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OF SOLAR-CELL POWER SYSTEMS FOR LOW EARTH ORBIT

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SUMMARY

The long-term operation of a large space station in low earth orbit will require the expenditure of tens of thousands of pounds of fuel by the maneuvering system in order to maintain the orbit energy in the presence of the small but continuous atmospheric drag. As the first step in reducing the space-station drag, the solar-electric power system might be designed to allow the large solar arrays to be "trailed" streamwise, rather than oriented for maximum solar collection. In the study described here, a preliminary engineering analysis was made to determine the effects of such a drag-reducing strategy on the size and weight of solar-electric power system required to satisfy a given constant-power demand. The power-system elements considered were the solar-cell array, the energy-storage batteries, and the maneuvering-system fuel required to offset the drag of the solar array; the measure of performance was taken to be the sum of the estimated weights of these three system elements for an assumed constant power demand of 150 kv. The study was based on unit-weight estimates for 1990 technology, and the drag of the solar array was calculated from free-molecule flow theory. It was found that for the trailing, minimum-drag system the required area of the solar array must be increased by about 80 percent over that for the conventional sun-pointing system, and that the capacity of the storage system must be increased by almost 30 percent. For reasonable orbit altitudes and lifetimes, however, the total system weight is dominated by the required drag-makeup fuel; consequently, the least total weight is obtained with the low-drag trailing-array system. For the 150-kw example system, operating for 20 years at an altitude of 220 n. mi., the total system weight with a trailing array was 35000 lb, compared with 92000 lb for the corresponding conventional sun-pointing system.

INTRODUCTION

The effort to develop a long-endurance space station has brought into focus the need for decreasing the atmospheric drag of orbiting satellites. The drag problem is particularly acute for a multipurpose, manned space station because of the large size of the vehicle and the frequency with which it must be reached by the Shuttle: if the space station were in a relatively low orbit, the Shuttle would be able to deliver more payload, but the extra drag of the denser atmosphere would require that a relatively large fraction of the payload on each trip be set aside for maneuvering-system fuel to make up the energy loss due to the space-station drag. Thus the effectiveness of the Space Transportation System itself stands to be improved significantly by a successful program to reduce the drag of the space station.

In undertaking a program of space-station drag reduction, an easy choice for a place to start is with the solar-cell-array surface for the solar-electric power system. Because of the anticipated need for supporting such power-intensive activities as materials processing, the solar-array surface can be expected to account for a large portion of the exposed surface area, and is therefore likely to dominate the atmospheric drag of the spacecraft. The study described in the present report was concerned with one particular strategy for reducing the drag of a solar-electric power system: that of adopting the operational procedure of "trailing" the solar-array surface "streamwise" so that the surface is always parallel to the flight path. The objective of the study was to determine the effect which such a strategy would have on the "performance" of a system of a given power-output capacity. Performance is measured here in terms of the total orbited weight required to install the system and maintain it over the lifetime of the space station. The orbited weight includes the solar array, the energy-storage system, and the maneuvering-system fuel required for drag makeup during the operating lifetime.

It should be emphasized that this study was intended to be no more than a "first cut" at the issue at hand, and that the ostensibly unequivocal results which are presented are based on a number of severe idealizations, and therefore carry a corresponding level of uncertainty. Nevertheless, it is believed that the essential features of the minimum-drag solar-array concept have been reasonably well represented and that the comparisons with the conventional approach are essentially valid.

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OVERVIEW OF THE ANALYSIS

Description of the Problem

A minimum-drag spacecraft with a trailing solar-cell array of the kind considered here is shown conceptually in Figure 1. Since a large, active space station would require perhaps 150 kw of continuous electric power, the required area for the solar-array surface can be expected to be very large, perhaps in excess of 10,000 square feet in area. It is clear that the atmospheric drag of such a solar-power system would depend strongly on the manner in which the system is operated: an array surface in the trailing position has (as will be shown) less than ten percent as much drag as it would have if it were oriented perpendicular to the flight path. Furthermore, since the frontal area of the spacecraft itself is small in comparison to the array area, it is clear that the array drag can contribute significantly to the amount of fuel that must be supplied to the spacecraft for maintaining orbit.

The question of whether it is advisable to design the solar-electric power system so as to allow the solar array to be trailed streamwise cannot, of course, be answered solely on the basis of the relative drag of an array in one orientation as compared with another. Since the trailing array becomes ineffective when the flight path becomes nearly parallel to the sun direction, a trailing array must be substantially larger than a sun-pointing array to achieve a given average power output; furthermore the sun-pointing array _is_ in trailing position when the sun is at the zenith position, and it could be trailed while the spacecraft is in the earth's shadow (about forty percent of the orbit, in the worst case), during which time its drag will be substantially lower than that of the full-time trailing array because of its smaller area. Another factor which must be considered is the weight of storage batteries for supplying power in the shadow. Since the power output of the trailing array drops below the nominal requirement well before the shadow is reached, the batteries must supply power for a longer time, and therefore must have higher capacity and higher weight than those for the sun-pointing system. Because of these various counteracting effects, it seems clear that a convincing comparison cannot be made without a reasonably careful integration of these effects over the course of the orbit.

A further complication in the problem results from the possibility that there exists a strong "optimum" design which lies somewhere between the two extremes of sun-pointing and trailing. An array which is trailed for most of the orbit might have better overall performance, for example, if it is never allowed to become parallel to the sun direction; if the array is oriented so that the angle between its normal and the sun direction is never allowed to exceed some specified value, say 75 degrees, then it collects appreciably more light but does so at some increase in drag. The possibilities for improving performance in this manner are explored in a superficial way in the present study by setting up a schedule by which the array incidence is to be varied as the spacecraft proceeds around the orbit. This schedule, which is described in detail in a later section, establishes a one-parameter family of operational cases which includes the two extreme cases of sun-pointing and trailing; the parameter used here is the maximum allowed angle of misalignment between the array-surface normal and the sun direction (zero for the sun-pointing case and 90 degrees for the trailing case). The power-system performance (i.e., the system weight) is then evaluated in terms of this parameter for the full range of design cases, that is, from zero to 90 degrees of maximum misalignment.

Another aspect of the problem that has been treated in a manner that might be characterized as an oversimplification is the matter of orbit inclination with respect to the direction of the sun. The study presented here considered only the case in which the orbit plane is parallel to the sun direction. The primary rationale for this simplification is that this case is the "worst case" from the point of view of sizing the power system, since the time in sunlight is at a minimum when the orbit is parallel to the sun direction. Additional justification is that the simplification reduces the number of independent variables, and, furthermore, restricts the problem to the case with the simplest geometry. While the implications of the larger problem have not been carefully considered, it is believed that the results of a more complete study would be substantially in agreement with the results presented here.

Outline of Computational Procedure

The process of computing the performance of a given case involves a number of steps. As an aid to the understanding of the analytical approach used here, the computational procedure is outlined below.

1. Select the case to be examined: orbit altitude, array orientation-schedule parameter, etc.

2. For a unit area of solar array, calculate the average power output by integrating over the duration of the orbit using the given orientation schedule.

3. Calculate the array area required to produce the desired average power; also calculate the array weight.

4. Calculate the energy-storage capacity required to maintain the desired average power continuously; also calculate the weight of batteries required to achieve this capacity.

5. For a unit area of the array, calculate the average drag over the duration of the orbit by integrating the drag variation along the orbit as the array incidence is varied according to the orientation schedule.

6. Calculate the average drag of the full-size array, using the area calculated in step 3. Also calculate the weight of fuel required by the maneuvering system to make up for the orbit-energy loss due to array drag.

7. Add the weights of the array, the energy-storage batteries, and the drag-makeup fuel required for the design lifetime, to get the total launch weight of the power system. The launch weight is taken to be the measure of the power-system performance.

Notation

A area of the solar array: length times width (one side only)

c_p normal-pressure coefficient for atmospheric drag force

c_τ shear-force coefficient (per unit area) for atmospheric drag force

C_d local drag coefficient

C_D overall drag coefficient

$\bar{C}_D$ average drag coefficient

$\bar{D}$ average drag force

E total energy demand for one orbit cycle

$\bar{I}$ average effective illumination for orbit cycle

I_{sp} specific impulse of sustainer-rocket system

k_p effective conversion factor for solar array (power available per unit area of intercepted sunlight)

ℓ length of solar array; for a trailing array, the dimension along the flight direction

m''_A weight of solar array per unit area

m'_Q weight of energy storage system per unit of capacity

P power available from solar array

$\bar{P}$ average power available over orbit cycle (design power demand)

q dynamic pressure of flow field

Q energy stored in storage system

Q_m maximum value of energy stored

t effective thickness of solar array, as represented by a flat plate for purposes of predicting drag

t_L lifetime of solar power system in orbit

T period of orbit

V orbital velocity (assumed to be flow velocity for calculating drag forces)

w width of solar array

W_A weight of solar-cell array

W_E weight of energy-storage system

W_F weight of fuel to maintain orbit

α angle of attack of flat plate

β angle between normal to solar array surface and sun direction (misalignment angle)

β_{max} maximum allowed misalignment for given orientation schedule

ψ orbit position angle (zero for sun at zenith)

ψ_s orbit position angle for edge of shadow

ψ_m orbit position angle at point of maximum energy stored.

Schedule for Varying Incidence

The schedule by which the solar array surface is to be oriented, as it proceeds around the orbit is shown in Figures 2 and 3. Figure 2 is a diagram of the orbit plane showing the array in edge view in a succession of orientations, as prescribed by the schedule: starting at $\psi = 0$ (sun at zenith) the array is first trailed streamwise until the misalignment angle β reaches the maximum allowed value β_{max}, at which point the array orientation becomes fixed with respect to the direction of the sun. Since the flight path is parallel to the sun direction at ψ = 90 degrees, the array can be rotated 180 degrees about the flight-path axis at that point, while maintaining $\beta = \beta_{max}$, as indicated. This 180-degree roll maneuver serves the purpose of allowing the angle of attack α to begin to decrease on the back side of the orbit ($\psi >$ 90 degrees), thereby providing the least possible drag consistent with the constraint of $\beta = \beta_{max}$. As the array passes into the shadow of the earth (at $\psi = \psi_s$, about 110 degrees for a 220 nautical-mile-altitude circular orbit), it is trimmed to the minimum-drag trailing position. The process is then reversed starting at the point at which the array breaks out of the shadow ($\psi = -\psi_s$), and continuing until the array reaches the zenith point ($\psi = 0$), after which the cycle is repeated.

The orientation schedule is plotted in Figure 3 for a half cycle. The heavy line shows the misalignment β as a function of ψ. Note that β increases linearly with ψ until β_{max} is reached, after which it remains constant until the point is reached at which the array can again be trailed at minimum drag. There is a minor complication here, in that two functionally distinct cases can occur: for schedules in which the array is trailed for almost all of the sunlit portion of the orbit (β_{max} greater than about 70 degrees, for the present case), the array will reach the trailing condition before it reaches the shadow; otherwise the final portion of the sunlit travel will take place at $\beta = \beta_{max}$.

Since β determines the rate at which sunlight is intercepted by the array at any time, and since ψ increases at a uniform rate with respect to time, the plot of β in Figure 3 can be interpreted as a plot of the angular misalignment of the array, with respect to the preferred sun-pointing direction, as a function of time. The $\beta(\psi)$ plot thus forms the basic input function for the integration over the orbit cycle of the energy produced per unit area of the array, as required to determine the average power production per unit area.

The dashed line in Figure 3 is a plot of the angle of attack α as a function of ψ. For the case in which the orbit plane is parallel to the sun direction (which is the case investigated here), the angle of attack is related to angles β and ψ by

$$\left.\begin{aligned} &\alpha = \psi - \beta, \quad \text{for} \quad 0 \leq \psi \leq 90^{\circ} \\ &\text{and} \\ &\alpha = (180^{\circ} - \psi) - \beta, \quad \text{for} \quad 90^{\circ} < \psi < \psi_s \end{aligned}\right\} \quad (1)$$

with corresponding relations for $\psi < 0$ (or $\psi >$ 180 degrees). For $\psi > \psi_s$, the array is in the shadow (by definition) and is therefore trimmed to $\alpha = 0$. The angle-of-attack function determines the variation in atmospheric drag of the array surface, and is used in integrating over the orbit to determine the average drag.

It should be recognized that the 180-degree roll maneuver described above is an idealization assumed for the convenience of modeling for the special case in which the orbit plane is parallel to the sun direction; for this case the condition at $\Psi = 90$ degrees gives rise to a singularity in the roll rate required to satisfy the constraints of minimum drag at the given angle of solar incidence. In practice, the roll rotation for this special case could be spread over twenty or thirty degrees of travel along the orbit with essentially no effect on the power output or the drag, as compared with more rapid roll rotation. In the general case, in which the orbit plane is not parallel to the sun direction, the roll attitude of the array must be changed <u>continuously</u> in order to satisfy the constraints. Thus there is no "requirement" for a stepwise adjustment in roll attitude except under the special case of orbit-plane orientation on which the present study was based.

DETERMINATION OF REQUIRED POWER-SYSTEM CAPACITY

Area of Solar-Cell Array

As the first step in determining the size of solar array required to produce power at a given average rate under a given orientation schedule, it is convenient to work with the average effective illumination, $\bar{I}$, which is a characteristic of the schedule. For the present purposes, $\bar{I}$ may be defined as the ratio of the radiation collected and converted to power by the array during the course of a complete orbit to that which would be collected and converted if the array were fully illuminated in the best orientation for the same period of time. Once $\bar{I}$ is determined for a given orientation schedule, the array size for a given average power can be found immediately by scaling up the array which would be required if it were fully illuminated. In general, $\bar{I}$ can be written as

$$\bar{I} = \frac{1}{\pi} \int_0^{\psi_s} f(\beta)\, d\psi \tag{2}$$

where $f(\beta)$ is the ratio of the amount of radiation converted into power when the array is misaligned by the angle β, to that for $\beta = 0$. (By definition, no energy is collected after the array enters the shadow, when $\psi_s < \psi < \pi$). Since β is determined by the orientation schedule, which has β_{max} as a parameter, then $\bar{I}$ is a function of β_{max}. The function $f(\beta)$ thus accounts for the decrease in projected area as the array is tilted. To be correct, it should also account for the decrease in the efficiency with which the intercepted radiation is collected and converted to electric power as the misalignment angle is increased. Because of a lack of available data on the performance of solar arrays under conditions of poor alignment, however, and furthermore because of the sensitivity of the off-design performance to a variety of parameters in the optical design, the likely decrease in efficiency due to misalignment was ignored in the present study. Thus the results shown here were based on the assumption that the average effective illumination, as defined above, can be adequately represented by

$$\bar{I} = \frac{1}{\pi}\int_0^{\Psi_s} \cos\beta \, d\psi \quad , \quad \beta = \beta(\psi; \beta_{max}) \qquad (3)$$

which accounts only for the loss of projected area as the array is inclined away from the sun direction.

For orientation schedules prescribing $\beta(\psi)$ in the form discussed previously (Figure 3), the integral is readily evaluated as a function of β_{max}. A plot of $\bar{I}(\beta_{max})$ for the assumed orbit parameters is shown in Figure 4. As shown by the figure, the trailing array has about 55 percent of the effectiveness of the sun-pointing array (under the stated assumptions) and therefore must be about 80 percent larger to produce the same average power. Corresponding comparisons can also be made for "intermediate" cases ($0 < \beta_{max} < 90°$) from this plot.

Given the average effective illumination $\bar{I}$, the array area which is required to supply the desired average power $\bar{P}$ can be calculated immediately:

$$A = \frac{\bar{P}}{k_p \bar{I}} \qquad (4)$$

where k_p is the power generated by a unit area of solar array in full illumination. For the present study k_p has been taken to be 225 watts per square meter. Figure 5 shows the solar array area required to produce an average power output of 150 kw, as a function of β_{max} .

