NASA CR- 170590

THE GEORGE WASHINGTON UNIVERSITY

(NASA-CR-170590) MODELING OF MICROWAVE SCATTERING FROM VEGETATED COVERED TERRAIN Final Report (George Washington Univ.) 116 p HC A06/MF A01 CSCL 20N

N83-36301

Unclas
G3/32 15198

STUDENTS FACULTY STUDY RESEARCH DEVELOPMENT FUTURE CAREER CREATIVITY COMMUNITY LEADERSHIP TECHNOLOGY FRONTIER DESIGN ENGINEERING APPLIED SCIENCE GEORGE WASHINGTON UNIVERSITY

SCHOOL OF ENGINEERING
AND APPLIED SCIENCE

FINAL REPORT

Modelling of Microwave Scattering from Vegetated Covered Terrain

by

Roger H. Lang
Department of Electrical Engineering and Computer Science
The George Washington University
Washington, D.C. 20052

July, 1980 - June, 1982

Prepared for:

National Aeronautics and Space Administration
Goddard Space Flight Center
Greenbelt, MD 20771

Grant NAG-5-83

Mr. Robert Meneghini - NASA Technical Officer

TABLE OF CONTENTS

Page

FORWARD

The work described in this final report is a continuation of the work started under NASA Grant 5288. The discrete approach to modeling vegetation originally developed under Grant 5288 has been extended and applied to a number of practical situations.

Electromagnetic backscattering from a layer of vegetation over a flat lossy ground is studied. The vegetated region is modeled by discrete lossy dielectric scatterers, for which the dyadic scattering amplitudes and orientation statistics are known. A method is developed to compute the backscattering coefficients from the vegetated layer. The technique is valid for scatterers having characteristic dimensions comparable to a wavelength.

The problem is solved by finding the mean field in the vegetated region and then using it in conjunction with the distorted Born approximation (first order multiple scattering) to calculate the backscattering coefficients. The mean field due to a plane wave obliquely incident on the vegetation is obtained by finding an approximate solution to the Foldy-Twersky mean equation in the limit of small fractional volume. This approximate solution is obtained by employing a two variable expansion procedure. An examination of the mean solution shows that from the viewpoint of the mean wave, the vegetated slab can be replaced by a deterministic anisotropic medium. The anisotropic medium becomes uniaxial when the scatterers are assumed to have orientation statistics that are independent of the azimuthal angle.

The backscattering coefficients are then calculated by employing single scattering theory, in which the scatterers are assumed to be embedded in the equivalent anisotropic bulk medium. The procedure is valid when the albedo of individual scatterers is small, that is, when the scatterers are highly absorbing. Formulas for copolarized and cross-polarized backscattering coefficients are given. Numerical calculations for the backscattering coefficients as a function of incidence angle are presented.

The material presented in the report has been published, in part, and has been presented at a number of technical meetings. Reference to this published material is contained at the end of the report.

CHAPTER I

INTRODUCTION

The objective of this dissertation is the investigation of electromagnetic backscattering from a layer of vegetation over a flat lossy homogeneous ground. The vegetated region, or canopy, is modeled by discrete lossy dielectric scatterers that have prescribed orientation statistics.

The motivation for this work has been the need to describe quantitatively the effects of the natural environment on electromagnetic wave propagation and scattering, and to relate radar return to the physical characteristics of the vegetation and the underlying ground. These characteristics can be used to determine the biomass and leaf area index of the vegetation and the moisture content of the ground. Information of this type is necessary as input data for crop yield models.

Models have been developed to serve the above applications for vegetated terrain. These models have been constructed by replacing the vegetated region with a random medium whose statistical characteristics are related to the physical quantities of the medium. The random modeling techniques divide naturally into two types: continuous and discrete. In the continuous case, the random medium is modeled by assuming that its permittivity $\epsilon(\underline{x})$ is a random process whose moments, such as the mean and correlation function, are known. In the discrete case, on the other hand, the medium is viewed as a collection of dielectric particles whose position and orientation statistics are given. In each, the medium statistics are used in conjunction with Maxwell's equations to calculate average quantities of physical interest.

For the continuous case, exact equations for the mean and correlation of the electric field can be obtained [Frisch, 1968]. These equations are known as the Dyson and Bethe-Salpeter equations respectively. Although almost impossible to solve even under the most ideal situations, they provide an exact formulation for the quantities of interest. Under appropriate physical circumstances, such as media with small correlation lengths or small fluctuations in the permittivity, perturbation theory can be used to simplify both the Dyson and Bethe-Salpeter equations to a tractable form. In the case of the mean field, perturbation methods have been used by Keller [1962 and 1964], Tartarskii and Gertsenshtein [1963], Keller and Karal [1966], Rosenbaum [1971] and others. In active remote sensing applications these approximate mean field solutions have been used along with first order renormalization or distorted Born approximation to obtain the backscattering coefficients, [Rosenbaum and Bowles, 1974; Stogryn, 1974; Hevenor, 1976; Fung and Fung, 1977; Fung and Ulaby, 1978; Fung, 1979; and Zuniga, et al., 1979, and many others].

Another technique used to obtain the scattered field from a continuous random medium is the radiative transport approach. Here the transport equations are obtained in terms of the statistics of $\varepsilon(\underline{x})$ [Tsang and Kong, 1978]. In the case where the medium correlation length is large compared to wavelength, de Wolf [1971], Ito and Adachi [1977] have developed multiple forward-single backscatter techniques.

In 1945 Foldy presented the first systematic probabilistic formulation of the multiple scattering of waves by collection of randomly distributed scatterers. In that paper he derived the mean field in a medium of scalar dipole scatterers. Lax [1951, 1952] generalized his treatment to resonant size scatterers and Twersky [1962, 1964, 1967, 1970, 1978] employed the Foldy-Lax method to find the mean field and effective dielectric constant for arbitrarily shaped dielectric particles. Ishimaru [1978a] has obtained an approximate equation for the mean field in both the scalar and vector cases. Ishimaru [1978b] also has found solutions to the correlation equation by employing the diffusion approximation.

The perturbation procedure applies when the fractional volume occupied by scatterers is small. The theory shows that the effective medium, as seen by the mean wave propagating through a collection of non-spherical particles, is anisotropic. Lang [1981], Lang and Sidhu [1983] used the Foldy-Lax method in conjunction with the distorted Born approximation to calculate the back-scattering coefficients from a slab of arbitrary shaped lossy dielectric scatterers.

Another work, using discrete scatterers to model vegetation, has been done by Du and Peake [1969], and they employed single scattering (Born approximation) without introducing an equivalent medium. Thus, they did not take into account the decay of the incident wave in the vegetation. This limits their theory to a much lower frequency and thin layers of vegetation. They also did not take into account the underlying ground.

Recently Tsang, et al, [1981], have used a vector radiative transport technique to analyze a slab of discrete scatterers. The results are similar to those Lang and Sidhu [1982] except that terms representing coherent wave effects are missing. The method is limited to low albedo particles where absorptive loss is the dominant mechanism. The two methods require different input quantities. For the continuous model the average permittivity of the medium is required in addition to the spatial correlation function of its fluctuations. In the discrete case, the scattering amplitude of the individual scatterers is required as well as the position and orientation statistics of the scatterers. The scattering amplitude can be obtained experimentally or by electromagnetic modeling of the individual scatterer. The latter modeling procedure relates the scattering amplitude to the physical dimensions and to the dielectric properties of the scatterer.

One of the advantages of the continuous modeling technique is that computed quantities of interest such as the backscattering coefficient are obtained directly in terms of the average dielectric constant of the medium and the correlation function of its permittivity fluctuations. In the case of the discrete approach, the average permittivity, the correlation function of its fluctuations and backscattering coefficients are all obtained in terms of orientation averages over the scattering amplitudes of the individual scatterers. The scattering amplitude, in turn, is then related to the electrical and physical characteristics of the individual scatterer. Although, computationally, the discrete approach is more complex than the continuous method, the discrete approach has certain important advantages. One of these is that the average permittivity of the scattering medium is now a derived quantity rather than one which is empirically determined. Another difference between the two approaches is that the continuous method does not permit cross polarized backscatter [see Tan, et al. 1980], to first order in the albedo, while the discrete theory predicts a first order contribution [Lang, 1981]. For these reasons, the discrete modeling technique may prove more successful in relating remote sensing signatures to actual physical characteristics of the medium.

The methodology which has been employed in this work models vegetation by dielectric discs (leaves) and uses discrete random media methods to calculate the scattering cross sections of interest. Lang [1981] assumed the canopy was thick enough so that ground reflection could be neglected.

The forest was thus modeled by a half space of dipole discs (400 MHz to 1 GHz regime). An equation for the mean field in the half space was derived and, from it, an equivalent dielectric constant for the leafy medium was obtained. Following this, the distorted Born approximation was used to calculate the backscattering cross sections of the leafy half space. Lang and Sidhu [1983] account for the effect of a flat ground by considering a layer of dipole discs over a lossy homogeneous half space. The distorted Born approximation was again used to find the backscattering coefficient of the layer of leaves. It was found that the backscattering coefficient could be decomposed into three terms: a contribution from direct backscatter; a return from a wave doubly reflected from the ground; and, finally, a direct-reflected component from a wave singly reflected from the ground. In this thesis, the method is extended to arbitrarily shaped scatterers having characteristic dimensions comparable to a wavelength (so-called resonance region). The development is generalized by using a matrix formulation.

This thesis has five chapters. Although the introduction to each chapter should provide the reader with an outline of its contents, we briefly summarize what will be done in each chapter. In Chapter II, Maxwell's equations are recast in an operator form. The coherent field equation obtained by employing Foldy's approximation will also be discussed. The correlation of field will be found by employing the distorted Born approximation.

In Chapter III, the mean field and Green's function of a plane wave obliquely incident upon the vegetation is obtained by finding an approximate solution to the Foldy-Twersky mean equation in the limit of small fractional volume. This approximate solution is obtained by employing a two variable expansion procedure. An examination of the mean solution shows that the vegetated slab can be replaced by a deterministic, anisotropic medium. The anisotropic medium becomes uniaxial when the scatterers are assumed to have orientation statistics that are independent of the azimuthal angle. Also, the average complex dielectric constant of the medium is determined from the average field in the medium.

In Chapter IV, the model is extended to obtain the transverse spectral density of the field, and the resonant backscattering coefficients of the vegetated layer. Simple expressions for the copolarized and cross polarized backscattering coefficients are obtained in terms of the dyadic scattering amplitude of an individual scatterer.

In Chapter V, a general discussion is presented and numerical results obtained by modeling a forest canopy as a collection of lossy dielectric discs. The results are examined.

CHAPTER II

GENERAL FORMULATION OF RESONANT BACKSCATTERING FROM VEGETATION

In this chapter we present the general formulation for scattering from discrete random medium in the limit of small fractional volume. We adopt an approach which has been originally developed by Lang [1981].

First, we consider the problem of scattering of a time harmonic electromagnetic wave from N discrete identical lossy dielectric scatterers which have random position and orientation. Scatterers are considered to be independent of one another, and as a result, neighboring particles are not necessarily aligned. Then we will consider single scatterers. In both cases we will obtain the necessary equations in terms of the transition operator. We will develop an approximate equation for the coherent field by employing the Foldy approximation [Foldy, 1945]. We will also obtain the macroscopic form of Maxwell's equations and the macroscopic permittivity operator which describes the average behavior of the equivalent medium.

Finally, the distorted Born approximation will be employed to obtain the correlation of the field in the equivalent anisotropic medium.

Problem Formulation

We consider the problem of scattering of time harmonic electromagnetic waves from N discrete scatterers located in a volume V as is shown in fig. 1. The particles are identical and each has a volume V_p, and a relative dielectric constant ϵ_r. We assumed that the relative dielectric constant of background medium is $\varepsilon_d(\underline{x})$. The only real restriction on the background medium is that it be constant inside V.

The location of the ith particle is specified by the vector $\underline{x}_i$, extending from an origin 0 to the center of that particle. The particle's center is located by the center of the smallest circumscribed sphere in which the particle can be placed. Although the particles are identical they have a rotation with respect to a fixed direction. The rotation for the ith particle is specified by $\underline{\Omega}_i=(\Theta_i,\Phi_i)$ where Θ_i and Φ_i are polar and azimuth angles, respectively, with $0\leq\Theta_i\leq\pi$ and $0\leq\Phi_i\leq 2\pi$.

The electric field $\underline{E}$ and the magnetic field $\underline{H}$ obey Maxwell's equations

$$
\begin{aligned}
-i\omega\varepsilon_0\,\varepsilon_r(\underline{x})\underline{E} - \nabla\times\underline{H} &= -J \\
\nabla\times\underline{E} - i\omega\mu_0\underline{H} &= -\underline{M}
\end{aligned}
\qquad (2.1)
$$

where a time dependence $e^{-i\omega t}$ has been assumed. In (2.1) $\underline{J}$ and $\underline{M}$ are, respectively, the vector electric current density and magnetic current density.

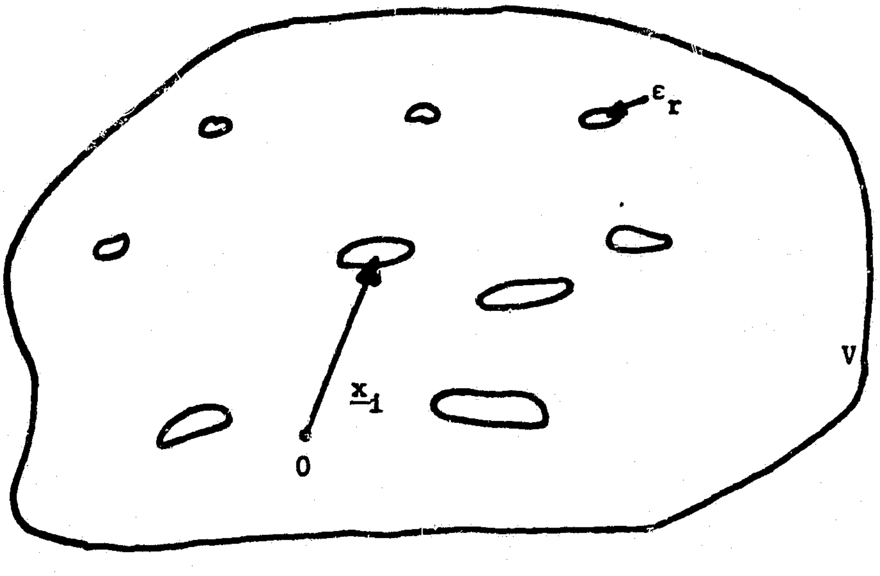


Fig. 1. Distribution of particles within volume V

Now let us assume that a particle located at the origin is characterized by the function $U(\underline{x})$, where

$$U(\underline{x}) = \begin{cases} 1, & \underline{x} \in V_p \\ 0, & \underline{x} \notin V_p \end{cases} \tag{2.2}$$

Using eq. (2.2), we express $\varepsilon_r(\underline{x})$ as

$$\varepsilon_r(\underline{x}) = \varepsilon_d(\underline{x}) + \Delta(\underline{x}) \sum_{i=1}^{N} U(\underline{x} - \underline{X}_i, \Omega_i) \quad , \quad \forall\, \underline{x} \in R_3 \tag{2.3a}$$

where

$$\Delta(\underline{x}) = \varepsilon_r(\underline{x}) - \varepsilon_d(\underline{x}) \tag{2.3b}$$

and

$$U(\underline{x}, \underline{\Omega}) = U(\underline{\underline{R}}(\underline{\Omega}) \cdot \underline{x}) \tag{2.3c}$$

In eq. (2.3c) $U(\underline{x},\underline{\Omega})$ is the function $U(\underline{x})$ rotated by $\underline{\Omega}$, and $\underline{\underline{R}}(\underline{\Omega})$ is a rotational dyadic.

Equations (2.1) can be written in matrix form,

$$\begin{bmatrix} -i\omega\varepsilon_0\varepsilon_r(\underline{x})\underline{\underline{I}} & -\nabla\times\underline{\underline{I}} \\ \nabla\times\underline{\underline{I}} & -i\omega\mu_0\underline{\underline{I}} \end{bmatrix} \cdot \begin{bmatrix} \underline{E} \\ \underline{H} \end{bmatrix} = - \begin{bmatrix} \underline{J}(\underline{x}) \\ \underline{M}(\underline{x}) \end{bmatrix} \tag{2.4}$$

Here $\underline{\underline{I}}$ is the unit dyadic and $\underline{\underline{I}}\cdot\underline{E} = \underline{E}$, $\underline{\underline{I}}\cdot\underline{H} = \underline{H}$.

Using (2.3a) in (2.4), we have for all $\underline{x}$

$$\left\{ \begin{bmatrix} -i\omega\varepsilon_0\varepsilon_d(\underline{x})\underline{\underline{I}} & -\nabla\times\underline{\underline{I}} \\ \nabla\times\underline{\underline{I}} & -i\omega\mu_0\underline{\underline{I}} \end{bmatrix} + \sum_{i=1}^{N} \begin{bmatrix} -i\omega\varepsilon_0\Delta(\underline{x})U(\underline{x}-\underline{X}_i,\underline{\Omega}_i)\underline{\underline{I}} & 0 \\ 0 & 0 \end{bmatrix} \right\} \cdot \begin{bmatrix} \underline{E} \\ \underline{H} \end{bmatrix} = - \begin{bmatrix} \underline{J}(\underline{x}) \\ \underline{M}(\underline{x}) \end{bmatrix} \tag{2.5}$$

Equation (2.5) can be represented in the operator form [Felsen and Marcuvitz, 1973; Harrington, 1968].

$$(L - \sum_{i=1}^{N} V_i)\Psi(\underline{x}) = g \tag{2.6}$$

where L is the operator descriptive of the field equations. The Ψ is a wave vector characterizing the field variables and the g is a wave vector describing the excitation.

Since L is an operator, it is necessary for uniqueness to state that Ψ lies in a prescribed domain of the operator L, a remark that is equivalent to the statement of initial and boundary conditions on the elements of Ψ. We list below the matrix form taken by the operator L and the wave vectors Ψ, g and V for the electromagnetic field.

$$L = \begin{bmatrix} -i\omega\varepsilon_0\varepsilon_d(\underline{x})\underline{\underline{I}} & -\nabla x\underline{\underline{I}} \\ \nabla x\underline{\underline{I}} & -i\omega\mu_0\underline{\underline{I}} \end{bmatrix} \tag{2.7}$$

$$\Psi = \begin{bmatrix} \underline{E}(\underline{x}) \\ \underline{H}(\underline{x}) \end{bmatrix} \tag{2.8}$$

$$g = -\begin{bmatrix} \underline{J}(\underline{x}) \\ \underline{M}(\underline{x}) \end{bmatrix} \quad , \quad V_i = \begin{bmatrix} i\omega\varepsilon_0\Delta U(\underline{x}-\underline{X}_i,\underline{\Omega}_i)\underline{\underline{I}} & 0 \\ 0 & 0 \end{bmatrix} \tag{2.9}$$

The matrix multiplication of equation (2.6) represents a dot product between the elements of the 2x2 matrix operators L, V_i and the wave vector Ψ. The elements of L and V_i are dyadics while the wave vector elements are vectors.

At times it will be convenient to write

$$\Psi(\underline{x}) = \Psi_0(\underline{x}) + \Psi_s(\underline{x}) \tag{2.10a}$$

where

$$\Psi_0(\underline{x}) = \begin{bmatrix} \underline{E}_0(\underline{x}) \\ \\ \underline{H}_0(\underline{x}) \end{bmatrix} \quad \text{and} \quad \Psi_s = \begin{bmatrix} \underline{E}_s(\underline{x}) \\ \\ \underline{H}_s(\underline{x}) \end{bmatrix} \tag{2.10b}$$

and $\Psi_0(\underline{x})$ is the solution to eq. (2.6) when no scatterers are present, i.e.,

$$L\Psi_0(\underline{x}) = g \tag{2.11}$$

and Ψ_s is the scattered field from the particles.

Single Scattering Equation and Transition Operator

Before proceeding with the N particle scattering problem, we will consider scattering from one particle located at the origin. Putting N=1 in eq. (2.6) with $\underline{x}_1=0$ and $\underline{\Omega}_1=\underline{\Omega}$, we obtain

$$(L-V)\psi = g \tag{2.12a}$$

where

$$\psi = \begin{bmatrix} \underline{e} \\ \\ \underline{h} \end{bmatrix} \tag{2.12b}$$

$$V = \begin{bmatrix} i\omega\varepsilon_0 \Delta U(\underline{x},\underline{\Omega})\underline{\underline{I}} & 0 \\ \\ 0 & 0 \end{bmatrix} \tag{2.12c}$$

The definition of L and g are given by (2.7) and (2.9) respectively. The eq. (2.12b) can be written as

$$\psi = \psi_0 + \psi_s \tag{2.13a}$$

where

$$\psi_0 = \begin{bmatrix} \underline{e}_0 \\ \\ \underline{h}_0 \end{bmatrix} \quad \text{and} \quad \psi_s = \begin{bmatrix} \underline{e}_s \\ \\ \underline{h}_s \end{bmatrix} \tag{2.13b}$$

and ψ_0 is the solution to eq. (2.6) when no scatterer is present, i.e.,

$$L\psi_0 = g \qquad . \tag{2.13c}$$

We have used the ψ notation instead of the Ψ for the field here to remind us that there is only one scatterer present.

If we use (2.13) in (2.12), we get

$$L\psi_s = V\psi \tag{2.14}$$

From (2.14) we see that the term on the right, $V\psi$, can be viewed as the source of the scattered field. We write

$$g_{eq} = V\psi \tag{2.15}$$

where g_{eq} is an equivalent source term. Since we know that V=0 when $\underline{x} \notin V_p$, the sources g_{eq}, exist only inside of the particle boundaries.

It is more natural to think of the equivalent sources as being induced by the incident field ψ_0. Because Maxwell's equations are linear, hence we an write

$$g_{eq} = T\psi_0 \tag{2.16}$$

The operator T in eq. (2.16) is known as the transition operator in the scattering literature [Lax, 1951]. Now using eq. (2.15) and eq. (2.16) in eq. (2.14) and multiplying through by L^{-1}, we have

$$\psi_s = L^{-1}\ T\ \psi_0 \tag{2.17}$$

Thus the knowledge of T completely characterizes the scattering properties of the particle and quantities of physical interest.

The transition operator is a linear bounded operator and, as a result, can be expressed in integral form

$$g_{eq}(\underline{x}) = T\psi_0 = \int d\underline{x}'t(x,\underline{x}')\psi_0(\underline{x}') \tag{2.18}$$

where the limits for the integral extend over all space and

$$t(\underline{x},\underline{x}') = \begin{bmatrix} \underline{\underline{t}}_{11} & \underline{\underline{t}}_{12} \\ \underline{\underline{t}}_{21} & \underline{\underline{t}}_{22} \end{bmatrix} \tag{2.19}$$

As shown in Appendix A, for a dielectric scatterer (relative permeability = 1)

$$\underline{\underline{t}}_{21} = \underline{\underline{t}}_{12} = \underline{\underline{t}}_{22} = 0$$

and (2.20)

$$\underline{\underline{t}}_{11} = \frac{-1}{i\omega\mu_0}\ \underline{\underline{t}}(\underline{x},\underline{x}')$$

The $\underline{\underline{t}}(\underline{x},\underline{x}')$ in eq. (2.20) is the same as the one Lang [1981] obtained when he solved the vector wave equation and $\frac{-1}{i\omega\mu_0}$ is the normalization constant associated with the source term.

One can show that $\underline{\underline{t}}$ is 0 when $\underline{x}$ or $\underline{x}'$ are outside the particle [Frisch, 1968], i.e.,

$$\underline{\underline{t}}(\underline{x},\underline{x}') = 0 \quad , \quad \underline{x} \notin V_p \quad \text{or} \quad \underline{x}' \notin V_p \tag{2.21}$$

The property follows directly from the fact that the equivalent sources for the scattered field are located within the particle boundaries.

The transition kernel $\underline{\underline{t}}$ can be expressed in terms of its plane wave representation $\tilde{\underline{\underline{t}}}$, i.e.,

$$\underline{\underline{t}}(\underline{x},\underline{x}') = \frac{1}{(2\pi)^3}\int d\underline{k}\, d\underline{k}'\, \tilde{\underline{\underline{t}}}(\underline{k},\underline{k}')e^{i(\underline{k}\cdot\underline{x} - \underline{k}'\cdot\underline{x}')} \tag{2.22}$$

or inverting eq. (2.22)

$$\tilde{\underline{\underline{t}}}(\underline{k},\underline{k}') = \frac{1}{(2\pi)^3}\int d\underline{x}\, d\underline{x}'\, \underline{\underline{t}}(\underline{x},\underline{x}')e^{-i(\underline{k}\cdot\underline{x} - \underline{k}'\cdot\underline{x}')} \tag{2.23}$$

In a similar manner, we can define the Fourier transform of $t(\underline{x},\underline{x}')$

$$\tilde{t}(\underline{k},\underline{k}') = \frac{1}{(2\pi)^3}\int d\underline{x}\, d\underline{x}'\, t(\underline{x},\underline{x}')e^{-i(\underline{k}\cdot\underline{x} - \underline{k}'\cdot\underline{x}')} \tag{2.24}$$

We have used the notation that $\tilde{\underline{h}}$ is the Fourier transform of $\underline{h}$. More specifically

$$\tilde{\underline{h}}(\underline{k}) = \int d\underline{x}\ \underline{h}(\underline{x}) e^{-i\underline{k}\cdot\underline{x}}$$

The dyadic scattering amplitude, $\underline{\underline{f}}(\underline{0},\underline{i})$, is defined in terms of the asymptotic expression for $\underline{\underline{e}}_s$ in the radiation zone. We have

$$\underline{\underline{e}}_s(\underline{x},\underline{i}) \sim \underline{\underline{f}}(\underline{0},\underline{i}) \frac{e^{ik_0|\underline{x}|}}{|\underline{x}|} \quad , \quad |\underline{x}| \to \infty \tag{2.25}$$

where $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$ is the free space wave number,

$\underline{0}$ is a unit vector in the $\underline{x}$ direction, $\underline{0} = \frac{\underline{x}}{|\underline{x}|}$

and $\underline{i}$ is a unit vector in the direction of the wave incident upon the scatterer.

The relationship between $\underline{\underline{f}}$ and $\tilde{\underline{\underline{t}}}$ can be found for large $|\underline{x}|$. (Appendix A). The result is

$$\underline{\underline{f}}(\underline{0},\underline{i}) = 2\pi^2(\underline{\underline{I}}-\underline{0}\,\underline{0}) \cdot \tilde{\underline{\underline{t}}}(k_0\underline{0};k_0\underline{i})\cdot(\underline{\underline{I}}-\underline{i}\,\underline{i}) \tag{2.26}$$

From this relationship, we see

$$\underline{0}\cdot\underline{\underline{f}} = 0 \quad , \quad \underline{\underline{f}}\cdot\underline{i} = 0 \tag{2.27}$$

Thus $\underline{\underline{f}}$ is a four component tensor--all combinations of two incident and two scattered polarizations. We also note that $\underline{\underline{f}}$ does not completely determine $\tilde{\underline{\underline{t}}}$ but rather only partially specifies it.

Before concluding this section, transition operators for particles that are not located at the origin will be needed. As we know, the equivalent sources $g_{eq}^{(i)}$ for a particle located at $\underline{x}_i$ can be related to the incident field. It follows that

$$g_{eq}^{(i)}(\underline{x}) = T_i\psi_0 = \int t_i(\underline{x},\underline{x}')\psi_0^{(i)}(\underline{x}')d\underline{x}' \tag{2.28}$$

By shifting the sources and the incident field to the origin, t_i can be related to t. One finds that

$$t_i(\underline{x},\underline{x}') = t(\underline{x}-\underline{X}_i,\underline{x}'-\underline{X}_i) \tag{2.29}$$

Note that throughout the discussion the dependence of t on rotations has been suppressed for convenience of notation.

Approximate Mean Field Equation

In this section, we will develop an approximate equation for the coherent field by employing the Foldy approximation [Foldy, 1945]. The equation is in terms of the transition operator and the particle density of the equivalent medium.

After the equation has been derived, it is pointed out that inside V, the mean field obeys Maxwell's equations with free space medium permeability and macroscopic permittivity that is inhomogeneous, anisotropic, and spatially dispersive.

