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Conservation and Pollution

A Joint Program for
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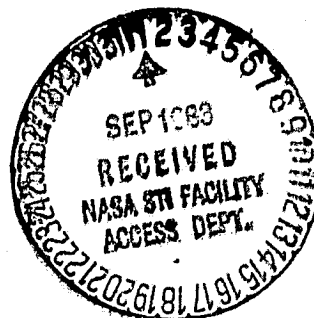## THE DESORPTIVITY MODEL OF BULK SOIL-WATER EVAPORATION

SOIL-WATER EVAPORATION (Maryland Univ.)
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ABSTRACT

To date, there are no generally accepted, physically based models of bulk evaporation from a bare-surfaced soil. The available models are difficult to apply to field conditions where evaporation is complicated by two main factors: rate-limiting climatic conditions and redistribution of soil moisture following infiltration. Both factors are included in the *desorptivity model*, as described herein. In the model the evaporation rate during the second stage (the soil-limiting stage) of evaporation is related to the desorptivity parameter, A. This parameter is derived from the similarity solution to the Richards equation, but in the past it had to be evaluated numerically.

In this paper, analytical approximations for A are presented. The approximations are independent of the surface soil moisture. However, calculations using the approximations indicate that both soil texture and soil moisture content at depth significantly affect A. Because the moisture content at depth decreases in time during redistribution, it follows that the A parameter also changes with time. Consequently, a method to calculate a representative value of A was developed. When applied to field data, the desorptivity model estimated cumulative evaporation well. The model is easy to calculate, but its usefulness is limited because it requires an independent estimate of the time of transition between the first and second stages of evaporation. The model shows that bulk evaporation after the transition to the second stage is largely independent of climatic conditions.

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PREFACE

In the past few years, interest in the physics of evaporation from unvegetated soil has been revitalized, due mostly to recent advancements in remote sensing. Typically, sensors measure moisture or thermal conditions in a thin layer beneath the surface, which is the region where evaporation occurs. Understanding the physics of evaporation becomes a key step in the interpretation and use of remotely sensed soil moisture. If these measurements are to be useful in solving hydrological problems they must be linked to the storage of water within the soil. The linkage must take the form of a mathematical model, one that includes the effects of evaporation.

The availability of infrared data from satellites combined with recent improvements in computer capability have tended to encourage the development of complex numerical models of evaporation that describe the simultaneous flow of moisture and energy in soil. Clearly, such complex models cannot be applied routinely on a watershed scale so the question arises -- how simply can soil-water evaporation be described and still retain both physical meaning and accuracy? Simplification is the guiding principle of the work reported here.

The focus of this report is on soil physics. The specialist in remote sensing interested in maps of brightness temperatures, Landsat scenes, mixed pixels and so forth will probably find little of interest in this report. Similarly, the micrometeorologist who enjoys the intricacies of radiation balances and advected heat may also be disappointed. I have encapsulated the climatic phenomena into the potential evaporation rate which is calculable by a great variety of methods. In fact, I am so disinterested in this aspect of evaporation (for the purposes of this research only) that in the model application described later I have used the measured soil-water evaporation -- when the soil was wet -- as an exact indicator of the PE rate. In the course

of this report, I arrive at the somewhat iconoclastic conclusion that once the evaporation rate falls below the potential rate then soil porperties alone control evaporation, i.e., that meteorological conditions no longer are important to the calcuation of bulk evaporation. The evidence for this conclusion comes from a model application using the now famous data set generated by the USDA-ARS at Phoenix nearly 10 years ago.

The choice of data sets is significant in two ways. First, it is the most complete assembly of information of its kind so that any candidate model of soil water evaporation must sooner or later be applied to it. Thus, in a sense it can be considered to be the standard. Second, past researchers have sought to explain these data in mostly energetic terms. That is to say, the observed evaporation at all stages of the process has been related to available energy. This approach is fundamentally different than that taken here, and therein lies the most interesting aspect of this work.

The conclusion that bulk evaporation can be calculated from soil properties alone (after an initial, climate-controlled stage) has some profound implications to the interpretation of surface moisture and temperature measurements. Unfortunately, these implications are beyond the scope of this report. Indeed, I have a second report, a sequel, which introduces a more complex model of evaporation; and in that report the problem of estimating the moisture profile is addressed directly. Of course, this problem must be solved if remotely sensed soil moistures are to be of benefit in hydrological simulation.

The key ideas in this paper did not originate with me. They were introduced by mainline soil physicists -- including J.R. Philip, W.R. Gardner and

D. Hillel -- during the fifties and sixties. Their work centered on analytical solutions to simplified problems for the obvious reason that computer resources were not as abundant as they are now. With the advent of ubiquitous large computers, their work has been mostly shelved. However, it is my viewpoint that their results can be modified to account for some of the more important complexities observed under field conditions, and thus they can serve in the development of useful hydrological tools.

The research reported herein was completed mostly at the University of Virginia in collaboration with Dr. George Hornberger, Chairman of the Department of Environmental Sciences. The work was supported by the Army Research Office as part of a larger project aimed at developing a simplified, efficient method of modeling the field water cycle. That project includes an investigation of the effects of parameter and input uncertainties to the calculated estimates of soil moisture. The project is continuing, but I have moved to the Remote Sensing Systems Laboratory in the Department of Civil Engineering, University of Maryland. The lab is under the direction of Dr. Robert Ragan. Despite the change in venue, the thrust of the work remains the same: the development of physically sound relationships for fundamental hydrological processes suitable to large scale applications.

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February 1983

CHAPTER 1

INTRODUCTION

Of all the components of the soil-water budget, evapotranspiration is the most difficult to calculate independently. Although the prospect for a reliable, mechanistic model of transpiration under field conditions is doubtful, given the complexity of the biological controls involved, the outlook for a model of soil-water evaporation from a bare-surfaced soil ought to be better. Yet to date, there are no generally accepted, field-tested models for bulk evaporation from a bare-surfaced soil that are based on fundamental soil-water properties. The pioneering efforts of Gardner and Hillel (1962), Gardner and Gardner (1969), and Gardner et al. (1970b) have not gained much acceptance because their models are difficult to apply to field situations where evaporation is complicated by variable climatic conditions and by changing moisture conditions within the soil caused by the redistribution of soil water that follows infiltration. For example, Black et al. (1969) showed that evaporation from a specific agricultural soil could be calculated from a theoretical model using typical moisture conditions measured 2 days after infiltration; however, they had no predictive method for determining this time interval.

In this and a subsequent paper, two models for soil-water evaporation are presented. They describe bulk flow in the same manner that the models of Green and Ampt (1911) and of Philip (1957a) describe bulk infiltration. Bulk-flow models are advantageous because they are easy to solve; they provide insight into processes; they can be easily incorporated into more comprehensive hydrological models; and in some cases, they can be applied to

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the inverse problem, i.e., the models can yield methods for determining fundamental hydraulic parameters using measurements of bulk flow. For evaporation, bulk flow is defined as the evaporation rate averaged over 24 hours, thus avoiding the complexities of the diurnal fluctuations observed in the field.

As with infiltration, soil-water evaporation is often conceptualized as a two-stage process. For an initially moist soil the evaporation rate is first limited by meteorological conditions and later by the soil's ability to transport water to the near-surface region where evaporation actually occurs. Stage I is often called the climate-limited stage, and stage II is called the soil-limited or the falling-rate stage. There can also be a third stage when the evaporation is maintained at a low, and perhaps steady, rate by the diffusion of water vapor within the soil. Field experiments by Idso et al. (1974) suggest that this latter stage occurs intermittently under very dry conditions.

These descriptions of the three stages of drying roughly conform to those of Philip (1957b), Jackson (1973) and Idso et al. (1974), but not to those of Idso et al. (1979); and the difference is potentially confusing. In the paper by Idso et al. (1979), stage II is a transitional interval when the soil surface exhibits both wet and dry patches; and according to them, the area-wide average evaporation rate is derived from both the climate-limited and the soil-limited rates. This "patchiness" is probably due to spatial variability in soil properties and/or microclimate. While this may be a significant phenomenon, it is not addressed in this study. According

to Idso et al. (1979) after the transitional stage comes their stage III which is equivalent to stage II as described above.

