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A Mathematical Model for the Doubly Fed Wound Rotor Generator

F. J. Brady
National Aeronautics and Space Administration
Lewis Research Center

Work performed for
U.S. DEPARTMENT OF ENERGY
Conservation and Renewable Energy
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ABS: A mathematical analysis of a doubly-fed wound rotor machine used as a constant frequency generator is presented. The purpose of this analysis is to derive a consistent set of circuit equations which produce constant stator frequency and constant stator voltage. Starting with instantaneous circuit equations, the necessary rotor voltages and currents are derived. The model, thus obtained, is assumed to be valid, since the resulting relationships between mechanical power and active volt-amperes agrees with the results of others. In addition, the model allows for a new interpretation of the power flow in the doubly-fed generator.

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A MATHEMATICAL MODEL FOR THE DOUBLY FED WOUND ROTOR GENERATOR

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Summary - Present electrical power generators dictate that the prime movers operate at constant speed. This is necessary because the power must be generated at constant frequency. However, some prime movers are, by their nature, not constant speed devices, such as, hydroturbines and wind turbines. Various schemes have been proposed that remove the constant speed requirement. The scheme investigated in this paper is a wound rotor generator, whose rotor is excited with variable frequency power.

This paper develops a mathematical model which predicts what the rotor excitation must be, if the stator is to operate at constant frequency and voltage. A machine with three-phase rotor and stator is analyzed. It is described by two sets of circuit equations; one set for the stator, another for the rotor. Under balanced conditions, these two sets reduce to only two equations. In practical application, these machines are operated with constant stator terminal voltage. When this constraint is applied to the two circuit equations, expressions for currents and voltages, on both rotor and stator are obtained.

These same circuit equations can be used to generate a set of volt-ampere equations. If the expressions for current and voltages are substituted into the volt-ampere equations, a relationship between active stator volt-ampere, active rotor volt-amperes, and shaft power is obtained. This relationship permits a new interpretation of the power flow.

INTRODUCTION

The doubly-fed wound rotor generator is capable of producing constant stator frequency while the shaft speed varies. In order to be of practical use, it must also maintain constant stator voltage at any speed and any load current. This paper develops a mathematical model for the wound rotor machine, which shows what the rotor excitation must be to satisfy the above needs. Expressions for power can be derived using the model. The power equations allow for a new interpretation to the power flow that takes place in the machine.

SYMBOLS

a	ratio of stator number of turns to rotor number of turns
i_R, i_S	instantaneous rotor and stator current, respectively
i_{Ro}	rotor current required to generate v_{So}
$i_{R\epsilon}$	rotor current required to compensate for stator impedance drop
k	coupling coefficient
L_R, L_S	self inductance, rotor and stator, respectively
n_R, n_S	number of turns
P_M	mechanical shaft power
R_R, R_S	resitance of winding, rotor and stator, respectively
S	slip
t	time
v_R, v_S	instantaneous terminal voltage, rotor and stator, respectively
v_{Ro}	rotor voltage required to establish i_{Ro}
$v_{R\epsilon}$	rotor voltage required to establish $i_{R\epsilon}$
$(vi)_S, (VI)_S$	instantaneous and mean value volt-amperes in the stator circuit, respectively
$(vi)_r, (VI)_r$	instantaneous and mean value volt-amperes induced in the rotor circuit, respectively
$\overline{x}$	the bar above a variable indicates it is a phasor
$\theta_R, \theta_S, \theta_L$	phase angle shift, rotor, stator, and load, respectively
$\rho\omega/2$	electrical angular frequency (rad/sec), where ρ = number of poles
φ_R, φ_S	instantaneous flux, rotor and stator, respectively
ω	mechanical angular frequency (rad/sec)
ω_S	synchronous electrical angular frequency (rad/sec)

DEVELOPMENT OF THE MODEL

Before proceeding to the development of the model, a few comments are necessary about the general approach taken in this paper. The device under consideration here is a machine with a three-phase winding on both the rotor and stator. When writing the elementary voltage equation for one of the phases, it must include five induced voltage terms because of coupling to the other five windings. The model and the solution to the equations depend heavily on how these flux terms are grouped. The grouping of terms used here differs from the textbook approach.