The total field $\Psi(\underline{x})$ can be thought of as a sum of the incident field Ψ_0 plus a sum of the fields scattered from each particle, $\Psi_s^{(i)}$. We have

$$\Psi = \Psi_0 + \sum_{i=1}^{N} \Psi_s^{(i)} \tag{2.30}$$

The total field incident on the ith particle is called the effective field and is denoted by $\Psi^{(i)}$. Thus $T_i\Psi^{(i)}$ represents the equivalent sources generated by the incident field in the ith particle and thus the scattered field by the ith particle is:

$$\Psi_s^{(i)} = L^{-1}\, T_i\, \Psi^{(i)} \tag{2.31}$$

Using eq. (2.31) in eq. (2.30) we have

$$\Psi(\underline{x}) = \Psi_0(\underline{x}) + \sum_{i=1}^{N} L^{-1}\, T_i\, \Psi^{(i)} \tag{2.32}$$

This is the equation that we wanted to obtain. Now, we average this equation and the result is

$$\langle\Psi\rangle = \Psi_0 + \sum_{i=1}^{N} L^{-1} \langle T_i \, \Psi^{(i)}\rangle \qquad (2.33)$$

To obtain an approximate equation for the mean, we follow [Foldy, 1945] and assume

$$\Psi^{(i)} \simeq \langle\Psi\rangle \qquad (2.34)$$

This means that the random quantity $\Psi^{(i)}$ is to first order, equal to a deterministic quantity. Using eq. (2.34) in eq. (2.33) and noting that

$$\langle T_i \Psi^{(i)}\rangle \simeq \langle T_i \langle\Psi\rangle\rangle = \langle T_i\rangle\langle\Psi\rangle \qquad (2.35)$$

We have the approximate equation for the mean fields

$$\langle\Psi\rangle = \Psi_0 + \sum_{i=1}^{N} L^{-1} \langle T_i\rangle\langle\Psi\rangle \qquad (2.36)$$

We need the dependence of average T_i upon $\underline{x}_i$ and $\underline{\Omega}_i$. For this, we assume that the position vectors $\underline{X}_i$ and rotation vectors $\underline{\Omega}_i$ are random variables that are specified by a 5N dimensional distribution function. Assuming the particles have identical distribution functions, we have:

$$P_{\underline{X}_i \underline{\Omega}_i}(\underline{x},\underline{\omega}) = P_{\underline{X}\,\underline{\Omega}}(\underline{x},\underline{\omega}) \qquad i = 1,2,\ldots,N \qquad (2.37a)$$

where $\underline{\omega}=(\theta,\phi)$. We assume that the particle's location and rotation are independent, thus

$$P_{\underline{X}\,\underline{\Omega}}(\underline{x},\omega) = P_{\underline{X}}(\underline{x}) P_{\underline{\Omega}}(\omega) \qquad (2.37b)$$

with the usual property:

$$\int_V P_{\underline{X}}(\underline{x}) d\underline{x} = 1 \qquad \int P_{\underline{\Omega}}(\underline{\omega}) d\underline{\omega} = 1 \qquad (2.38)$$

The particle density is defined by

$$\rho(\underline{x}) = N P_{\underline{X}}(\underline{x}) \qquad (2.39)$$

so that

$$\int_V \rho(\underline{x}) d\underline{x} = N \qquad (2.40)$$

Now, we have

$$\langle T_i \rangle = \langle T(\underline{x}_i,\underline{\Omega}_i)\rangle = \int_V d\underline{s} \int_{4\pi} d\underline{\omega}\; P_{\underline{x}\,\underline{\Omega}}(\underline{s},\underline{\omega}) T(\underline{s},\underline{\omega})$$

$$= \int_V d\underline{s}\; P_{\underline{x}}(\underline{s}) \overline{T}(\underline{s}) \tag{2.41}$$

where

$$\overline{T}(\underline{s}) = \int_{4\pi} d\underline{\omega}\; P_{\underline{\Omega}}(\underline{\omega}) T(\underline{s},\underline{\omega}) \tag{2.42}$$

In (2.42) the bar over T has been used to indicate only an average of angular variables. By substituting eq. (2.41) in eq. (2.36), noting that the scattered terms are identical, and by using eq. (2.39) we obtain

$$\langle \Psi \rangle = \Psi_0 + \int_V d\underline{s}\; \rho(\underline{s}) L^{-1} \overline{T}(\underline{s}) \langle \Psi \rangle \tag{2.43}$$

Multiplying from the left by L and using eq. (2.11), we obtain

$$\mathscr{L} \langle \Psi \rangle = g \tag{2.44}$$

where

$$\mathscr{L} = L - \int_V d\underline{s}\; \rho(\underline{s}) \overline{T}(\underline{s}) \tag{2.45}$$

This is the equation for the mean field which has been obtained essentially by assuming that the effective field is approximately equal to the mean field (equation 2.34). Equation (2.34) is only valid when the fractional volume occupied by the particles is small compared to the total volume [Twersky, 1978]. We shall refer to a distribution of scatterers satisfying this condition as a sparse distribution.

We now write the equation for the mean in a more explicit form. Using eqs. (2.28) and (2.29) in eqs. (2.44) and (2.45) we obtain the macroscopic form of

Maxwell's equation:

$$L\langle\Psi(\underline{x})\rangle - \int_V d\underline{s} \int d\underline{x}' \rho(\underline{s}) \bar{t}(\underline{x}-\underline{s},\underline{x}'-\underline{s}) \langle\Psi(\underline{x}')\rangle = g \tag{2.46}$$

where

$$\bar{t}(\underline{x},\underline{x}') = \int_{4\pi} d\underline{\omega} p_\Omega(\underline{\omega}) t(\underline{x},\underline{x}',\underline{\omega}) \tag{2.47}$$

Here the kernel $t(\underline{x},\underline{x}',\underline{\omega})$ is the same as eq. (2.29), however, we have explicitly shown its dependence on the angular coordinate $\underline{\omega}$.

By writing eq. (2.46) out in vector components, we explicitly obtain Maxwell's equations.

The macroscopic form of Faraday's law:

$$\nabla x \langle\underline{E}(\underline{x})\rangle = i\omega\mu_0 \langle\underline{H}(\underline{x})\rangle - \langle\underline{M}(\underline{x})\rangle \tag{2.48}$$

and the macroscopic form of Ampere's law with displacement current:

$$\nabla x \langle\underline{H}(\underline{x})\rangle = -i\omega\langle\underline{D}(\underline{x})\rangle + \langle\underline{J}(\underline{x})\rangle \ , \ \langle\underline{D}\rangle = \varepsilon_0 \underline{\underline{\varepsilon}}_e \langle\underline{E}\rangle \tag{2.49}$$

where $\underline{\underline{\varepsilon}}_e$ is a macroscopic permittivity operator which describes the average behavior of the medium. We have

$$\underline{\underline{\varepsilon}}_e(\underline{x}) = \underline{\underline{I}}\varepsilon_d(\underline{x}) + \frac{i}{k_0^2} \int_V d\underline{s} \int d\underline{x}' \rho(\underline{s}) \underline{\underline{\bar{t}}}(\underline{x}-\underline{s},\underline{x}'-\underline{s}) \cdot \tag{2.50}$$

This expression simplifies to $\underline{\underline{I}}\varepsilon_d(\underline{x})$ when $\underline{x} \notin V$. To see this we note that when $\underline{x} \notin V$, we have $\underline{x}-\underline{s} \notin V_p$, since $\underline{s} \in V$ except for a small region near the boundary. Now using eq. (2.21) we have $\underline{\underline{\bar{t}}}=0$. When $\underline{x} \in V$, eq. (2.44) does not simplify in general. It describes anisotropic, inhomogeneous, spatially dispersive medium.

Lang [1981] showed that eq. (2.50) reduces to some more familiar expressions in certain special situations.

The Correlation Field Equation

In this section we will calculate the correlation of the field. The distorted Born approximation will be employed. It is a single scattering approximation where the scatterers are assumed to be embedded in the equivalent anisotropic bulk medium. The procedure is valid when the albedo of a single particle is small. The latter condition implies that the energy absorbed by a particle must be much larger than the energy scattered by it.

We start by considering a volume V of equivalent medium which was mentioned before. There are N particles embedded in V as shown in fig. 1. The scattered field due to the ith particle can be calculated by modifying eq. (2.31). We assume that the incident field on the ith particle is the mean field $\langle\Psi\rangle$ and that the operator L is replaced by the equivalent medium operator $\mathscr{L}$ as given in eq. (2.45).

We have

$$\Psi_s = \sum_{i=1}^{N} \Psi_{se}^{(i)}$$

where (2.51)

$$\Psi_{se}^{(i)} = \mathscr{L}^{-1} T_i \langle\Psi\rangle$$

Before proceeding, we point out that our main interest in finding the correlation of the field is to use it to calculate backscattering cross sections. We define the correlation of the field fluctuations as

$$\Psi_f = \Psi_s - \langle\Psi_s\rangle \qquad \langle\Psi_f\rangle = 0 \tag{2.52}$$

Now computing the correlation of the fluctuation component of the scattered field, we obtain

$$\langle\Psi_f(\underline{x})\Psi_f^\dagger(\hat{\underline{x}})\rangle = \langle\Psi_s(\underline{x})\Psi_s^\dagger(\hat{\underline{x}})\rangle - \langle\Psi_s(\underline{x})\rangle\langle\Psi_s^\dagger(\hat{\underline{x}})\rangle \tag{2.53}$$

where $\Psi^\dagger$ denotes the conjugate transpose of Ψ.

In eq. (2.53) ψ is conformable to $\Psi^{\dagger}$ for matrix multiplication. The elements of Ψ and $\Psi^{\dagger}$ are vectors. Therefore, eq. (2.53) represents a 2x2 matrix with elements of dyadics.

Putting eq. (2.51) in eq. (2.53) and noting that a portion of $<\Psi_s\Psi_s^{\dagger}>$ cancels the term $<\Psi_s><\Psi_s^{\dagger}>$. We also assume that N>>1. Then, we obtain

$$<\Psi_f(\underline{x})\Psi_f^{\dagger}(\hat{\underline{x}})> = \int_{4\pi} d\underline{\omega} p_{\underline{\Omega}}(\underline{\omega}) <\Psi_f(\underline{x})\Psi_f^{\dagger}(\hat{\underline{x}})>_{\underline{\omega}} \tag{2.54}$$

where

$$<\Psi_f(\underline{x})\Psi_f^{\dagger}(\hat{\underline{x}})>_{\underline{\omega}} = \int_V d\underline{\eta} \rho(\underline{s}) \Psi_{s_e}(\underline{x},\underline{s}) \Psi_{s_e}^{\dagger}(\hat{\underline{x}},\underline{s}) \tag{2.55}$$

with

$$\Psi_{s_e}(\underline{x},\underline{s}) = \mathscr{L}^{-1}\, T(\underline{s}) <\Psi> \tag{2.56}$$

Here we have separated the average into rotation and coordinate space averages, and thus we introduce the conditional expectation, $<\Psi_f\Psi_f^{\dagger}>_{\underline{\omega}}$, with respect to given $\underline{\omega}$.

The field Ψ_{s_e} defined in eq. (2.56) represents the field scattered by a single particle located in the equivalent medium at $\underline{s}$. V is the volume of the medium. We see that equation (2.55) is just an incoherent addition of the single scattering contributions from each particle. This is a result of the assumed independence of particle statistics.

The correlation of the fluctuation field $\Psi_f(\underline{x})$ is calculated instead of the correlation of $\Psi_s(\underline{x})$, so that coherent effects will be eliminated from the backscatter coefficient expressions.

The backscattering coefficients are then directly related to the transverse Fourier transform of equation (2.55) with respect to $\underline{x}_t$ and $\hat{\underline{x}}_t$ evaluated at the upper interface (z=0). This work will be done in chapter IV.

To write equation (2.56) more explicitly, we introduce the Green's function $G(\underline{x},\underline{x}')$ for the operator $\mathcal{L}$. It satisfies

$$\mathcal{L}G(\underline{x},\underline{x}') = I\delta(\underline{x}-\underline{x}') \tag{2.57}$$

where $\mathcal{L}$ is given by eq. (2.45). The I is a square diagonal matrix of the unit dyadics,

$$I = \begin{bmatrix} \underline{\underline{I}} & 0 \\ 0 & \underline{\underline{I}} \end{bmatrix} \tag{2.58}$$

and

$$G(\underline{x},\underline{x}') = \begin{bmatrix} \underline{\underline{G}}_{11}(\underline{x},\underline{x}') & \underline{\underline{G}}_{12}(\underline{x},\underline{x}') \\ \underline{\underline{G}}_{21}(\underline{x},\underline{x}') & \underline{\underline{G}}_{22}(\underline{x},\underline{x}') \end{bmatrix} \tag{2.59}$$

where $\underline{\underline{G}}_{11}$, $\underline{\underline{G}}_{12}$, $\underline{\underline{G}}_{21}$ and $\underline{\underline{G}}_{22}$ are dyadic Green's functions. [See Felsen and Marcuvitz, 1973]. Now equation (2.56) becomes

$$\Psi_{s_e}(\underline{x},\underline{s}) = \int d\underline{x}' G(\underline{x},\underline{x}') \int d\underline{x}'' t(\underline{x}'-\underline{s},\underline{x}''-\underline{s}) \langle \Psi(\underline{x}'') \rangle \tag{2.60}$$

Equation (2.60) simplifies in the low frequency or Rayleigh limit.

Before proceeding to a specific application, we would like to point out the relationship between the distorted Born approximation as presented here and the Twersky equation for the correlation. Basically, the vector analog of the scalar Twersky equation [Twersky, 1964] can be found by applying the method of smoothing when the fractional volume is small. If one then solves this equation under the assumption of small albedo, the result should be the same as the distorted Born method we have employed.

CHAPTER III

MEAN FIELD AND GREEN'S FUNCTION IN THREE MEDIA

The mean field and Green's function in three layered media are developed in this chapter. We begin first by obtaining the transverse wave vector, then we obtain a fluctuation equation. Using the two-variable expansion procedure, the equation will be solved approximately. The two-variable expansion procedure is a perturbation technique for approximating solutions to differential equations, and it is the principal mathematical technique used in this study. Some particular applications of this method has been made by Tsang and Kong [1976], Tan and Fung [1979], Tsang and Kong [1979], and Zuniga and Kong [1981].

We conclude the mean field study by obtaining the mean field of three media in a small volume limit ($\rho V_p \ll 1$). The small volume limit or sparse distribution is natural for vegetation where leaves and stems make up a relatively small percentage of the volume. This in turn leads to the concept that the mean field acts as though it propagates in an equivalent medium which is anisotropic. Because of this condition, we expect interface effects to be relatively unimportant. The most important effect is the decay of the mean field as it progresses through and reflects from the ground.

Finally, we will compute the transverse dyadic Green's function in three layered media but will only give explicit expressions for it in the slab region.

The Evaluation of the Eigenvectors

A. Transverse Fourier Transform

To illustrate the application of the method developed in the previous sections, we will calculate the mean field in a three layered medium. As shown in fig. 2, this consists of free space at the top, the vegetation layer and the ground. A plane wave is assumed to be incident at an angle θ_0 and scattered at an angle θ_s. The unit polarization vectors for incident and scattered wave are shown in fig. 2. The interface between the ground and the vegetation is taken to be smooth.

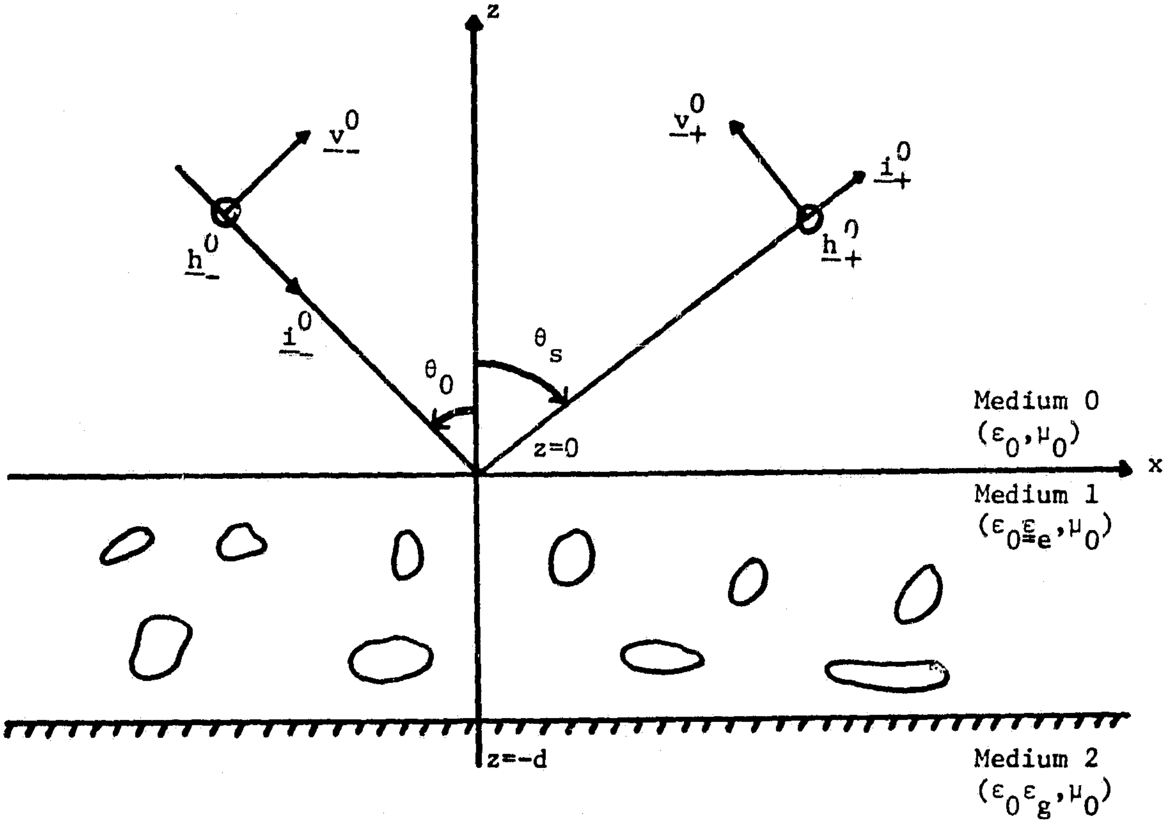


Fig. 2. Geometrical configuration of the problem

Medium 0 is free space having permeability μ_0 and permittivity ε_0. The scattering particles are all assumed to be identical and each has a volume V_p, relative dielectric constant ε_r, and free space permeability μ_0 in Medium 1. Medium 2 is the ground underlying the vegetation with relative dielectric constant ε_g and permeability μ_0.

The equations satisfied by the mean fields in the three media, which are mentioned above, have been derived in chapter II. There it was shown that the average electric field, $\langle \underline{E}(\underline{x})\rangle$, and the average magnetic field, $\langle \underline{H}(\underline{x})\rangle$, obey Maxwell's equations in an equivalent medium having relative permeability 1 and relative permittivity $\underline{\underline{\varepsilon}}_e(\underline{x})$. The relative permittivity $\underline{\underline{\varepsilon}}_e(\underline{x})$ is dyadic since the scatterers are arbitrarily shaped.

In this section we consider first the abstract formulation of the guided-mode wavevector representation of Maxwell's equations. We start by writing the macroscopic form of Maxwell's equations which were obtained in chapter II. Assuming g=0, we have

$$L\langle\Psi(\underline{x})\rangle = \int_V d\underline{s}\int d\underline{x}'\rho(\underline{s})\bar{\bar{t}}(\underline{x}-\underline{s},\underline{x}'-\underline{s})\langle\Psi(\underline{x}')\rangle \tag{3.1}$$

where

$$L = \begin{bmatrix} -i\omega\varepsilon_0\varepsilon_d(\underline{x})\underline{\underline{I}} & -\nabla\times\underline{\underline{I}} \\ \nabla\times\underline{\underline{I}} & -i\omega\mu_0\underline{\underline{I}} \end{bmatrix} \text{ with } \varepsilon_d(\underline{x}) = \varepsilon_d(z) = \begin{cases} 1 & , \quad z>-d \\ \varepsilon_g & , \quad z<-d \end{cases} \tag{3.2}$$

and

$$\langle\Psi(\underline{x})\rangle = \begin{bmatrix} \langle\underline{E}(\underline{x})\rangle \\ \langle\underline{H}(\underline{x})\rangle \end{bmatrix} \tag{3.3}$$

$$\bar{\bar{t}} = \frac{-1}{i\omega\mu_0}\begin{bmatrix} \underline{\underline{t}} & 0 \\ 0 & 0 \end{bmatrix} \tag{3.4}$$

The left side of eq. (3.1) becomes zero when $\underline{x}\notin V$. To see this we note that when $\underline{x}\notin V$, we have $\underline{x}-\underline{s}\notin V_p$ since $\underline{s}\in V$. Now using eq.(2.21), we have $\underline{\underline{t}}=0$. Thus the integral over V in eq. (3.1), can be replaced by infinite limit if $\underline{x}$ is restricted to lie in the slab region.

$$L\langle\Psi(\underline{x})\rangle = \left\{\int d\underline{s}\int d\underline{x}'\rho(\underline{s})\overline{t}(\underline{x}-\underline{s},\underline{x}'-\underline{s})\langle\Psi(\underline{x}')\rangle\right\} \text{rect}_d(z) \tag{3.5}$$

where

$$\text{rect}_d(z) = \begin{cases} 0 & , z>0 \quad \text{(free space)} \\ 1 & , -d<z<0 \quad \text{(slab)} \\ 0 & , z<-d \quad \text{(ground)} \end{cases} \tag{3.6}$$

The general field in the vegetation can be described either in terms of a first-order system of field equations, or it can be reduced to a higher-order equation in terms of a particular field component. The reduced field formulation (eq. (3.5)) frequently leads to analytical complexities in identification of energy expressions, reciprocity properties, eigen modes, etc.. The first-order formulation avoids many of these difficulties. In this work the first-order formulation will be used. Also this formulation will be more suitable to apply to the two-variable expansion procedure.

In order to simplify eq. (3.5), we should like to take the transverse Fourier transform of it. For this purpose, we assume ρ has no transverse variation, i.e.,

$$\rho(\underline{s}) = \rho(s) \tag{3.7}$$

where

$$\underline{s} = \underline{s}_t + s\underline{z}^0$$

Also we define

$$\overline{\chi}(\underline{x}_t,\underline{x}_t',z-s,z'-s) = \int d\underline{s}_t\overline{t}(\underline{x}_t-\underline{s}_t,\underline{x}_t'-\underline{s}_t,z-s,z'-s) \tag{3.8a}$$

and let $\quad \hat{\underline{s}}_t = \underline{s}_t - \underline{x}_t \qquad d\hat{\underline{s}}_t = d\underline{s}_t \qquad (3.8b)$

Using eq. (3.8b) in (3.8a) we have

$$\overline{\chi}(\underline{x}_t,\underline{x}_t';z-s,z'-s)=\int d\hat{\underline{s}}_t\overline{t}(-\hat{\underline{s}}_t,\underline{x}_t'-\underline{x}_t-\hat{\underline{s}}_t,z-s,z'-s)$$

$$= \overline{\chi}(\underline{x}_t-\underline{x}_t';z-s,z'-s) \tag{3.8c}$$

As seen in eq. (3.8c) we suppress the $\underline{s}_t$ dependence by using eq. (3.8a). Substituting (3.8c), and (3.7) into (3.5) we have

$$L\langle\Psi(\underline{x})\rangle = \left\{\int ds \int d\underline{x}'\rho(s)\bar{\chi}(\underline{x}_t-\underline{x}_t';z-s,z'-s)\langle\Psi(\underline{x}')\rangle\right\} \mathrm{rect}_d(z) \tag{3.9}$$

In eq. (3.9) we have written the difference in transverse coordinates because the particle density ρ does not depend on $\underline{s}_t$. Because of the exponential dependence of the incident wave on $\underline{x}$ and the invariance of the mean equation in the transverse direction, we assume

$$\langle\Psi(\underline{x})\rangle \equiv \langle\Psi(\underline{x}_t,z)\rangle = \langle\tilde{\Psi}(\underline{k}_t,z)\rangle e^{i\underline{k}_t\cdot\underline{x}_t} \tag{3.10}$$

where

$$\underline{k}_t = k_t\underline{x}^0 \quad , \quad k_t = k_0\sin\theta \tag{3.11}$$

The system of ordinary differential equations is obtained by substituting eq. (3.10) into eq. (3.9). This is equivalent to taking the transverse Fourier transform of eq. (3.9). We find:

$$\tilde{L}\langle\tilde{\Psi}(\underline{k}_t,z)\rangle = \left\{\int ds \int dz'\rho(s)\tilde{\bar{\chi}}(\underline{k}_t,z-s,z'-s)\langle\tilde{\Psi}(\underline{k}_t,z')\rangle\right\} \mathrm{rect}_d(z) \tag{3.12}$$

where

$$\langle\tilde{\Psi}(\underline{k}_t,z)\rangle = \begin{bmatrix} \langle\underline{\tilde{E}}(\underline{k}_t,z)\rangle \\ \langle\underline{\tilde{H}}(\underline{k}_t,z)\rangle \end{bmatrix} \tag{3.13}$$

The transverse Fourier transform of the operator L is

$$\tilde{L} = -iK + \Gamma\frac{d}{dz} \tag{3.14}$$

with the component operators K and Γ defined by

$$K = \begin{bmatrix} \omega\epsilon_0\epsilon_d(z)\underline{\underline{I}} & \underline{k}_t\times\underline{\underline{I}} \\ -\underline{k}_t\times\underline{\underline{I}} & \omega\mu_0\underline{\underline{I}} \end{bmatrix} \quad \text{and} \quad \Gamma = \begin{bmatrix} 0 & -\underline{z}^0\times\underline{\underline{I}} \\ \underline{z}^0\times\underline{\underline{I}} & 0 \end{bmatrix} \tag{3.15}$$

From eqs. (2.20), (3.8a) and (3.11) we see that

$$\tilde{\chi}(\underline{k}_t, z-s, z'-s) \equiv \tilde{\chi}(z-s, z'-s) = \begin{bmatrix} \tilde{\chi}_{11}(z-s, z'-s) & 0 \\ 0 & 0 \end{bmatrix} \tag{3.16}$$

B. Eigen-Functions for the Particle Free Medium

The complete solution to eq. (3.12) is obtained by a perturbation technique under the assumption that the fractional volume is small. As a first step in implementing this procedure the homogeneous solutions to $\tilde{L}\langle\tilde{\Psi}\rangle=0$ are found. The homogeneous solutions are of the form:

$$\langle\tilde{\Psi}\rangle = \Psi_\alpha e^{i\kappa_\alpha z} \tag{3.17}$$

where Ψ_α and κ_α will depend upon whether $z>-d$ or $z<-d$. Substituting eq. (3.17) into the homogeneous equation $\tilde{L}\langle\tilde{\Psi}\rangle=0$ yields

$$\tilde{L}\Psi_\alpha = (K-\kappa_\alpha\Gamma)\Psi_\alpha = 0$$

or

$$K\Psi_\alpha = \kappa_\alpha\Gamma\Psi_\alpha \tag{3.18}$$

where K and Γ are defined by eq. (3.15). It is seen that the κ_α and Ψ_α are eigen values and eigen vectors respectively of the operator K.

To establish the orthogonality of the eigen vectors, we introduce the product

$$(\Psi_\alpha, \Psi_\beta) = \Psi_\alpha^T \Psi_\beta = \underline{E}_\alpha \cdot \underline{E}_\beta + \underline{H}_\alpha \cdot \underline{H}_\beta \tag{3.19}$$

where Ψ^T is the transpose of Ψ. The adjoint operator L^+ is defined by

$$(L\,\Psi_\alpha, \Psi_\beta) = (\Psi_\alpha, L^+\,\Psi_\beta)$$

For the product defined in eq. (3.19), we conclude

$$(L\,\Psi_\alpha, \Psi_\beta) = (L\,\Psi_\alpha)^T\,\Psi_\beta = \Psi_\alpha^+\,L^T\,\Psi_\beta = \Psi_\alpha^T\,L^+\,\Psi_\beta$$

thus

$$L^{+} = L^{T}$$

Now the adjoint eigen function problem for eq. (3.18) can be defined as

$$K^{+}\Psi_{\alpha}^{+} = \kappa_{\alpha}^{+}\Gamma^{+}\Psi_{\alpha}^{+}$$

Since $K^{+} = K^{T} = K$ and $\Gamma^{+} = \Gamma^{T} = \Gamma$ the adjoint problem is the same as the original eigen value problem; thus $\Psi_{\alpha}^{+} = \Psi_{\alpha}$ and $\kappa_{\alpha}^{+} = \kappa_{\alpha}$. As a result we say the eigen value problem is self adjoint.

We can now show that two eigen vectors having different eigen functions are orthogonal. Consider eigen vectors Ψ_{α} and Ψ_{β} and their corresponding eigen values λ_{α} and λ_{β} . We have

$$(K\Psi_{\alpha},\Psi_{\beta}) - (K\Psi_{\alpha},\Psi_{\beta}) = 0$$

$$(K\Psi_{\alpha},\Psi_{\beta}) - (\Psi_{\alpha},K\Psi_{\beta}) = 0$$

$$(\lambda_{\alpha}\Gamma\Psi_{\alpha},\Psi_{\beta}) - (\Psi_{\alpha},\lambda_{\beta}\Gamma\Psi_{\beta}) = 0$$

$$\lambda_{\alpha}(\Gamma\Psi_{\alpha},\Psi_{\beta}) - \lambda_{\beta}(\Psi_{\alpha},\Gamma\Psi_{\beta}) = 0$$

$$(\lambda_{\alpha} - \lambda_{\beta})(\Gamma\Psi_{\alpha},\Psi_{\beta}) = 0$$

If $\lambda_{\alpha} \neq \lambda_{\beta}$ then

$$(\Gamma\Psi_{\alpha},\Psi_{\beta}) = 0 \qquad (3.20)$$

The eigen vectors Ψ_{α} and Ψ_{β} are orthogonal to each other as defined by eq. (3.20).

Actual computation of the eigen vectors and values proceeds in a direct manner from eq. (3.18). The calculation follows the method of Felsen and Marcuvitz, 1973 and thus will not be repeated here. The results of the calculation shows that there are four eigen vectors and four eigen values. The eigen values are given by

$$\kappa_{\pm}(z) = \pm\kappa(z) \qquad (3.21a)$$

where

$$\kappa(z) = \begin{cases} \kappa & , \quad z>-d \\ \kappa_g & , \quad z<-d \end{cases} \tag{3.21b}$$

with

$$\kappa = k_0\cos\theta \quad , \quad \kappa_g = \sqrt{k_g^2-k_t^2} \quad , \quad k_g = k_0\sqrt{\varepsilon_g} \tag{3.22}$$

and k_t is given in eq.(3.11). We note that there are only two distinct eigen values; each has multiplicity two. Thus two eigen vectors correspond to each distinct eigen value. The eigen vectors are:

$$\Psi_{\pm 1}(z) = \begin{bmatrix} \underline{v}_{\pm}^0(z) \\ -\eta(z)\underline{h}^0 \end{bmatrix} \qquad \Psi_{\pm 2}(z) = \begin{bmatrix} \underline{h}^0 \\ \eta(z)\underline{v}_{\pm}^0\ (z) \end{bmatrix} \tag{3.23}$$

where the unit polarization vectors are given by

$$\underline{h}^0 = \underline{h}_{\pm}^0 = \underline{h}_{g\pm}^0 = \underline{y}^0 \tag{3.24}$$

$$\underline{v}_{\pm}^0(z) = \begin{cases} \underline{v}_{\pm}^0 & , \quad z>-d \\ \underline{v}_{g\pm}^0 & , \quad z<-d \end{cases} \tag{3.25a}$$

with

$$\underline{v}_{\pm}^0 = (\mp\kappa\underline{x}^0 + k_t\underline{z}^0)/k_0 \tag{3.25b}$$

$$\underline{v}_{g\pm}^0 = (\mp\kappa_g\underline{x}^0 + k_t\underline{z}^0)/k_g \tag{3.25c}$$

and

$$\eta(z) = \sqrt{\frac{\varepsilon_0\varepsilon_d(z)}{\mu_0}} \tag{3.26}$$

An examination of the eigen vectors shows that they obey the following orthogonality relationship

$$(\Psi_\alpha,\Psi_\beta) = 2N_\alpha\delta_{\alpha\beta} \quad , \quad \alpha,\beta \in \{\pm 1,\pm 2\} \tag{3.27a}$$

where $\delta_{\alpha\beta}$ is the Kronecker delta function and N_α is a normalization coefficient given by

$$N_{\pm i} = \pm N_T(z) \quad , \quad N_T(z) = \frac{\eta\kappa(z)}{k_0} \quad , \quad i=1,2 \tag{3.27b}$$

This result embodies the orthogonality relationship given in eq. (3.20), plus the fact the eigen functions corresponding to the same eigen value were choosen to be orthogonal.

