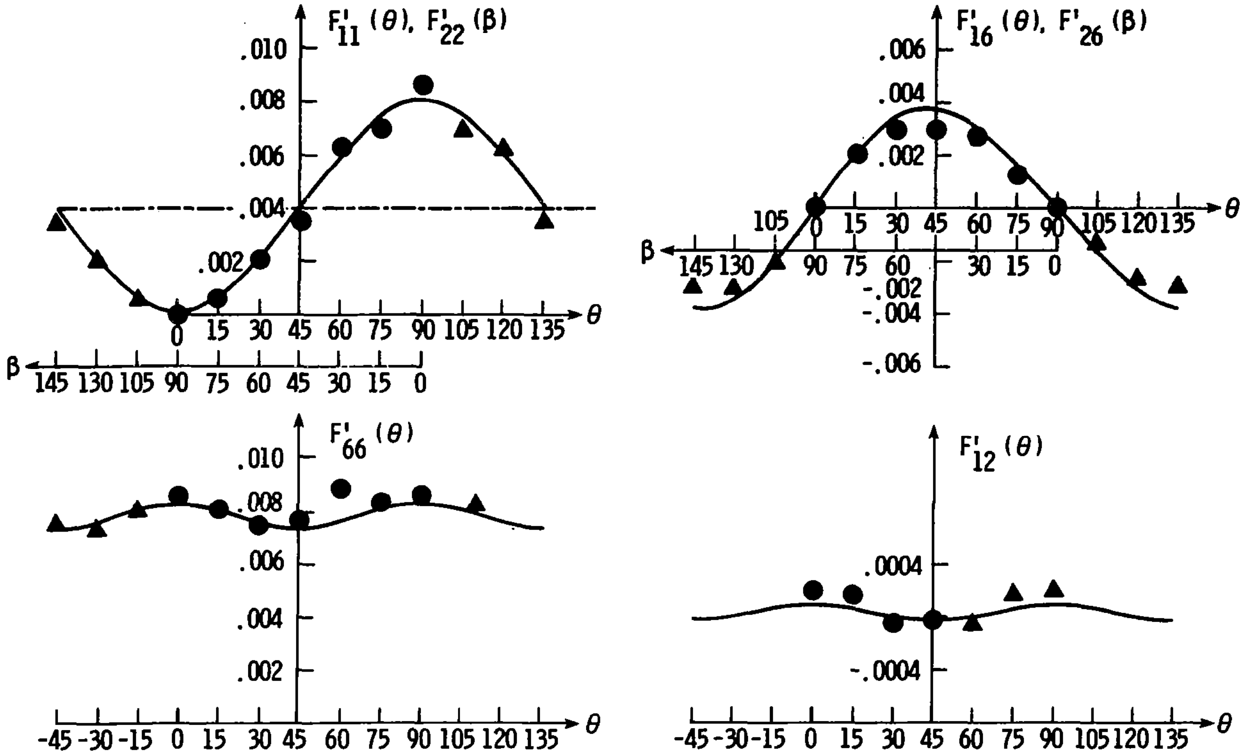
0°
0°
θ= 15°
θ= 30°
θ= 45°
θ= 60°
θ= 75°
θ= 90°
2'
1'
θ



DATA INDICATES STRENGTH COUPLING



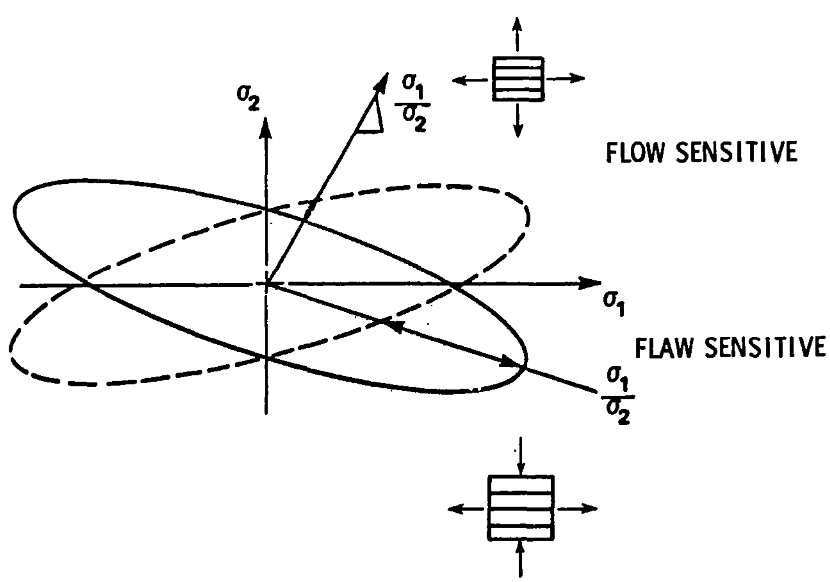




SENSITIVITY OF COMBINED STRESS EXPERIMENTS

$$F_{12} = (1 - F_1 \tilde{\sigma}_1 - F_2 \tilde{\sigma}_2 - F_{11} \tilde{\sigma}_1^2 - F_{22} \tilde{\sigma}_2^2)(1/2_{\tilde{\sigma}_1 \tilde{\sigma}_2})$$

$$F_{23} = (1 - F_2 \tilde{\sigma}_2 - F_3 \tilde{\sigma}_3 - F_{22} \tilde{\sigma}_2^2 - F_{33} \tilde{\sigma}_3^2)(1/2_{\tilde{\sigma}_2 \tilde{\sigma}_3})$$