As stated above, the curves of Figures 4 and 5 are based on the assumption that no power is lost by reflection from the array surface. Since the reflection losses can be expected to be significant where the alignment is poor, this assumption gives results that are somewhat optimistic for a simple plane array. To correct the deficiency in the predicted power-generation capability the array area could be increased by the appropriate amount (which would thereby also increase the array weight and drag proportionally). Alternatively, the collector could be slatted in the manner of a venetian blind, as sketched in Figure 6, so as to allow the multiple surfaces of the array to remain perpendicular to the incident sunlight. With the proper control, a slatted collector would have no additional losses due to reflection under conditions of poor alignment, and would therefore satisfy the requirements of equations (3) and (4).

Because of the increased frontal area of the turnable slats, the slatted collector would have somewhat greater drag in the minimum drag orientation than the plane collector. In principle, of course, the additional drag could be made to be arbitrarily small by making the slats sufficiently narrow; it can be shown, for example, that if the collector is divided into a hundred slats, the average drag would be increased by a few percent. Since no design studies have been made on this concept, it remains to be seen whether the increased weight and mechanical complexity would be justified by the predicted improvement in the collector efficiency.

Capacity of Energy-Storage System

For a space system which is powered by a sun-pointing solar-cell array, the requirements for energy storage are readily established: the storage system must supply the power required for operation in the shadow under the design conditions. If the anticipated power use is constant at some nominal level, for example, then for the sun-pointing system, there must be enough energy-storage capacity to supply that nominal power for the time during which the vehicle is in the shadow. For the trailing-array system, however, the requirements are less easily stated, but it is clear that more storage capacity is required than for the sun-pointing system because the power output of the array drops below the demand well before the vehicle enters the shadow. For "intermediate" systems additional storage capacity is required only if the maximum misalignment angle β_{max} is above a certain threshhold value, as will be shown.

Diagrams of the required power schedules corresponding to several different orientation schedules are plotted in Figure 7, which shows the relative power available as a function of orbit position for several values of the parameter β_{max}. The relative heights of the curves at $\Psi = 0$ reflect the relative array-surface areas required to produce enough energy over the course of the orbit to serve a given average power demand. Thus the area under each curve from $\Psi = 0$ to $\Psi = \Psi_S$ is the same as the area under the "average power demand" line from $\Psi = 0$ to $\Psi = 180$ degrees.

The nature of the energy-storage requirement for the given system of orientation scheduling can be seen by examining the shapes of the various curves in the figure. Each curve has a portion which is proportional to cos Ψ and a flat portion, corresponding to operation at β_{max}. If the flat portion lies above the average demand, then the system is not required to supply power from storage until the shadow is entered, and therefore the storage requirement is the same as for the sun-pointing array. Additional storage is required, however, for the higher values of β_{max}. As an example, the case of β_{max} = 75 degrees is shown shaded. For this case extra storage is required beyond Ψ = 69 degrees, which represents an increase of almost 60 percent in the time during which power must be supplied. Nevertheless, since the array continues to supply some power until the shadow is reached, the additional storage-capacity requirement here is only about 15 percent above that for the sun-pointing array.

A curve of the relative energy-storage capacity which is required to supply power continuously is shown plotted in Figure 8 as a function of β_{max}. The ordinate here is the ratio of the maximum energy stored to the total energy used during the one-orbit cycle. The plotted curve displays the abrupt change in functional form which distinguishes those cases for which the power output drops below the average demand before the shadow is reached. As shown by the curve, the trailing-array system requires energy-storage capacity equal to about half of the energy used in the orbit cycle, which represents an increase of about 28 percent over that required for the sun-pointing array.

Derivations for the relationships which are plotted in Figure 8 are given in Appendix A.

CALCULATION OF THE DRAG OF THE SOLAR ARRAY

Drag Coefficient of a Thin Flat Plate

In the calculation of atmospheric drag forces acting on a solar-cell array in low earth orbit, the array can be characterized as a thin flat plate in "free-molecule" flow. The free-molecule flow regime, which is discussed at length in Reference 1 admits a rational analysis of the local flow-induced forces on a surface exposed to the flow, without considerations of the overall geometry of the flow field (in marked contrast to the corresponding calculations required for higher-density flows). Thus formulas for the shear force and normal-pressure force acting on an element of the surface have been derived on the basis of first principles, together with some basic assumptions concerning the statistical properties of the molecular motion of the rarefied flow medium before and after the interaction with the surface. The derivations are discussed in Reference 1 and the formulas are reproduced for convenience in Appendix B.

It should be recognized that because it has not been possible to make measurements of drag coefficient at orbital velocities and densities, the formulas given in Reference 1 represent the best available means of estimating the drag of satellites of unusual shape. In this connection, it may be recalled that measurements of satellite motion yield drag force but cannot provide accurate drag-coefficient data because of the unknown (and apparently unknowable) atmospheric density, which varies widely and unpredictably with latitude, longitude, and time, as well as with altitude. Since the drag coefficient is known with good confidence to within about twenty percent, while the density at a given time and place may be unknown within a factor of ten or more, the satellite trajectory measurements are used as a means of determining density.

It is probably worthwhile to introduce a second point of background here concerning the matter of estimating satellite drag. For many purposes the drag of satellites of conventional shape can be estimated satisfactorily by assuming the drag to be proportional to the frontal area of the satellite, with an effective drag coefficient of about 2.2. This procedure is readily justified by observing that the theoretically derived drag coefficients for a variety of shapes (e.g., sphere, cylinder, and flat plate perpendicular to the flow), for typical conditions all lie within one percent or so of each other and within a few percent of the assumed value of 2.2. For the purposes of the present study, however, it is clearly not adequate to estimate drag on the basis of frontal area, since the drag of a thin flat plate aligned with the flow is caused primarily by shear effects on the large lateral area of the plate rather than by pressure on the thin leading edge. In the case of a solar-cell array of typical dimensions in a streamwise-trailing attitude, as will be shown, the drag contribution of the leading edge would be no more than a few percent of the total drag of the array.

As discussed in Reference 1, the results from the theory of free-molecule flow can be applied to those cases for which the Knudsen number is substantially greater than one (i.e., where the length of the mean free path in the undisturbed gas is substantially greater than the characteristic dimension of the body). This condition is easily satisfied for all practical cases of interest involving solar arrays in low earth orbit, since even at an altitude of 150 n. mi. (280 km) the mean free path is more than 1 km.

The aerodynamic-force coefficients for shear c_τ and normal pressure c_p, as given by the formulas in Appendix B, are plotted in Figure 9 as functions of angle of attack α. The numerical values for the atmosphere-dependent constants were evaluated for an orbit altitude of 220 n. miles (407 km), using the US Standard Atmosphere 1962 (Ref. 2). Since all subsequent drag calculations are based on these coefficient functions, this figure can be considered as the basic description of the drag properties of flat plates, as applied to the present study.

The shear and normal-pressure coefficients of Figure 9 can be combined to give the resultant force coefficient in the flow direction, which is to say, the local drag-force coefficient, as follows:

$$C_d(\alpha) = c_p \sin\alpha + c_\tau \cos\alpha \qquad (5)$$

For a body of arbitrary shape, this local drag-force coefficient can be integrated over the surface to determine the overall drag coefficient. For a thin flat plate, the drag coefficient based on the nominal area of the array (one side of the plate) can be shown to be

$$C_D(\alpha) = C_d(\alpha) + C_d(-\alpha) + \frac{t}{l} C_d\left(\frac{\pi}{2} - \alpha\right) \qquad (6)$$

where the second term represents the contribution from the "back" (downstream-facing) side of the plate, and the third term accounts for the leading edge. As will be seen, for plates in which the thickness is less than about one percent of the length, the leading-edge drag is negligible, even for the trailing orientation ($\alpha = 0$). Since solar-cell arrays and their stiffening structures are typically very thin compared to the array lengths, the drag calculations presented here are based on the assumption that the drag contribution from thickness is negligible.

A plot of the drag coefficient for a very thin plate is shown in Figure 10. This plot was constructed by using equations 5 and 6 to combine the contributions from shear and normal pressure shown in the previous figure. Except for small angles of attack ($\alpha < 8^\circ$), the curve lies very close to the curve which would be predicted by the previously described approximation based on frontal area (which would give $C_D = (C_D)_{90^\circ} \cdot \sin \alpha$). Since the trailing array is presumed to operate in the neighborhood of $\alpha = 0$, however, the more complex relation must be retained here.

The justification for neglecting the drag contribution due to the leading edge of the array can also be seen from Figure 10. The drag coefficient for the plate flat-wise to the flow is seen to be about 13 times as great as that for the edge-wise orientation. Thus if the thickness were one percent of the length, the contribution of the leading edge to the drag of the plate edge-wise would be about 13 percent. Since large solar arrays tend to be much thinner than one percent of the length, the drag contribution from thickness can be safely neglected here.

Because of the importance of the drag behavior near zero angle of attack, the curves of Figure 10 are replotted near the origin to an expanded scale in Figure 11. The plot shows that the drag coefficient is relatively insensitive to changes in angle of attack when the array is in the trailing position: to increase the drag by ten percent from that for $\alpha = 0$ requires an attitude change of about two degrees, which represents a relatively large attitude-control error. Thus the predicted low-drag performance of the trailing array should be relatively easily realized without resort to sophisticated control systems for minimizing spurious deviations in alignment due to control error or structural deformation.

Average Drag of the Solar Array

In order to compute the fuel required to make up the orbital energy loss due to drag, it is necessary first to compute the average drag coefficient of the array as it proceeds around the orbit under the given orientation schedule. For this study, the average drag coefficient was computed for a given orientation schedule by numerically evaluating the integral

$$\bar{C}_D = \frac{1}{\pi} \int_0^{\pi} C_D \, d\psi \tag{7}$$

where

$$C_D = C_D(\alpha) , \qquad \alpha = \alpha(\psi, \beta_{max})$$

Here the angle of attack function $\alpha(\psi, \beta_{max})$ is associated with a given orientation schedule through equation (1), as indicated in Figure 3, and $C_D(\alpha)$ is represented by Figure 10.

The manner in which C_D varies with ψ, for several different schedules (i.e., different values of β_{max}), is plotted in Figure 12. These curves display the large differences in their average values which might be anticipated for the extreme cases of sun-pointing and trailing systems.

The average drag coefficient, $\bar{C}_D$, is shown in Figure 13 as a function of β_{max}. This plot was obtained as indicated by equation (7), using the set of functions $C_D(\psi, \beta_{max})$ which are represented by Figure 12. The basis for the shape of the $\bar{C}_D$ curve (i.e., large slope near $\beta_{max}=0$ and small slope near $\beta_{max}=90$ degrees) can be understood by examining Figure 12 and noting that a small change in β_{max} near the sunpointing condition makes a much larger change in the area under the drag curve than a similar change near the trailing condition.

The average drag of an array operated under a given orientation schedule is

$$\bar{D} = q \bar{C}_D A \tag{8}$$

where q is the dynamic pressure, and A is the area of the array. As was discussed previously (e.g., Figure 5), A also depends on the schedule parameter β_{max}, but in a manner which is inverse to that of $\bar{C}_D$ (low β_{max} is associated with low A and high $\bar{C}_D$, while high β_{max} has high A and low $\bar{C}_D$). The product $\bar{C}_D A$, by which the average drag associated with a given schedule can be judged, is shown plotted against β_{max} in Figure 14. The plotted values are normalized here by the value of $\bar{C}_D A$ for $\beta_{max} = 0$. Thus the curve shows the drag for a solar array system designed to operate under a given orientation schedule, in relation to the drag for a sun-pointing array of the size necessary to satisfy the same average power-demand. The figure shows that the drag of a trailing array is about 30 percent of the drag of an equivalent sun-pointing array. The figure also shows that, for the family of orientation schedules considered here, the minimum-drag case is that of the trailing array.

CALCULATION OF SYSTEM WEIGHT

Weight of Solar-Cell Array

For the purposes of making an initial estimate of the weight associated with the solar array, it has been assumed that the weight is proportional to the array area. This assumption implies that the weight of the associated structure, deployment mechanisms, etc., is small enough compared to the weight of the array surface that adverse size effects can be safely neglected. As will be shown, however, the assumption can be justified on the broader grounds that the weight of the solar array is only a small part of the total system weight.

The solar array weight is thus taken to be

$$W_A = m''_A \cdot A \tag{9}$$

where m''_A is the weight per unit area and A is given by equation (4) as $\overline{P}/k_p\overline{I}$. With this substitution,

$$W_A = \frac{m''_A \cdot \overline{P}}{k_p \, \overline{I}} \tag{10}$$

The function $\overline{I}$ is defined by equation (3) and plotted in Figure 4. Table I gives the values which have been used for the remaining terms in determining the system weights for the example cases discussed here.

Weight of Energy-Storage System

The energy-storage system weight has been taken to be proportional to the required system capacity:

$$W_E = m'_Q \cdot Q_m \tag{11}$$

where m'_Q is the weight per unit of stored energy and Q_m is the energy-storage requirement. For the present study, m'_Q is represented by an estimate of the weight per unit capacity for 1990-technology batteries which would be suitable for use in the primary power-supply system for a space station. The storage requirement Q_m is plotted as a function of β_{max} in Figure 8, in the form of (Q_m/E), where $E = \bar{P}T$, or the total energy demand per orbit. Thus

$$W_E = m'_Q \bar{P} T \cdot (Q_m/E) \tag{12}$$

where (Q_m/E) is given by Figure 8. Pertinent values for the remaining terms are listed in Table I.

Weight of Fuel

The fuel required to overcome the solar array drag in order to maintain the orbit altitude has been taken to be proportional to the product of the average drag $\bar{D}$ times the design lifetime, t_L :

$$W_F = \frac{\bar{D} \cdot t_L}{I_{sp}} \tag{13}$$

where I_{sp} is the specific impulse of the maneuvering system used for orbit-energy maintenance. The average drag is given by equation (8) as $q\bar{C}_D A$, where $\bar{C}_D A$ is plotted against β_{max} in Figure 14, and the dynamic pressure q is

$$q = \left(\frac{\rho}{\rho_0}\right) \cdot \frac{1}{2} \rho_0 V^2 \tag{14}$$

Note that the velocity V and the density ratio (ρ/ρ_0) are functions of orbit altitude. Atmospheric properties used here were taken from Reference 2. The various values for the numerical cases to be considered are summarized in Table I.

Total Orbited Weight for the Solar-Power System

The total weight which must be placed in orbit in order to establish and maintain the power system is taken here to be the sum of the weights of the array, the energy-storage batteries, and the sustaining fuel, as estimated by the procedures described above. Thus

$$W_S = W_A + W_E + W_F \qquad (15)$$

All of these weights depend on the orientation schedule parameter β_{max}; the fuel weight W_F also depends on the orbit lifetime t_L. The system weights are plotted in Figure 15 as functions of β_{max}, with lifetime as a parameter, for the particular case of a 220 n. mi. altitude, as defined in Table I. Since the total system weight is considered here to be the primary measure of system performance, Figure 15 can be considered the principal result of the present study.

The curves of Figure 15 show several important features of the system which has been studied:

- The fuel weight required for orbit maintenance completely dominates the total weight of the system, even for relatively short lifetimes.
- The trailing-array system (β_{max}= 90°) offers very significant savings in total weight, even though its fixed weight is about fifty percent higher than that of the sunpointing array.
- There is no strong minimum-weight case for orbit schedules which are "intermediate" between trailing and sunpointing; the trailing case

provides minimum weight for greater lifetimes and near-minimum weight for shorter lifetimes.