In this paper, the *desorptivity model* for stage-II evaporation is presented and applied to a field situation. The model is derived from theory, but its predictive value is limited because it requires an independent estimate of the time of transition between stages I and II. This requirement is waived in a second model presented in a following paper. The second one I call the *continuous similarity model*. It is analogous to the Green-Ampt infiltration model.

The first objective of this report is to present analytical approximations to the desorptivity parameter, *A*, based on two commonly used diffusivity functions. The approximations are useful because they are easy to solve and allow one to determine the sensitivity of *A* to fundamental parameters directly. They were developed using nonstandard techniques, but they are judged to be sufficiently accurate for all field applications and for the investigations of *A* reported herein.

The second objective is to explore the sensitivity of *A* to the boundary conditions required of the similarity solution and to the diffusivity functions that describe the hydraulic behavior of typical soils representing several different textural classes.

The third objective is to show how the desorptivity model may be applied under field conditions. The application is not straightforward

because redistribution of soil moisture following infiltration causes the desorptivity parameter to vary in time. Consequently, the fundamental problem in applying the model becomes the specification of a method to calculate a representative value of desorptivity, i.e., a value that gives the best estimate of evaporation for a given interval of stage-II evaporation.

CHAPTER 2

BACKGROUND

The dynamics of stage-II evaporation can be represented mathematically as a desorption phenomenon. The standard desorption problem and its solution using the so-called similarity transformation provide the theoretical base for the subsequent model development. Soil physicists have applied the similarity transformation to a variety of problems in the unsaturated zone, but they have concentrated mostly on the analogous process of water sorption into the soil (e.g., Gardner, 1959; Childs, 1966; Remson et al., 1971). The desorption problem is central to all of the work reported later; for this reason it is reviewed briefly here.

2.1 Desorption

Evaporation from a semi-infinite, homogeneous column where gravity flow is negligible is described by the Richards equation with the conventional diffusivity transformation:

$$\frac{\partial \theta}{\partial t} = \frac{\partial (D \frac{\partial \theta}{\partial z})}{\partial z} \tag{1}$$

where θ is the volumetric moisture content, z is depth oriented positively downward, and the diffusivity, D, is dependent on θ. For the desorption problem, the initial and boundary conditions are:

$$t = 0, \; z = 0, \; \theta = \theta_1; \tag{2a}$$

$$t > 0, \ z = 0, \ \theta = \theta_0; \tag{2b}$$

$$t > 0, \ z \to \infty, \ \theta = \theta_1; \tag{2c}$$

where θ_0 and θ_1 are the moisture contents at the surface and at depth, respectively.

It is often assumed that the surface dries to some low, equilibrium value designated as the air-dry moisture content, θ_a. This idea is the "air-dry assumption," and it implies that $\theta_0 = \theta_a$. However, Jackson (1973) showed that evaporation may be soil-limited when the surface exhibits an intermediate degree of wetness, thus in this study I differentiate between the actual surface moisture content (θ_0) and θ_a.

With respect to conditions at depth, the requirement that θ_1 remain steady was termed the "field capacity assumption" by Gardner et al. (1970a). It is this assumption that is violated by redistribution of moisture that follows infiltration, as discussed later.

To solve (1) subject to (2), the Boltzmann similarity variable, y, is defined as

$$y = z \ (t - t_0)^{-1/2} \tag{3}$$

where the time delay, t_0, is an empirical adjustment that accounts for stage-I evaporation. For the standard desorption problem, the instanteous drying of the surface implies that stage I is nonexistent, therefore $t_0 = 0$. The substitution of (3) into (1) yields the ordinary differential equation:

$$\frac{-y}{2} \cdot \frac{d\theta}{dy} = \frac{d(D\frac{d\theta}{dy})}{dy} \qquad (4)$$

subject to

$$y = 0, \ \theta = \theta_0; \qquad (5a)$$

$$y \to \infty, \ \theta = \theta_1. \qquad (5b)$$

For any diffusivity function, (4) and (5) yield a unique function

$$\theta(y) \quad \text{or} \quad \theta(z,t)$$

describing the moisture profile, as illustrated in Figure 1. The complete

solution to (4) is usually generated from the integrated form:

$$-1/2 \int_{\theta}^{\theta_1} y \ d\alpha = -D \ d\theta/dy \qquad (6)$$

but the evaluation of (6) requires a numerical approximation. When the

lower bound of the integral is equal to θ_0, the integral itself defines the

soil's desorptivity, A. Multiplying (6) by the derivative of (3) yields

$$1/2 \ A(t-t_0)^{-1/2} = D \ d\theta/dz|_{z=0}$$

where the r.h.s. is equivalent to the evaporation rate. Expressing that

rate as a positive value,

$$dE_2/dt = 1/2 \ A(t-t_0)^{-1/2} \qquad (7)$$

which is the fundamental expression of the desorptivity model. The subscript

2 indicates that the evaporation rate applies only to stage-II.

Under laboratory conditions, Rose (1966) showed that Eq. (7) is valid;

however, (7) has never been applied to field conditions for an *a priori*

estimate of stage-II evaporation using a physically-based evaluation of the
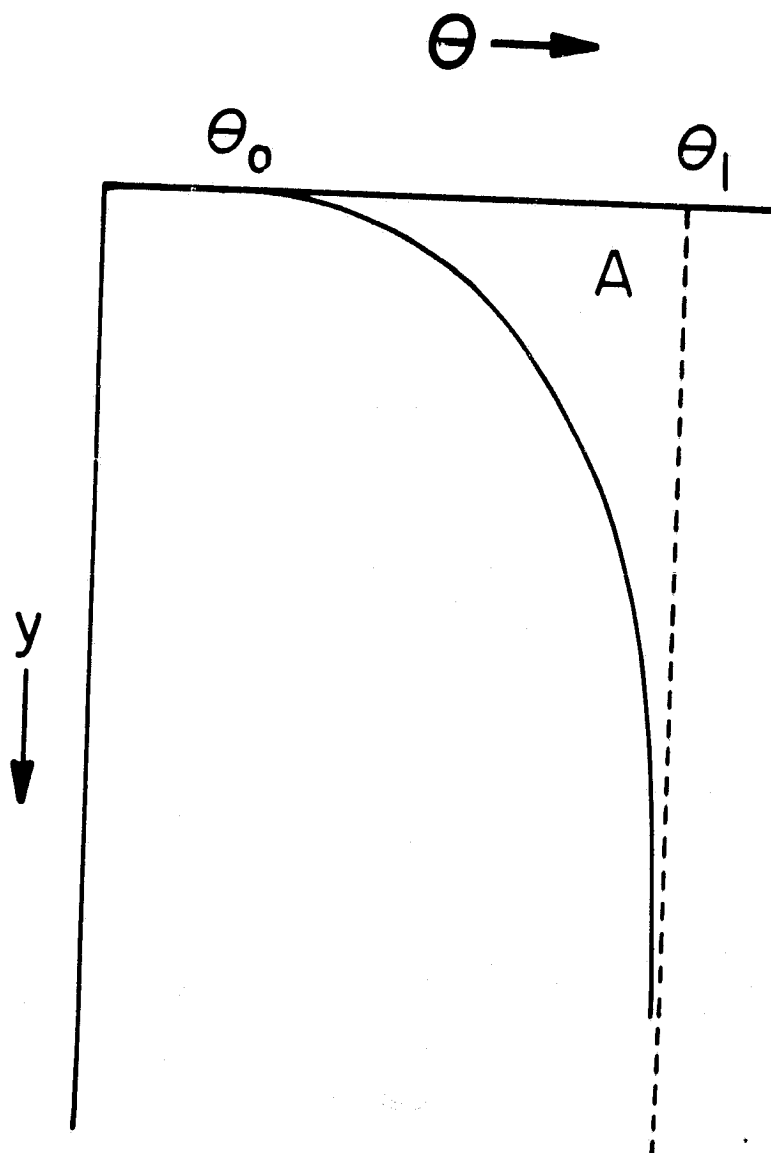
FIGURE 1. Definition sketch for the desoprtion problem. The desorptivity parameter A corresponds to the area bounded by the solid curve and the broken line.

coefficient A. The two main reasons are the difficulty in evaluating A from the diffusivity function and the uncertainty in meeting the assumptions of the similarity solution. Thus, in practice, A and t_0 are usually fitted parameters (e.g. Ritchie, 1972).