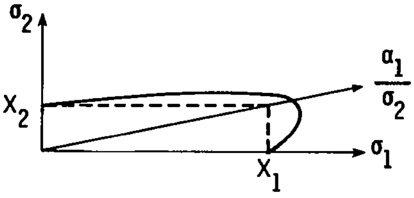
DETERMINATION OF COUPLING COEFFICIENTS

- EXPERIMENTAL MEASURMENT AT OPTIMIZED STRESS RATIO $\frac{\sigma_1}{\sigma_2}$ (REF. 5)

- SIMPLIFIED STRESS RATIO:

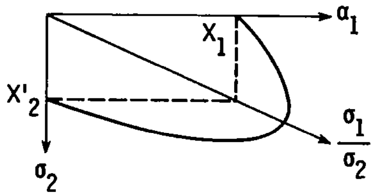
FOR DUCTILE MATRIX

MEASURE AT $\frac{\sigma_1}{\sigma_2} \sim \frac{X_1}{X_2}$



FOR BRITTLE MATRIX

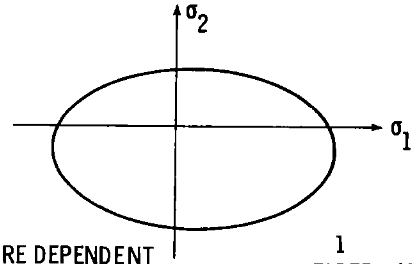
MEASURE AT $\frac{\sigma_1}{\sigma_2} \sim \frac{X_1}{-X'_2}$



- HEURISTIC ESTIMATION - BOUND $F_{12}^2 \leq F_{11} F_{22}$

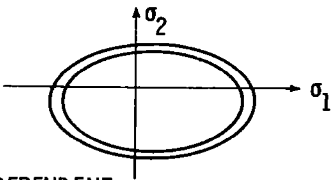
IF X_1 HAS SMALL SCATTER THEN $F_{12} \longrightarrow$ SMALL $\longrightarrow 0$ } WITHIN BOUND
IF X_1 HAS LARGE SCATTER THEN $F_{12} \longrightarrow$ LARGE

COMBINED STRESS, ANISOTROPIC STRENGTH

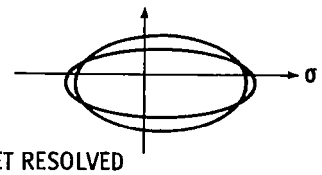


STRENGTHS ARE DEPENDENT

1 FIBER
2 MATRIX



STRENGTHS ARE INDEPENDENT



ISSUE NOT YET RESOLVED

SUMMARY

- MODELING OF SEQUENTIAL FAILURE - RECENT PROGRESS
- SHAPE OF COMBINED STRESS FAILURE SURFACE - LONGITUDINAL AND TRANSVERSE COUPLING EXPECTED AND OBSERVED
- FOR STRENGTH-COUPLED FAILURE SURFACE, TENSOR POLYNOMIAL IS OPERATIONALLY ATTRACTIVE:

  COUPLING COEFFICIENTS (F_{12}) CAN BE MEASURED OR HEURISTICALLY ESTIMATED

  READILY EXTENDABLE TO 3-D AND HIGHER ORDER

  MOST ISSUES IN PROPER ORDER
- PROBABLISTIC REPRESENTATION OF FAILURE SURFACE NOT YET RESOLVED

REFERENCES

1. Rosen, B. W.: Tensile Failure of Fibrous Composites. AIAA Journal, vol. 2, no. 11, November 1964, pp. 1985-1991.

2. Zweben, C.; and Rosen, B. W.: A Statistical Theory of Material Strength With Application to Composite Materials. J. Mech. Physics of Solids, vol. 18, 1970, pp. 189-206.

3. Harlow, E. G., and Phoenix, L.: The Chain-of-Bundles Probability Model for the Strength of Fibrous Materials. Part I and II. J. of Composite Materials, vol. 12, April 1978, pp. 195-214.

4. Chamis, C. C.; and Sinclair, J. H.: Ten-Deg. Off-Axis Test for Shear Properties in Fiber Composites. Experimental Mechanics, vol. 17, no. 9, September 1977, pp. 339-346.

5. Wu, E. M.: Phenomenological Anisotropic Failure Criterion. Composite Materials, vol. 2 (edited by G. P. Sendeckyj), Academic Press, New York, 1974, pp. 354-431.

6. Malmeister, A. K.: Geometry of Endurance Theory. Mekhanika Polimerov, vol. 2, no. 4, 1966, pp. 519-534.

7. Puck, A.: Calculating the Strength of Glass-Fibre Plastic Laminates Under Combined Load. Kunstoffe, vol. 59, no. 11, pp. 780-787.