- The weight of batteries required is roughly equal to the weight of the solar array.

Because of the fact that there is no particularly interesting intermediate case between sunpointing and trailing, the principal results of the study can be displayed somewhat more clearly by eliminating β_{max} as a variable and then comparing the weights for the two extreme cases as functions of time. This comparison is shown in Figure 16. The figure shows that the fuel saved by trailing the array at minimum drag becomes greater than the initial weight gain (due to inefficient use of the solar array) after only one year in orbit; after ten years the total weight for the trailing array system is less than half of that for sunpointing, and after twenty years it is less than forty percent. For the example which was worked here (150 kw system at 220 n. mi), the predicted saving in orbited weight after twenty years is about 57,000 lb, which is roughly equivalent to the total nominal payload for one trip of the Shuttle to the assumed altitude. This weight saving can be considered to be the result of an "investment in drag reduction" of about 3000 lb (i.e., the trailing system has a fixed weight of about 9000 lb while that for the more-effective sunpointing system is only about 6000 lb). Thus the "payback ratio" from the drag reduction program, for the example worked here, is about 20:1 after twenty years in orbit.

The impact of these results is shown in Figure 17 in a somewhat different perspective. The figure shows a plot of the sustainer-motor fuel required for twenty years in orbit (in units of "Shuttle payloads" of 60,000 lb) as a function of size (in terms of projected area) for several possible space-station components: three cases for 150 kw solar arrays and two cases for space-station habitat modules (15-ft-dia cylinders, 60 ft long). These cases are plotted for an orbit altitude of 220 n. mi. The two lines represent flat plates oriented perpendicular to the flow, at orbit altitudes of 220 n. mi. and 150 n. mi. The lower line (for 220 n. mi.) is intended to serve as a reference for the plotted points; for similar components of different areas, the fuel requirements can be found by translating the points parallel to the line. The upper line can be used in a similar way as the reference line to find the fuel requirements for the same components in a 150-n. mi. orbit, since the pattern of points for the new altitude would bear the same relationship to the reference line for that altitude. As might be expected, since the drag coefficients are almost independent of orbit altitude, the fuel requirements are proportional to atmospheric density, which is about 10 times as great at 150 n. mi. as at 220 n. mi.

For the sunpointing 150-kw solar array system, Figure 17 shows two points which represent the cases in which the array is either trimmed for minimum drag in shadow, or not trimmed in the shadow. The untrimmed system requires the equivalent of about two full Shuttle flights of fuel to maintain a 220 n. mi. orbit for twenty years, which is about 40 percent more than the trimmed system. The larger-area trailing system requires only 30 percent as much fuel as the trimmed sunpointing system, as was discussed previously, which amounts to a difference of about one Shuttle payload. Note that for a 150 n. mi. orbit, the corresponding difference would amount to about ten Shuttle flights of fuel.

Figure 17 also indicates the relative importance of working toward reducing the drag of the cylindrical modules of the space station. The drag of the cylinder moving lengthwise is slightly less than a third of the drag crosswise; for a single cylinder the difference amounts to about one-sixth of a Shuttle payload of fuel, or about 10,000 lbs. For multiple cylinders, or for lower orbits, the difference in fuel requirements will, of course, be multiplied.

It can be shown that about 60 percent of the drag of the single cylinder moving endwise is due to the pressure drag on the end, with the remainder due to shear drag on the wall. In view of this, it is clear that for long-lifetime, multiple-cylinder installations, very large amounts of sustainer fuel could be saved if the cylinders could be arranged in an end-to-end string moving lengthwise. It remains to be seen, of course, whether such an arrangement might be made to be acceptable from the point of view of the overall space-station system.

SUMMARY OF RESULTS

For the conditions which have been assumed, the simplified analysis has produced the following results:

1. The solar energy intercepted by the sunpointing array is about 61 percent of that which would be intercepted if the array were illuminated full time. For the trailing array, the figure is 34 percent. Thus a trailing array must have about 80 percent more area than an equivalent sunpointing array.

2. The trailing-array system requires about 28 percent more energy storage to serve a constant power demand, because the array output drops below the demand about 40 degrees of orbit position before the shadow is reached.

3. The weight of fixed equipment (which is divided roughly equally between array and batteries) is about 50 percent greater for the trailing-array system than for the sunpointing system.

4. The atmospheric drag of solar arrays in low earth orbit can be estimated by the formulas developed for thin flat plates in free-molecule flow. For a trailing array, the drag is dominated by shear forces on the lateral surface, rather than by pressure forces on the thin leading edge. The drag on a trailing array was found to be about 7 percent of the drag which would occur with the plane of the array perpendicular to the flow.

5. The frontal area (or "projected area") of a thin flat array is an acceptable basis for estimating the drag if the angle of attack of the plane of the array is greater than about 8 degrees.

6. For a trailing-array system, in which the plane of the array is nominally aligned with the flow, the drag is not sensitive to small variations in angle of attack less than about ± 2 degrees.

7. In the family of orientation schedules between sunpointing and trailing, the schedule which provides the minimum average drag for a given average power output is that of the "pure" trailing array.

8. The sunpointing-array system has an average drag coefficient which is about 45 percent of the drag coefficient perpendicular to the flow, if it is trimmed for minimum drag while it is in shadow. If it is not trimmed in shadow (but left in the sunpointing orientation) the average drag coefficient rises to about 64 percent of that when perpendicular.

9. The average drag coefficient of the trailing array is about 17 percent of that of the sunpointing array, but, because of its larger area, the average total drag for the trailing array is about 30 percent of that of the equivalent sunpointing array.

10. The weight of maneuvering system fuel for sustaining the orbit energy is much greater than the fixed-system weight (array plus batteries) for reasonably long operating lifetimes.

11. Because of its lower drag, the trailing array has lower total orbited weight (including sustainer fuel) for lifetimes in excess of about one year; after twenty years the trailing-system weight is less than 40 percent of the sunpointing system.

12. For equivalent 150-kw systems, after twenty years, the difference in the sustainer-fuel requirement is equal to about 60,000 lbs, or one full payload of the Shuttle.

13. The saving in fuel for the low-drag configuration in the example worked was about twenty times as great as the increase in the fixed weight which was required to achieve the low drag. This factor appears to be far greater than the uncertainty in the weight estimation, even for the simplified analysis used here. It may therefore be concluded that a program of space-station drag reduction can be expected to save operating weight in the long term.

DISCUSSION

Some Considerations for the Design of Solar-Array Systems

The foregoing analysis has shown that there is a strong basis for designing space-station solar-cell arrays to be trailed at minimum drag. The analysis makes use of several simplifying idealizations which can be expected to influence the results to some extent. The likely effects of these simplifications are discussed briefly in the following paragraphs, along with some observations concerning the design of trailing solar power systems.

The major shortcoming of the analysis is believed to be the failure to account for the power loss due to reflection from the trailing array under conditions of poor alignment. It seems clear, however, that these losses cannot be great enough to change the conclusions of the study, since it would require a doubling of the trailing-array area before the total system weight would approach that of the sunpointing system, for a ten-year lifetime. Furthermore, as has been discussed, the reflection losses could be eliminated, at some cost in weight and complexity, by the use of a slatted collector.

Another deficiency in the analysis concerns the shear drag of the trailing array. The analysis was based on the assumption of "perfect accommodation" (see Appendix B), which is to say that the impacting air molecules are assumed to stick to the surface momentarily, losing all momentum in the process and thereby producing the highest possible value of drag. There is reason to believe that, for the very-high-velocity grazing impacts of concern here, this assumption may be substantially in error, and that furthermore it may be possible to reduce the drag substantially through appropriate surface treatment. Thus it may develop that the drag of advanced-design trailing arrays can be reduced to a fraction of that which would be predicted by the foregoing analysis. Such a development would tend to favor the use of a larger single-plane array, rather than a slatted array, to account for reflection losses. It is conceivable that this effect may reduce the total drag of the trailing array by more than enough to offset the increased area, so that when the effects of shear-drag reduction and reflection are accounted for, the comparisons shown here can reasonably be expected to remain valid. (It should perhaps be mentioned that no corresponding reduction in drag is possible with the sunpointing array, since most of the drag loss occurs with the array at high angles of attack, for which the assumption of perfect accommodation gives the least possible drag.)

A third deficiency in the comparisons made here lies in neglecting to account for the effects of precession of the plane of the orbit, which will produce a continuous change in the angle of incidence of the sunlight on the orbit plane. The case which was assumed (orbit plane parallel to sun direction) gives the least power output and greatest drag, and therefore the estimations of the long-term performance are pessimistic to some degree. A minor factor, which can be reasonably discounted for the kind of orbit which is likely for early space stations (low inclination, low altitude), is the time in sunlight. According to curves presented in Reference 3, such orbits never have more than about ten percent more sunlight than the minimum; the long-term average is typically about four percent more than the minimum. Since all the array designs are affected equally by this factor, there will be no significant effect on the comparison between equivalent designs. (Since the maximum time in sunlight increases rapidly as the orbit inclination is increased above about 25 degrees, however, this conclusion is not valid for near-polar orbits.)

A more important effect of the change in orbit plane concerns the drag of the sunpointing array. It can be shown that the average drag coefficient over a given orbit is proportional to the cosine of the angle between the sun direction and the orbit plane. For low-inclination orbits the maximum sun angle is about 45 degrees (adding the orbit inclination and the tilt of the equatorial plane); in this "best case" the drag would be about seventy percent of the worst-case drag. The long-term average drag would therefore be somewhat less than the worst-case drag, and the fuel used to maintain the orbit of the sunpointing array would therefore be less than that estimated for Figures 15 - 17. While a careful prediction of the long-term average drag coefficient would require detailed knowledge of the orbit, it appears that a reasonable estimate for most cases could be obtained by assuming the average drag to be about ten percent less than that for the worst case presented here.

For the trailing array, which is by definition always trimmed to the minimum-drag attitude, the sun angle has no effect on the drag coefficient. Furthermore, if the array size is determined by average power requirements which must be met under all conditions, as was assumed for the present study, the power system would be designed in the manner indicated previously. It is of interest to note, however, that the power available over the orbit cycle increases rapidly as the angle of solar incidence on the orbit plane increases, since the minimum illumination (at the lateral extremities of the orbit) is proportional to the sine of the solar angle. It can be shown, for example, that with the sun at 45 degrees to the orbit plane, the total energy per orbit available from a trailing array is increased by more than fifty percent over that for the sun-in-plane case, while the long-term average power output for low-inclination orbits would be increased by almost one fourth. For the sunpointing array, of course, there can be no increase in power output due to improved solar alignment since the array is, by definition, always maintained at the best orientation with respect to the sun.

To summarize the effects of changes in the alignment of the orbit plane with the sun direction:

- Time-in-sunlight increases very slightly with increases in the angle of incidence of the sun on the orbit plane.
- The sustainer fuel required for long-term operation of sunpointing arrays would be about ten percent less than that estimated on the basis of the worst-case solar alignment.
- The power-output capacity of the trailing array under the best conditions would be as much as fifty percent greater than for the worst case, with a long-term average increase of about one fourth.

As a final supplementary note to the comparisons presented for the trailing array vs. the sunpointing array, it is appropriate to consider briefly the problem of controlling the orientation of the unconventional low-drag system. For the low-drag space-station configuration which has been proposed, it is not possible to stabilize the attitude of the spacecraft in the conventional manner by means of the gravity-gradient field;

in fact, the attitude of the horizontally-aligned structure is unstable in the absence of some additional stabilizing influence. In view of this, it is important to establish whether the new and unusual control problems can be managed satisfactorily without the constant use of a reaction-jet maneuvering system which might negate some or all of the apparent gains in fuel economy.

The problem of attitude stability and control for a large space station of the kind which is sketched in Figure 1 is discussed briefly in Appendix C. The results of the preliminary investigation on which the discussion is based can be summarized as follows:

- The attitude control and stability of large, low-drag space station can be assured by means of a conventional reaction-wheel control system of small size and weight.
- The aerodynamic forces which act on the trailing array serve to improve the stability of the spacecraft attitude and to relieve the requirements on the control system.
- The operation of a low-drag space station with a trailing array should require no maneuvering-system fuel beyond that which is required for drag makeup.

Recommendations

The results of the study indicate that the concept of the minimum-drag trailing solar array should be developed for use in space stations. In order to get this development on a firm foundation, there are several areas in which the early work should be concentrated. Accordingly, it is recommended that priority be given to the following tasks:

1. Develop the technology of reducing shear drag at grazing incidence on smooth flat surfaces. This technology can be used to advantage on the back side of the array and on aerodynamic control surfaces; some treatment may also be possible for the sunlit side of the solar cells if the obvious optical constraints can be met. Final demonstration of the low-drag technology will require flight testing at orbital speed and altitude, and therefore an important part of this task is to develop plans for a Shuttle-borne experiment.

2. Develop a design for a solar-cell array that will give good performance under the operating conditions of the trailing-array system, which include poor solar alignment for a large fraction of the time. It is important to understand the manner in which these reflection losses can be influenced by the use of optical coatings, how the losses affect the required size of the array, and whether it is reasonable to add the weight and complication associated with slatting the collector in order to avoid reflection losses.

3. Develop candidate structural arrangements for trailing-array designs, taking account of the requirements for deployability, attitude

control and drag. Eventually the structural design should incorporate the developments indicated in 1. and 2. above, but early configuration studies are needed to guide those developments.

4. Develop candidate spacecraft configurations in which the trailing array concept is integrated as a basic feature of the design. The spacecraft configuration studies should be closely correlated with the solar-array design studies mentioned in 3. above.

MAJOR CONCLUSIONS

The major conclusions of the study are as follows:

1. The "pure" trailing array gives better overall performance than any other kind of system examined in this study.

2. Compared to a conventional sunpointing solar-cell array system the trailing-array system

- has 1.8 times as much area
- requires 1.28 times as much energy storage
- has only 0.30 times as much average drag.

3. For long lifetimes in orbit at altitudes of 220 n. mi. or less, the weight of sustainer-motor fuel required to overcome the drag of a solar-cell power system is much greater than the weight of fixed equipment such as solar arrays and batteries.

4. For the systems investigated here, for a twenty-year lifetime, the weight of sustainer-motor fuel which would be <u>saved</u> by the use of the low-drag trailing system, instead of the conventional sunpointing system, is about 20 times as great as the additional weight of fixed equipment needed because of the decreased effectiveness of the low-drag system.

5. For a space station in low earth orbit, low-drag configurations offer the potential of providing large reductions in operating cost because of the reduction in fuel required to maintain the orbit.

APPENDIX A

Derivation of Relations for Energy-Storage Requirements

The storage requirement can be expressed quantitatively as

$$Q(\psi) = \frac{1}{\Omega} \int_{-\psi}^{\psi} [P(\psi_i) - \bar{P}]\, d\psi_i \tag{A 1}$$

where $Q(\psi)$ is the energy stored during the symmetrical power "peak" of width 2ψ, $P(\psi)$ and $\bar{P}$ are the instantaneous power and average power, and the subscript i denotes the dummy variable of integration. The power is assumed to vary with ψ according to

$$P(\psi) = k_p A \cos\beta(\psi) \tag{A 2}$$

Recalling from equation (4) in the text that

$$A = \frac{\bar{P}}{k_p \bar{I}} \tag{A 3}$$

and, further, observing that

$$\frac{k_p \bar{I}}{\Omega} \cdot \frac{T}{T} = \frac{E}{2\pi} \tag{A 4}$$

where E is the total energy required for the period T of the orbit, then it can be shown that

$$\frac{1}{E} Q(\psi) = \frac{1}{\pi \bar{I}} \int_{0}^{\psi} (\cos\psi_i - \bar{I})\, d\psi_i \tag{A 5}$$

where Ψ has been substituted for β since $\beta = \Psi$ for $\Psi < \beta_{max}$. The maximum value of the integral is reached when the integrand decreases to zero. Denoting this value of the orbit angle by Ψ_m, then

$$\cos \Psi_m = \bar{I} \qquad \text{(A 6)}$$

After the integration is carried out using Ψ_m as the upper limit, the required storage capacity is seen to be

$$\frac{Q_m}{E} = \frac{1}{\pi}\left[\frac{\sin \Psi_m}{\bar{I}} - \Psi_m\right] \qquad \text{(A 7)}$$

where Ψ_m is found from (A 6). Note that since Ψ_m is determined by $\bar{I}$, and and $\bar{I}$ is determined by β_{max}, then Q_m/E is a function of β_{max}.