The eqs. (3.23) represent the eigen vectors for z>-d and for z<-d. For further use we write these eigen vectors explicitly in each region. We have

$$\Psi_{\pm 1} = \begin{bmatrix} \underline{v}^0_{\pm} \\ -\eta \underline{h}^0 \end{bmatrix}, \quad \Psi_{\pm 2} = \begin{bmatrix} \underline{h}^0 \\ \eta \underline{v}^0_{\pm} \end{bmatrix} \quad \text{for } z>-d \tag{3.28a}$$

with $\eta = \sqrt{\frac{\varepsilon_0}{\mu_0}}$,

and

$$\Psi_{1g} = \begin{bmatrix} \underline{v}^0_{g} \\ -\eta_g \underline{h}^0 \end{bmatrix}, \quad \Psi_{2g} = \begin{bmatrix} \underline{h}^0 \\ \eta_g \underline{v}^0_{g} \end{bmatrix} \quad \text{for } z<-d \tag{3.28b}$$

with $\eta_g = \eta\sqrt{\varepsilon_g}$.

In obtaining eq. (3.28b), we have used $\underline{v}^0_g = \underline{v}^0_{g-}$. Due to radiation condition, $\underline{v}^0_{g+}$ will not come into our development.

<u>The Solution of the Mean Wave Equation</u>

A. Reduction to Slowly Varying Coefficients:

We start with substituting eqs. (3.23) in eq. (3.17), then we obtain

$$\langle\tilde{\Psi}(z)\rangle = (\Psi_{-1}a' + \Psi_{-2}a'')e^{-i\kappa z} + (\Psi_{+1}b' + \Psi_{+2}b'')e^{+i\kappa z} \tag{3.29}$$

where the form of the eigen values and vectors will depend on whether z<-d or z>-d.

The problem is now reformulated in terms of slowly varying parameters [Kohler and Papanicolaou ,1973]. These parameters are defined by the transformation

$$\begin{aligned} \underline{A}(z) &= a'(z)\underline{v}^0_-(z) + a''(z)\underline{h}^0 \\ \underline{B}(z) &= b'(z)\underline{v}^0_+(z) + b''(z)\underline{h}^0 \end{aligned} \tag{3.30}$$

Here $\underline{A}(z)$ and $\underline{B}(z)$ are the slowly varying wave amplitudes associated incident and scattered wave, respectively.

We also define

$$y^-(z) = \Psi_{-1}\underline{v}^0_-(z) + \Psi_{-2}\underline{h}^0$$
$$y^+(z) = \Psi_{+1}\underline{v}^0_+(z) + \Psi_{+2}\underline{h}^0 \tag{3.31}$$

where $\Psi_{\pm 1}$ and $\Psi_{\pm 2}$ are defined by eq. (3.23). The Ψ's are 2x1 matricies whose elements are vectors, therefore y^- and y^+ are 2x1 matricies where each element is a dyadic.

Using eq. (3.30) and (3.31) in (3.29) we obtain

$$<\tilde{\Psi}(z)> = y^-\underline{A}e^{-i\kappa z} + y^+\underline{B}e^{i\kappa z} \tag{3.32}$$

Defining the vector $\phi(z)$ by

$$\phi(z) = \begin{bmatrix} \underline{A}(z) \\ \\ \underline{B}(z) \end{bmatrix} \tag{3.33}$$

and the 2x2 matrix $y(z)$ of dyadics by

$$y(z) = [y^-(z)e^{-i\kappa z}, y^+(z)e^{i\kappa z}] \tag{3.34}$$

We cast eq. (3.32) in matrix form as follows:

$$<\tilde{\Psi}(z)> = y(z)\phi(z) \tag{3.35}$$

where we denote the matrix $y(z)$ in the two regions by

$$y(z) = \begin{cases} y \quad , & z>-d \\ \\ y_g \quad , & z<-d \end{cases}$$

The equation (3.35) represents the transformation from field quantities to slowly varying amplitudes which we wanted to obtain.

Using eq. (3.23) in eq. (3.31) we obtain

$$y^-(z) = \begin{bmatrix} \underline{\underline{I}}^-(z) \\ \\ \eta(z)\underline{i}^0_-\times\underline{\underline{I}}^-(z) \end{bmatrix} \qquad y^+(z) = \begin{bmatrix} \underline{\underline{I}}^+(z) \\ \\ \eta(z)\underline{i}^0_+\times\underline{\underline{I}}^+(z) \end{bmatrix} \tag{3.36}$$

where $\underline{i}^0_\pm(z)$ are unit vectors in the direction of the up and down going waves. They are defined by

$$\underline{i}^0_{\pm}(z) = \begin{cases} (\underline{k}_t \pm \kappa \underline{z}^0)/k_0 & , \quad z>-d \\ (\underline{k}_t \pm \kappa_g \underline{z}^0)/k_g & , \quad z<-d \end{cases} \tag{3.37a}$$

and

$$\underline{\underline{I}}^{\pm}(z) = \underline{h}^0 \underline{h}^0 + \underline{v}^0_{\pm}(z) \underline{v}^0_{\pm}(z) \tag{3.37b}$$

Here $\underline{\underline{I}}^{\pm}(z)$ have the property $\underline{\underline{I}}^{-}(z) \cdot \underline{A}(z) = \underline{A}(z)$ and $\underline{\underline{I}}^{+}(z) \cdot \underline{B}(z) = \underline{B}(z)$.

The $\underline{\underline{I}}^{-}(z)$ and $\underline{\underline{I}}^{+}(z)$ are unit dyadic tensors associated with up and down going waves respectively.

Using eq. (3.36) in eq. (3.34) we obtain the y matrix as

$$y(z) = \begin{bmatrix} \underline{\underline{I}}^{-}(z) e^{-i\kappa z} & \underline{\underline{I}}^{+}(z) e^{i\kappa z} \\ \eta(z)((\underline{i}^0_{-} x \underline{\underline{I}}^{-}(z)) e^{-i\kappa z} & \eta(z)((\underline{i}^0_{+} x \underline{\underline{I}}^{+}(z)) e^{i\kappa z} \end{bmatrix} \tag{3.38}$$

One can define dyadic inner product similar to eq. (3.19) [Friedman, 1962], as

$$(y^{\pm}, \Gamma y^{\pm}) = (y^{\pm})^T \Gamma (y^{\pm}) \tag{3.39}$$

Substituting eqs. (3.36) and (3.15) into eq. (3.19) and proceeding with the matrix multiplication, we obtain the results

$$(y^{\pm})^T \Gamma (y^{\pm}) = \pm 2N_T \underline{\underline{I}}^{\pm} \tag{3.40}$$

with $(y^{\pm})^T \Gamma (y^{\mp}) = 0$. The normalization coefficient N_T is given by eq. (3.27b).

Now we define the inverse matrix, y^{-1}, as:

$$y^{-1}(z) = [\frac{-1}{2N_T} y^{-}(z) e^{i\kappa z} \ , + \frac{1}{2N_T} y^{+}(z) e^{-i\kappa z}]^T \tag{3.41}$$

or

$$y^{-1}(z) = \frac{1}{2N_T}\begin{bmatrix} -\underline{\underline{I}}^-(z)e^{i\kappa z} & -\eta(z)(\underline{i}_-^0 x \underline{\underline{I}}^-(z))e^{i\kappa z} \\ \underline{\underline{I}}^+(z)e^{-i\kappa z} & \eta(z)(\underline{i}_+^0 x \underline{\underline{I}}^+(z))e^{-i\kappa z} \end{bmatrix} \tag{3.42}$$

The inverse matrix has the property that

$$y^{-1}\Gamma y = I_\phi \tag{3.43}$$

where

$$I_\phi = \begin{bmatrix} \underline{\underline{I}}^- & 0 \\ 0 & \underline{\underline{I}}^+ \end{bmatrix} \tag{3.44}$$

Here I_ϕ is a identity matrix in the ϕ space where $I_\phi \phi = \phi$.

Next we substitute eq. (3.35) into the transverse mean wave eq. (3.12). We obtain

$$\tilde{L}y(z)\phi(z) = \left(\int ds \int dz' \rho(s)\tilde{\tilde{\chi}}(z-s,z'-s)y(z')\phi(z')\right) \text{rect}_d(z) \tag{3.45}$$

where

$$\tilde{L}y(z)\phi(z) = [(-iK+\Gamma\frac{d}{dz})y(z)]\phi(z) = (-iKy+\Gamma\frac{d}{dz}y)\phi+\Gamma y\frac{d\phi}{dz} \tag{3.46}$$

Using eq. (3.34) we obtain

$$(-iK+\Gamma\frac{d}{dz})y = (-iK+\Gamma\frac{d}{dz})(y^-e^{-i\kappa z},\ y^+e^{+i\kappa z})$$

$$= [(-iK-i\kappa\Gamma)y^-e^{-i\kappa z},\ (-iK+i\Gamma\kappa)y^+e^{+i\kappa z}] \tag{3.47}$$

Using eq. (3.31) in eq. (3.47) and proceeding the matrix multiplication, we obtain

$$(-iK+\Gamma\frac{d}{dz})y(z) = 0 \tag{3.48}$$

Hence eq. (3.45) becomes

$$\Gamma y\frac{d\phi(z)}{dz} = \left(\int ds \int dz' \rho(s)\tilde{\tilde{\chi}}(z-s,z'-s)y(z')\phi(z')\right) \text{rect}_d(z) \tag{3.49}$$

Multiplying both sides of eq. (3.49) by $y^{-1}(z)$ and using (3.43) we find the fluctuation equation to be

$$\frac{d}{dz}\phi(z) = \left(\int ds \int dz' \rho(s) y^{-1}(z) \bar{\bar{X}} y(z') \phi(z') \right) \text{rect}_d(z) \tag{3.50}$$

with boundary conditions: i) tangential components continuous, i.e., $\Gamma y(z)\phi(z)$ is continuous at $z = 0$ and $z=-d$, and ii) radiation condition at $\pm\infty$. We also have

$$y^{-1}(z)\bar{\bar{X}}y(z') = \bar{\bar{X}}_\phi(z,z',s) = \begin{bmatrix} \bar{\bar{X}}_\phi^{(-,-)} & \bar{\bar{X}}_\phi^{(-,+)} \\ \bar{\bar{X}}_\phi^{(+,-)} & \bar{\bar{X}}_\phi^{(+,+)} \end{bmatrix} \tag{3.51}$$

with

$$\bar{\bar{X}}^{(\alpha,\beta)}(z,z',s) = \frac{(\alpha)}{2N_T} \bar{\bar{I}}^{\alpha} \cdot \bar{\bar{X}}_{11}(\underline{k}_{t_0}, z-s, z'-s) \cdot \bar{\bar{I}}^{\beta} e^{-i\kappa(\alpha z - \beta z')} ,$$

$$\alpha,\beta \in \{+,-\}$$

B. Application of the Two-variable Procedure:

The eq. (3.50) can be solved along with the appropriate boundary conditions by using exact or approximate methods. The two-variable perturbation procedure will allow us to determine the approximate solution to eq. (3.50) directly without having to find the more complicated exact solution first.

The main advantage of the two-variable procedure is the simplicity of the formalism. The higher approximations are more easily calculated. A disadvantage of the method, however, is that the proper choice of fast and slow variables are not always obvious [Cole, 1968].

We assume a slow variable $\bar{z}=\delta z$, and then we assume $\rho(z)=\delta\bar{\rho}(\bar{z})$. Thus $\bar{\rho}(\bar{z})$ is a slowly varying function. Next we consider $\phi(z)$ to be both a function of z and $\bar{z}$. We have

$$\phi(z) \equiv \phi(z,\bar{z},\delta) = \sum_{n=0}^{\infty} \phi^{(n)}(z,\bar{z})\delta^n = \phi^{(0)}(z,\bar{z}) + \delta\phi^{(1)}(z,\bar{z}) \ldots \tag{3.52}$$

If we proceed one step further by expanding $\bar{\rho}(\bar{s})$ in power series around $\bar{s}=\bar{z}$, then we have

$$\bar{\rho}(\bar{s}) = \sum_{m=0}^{\infty} \delta^{m}\bar{\rho}^{(m)}(\bar{z}) \frac{(s-z)^{m}}{m!} = \bar{\rho}(\bar{z}) + \ldots \tag{3.53}$$

where

$$\bar{\rho}^{\,m} = \frac{d^{m}\bar{\rho}}{ds^{m}} \quad , \qquad \bar{z} = \delta z$$

with $\delta<<1$, and total derivative with respect to z can be written as a sum of partial derivatives,

$$\frac{d}{dz} = \frac{\partial}{\partial z} + \delta\frac{\partial}{\partial \bar{z}} \tag{3.54}$$

We substitute eqs. (3.54), (3.53) and (3.52) into eq. (3.50). Using the fact that $\bar{\rho}(\bar{s})$ is slowly varying over the support of $\tilde{X}$, it can be taken in front of the integral as $\bar{\rho}(\bar{z})$. Thus eq. (3.50) takes the following form

$$\frac{\partial}{\partial z}+\delta\frac{\partial}{\partial \bar{z}} \sum_{n=0}^{\infty}\phi^{(n)}(z,\bar{z})\delta^{n} = \left\{\sum_{m=0}^{\infty}\delta^{m+1}\bar{\rho}^{(m)}(\bar{z})\int ds\int dz' \frac{(s-z)^{m}}{m!} y^{-1}\tilde{X}y \sum_{n=0}^{\infty}\phi^{(n)}(z',\bar{z}')\delta^{n}\right\}\mathrm{rect}_{d}(z) \tag{3.55}$$

We also need to expand $\phi^{(n)}(z',\bar{z}')$ around $\bar{z}'=\bar{z}$. Doing that we get

$$\phi^{(n)}(z',\bar{z}') = \phi^{(n)}(z',\bar{z}) + \delta\frac{\partial\phi}{\partial\bar{z}}(z',\bar{z})(z'-z) + \ldots \tag{3.56}$$

Using eq. (3.56) in (3.55) and by equating coefficients of δ, we find

$$\delta^{(0)}: \frac{\partial}{\partial z}\phi^{(0)}(z,\bar{z}) = 0 \tag{3.57a}$$

therefore $\phi^{(0)}(z,\bar{z}) = \phi^{(0)}(\bar{z})$ (3.57b)

$$\delta^{(1)}: \frac{\partial}{\partial z}\phi^{(1)}(z,\bar{z}) + \frac{\partial}{\partial\bar{z}}\phi^{(0)}(z,\bar{z}) = \bar{\rho}(\bar{z})\left\{\int ds\int dz' y^{-1}(z)\tilde{X}y(z')\phi^{(0)}(z',\bar{z})\right\}\mathrm{rect}_{d}(z) \tag{3.58a}$$

From eq. (3.58a), we obtain by using (3.57b)

$$\phi^{(1)}(z,\bar{z}) = -\frac{d\phi^{(0)}(\bar{z})}{d\bar{z}}z + \bar{\rho}(\bar{z})\left\{\int_{0}^{z}dz'\int ds\int dz'' y^{-1}(z')\tilde{X}y(z'')\right\}\phi^{(0)}(\bar{z})\mathrm{rect}_{d}(z) + \phi^{(1)}(\bar{z}) \tag{3.58b}$$

Following the two-variable procedure, which requires that the $\phi^{(1)}$ term does not grow as fast as z, i.e.,

$$\lim_{z\to\infty} \frac{1}{z}\,\phi^{(1)}(z,\bar{z}) = 0 \tag{3.59}$$

In Appendix B, the condition given by eq. (3.59) is evaluated. The result leads to the following equation for $\phi^{(0)}(\bar{z})$:

$$\frac{d}{dz}\phi^{(0)}(\bar{z}) = \bar{\rho}(\bar{z})M\phi^{(0)}(\bar{z})\mathrm{rect}_d(z) \tag{3.60}$$

where

$$M = \begin{bmatrix} \underline{\underline{m}}^- & 0 \\ 0 & \underline{\underline{m}}^- \end{bmatrix} \tag{3.61a}$$

and employing

$$\underline{\underline{m}}^\beta = \frac{\beta i 4\pi^3}{\kappa}\,\underline{\underline{I}}^\beta\cdot\bar{\underline{\underline{t}}}(k_0\underline{i}^0_\beta, k_0\underline{i}_\beta)\cdot\underline{\underline{I}}^\beta \quad , \quad \beta\ \varepsilon\{+,-\} \tag{3.61b}$$

If we use eq. (2.26) along with the fact that $\underline{\underline{I}}^\pm = \underline{h}^0\underline{h}^0 + \underline{v}^0_\pm\underline{v}^0_\pm + \underline{i}^0_\pm\underline{i}^0_\pm$ in eq. (3.61b), we have

$$\underline{\underline{m}}^\beta = \frac{\beta 2\pi i}{\kappa}\,\bar{\underline{\underline{f}}}(\underline{i}^0_\beta, \underline{i}^0_\beta) \tag{3.62}$$

Equation (3.60) can now be written as:

$$\frac{d}{dz}\phi^0(\bar{z}) = -\frac{2\pi i}{\kappa}\,\rho(\bar{z})\;\bar{f}\phi^0(\bar{z}) \tag{3.63}$$

where

$$\phi^{(0)}(\bar{z}) = \begin{bmatrix} \underline{A}(\bar{z}) \\ \underline{B}(\bar{z}) \end{bmatrix}$$

and

$$\bar{f} = \begin{bmatrix} \bar{\underline{\underline{f}}}(\underline{i}^0_-, \underline{i}^0_-) & 0 \\ 0 & -\bar{\underline{\underline{f}}}(\underline{i}^0_+, \underline{i}^0_+) \end{bmatrix} \tag{3.64}$$

Equation (3.63) represents two independent equations, namely

$$\frac{d\underline{A}^0(\bar{z})}{d\bar{z}} = -\frac{2\pi i}{\kappa}\,\bar{\rho}(\bar{z})\,\underline{\underline{f}}\,(\underline{i}_-^0,\underline{i}_-^0)\cdot\underline{A}^0(\bar{z}) \tag{3.65}$$

$$\frac{d\underline{B}^0(z)}{d\bar{z}} = \frac{2\pi i}{\kappa}\,\bar{\rho}(\bar{z})\,\underline{\underline{f}}\,(\underline{i}_+^0,\underline{i}_+^0)\cdot\underline{B}^0(\bar{z}) \tag{3.66}$$

The equations (3.65) and (3.66) are two first order systems of differential equations for slowly varying coefficients.

C. The solution of the slowly varying equations:

Here we will solve the slowly varying differential equations in order to obtain the mean field in the air, the ground and the vegetated slab as shown in fig. 3. We assume the transverse mean field is excited by a plane wave

$$\langle\tilde{\Psi}_i(z)\rangle = y^-\underline{q}_-^0\, e^{-i\kappa_0 z} \qquad q\varepsilon\{h,v\}\ ,\quad z>0 \tag{3.67}$$

having polarization $\underline{q}^0$ and propagation constant $\kappa_0=k_0\cos\theta_0$.

We can write the mean field in the three regions using eq. (3.32) and eq. (3.67). It is:

$$\langle\tilde{\Psi}(z)\rangle = y^-\cdot\underline{q}_-^0 e^{-i\kappa_0 z} + y^+\cdot\underline{\underline{\Gamma}}_g\cdot\underline{q}_-^0 e^{i\kappa_0 z} \qquad z>0 \tag{3.68}$$

$$\langle\tilde{\Psi}(z)\rangle = y^-\cdot\underline{A}(\bar{z})e^{-i\kappa_0 z} + y^+\cdot\underline{B}(\bar{z})e^{i\kappa_0 z} \qquad 0>z>-d \tag{3.69}$$

$$\langle\tilde{\Psi}(z)\rangle = y_g^-\cdot\underline{A}_g e^{-i\kappa_{g_0} z} \qquad z<-d \tag{3.70}$$

where $\underline{\underline{\Gamma}}_g$ is the dyadic reflection coefficient and $\underline{A}_g$ is constant vector. The subzero on quantities such as κ_0 and κ_{g_0} which indicate an angle of incidence θ_0 will be suppressed through the rest of section C for notation convenience, and κ_{g_0} is given by

$$\kappa_{g_0} = k_0(\varepsilon_g - \sin^2\theta_0)^{1/2} \tag{3.71}$$

The y and y^+ in eqs. (3.68)-(3.70) are given by eq. (3.36) and $\underline{A}(\bar{z})$,

$\underline{B}(\bar{z})$ are the slowly varying coefficients. Assuming constant ρ, from eqs. (3.66) and (3.67), we find

$$\underline{A}(\bar{z})=e^{-i\frac{2\pi}{\kappa}\bar{\rho}\ \bar{\underline{\underline{f}}}\ (\underline{i}_-^0,\ \underline{i}_-^0)\bar{z}}\cdot\underline{A}_0\ ,\ \underline{B}(\bar{z})=e^{i\frac{2\pi}{\kappa}\bar{\rho}\ \bar{\underline{\underline{f}}}\ (\underline{i}_+^0,\underline{i}_+^0)\bar{z}}\cdot\underline{B}_0 \tag{3.72}$$

where $\underline{A}_0$ and $\underline{B}_0$ are constant vectors. The mean field eqs. (3.68)-(3.70) can be expressed as

$$<\tilde{\Psi}> = \begin{cases} y\phi_f & , \quad z>0 \\ y\phi_s & , \quad 0>z>-d \\ y_g\phi_g & , \quad z<-d \end{cases} \tag{3.73}$$

where

$$\phi_f = \begin{bmatrix} \underline{q}^0 \\ \underline{\underline{\Gamma}}_s\cdot\underline{q}^0 \end{bmatrix} \quad , \quad \phi_s = \begin{bmatrix} \underline{A}(\bar{z}) \\ \underline{B}(\bar{z}) \end{bmatrix} \quad , \quad \phi_g = \begin{bmatrix} \underline{A}_g \\ 0 \end{bmatrix} \tag{3.74}$$

Boundary Conditions:

(i) At $z=\bar{z}=0$, the tangential field has to be continuous, hence

$$\Gamma<\tilde{\Psi}_f> = \Gamma<\tilde{\Psi}_s> \tag{3.75}$$

where $<\tilde{\Psi}_f>=y\phi_s$ and $<\tilde{\Psi}_s>=y\phi_s$ represent the mean fields for $z>0$ and $0>z>-d$, respectively.

After substituting eq. (3.73) into (3.75), we multiply both sides by y^{-1}. Then using eq. (3.43), we obtain

$$\phi_f = \phi_s\Big|_{z=\bar{z}=0} \tag{3.76}$$

Using eqs. (3.74) in (3.76) we conclude that

$$\underline{q}^0 = \underline{A}(0) = \underline{A}_0 \tag{3.77}$$

$$\underline{\underline{\Gamma}}_s\cdot\underline{q}^0 = \underline{B}(0) = \underline{B}_0 \tag{3.78}$$

(ii) At $z=-d$, $\bar{z}=-\bar{d}$, the tangential field has to be continuous, hence

$$\Gamma<\tilde{\Psi}_s> = \Gamma<\tilde{\Psi}_g> \tag{3.79}$$

where $<\tilde{\Psi}_g>=y_g\phi_g$ represents the mean field for $z<-d$.

After substituting eq. (3.73) in (3.79), we multiply both sides by y^{-1}. Then using (3.43), we obtain

$$\phi_s(-\bar{d}) = (y^{-1}\Gamma y_g)\phi_g \tag{3.80}$$

Substituting ϕ_s, ϕ_g and $(y^{-1}\Gamma y_g)$ into eq. (3.80), we obtain

$$\underline{A}(-\bar{d}) = \frac{-1}{2N_T}[(y^-)^T\Gamma y_g^-]\underline{A}_g e^{-i(\kappa-\kappa_g)d} \tag{3.81}$$

$$\underline{B}(-\bar{d}) = \frac{1}{2N_T}[(y^+)^T\Gamma y_g^-]\underline{A}_g e^{i(\kappa+\kappa_g)d} \tag{3.82}$$

where $N_T=\eta\kappa/k_0$ and y_g^- is given by eq. 3.36 for $z<-d$.

Now we introduce

$$\underline{A}_g = \underline{\underline{T}}\cdot\underline{A}(-\bar{d}) \tag{3.83}$$

and

$$\underline{B}(-\bar{d}) = \underline{\underline{\Gamma}}\cdot\underline{A}(-\bar{d}) \tag{3.84}$$

where $\underline{\underline{T}}$ and $\underline{\underline{\Gamma}}$ are dyadic transmission and reflection coefficients at the ground, respectively.

Using eqs. (3.83), (3.84) in eqs. (3.81) and (3.82) we obtain

$$\underline{\underline{T}} = \underline{\underline{T}}_g e^{i(\kappa-\kappa_g)d} \tag{3.85}$$

where

$$\underline{\underline{T}}_g = T_{gh}\ \underline{h}^0\underline{h}^0 + T_{gv}\ \underline{v}^0_{g-}\ \underline{v}^0_- \tag{3.86}$$

In eq. (3.86) T_{gh} and T_{gv} are Fresnel transmission coefficients which are given by

$$T_{gh} = \frac{2\kappa}{\kappa+\kappa_g} \qquad\qquad T_{gv} = \frac{2\sqrt{\varepsilon_g}\kappa}{\varepsilon_g\kappa+\kappa_g} \tag{3.87}$$

These calculations have been carried out in Appendix C.

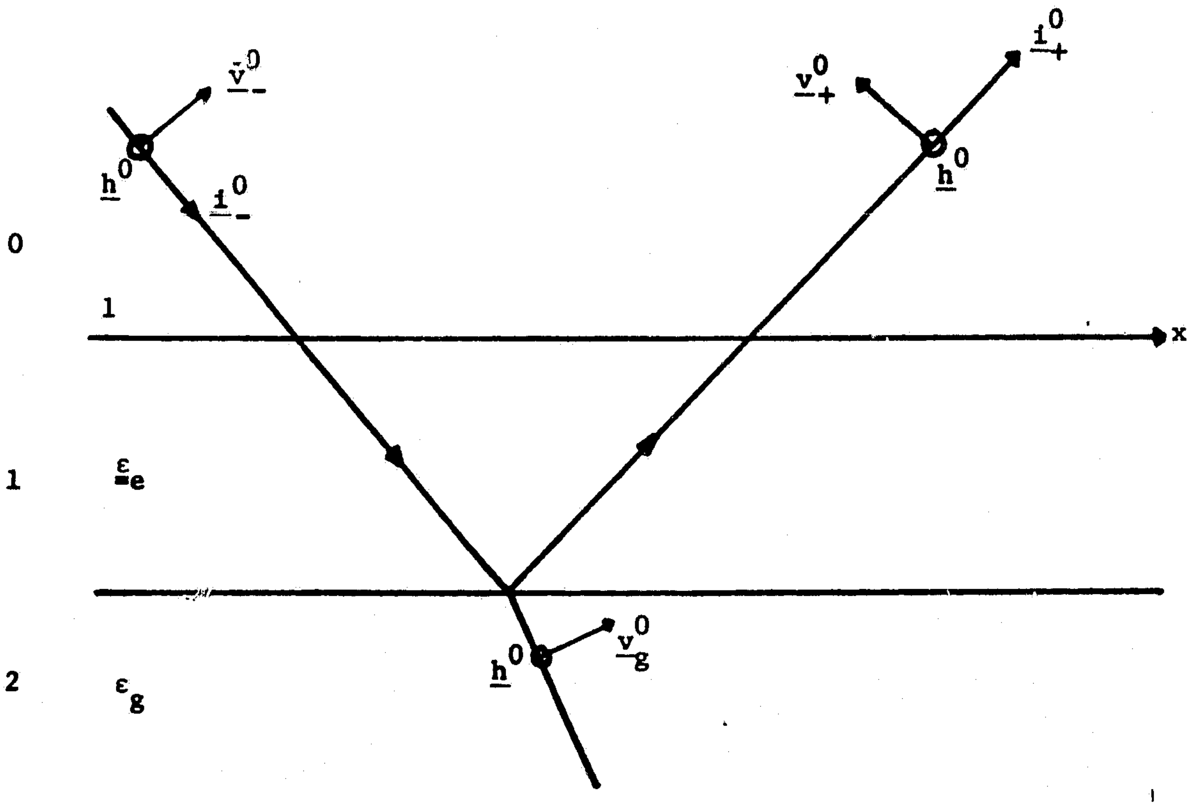


Fig. 3. Mean wave in equivalent medium

We obtain for dyadic reflection coefficient

$$\underline{\underline{\Gamma}} = \underline{\underline{\Gamma}}_g \, e^{2i\kappa d} \tag{3.88}$$

where

$$\underline{\underline{\Gamma}}_g = \Gamma_{gh}\, \underline{h}^0 \underline{h}^0 + \Gamma_{gv}\, \underline{v}^0_+ \underline{v}^0_- \tag{3.89}$$

In eqs. (3.89) Γ_{gh} and Γ_{gv} are Fresnel reflection coefficients which are given by

$$\Gamma_{gh} = \frac{\kappa - \kappa_g}{\kappa + \kappa_g} \quad , \qquad \Gamma_{gv} = \frac{\varepsilon_g \kappa - \kappa_g}{\varepsilon_g \kappa + \kappa_g} \tag{3.90}$$

We also obtain using eqs. (3.72), (3.77) and (3.78)

$$\underline{\underline{\Gamma}}_s = e^{i\underline{\underline{\kappa}}^+ d} \cdot \underline{\underline{\Gamma}}_g \cdot e^{i\underline{\underline{\kappa}}^- d} \tag{3.91}$$

where

$$\underline{\underline{\kappa}}^{\pm} = \kappa \underline{\underline{I}}^{\pm} + \frac{2\pi\rho}{\kappa}\, \overline{\underline{\underline{f}}}\, (\underline{i}^0_{\pm}, \underline{i}^0_{\pm}) \tag{3.92}$$

For the final solution, we write the mean field in three regions as

$$\langle \tilde{\Psi}(z,q) \rangle = \begin{bmatrix} \underline{q}^0_- \\ \eta(\underline{i}^0_- \times \underline{\underline{I}}^-)\underline{q}^0_- \end{bmatrix} e^{-i\kappa z} + \begin{bmatrix} \underline{\underline{\Gamma}}_s \cdot \underline{q}^0_- \\ \eta(\underline{i}^0_+ \times \underline{\underline{I}}^+)\underline{\underline{\Gamma}}_s \underline{q}^0_- \end{bmatrix} e^{i\kappa z} \quad , \quad z>0 \tag{3.93}$$

and

$$\langle \tilde{\Psi}(z,q) \rangle = \begin{bmatrix} \underline{\underline{I}}^- \\ \eta(\underline{i}^0_- \times \underline{\underline{I}}^-) \end{bmatrix} \cdot e^{-i\underline{\underline{\kappa}}^- z} \cdot \underline{q}^0_- + \begin{bmatrix} \underline{\underline{I}}^+ \\ \eta(\underline{i}^0_+ \times \underline{\underline{I}}^+) \end{bmatrix} \cdot e^{i\underline{\underline{\kappa}}^+ z} \cdot \underline{\underline{\Gamma}}_s \cdot \underline{q}^0_- \quad , \quad 0>z>-d \tag{3.94}$$

and

$$\langle \tilde{\Psi}(z,q) \rangle = \begin{bmatrix} \underline{\underline{T}}_g \\ \eta_g \underline{i}^0_g \times \underline{\underline{T}}_g \end{bmatrix} \cdot e^{i\underline{\underline{\kappa}}^- d} \cdot \underline{q}^0_-\, e^{-i\kappa_g (z+d)} \quad , \quad z<-d \tag{3.95}$$

where $\underline{i}^0_g$ is $\underline{i}^0_-(z)$ for $z<-d$ as given in eq. (3.37a).