To illustrate this, the voltage equation for any phase may be used; they all have the same form, since balanced conditions are assumed. Using figure 1, and writing the equation for stator phase 1 gives,

$$\overline{v}_{S1} = R_{S1}\overline{i}_{S1} + \left[L_{S1}\frac{d}{dt}\overline{i}_{S1} - n_{S1}\frac{d}{dt}(k\overline{\varphi}_{S2}) - n_{S1}\frac{d}{dt}(k\overline{\varphi}_{S3})\right] + \left[n_{S1}\frac{d}{dt}(k\overline{\varphi}_{R1}) + n_{S1}\frac{d}{dt}(k\overline{\varphi}_{R2}) + n_{S1}\frac{d}{dt}(k\overline{\varphi}_{R3})\right] \qquad (1-1)$$

The brackets indicate the grouping of terms used in this paper. The first bracket collects the stator fluxes, the second collects the rotor fluxes. This differs from the textbook approach, which combines all five flux terms into one, referred to as mutual flux.

The five induced voltage terms contain the factor k, which will be explained further. The second term in the first bracket is the voltage induced in stator phase 1 by the current in stator phase 2. $\overline{\varphi}_{S2}$ is the total flux generated by current $\overline{i}_{S2}$. $\overline{\varphi}_{S2}$ is the sum of the useful flux that couples phase 1 winding and the wasted (leakage) flux. The term $k\varphi_{S2}$ symbolizes the useful flux that couples stator phase 2 to phase 1. A similar explanation holds for the third term in the first bracket.

The same k factor appears in the flux terms of the second bracket. It can be argued that the k factor in the first bracket should be different from the k factor in the second bracket. The first k factor is the coefficient of coupling between stator windings; the second, is the coefficient of coupling between any rotor winding and a stator winding. If two distinct k are used, the results of this paper are only slightly changed. However, for simplicity, the k's are assumed to be equal.

The two brackets can now be simplified. If balanced conditions (Appendix A) are assumed, the first bracket becomes

$$L_S \frac{d}{dt}\left[1 - k\, e^{-j(3/2)\pi} - k\, e^{-j(4/3)\pi}\right]\bar{i}_{S1}$$

The second bracket can be written as,

$$k n_{S1} \frac{d}{dt}\left[\bar{\varphi}_{R1} + \bar{\varphi}_{R2} + \bar{\varphi}_{R3}\right]$$

It is assumed in this paper that the magnetic circuit is linear; this allows the rotor fluxes to be added. The previous expression becomes, $kn_{S1}(d/dt)(\bar{\varphi}_R)$. This expressions is evaluated in Appendix B.

Equation (1-1) reduces to the following.

$$\bar{v}_{S1} = R_S\bar{i}_{S1} + (1 + k)L_S \frac{d}{dt}\bar{i}_{S1} + n_S k \frac{d}{dt}\bar{\varphi}_R \qquad (1\text{-}2)$$

If the rotor is represented in a manner similar to Fig. 1, the instantaneous rotor voltage equation will be similar to equation (1-1) and for balanced conditions it is,

$$\bar{v}_{R1} = R_R\bar{i}_{R1} + (1 + k)L_R \frac{d}{dt}\bar{i}_{R1} + n_R k \frac{d}{dt}\bar{\varphi}_S \qquad (1\text{-}3)$$

The terms $\bar{\varphi}_R$ and $\bar{\varphi}_S$ that appear in equations (1-2) and (1-3) are fluxes, each produced by a distributed three-phase winding (Appendix B).

If these flux expressions are substituted into equations (1-2) and (1-3), the resulting per phase equations are as follows,

$$\bar{v}_S = R_S\bar{i}_S + (1 + k)L_S \frac{d}{dt}\bar{i}_S + \frac{3}{2}akL_R \frac{d}{dt}\left[e^{j(\rho\omega t/2)}\,\bar{i}_R\right] \qquad (1\text{-}4)$$

$$\bar{v}_R = R_R\bar{i}_R + (1 + k)L_R \frac{d}{dt}\bar{i}_R + \frac{3}{2}\frac{kL_S}{a} \frac{d}{dt}\left[e^{-j(\rho\omega t/2)}\,\bar{i}_S\right] \qquad (1\text{-}5)$$

These two equations are the basic model of the wound rotor machine. All expressions that follow will be derived from them.