For those cases in which $\cos \beta_{max} > \bar{I}$, the array is capable of satisfying the power demand as long as it is illuminated, and consequently the above procedure for finding the maximum value of the integral does not apply. In those cases, the storage requirement is simply the demand times the time in shadow:

$$Q_m = \bar{P}\, T \left(\frac{\pi - \Psi_s}{\pi}\right) \qquad \text{(A 8)}$$

or

$$\frac{Q_m}{E} = 1 - \frac{\Psi_s}{\pi} \qquad \text{(A 9)}$$

APPENDIX B

Aerodynamic Forces in Free-Molecule Flow

The shear and normal-pressure force-coefficients, c_τ and c_p, which form the basis for the drag calculations in this study, were obtained by evaluating formulas presented in Section 10.3 of Hypersonic Flow Theory by Hayes and Probstein (Ref 1). These formulas are reproduced below, without derivation, for the convenience of the interested reader. The derivations, which are outlined and discussed in detail in the reference, are far too involved to be presented here.

The normal-pressure coefficient is defined in the present study as

$$c_p = \frac{p}{q} \qquad \text{(B-1)}$$

where q corresponds to the "dynamic pressure" of the usual aerodynamic calculations, and p is the aerodynamic force per unit area acting normal to the surface of the body at the point in question. The reference gives the following formula for p :

$$p = \frac{\rho_\infty V^2}{2 s^2}\left\{\left[\frac{2-f_n}{\sqrt{\pi}}(s \sin\theta) + \frac{f_n}{2}\sqrt{\frac{T_b}{T_\infty}}\right]\exp\left(-s^2 \sin^2\theta\right) + \left[(2-f_n)\left(s^2\sin^2\theta + \frac{1}{2}\right) + \frac{f_n}{2}\sqrt{\pi}\sqrt{\frac{T_b}{T_\infty}}\,(s \sin\theta)\right]\left[1 + \operatorname{erf}(s\sin\theta)\right]\right\}$$

(B-2)

In similar fashion the shear-force coefficient is defined as

$$c_\tau = \frac{\tau}{q} \tag{B-3}$$

where τ is the aerodynamic force per unit area acting parallel to the surface of the body at the point in question. In the reference, τ is given as

$$\tau = \frac{f_\tau \rho_\infty V^2 \cos\theta}{2\sqrt{\pi}\, s} \Big\{ \exp\left(-s^2 \sin^2\theta\right) + \sqrt{\pi}\,(s \sin\theta)\left[1 + \operatorname{erf}(s \sin\theta)\right] \tag{B-4}$$

In developing the aerodynamic drag relations used in the study, the above formulas were, of course, programmed for evaluation by digital computer. The computer formulations were carefully checked against a variety of plotted results given in the reference.

The notation used in the above relations is defined below:

c_p	normal-pressure coefficient $(= p/q)$
c_τ	shear-force coefficient $(= \tau/q)$
f_n	accommodation coefficient for normal forces
f_τ	accommodation coefficient for shear forces
p	normal pressure acting on surface
τ	shear force per unit area acting on surface
q	dynamic pressure $(= \rho_\infty V^2/2)$

ρ_∞ density in the free stream

V flow velocity (flight speed)

θ inclination of local surface to flow (for the lateral area of a flat plate, θ is the angle of attack of the plate)

s molecular speed ratio: the flight speed divided by the most-probable random speed in the undisturbed atmosphere $\left(= V/\sqrt{2RT_\infty}\right)$

T_b surface temperature of body

T_∞ free-stream temperature

In the numerical calculations, it was assumed that the body-surface temperature T_b was 340° K. This assumption was found to be noncritical, since the computed value for c_p is not sensitive to T_b in the range of interest. Values for properties of the atmosphere were taken from Reference 2.

The accommodation coefficients f_n and f_τ were both taken to be 1.0, as recommended in Reference 1, in the absence of better information. These coefficients represent ratios of momentum transfer in the two directions (normal and tangential) for the molecular impacts which produce the forces. Values of unity represent "perfect accommodation", in which the impacting molecules momentarily stick to the surface, thereby giving up all their initial momentum relative to the surface, and are then re-emitted in random directions with a random velocity distribution scaled by the surface temperature; values of zero represent specular reflection (as for light) with no momentum exchange except that caused by reversing the component of relative velocity normal to the surface. It can be shown that values for f_n less than 1.0 serve to increase the normal-pressure coefficient (almost doubling it, to a value of 4, for $f_n = 0$); decreasing f_t below 1.0, on the other hand, produces a proportional decrease in the shear coefficient. The assumption of unity for both coefficients, to the extent it is in error, thus tends to give optimistic predictions for the sunpointing system and pessimistic predictions for the trailing system.

APPENDIX C

Attitude Stability and Control of the Low-Drag Space Station with a Trailing Solar Array

For the sake of completeness in the estimation of the long-term fuel requirements for the trailing solar array, it is important to include an estimate of the fuel required for the control system which maintains the desired orientation of the array. This task is complicated somewhat by the facts that a large trailing array would most likely find application as part of an unconventional, low-drag space station, and that the problems of attitude stability and control of the large solar array cannot be isolated from those of the space station itself. The principal new factors in the attitude-control problem are a), the potential for utilizing the aerodynamic forces on the array, and b), the likelihood that the minimum-drag attitude of the vehicle will be unstable in the gravity-gradient field. As will be discussed, a preliminary investigation indicates that a low-drag space station with a trailing array should not require additional fuel (beyond that required for drag makeup) for control of either the main space-station structure or the array, and that the aerodynamics of the array can be used to advantage in a variety of ways in the control of the spacecraft.

For the purposes of this brief study it has been assumed that the space-station configuration is basically as indicated in Figure 1: a long, slender structure which is to be operated with its longitudinal axis aligned with the flight path, and which has the trailing solar array structure attached in such a way that it lies to the rear of the main structure, with its axis of symmetry along the flight-path axis. Placing the array in this position assures that the aerodynamic forces will help to stabilize the space station in the low-drag attitude. It also produces the lowest set of gravity-gradient torques consistent with the constraints of minimum drag and the best allowable orientation for solar collection.

The gravity-gradient torques which act on the spacecraft tend to amplify any misalignment between the longitudinal axis of the slender structure and the horizontal flight path, since a slender mass orbiting in a horizontal attitude is in unstable equilibrium and tends to rotate toward the vertical under the influence of the gravity-gradient field. For small misalignments of a slender mass, the destabilizing torque is proportional to the misalignment, so that the rotational motion which is produced is a divergent exponential (as long as the amplitudes remain small). Since the torque is also proportional to the moment of inertia, the divergent motion of a slender mass starting from rest in a near-horizontal position is not dependent on the mass or its distribution along the axis. Thus the rotational motion is the same for all such bodies, except for a weak dependence on orbit altitude; for small amplitudes, the time required to double the amplitude of the misalignment is about six minutes. Because of this relatively rapid response, attitude control must be either continuous or nearly so; intermittent control would have to be applied at least about ten times per orbit in order to be effective in maintaining small misalignments. In contrast, the control for orbit-energy maintenance (drag makeup) can be effective if applied only about once per orbit.

There are basically three kinds of control mechanisms which appear to be readily applicable to the attitude-control problem considered here: the inertial-reaction flywheel; aerodynamic forces on the trailing array; and reaction-jet motors as used for the maneuvering system. Because of the nature of this particular control problem it appears that the mechanism which is best suited for the primary control system is the inertial-reaction flywheel ("reaction wheel"). The basis for this claim is that the gravity-gradient torques are small, even for a large space station, and furthermore the space-station attitude can be managed so as to make the time integral of the torque vanish over the course of the orbit. As an indication of the size of the gravity gradient torque which must be resisted, a large, slender spacecraft (300 ft long; 100,000 lbm) would experience a torque of about two lbf-ft per degree of misalignment. The reaction wheels can therefore be lightweight and compact, and can be operated in such a way that the rotation speed is low most of the time.

The attitude of a spacecraft with a trailing array could also be controlled by the use of aerodynamic forces. In acting to stabilize the attitude of the spacecraft, the trailing array behaves in the manner of a tail surface on an airplane, except that its lift-to-drag ratio is extremely poor (by aircraft standards) at small angles of incidence. Because of the location of the array with respect to the center of mass of the spacecraft, the normal-pressure force due to a small misalignment of the plane of the array with the flight direction tends to reduce the misalignment. Similarly, if the misalignment involves a "sideslip" angle by which the array is shifted laterally in-plane so as to misalign the drag force with the flight path of the center of mass, the resulting in-plane moment will also tend to reduce the misalignment. For a trailing array with aerodynamics as assumed, the in-plane drag moment is about a third of the lift moment for equal misalignments (for aircraft it is about one tenth of one percent), and is therefore large enough to be significant whenever the lift-direction moment must be considered (the drag moment can always be increased, of course, by adding drag, but that would defeat the purpose here). Thus the single-plane array can be used to provide stabilizing moments both in the plane of the array and perpendicular to the plane.

For a large space station in a 220-n.mi orbit, the forces developed by the array would not be sufficient for attitude stability unless the array surface were somehow operated as an active control surface. Since the required control system would probably not be as lightweight or compact as the reaction-wheel system, or as easily certified in pre-launch checkout, active aerodynamic control would probably not be used as the primary control system. The aerodynamic forces which are available passively would serve, however, to relieve the requirements on the control system, by preventing tumbling in the case of a temporary failure of the primary system and by reducing the magnitude of the net torque acting on the vehicle with any given amount of misalignment.

Since the aerodynamic forces are proportional to atmospheric density, the usefulness of aerodynamic stabilization increases as the orbit altitude is reduced. Furthermore, because of the fact that, as the size of the spacecraft is scaled down, the gravity-gradient torques decrease more rapidly than the aerodynamic torques, it may be possible with smaller spacecraft to provide attitude stabilization passively with a trailing array. In the case of smaller spacecraft in relatively low orbits, the aerodynamic stabilization provided by a large trailing solar array may easily be sufficient without an active control system.

Where the forces on the array are not sufficient for control as a passive, static surface, the effectiveness of the control action can be increased by actively controlling the alignment of the array. The control system might make use of a power-actuated pivot at the point at which the array is attached to the main structure of the spacecraft. If the alignment of the array with the structure were always maintained so that the angle of incidence of the array were several times as great as the misalignment angle of the longitudinal axis of the space craft, the aerodynamic forces would become correspondingly larger in relation to the gravity-gradient forces. A "tail-wagging" system of this kind should be capable of increasing the effectiveness of the array surface by an order of magnitude, which should suffice for a large, low-drag space station of the kind considered here. It should be understood that the motions of the array which would be required for control are small enough that there would be no significant effect either on the solar-energy collection or on the drag of the surface.

Because an aerodynamic surface is capable of producing a steady lateral force for an indefinite length of time, it is conceivable that the actively controlled array might best be used in a manner which complements a reaction-wheel system. In the event of a spurious rotational impulse, such as might occur in docking or with misaligned drag-makeup thrusters, the reaction-wheel system must absorb the acquired angular momentum quickly and then continue to maintain the alignment of the spacecraft. An aerodynamic control surface would be useful in providing the steady torque necessary to unload the reaction wheel in order to bring the system back to the desired condition of near-zero angular momentum.

Attitude stabilization could also be accomplished by the use of a reaction-jet maneuvering system. In principle, reaction-jet control would be available at essentially no additional fuel cost, because the jet propulsion is needed, in any case, for counteracting the orbit-energy loss due to atmospheric drag. For this purpose, the rearward-pointing drag-makeup jets could be gimballed, to allow the jets to be directed at small angles to the flight path, in order to develop the necessary small, lateral, attitude-control forces. Since the control-force components could be made to be very small compared to the drag force, the control function would add very little to the thrust which is required for sustaining the orbit. A major difficulty with this scheme is that while the drag-makeup impulse could be supplied satisfactorily by a firing once or twice per orbit, the control system must be in operation essentially continuously to avoid large build-up of the unstable motion. Since rapidly sequencing or continuous firings of the reaction motors would probably be undesirable for a variety of reasons (including poor fuel economy, and increase of probability of failure in a given lifetime), it may be concluded that the use of the drag-makeup jets for primary attitude control is undesirable.

In discussing the specific requirements for an attitude control system for a low-drag spacecraft, it is convenient to make use of a reference system aligned with the flight path direction and the direction of the local vertical. Angular motion of the spacecraft can then be considered to be either pitch (rotation in the vertical plane aligned with the flight path), yaw (rotation in the horizontal plane), or roll (rotation about the longitudinal axis). In this reference system, attitude control of the spacecraft is a matter of maintaining the pitch and yaw displacement angles at essentially zero (i.e. keeping the longitudinal axis aligned with the flight path), while maintaining the solar array at the desired roll angle (the angle for best solar collection). Since the effects of both gravity gradient and aerodynamics are different in each direction, the control problem for each axis will be considered separately.

With regard to pitch motion, the vehicle is nominally in unstable equilibrium: there is nominally no gravity-gradient torque acting on the space-station mass, but a small displacement either nose-up or nose-down would tend to start it rotating toward a vertical alignment, in which orientation it would be stable (but would have higher drag). A reaction wheel in the pitch plane could be used to stabilize the horizontal attitude. Since the gravity-gradient torque in the desired attitude is nominally zero, with excursions equally likely on either side of zero, the attitude-control system for pitch could be managed so as to maintain a net change in angular momentum which is essentially zero at all times. Thus the reaction wheel for pitch could be a very small, light-weight unit; preliminary estimates indicate that the mass of the wheel could be well under one-tenth of one percent of the space-station mass if the pitch excursions are limited to a few degrees. Since the pitch wheel is in the plane of the orbit, and does not change plane as the spacecraft moves around the orbit, there are no gyroscopic precession torques imposed by the rotating wheel on the spacecraft.

The aerodynamic forces which develop when the spacecraft is disturbed in the pitch plane depend on the orientation of the array (i.e. the roll attitude). Control by aerodynamic forces is, of course, most easily achieved when the array is in the horizontal plane (with the sun in the plane of the orbit), for which case the lift-direction forces are in the pitch plane. For other solar angles, however, an aerodynamic torque in the pitch plane would require components both in the lift direction and in the drag direction. In the case of a spacecraft with a passive (uncontrolled) array oriented at an angle to the horizontal, a small pitch excursion would produce a lift force with a horizontal component, which in turn would produce a small displacement in the yaw direction. Depending on the system, this kind of yaw motion may or may not have to be actively corrected.

With regard to yaw motion, the rear-mounted trailing array tends to stabilize the spacecraft in the desired alignment with the flight path. Since there are no destabilizing yaw moments acting, passive aerodynamic stabilization may be sufficient. Thus, depending on the application, it may not be necessary to provide reaction-wheel control for the yaw axis of a low-drag spacecraft with a trailing array.