The first term in eq. (3.93) is the incoming wave and the second term represents the reflected wave after travelling through the equivalent medium and having been reflected from underlying ground. The same is true inside the equivalent medium except that propagation constant now is $\underline{\underline{\kappa}}$ as defined by eq. (3.92). The original wave after decaying through the equivalent medium is partially transmitted into the ground where it travels with a propagation constant κ_g. These results, in the case of low frequency (Rayleigh regime), are identical to those obtained by Lang and Sidhu [1982].

Transverse Dyadic Green's Function in Three Media

To compute the backscattering coefficients we have to evaluate the transverse spectral density. To evaluate it we will need the transverse Green's function in the equivalent medium which gives the scattered field at $\underline{x}$ in response to a unit dipole located at $\underline{x}'$. The Green's function will be calculated in the equivalent medium and thus, is an averaged quantity. We have defined the Green's function problem at the end of chapter II. Now, if we take the transverse Fourier transform of eq. (2.57) and proceed in the same way we did in the mean wave case, we obtain

$$\tilde{L}\tilde{G}(\underline{k}_t,z,z') = I\delta(z-z') + \left\{\int ds \int dz''\rho(s)\tilde{\tilde{\chi}}(\underline{k}_t,z-s,z''-s)\tilde{G}(\underline{k}_t,z'',z')\right\} \mathrm{rect}_d(z) \tag{3.96}$$

$$+ \text{ radiation condition}$$

The definition of $\tilde{\tilde{\chi}}$, I and $\tilde{L}$ are given by eqs. (3.16), (2.58) and (3.14), respectively.

We write

$$\tilde{G}(\underline{k}_t,z,z') = \tilde{G}(z,z') = y(z)\hat{G}(z,z') \tag{3.97}$$

Substituting eq. (3.97) into eq. (3.96) and using the identical procedure as we did in the mean wave case, we find

$$\frac{d}{dz}\hat{G}(z,z') = y^{-1}(z')\delta(z-z') + \left\{\int ds \int dz''\rho(s)y^{-1}(z)\tilde{\tilde{\chi}}(z-s,z''-s)y(z'')\hat{G}(z'',z')\right\} \cdot \mathrm{rect}_d(z) \tag{3.98}$$

We would like to put eq. (3.98) into a more familiar form by multiplying from the right side by $\Gamma y(z')$, and using $y^{-1}\Gamma y = I_\phi$ we obtain

$$\frac{d}{dz}\hat{\hat{G}}(z,z') = I_\phi \delta(z-z') + \left\{\int ds \int dz'' \rho(s) y^{-1}(z) \tilde{\bar{\chi}}(z-s,z''-s) y(z'') \hat{\hat{G}}(z'',z')\right\} \cdot \text{rect}_d(z) \qquad (3.99)$$

where

$$\hat{\hat{G}}(z,z') = \hat{G}(z,z')\Gamma y(z') \qquad (3.100a)$$

From eq. (3.100a) we have

$$\hat{G}(z,z') = \hat{\hat{G}}(z,z')y^{-1}(z') \qquad (3.100b)$$

Using eq. (3.100b) in eq. (3.97) we obtain that the transverse Fourier transform of Green's function is in the form of similarity transformation, i.e.

$$\tilde{G}(z,z') = y(z)\hat{\hat{G}}(z,z')y^{-1}(z') \qquad (3.100c)$$

We will solve eq. (3.99) approximately by utilizing the two variable method. We expand the Green's function in a power series and assume that the unit source is inside the slab, i.e. $0>z'>-d$. We have

$$\hat{\hat{G}}(z,z') = \hat{\hat{G}}(z,\bar{z};z',\bar{z}',\delta) = \sum_{n=0}^{\infty} \hat{\hat{G}}^{(n)}(z,\bar{z},z',\bar{z}')\delta^n$$

where $\bar{z} = \delta z$ and $\bar{z}' = \delta z'$ with $\delta \ll 1$.

Proceeding in the same manner as in the mean wave, we obtain

$$\delta^{(0)}: \quad \frac{\partial}{\partial z}\hat{\hat{G}}^{(0)}(z,\bar{z};z',\bar{z}') = I_\phi \delta(z-z') \qquad (3.101a)$$

From eq. (3.101a) we have for $z \neq z'$

$$\hat{\hat{G}}^{(0)}(z,\bar{z};z',\bar{z}') = \hat{G}^{(0)}(\bar{z};z',\bar{z}')$$

$$\delta^{(1)}: \quad \frac{\partial}{\partial z}\hat{\hat{G}}^{(1)}(z,\bar{z};z',\bar{z}') + \frac{\partial}{\partial \bar{z}}\hat{G}^{(0)}(\bar{z};z',\bar{z}') = \bar{\rho}(\bar{z})\left\{\int ds \int dz'' y^{-1}\bar{\bar{\chi}} y\right\}\hat{\hat{G}}^{(0)}(\bar{z};z',\bar{z}')\,\text{rect}_d(z) \qquad (3.101b)$$

Following the two variable procedure which requires that the $\hat{G}^{(1)}$ term does not grow as fast as z, i.e.,

$$\lim_{z\to\infty} \frac{1}{z}\,\hat{G}^{(1)}(z,\bar{z};z',\bar{z}') = 0$$

then we obtain

$$\frac{d}{d\bar{z}}\,\hat{\hat{G}} = -\,\frac{2\pi i}{\kappa}\,\bar{\rho} f \hat{\hat{G}} \tag{3.102}$$

where f is given by eq. (3.64) and

$$\hat{\hat{G}} = \hat{G}^{(0)}(\bar{z},z',\bar{z}')$$

In eq. (3.102), $\hat{\hat{G}}$ represents slowly varying terms for three media, and in each medium is noted by

$$\hat{\hat{G}} = \begin{cases} \hat{\hat{G}}_f \quad , & z>0 \\ \hat{\hat{G}}_s^{+} \quad , & 0>z>z' \\ \hat{\hat{G}}_s^{-} \quad , & z'>z>-d \\ \hat{\hat{G}}_g \quad , & z<-d \end{cases} \tag{3.103a}$$

with

$$\hat{\hat{G}}_f = \begin{bmatrix} 0 & 0 \\ \underline{\underline{B}}_{f_1}(\bar{z}) & \underline{\underline{B}}_{f_2}(\bar{z}) \end{bmatrix}, \quad \hat{\hat{G}}_s^{\pm} = \begin{bmatrix} \underline{\underline{A}}_{s_1}^{\pm}(\bar{z}) & \underline{\underline{A}}_{s_2}^{\pm}(\bar{z}) \\ \underline{\underline{B}}_{s_1}^{\pm}(\bar{z}) & \underline{\underline{B}}_{s_2}^{\pm}(\bar{z}) \end{bmatrix}, \quad \hat{\hat{G}}_g = \begin{bmatrix} \underline{\underline{A}}_{g_1}(\bar{z}) & \underline{\underline{A}}_{g_2}(\bar{z}) \\ 0 & 0 \end{bmatrix} \tag{3.103b}$$

Now the boundary conditions to be used are that $\Gamma\tilde{G}(z,z',\delta)$, has to be continuous at z=0 and z=-d. In addition, Green's function must satisfy the jump condition at the source.

The boundary condition at z=0 gives us

$$\underline{\underline{A}}_{s_i}^{+}(0) = 0 \;, \qquad i=1,2 \tag{3.104}$$

and

$$\underline{\underline{B}}^{+}_{s_i}(0) = \underline{\underline{B}}_{f_i} \quad , \qquad i=1,2 \tag{3.105}$$

The boundary condition at $z=-d$, $\bar{z}=-\bar{d}$ gives us

$$\underline{\underline{A}}^{-}_{s_i}(-\bar{d}) = \frac{-1}{2N_T}[y^{-}\Gamma y^{-}_{g}]^{T} \cdot \underline{\underline{A}}_{g_i} e^{i(\kappa_g-\kappa)d} \qquad i=1,2 \tag{3.106}$$

$$\underline{\underline{B}}^{-}_{s_i}(-\bar{d}) = \frac{1}{2N_T}[y^{+}\Gamma y^{-}_{g}]^{T} \cdot \underline{\underline{A}}_{g_i} e^{i(\kappa+\kappa_g)d} \qquad i=1,2 \tag{3.107}$$

Equations (3.106) and (3.107) are the same as the ones we obtained in the mean wave case. Proceeding in the same way

$$\underline{\underline{A}}_{g_i} = \underline{\underline{T}}_i \cdot \underline{\underline{A}}^{-}_{s_i}(-\bar{d}) \qquad i=1,2 \tag{3.108}$$

and

$$\underline{\underline{B}}^{-}_{s_i}(-\bar{d}) = \underline{\underline{\Gamma}}_i \cdot \underline{\underline{A}}^{-}_{s_i}(-\bar{d}) \qquad i=1,2 \tag{3.109}$$

Therefore following Appendix C, we obtain

$$\underline{\underline{T}}_1 = \underline{\underline{T}}_2 = \underline{\underline{T}} = \underline{\underline{T}}_g \, e^{i(\kappa-\kappa_g)d} \tag{3.110}$$

$$\underline{\underline{\Gamma}}_1 = \underline{\underline{\Gamma}}_2 = \underline{\underline{\Gamma}} = \underline{\underline{T}}_g \, e^{2i\kappa d} \tag{3.111}$$

where $\underline{\underline{T}}_g$ and $\underline{\underline{\Gamma}}_g$ are given in eqs. (3.86) and (3.89) respectively .

Integrating eq. (3.101a) from $z'-\varepsilon$ to $z'+\varepsilon$ and letting $\varepsilon\to 0$ yields the jump condition across the delta function. It is:

$$\hat{G}^{+}_{s}(\bar{z}') - \hat{G}^{-}_{s}(\bar{z}') = I_{\phi} \tag{3.112}$$

From eq. (3.112) we have

$$\underline{\underline{A}}^{+}_{s_i}(\bar{z}') - \underline{\underline{A}}^{-}_{s_i}(\bar{z}') = \underline{\underline{J}}_{A_i} \qquad i=1,2 \tag{3.113}$$

and

$$\underline{\underline{B}}^{+}_{s_i}(\bar{z}') - \underline{\underline{B}}^{-}_{s_i}(\bar{z}') = \underline{\underline{J}}_{B_i} \qquad i=1,2 \tag{3.114}$$

where

$$\underline{\underline{J}}_{A_1} = \underline{\underline{I}}^{-} \quad ; \quad \underline{\underline{J}}_{A_2} = 0 \tag{3.115}$$

$$\underline{\underline{J}}_{B_1} = 0 \quad ; \quad \underline{\underline{J}}_{B_2} = \underline{\underline{I}}^{+} \tag{3.116}$$

We obtained from eqs. (3.102) and (3.103)

$$\underline{\underline{A}}^{+}_{s_i}(\bar{z}) = e^{-i\frac{2\pi\bar{\rho}}{\kappa}\bar{\underline{\underline{f}}}^{-}\bar{z}} \cdot \underline{\underline{A}}^{+}_{0_i} \quad ; \quad \underline{\underline{B}}^{+}_{s_i}(\bar{z}) = e^{i\frac{2\pi\bar{\rho}}{\kappa}\bar{\underline{\underline{f}}}^{+}\bar{z}} \cdot \underline{\underline{B}}^{+}_{0_i} \quad , \quad i=1,2 \quad z'<z<0$$

$$\underline{\underline{A}}^{-}_{s_i}(\bar{z}) = e^{-i\frac{2\pi\bar{\rho}}{\kappa}\bar{\underline{\underline{f}}}^{-}\bar{z}} \cdot \underline{\underline{A}}^{-}_{0_i} \quad ; \quad \underline{\underline{B}}^{-}_{s_i}(\bar{z}) = e^{i\frac{2\pi\bar{\rho}}{\kappa}\bar{\underline{\underline{f}}}^{+}\bar{z}} \cdot \underline{\underline{B}}^{-}_{0_i} \quad , \quad i=1,2 \quad -d<z<z' \tag{3.117}$$

where $\bar{\underline{\underline{f}}}^{\pm} = \bar{\underline{\underline{f}}}(\underline{i}^{0}_{\pm}, \underline{i}^{0}_{\pm})$.

We assume $\underline{\underline{A}}_{0_i}$ and $\underline{\underline{B}}^{+}_{0_i}$ are known, then the rest of the terms will be expressed in terms of $\underline{\underline{A}}^{-}_{0_i}$ and $\underline{\underline{B}}^{+}_{0_i}$. After that, using jump condition $\underline{\underline{A}}^{-}_{0_i}$ and $\underline{\underline{B}}^{+}_{0_i}$ will be determined. We have:

$$\underline{\underline{A}}^{-}_{s_i}(\bar{z}) = e^{-i\frac{2\pi\bar{\rho}}{\kappa}\bar{\underline{\underline{f}}}^{-}\bar{z}} \cdot \underline{\underline{A}}^{-}_{0_i} \quad , \quad \underline{\underline{B}}^{-}_{s_i}(\bar{z}) = e^{i\frac{2\pi\bar{\rho}}{\kappa}\bar{\underline{\underline{f}}}^{+}\bar{z}} \cdot \underline{\underline{\Gamma}}_{s} \cdot \underline{\underline{A}}^{-}_{0_i} \quad , \quad i=1,2 \tag{3.118}$$

$$\underline{\underline{A}}_{g_i} = \underline{\underline{T}}_i \cdot e^{i\frac{2\pi\bar{\rho}}{\kappa}\bar{\underline{\underline{f}}}^{-}\bar{d}} \cdot \underline{\underline{A}}^{-}_{0_i} \quad , \quad \underline{\underline{A}}^{-}_{0_i} = -e^{i\frac{2\pi\bar{\rho}}{\kappa}\bar{\underline{\underline{f}}}^{-}\bar{z}'} \cdot \underline{\underline{J}}_{A_i} \quad , \quad i=1,2 \tag{3.119}$$

and

$$\underline{\underline{B}}^{+}_{0_i} = e^{-i\frac{2\pi\bar{\rho}}{\kappa}\bar{\underline{\underline{f}}}^{+}\bar{z}'} \cdot \underline{\underline{J}}_{B_i} + \underline{\underline{\Gamma}}_{s} \cdot \underline{\underline{A}}^{-}_{0_i} \quad , \quad i=1,2 \tag{3.120}$$

Where $\underline{\underline{J}}_{A_i}$, $\underline{\underline{J}}_{B_i}$ and $\underline{\underline{T}}_i$ are defined by eqs. (3.115), (3.116) and (3.110), respectively.

We also obtain for $\underline{\underline{\Gamma}}_s$ the following expression

$$\underline{\underline{\Gamma}}_s = e^{i\underline{\underline{\kappa}}^{+}d}\,\underline{\underline{\Gamma}}_g\,e^{i\underline{\underline{\kappa}}^{-}d} \quad , \qquad i=1,2 \tag{3.121}$$

where $\underline{\underline{\kappa}}^{\pm}$ and $\underline{\underline{\Gamma}}_g$ are given by eqs. (3.92) and (3.89).

Substituting $\underline{\underline{A}}$'s and $\underline{\underline{B}}$'s into the eqs. (3.103) and using eq. (3.100c) we can write the Green's functions in three media. It will only be needed in the region 0>z>z´.

$$\tilde{G}_s(\underline{k}_t,z,z') = y(z)\hat{\hat{G}}_s(\bar{z},\bar{z}')y^{-1}(z') \tag{3.122}$$

where the slowly varying term $\hat{G}$ is given by

$$\hat{\hat{G}}_s^{+}(\bar{z},\bar{z}') = \begin{bmatrix} 0 & 0 \\ e^{i\frac{2\pi\bar{\rho}}{\kappa}\underline{\underline{f}}^{+}\bar{z}}\cdot\underline{\underline{B}}^{+}_{0_1} & e^{i\frac{2\pi\bar{\rho}}{\kappa}\underline{\underline{f}}^{+}\bar{z}}\cdot\underline{\underline{B}}^{+}_{0_2} \end{bmatrix} \tag{3.123}$$

and $\underline{\underline{B}}^{+}_{0_1}$ and $\underline{\underline{B}}^{+}_{0_2}$ are defined by eq. (3.120).

Substituting y(z), $y^{-1}(z')$ and $\hat{G}_s$ into eq. (3.123), we obtain

$$\tilde{G}_s(\underline{k}_t,z,z') = \begin{bmatrix} \tilde{\underline{\underline{G}}}_{11} & \tilde{\underline{\underline{G}}}_{12} \\ \tilde{\underline{\underline{G}}}_{21} & \tilde{\underline{\underline{G}}}_{22} \end{bmatrix} \tag{3.124}$$

where

$$\tilde{\underline{\underline{G}}}_{11} = \frac{\omega\mu_0}{2\kappa}\left\{ e^{i\underline{\underline{\kappa}}^{+}(z-z')} + e^{i\underline{\underline{\kappa}}^{+}z}\cdot\underline{\underline{\Gamma}}_s\cdot e^{i\underline{\underline{\kappa}}^{-}z'} \right\} \tag{3.125}$$

$$\tilde{\underline{\underline{G}}}_{12} = \frac{k_0}{2\kappa}\left\{ e^{i\underline{\underline{\kappa}}^{+}(z-z')}\cdot(\underline{i}_{+}^{0}\times\underline{\underline{I}}^{+}) + e^{i\underline{\underline{\kappa}}^{+}z}\cdot\underline{\underline{\Gamma}}_s\cdot e^{i\underline{\underline{\kappa}}^{-}z'}\cdot(\underline{i}_{-}^{0}\times\underline{\underline{I}}^{-}) \right\} \tag{3.126}$$

$$\tilde{\underline{\underline{G}}}_{21}=\frac{k_0}{2\kappa}\left\{(\underline{i}_+^0 x \underline{\underline{I}}^+)e^{i\underline{\underline{\kappa}}^+(z-z')}+(\underline{i}_+^0 x \underline{\underline{I}}^+)e^{i\underline{\underline{\kappa}}^+ z}\cdot\underline{\underline{\Gamma}}_s\cdot e^{i\underline{\underline{\kappa}}^- z'}\right\} \tag{3.127}$$

$$\tilde{\underline{\underline{G}}}_{22}=\frac{\omega\varepsilon_0}{2\kappa}\left\{(\underline{i}_+^0 x \underline{\underline{I}}^+)e^{i\underline{\underline{\kappa}}^+(z-z')}\cdot(\underline{i}_+^0 x \underline{\underline{I}}^+)+(\underline{i}_+^0 x \underline{\underline{I}}^+)e^{i\underline{\underline{\kappa}}^+ z}\cdot\underline{\underline{\Gamma}}_s\cdot e^{i\underline{\underline{\kappa}}^- z}\cdot(\underline{i}_-^0 x \underline{\underline{I}}^-)\right\} \tag{3.128}$$

$$, \qquad 0>z>z'>-d$$

with

$$G_s(\underline{x},\underline{x}') = \int\tilde{G}_s(\underline{k}_t,z,z')e^{i\underline{k}_t\cdot(\underline{x}_t-\underline{x}_t')}d\underline{k}_t$$

CHAPTER IV

ELECTROMAGNETIC BACKSCATTERING COEFFICIENTS FROM A LAYER OF VEGETATION

In this chapter we will illustrate the application of the methods developed in the previous chapters by calculating the backscattering coefficients from a vegetated slab of scatterers that are comparable in size to a wavelength. The physical configuration is shown in fig. 2.

First, we will evaluate the transverse Fourier transform of the scattered field. The knowledge of the transverse mean field due to an incident plane wave and the transverse Green's function in the equivalent medium, will be used in conjunction with the spectral density to obtain the general form of backscattering coefficients.

Then we will obtain explicit expressions of the electromagnetic backscattering coefficients in terms of the bistatic scattering cross section of an individual scatterer. After determining the backscattering cross sections, their behavior will be studied by associating each term in the total backscattering cross section with a physically understandable scattering process.

The Transverse Fourier Transform of the Scattered Field

In chapter II we found, by employing the distorted Born approximation, that the correlation of the fluctuating component of the scattered field is:

$$\langle \Psi_f(\underline{x})\Psi_f^\dagger(\hat{\underline{x}})\rangle_{\underline{\omega}} = \int_V d\underline{s}\rho(\underline{s})\Psi_{s_e}(\underline{x},\underline{s})\Psi_{s_e}^\dagger(\hat{\underline{x}},\underline{s}) \qquad (4.1)$$

where V is the volume of the slab of vegetation and $\Psi_{s_e}(\underline{x},\underline{s})$ is the scattered field at $\underline{x}$ in the effective medium. It is given by

$$\Psi_{s_e}(\underline{x},\underline{s}) = \int d\underline{x}' G(\underline{x},\underline{x}') \int d\underline{x}'' t(\underline{x}'-\underline{s},\underline{x}''-\underline{s}) \langle\Psi(\underline{x}'')\rangle \tag{4.2}$$

From chapter II, we have

$$t(\underline{x},\underline{x}') = \begin{bmatrix} \underline{\underline{t}}_{11} & 0 \\ 0 & 0 \end{bmatrix} \tag{4.3}$$

with $\underline{\underline{t}}_{11} = \frac{-1}{i\omega\mu_0}\, \underline{\underline{t}}(\underline{x},\underline{x}')$

and $$\underline{\underline{t}}(\underline{x},\underline{x}') = \frac{1}{(2\pi)^3} \int d\underline{k}\, d\underline{k}'\, \tilde{\underline{\underline{t}}}(\underline{k},\underline{k}') e^{i(\underline{k}\cdot\underline{x} - \underline{k}'\cdot\underline{x}')} \tag{4.4}$$

We have written in chapter III for the mean wave

$$\langle\Psi(\underline{x}'')\rangle = \langle\tilde{\Psi}(\underline{k}_{t_0}, z'') \, e^{i\underline{k}_{t_0}\cdot\underline{x}_t''} \quad , \quad \underline{k}_{t_0} = k_0 \sin\theta_0 \underline{x}^0 \tag{4.5}$$

where

$$\langle\tilde{\Psi}(\underline{k}_{t_0}, z'')\rangle = y(z'')\phi_s(\overline{z}'') \tag{4.6}$$

The Green's function used in eq. (4.2) can be written as

$$G(\underline{x},\underline{x}') = G(\underline{x}_t - \underline{x}_t', z, z') \tag{4.7}$$

The Green's function can be represented by its transverse Fourier transform:

$$G(\underline{x}_t - \underline{x}_t'; z, z') = \frac{1}{(2\pi)^2} \int d\underline{k}_t\, \tilde{G}(\underline{k}_t, z, z') e^{i\underline{k}_t\cdot(\underline{x}_t - \underline{x}_t')} \tag{4.8}$$

In order to obtain the transverse Fourier transform of eq. (4.2), we substitute eqs. (4.5) and (4.8) into eq. (4.2), and also use

$$t(\underline{x}',\underline{x}'') = \frac{1}{(2\pi)^2} \int d\underline{k}_t' d\underline{k}_t'' \tilde{t}(\underline{k}_t', z'; \underline{k}_t'', z'') e^{i(\underline{k}_t'\underline{x}_t' - \underline{k}_t''\underline{x}_t'')} \tag{4.9}$$

In eq. (4.9) $\tilde{t}(\underline{k}_t', z'; \underline{k}_t'', z'')$ is different from $\tilde{t}(\underline{k}', \underline{k}'')$. We have used the same symbol for convenience of notation.

Now we have

$$\tilde{\Psi}_{s_e}(\underline{k}_t,z,\underline{s}) = (2\pi)^2\int dz' dz'' \tilde{G}(\underline{k}_t,z,z')\tilde{t}(\underline{k}_t,z'-s;\underline{k}_{t_0},z''-s)\langle\tilde{\Psi}(\underline{k}_{t_0},z'')\rangle \cdot e^{-i(\underline{k}_t-\underline{k}_{t_0})\cdot\underline{s}_t} \tag{4.10}$$

From eq. (3.122), we have

$$\tilde{G}(\underline{k}_t,z,z') = y(\underline{k}_t,z)\hat{\hat{G}}(\underline{k}_t,\overline{z},\overline{z}')y^{-1}(\underline{k}_t,z') \tag{4.11}$$

where we have explicitly shown the dependence on $\underline{k}_t$ in y, y^{-1} and $\hat{\hat{G}}$. We now use eqs.(4.6) and (4.11) in eq.(4.10). The Green's function $\hat{\hat{G}}(\underline{k}_t,\overline{z},\overline{z}')$ and the mean field $\phi_s(\underline{k}_{t_0},\overline{z}'')$ are replaced by $\hat{\hat{G}}(\underline{k}_t,\overline{z},\overline{s})$ and $\phi_s(\underline{k}_{t_0},\overline{s})$. This is a good approximation to first order in δ due to the behavior of $\tilde{t}$ about $z' = s$ and $z'' = s$. The result is:

$$\tilde{\Psi}_{s_e}(\underline{k}_t,z,\underline{s}) = \hat{\hat{\Psi}}_{s_e}(\underline{k}_t,z,s)e^{-i(\underline{k}_t-\underline{k}_{t_0})\cdot\underline{s}_t} \tag{4.12a}$$

where

$$\hat{\hat{\Psi}}_{s_e}(\underline{k}_t,z,s) = (2\pi)^2 y(\underline{k}_t,z)\hat{\hat{G}}(\underline{k}_t,\overline{z},\overline{s})p(\underline{k}_t,\underline{k}_{t_0},s)\phi_s(\underline{k}_{t_0},\overline{s}) \tag{4.12b}$$

with $\overline{s} = \delta s$ and

$$p(\underline{k}_t,\underline{k}_{t_0},s) = \int dz' dz'' y^{-1}(\underline{k}_t,z')\tilde{t}(\underline{k}_t,z'-s;\underline{k}_{t_0},z''-s)y(\underline{k}_{t_0},z'') \tag{4.13}$$

The Formulation of the General Form of the Backscattering Coefficients

The backscattering coefficients are directly related to the transverse Fourier transform of eq. (4.1) with respect to $\underline{x}_t$ and $\hat{\underline{x}}_t$ evaluated at the upper interface (z=0). We start by taking the transverse Fourier transform with respect to $\underline{x}_t$ and $\hat{\underline{x}}_t$ of eq. (4.1). We have

$$<\tilde{\Psi}_f(\underline{k}_t,z)\tilde{\Psi}_f^\dagger(\hat{\underline{k}}_t,\hat{z})> = \rho\int_V d\underline{s}\left\{\tilde{\Psi}_{s_e}(\underline{k}_t,z,\underline{s})\tilde{\Psi}^\dagger_{s_e}(\hat{\underline{k}}_t,\hat{z},\underline{s})\right\} \qquad (4.14)$$

where ρ is the constant particle density.

Now, by putting eq. (4.12) into eq. (4.14) and by integrating over $\underline{s}_t$ and by setting $z=\hat{z}=0$, we have

$$<\tilde{\Psi}_f(\underline{k}_t,0)\tilde{\Psi}^\dagger(\hat{\underline{k}}_t,0)> = S(\underline{k}_t,q|\underline{\omega})\delta(\underline{k}_t-\hat{\underline{k}}_t) \quad , \qquad q \in \{h,v\} \qquad (4.15)$$

where

$$S(\underline{k}_t,q|\omega) = \rho\int_{-d}^{0} ds\hat{\tilde{\Psi}}_{s_e}(\underline{k}_t,0,s)\hat{\tilde{\Psi}}^\dagger_{s_e}(\underline{k}_t,0,s) \qquad (4.16)$$

Here $S(\underline{k}_t,q|\omega)$ is the transverse spectral density of $\Psi_f(\underline{x})$ at the interface. The symbol q represents the polarization of the incident wave and $\underline{\omega}$ the orientation of a typical scatterer. The spectral density S is a 2x2 matrix of dyadics. We write

$$S(\underline{k}_t,q|\omega) = \begin{bmatrix} \underline{\underline{S}}_{11} & \underline{\underline{S}}_{12} \\ \underline{\underline{S}}_{21} & \underline{\underline{S}}_{22} \end{bmatrix} \qquad (4.17)$$

where $\underline{\underline{S}}_{11}$ and $\underline{\underline{S}}_{22}$ are the dyadic spectral densities of electric and magnetic fields respectively. In our development, we will concern ourselves only with $\underline{\underline{S}}_{11}$.