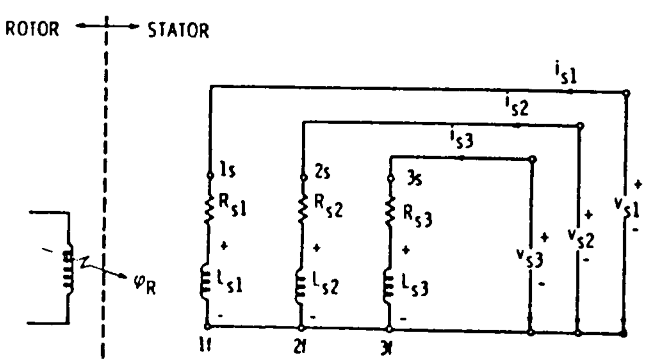

Figure 1. - Stator circuit diagram

In a practical application of this machine, it would be operated at constant stator terminal voltage; therefore, the model should include this constraint. That is, the stator terminal voltage must remain the same for both open- and closed-circuit. This voltage will be designated, $\bar{v}_S$. The rotor terminal voltage is assumed to have two components; one for open-circuited stator, $\bar{v}_{Ro}$, and one for closed-circuited stator, $\bar{v}_{R\epsilon}$. The rotor currents produced by these voltages are designated $\bar{i}_{Ro}$ and $\bar{i}_{R\epsilon}$, respectively. If $\bar{v}_R = \bar{v}_{Ro} + \bar{v}_{R\epsilon}$ and $\bar{i}_R = \bar{i}_{Ro} + \bar{i}_{R\epsilon}$ are substituted into equations (1-4) and (1-5), and then two stator conditions are considered; namely, $\bar{i}_S = 0$ and $\bar{i}_S \neq 0$, the following equations are obtained. The condition $\bar{i}_S = 0$ gives,

$$\bar{v}_S = \frac{3}{2}akL_R \frac{d}{dt}\left[e^{j(\rho\omega t/2)}\,\bar{i}_{Ro}\right] \qquad (1\text{-}6)$$

$$\bar{v}_{Ro} = R_R\bar{i}_{Ro} + (1 + k)L_R \frac{d}{dt}\bar{i}_{Ro} \qquad (1\text{-}7)$$

The condition $\bar{i}_S \neq 0$ gives,

$$0 = R_S\bar{i}_S + (1 + k)L_S \frac{d}{dt}\bar{i}_S + \frac{3}{2}akL_R \frac{d}{dt}\left[e^{j(\rho\omega t/2)}\,\bar{i}_{R\epsilon}\right] \qquad (1\text{-}8)$$

$$\bar{v}_{R\epsilon} = R_R\bar{i}_{R\epsilon} + (1 + k)L_R \frac{d}{dt}\bar{i}_{R\epsilon} + \frac{3}{2}\frac{k}{a}L_S \frac{d}{dt}\left[e^{-j(\rho\omega t/2)}\,\bar{i}_S\right] \qquad (1\text{-}9)$$

Equation (1-8) says that, if constant stator voltage is to be achieved for all stator currents, then the induced voltage produced by $\bar{i}_{R\epsilon}$ must compensate for the stator impedance drop. Equation (1-9) says that $v_{R\epsilon}$ is the source voltage required to overcome the rotor impedance drop caused by $\bar{i}_{R\epsilon}$ and the induced voltage drop produced by $\bar{i}_S$.

Equations (1-6) through (1-9) represent the constant voltage model of a wound rotor machine. These differential equations can now be solved for instantaneous values of voltages and currents. These solutions have practical significance because they will show what the rotor voltages and currents must be in order to obtain the desired stator effects.

DERIVATION OF ROTOR EXCITATION

The desired stator effects are; one, the terminal voltage must be independent of slip; and second, it must remain constant for all values of load current. The solutions that follow will show that the independence from slip is controlled by $\bar{v}_{Ro}$ and $\bar{i}_{Ro}$, while the second effect is regulated by $\bar{v}_{R\epsilon}$ and $\bar{i}_{R\epsilon}$.