With regard to roll motion, the low-drag array tends to produce a steady gravity-gradient roll torque about the horizontal axis of symmetry whenever the plane of the array is not horizontal or vertical. This torque is maximum when the array is inclined at 45 degrees to the horizontal, and has a value of about an eighth of one lbf-ft for the 150 kw array considered previously. In order to counteract the roll torque over the sunlit portion of the orbit, a reaction wheel would have to absorb about 250 lbf-ft-sec of torque impulse, which could be handled by a very small, lightweight system weighing no more than perhaps twenty pounds total. It should be understood that the torque can be reversed during the dark side of the orbit, so that there is no need to accumulate angular momentum beyond that required for a single orbit.

There is also a small amount of roll-yaw coupling caused by the fact that the roll-axis reaction wheel is continuously changing plane due to the orbit curvature. When the roll-axis wheel has maximum angular momentum, a yaw-axis torque of about one sixth of one lbf-ft is produced. If this torque were applied indefinitely it would eventually produce a small yaw rotation (about three degrees in the worst case for the example considered) before it was equilibrated by the tail-fin action of the array (assuming no active control). Since the torque is periodic (the roll wheel changes direction of rotation every half orbit), however, and since the space vehicle has a very large moment of inertia in yaw, the maximum yaw deflection would be very small even in the case of a vehicle with no active control on the yaw axis (less than half a degree in the example).

Control torque for the roll axis can also be generated by aerodynamic forces on the array, if an active control system is used. The required surface deflections could be produced by twisting the array or by actuating control tabs along the trailing edge. The availability of aerodynamic torque would allow a steady roll torque to be applied to the spacecraft for an indefinite length of time, as might be required to counteract some systematic roll-torque bias due to misalignments of thrusters, out-gassing, or perhaps even unsymmetrical aerodynamic forces on the solar array itself. If the aerodynamic torque were used as a trimming mechanism in conjunction with a reaction-wheel primary system, the control system for the array could be integrated into the array structure at very little cost in weight and complexity.

As the arguments presented above indicate, the attitude control of a low-drag spacecraft with a trailing solar array is a well-defined matter of using conventional control elements, and involves levels of torque and total impulse which are small enough to be provided by "hand-portable" reaction-wheel units, even for very large space stations. Since the nominal attitude of the spacecraft is invariant, and the relative orientation of the array is unchanging except for the slow periodic rotation in roll about the longitudinal axis of symmetry, the general control problem is extremely simple by the standards of conventional spacecraft with sunpointing arrays. The presence of aerodynamic forces on the trailing array provides ultimate stability in a near-correct attitude, and prevents tumbling in the event of catastrophic control-system failure. These considerations should permit the attitude of the spacecraft and array to be controlled reliably, with a simple, light-weight control system, and without the use of reaction-jet fuel except for that which is required to maintain the orbit energy.

To complete the comparison between the trailing and the sunpointing systems, it may be worthwhile to include a very brief discussion of the problem of maintaining the alignment of a sunpointing solar array on a "conventional" space station. In the usual case, the sunpointing array is supported on a cluster of structural modules which are allowed to "hang" in stable equilibrium in the gravity-gradient field (with the axis of minimum rotational inertia aligned with the vertical). Since the inertia of the main structure dominates the inertia of the spacecraft/array combination, the main structure will continue to hang with its own axis nearly vertical as the array is continuously repositioned to remain aligned with the sun. The gravity-gradient torques on the main structure therefore provide a stable reference against which the array can be controlled in the pitch and roll planes (i.e. in elevation).

For control of the yaw (azimuth) attitude, however, there is no corresponding reference; the spacecraft orientation about the vertical axis must be artificially controlled and stabilized by some means such as the maneuvering-system jets or a reaction wheel. The yaw-control problem is complicated by the fact that the array must be kept out of the shadow of the space station, and the fact that there may be spurious torques (such as those associated with aerodynamic forces on the solar array) which act on the structure in one direction for extended periods of time. Since a reaction-flywheel system can be quickly saturated by small but continuous torques, the yaw-attitude control system must make use of the reaction-jet maneuvering system to "unload" the flywheel. Depending on the system, it may be possible to apply the necessary corrective yaw torque through the use of the drag-makeup thrusters, which must be fired about once per orbit in any case.

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Table I

h	altitude of circular orbit: 220 n. mi. (407 km)
I_{sp}	specific impulse of sustainer-jet motors: 300 lbf sec/lbm
k_p	power output of solar cell array at normal incidence: 225 watts/m^2
m''_A	mass per unit area of solar array: 0.21 lbm/ft^2 (1.0 kg/m^2)
m'_Q	specific mass of energy storage system: 40 lbm/kw hr (55 watt hrs/kg)
$\bar{P}$	average power demand: 150 kw
T	orbit period: 1.544 hr
v	orbit velocity: 25144 ft/sec
ρ/ρ_0	density ratio at orbit altitude: 4.7×10^{-12}
ψ_s	orbit angle at edge of shadow: 110 degrees

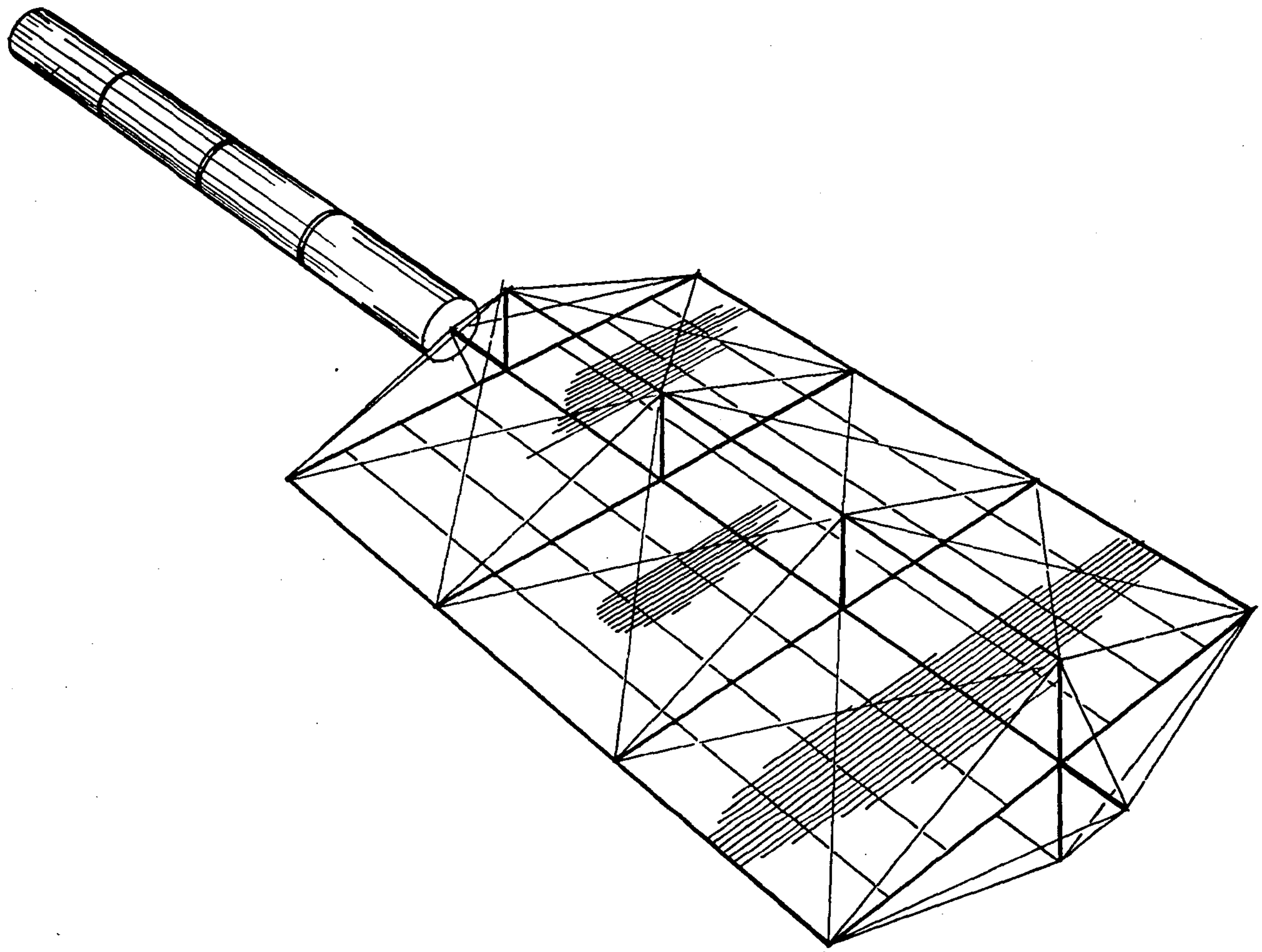

Figure 1. - Proposed configuration for low-drag space station with trailing solar array.

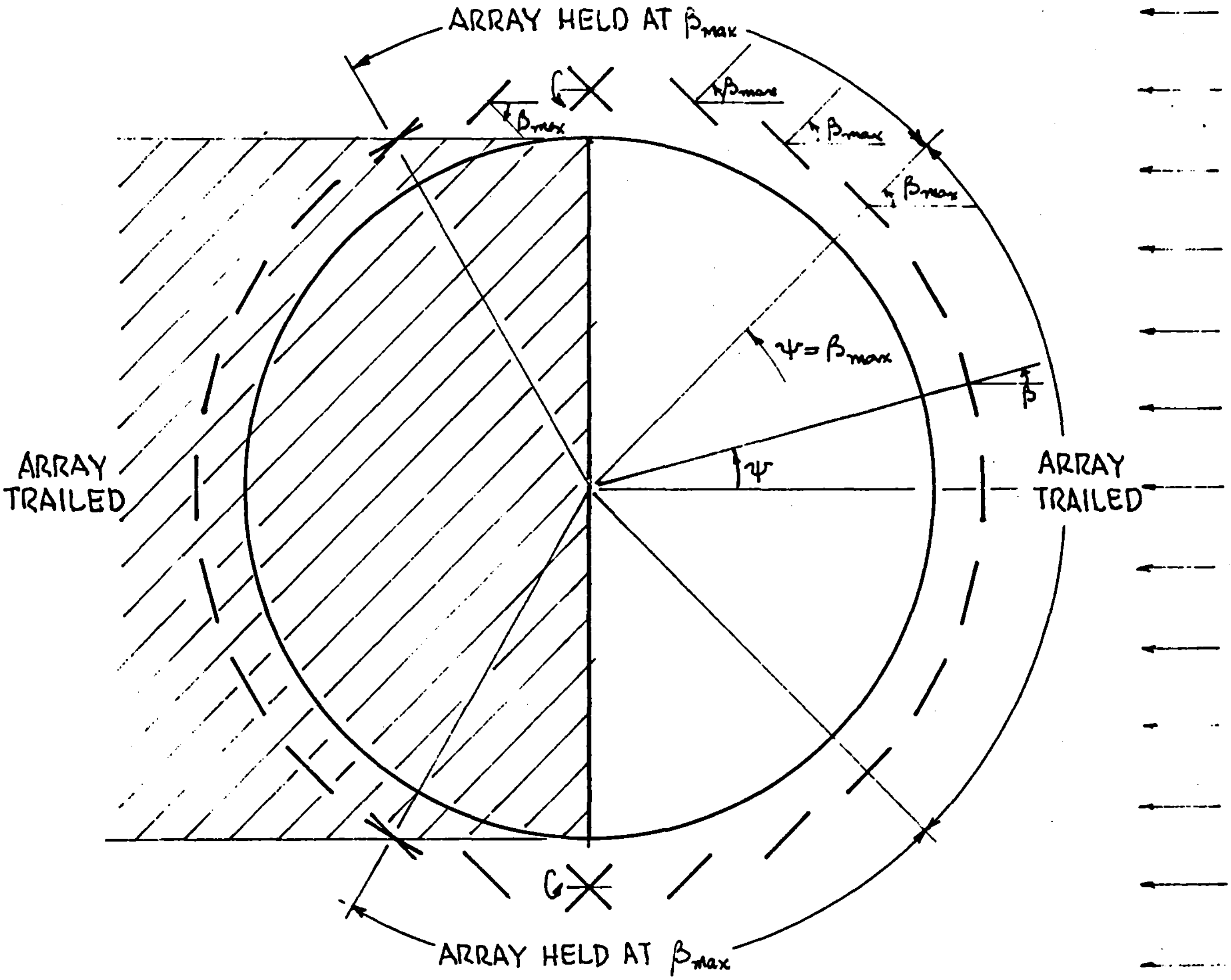

Figure 2. - Orientation schedule assumed for operation of solar array (sun in plane of orbit).

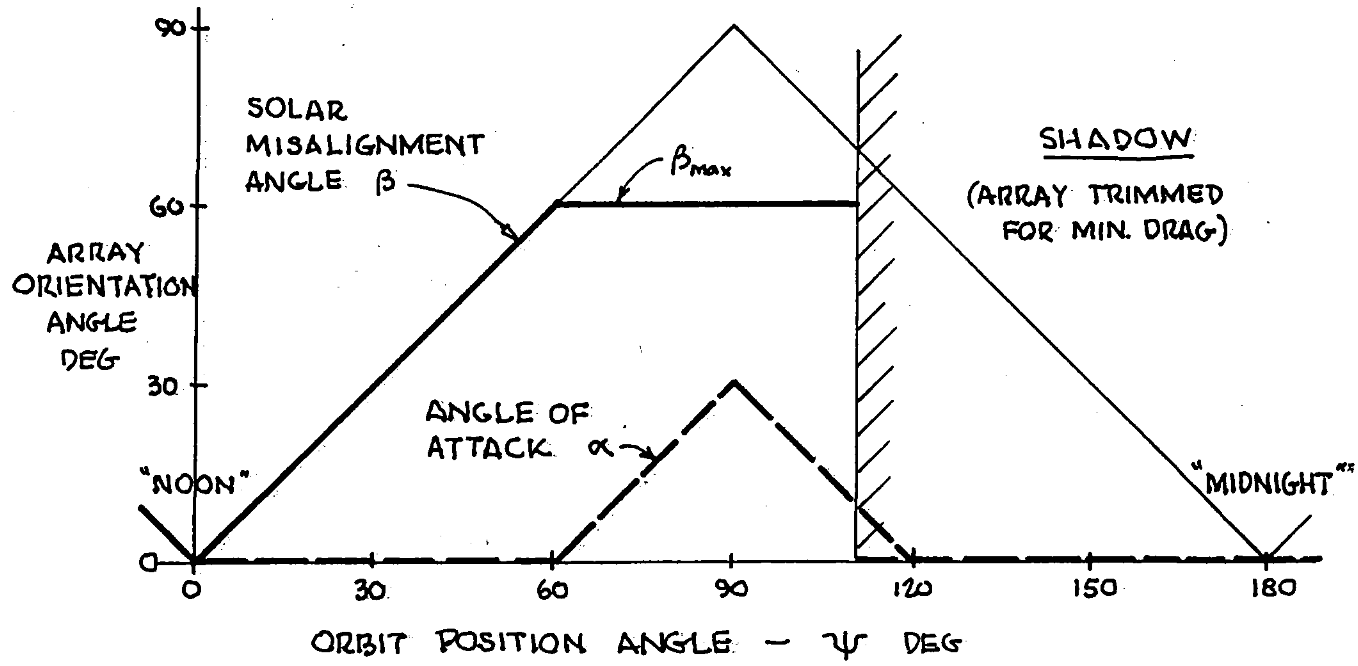

Figure 3. - Plot of typical orientation schedule for solar array (β_{max} = 60°).