It is shown by Lang [1981] that $\underline{\underline{S}}_{11}$ (Lang has used $\underline{\underline{S}}$ for $\underline{\underline{S}}_{11}$) is directly related to the backscattering coefficients σ^0_{pq} by

$$\sigma^0_{pq} = \frac{k_0^2\cos^2\theta_0}{4\pi^3}\underline{p}^0\cdot\overline{\underline{\underline{S}}_{11}(-\underline{k}_{t_0},q|\omega)}\cdot\underline{p}^0 \quad , \qquad p,q \in \{h,v\} \qquad (4.18)$$

where θ_0 is the angle of incidence of the plane wave with respect to the slab normal, and $\underline{p}^0,\underline{q}^0\epsilon\{\underline{h}^0,\underline{v}^0\}$ are unit vectors indicating the polarization of the backscattered wave.

To simplify the remaining computation for $\underline{\underline{S}}_{11}$, we note from eq. (4.18) that $S(\underline{k}_t,q|\omega)$ will only be required for $\underline{k}_t = -\underline{k}_{t_0} = -k_0\sin\theta_0\underline{x}^0$. The transverse transition operator $\tilde{t}$ in eq. (4.13) can be expressed

$$\tilde{t}(-\underline{k}_{t_0},z'-s;\underline{k}_{t_0},z''-s) = \frac{1}{2\pi}\int \tilde{t}(-\underline{k}_{t_0},\kappa',\underline{k}_{t_0},\kappa'')e^{i\kappa'(z'-s)-i\kappa''(z''-s)}d\kappa'd\kappa'' \tag{4.19}$$

Using eq. (4.19), y^{-1} and y from chapter III, we compute eq. (4.13); then we substitute this value of p, y, y^{-1}, $\hat{\hat{G}}$ and ϕ_s from chapter III in eq. (4.16). We find that the spectral density is:

$$\underline{\underline{S}}_{11}(-\underline{k}_{t_0},q|\omega) = \frac{(2\pi)^4}{\kappa^2}\rho\int_{-d}^{0}\sum_{i,j=1}^{4}(\underline{W}_i)(\underline{W}_j^*)ds \tag{4.20}$$

where

$$\underline{W}_1 = e^{-i\underline{\underline{\kappa}}_0^- s}\cdot\underline{\underline{f}}_I\cdot e^{-i\underline{\underline{\kappa}}_0^- s}\cdot\underline{q}_-^0 \qquad \underline{W}_2 = \hat{\underline{\underline{\Gamma}}}_{s_0}\cdot e^{i\underline{\underline{\kappa}}_0^+ s}\cdot\underline{\underline{f}}_{II}\cdot e^{i\underline{\underline{\kappa}}_0^+ s}\cdot\underline{\underline{\Gamma}}_{s_0}\cdot\underline{q}_-^0$$

$$\underline{W}_3 = \hat{\underline{\underline{\Gamma}}}_{s_0}\cdot e^{i\underline{\underline{\kappa}}_0^+ s}\cdot\underline{\underline{f}}_{III}\cdot e^{-i\underline{\underline{\kappa}}_0^- s}\cdot\underline{q}_-^0 \qquad \underline{W}_4 = e^{-i\underline{\underline{\kappa}}_0^- s}\cdot\underline{\underline{f}}_{IV}\cdot e^{i\underline{\underline{\kappa}}_0^+ s}\cdot\underline{\underline{\Gamma}}_{s_0}\cdot\underline{q}_-^0 \tag{4.21}$$

with

$$\underline{\underline{f}}_I = \underline{\underline{f}}(-\underline{k}_{t_0},\kappa_0;\underline{k}_{t_0},-\kappa_0) \qquad \underline{\underline{f}}_{II} = \underline{\underline{f}}(-\underline{k}_{t_0},-\kappa_0;\underline{k}_{t_0},\kappa_0) \tag{4.22}$$

$$\underline{\underline{f}}_{III} = \underline{\underline{f}}(-\underline{k}_{t_0},-\kappa_0;\underline{k}_{t_0},-\kappa_0) \qquad \underline{\underline{f}}_{IV} = \underline{\underline{f}}(-\underline{k}_{t_0},\kappa_0;\underline{k}_{t_0},\kappa_0) \tag{4.23}$$

In obtaining eqs.(4.20) through (4.23), we have used the relationsihp between $\underline{\underline{f}}$ and $\underline{\underline{t}}$ given in eq.(2.26). In addition, $\underline{\underline{\kappa}}_0^{\pm}$ and $\underline{\underline{\Gamma}}_{s_0}$ are $\underline{\underline{\kappa}}^{\pm}$ (as given in eq. (3.92)) and $\underline{\underline{\Gamma}}_s$ (as given in eq.(3.91)) respectively with $\underline{k}_t$ replaced by $\underline{k}_{t_0}$. Finally $\hat{\underline{\underline{\Gamma}}}_{s_0}(\underline{k}_{t_0}) = \underline{\underline{\Gamma}}_{s_0}(-\underline{k}_{t_0}) = e^{i\underline{\underline{\kappa}}_0^- d}\cdot\hat{\underline{\underline{\Gamma}}}_s\cdot e^{i\underline{\underline{\kappa}}^+ d}$ and $\hat{\underline{\underline{\Gamma}}}_s = \Gamma_{sh}\underline{h}^0\underline{h}^0 + \Gamma_{sv}\underline{v}_-^0\underline{v}_+^0$.

Substituting eq. (4.20) into (4.18) we have the general form of back-scattering cross section as

$$\sigma^0_{pq} = 4\pi\rho \overline{\int_{-d}^{0} \underline{p}^0_- \cdot \left\{ \sum_{i,j=1}^{4} (\underline{W}_i)(\underline{W}^*_j) \right\} \cdot \underline{p}^0_- \, ds} \qquad (4.24)$$

Now substituting eq. (4.21) into eq. (4.24), we see that there will be sixteen exponential integral terms to evaluate. We examine these terms and find that phase cancellation gives $\delta^{(0)}$ order terms and phase accumulation gives $\delta^{(1)}$ order terms.

Omitting the $\delta^{(1)}$ order terms, which are much smaller than $\delta^{(0)}$, eq. (4.24) can be written as

$$\sigma^0_{pq} = \sigma^0_{pqd} + \sigma^0_{pqr} + \sigma^0_{pqdr} \qquad (4.25)$$

where

$$\sigma^0_{pqd} = 4\pi\rho \left\{ \overline{\int_{-d}^{0} [\underline{p}^0_- \cdot (e^{-i\underline{\underline{\kappa}}^-_0 s} \underline{\underline{f}}_I e^{-i\underline{\underline{\kappa}}^-_0 s} \underline{q}^0_-) \underline{p}^0_- \cdot (e^{i\underline{\underline{\kappa}}^{-*}_0 s} \underline{\underline{f}}^*_I e^{i\underline{\underline{\kappa}}^{-*}_0 s} \underline{q}^0_-)] ds} \right\} \qquad (4.26)$$

$$\sigma^0_{pqr} = 4\pi\rho \left\{ \overline{\int_{-d}^{0} [\underline{p}^0_- \cdot (\hat{\underline{\underline{\Gamma}}}_{s_0} \cdot e^{i\underline{\underline{\kappa}}^+_0 s} \underline{\underline{f}}_{II} e^{i\underline{\underline{\kappa}}^+_0 s} \underline{\underline{\Gamma}}_{s_0} \underline{q}^0_-) \, \underline{p}^0_- \cdot (\hat{\underline{\underline{\Gamma}}}^*_{s_0} \cdot e^{-i\underline{\underline{\kappa}}^{+*}_0 s} \underline{\underline{f}}^*_{II} e^{-i\underline{\underline{\kappa}}^{+*}_0 s} \underline{\underline{\Gamma}}^*_{s_0} \underline{q}^0_-)] ds} \right\} \qquad (4.27)$$

and

$$\sigma^0_{pqdr} = 4\pi\rho \left\{ \int_{-d}^{0} \left\{ \overline{[\underline{p}^0_- \cdot (e^{-i\underline{\underline{\kappa}}^-_0 s} \underline{\underline{f}}_{IV} e^{i\underline{\underline{\kappa}}^+_0 s} \underline{\underline{\Gamma}}_{s_0} \underline{q}^0_-) \underline{p}^0_- \cdot (e^{i\underline{\underline{\kappa}}^{-*}_0 s} \underline{\underline{f}}^*_{IV} e^{-i\underline{\underline{\kappa}}^{+*}_0 s} \underline{\underline{\Gamma}}^*_{s_0} \underline{q}^0_-)]} \right. \right.$$

$$+ \overline{[\underline{p}^0_- \cdot (\hat{\underline{\underline{\Gamma}}}_{s_0} \cdot e^{i\underline{\underline{\kappa}}^+_0 s} \underline{\underline{f}}_{III} e^{-i\underline{\underline{\kappa}}^-_0 s} \underline{q}^0_-) \, \underline{p}^0_- \cdot (\hat{\underline{\underline{\Gamma}}}^*_{s_0} \cdot e^{-i\underline{\underline{\kappa}}^{+*}_0 s} \underline{\underline{f}}^*_{III} e^{i\underline{\underline{\kappa}}^{-*}_0 s} \underline{q}^0_-)]}$$

$$+ \overline{[\underline{p}_-^0 \cdot (e^{-i\kappa_0^- s} \underline{\underline{f}}_{IV} \cdot e^{i\kappa_0^+ s} \underline{\underline{\Gamma}}_{s_0} \cdot \underline{q}_-^0) \; \underline{p}_-^0 \cdot (\hat{\underline{\underline{\Gamma}}}^*_{s_0} \cdot e^{-i\kappa_0^{+*} s} \underline{\underline{f}}^*_{III} \cdot e^{i\kappa_0^{-*} s} \underline{q}_-^0)]}$$

$$+ \overline{[\underline{p}_-^0 \cdot (\hat{\underline{\underline{\Gamma}}}_{s_0} \cdot e^{i\kappa_0^+ s} \underline{\underline{f}}_{III} \cdot e^{-i\kappa_0^- s} \underline{q}_-^0) \underline{p}_-^0 \cdot (e^{i\kappa_0^{-*} s} \underline{\underline{f}}^*_{IV} \cdot e^{-i\kappa_0^{+*} s} \underline{\underline{\Gamma}}^*_{s_0} \cdot \underline{q}_-^0)]} \Big\} ds \Big\} \tag{4.28}$$

$$p\ ,q\ \varepsilon\{h\ ,v\ \}$$

Here σ^0_{pqd}, σ^0_{pqr} and σ^0_{pqdr} represent the direct, reflected, and direct-reflected backscattering contributions as shown in figs. (4a), (4b) and (4c-4d), respectively.

The Calculation of Backscattering Coefficients

In this section, we simplify the expressions for the backscattering coefficients by assuming the scatterers have no average depolarization at the level of the mean, i.e.,

$$\overline{f_{hv_\pm}} = \overline{f_{v_\pm h}} = 0 \tag{4.29}$$

It was shown by Lang, 1981 that dipole discs having a uniform distribution in the ϕ coordinate (azimuth symmetry) obey eq. (4.29). It is conjectured that eq. (4.29) holds for all scatterers having azimuthal symmetry. It should be noted the eq. (4.29) also implies that the average or bulk medium is uniaxial.

Using eq. (4.29), the scattering amplitude can be written as

$$\overline{\underline{\underline{f}}}^\pm = \overline{f}^\pm_{hh}\, \underline{h}^0 \underline{h}^0 + \overline{f}^\pm_{vv}\, \underline{v}^0_\pm \underline{v}^0_\pm \tag{4.30}$$

where

$$f^\pm_{pp} = \underline{p}^0_\pm \cdot \underline{\underline{f}}(\underline{i}^0_\pm, \underline{i}^0_\pm) \cdot \underline{p}^0_\pm \quad , \qquad p\ \varepsilon\{h,v\} \tag{4.31}$$

(a) $\underline{\underline{f}}_{I} = \underline{\underline{f}}(-\underline{k}_{t_0}, \kappa_0; \underline{k}_{t_0}, -\kappa_0)$

$= \underline{\underline{f}}(-\underline{i}^{0}_{-}, \underline{i}^{0}_{-})$

(b) $\underline{\underline{f}}_{II} = \underline{\underline{f}}(-\underline{k}_{t_0}, -\kappa_0; \underline{k}_{t_0}, \kappa_0)$

$= \underline{\underline{f}}(-\underline{i}^{0}_{+}, \underline{i}^{0}_{+})$

(c) $\underline{\underline{f}}_{III} = \underline{\underline{f}}(-\underline{k}_{t_0}, -\kappa_0; \underline{k}_{t_0}, \kappa_0)$

$= \underline{\underline{f}}(-\underline{i}^{0}_{+}, \underline{i}^{0}_{-})$

(d) $\underline{\underline{f}}_{IV} = \underline{\underline{f}}(-\underline{k}_{t_0}, \kappa_0; \underline{k}_{t_0}, \kappa_0)$

$= \underline{\underline{f}}(-\underline{i}^{0}_{-}, \underline{i}^{0}_{+})$

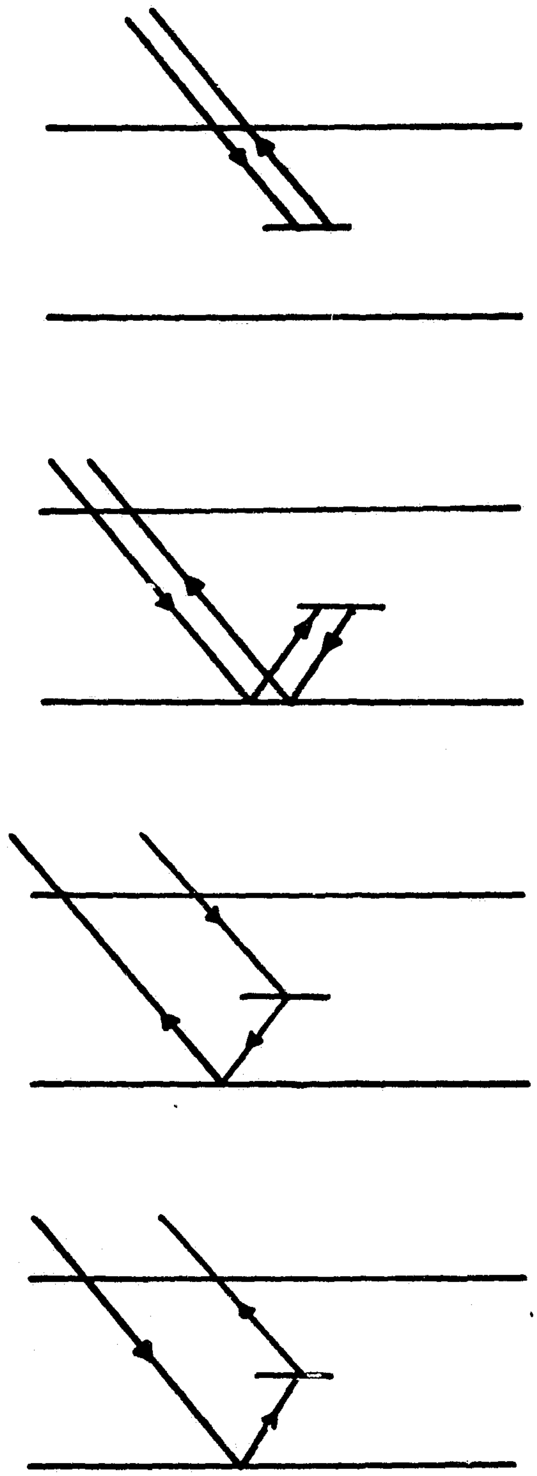


Fig. 4. Fundamental backscattering contributions of scattering amplitudes

Next a simplified expression for $\underline{\underline{\kappa}}_0^{\pm}$ can be obtained by employing eq. (4.30) in eq. (3.92). It is

$$\underline{\underline{\kappa}}_0^{\pm} = \kappa_h^{\pm}\, \underline{h}^0\underline{h}^0 + \kappa_v^{\pm}\, \underline{v}_{\pm}^0\underline{v}_{\pm}^0 \tag{4.32}$$

where

$$\kappa_p^{\pm} = \kappa_0 + \frac{2\pi\rho}{\kappa_0}\,\overline{f}_{pp}^{\pm} \quad , \qquad \kappa_0 = k_0\sin\theta_0 \tag{4.33}$$

Since $\underline{\underline{\kappa}}_0^{\pm}$ has no off diagonal terms, the exponential terms appearing in eqs. (4.26)-(4.28) can be written as

$$e^{i\underline{\underline{\kappa}}_0^{\pm}z} = e^{i\kappa_h^{\pm}z}\,\underline{h}^0\underline{h}^0 + e^{i\kappa_v^{\pm}z}\,\underline{v}_{\pm}^0\underline{v}_{\pm}^0 \tag{4.34}$$

A. Calculation of like polarization cross sections:

From eq. (4.24) we have

$$\sigma_{pp}^0 = 4\pi\rho\left[\overline{\int_{-d}^{0}\underline{p}_-^0\cdot\left\{\sum_{i,j=1}^{4}(\underline{W}_i)(\underline{W}_j^*)\right\}\cdot\underline{p}_-^0\,ds}\right] , \qquad p\ \epsilon\{h\ ,v\ \} \tag{4.35}$$

where

$$\sigma_{pp}^0 = \sigma_{ppd}^0 + \sigma_{ppr}^0 + \sigma_{ppdr}^0 \tag{4.36}$$

We proceed by dotting both sides of eqs. (4.26), (4.27) and (4.28) with $\underline{q}_-^0$, using eq. (4.34) and taking the average over all particle orientations gives

$$\sigma_{ppd}^0 = 4\pi\rho\,\overline{|\underline{p}_-^0\cdot\underline{\underline{f}}_I\cdot\underline{p}_-^0|^2}\,\frac{1-e^{-4\mathrm{Im}\kappa_p^- d}}{4\mathrm{Im}\kappa_p^-} \quad , \qquad p\ \epsilon\{h,v\} \tag{4.37}$$

$$\sigma_{ppr}^0 = 4\pi\rho\,\overline{|\underline{p}_+^0\cdot\underline{\underline{f}}_{II}\cdot\underline{p}_+^0|^2}\,\frac{e^{4\mathrm{Im}\kappa_p^+ d}-1}{4\mathrm{Im}\kappa_p^+}\,|\Gamma_{sp}|^4 \tag{4.38}$$

The σ_{ppdr}^0 has three components:

$$\sigma^0_{ppdr} = \sigma^0_{ppdr_1} + \sigma^0_{ppdr_2} + \sigma^0_{ppdr_3} \tag{4.39}$$

where

$$\sigma^0_{ppdr_1} = 4\pi\rho \overline{|\underline{p}^0_{+}\cdot\underline{\underline{f}}_{III}\cdot\underline{p}^0_{-}|^2}\; \frac{e^{2\mathrm{Im}(\kappa_p^+-\kappa_p^-)d} - 1}{2\mathrm{Im}(\kappa_p^+-\kappa_p^-)} |\Gamma_{sp}|^2 \tag{4.40}$$

$$\sigma^0_{ppdr_2} = 4\pi\rho \overline{|\underline{p}^0_{-}\cdot\underline{\underline{f}}_{IV}\cdot\underline{p}^0_{+}|^2}\; \frac{e^{2\mathrm{Im}(\kappa_p^+-\kappa_p^-)d} - 1}{2\mathrm{Im}(\kappa_p^+-\kappa_p^-)} |\Gamma_{sp}|^2 \tag{4.41}$$

and

$$\sigma^0_{ppdr_3} = 8\pi\rho \;\mathrm{Re}\left\{\overline{(\underline{p}^0_{+}\cdot\underline{\underline{f}}_{III}\cdot\underline{p}^0_{-})(\underline{p}^0_{-}\cdot\underline{\underline{f}}^*_{IV}\cdot\underline{p}^0_{+})}\right\} \frac{e^{2\mathrm{Im}(\kappa_p^+-\kappa_p^-)d} - 1}{2\mathrm{Im}(\kappa_p^+-\kappa_p^-)} |\Gamma_{sp}|^2 \tag{4.42}$$

with

$$\Gamma_{sp} = \Gamma_{gp}\, e^{i(\kappa_p^+ + \kappa_p^-)d} \quad , \qquad p \in \{h,v\} \tag{4.43}$$

We note that if $\kappa_p^+ = \kappa_p^-$ then eq.(4.40)-(4.42) changes as follows:

$$\frac{e^{2\mathrm{Im}(\kappa_p^+ - \kappa_p^-)d} - 1}{2\mathrm{Im}(\kappa_p^+ - \kappa_p^-)} \to d \tag{4.44}$$

The Fresnel reflection coefficients appearing in eq.(4.43) are given by

$$\Gamma_{gh} = \frac{\kappa_0 - \kappa_g}{\kappa_0 + \kappa_g} \quad , \qquad \Gamma_{gv} = \frac{\varepsilon_g\kappa_0 - \kappa_g}{\varepsilon_g\kappa_0 + \kappa_g} \tag{4.45}$$

$$\kappa_0' = k_0 \cos\theta_0 \qquad (4.46)$$

and

$$\kappa_g = k_0(\varepsilon_g - \sin^2\theta_0)^{1/2} \qquad (4.47)$$

The σ^0_{ppd} represents the direct backscattering contribution as shown in fig. (4a). The incoming wave propagates into the vegetation and is scattered directly back to the observer. When summed over all scatterers, eq. (4.37) results. The backscattering coefficient σ^0_{ppr} given by eq. (4.38) represents the sum of all waves which are first reflected from the ground, then scattered, and finally reflected again by the ground towards the observer as shown in fig. (4b). The reflection coefficient $|\Gamma_{sp}|^4$ appears in the equation since the wave in this case has been reflected by the ground twice.

The third term σ^0_{ppdr} as given by eq. (4.39), results from two different but similar mechanisms as shown in fig. (4c) and fig. (4d). In one case, the wave is scattered and then reflected toward the observer; whereas in the second case, the wave is first reflected from ground interface and then scattered toward the observer. The third term, eq. (4.42), occurs due to a combination of both cases. These terms interfere coherently. The $|\Gamma_{gp}|^2$ factor represents single reflection from the ground.

The coefficients $\overline{|\underline{p}^0_- \cdot \underline{\underline{f}}_I \cdot \underline{p}^0_-|^2}$ and $\overline{|\underline{p}^0_+ \cdot \underline{\underline{f}}_{II} \cdot \underline{p}^0_+|^2}$ represent the backscattering cross sections in $\underline{p}^0_-$ and $\underline{p}^0_+$ directions, respectively. The coefficients $\overline{|\underline{p}^0_- \cdot \underline{\underline{f}}_{III} \cdot \underline{p}^0_+|^2}$ and $\overline{|\underline{p}^0_+ \cdot \underline{\underline{f}}_{IV} \cdot \underline{p}^0_-|^2}$ represent the bistatic cross sections in different directions. Note that the bar over the expression represents the average over angular variables.

B. Calculation of cross polarization cross sections:

In this case the incoming and reflected waves have different polarizations. Thus the equation for σ^0_{pqdr} cannot be reduced to a simple term as in the previous two cases. But otherwise, following a similar procedure, we obtain

$$\sigma^0_{qp} = \sigma^0_{pq} = \sigma^0_{qpd} + \sigma^0_{qpr} + \sigma^0_{qpdr} \quad , \qquad p \neq q \tag{4.48}$$

where

$$\sigma^0_{qpd} = 4\pi\rho \overline{|\underline{p}^0_{-}\cdot\underline{\underline{f}}_{I}\cdot\underline{q}^0_{-}|^2} \; \frac{1 - e^{-2\mathrm{Im}(\kappa^-_q+\kappa^-_p)d}}{2\mathrm{Im}(\kappa^-_q+\kappa^-_p)} \tag{4.49}$$

$$\sigma^0_{qpr} = 4\pi\rho \overline{|\underline{p}^0_{+}\cdot\underline{\underline{f}}_{II}\cdot\underline{q}^0_{+}|^2} \; \frac{e^{2\mathrm{Im}(\kappa^+_q+\kappa^+_p)d} - 1}{2\mathrm{Im}(\kappa^+_q+\kappa^+_p)} \, |\Gamma_{sq}|^2 |\Gamma_{sp}|^2 \tag{4.50}$$

The σ^0_{pqdr} has three components:

$$\sigma^0_{qpdr} = \sigma^0_{qpdr_1} + \sigma^0_{qpdr_2} + \sigma^0_{qpdr_3} \tag{4.51}$$

where

$$\sigma^0_{qpdr_1} = 4\pi\rho \overline{|\underline{p}^0_{+}\cdot\underline{\underline{f}}_{III}\cdot\underline{q}^0_{-}|^2} \; \frac{1 - e^{-2\mathrm{Im}(\kappa^-_q-\kappa^+_p)d}}{2\mathrm{Im}(\kappa^-_q-\kappa^+_p)} \, |\Gamma_{sp}|^2 \tag{4.52}$$

$$\sigma^0_{qpdr_2} = 4\pi\rho \overline{|\underline{p}^0_{-}\cdot\underline{\underline{f}}_{IV}\cdot\underline{q}^0_{+}|^2} \; \frac{e^{2\mathrm{Im}(\kappa^+_q-\kappa^-_p)d} - 1}{2\mathrm{Im}(\kappa^+_q-\kappa^-_p)} \, |\Gamma_{sq}|^2 \tag{4.53}$$

and

$$\sigma^0_{qpdr_3} = 8\pi\rho \, \mathrm{Re} \left\{ \overline{(\underline{p}^0_+\cdot\underline{\underline{f}}_{III}\cdot\underline{q}^0_-)(\underline{p}^0_-\cdot\underline{\underline{f}}^*_{IV}\cdot\underline{q}^0_+)} \right.$$

$$\left. \left[\Gamma^*_{sq}\Gamma_{sp} \frac{1 - e^{i(\kappa^-_q - \kappa^+_p + \kappa^{+*}_q - \kappa^{-*}_p)d}}{i(-\kappa^-_q + \kappa^+_p - \kappa^{+*}_q + \kappa^{-*}_p)} \right] \right\} \tag{4.54}$$

Note that if $\kappa^+_p = \kappa^-_p$ or $\kappa^-_q - \kappa^+_p + \kappa^{+*}_q - \kappa^{-*}_p = 0$ then one obtains the correct form of eqs. (4.52)-(4.54) by taking the limit as those equalities are approached.

It is interesting to note that only the imaginary part of the effective permittivity enters the expression for σ^0_{pq}. This is a result of the fact that σ^0_{pq} represents an incoherent sum of the scattered power from individual particles embedded in the effective medium.

If the particles' sizes are small in comparison to the wavelength, these results reduce to the results obtained by Lang and Sidhu [1983].

CHAPTER V

NUMERICAL RESULTS AND DISCUSSION

Our main interest in this chapter is to obtain the numerical results for the theory we have developed in the previous chapters. For this purpose, we use our method to model a forest canopy by a collection of lossy dielectric discs, which are assumed to have radius a, thickness T and relative dielectric constant ε_r. Our formulation is applicable for arbitrarily shaped scatterers in the Rayleigh, resonant and geometric optics regimes; however, because of the availability of Rayleigh and geometric optic scattering amplitude algorithms we will limit our calculation to those regimes.

Using the parameters encountered in active remote sensing of vegetation layers, we will present the numerical results for the skin depth and for the backscattering cross sections in the Rayleigh and geometric optics regimes. Before proceeding with this calculation, we will discuss the relative dielectric constant of the leaves and the scattering albedo of scatterers.

The Relative Dielectric Constant of the Leaf

Our calculation of the relative dielectric constant follows the model of de Loor [1968], and Fung and Ulaby [1978]. We conclude, on the basis of measurements conducted by Broadhurst [1970] that this model is not suitable for frequencies below 1 GHz due to its failure to account for conductive (ohmic) losses within the leaves; however, we will use this model below 1 GHz in order to recover results obtained by Lang and Sidhu [1983]. The dielectric formula is given for the real and the imaginary parts of the relative dielectric permittivity below.

$$\varepsilon_r' = 5.5 + \frac{\varepsilon_m - 5.5}{1+(f\tau)^2} \qquad (5.1)$$

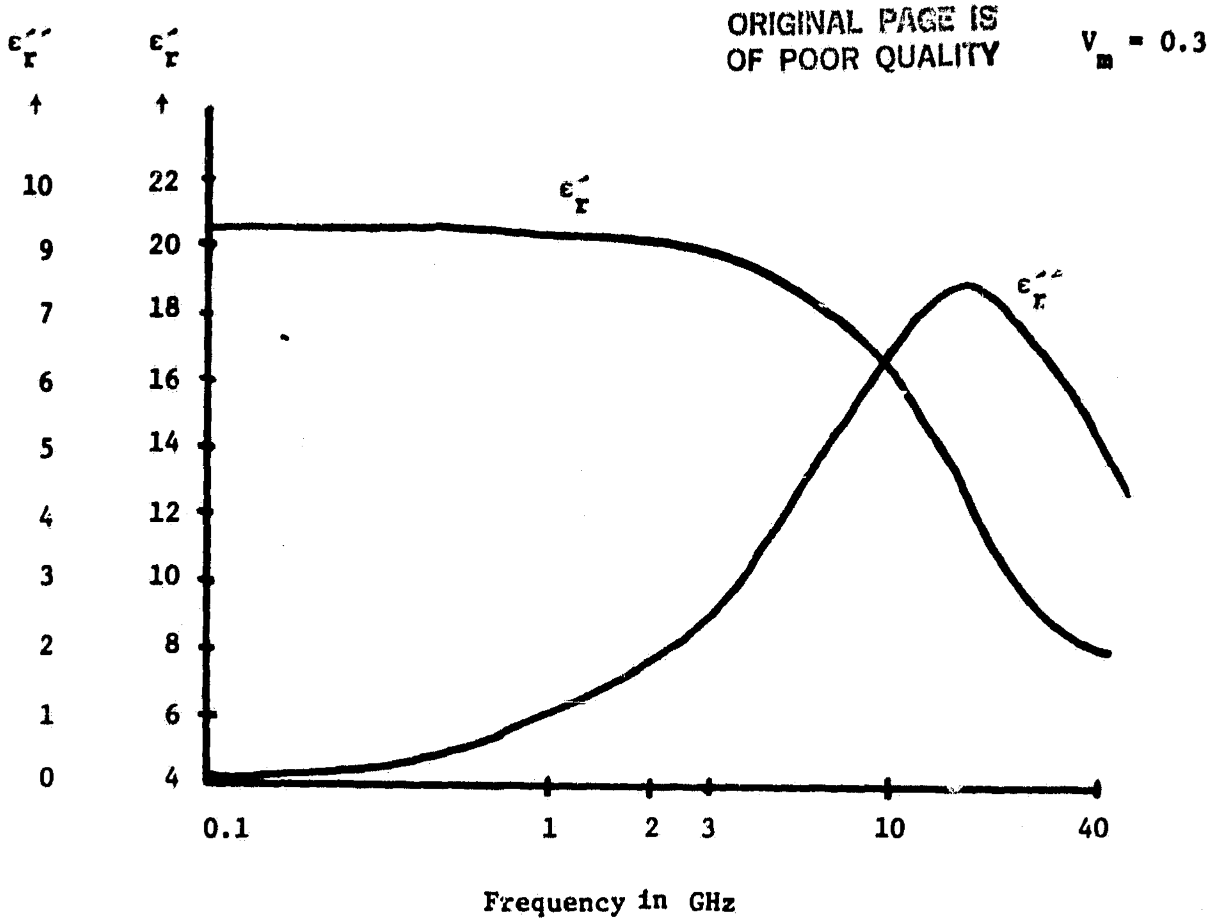


Fig. 5. Real and Imaginary part of dielectric constant vs frequency

$$\varepsilon_r'' = (\varepsilon_m - 5.5)\,\frac{f\tau}{1+(f\tau)^2} \tag{5.2}$$

with $\varepsilon_r = \varepsilon_r' + i\varepsilon_r''$. The letter τ is the relaxation time of water which depends on the temperature and f is the frequency. At 20°C, $f\tau$ is approximately equal to $1.85/\lambda$, where λ is the free space wavelength in centimeters. The ε_m appearing in eqs. (5.1) and (5.2) is the relative macroscopic static permittivity of a leaf and given by

$$\varepsilon_m = 5 + 51.56\, V_m \tag{5.3}$$

where V_m is the volume filling factor which lies between 0.1 and 0.6. For illustrative purposes, we plot a real and imaginary part of ε_r as a function of the frequency as shown in fig. 5 with $V_m = 0.3$.