For nonlinear diffusivity functions, A can be approximated by three methods: evaluating Eq. (6), estimating an average diffusivity, or using a procedure reported by Eagleson (1978). The first two methods involve numerical approximations, and the third one depends on restrictive assumptions. All three methods are cumbersome to apply routinely; therefore simple approximations to A were sought by developing expressions for an average diffusivity.

Eq.(4) can be rewritten in the following form:

$$dE_2/dt = (\theta_1 - \theta_0)\left[\frac{D^*}{\pi(t-t_0)}\right]^{1/2} \tag{8}$$

where D^* is the mean weighted diffusivity. For $c_0 = 0$, D^* can be approximated from the expression of Crank (1956):

$$D^* = \frac{1.85}{(\theta_1 - \theta_0)^{1.85}} \int_{\theta_0}^{\theta_1} D \ (\theta_1 - \theta)^{.85} \, d\theta \tag{9}$$

which applies to desorption and assumes D to be a monotonically increasing function of θ. Crank reported an analogous expression for sorption to be accurate within 1%. The disadvantage is that for most nonlinear diffusivity functions (9) must be solved numerically.

An approximation to D^* requires a functional representation of diffusivity, and in this study, two alternative functions are specified

$$D = D_s \ (\theta/\theta_s)^c \tag{10}$$

$$D = D_o \exp (\alpha \ \theta), \tag{11}$$

identified herein as power D and exponential D, respectively. The coefficients

D_s and D_o, and the exponents c and a can be empirically fit to data or derived from other hydraulic functions. The exponent c is related to the exponent b in the power curve used by Campbell (1974) to approximate the moisture characteristic curve. The relationship is

$$c = b + 2. \tag{12}$$

Clapp and Hornberger (1978) showed that b is related to soil texture, specifically to the clay fraction of the soil.

It is difficult to say whether one function for diffusivity is better than the other. Reviewing the soil physics literature, one tends to see the power curve applied to measurements in the high range of θ (hence it is the preferred choice in inflitration studies), whereas the exponential function is usually fitted to low and medium values of θ (hence it is used mostly in evaporation studies). Comparative studies of (10) and (11) have not been done probably because there are so few physically based models covering the entire water cycle within a field.

For power D and exponential D, two approximations to Crank's formula were developed by trial and adjustment. In essence, the procedure went as follows. A set of D* values was generated by solving (9) using numerical integration. For realistic values of D*, the independent variables were selected from ranges that were judged to be reasonable for naturally occurring soils. Next, several trial functions, essentially simplifications of (9), were formulated. In turn, values from those functions were compared to D* calculated from (9) by linear regression. Among those tested, the "best" approximations for D* are:

$$D_p^* = \frac{3D_s \; \theta_s^{\;2}}{(c+1) \; (c+4) \; (\theta_1 - \theta_0)^2} \left(\frac{\theta_1}{\theta_s} \right)^{c+2} \tag{13}$$

and

$$D_e^* = \frac{2.83 \; \theta_1 \; D_0 \; \exp \; (a \, \theta_1)}{a \; (a\theta_1 + 1.85) \; (\theta_1 - \theta_0)^2} \tag{14}$$

for power D and exponential D, respectively. Eqs. (13) and (14) are neither rigorously derived nor are they considered to be optimal in any meaningful sense. Nevertheless, the errors of the approximations relative to (9) were always less than 10%. Moreover, it was found that the accuracy of (13) and (14) is limited mostly by θ_0.

To explore the relative error caused by variations in θ_0, some test calculations were made. The results shown in Figure 2 indicate that for $\theta_0 = 0$ both approximations are good estimators, but as θ_0 increases the error in the approximated D* also increases. With this problem in mind, it would be helpful to have a criterion indicating when to use the approximations. It seems reasonable to specify a critical moisture content, θ_c, where if $\theta_0 < \theta_c$ then D* from the approximations is considered to be acceptably accurate. I have defined θ_c as the θ_0 that yields a 10% error in the approximated D*. In this discussion, an error of 10% seems tolerable because it implies an error of only 5% in A (hence in the evaporation rate itself) due to the square root of D* in (8). As shown in Table 1, θ_c varies with θ_1 and with soil type, through either exponent c or a.
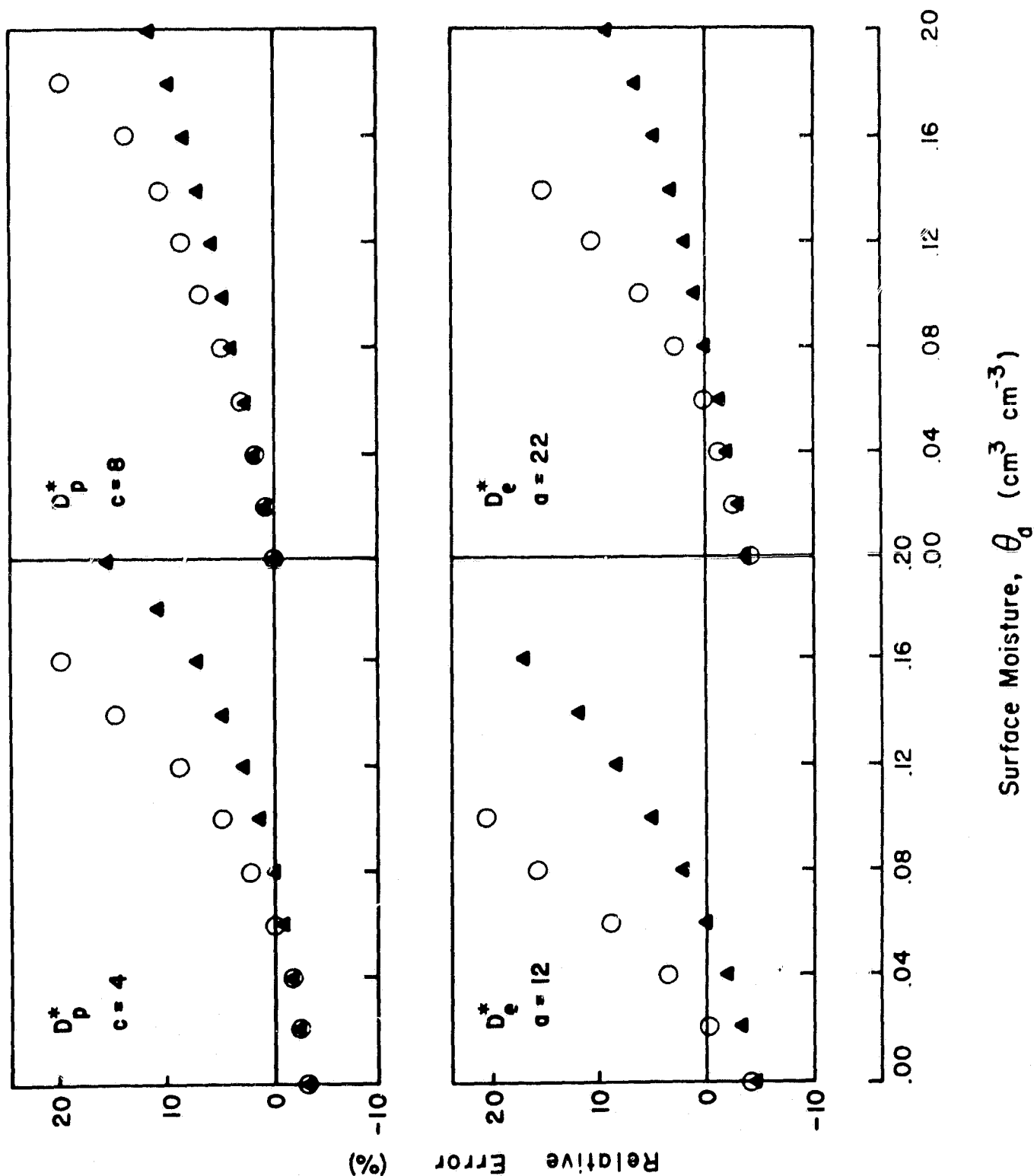
FIGURE 2. Relative error of the apprxoimated D* for θ_1 = .42 cm^3 cm^{-3} (triangles) and for θ_1 = .30 cm^3 cm^{-3} (circles).