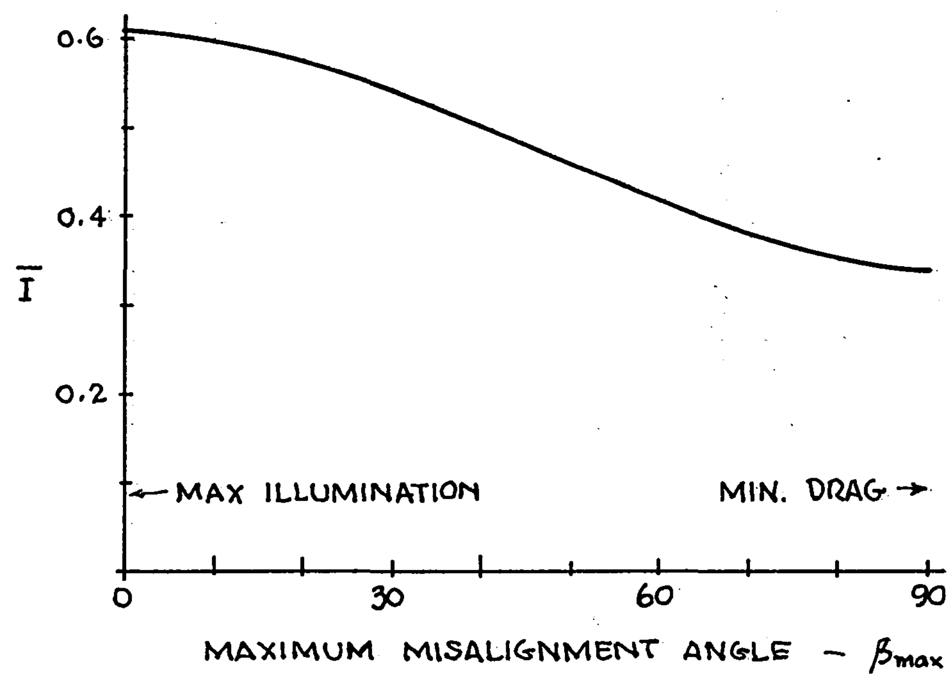

Figure 4. - Variation of average effective illumination as a function of maximum misalignment angle.

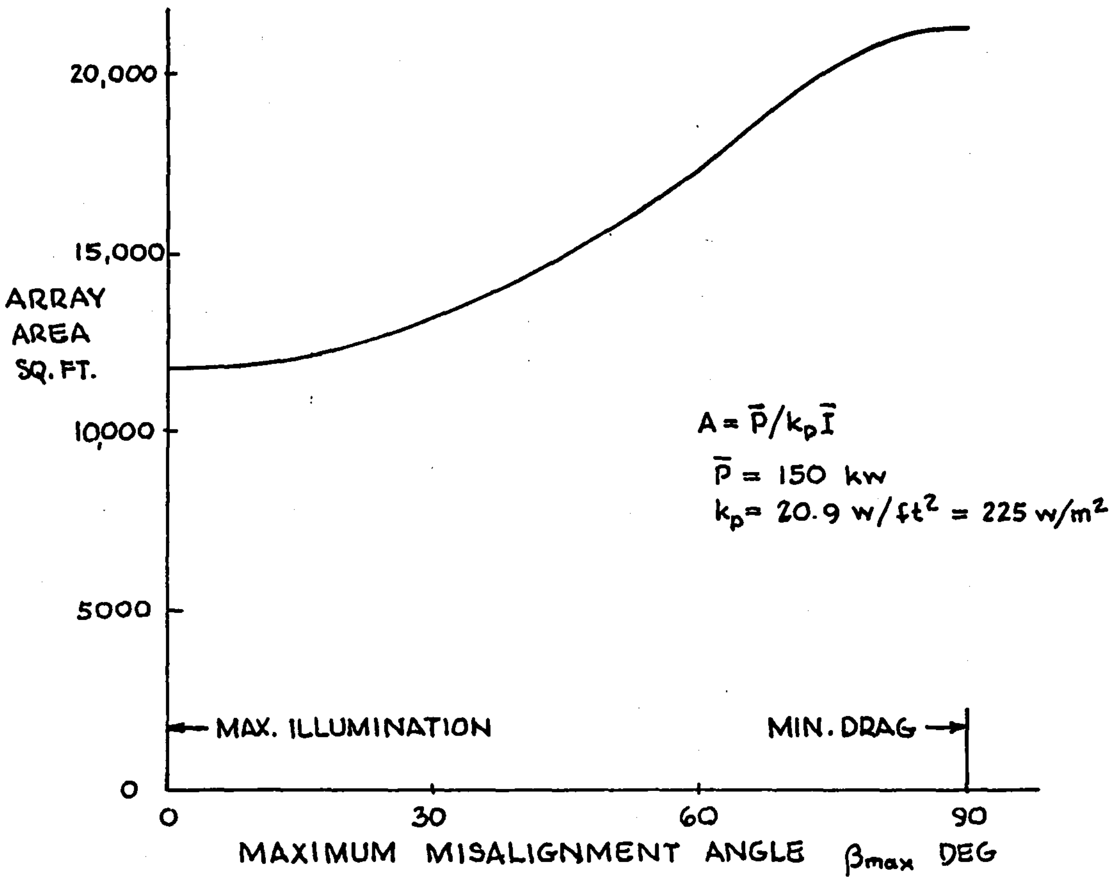

Figure 5. - Variation of required area of solar array as a function of maximum misalignment angle (for average power of 150 kw).

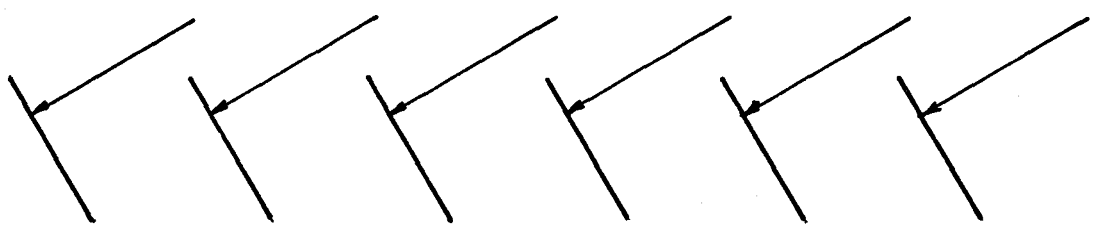

Figure 6. - Slatted-collector concept for reducing losses due to reflection under conditions of poor solar alignment.

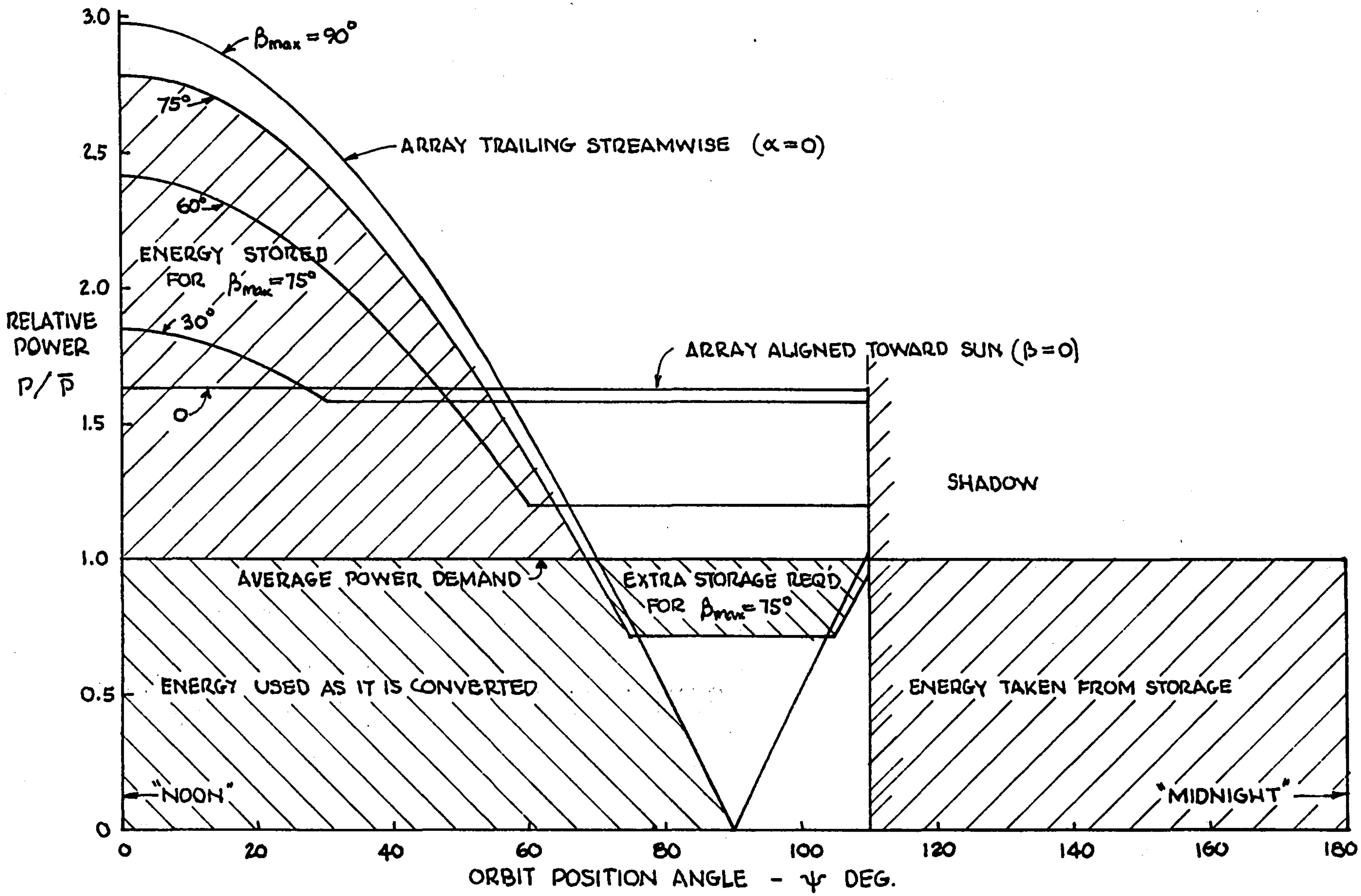

Figure 7. - Power schedules for several different orientation schedules (shaded areas apply for β_{max} = 75°).

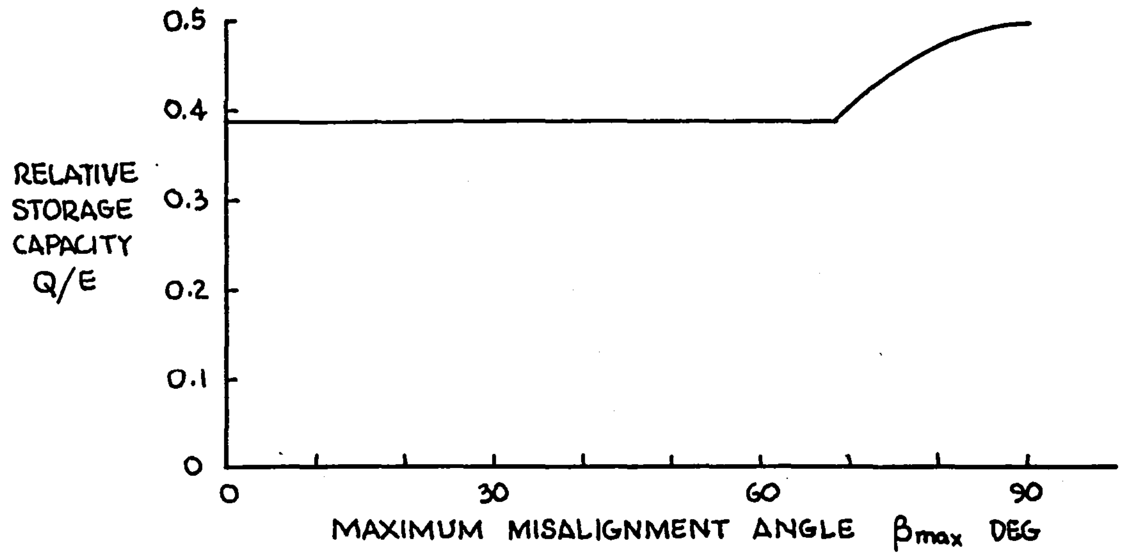

Figure 8. - Variation of the required energy-storage capacity with maximum misalignment angle β_{max} .

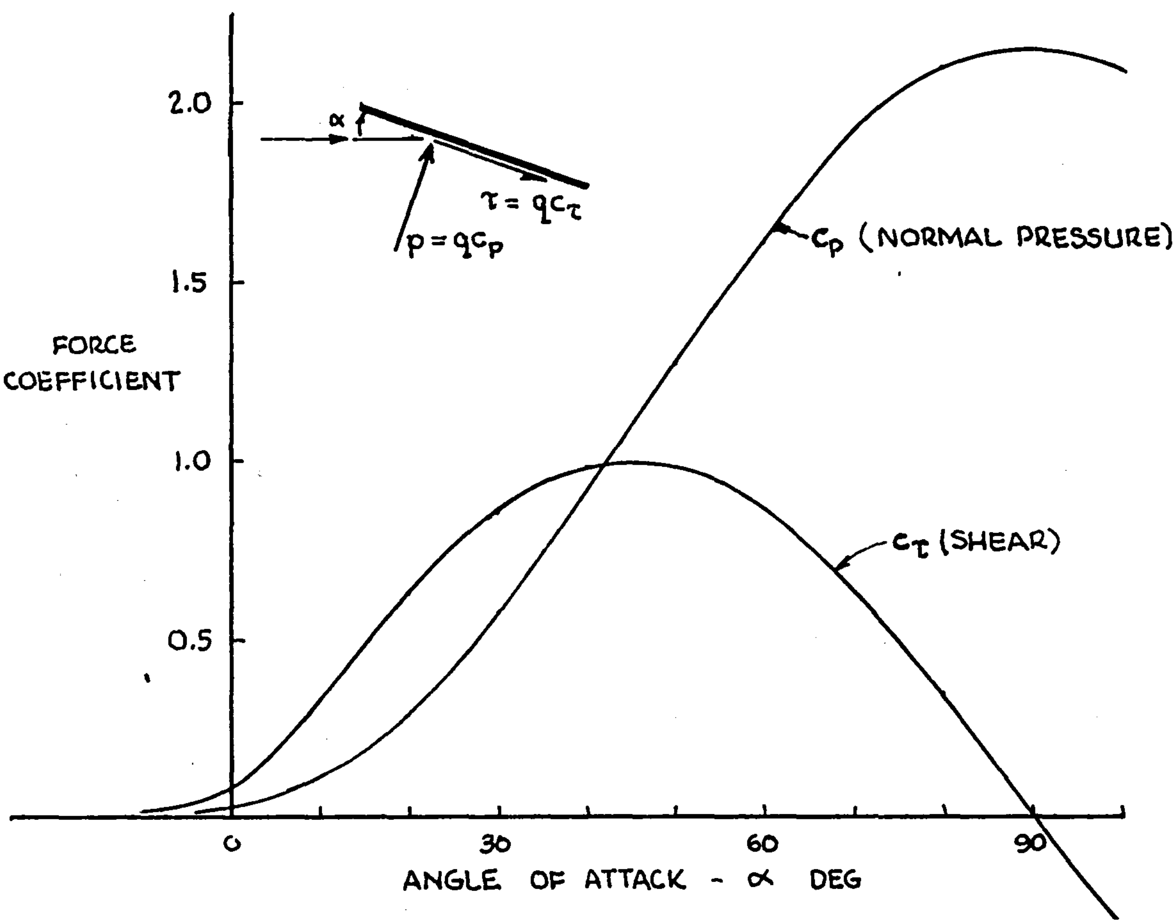

Figure 9. - Aerodynamic shear and pressure coefficients for a flat-plate surface in free-molecule flow (circular orbit at 220 n.mi. -- see Table I).