Equation (1-7) is solved by assuming that the applied rotor voltage is $\bar{v}_{Ro} = V_R\, e^{j(S\omega_S t)}$, where $S\omega_S = \omega_S - (\rho\omega/2)$ is the slip frequency. When this expression for v_{Ro} is substituted into equation (1-7) and the resulting expression is integrated, the result is,

$$\bar{i}_{Ro} = \frac{V_R}{|Z_R|}\, e^{j(S\omega_S t - \theta_R)} \qquad (2\text{-}1)$$

where

$$Z_R = \sqrt{R_R^2 + \left[(1 + k)SX_R\right]^2}, \; X_R = \omega_S L_R$$

When equation (2-1) is substituted in (1-6), and the differentiation performed, the result is,

$$\overline{v}_S = \frac{3}{2} ak \frac{X_R}{|Z_R|} V_R e^{j(\omega_S t - \theta_R + \pi/2)} \tag{2-2}$$

The two requirements on the stator terminal voltage are, first, that it be constant frequency and second, that its amplitude be constant. Equation (2-2) shows that the first is satisfied. The second is satisfied if

$$V_R = \frac{2}{3} \frac{(1 + k)}{ak} V_S \sqrt{\left[\frac{R_R}{(1 + k)X_R}\right]^2 + S^2} \tag{2-3}$$

where V_S is the desired peak value of stator voltage.

Equation (2-3) indicates how the rotor voltage must be regulated in order to compensate for slip. Substituting this same equation into the expression for $\overline{v}_{Ro}$ and $\overline{i}_{Ro}$, yield the following,

$$\overline{v}_{Ro} = \left\{\frac{2}{3} \frac{(1 + k)}{ak} V_S \sqrt{\left[\frac{R_R}{(1 + k)X_R}\right]^2 + S^2} \; e^{jS\omega_S t}\right\} \tag{2-4}$$

and

$$\overline{i}_{Ro} = \left(\frac{2}{3} \frac{1}{ak} \frac{V_S}{X_R}\right) e^{j(S\omega_S t - \theta_R)} \tag{2-5}$$

Comparison of these two equations, yields a significant result. That is, the stator terminal voltage can be made independent of slip in either of two ways. One, a voltage regulator must sense slip and produce an AC output, $\overline{v}_{Ro}$. The other, would be to supply the rotor through a constant current AC source, $\overline{i}_{Ro}$.

To show how the stator terminal voltage is made independent of load current, the expressions for $\overline{v}_{R\epsilon}$ and $\overline{i}_{R\epsilon}$ must be solved. Substituting equation (2-3) into (2-2) gives,

$$\overline{v}_S = V_S e^{j(\omega_S t - \theta_R + \pi/2)} \tag{2-6}$$

Because $\overline{v}_S$ remain constant, stator current is a function of load only. For lagging power factor loads the equation is

$$\overline{v}_S = R_L \overline{i}_S + L_L \frac{d}{dt}(\overline{i}_S) \tag{2-7}$$

Substituting equation (2-6) into (2-7) and solving for $\overline{i}_S$, gives

$$\overline{i}_S = I_S e^{j(\omega_S t - \theta_R - \theta_L + \pi/2)} \tag{2-8}$$

where

$$I_S = \frac{V_S}{|Z_L|}$$

and

$$|Z_L| = \sqrt{R_L^2 + X_L^2}, \; X_L = \omega_S L_L$$

$\overline{i}_{R\epsilon}$ is obtained by substituting equation (2-8) into (1-8). Integrating the resulting expression gives,

$$\overline{i}_{R\epsilon} = \frac{2}{3} \frac{1}{ak} \frac{|Z_S|}{X_R} I_S e^{j(S\omega_S t + \theta_S - \theta_R - \theta_L)} \tag{2-9}$$

$\overline{v}_{R\epsilon}$ is obtained by substituting equations (2-9) and (2-8) into (1-9). After differentiation the result is,

$$\overline{v}_{R\epsilon} = \frac{1}{a}\left[\frac{2}{3} \frac{1}{k} \frac{|Z_R||Z_S|}{X_R} I_S e^{j(S\omega_S t + \theta_S - \theta_L)} + \frac{3}{2} k X_S S I_S e^{j(S\omega_S t - \theta_R - \theta_L)}\right] \tag{2-10}$$

Comparison of equations (2-9) and (2-10) shows that the stator voltage is made independent of load current by either of two means. If a voltage regulator is used, it must sense $\overline{i}_S$, as well as slip, and produce $\overline{v}_{R\epsilon}$, as shown by equation (2-10). The same effect is obtained by use of a constant current regulator, described by equation (2-9), which senses only $\overline{i}_S$.