TABLE 1. Critical Moisture Content (θ_c, $cm^3 cm^{-3}$)

Diffusivity Function:		Power		Exponential	
	c:	4	8	a: 12	22
θ_1 $(cm^3\ cm^{-3})$					
.42		.175	.185	.130	.200
.30		.125	.125	.065	.120

How does θ_c compare to surface moisture conditions during stage II?
I offer no definitive answer, however it is possible to evaluate the air-dry
assumption. Although values of θ_a are not generally available, for Avondale
loam (the soil type used in the model application reported later) Jackson (1973)
determined θ_a from measurements of the soil's specific surface. For this soil,
$\theta_a = .06$ cm^3 cm^{-3} which is considerably less than .16 cm^3 cm^{03}, the computed θ_c.
(For this calculation, $\theta_1 = .28$ cm^3 cm^{-3}, typical of moisture conditions 4 days
following irrigation.) Avondale loam is a rather fine-textured soil, indeed it
was formerly called Adelanto clay loam. For coarser soils both θ_a and θ_c will
be lower, but in any case θ_a is expected to be substantially less than θ_c.
Therefore, errors in the approximated D* using (13) or (14) due to θ_0 are
probably not significant, and these equations are judged to be acceptably
accurate for field applications.

3.1   Eagleson's Approximation for D*

Eq. (13) leads to a simplification of the solution for D* reported by
Eagleson (1978). In his series of papers on hydroclimatology, he used the
relationship

$$D^* = \phi \, D_1$$

where D_1 is diffusivity at θ_1 and ϕ is a function dependent on the hydraulic
parameters of the soil. He described the hydraulic properties of the soil using
the relationships of Brooks and Corey (1966) which are similar to those of
Campbell (1974). Eagleson approximated ϕ from a power series solution to
(9); however, the equation is tedious to solve because ϕ is only presented
as a graphical function of c. Because of the similarities in the treatment
of the hydraulic properties, (13) can be factored to yield ϕ explicitly.

In achieving this result, it is assummed that the residual moisture content (the minimum moisture content in the Brooks-Corey system) is equal to θ_0. This assumption is equivalent to setting θ_0 equal to zero in (7). Campbell's b from (6) is then equal to the reciprocal of the shape factor of Brooks and Corey. With these substitutions,

$$\phi = \frac{3}{b^2 + 9\,b + 18}.$$

This expression conforms to the graphical results presented by Eagleson (1978, p. 727).

CHAPTER 4

DESORPTIVITY

The approximations to desorptivity are generated by substituting (13) and (14) into (8). The terms containing θ_0 cancel, and for power D,

$$A_p = \left[\frac{12 \, D_s \, \theta_s^2}{\pi(c+1)(c+4)} \left(\frac{\theta_1}{\theta_s} \right)^{c+2} \right]^{1/2} \tag{15}$$

and for exponential D,

$$A_e = \left[\frac{11.3 \, D_o \, \theta_1 \exp(\alpha \, \theta_1)}{\alpha\pi \, (\alpha\theta_1 + 1.85)} \right]^{1/2} \tag{16}$$

Both expressions are independent of θ_0, and they are considered to be accurate to $\pm$ 5% where $\theta_0 < \theta_c$. Because θ_c is fairly large, the assumption of a truly air-dry surface is overly restrictive, especially where diffusivity is a power function.

In qualitative terms, evaporation during stage II is insensitive to θ at the surface because the moisture gradient beneath the surface is always large. In other words, the soil pores represented by the dry range of θ (i.e., $\theta < \theta_c$) never dry to any significant depth. This conclusion is true where diffusivity is either a power or exponential function. In reality, soil-water diffusivity exhibits an increase at low θ due to vapor diffusion, thus qualifying this conclusion somewhat under very dry conditions.

For purposes of illustration, the evaporation from a 100-cm column of Pachappa sandy loam was calculated using data published by Garnder (1959). Using (16) with the parameters of Table 2 and specifying θ_1 = .332 cm^3 cm^{-3}, the resulting A = 24.5 mm $d^{-1/2}$. Because the soil was exposed to intensely evaporative conditions, there was no stage I, and t_0 = 0. Substitution into (7) yielded the evaporation graphed in Figure 3. As shown, the square-root relationship is valid for about 25 days at which point the observed evaporation falls off. This deviation occurs because the moisture available for evaporation is limited by the impervious bottom boundary of the column, invalidating the assumption of a semi-infinite medium. It follows that the rate of evaporation is sensitive to changes in the moisture content at depth. In the next section, the sensitivity of A to θ_1 is explored.

Table 2. Hydraulic Parameters

Exponential Diffusivity

Soil	Clay fraction	θ_s ($cm^3\ cm^{-3}$)	D_o ($mm^2 d^{-1}$)	a
Packappa sandy loam*	.08	.45	167.	18.3
Avondale loam**	.22	.39	.605	37.4

Power Diffusivity for "Typical" Soil Textures***

Soil	Clay fraction	θ_s $cm^3\ cm^{-3}$	b	ψ_s mm	K_s $mm\ d^{-1}$
Sand	.03	.40	4.0	35	15000
Silt loam	.14	.49	5.3	570	600
Loam	.19	.45	5.4	150	600
Silty clay loam	.34	.48	8.5	360	210
Clay	.63	.48	11.0	190	100

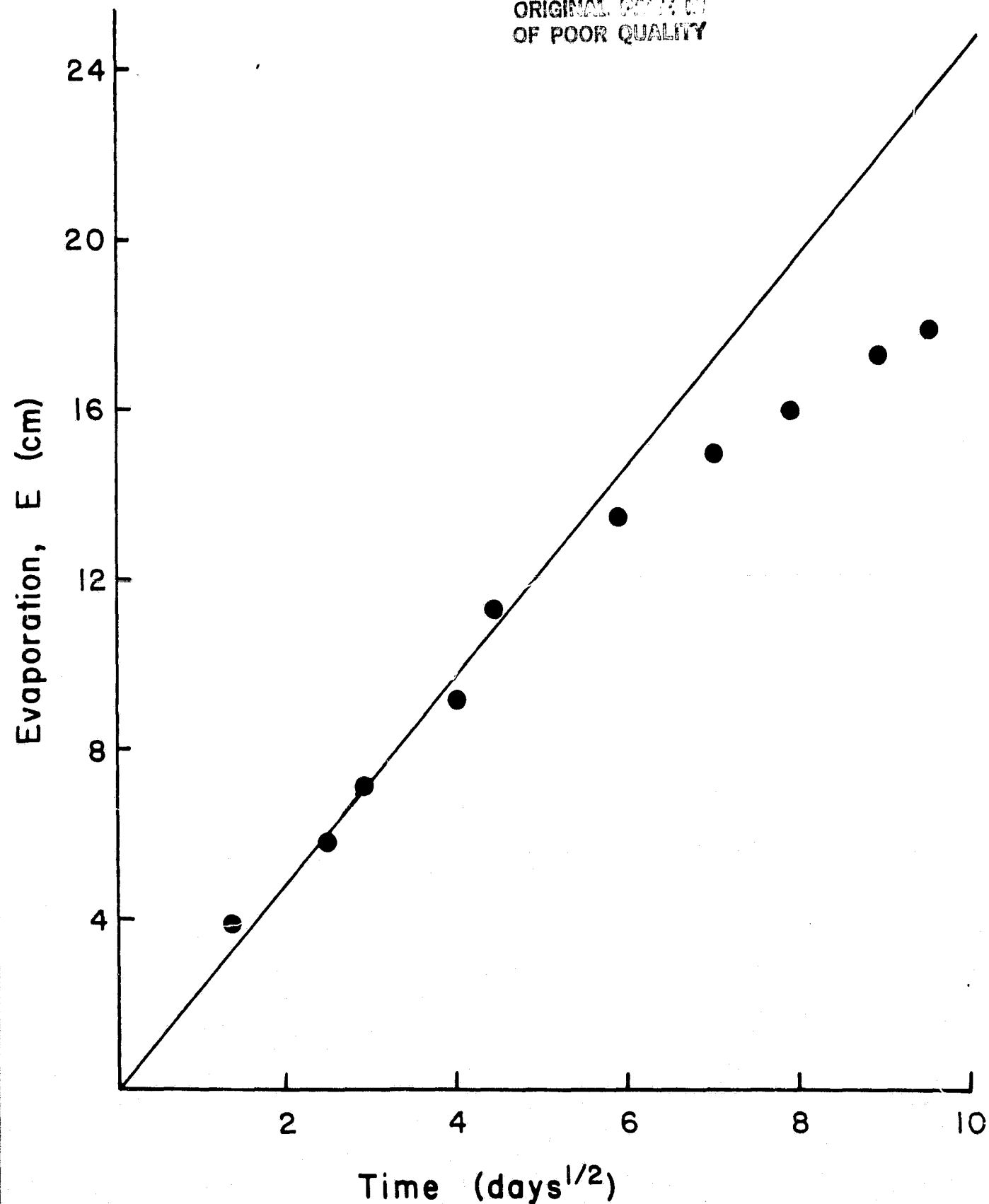*Gardner (1959), **Jackson (1973), ***Clapp and Hornberger (1978)