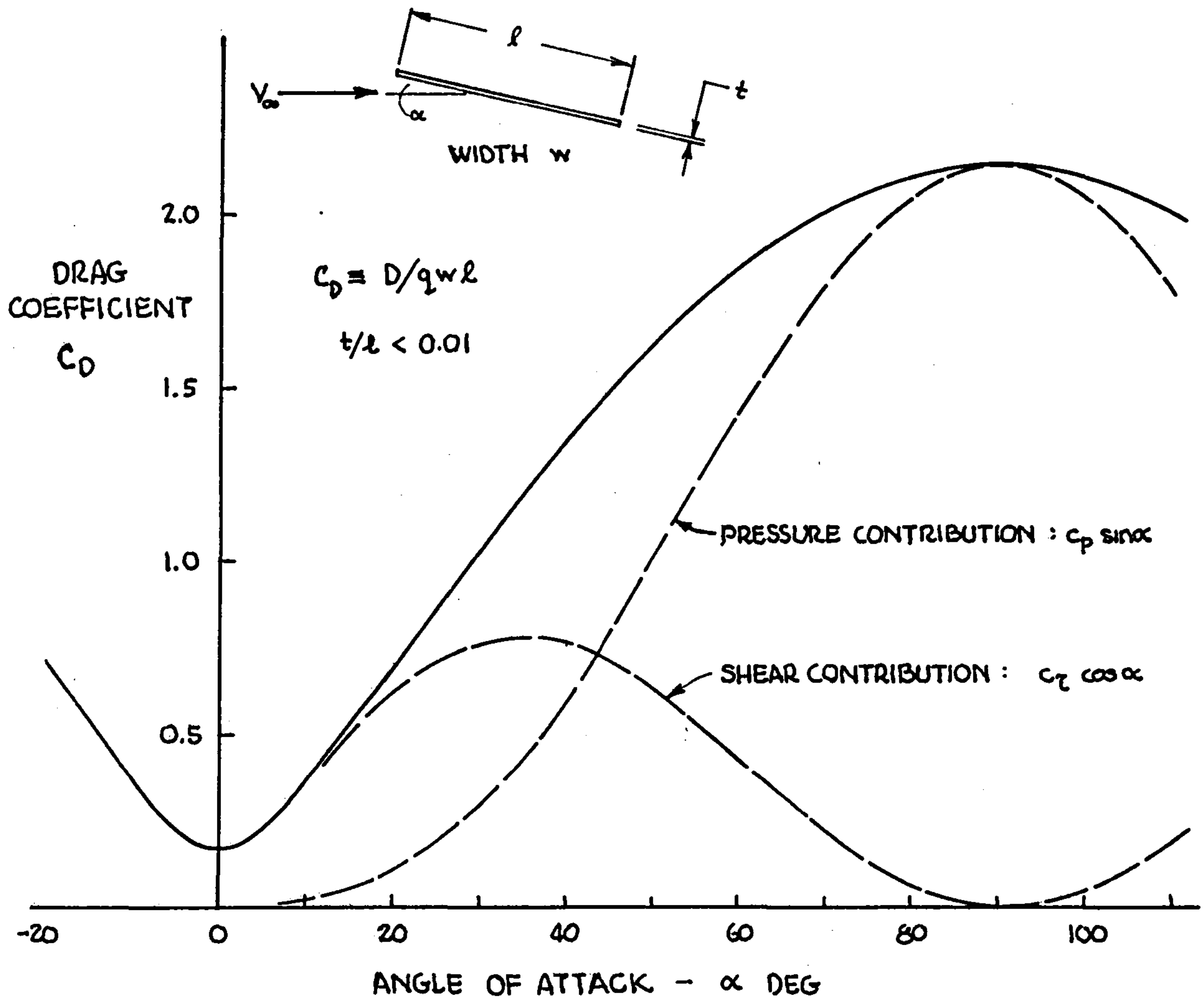

Figure 10. - Drag coefficient for a thin, flat-plate structure in free-molecule flow (based on Figure 9).

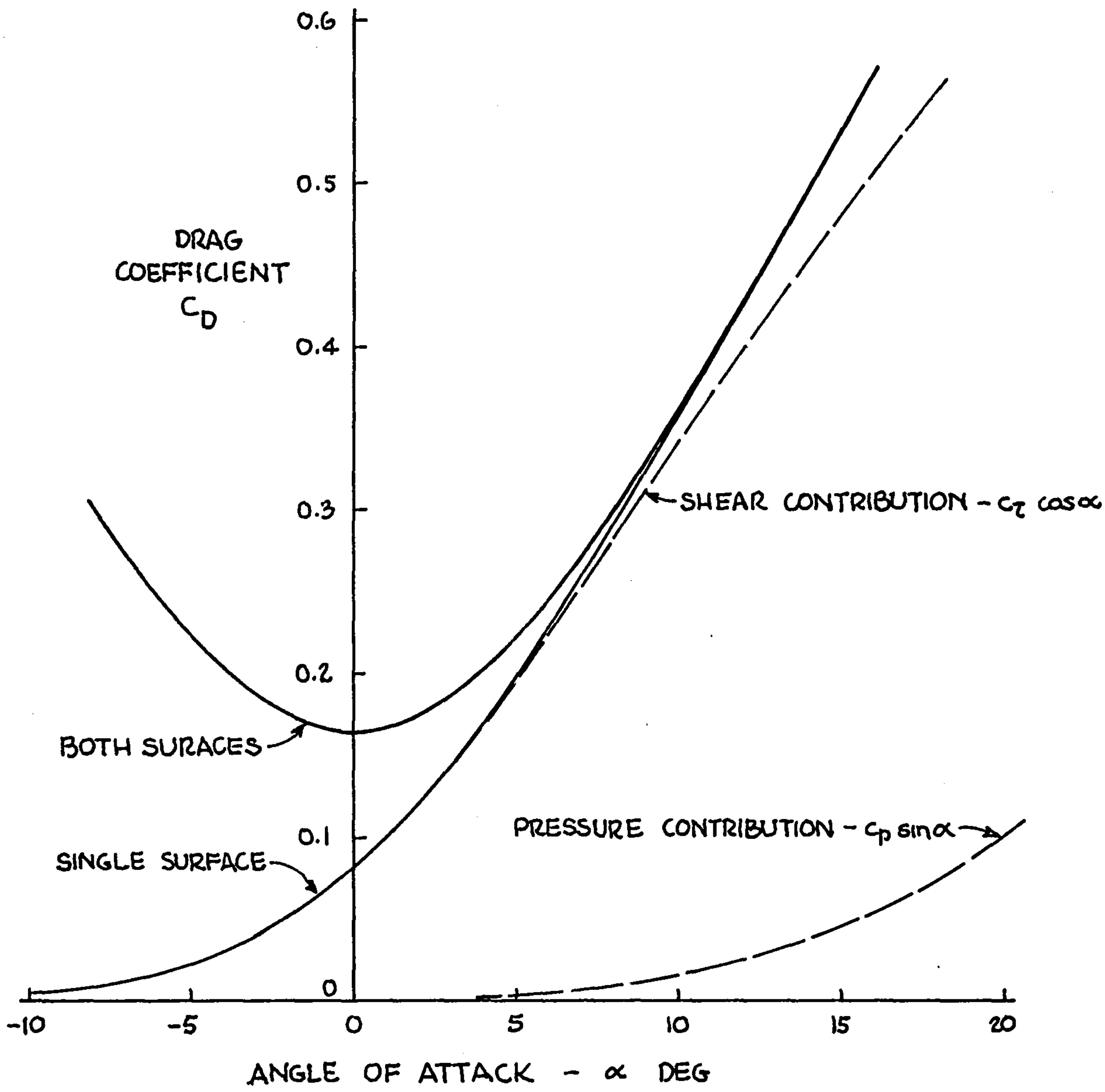

Figure 11. - Drag coefficient for a thin, flat-plate structure in free-molecule flow, near zero angle of attack (expanded from Figure 10).

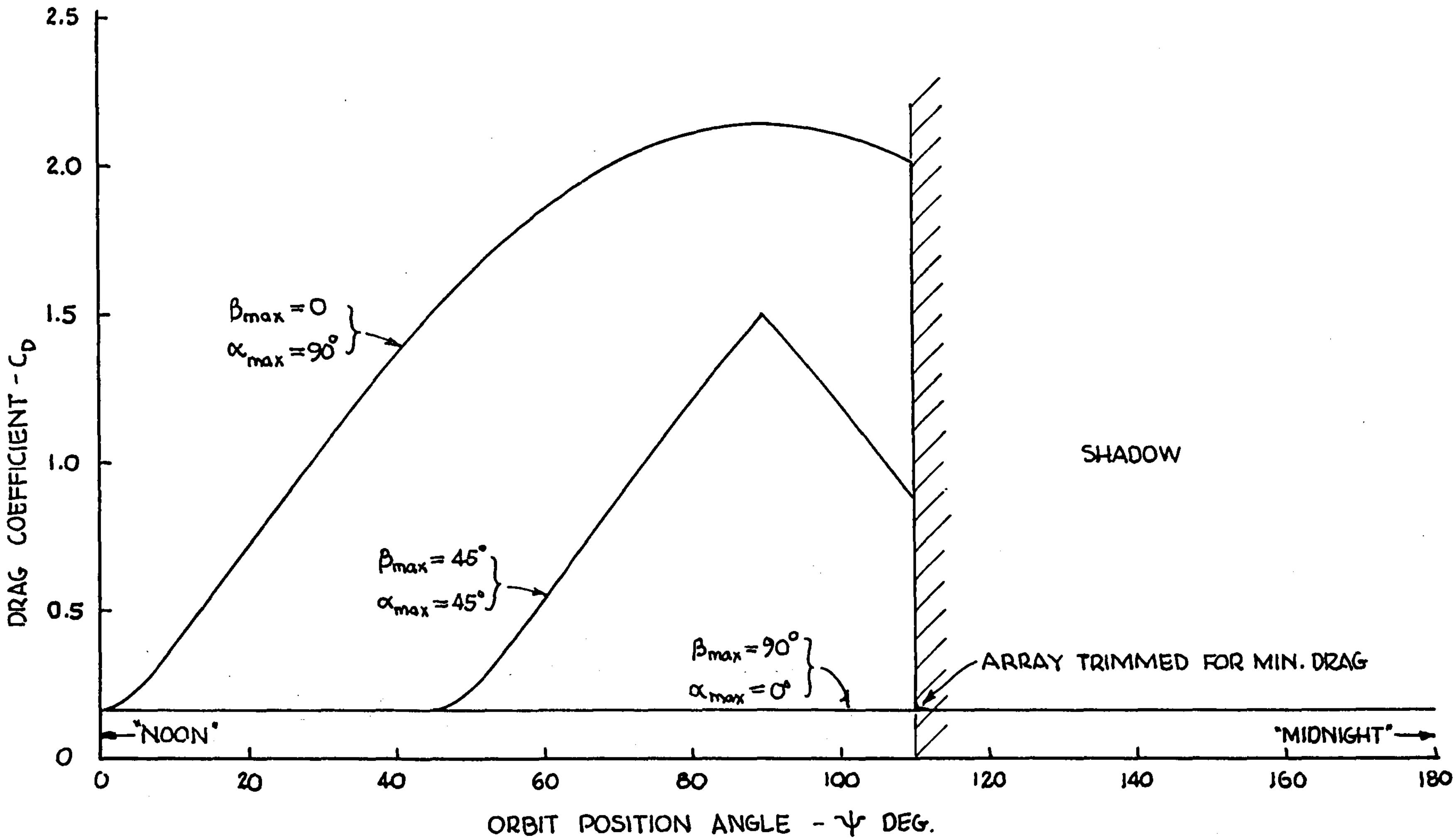

Figure 12. - Typical profiles of drag coefficient as functions of orbit position, for several values of maximum misalignment angle β_{max}.

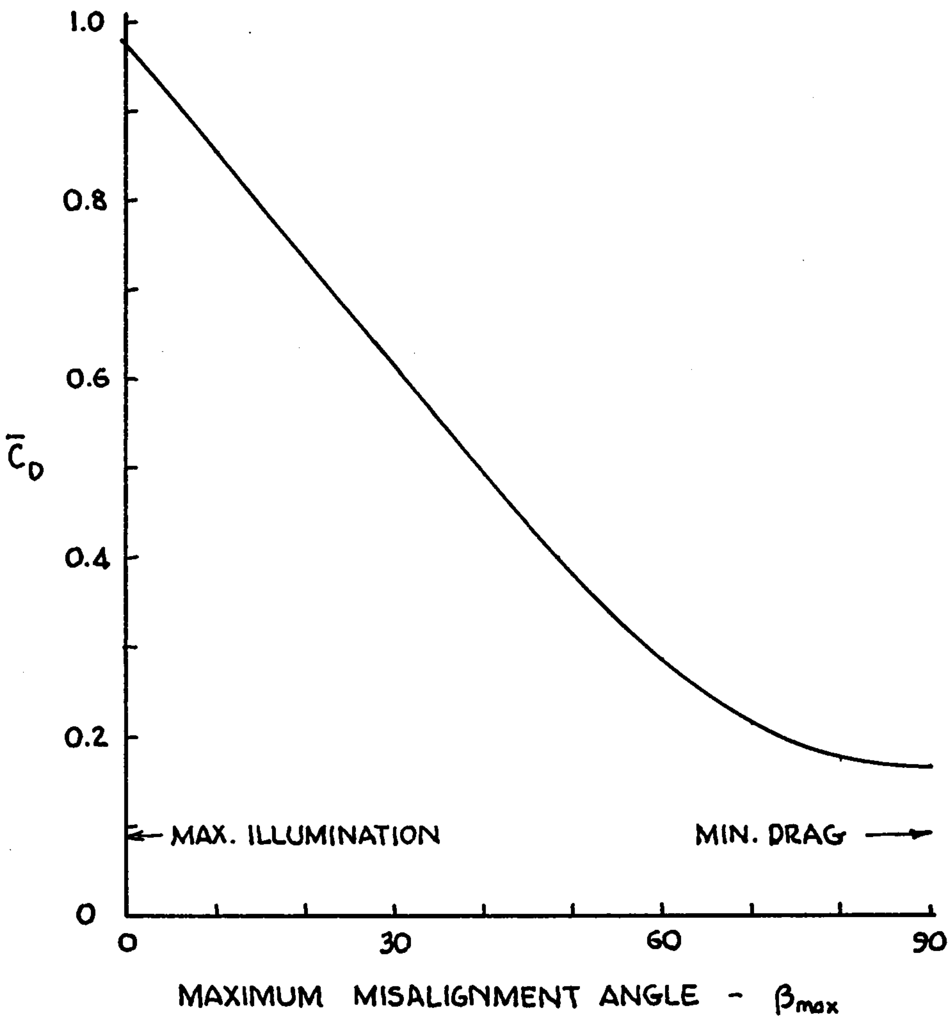

Figure 13. - Average drag coefficient $\bar{C}_D$ for a complete orbit, as a function of maximum misalignment angle β_{max}.