We observe that ε'' has a maximum around 15 GHz and ε' is constant until 1 GHz, then it decreases monotonically. We also have observed that changing parameters did not effect the overall shape of the curves.

The Albedo of a Single Leaf

In this section we will calculate the albedo of a single dielectric disc. This is necessary due to the fact that our theory is limited to low albedo particles where absorptive loss is then the dominant mechanism. First, the dyadic scattering amplitude for a lossy dielectric scatterer is required. In the Rayleigh regime, [see Lang 1981], the dyadic scattering amplitude $\underline{\underline{f}}$ is related to the polarizability, $\underline{\underline{\alpha}}$, by

$$f_{pq} = \underline{p}^0 \cdot \underline{\underline{f}} \cdot \underline{q}^0 = \left(\frac{k_0^2}{4\pi}\right) \alpha_{pq} \quad , \quad p,q \varepsilon \{h,v\} \qquad (5.4)$$

Ishimaru [1978] then gives expressions for the polarizability of a lossy dielectric disc.

The dyadic scattering amplitude for a lossy dielectric disc in the geometric optics regimes has been calculated by LeVine, et al. [1982]. An approximate expression for the dyadic scattering amplitude is obtained by considering a plane wave incident on an infinite dielectric slab. The internal fields in the slab are calculated exactly. Then the equivalent sources generated by these fields in the region of the slab corresponding to the disc are used to calculate the scattered field. The approximation requires that the leaf be many wavelengths in diameter and have a thickness, T, small compared to the diameter.

From LeVine, et al., we have the following expression for the dyadic scattering amplitude in the geometric optics regime:

$$\underline{\underline{f}} = \left(\frac{k_0^2}{4\pi}\right)\Delta T\ \tilde{S}\ (\underline{\nu}_t)\ \underline{\underline{F}} \tag{5.5}$$

where $\Delta = \varepsilon_r - 1.$, and the Fourier transform of the cross sectional shape of the disc is given by

$$\tilde{S}(\underline{\nu}_t) = \int_{R_2} S(\underline{x}_t')e^{i\underline{\nu}_t \cdot \underline{x}_t'}\, d\underline{x}_t' \tag{5.6a}$$

with

$$S(\underline{x}_t') = \begin{cases} 1 & , \ \underline{x}_t' \ \varepsilon \text{ disk face} \\ 0 & , \ \underline{x}_t' \notin \text{disk face} \end{cases}$$

where the prime quantities are coordinates in the face of disk and $\underline{\nu}_t = k_0[\underline{i}_t - \underline{0}_t]$. Here $\underline{i}_t$ and $\underline{0}_t$ are the projections of $\underline{i}$ and $\underline{0}$ of fig. 6 onto the face of the disk. In eq. (5.5), $\underline{\underline{F}}$ is related to an integral of the induced charge in the infinite dielectric slab. It is given by LeVine, et al, [1982].

For a circular disk of radius a eq. (5.6a) becomes

$$\tilde{S}(\nu_t) = \frac{2\pi a}{\nu_t} J_1(\nu_t a) \tag{5.6b}$$

where $J_1(\zeta)$ is the Bessel function of first order and $\nu_t = |\underline{\nu}_t|$.

From Ishimaru [1978], we have the following expressions for total, scattering, and absorption cross sections.

$$\sigma_t^{(q)} = \left(\frac{4\pi}{k_0}\right) \mathrm{Im}[\underline{\underline{f}}(\underline{i}^0, \underline{i}^0; q) \cdot \underline{q}^0] \quad , \quad q \varepsilon \{\underline{h}\ , \underline{v}\ \} \tag{5.7}$$

$$\sigma_s^{(q)} = \int_{4\pi} |\underline{f}(\underline{0}^0, \underline{i}^0; q)|^2 d\Omega \tag{5.8}$$

and

$$\sigma_a^{(q)} = \int_V k_0 \varepsilon_r''(\underline{x}') |\underline{E}(\underline{x}')|^2 dV' \tag{5.9}$$

where $\underline{f}(\underline{0}^0, \underline{i}^0, q)$ is the vector scattering amplitude due to an incident plane wave with polarization q. These vector scattering amplitudes are related to the dyadic scattering amplitude by

$$\underline{f}(\underline{0}^0,\underline{i}^0) = \underline{f}(\underline{0}^0,\underline{i}^0;h)h^0 + \underline{f}(\underline{0}^0,\underline{i}^0;v)\underline{v}^0 \tag{5.10}$$

In addition, in eqs. (5.7)-(5.9), $d\Omega$ is the differential solid angle, dV is the differential volume element, and $E(\underline{x}')$ is the electric field inside the particle. Using the total scattering cross section, $\sigma_t^{(q)}$ and the scattering cross section, $\sigma_s^{(q)}$, we can define the scattering albedo $W_0^{(q)}$ by

$$W_0^{(q)} = \frac{\sigma_s^{(q)}}{\sigma_t^{(q)}} \tag{5.11}$$

The total cross section $\sigma_t^{(q)}$ represents the total power loss from the incident wave due to the scattering and absorption of the wave by the scatter. One can show that [Born and Wolf, 1964],

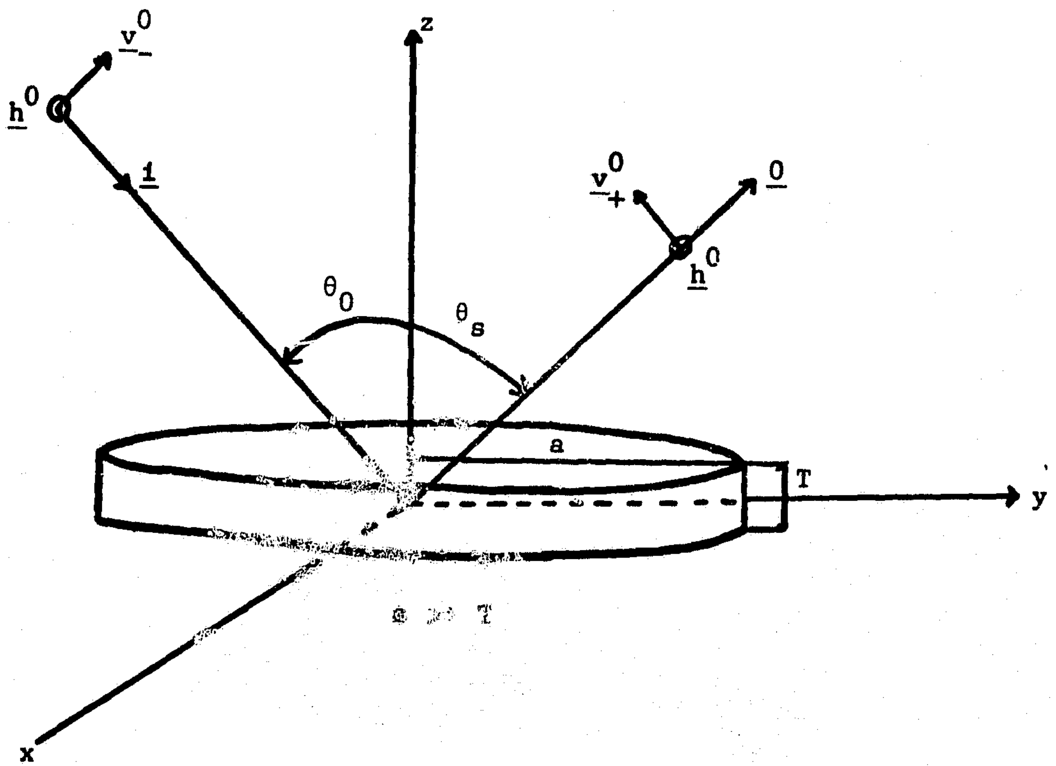


Fig. 6. The geometric configuration of a leaf

$$\sigma_t^{(q)} = \sigma_a^{(q)} + \sigma_s^{(q)} \qquad q\epsilon\{h,v\} \tag{5.12}$$

The geometric configuration of a leaf for an albedo calculation is shown in fig. 6.

We evaluate eqs. (5.8) and (5.9) for the horizontal and vertical polarization cases. The results at low frequency (Rayleigh regime) are:

$$\sigma_a^{(h)} = k_0\varepsilon_r''V \tag{5.13}$$

$$\sigma_a^{(v)} = k_0\varepsilon_r''V\left(\cos^2\theta_0 + \frac{\sin^2\theta_0}{|\varepsilon_r|^2}\right) \tag{5.14}$$

$$\sigma_s^{(h)} = \frac{k_0^4V^2}{6\pi}\,|\Delta|^2 \tag{5.15}$$

and

$$\sigma_s^{(v)} = \frac{k_0^4V^2\,|\Delta|^2}{6\pi}\left(\cos^2\theta_0 + \frac{\sin^2\theta_0}{|\varepsilon_r|^2}\right) \tag{5.16}$$

where

$$V = \frac{2}{3}\,\pi a^2 T \tag{5.17}$$

$$\Delta = \varepsilon_r - 1 \tag{5.18}$$

In fig. 7 we plot eqs. (5.13)-(5.16) as a function of the frequency at zero incident angle with a=7 cm, T=0.3 mm and V_m=0.1. Typical dimensions of a leaf having radii of one to several centimeters and thicknesses of tenths of a millimeter have been used.

At zero incident angle, using the above eqs. we conclude that $\sigma_s^{(h)}=\sigma_s^{(v)}\equiv\sigma_s$ and $\sigma_a^{(h)}=\sigma_a^{(v)}\equiv\sigma_a$ as seen in fig. 7. The curves show that absorption and scattering cross sections increase with the frequency. For large values of frequency σ_s is greater than σ_a.

Following the LeVine, et al [1982] work, we evaluate σ_a and σ_s for the horizontal and vertical polarization cases at zero incident angle in the geometric optic regime.

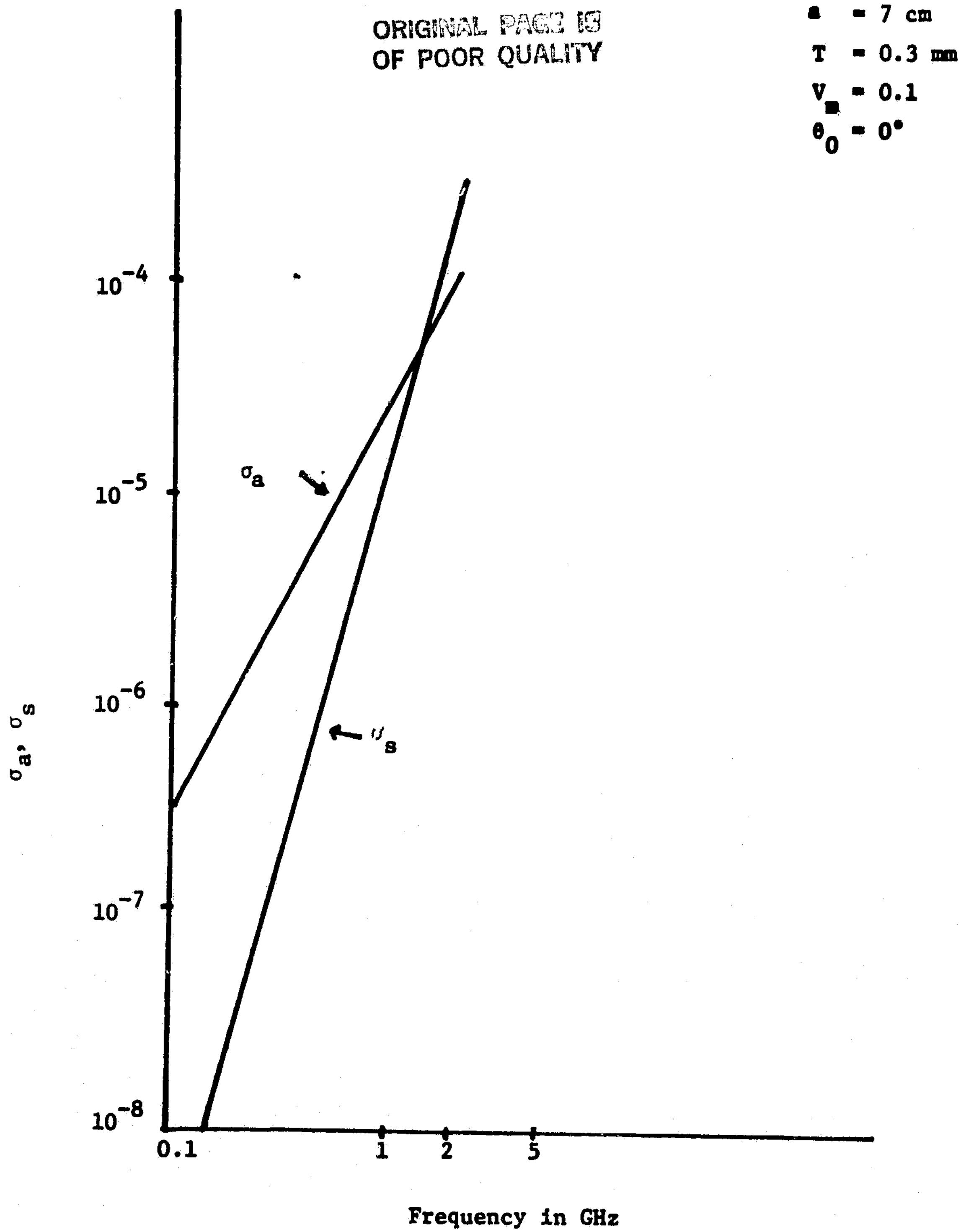


Fig. 7. Scattering and absorbtion cross sections vs frequency for Rayleigh scatterers

The absorption and scattering cross sections are given by

$$\sigma_a^{(q)} = k_0 \varepsilon_r'' \tilde{S}_0 \Bigg\{ (|e_q^-|^2 + |e_q^+|^2) \frac{\sinh(\kappa_i T)}{\kappa_i} + 2\mathrm{Re}\left[e_q^- e_q^{+*} (\underline{q}^0_{\varepsilon -} \cdot \underline{q}^{0*}_{\varepsilon +}) \frac{\sin(\kappa_r T)}{\kappa_r} \right] \Bigg\} \tag{5.19}$$

and

$$\sigma_s^{(q)} = \frac{(2\pi)^2 C \tilde{S}_0}{k_0 \cos\theta_0} [W^{(q)}(\theta_r, \phi_r) + W^{(q)}(\theta_t, \phi_t)] \tag{5.20}$$

where

$$W^{(q)}(\theta,\phi) = |\mathrm{sinc}(\theta^+) e_q^+ \underline{q}^0_{\varepsilon +} + \mathrm{sinc}(\theta^-) e_q^- \underline{q}^0_{\varepsilon -}|^2 \tag{5.21}$$

with $\tilde{S}_0 = \tilde{S}(0) = \pi a^2$, $C = |\frac{\Delta}{4\pi} k_0^2 T|^2$ and

$$\theta^{\pm} = [\kappa \pm k_0 0_z']T/2 \quad , \; 0_z' = \underline{n}^0 \cdot \underline{0}^0 \tag{5.22}$$

Quantities not explicitly defined in eqs. (5.19)-(5.22) are defined in the LeVine, et al, reference. We note that the $e_q^{\pm}$ are forward and backward going wave amplitudes in the dielectric slab for polarization q. The propagation constant in the z direction inside the slab is $\kappa = \kappa_r + i\kappa_i$.

Using the same parameters we used for the Rayleigh regime, i.e. a=7 cm, T= 0.3 mm, and V_m= 0.1, we plot σ_a and σ_s as a function of the frequency at zero incident angle (fig. 8). As we see from this figure, at a low frequency of around 1 GHz both of the curves are linear; however as the frequency

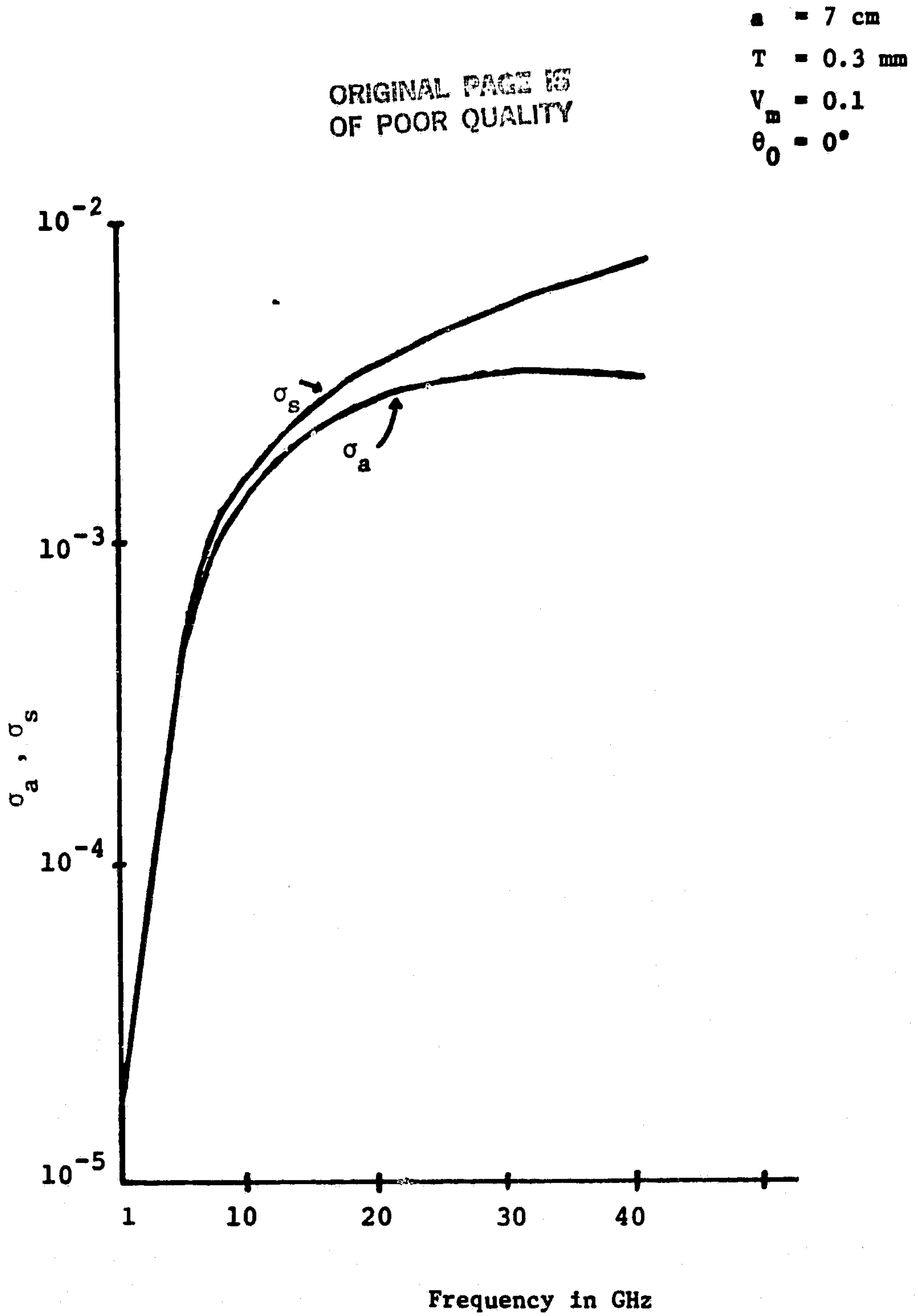


Fig. 8. Scattering and absorbtion cross sections vs frequency in geometric optics regime

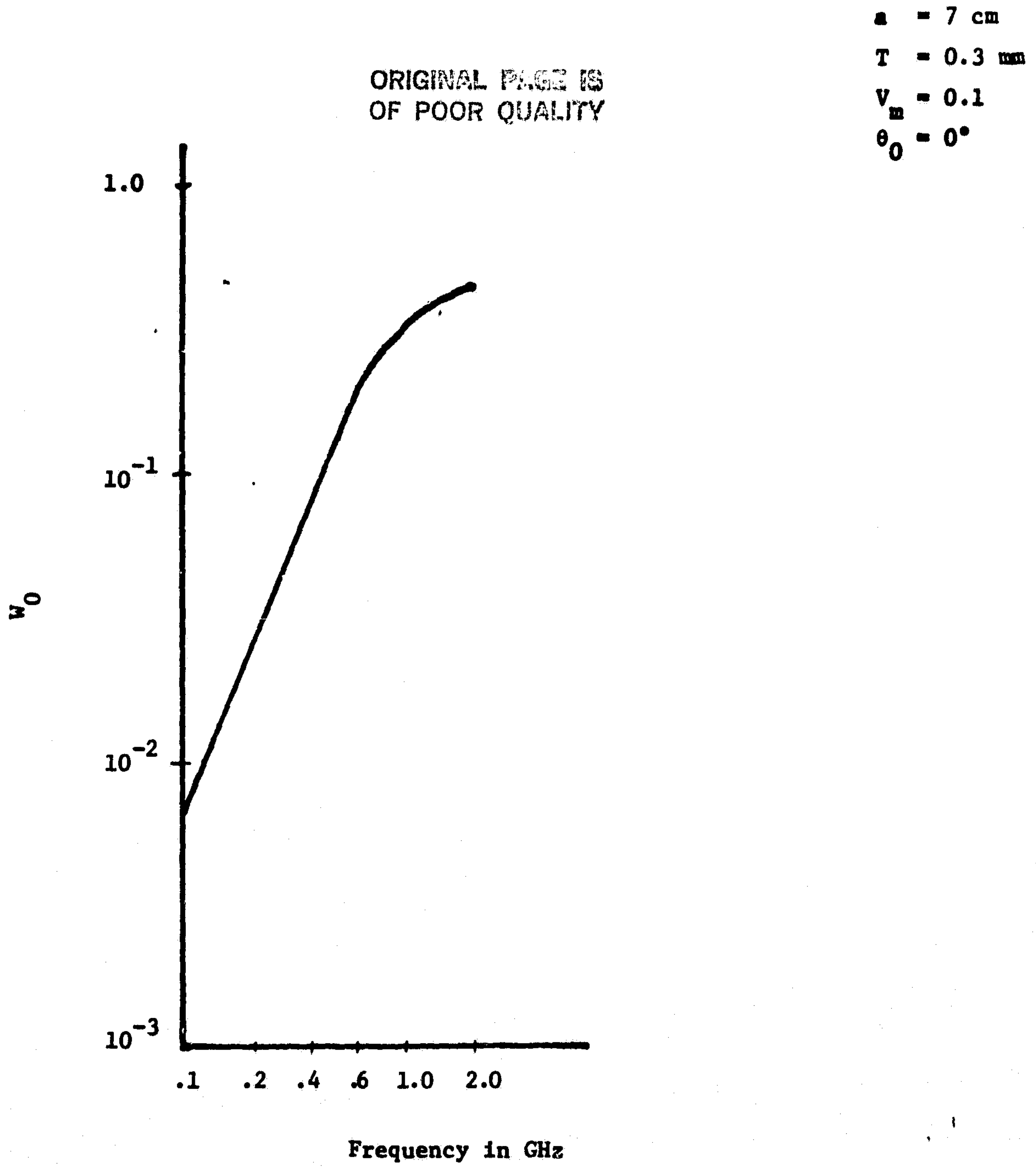


Fig. 9. Albedo vs frequency for Rayleigh regime

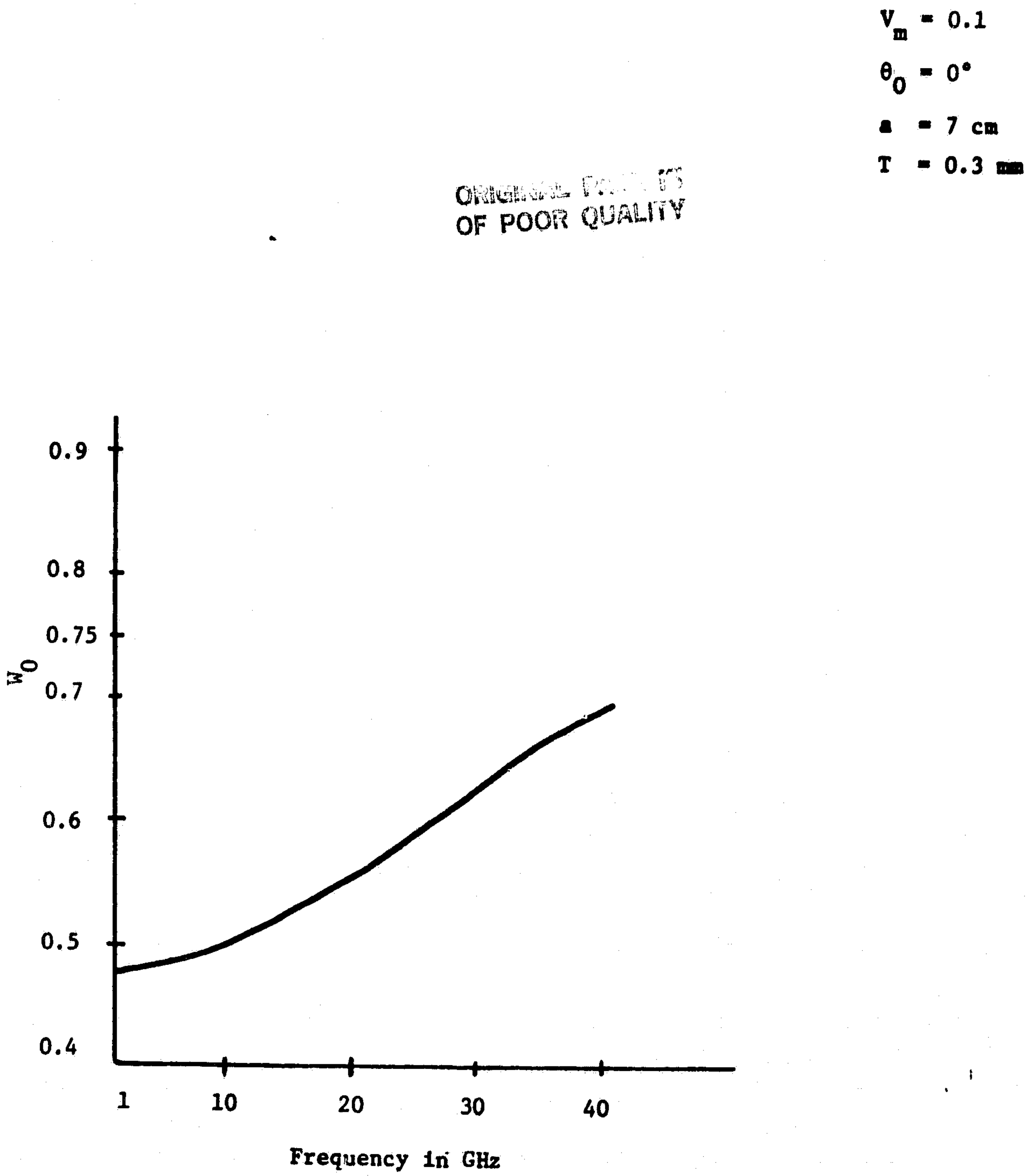


Fig. 10. Albedo vs frequency at geometric optics regime

increases the σ_s curve becomes greater than the σ_a curve.

Now, substituting σ_s and σ_t into the eq. (5.11) a low albedo check is made for both regimes. We plot the albedo as a function of the frequency in the Rayleigh regime and geometric optic regime, as shown in figs. (9) and (10) with the same parameters (a=7 cm, T=0.3 mm, V_m=0.1 and θ_0=0°). In these figures it was observed that albedo increases almost linearly in the Rayleigh regime and monotonically in the geometric optic regime. The expression for the geometric-optic albedo obtained by using eqs. (5.19) and (5.20) in eq. (5.11) can be substantially simplified when the phase variation of the internal field across the slab thickness is small (κ T<<1). The simplified expression for the albedo is

$$W_o = \frac{1}{1 + \sigma_a/\sigma_s} \tag{5.23}$$

where

$$\frac{\sigma_a}{\sigma_s} = \frac{2\varepsilon_r'' \cos\theta_0}{k_0 T |\Delta|^2} \tag{5.24}$$

From eq. (5.24), we conclude that the albedo is independent of the leaf radius, but very sensitive to the thickness, incident angle, and water content of the leaf.