DERIVATION OF POWER EQUATIONS

The approach used in this paper has not been used previously. Therefore, the model that has been derived must agree with previous information if it is to be valid. A great deal of past effort has been spent determining the relationship between real power and active volt-amperes on the rotor and stator. The derivations that follow will show that the model developed here agrees with accepted theory.

Stator volt-amperes is obtained by multiplying equation (1-4) by $\overline{i}_S$. The last term in the resulting expression, represents the instantaneous volt-amperes flowing to the stator, that is,

$$\overline{(vi)_S} = \frac{3}{2} akL_R\left\{\overline{i}_S \cdot \frac{d}{dt}\left[e^{j(\rho\omega t/2)} (\overline{i}_{Ro} + \overline{i}_{R\epsilon})\right]\right\} \tag{3-1}$$

If the differentiation is carried out, two terms are generated. Equation (3-1) becomes,

$$\overline{(vi)_S} = \frac{3}{2} akL_R\left\{\left[\overline{i}_S \cdot \frac{\rho\omega}{2} e^{j(\rho\omega t/2 + \pi/2)} (i_{Ro} + i_{R\epsilon})\right] + \left[\overline{i}_S \cdot e^{j(\rho\omega t/2)} \frac{d}{dt}(\overline{i}_{Ro} + \overline{i}_{R\epsilon})\right]\right\} \tag{3-2}$$

The first bracket will be labeled $\overline{(vi)_\omega}$, signifying it originates because of a changing rotation phasor. The second bracket will be labeled $\overline{(vi)_X}$ since it originates because of a transformer action. Using this notation, equation (3-2) becomes,

$$\overline{(vi)_S} = \overline{(vi)_\omega} + \overline{(vi)_X} \qquad (3\text{-}3)$$

The active volt-amperes flowing to the stator is found by calculating the mean value of equation (3-2). This is done by converting the phasor quantities to real ones, substituting appropriate values of current, then integrating the whole expression over one period. The results of this is the following:

The active stator volt-amperes due to rotation, $(VI)_\omega$ is,

$$(VI)_\omega = (1 - S)\left(\frac{V_S I_S}{2}\cos\theta_L + R_S\frac{I_S^2}{2}\right) \qquad (3\text{-}4)$$

The active volt-amperes due to transformation, $(VI)_X$ is,

$$(VI)_X = S\left(\frac{V_S I_S}{2}\cos\theta_L + R_S\frac{I_S^2}{2}\right) \qquad (3\text{-}5)$$

The total active volt-amperes, $(VI)_S$ is,

$$(VI)_S = (VI)_\omega + (VI)_X$$

$$= \left(\frac{V_S I_S}{2}\cos\theta_L + R_S\frac{I_S^2}{2}\right) \qquad (3\text{-}6)$$

If we examine $\overline{(vi)_\omega}$ in equation (3-2), it is directly proportional to ω, and originates because of a change in the rotation phasor. Based on this, the shaft power is equal to the active volt-amperes due to rotation; that is,

$$(VI)_\omega = \frac{1}{3}P_n \qquad (3\text{-}7)$$

Therefore, the real (shaft) power is

$$P_n = 3(1 - S)\left(\frac{V_S I_S}{2}\cos\theta_L + R_S\frac{I_S^2}{2}\right) \qquad (3\text{-}8)$$

This result agrees with other papers [1,2].

The active volt-amperes flowing to the rotor can be found by multiplying equation (1-5) by $\bar{i}_R$. The last term in this equation is the volt-amperes flowing to the rotor,

$$\overline{(vi)_r} = \frac{3}{2}\frac{k}{a}L_S\left\{(\bar{i}_{Ro} + \bar{i}_{R\epsilon}) \cdot \frac{d}{dt}\left[e^{-j(\rho\omega t/2)}\,\bar{i}_S\right]\right\} \qquad (3\text{-}9)$$

When the phasor quantities are converted to real ones, and appropriate values of current are substituted, and then the whole expression integrated over one period, the result is the active volt-amperes flowing to the rotor, $(VI)_r$. It is,

$$(VI)_r = \frac{n_R^2 L_S}{n_S^2 L_R} S\left(\frac{V_S I_S}{2}\cos\theta_L + \frac{R_S I_S^2}{2}\right) \qquad (3\text{-}10)$$

This result, also, agrees with other papers [1,2]. A power balance equation can now be written. Using equations (3-6) and (3-10) and equating rotor power to stator power gives,

$$P_n + S(VI)_S = (1 - S)(VI)_S + S(VI)_S \qquad (3\text{-}11)$$

The reason for expressing the power equation in this form, is that, it shows that there is volt-amperes transformed between the rotor and stator windings; namely, $S(VI)_S$. This is a new interpretation to the power flow.