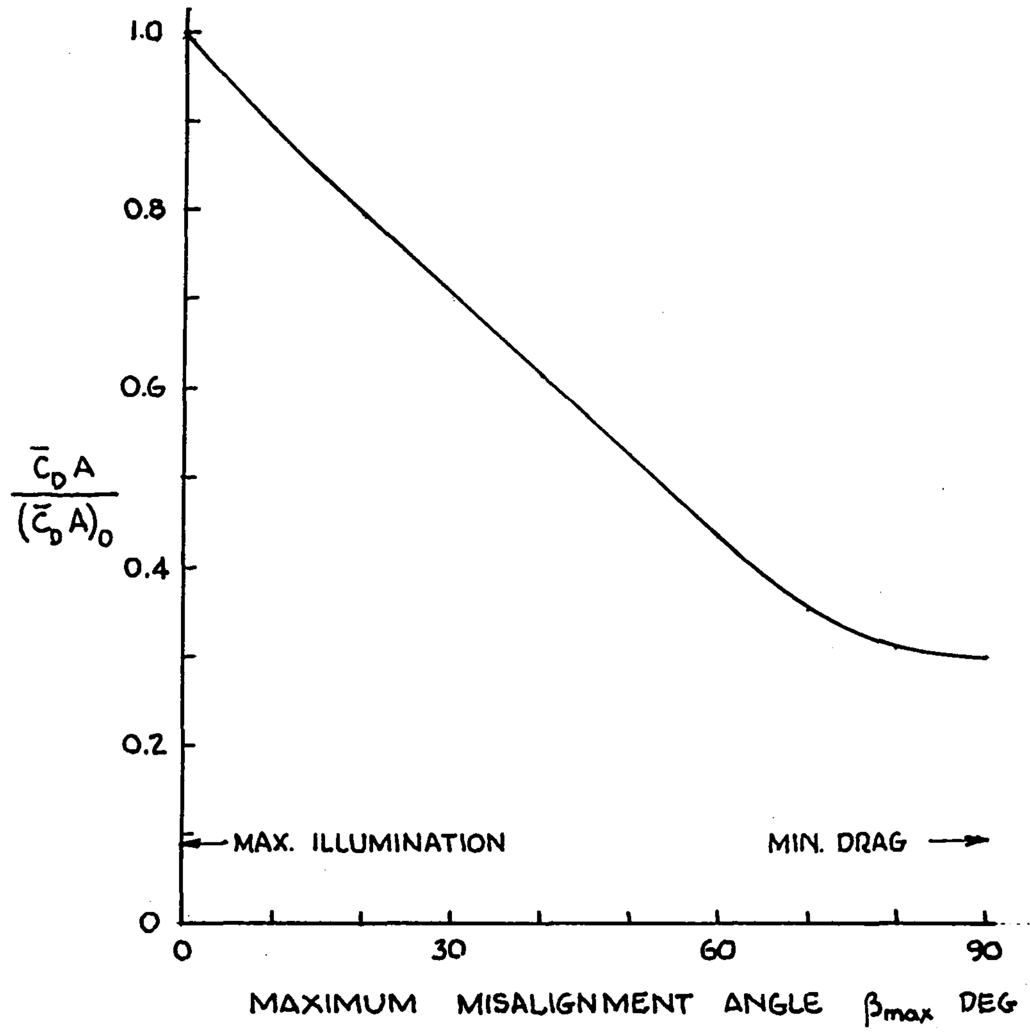

Figure 14. - Average effective drag area $\bar{C}_D A$ for equivalent solar array designs, as a function of maximum misalignment angle β_{max} .

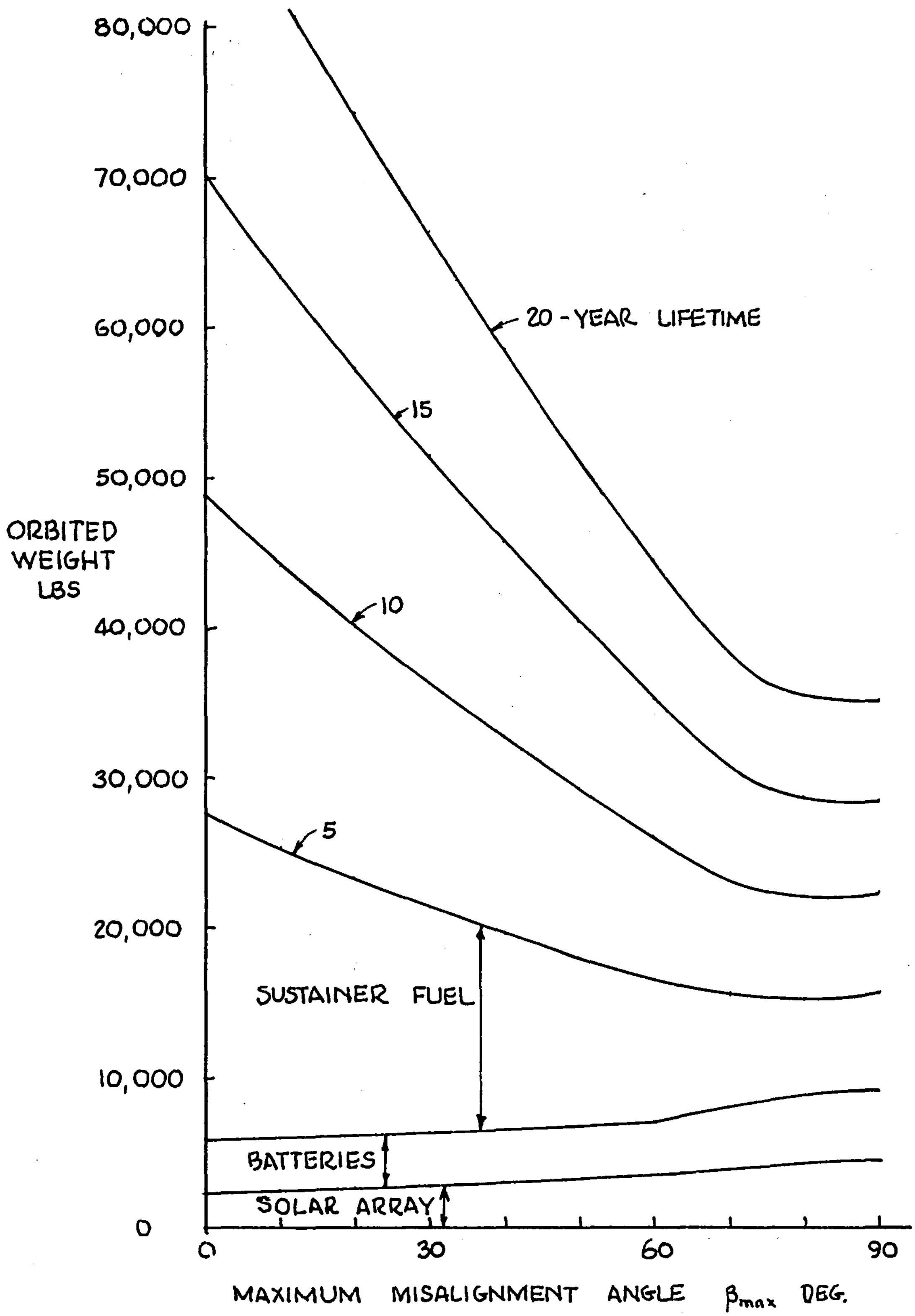

Figure 15. - Total orbited weight of 150-kw solar power systems for several values of operational lifetime, as functions of the maximum misalignment angle (system parameters given in Table I).

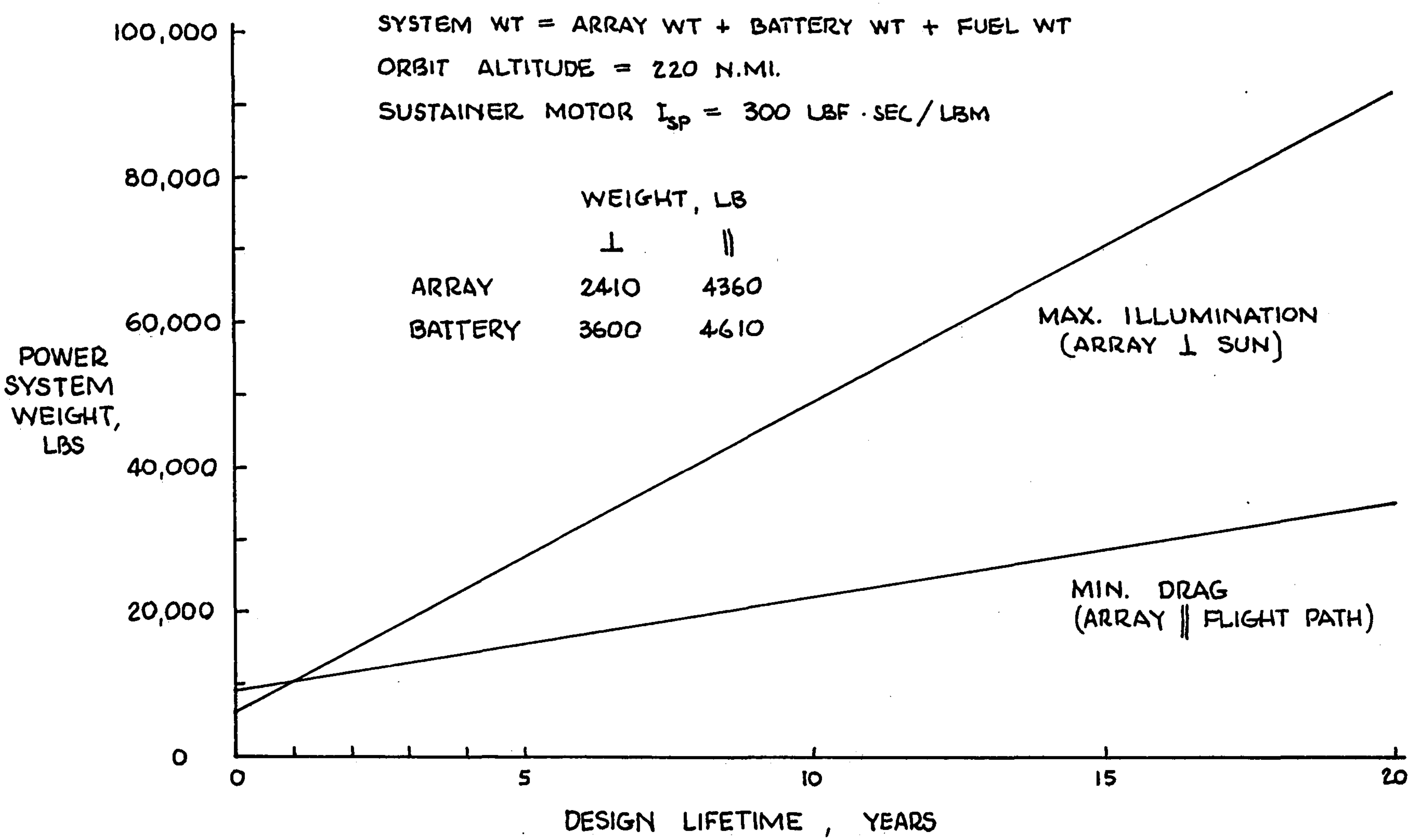

Figure 16. - Comparison of total orbited weights vs design lifetime for two types of 150 kw solar power systems: sunpointing and minimum-drag.

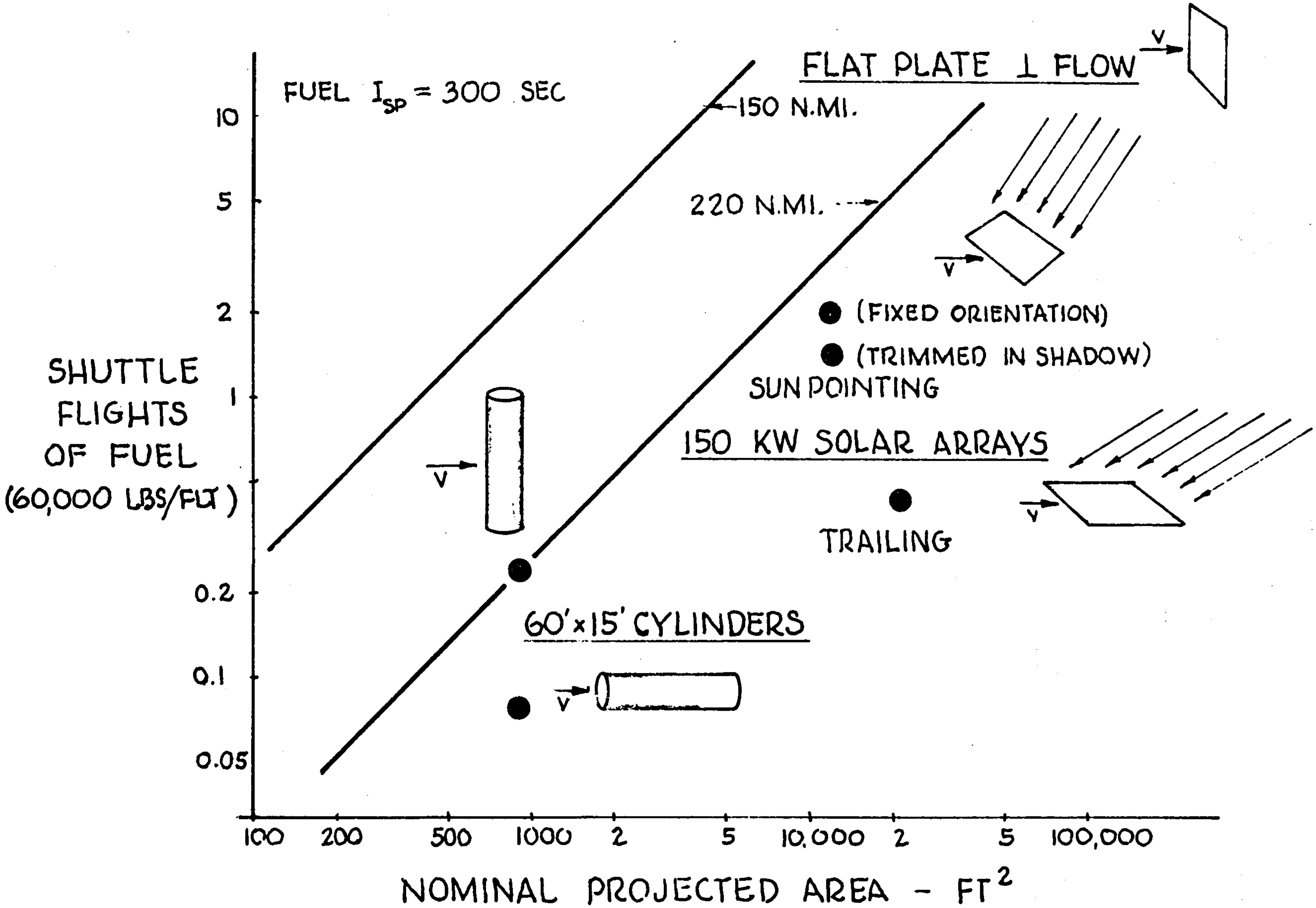

Figure 17. - Drag-makeup fuel required for 20-year lifetimes for several representative space-station elements.

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15. Supplementary Notes

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16. Abstract

A preliminary engineering study was made to determine the feasibility of reducing the atmospheric drag of low-orbit solar-powered satellites by operating the solar-cell array in a minimum-drag attitude (i.e. trailing streamwise), rather than in the conventional sun-pointing attitude. The study took into account the weights of the solar array, the energy-storage batteries, and the fuel required to overcome the drag of the solar array for a range of design life times in orbit.

The drag of the array was estimated by free-molecule flow theory, and the system weights were calculated from unit-weight estimates for 1990 technology. Although the trailing, minimum-drag system was found to require 80 percent more solar-array area, and 30 percent more battery capacity, the system weights for reasonable life times were found to be dominated by the thruster fuel requirements. For an example case of a space station orbiting at 220 n.mi. for 20 years, with a specific impulse of 300 sec for the drag-makeup thrusters, the total system weight with a trailing array was 35,000 lb compared with 92,000 lb for the sun-pointing system.

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