FIGURE 3. Calculated (solid line) and measured (symbols) evaporation from a
column of Pachappa sandy loam (data from Gardner, 1959).

CHAPTER 5

THE FIELD CAPACITY ASSUMPTION

The dependence of A on θ_1 is plotted in Figure 4a for Pachappa sandy loam, Avondale loam, and 5 "typical" soils representing different textural classes as specified by the parameters listed in Table 2. For the specific soils, desorptivity was calculated using A_e from Eq. (16), whereas for the typical soils, A_p from Eq. (15) was used.

As shown in Figure 4a, for all soils A is very sensitive to θ_1. In fact, A varies by about 2 - 2.5 orders of magnitude. Comparing approximations, A_p is more sensitive to θ_1 than is A_e. Comparing textures, A at a given θ_1 generally decreases for increasing clay fraction.

To apply the desorptivity model, it is obvious that θ_1 must be specified, but under field conditions this task is difficult. As a rule of thumb, soils are said to drain to "field capacity" which is usually designated as θ at a suction of 1/3 bar. However useful the concept of field capacity may be, it is not considered to be an accurate conceptualization because soil-water drainage is a continuous process (Hillel, 1971). After infiltration, soil water redistributes so that suction becomes uniform with depth, and thereafter suction increases at a declining rate.

Consequently, it may be useful to consider desorptivity as a function of the soil-water suction at depth, ψ_1, where ψ_1 is related to θ_1 by the soil's characteristic curve. Using Campbell's (1974) approximations of the fundamental soil hydraulic properties, the moisture characteristic is given by the expression:
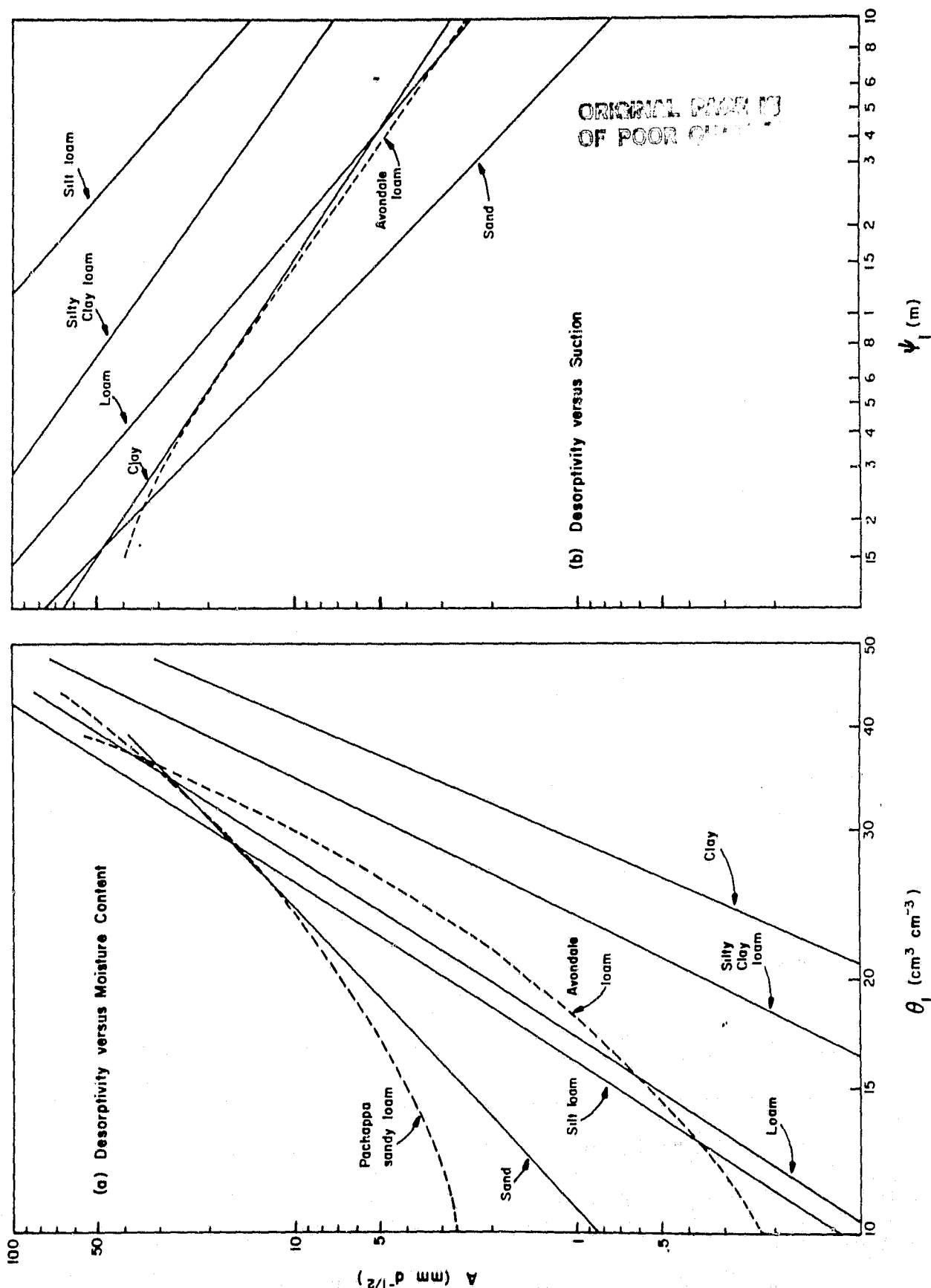
FIGURE 4. Desorptivity as a function of moisture conditions at depth. In 3a, the solid and dashed lines represent approximations A_p and A_e, respectively.

$$\psi = \psi_s \ (\theta/\theta_s)^{-b}$$

where the exponent b is the same as that in (12) and ψ_s is the hypothetical suction at saturation. Furthermore, D_s in (10) is given by the expression

$$D_s = K_s \ \psi_s \ b/\theta_s$$

where K_s is the hydraulic conductivity at saturation.

Substitution into (15) yields

$$A_p = \left[\frac{12 \ K_s \ \psi_s \ b \ \theta_s}{(b+6) \ (b+3) \ \pi} \left(\frac{\psi_s}{\psi_1}\right)^{1+4/b} \right]^{1/2} \tag{17}$$

There is no similar expression based on exponential D because there are no exponential relationships between ψ, θ and K like the power functions of Campbell.

The dependence of A on ψ_1 using (17) is graphed in Figure 4b for the 5 typical soils. In addition, the A - ψ_1 curve for Avondale loam is shown; it was generated using the A - θ_1 relationship in Figure 4a and the graph of the moisture characteristic reported by Jackson (1973). A similar curve for Pachappa sandy loam is not shown because no desorption data were available.