The Skin Depth

A knowledge of the skin depth gives a better understanding of the mean wave's behavior inside the vegetation layer. The skin depth is defined by the depth of penetration at which the wave's amplitude decreases to e^{-1} of its initial value, we write

$$\text{Skin Depth} = \frac{1}{\text{Im}\,\kappa_p} \quad , \quad p\varepsilon\{h,v\} \tag{5.25}$$

f = 400 MHz
ρ = 500/m^3
ε_r = 30.8 + i0.62
a = 7.5 cm
T = 0.5 mm
$\Delta\theta_{||}$ = 30°

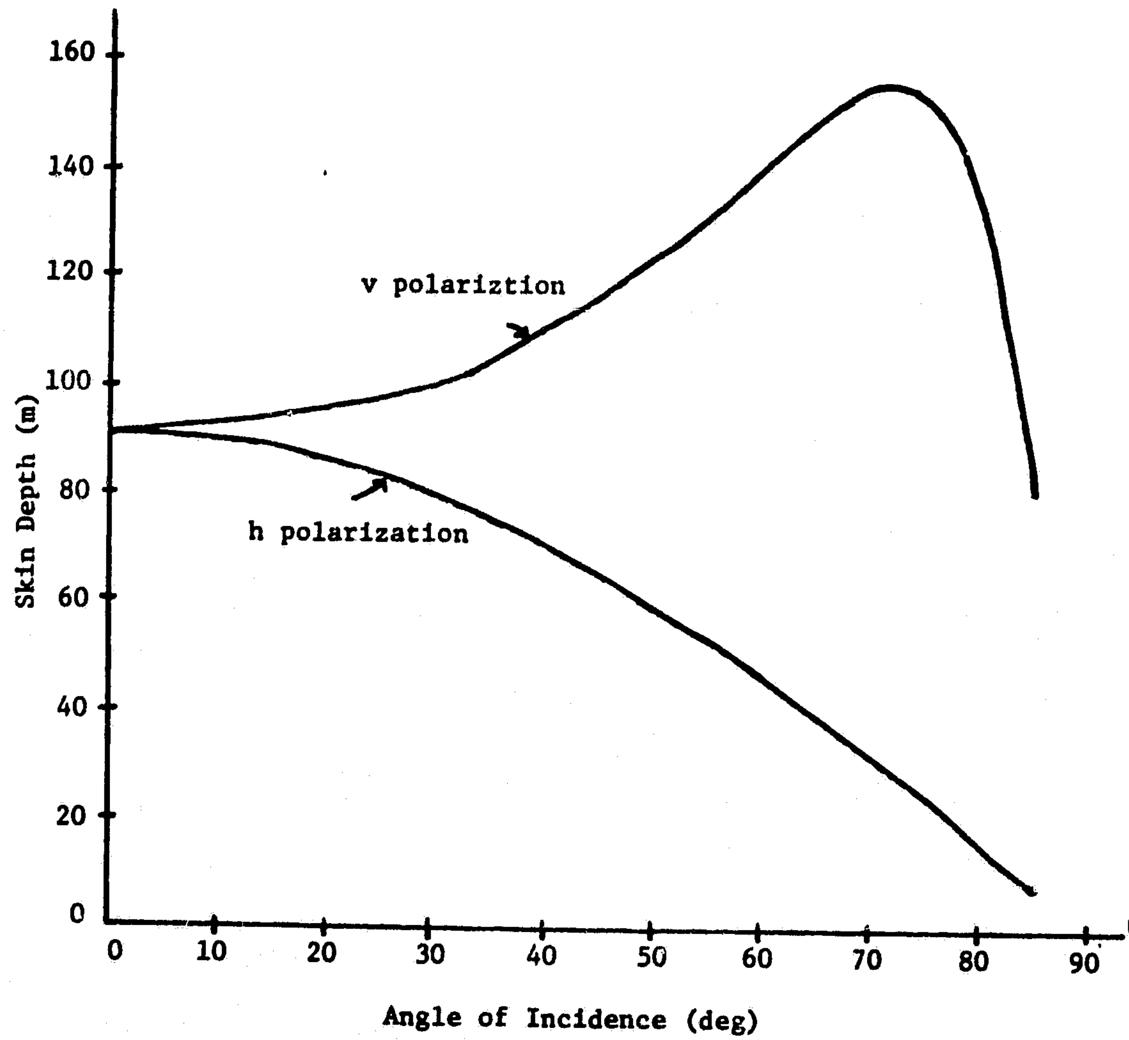


Fig. 11. Skin depth vs angle of incidence for Rayleigh regime

$f = 5$ GHz
$\rho = 500/m^3$
$\epsilon_r = 28.6 + i7.12$
$a = 7.5$ cm
$T = 0.5$ mm
$\theta = 30°$

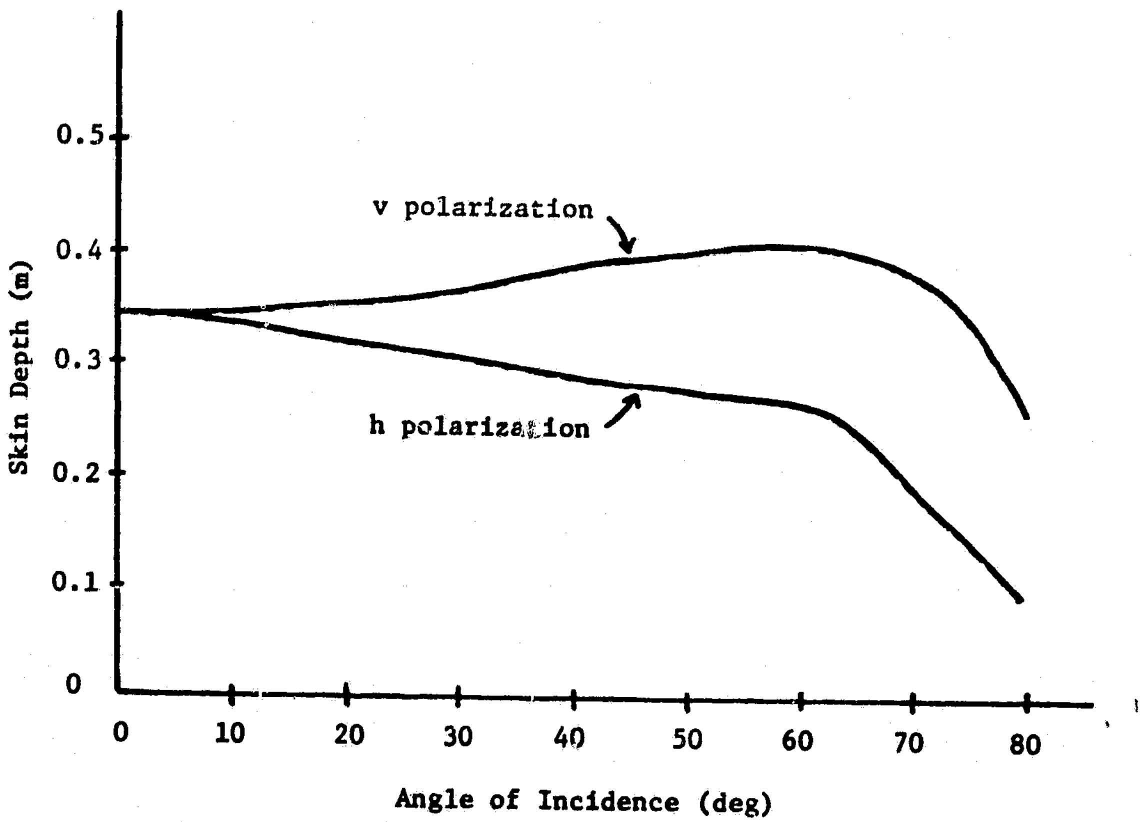


Fig. 12 Skin depth vs angle of incidence for geometric optics regime

where from eq. (4.33), we have

$$\kappa_p = k_0\cos\theta_0 + \frac{2\pi\rho}{k_0\cos\theta_0}\,\overline{f}_{pp}(\underline{i}^0,\underline{i}^0) \qquad p\epsilon\{h,v\} \qquad (5.26)$$

We have used the fact the $\overline{f}_{vh}=\overline{f}_{hv}=0$ and $\kappa_p^+=\kappa_p^-=\kappa_p$. These results can be obtained from (Lang, 1981) for Rayleigh discs and from Appendix D for geometric-optic discs.

We can see from eq. (5.25) that if the vegetation depth is large compared to the skin depth, then the effect of the ground reflections are negligible, whereas, when the skin depth is large compared to layer thickness, the characteristics of the ground become important in interpreting radar backscatter information.

An examination of eq. (5.26) shows that the skin depth for both h and v polarizations contains a factor of $\cos\theta_0$ in the numerator. This forces the skin depth to zero as the angle of incidence approaches grazing. Due to the increasing angle of incidence, the effective depth of the propagation keeps decreasing monotonically as seen in the following figures. In fig. (11) and (12), the skin depth is plotted as a function of the incidence angle for both polarizations in Rayleigh and geometric optic regimes, respectively. In figs. (13) and (14) the skin depth is plotted as a function of the frequency for two different regimes.

Now, let us examine figs. (11) and (12) in more detail. In the case of horizontal polarization, the electric field is approximately parallel to the leaves at all angles of incidence and thus the skin depth has only $\cos\theta_0$ dependence. The same thing is true for vertical polarization, but only when the angles of incidence are small. Thus, both have the same skin depth at small angles. As θ_0 becomes larger, the electric field tends to become perpendicular to the leaves and each leaf absorbs less energy. This explains the increasing skin depth as a function of the incidence angle for vertical

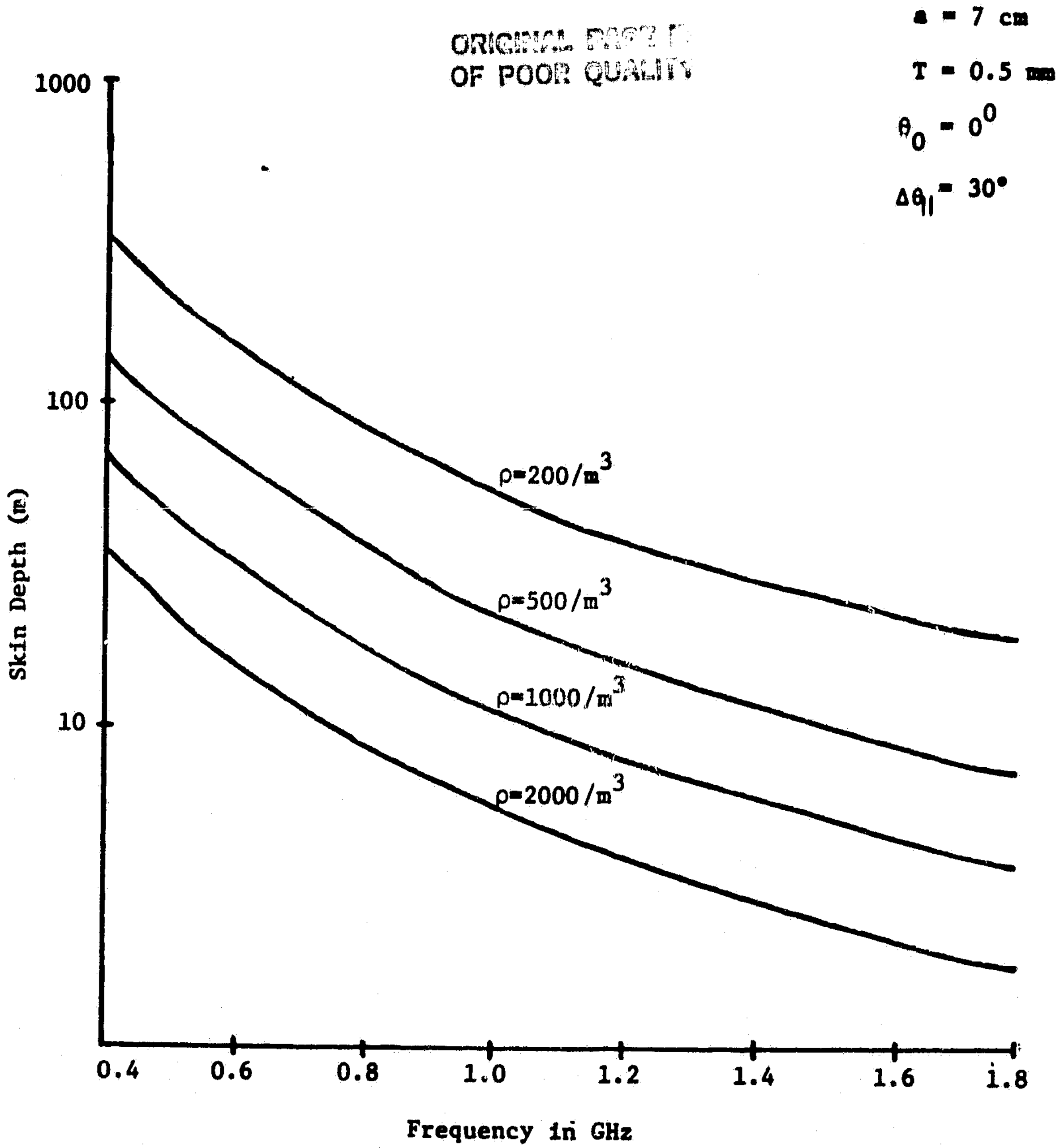


Fig. 13. Frequency vs skin depth for Rayleigh regime at different ρ

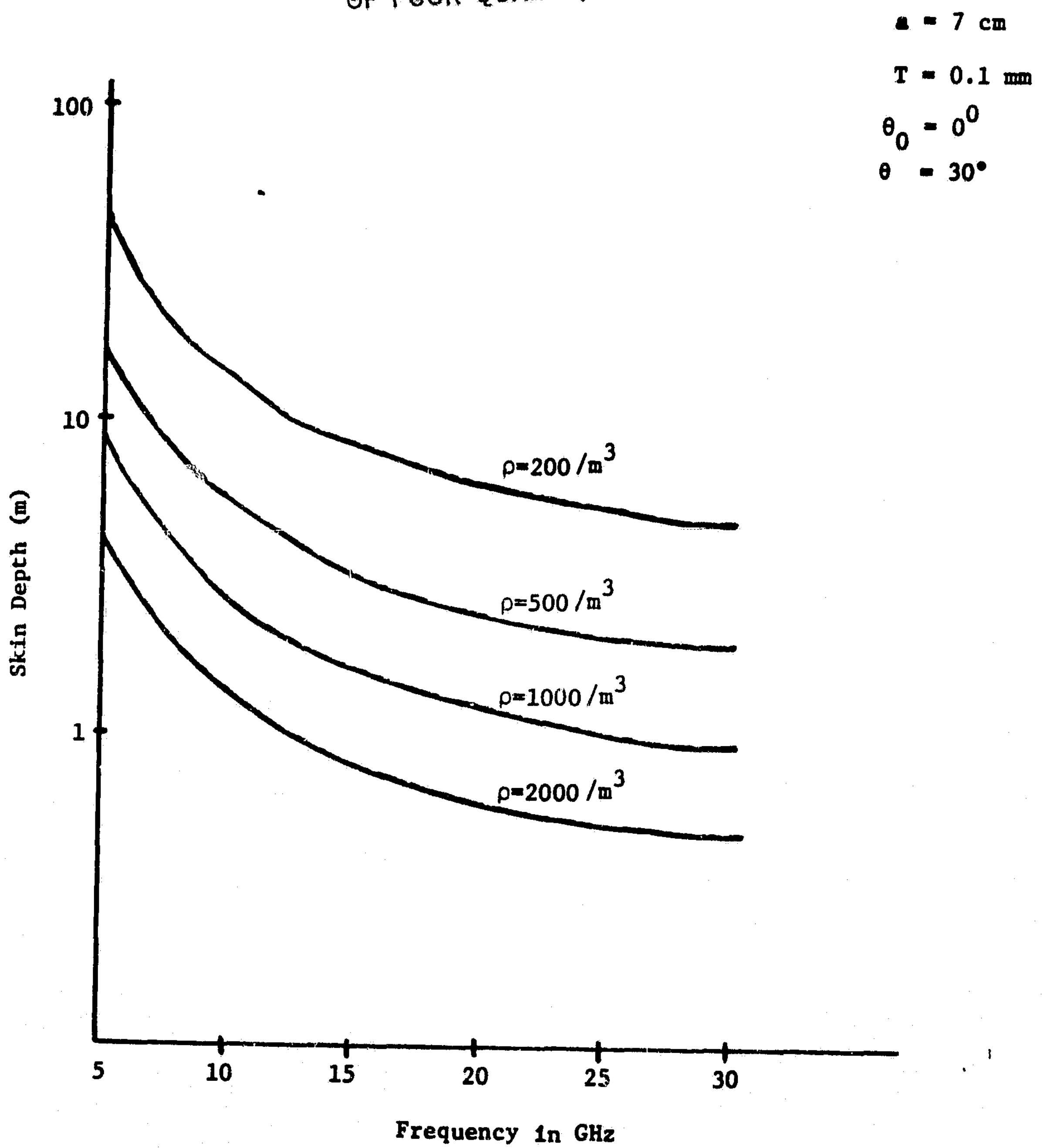


Fig. 14. Frequency vs skin depth for geometric optics regime at different ρ

polarization. Eventually, the skin depth decreases due to the $\cos\theta_0$ in the numerator in the vertical case.

The curves of Rayleigh and geometric optics regimes have identical shape as seen from figs. (11), (12), (13) and (14). The only real differences are that the curves for Rayleigh regime decrease more rapidly with increasing frequency and the skin depth is much higher than values obtained for the geometrical optic regime. These results were expected due to the fact that the physical size of the scatterers become an important factor in the high frequency regime.

Both figs. (11) and (12) have the same parameter: a=7 cm, T=.5 mm and ρ=500/m^3. In figs.(13) and (14), we have choosen a=7 cm. A value of T=.5 mm was picked for fig. 13 to recover Lang and Sidhu's result, but in fig. 14 we have used T=.1 mm in order to obtain a low albedo. The Rayleigh results have used scatterers with polar angle θ distributed uniformly between 0 and $\Delta\theta_{||}$=30° (see Lang, 1981) and the geometric optic results have used scatterers with a fixed θ=30°.

The Curves of the Backscattering Coefficients

We developed a computer program that provides numerical solutions for the backscattering cross sections which we have obtained in chapter IV. In the Rayleigh region the results are identical to those obtained by Lang and Sidhu[1983]. For purposes of completeness, we present some of their results in through figs. (15), (16), (17) and (18). These figures have common parameters: a=7.5cm, T=0.5mm, ρ=500/m^3, f=400MHz, ε_r=30.8+i0.62 and ε_g=12+i3.

In fig. (15), σ^0_{hhd}, σ^0_{hhdr}, and σ^0_{hhr} are shown as a function of θ_0. In this case, the term σ^0_{hhdr} is greater than σ^0_{hhd} for all angles of incidence. This means that the energy backscattered from the ground is a significant part of the total backscattered energy. Since the skin depth is large

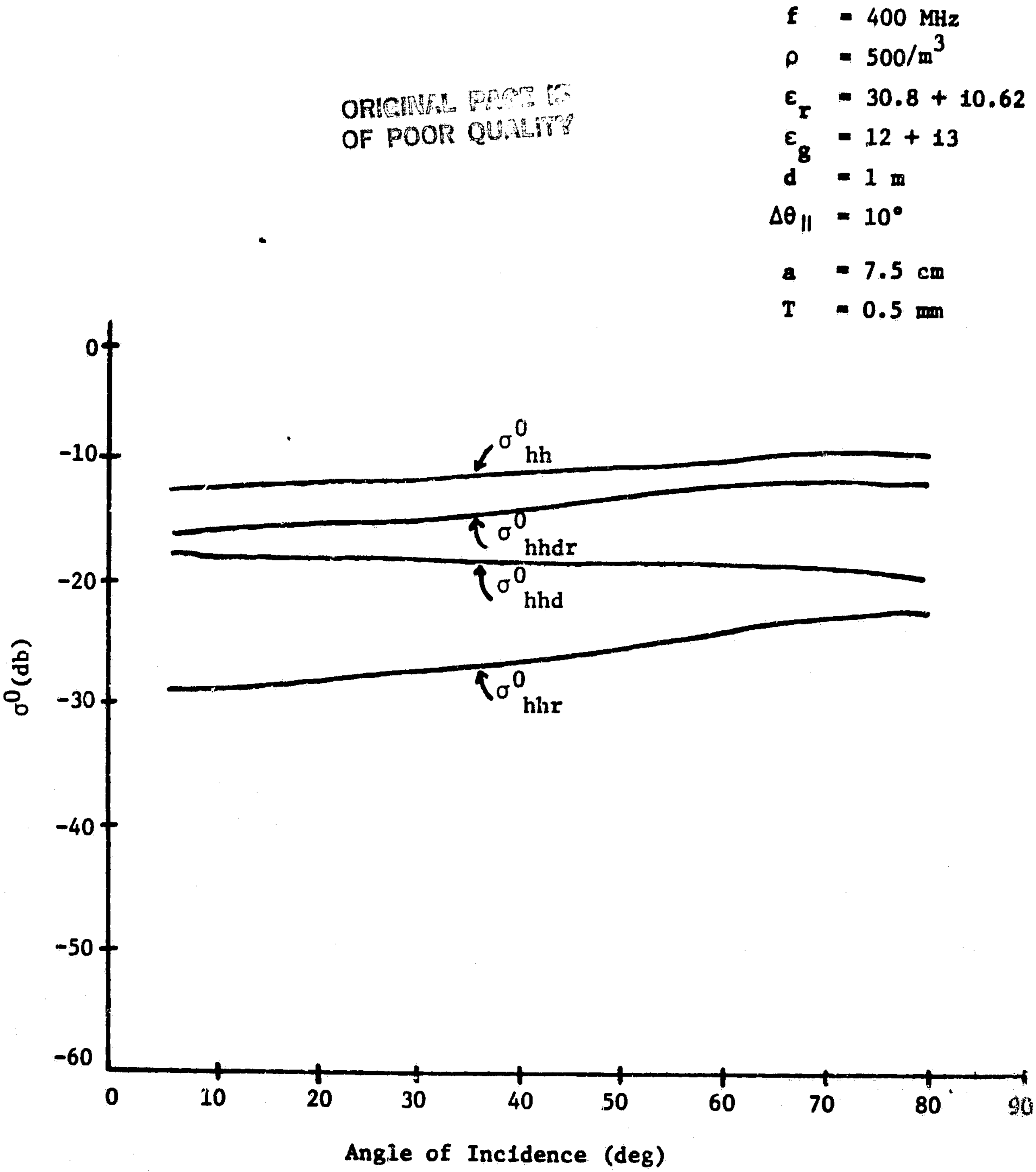


Fig. 15. Horizontal copolarized backscattering cross sections vs angle of incidence

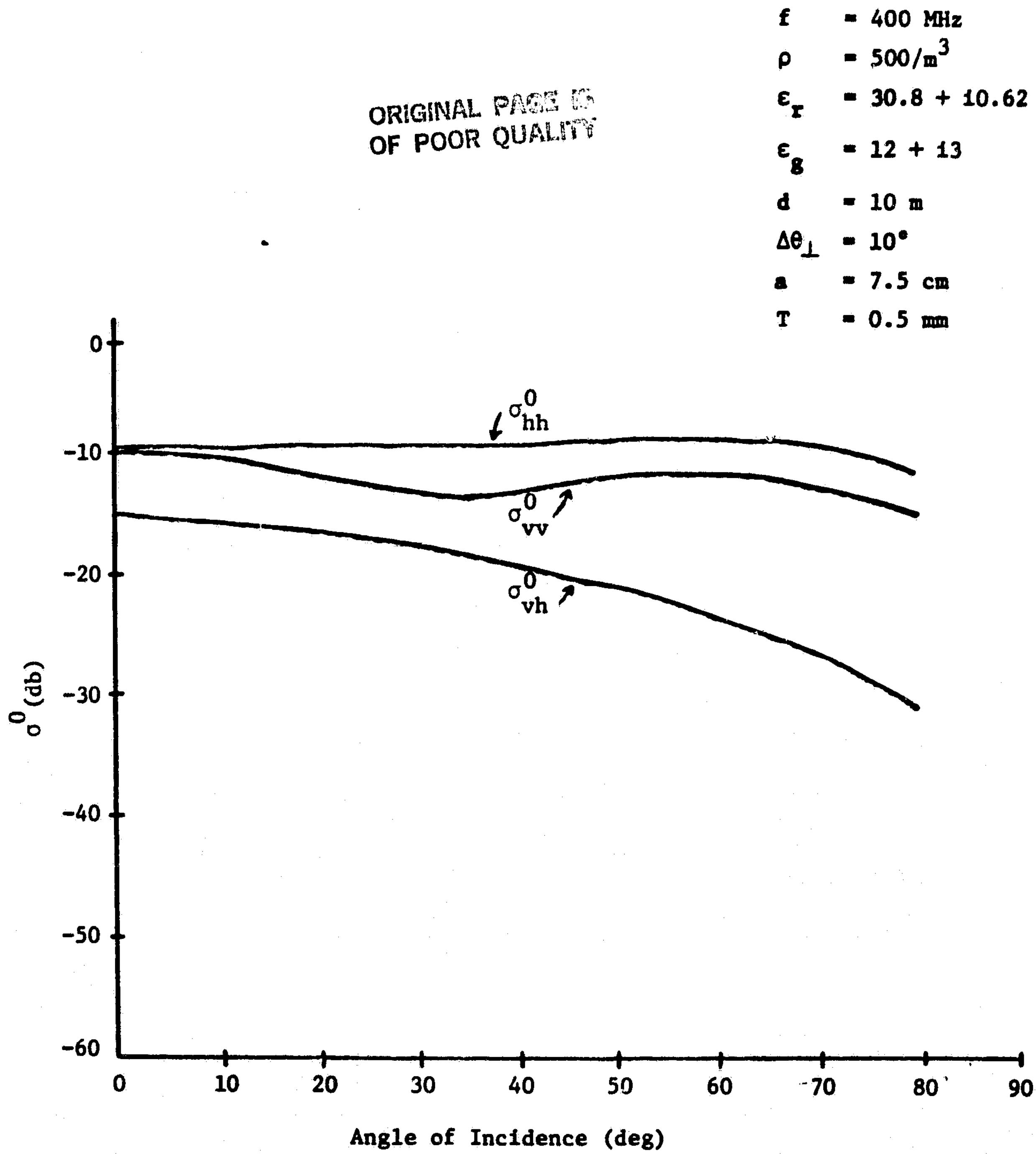


Fig. 16. Backscattering coefficients vs angle of incidence for Rayleigh regime

f = 400 MHz
ρ = $500/m^3$
ε_r = 30.8 + i0.62
ε_g = 12 + i3
d = 10 m
$\Delta\theta_{||}$ = 10°
a = 7.5 cm
T = 0.5 mm

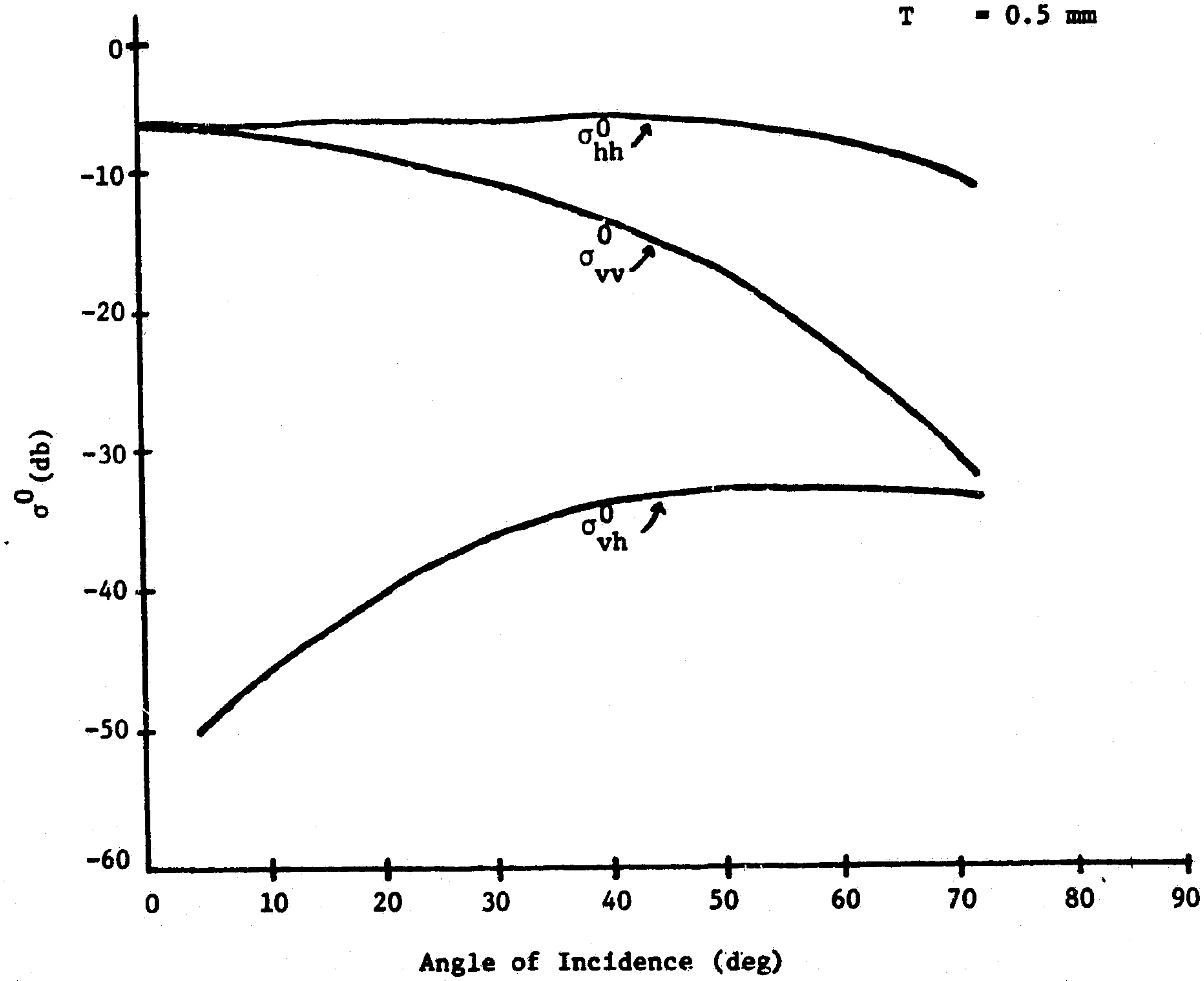


Fig. 17. Backscattering coefficients vs angle of incidence for Rayleigh regime

compared to the layer thickness for all angles of incidence, we are receiving scattering back from the same number of scatterers for each angle of incidence, resulting in a flat behavior for σ^0_{hhd}. The reflected terms σ^0_{hhdr} and σ^0_{hhr} slopes up with an increasing angle of incidence with the increasing reflection constant.