SUMMARY OF RESULTS

A model for the doubly-fed wound rotor generator is developed. It incorporates the provision for constant voltage operation. The rotor voltages and currents, necessary to achieve this, are derived. To show that the model has validity, power equations are, also, derived. These results agree with those obtained by others. In addition, a new interpretation is given to the power flow, by showing that volt-amperes are transformed between rotor and stator.

APPENDIX A

When solving the circuit voltage equations and those for a distributed three-phase flux, balanced conditions are assumed. These conditions are as follows. The inductances in each phase is the same, $L_S = L_{S1} = L_{S2} = L_{S3}$ and $L_R = L_{R1} = L_{R2} = L_{R3}$. The number of turns in each phase is the same, $n_S = n_{S1} = n_{S2} = n_{S3}$ and $n_R = n_{R1} = n_{R2} = n_{R3}$. That the flux for each phase is directly proportional to the current that produced it; namely,

$$\bar{\varphi}_{S1} = \frac{L_{S1}}{n_{S1}}\bar{i}_{S1}$$

The stator currents are balanced, that is,

$$\bar{i}_{S2} = e^{-j(2/3)\pi}\,\bar{i}_{S1}$$

$$\bar{i}_{S3} = e^{-j(4/3)\pi}\,\bar{i}_{S1}$$

and likewise for the rotor.

APPENDIX B

In order to solve the circuit equations, an expression for the flux produced by a distributed three-phase winding must be derived. The magnetic circuit is assumed to be linear; therefore, the flux from each phase can be added algebraically to produce a composite flux. For the stator

$$\bar{\varphi}_S = \bar{\varphi}_{S1} + \bar{\varphi}_{S2} + \bar{\varphi}_{S3} \qquad (B\text{-}1)$$

Each flux is a function of the relative position of the winding as well as the current in that winding. The total flux can be written as,

$$\bar{\varphi}_S = \frac{L_{S1}}{n_{S1}}\cos\left(\frac{\rho\omega t}{2}\right)\cdot\bar{i}_{S1} + \frac{L_{S2}}{n_{S2}}\cos\left(\frac{\rho\omega t}{2} - \frac{2}{3}\pi\right)\cdot\bar{i}_{S2} + \frac{L_{S3}}{n_{S3}}\cos\left(\frac{\rho\omega t}{2} - \frac{4}{3}\pi\right)\cdot\bar{i}_{S3} \qquad (B\text{-}2)$$

By converting the cos to exponentials and using balanced conditions from Appendix A, the total stator flux, expressed in terms of phase 1 current, becomes

$$\bar{\varphi}_S = \frac{3}{2}\frac{L_S}{n_S} e^{-j(\rho\omega t/2)} \cdot \bar{i}_{S1} \tag{B-3}$$

The total rotor flux can be expressed as,

$$\bar{\varphi}_R = \frac{L_{R1}}{n_{R1}} \cos\left(\frac{\rho\omega t}{2}\right) \cdot \bar{i}_{R1} + \frac{L_{R2}}{n_{R2}} \cos\left(\frac{\rho\omega t}{2} + \frac{2}{3}\pi\right) \cdot \bar{i}_{R2}$$

$$+ \frac{L_{R3}}{n_{R3}} \cos\left(\frac{\rho\omega t}{2} + \frac{4}{3}\pi\right) \cdot \bar{i}_{R3} \tag{B-4}$$

Using balanced conditions, the total rotor flux, expressed in terms of phase 1 current becomes,

$$\bar{\varphi}_R = \frac{3}{2}\frac{L_R}{n_R} e^{j(\rho\omega t/2)} \bar{i}_{R1} \tag{B-5}$$

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16. Abstract

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