Figure 4b shows that A also varies significantly with ψ_1, but the

pattern among different textural groups is more complex than it is for the $A - \theta$ function. Over the interval of 4 shown in Figure 4b, A is least for sand, intermediate for both loam and clay, and greatest for silty clay loam and silt loam. These results indicate that bare-surfaced silty loams exposed to appreciable PE rates are expected to be very dry indeed.

The fact that under field conditions ψ is expected to increase in time implies that A is expected to decrease, but Eq. (7) requires a fixed value for A. In order to use the desorptivity model, a method must be devised that will yield a representative A value for a given interval of stage-II evaporation. For example, Ritchie (1973) suggested that A might be calculated from the hydraulic properties of the soil at $\psi = 1$ m; however, the sensitivity of A to ψ_1 implies a large uncertainty in A determined at any fixed ψ_1.

CHAPTER 6

DESORPTIVITY WITH SIMULTANEOUS

REDISTRIBUTION

To investigate methods for estimating A under conditions of simultaneous redistribution following infiltration it is useful to examine previously reported values for A derived from field studies. In each case cited below A was fit to evaporation data measured by lysimeter. The desorptivities for 4 different soils in Table 3 were reported by Ritchie (1973). In each case, Ritchie reported no soil-water data but he judged the soil to be at field capacity or wetter. In contrast, the desorptivities in Table 4 (Column 3), were determined for a single soil at different times of the year. A comparison of values in Tables 3 and 4 reveals a seasonal variation in A for a single soil, Avondale loam, that is greater than the variation in A for a set of soils characterized by widely differing textures. At first glance this comparison does not augur well for any predictive capability. Somewhat reassuringly, the single desorptivity for Avondale loam from Ritchie's analysis was based on data gathered during October (van Bavel and Reginato, 1965), and it fits well between desorptivities for September and December in Table 4.

The remainder of this section is devoted to "explaining" the seasonal variation in A shown in Table 4. Those parameters were derived from a series of experiments conducted by the Agricultural Research Service located near Phoenix, Arizona. I refer to these collectively as the Phoenix experiments, and in each one evaporation and moisture conditions were monitored in a lysimeter and a nearby test plot that were initially

TABLE 3. Field Values of Desorptivity*

Soil	A mm $d^{-1}/2$
Avondale loam	5.08
Yolo loam	4.04
Houston black clay	3.5
Plainfield sand	3.34

*Ritchie (1972)

Table 4. Desorptivity for Avondale Loam

	Stage II:		Optimal*	A by method:			
	Start	End		I	II	III	IV
	m	n	A				
	(d)	(d)	$(\text{mm } d^{-1/2})$	$(\text{mm } d^{-1/2})$	$(\text{mm } d^{-1/2})$	$(\text{mm } d^{-1/2})$	$(\text{mm } d^{-1/2})$
July	1.5	7	8.0	8.1	7.3	6.4	6.1
September	2.5	14	6.1	5.8	5.2	4.5	4.2
March	3.5	14	5.6	5.0	4.7	4.3	4.1
December	9.5	14	3.8	3.6	3.6	3.5	3.5

*derived from Figure 2 in Jackson et al. (1976).

irrigated with about 10 cm of water. Because the experiments were conducted at different times of the year, the varying factor among them was climate. The ultimate question, therefore, becomes: By what mechanism does climate affect A and, by extension, the evaporation rate? The ensuing analysis indicates that the representative value of A for a specific interval of soil-limited evaporation results from the interaction between climate and the redistribution of soil moisture within the soil column. However, it appears that redistribution affects A most directly.

To estimate A during redistribution, the variation of θ_1 through time must be estimated. For the Phoenix data this was done using daily averages of the moisture profiles measured during the March experiment. From these profiles, θ_1 was specified as the maximum θ above the wetting front that continued to move downward through the soil column long after irrigation had ceased. The redistribution function:

$$\theta_1 = .3216 \ t^{-.1102} \qquad (r = .995) \qquad (18)$$

was generated by log-log regression with t in days starting at midnight following the initial irrigation. The power function used in (18) has a theoretical basis (Gardner et al., 1970a). Profile measurements for the other experiments during the year were not sufficient to generate more redistribution functions; consequently (18) was assumed to be applicable to all the experiments. This assumption is reasonable because where initial infiltration is relatively large, evaporation does not affect redistribution significantly (Gardner et al., 1970b).

Because θ_1 varies in time the starting and ending times for stage II are important to the determination of a representative desorptivity. For each experiment the first day of stage II was determined by changes in albedo (Idso et al., 1974). For this study, it was assumed that profile control of evaporation started at noon of that day thereby yielding the start times designated by m in Table 4. The corresponding end times, designated by n, refer to the last day of the period used in the statistical fit of parameter A.

Representative desorptivities were calculated by 4 methods:

$$\text{I} \qquad [A \, (^{m}\theta_1) + A (^{n}\theta_1)]/2$$

$$\text{II} \qquad A \, [\, (^{m}\theta_1 + {}^{n}\theta_1)/2]$$

$$\text{III} \qquad A \, [\, \int_{m}^{n} \theta_1 \, dt/(m-n) \,]$$

$$\text{IV} \qquad A \, [^{(m+n)/2}\theta_1]$$

where A is given as a function of θ_1 using Eq. (16), and the superscripts of θ_1 indicate t in (18).

Due to the considerable nonlinearity in both $A(\theta_1)$ and $\theta_1(t)$, the 4 methods yield different results, but not so different that the results from any particular method seem unreasonable. This is not surprising since averaging procedures for nonlinear functions can often be found. The problem

is specifying the best procedure. As an example, for ratios and percentages the geometric mean is the most appropriate average. Consequently, selection of one of the averaging methods based on agreement with observed parameters does not mean that theory has been abandoned or that the desorptivity model is simply an empirical model.

Method I gives the largest values, and they agree well with the empirical desorptivities, as shown in Figure 5. Based on this agreement Method I is judged to be best, and it is concluded that stage-II evaporation is strongly conditioned by redistribution. Methods other than those tried here may improve the agreement, but further analysis seems unjustified given the uncertainties, especially in the assumed redistribution function.

6.1 Cumulative Evaporation

The components of the desorptivity model are now fully described and can be used to simulate the cumulative evaporation E. Such calculations are not a strict test of the model because the model depends on Method I, and it was chosen based on its ability to match parameters that were fit statistically to the data. Because the representative A values match the optimal ones so well, a good agreement between calculated and observed E is almost guaranteed. However, the calculations of E are not a trivial excercise in circular reasoning either. They serve to show how the model might be applied to field data, and the results provide insight into the role of climate in the evaporative process.
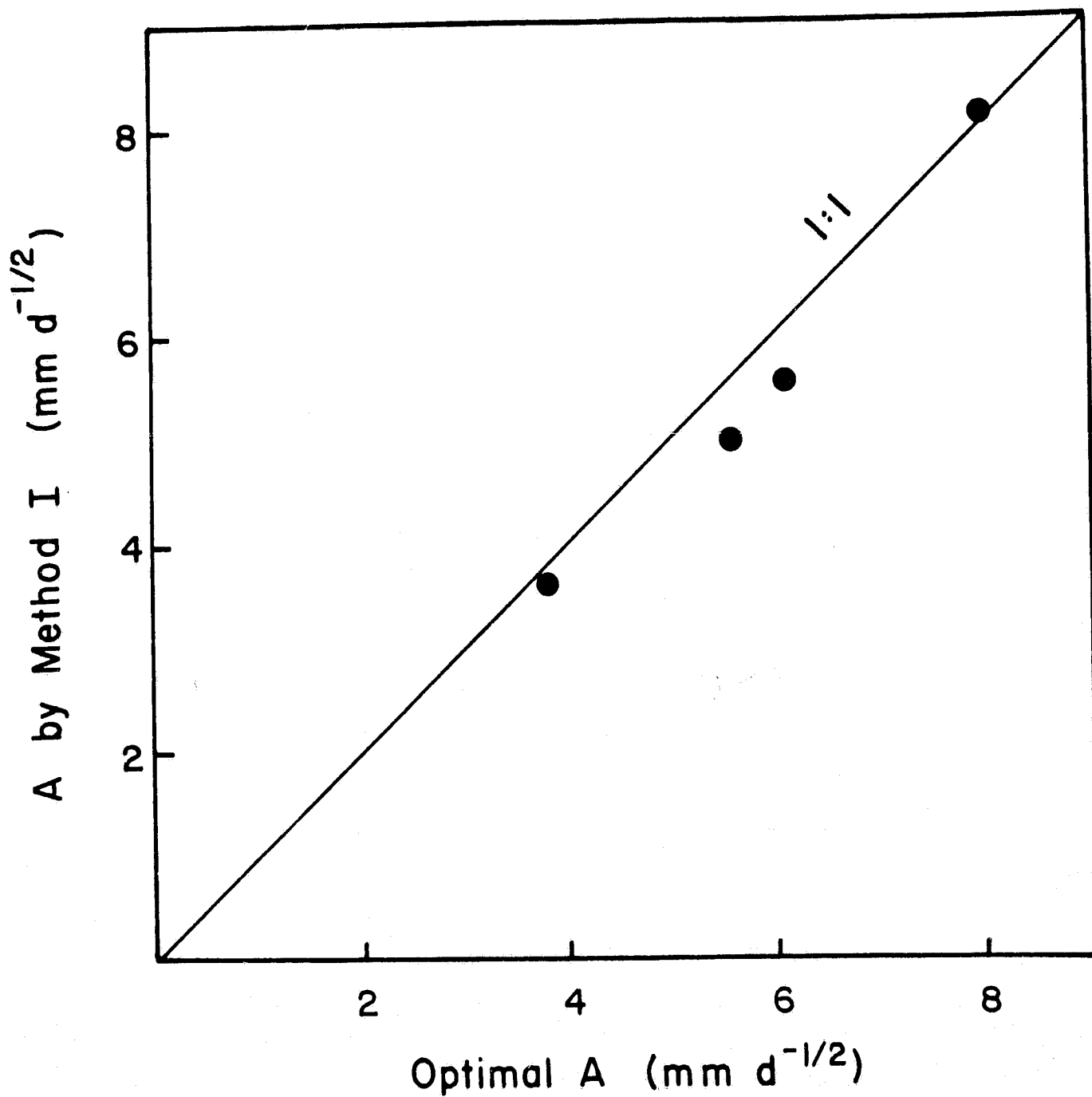
FIGURE 5. Calculated and statistically derived desorptivity.

Cumulative evaporation can be calculated if the PE rate during stage I is also known. For each experiment listed in Table 5, the average PE rate was obtained from the observed daily evaporations prior to time m. Using measured amounts is equivalent to having perfect knowledge of PE, but it is appropriate in this analysis because it allows a more specific examination of the model's capability to simulate stage II. With PE known, first t_0 was backcalculated by equating dE_2/dt in (7) to PE at time m. Next, E was calculated as the sum of evaporation for stages I and II:

$$E = E_1 + E_2 = m \cdot PE + A \: [(m-t_0)^{1/2} - (n-t_0)^{1/2}] \qquad (19)$$

where A was computed by Method I.

As expected, the calculated evaporations in Table 6 compare well with the observed amounts, and as an illustration, daily E values for the September experiment are plotted in Figure 6. For September and December the observed data were abstracted from graphs published by Jackson et al. (1976). Because of the possible bias in these data and because of the inclusion of measured data in the calculations, definitive statements about model accuracy are unwarranted.

In review, the desorptivity model of stages I and II for soil-water evaporation consists of the following components: Eq. (19) for E, either Eq. (15) or (16) for A, Method I to estimate an optimal A, and a redistribution function. The redistribution function can be generated using the equations reported by Gardner et al. (1970a) although this point

Table 5.   Estimated Evaporation

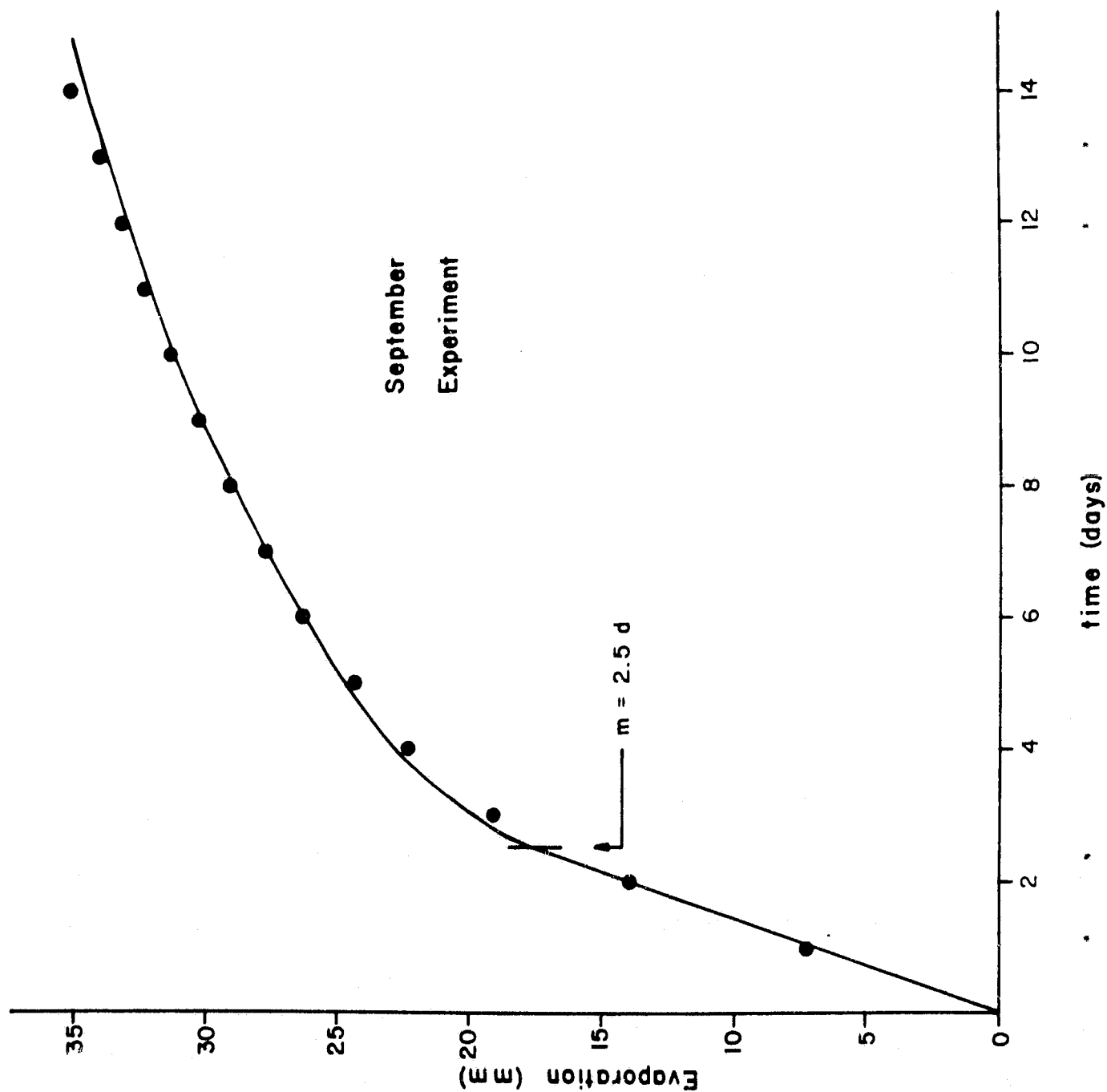|	Ave. PE $(mm\ d^{-1})$	End time n (d)	Calculated E (mm)	Observed E (mm)	Difference (mm)
July	9.1	7	29.3	30.6	-1.3
"	"	14	36.9		
Sept.	7.0	14	34.9	35.1	-0.2
March	4.55	14	29.7	29.2	0.5
Dec.	2.1	14	25.1	23.5	1.6

FIGURE 6.   Calculated and observed evaporation.

requires further investigation. In addition, two variables must be measured or estimated: an initial, average PE rate and the time of transition. From a practical standpoint, inability to predict the time of transition is the main factor that limits the predictive capability of the model.

CHAPTER 7

DISCUSSION

In the desorptivity model the seasonal variation in the A parameter results from the interaction of the evaporative capability of the atmosphere and the simultaneous redistribution of soil moisture. By means that are beyond the scope of this paper to discuss, atmospheric conditions influence the time (m) of transition to stage II, but from this time onward, bulk evaporation is mainly limited by conditions in the soil. In essence, a large PE rate leads to an early transition, and because θ_1 is large at that time the resulting values of A -- both the instantaneous A and the time-averaged, representative A -- are also large. Most importantly, A is not affected by climatic conditions following the transition.

Jackson et al. (1976) provided a different explanation and concluded that A is directly related to climatic conditions, specifically to temperature. According to them, the variation in A can be explained by the temperature dependence of the diffusivity function. In simple terms, during the summer temperatures are higher, hence the diffusivity of the soil is enhanced, hence evaporation is increased. However this explanation is unsatisfactory for three reasons. (1) As shown in (8), A is related to the square root of diffusivity, not to diffusivity, itself. Thus, although Jackson et al. observed that A and the temperature-dependent coefficient for diffusivity both increased by a factor of 2 between December and July (see Table 4), in theory A could increase only by $\sqrt{2}$, or 1.4, due to temperature-related effects. (2) They used daytime air temperature as a

surrogate for soil temperature. However, bulk evaporation depends on conditions in the entire zone of drying, from the surface to the zero-flux depth. This idea is inherent in the desorptivity model, and it is specifically discussed in the next paper. In this zone, the extreme variation in seasonal temperatures is undoubtedly damped relative to that measured in the air, thus the temperature-related factor is expected to be less than 1.4. (3) Finally, the desorptivity model, which is based on physical theory, adequately describes the data.

Although the July experiment lasted only 7 days, the desorptivity model was used to calculate the cumulative evaporation after 14 days, and this amount is listed with the other calculations in Table 5. As shown in that table, the model predicts increased cumulative evaporation for increased PE rates, precisely the trend cited by other researchers (i.e., Hillel, 1971, Ch. 9; Jackson, 1973). However, this model for soil-water evaporation is an isothermal model. Once stage II has begun, surface temperatures, humidity and wind distributions, and solar radiation are not necessary to the estimation of bulk evaporation. Thus the approach presented here contradicts the conventional concepts of evaporation that lead to the practice of scaling actual evaporation rates to PE rates; yet this approach generates the same basic result, i.e., increased evaporation for increased PE rates. Aside from the results reported here, the idea of scaling evaporation is suspect to me because both the meaning and accuracy of a PE rate -- either measured as pan evaporation or computed from climatic variables -- when the surrounding environment is dry is questionable. Consequently, this approach in which dE/dt is independent of PE after time m seems even more reasonable.

CHAPTER 8

SUMMARY AND CONCLUSIONS

In this paper, the desorption model was developed from fundamental principles. The model includes an empirical time delay to account for the initial, climate-limited stage of evaporation, and it relates the evaporation rate directly to the desorptivity parameter, A. Analytical approximations to A, developed herein, were used to investigate the sensitivity of stage-II evaporation to differing soil types and to the boundary conditions of the model. From these investigations, the air-dry surface assumption was shown to be overly restrictive, i.e., the surface need not be air dry for evaporation to be soil-limited. Indeed, the surface moisture can be quite large without affecting the evaporation rate. The upper limit for this independence can be assessed using the critical moisture content, defined herein. These conclusions arise from the theoretical equations and should be accepted cautiously because in the equations there is no allowance for vapor flow.

Stage-II evaporation is sensitive to moisture conditions at depth, and because those conditions are expected to vary in time a method for calculating a representative A was also developed using the Phoenix data. Though the model has not been tested in any strict sense, applying it to the Phoenix data served to indicate how it might be applied in the future. Because three important items -- the redistribution function, the initial PE rate, and the time of transition -- were obtained directly from experimental observations, the inherent uncertainty in any predictive version of the

model has not be explored, and this should be done in the future. *A priori* knowledge of the time to transition is the severest limitation on the model's predictive capability.

The model application to the Phoenix data is significant in that it shows that after the transition to stage II the bulk evaporation rate is essentially independent of climatic factors. While there are ramifications to our present practices in measuring and modeling hydrological processes at the soil surface, they are too complex for this report and will be discussed elsewhere. With respect to the desorptivity model itself, from a conceptual standpoint it serves as a link between the minimal parameters that describe a soil's hydraulic properties and the process of bulk evaporation; and in so doing, it accounts for the two complexities that confound evaporation estimates: climate and redistribution. From a practical standpoint, the model is a step towards developing a useful hydrological tool applicable where complex numerical models are inappropriate.

APPENDIX

APPROXIMATIONS TO D*

Simple, analytic approximations to the mean weighted diffusivity were sought because the approximation given by (9) requires a numerical solution. Approximations for both power D and exponential D were developed. The method consisted of first postulating a set of candidate functions which were, in essence, simplifications of (9). Each function contained an "undetermined" variable, T, which was assumed to depend only on soil properties, specifically on the exponent in the diffusivity function. Subsequently, the variable T was evaluated by linear regression, and the candidate function yielding the best correlation coefficient was selected. Details of the procedure are given below.

The general form of the function that was finally selected is

$$D^* = \frac{1}{T} \frac{\theta_1}{(\theta_1 - \theta_0)^2} \int_x^{\theta_1} D \, d\theta \qquad (A.1)$$

where the lower bound x is specified as zero for power D and as $-\infty$ for exponential D. To evaluate the T term, values for D* were generated by solving (9) using an iterative application of Simpson's rule. The maximum error of these numerical integrations was judged to be 0.1%. The D* values were calculated for a group of "synthetic" soils, each described by a particular set of independent variables. For power D, the independent variables are

$$\theta_s, \ \theta_1, \ \theta_0, \ c, \ D_s,$$

and for exponential D, they are

$$\theta_1, \ \theta_0, \ a, \ D_0.$$

For realistic values of D*, these variables were chosen from uniform distributions, each having ranges that are reasonable for naturally occurring soils:

1. For all of the synthetic soils, θ_s was maintained at .46 cm^3 cm^{-3}, and 5 evenly distributed values of θ_1 ranging from 1/2 θ_s to θ_s were specified.

2. As stated in the main text, the exponent c is related to the b exponent in Campbell's model (1974) of the moisture characteristic by the expression

    $$c = b + 2$$

    where b is statistically related to clay content and ranges from 2 to 10 for sandy soils to clayey ones (Clapp and Hornberger, 1978). Consequently, 5 values of c ranging from 4 to 12 were selected.

3. For exponential D, the exponent a replaces c, and a is roughly equivalent to c/θ_1, hence this variable ranged from 7 to 35.

4. As a first approximation, θ_0 is equivalent to θ_a, the air-dry value. θ_a is expected to increase with clay content so a simple empiricism relating θ_a to b was devised:

    $$(.5\ b\ i)/\ 100\ cm^3\ cm^{-3} \quad i = 0, 1, 2, 3, 4.$$

    It follows that most values were less than .10 cm^3 cm^{-3}.

5. Because both D_s and D_0 are simple coefficients, they were maintained at unity.

To generate the D* values, all combinations of the above parameters were used. From D*, T was calculated by rearranging the candidate functions, i.e., for the selected function,

$$T = \frac{\theta_1}{D^* (\theta_1 - \theta_0)^2} \int_x^{\theta_1} D \, d\theta.$$

Next, T was assumed to have the forms:

$$T = f_1 c + f_2$$

for power D, and

$$T = f_3 (a \theta_1) + f_4$$

for exponential D. The f coefficients were determined by linear regression, and the correlation coefficients for all of the candidate functions were compared. For the selected function, the r values were .98 and .97 for the power and the exponential D functions, respectively. With the linear approximations substituted into (A.1), the integral in (A.1) evaluated, and with slight rounding of the coefficients, the final approximations for D* appear as Eqs. (13) and (14) in the main text.

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