Figure (16) shows the total backscattering coefficients plotted for a perpendicular distribution of leaves. At small angles of incidence σ^0_{vv} and σ^0_{hh} are equal, and as the angle of incidence increases, σ^0_{vv} decreases because the vertically polarized wave becomes parallel to the leaves and thus gets absorbed. On the other hand, σ^0_{hh} has a flat response because the changing angle of incidence does not affect the polarization orientation with respect to the leaves.

Figure (17) shows the total backscattering coefficients plotted for a parallel distribution of leaves. In this case as the angle of incidence is increasing, the horizontal polarization is not affected since the leaves are parallel to the polarization at all angles. The backscattering cross section is higher as compared to the perpendicular distribution at small angles of incidence. Comparing σ^0_{vh} in figs. (17) and (16), we see that the vertically inclined leaves give rise to more depolarization as compared to the horizontally inclined leaves.

To see the effect of moisture in the underlying ground, we have selected two different dielectric constants. For dry ground, we have used $\varepsilon_g = 12+i3$, while for very wet ground we have used the dielectric constant of water obtained from Debye's formulation [1929], i.e.,

$$\varepsilon_g = \varepsilon_g' + i\varepsilon_g''$$

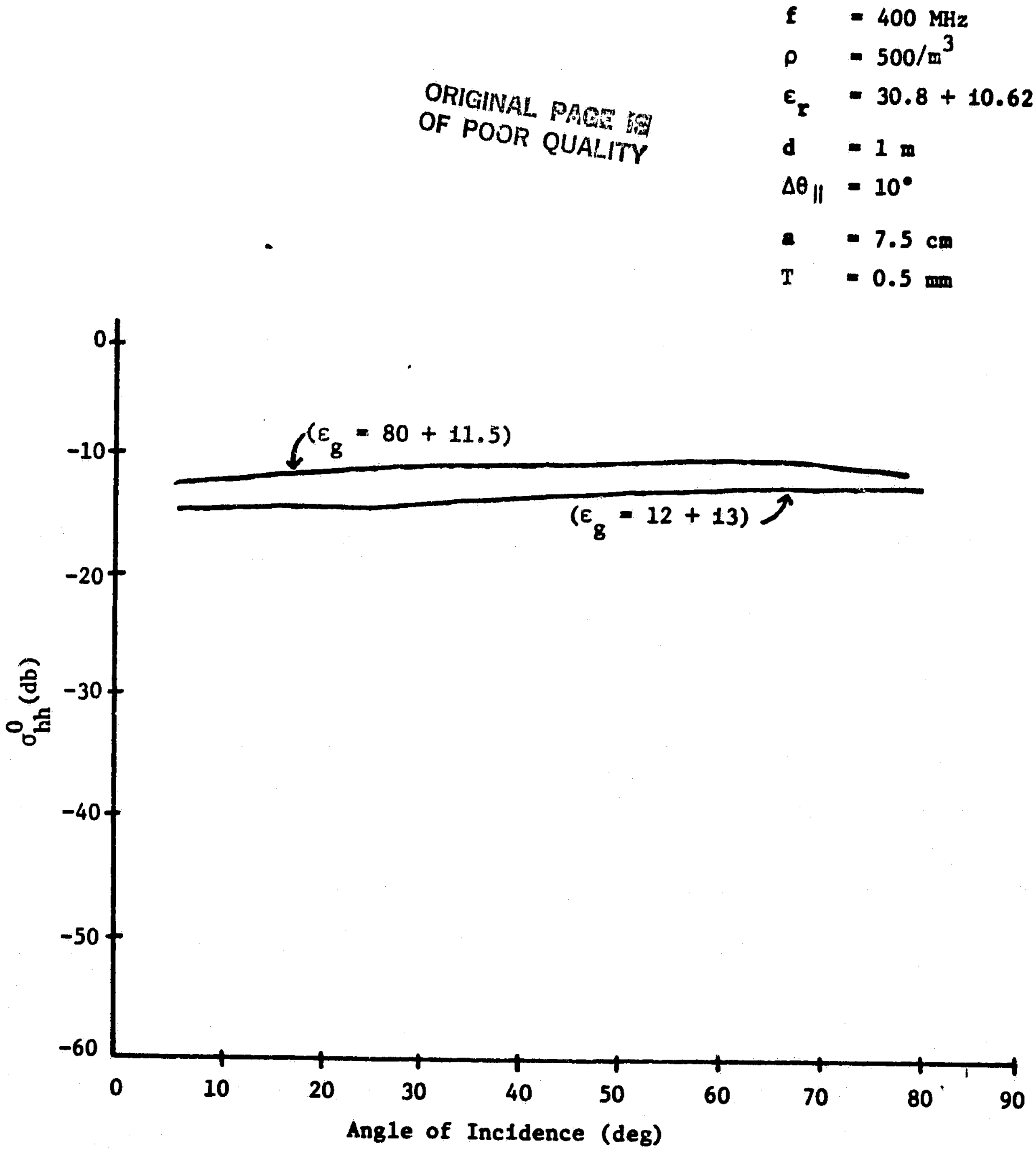


Fig. 18. Comparison of horizontal backscattering cross sections for different ground wetnesses

with

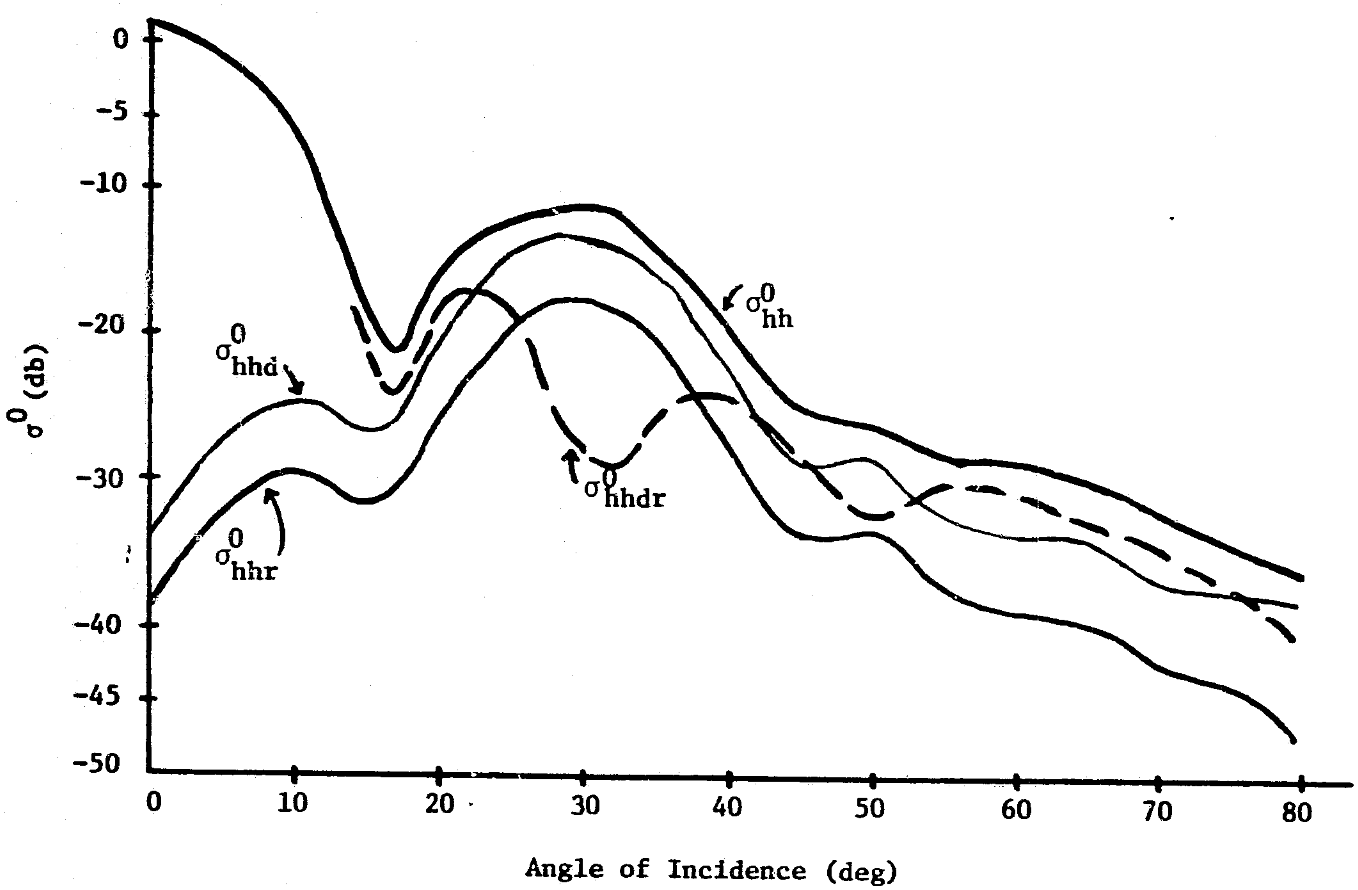
$$\varepsilon_g'' = \frac{(75)\left(\frac{1.85}{\lambda}\right)}{1 + \left(\frac{1.85}{\lambda}\right)^2} \tag{5.27}$$

and

$$\varepsilon_g' = 5 + \varepsilon_g''\left(\frac{\lambda}{1.85}\right) \tag{5.28}$$

where λ is the wavelength in centimeters. From eqs. (5.27) and (5.28) we have ε_g=80+i1.5 at f=400 MHz. Figure 18 shows σ^0_{hh} plotted for these two different ground conditions. From this figure we see that there is a large difference in the backscattering coefficients for different ground moistures because the ground reflected terms are dominant. When the layer thickness is increased, the ground reflected terms become small. Hence their effect is masked and only the direct component, σ^0_{ppd}, $p\epsilon\{h,v\}$ is important.

Next, we present the backscattering curves for the geometric optics regime. All of them have the same parameters: f=5 GHz, a=7 cm, T=0.1 mm, and ε_r=9.75+i1.31. An example of the results is shown in fig. (19) for h polarization. In this computation the discs all have an inclination angle of 30° with respect to the slab normal (θ=30°), however they are uniformily distributed in the azimuth coordinate. To aid in the interpretation of the results, the direct, reflected, and direct-reflected components have also been plotted. An examination of fig. (19) shows that σ^0_{hh} has two major peaks: one at an incidence angle of 0° and the other at 30°. The peak at 30° comes from the direct and reflected components and is due to specular reflection from the front and back faces of correctly oriented leaves. The larger term at 0° is caused by the direct-reflected components of the backscattering coefficient and is caused by the transmitted energy through



$f = 5$ GHz
$\rho = 500/m^3$
$\varepsilon_r = 9.75 + i1.31$
$\varepsilon_g = 73.5 + i21.1$
$d = 1$ m
$\theta = 30°$
$a = 7$ cm
$T = 0.1$ mm

Fig. 19. σ^0_{hh} and its components vs incidence angle for geometric optics regime

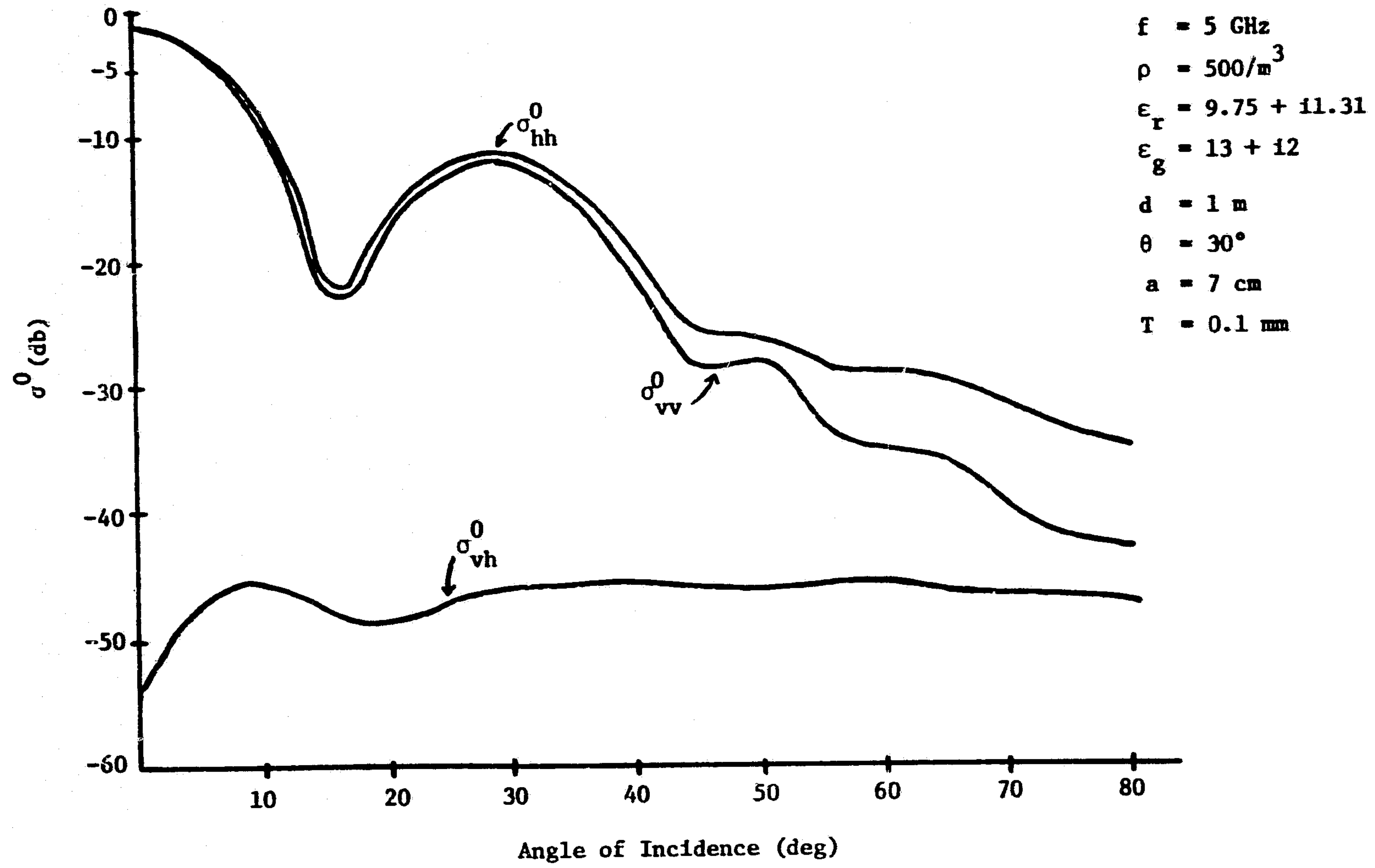


Fig. 20. Backscattering coefficients vs incident angle for geometric optics regime

the leaves that are reflected from the ground. The width of each peak is related to the radiation pattern or the beam-width of an individual leaf. The leaves considered have been chosen to be thin. This was necessary to keep their albedo small so that the distorted Born results could be applied. It is anticipated that by introducing a distribution of leaves, which have an inclination angles that depart somewhat from 30°, a considerable smoothing of the curve will occur. The parameters which we used in fig. (19) are $\rho=500/m^3$, d=1 m, $\theta=30°$, and $\epsilon_g=73.5+i21.1$.

We again obtain ϵ_g from Debye's formula to see the effect of moisture in the underlying ground. We use the parameters of fig. (19) in order to plot fig. (20) with a new ground dielectric constant $\epsilon_g=13+i2$. This value corresponds to volumetric water content (cm^3/cm^3) of 0.3, which is relatively dry ground, Wang and Schmugge [1980].

From figs. (19) and (20), we can see that, except for small incident angles, there is almost no difference in the backscattering coefficients for different ground moistures. This result differs from the result obtain in the Rayleigh regime as is shown in fig. (18). The difference is due to the physical optic scatterers which we are employing. They have a gain in the direction of the incident wave. It is this gain that was essentially compensating for the reflection loss at the interface. In fig. (20) we also plot σ^0_{vv} and σ^0_{vh} for illustrative purposes.

In fig. (21) we use the same parameters as in fig. (20) except ρ and d are increasing to values of $\rho=2000/m^3$ and d=10 m. We plot σ^0_{hh}, σ^0_{hv} and σ^0_{vv} in this figure. A comparison of the two figures shows that they are the same except that fig. (21) no longer has the peak at $\theta= 0°$. This is because increasing ρ and d made the skin depth substantially smaller. Thus the incident mean wave never reached the ground and there was no dr component in

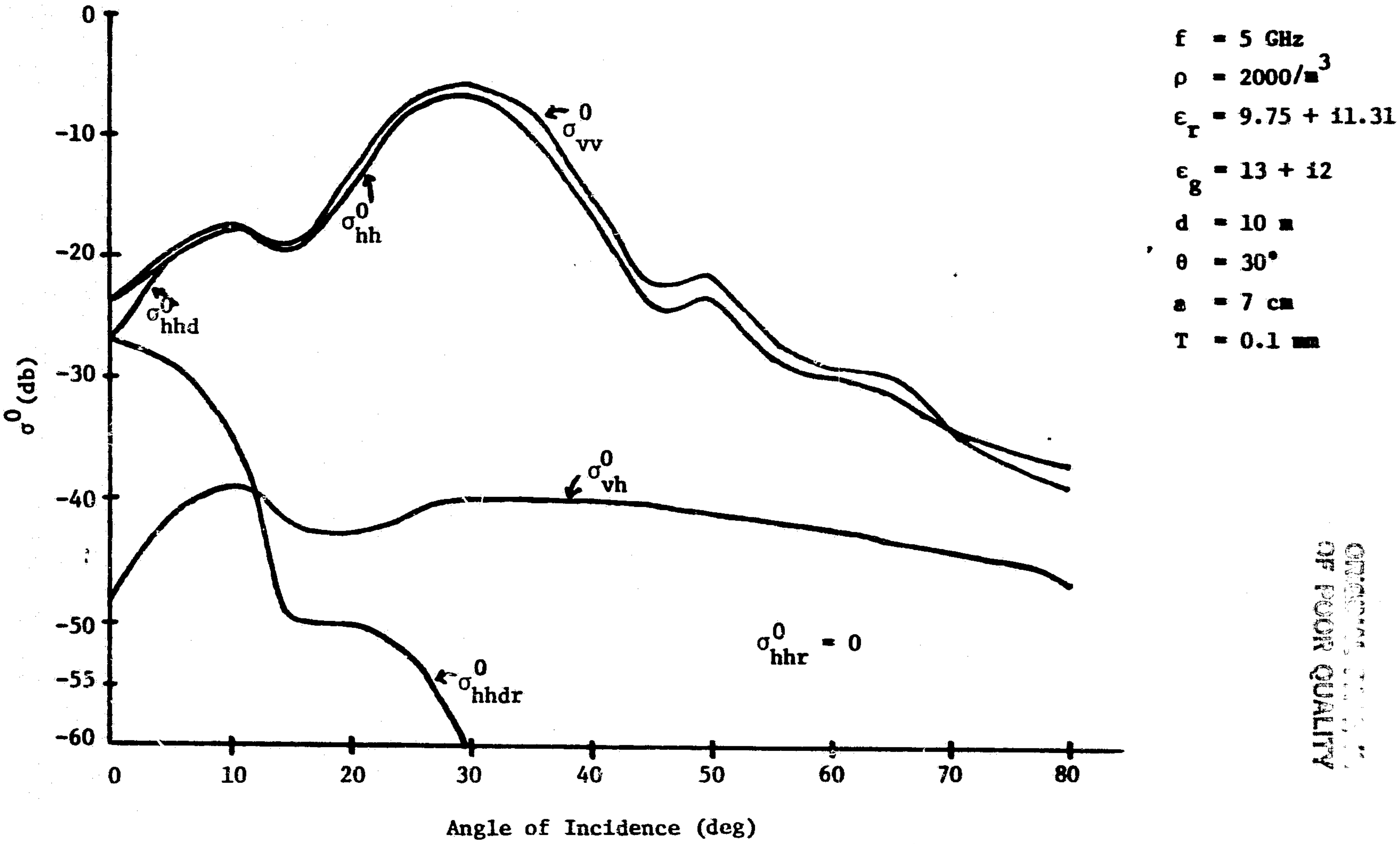


Fig. 21. Backscattering coefficients vs incidence angle for geometric optics regime

the backscatter. The layer essentially looks like a half space.

In fig. (22), we illustrate the effect of θ on the backscattering cross section in the geometric optics case. Since θ=90°, this figure corresponds to the leaves perpendicular to the ground. We also plot σ^0_{vv} and σ^0_{vh} on the same figure. In this case, we have $\sigma^0_{ppd} \approx \sigma^0_{ppr} \approx 0$ and $\sigma^0_{pp} \approx \sigma^0_{ppdr}$. We see that σ^0_{pp} has flat response due to the fact that the leaves act like a corner reflector.

At zero angle of incidence the value of σ^0_{vv} and σ^0_{hh} are equal, but as the angle of incidence increases, σ^0_{vv} decreases because the vertically polarized wave gets absorbed. It was observed that figs. (16) and (22) follow almost the same trend, except the depolarization is smaller in the geometric optics case.

So far, we have shown some backscattering coefficient curves. These curves are not smooth as obtained in the Rayleigh regime. This is due to the radiation pattern of the scatterers. In the Rayleigh regime, the dipole radiation pattern has a large beamwidth which in turn gives a smooth response. But in the high frequency domain, the radiation pattern has a smaller beamwidth compared to the Rayleigh regime, and the scatterers behave as a strong radiator. This in turn gives us the backscattering coefficients as we have shown throughout figs. (19)-(22). This is the reason why we observe a peak at 30° in the geometric optic regime.

We have analyzed numerically discrete scattering model for vegetation. We have calculated the skin depth and backscattering coefficients explicitly in terms of the scattering amplitude. The scattering albedo was calculated to make sure that it is small. It was found that the leaves had a much higher albedo than was previously anticipated at microwave frequency above 5 GHz.

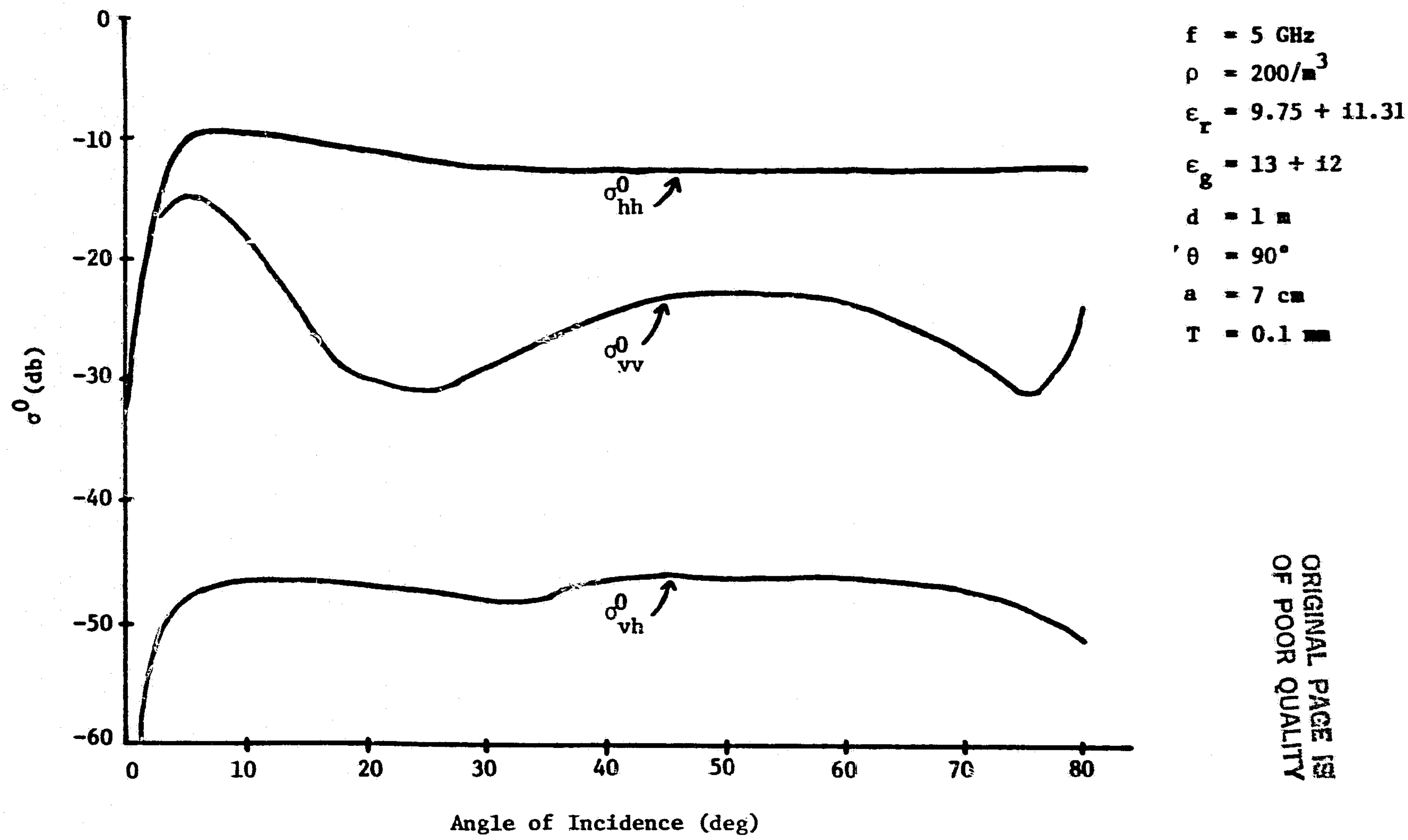


Fig. 22. Backscattering coefficients vs incidence angle for geometric optics regime

It was found that for vegetation layers less than one skin depth thick and for moist ground that σ^0_{pqdr} was the dominant term. It was also observed that thin layers had backscattering coefficients that were almost flat functions of angle of incidence in the Rayleigh regime. In the geometric optics regime this was not the case because the scatterers which we were employing had gain in the direction of the incident wave. It was this gain that was essentially compensating for the reflection loss at the interface.

C-2

APPENDIX A

RELATIONSHIP BETWEEN THE TRANSITION OPERATOR AND THE SCATTERING AMPLITUDE

In this Appendix we will find the relationship between $\underline{\underline{f}}$ and $\tilde{\underline{\underline{t}}}$. We start with equation (2.14). Using (2.15) and (2.16), we can write (2.14) in terms of the transition operator and incident field for N particles

$$L\Psi_s = T\Psi_0 = \sum_{N=1}^{\infty} L(L^{-1}V)^N \Psi_0 \tag{A.1}$$

From (A.1) we can write

$$T = \sum_{N=1}^{\infty} L(L^{-1}V)^N \tag{A.2}$$

From eq. (A.2), we have

$$T\Psi = V\Psi + VL^{-1}V\Psi + VL^{-1}VL^{-1}V\Psi + \ldots \tag{A.3}$$

Substituting eq. (2.8) for Ψ, (2.12c) for V and (2.19) for T in eq. (A.3) along with the free space dyadic Green's function for L^{-1}, we obtain

$$T\Psi = \begin{bmatrix} \underline{\underline{T}}_{11} & \underline{\underline{T}}_{12} \\ \underline{\underline{T}}_{21} & \underline{\underline{T}}_{22} \end{bmatrix} \begin{bmatrix} \underline{E} \\ \underline{H} \end{bmatrix} = \begin{bmatrix} \underline{\underline{\gamma}}(\underline{x}) & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{E} \\ \underline{H} \end{bmatrix} + \begin{bmatrix} \underline{\underline{\gamma}}(\underline{x}) & 0 \\ 0 & 0 \end{bmatrix} \int \begin{bmatrix} \underline{\underline{G}}_{11} & \underline{\underline{G}}_{12} \\ \underline{\underline{G}}_{21} & \underline{\underline{G}}_{22} \end{bmatrix} \begin{bmatrix} \underline{\underline{\gamma}}(\underline{x}) & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{E}(\underline{x}') \\ \underline{H}(\underline{x}') \end{bmatrix} d\underline{x}'$$

$$+ \iint d\underline{x}' d\underline{x}'' \begin{bmatrix} \underline{\underline{\gamma}}(\underline{x}) & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{\underline{G}}_{11} & \underline{\underline{G}}_{12} \\ \underline{\underline{G}}_{21} & \underline{\underline{G}}_{22} \end{bmatrix} \begin{bmatrix} \underline{\underline{\gamma}}(\underline{x}') & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{\underline{G}}_{11} & \underline{\underline{G}}_{12} \\ \underline{\underline{G}}_{21} & \underline{\underline{G}}_{22} \end{bmatrix} \begin{bmatrix} \underline{\underline{\gamma}}(\underline{x}'') & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{E} \\ \underline{H} \end{bmatrix} + \ldots \tag{A.4}$$

where $\underline{\underline{\gamma}}(\underline{x}) = i\omega\varepsilon_0 \Delta U(\underline{x})\underline{\underline{I}}$

Using eq. (2.18), (A.4) can be expressed as

$$\int \begin{bmatrix} \underline{\underline{t}}_{11}(\underline{x},\underline{x}') & \underline{\underline{t}}_{12}(\underline{x},\underline{x}') \\ \underline{\underline{t}}_{21}(\underline{x},\underline{x}') & \underline{\underline{t}}_{22}(\underline{x},\underline{x}') \end{bmatrix} \begin{bmatrix} \underline{E}(\underline{x}') \\ \underline{H}(\underline{x}') \end{bmatrix} d\underline{x}' = \int d\underline{x}' \Bigg[\begin{pmatrix} \underline{\underline{\gamma}}(\underline{x}') & 0 \\ 0 & 0 \end{pmatrix} \delta(\underline{x}-\underline{x}') + \begin{pmatrix} \underline{\underline{\gamma}}(\underline{x})\cdot\underline{\underline{G}}_{11}\cdot\underline{\underline{\gamma}}(\underline{x}') & 0 \\ 0 & 0 \end{pmatrix} + \int d\underline{x}'' \begin{pmatrix} \underline{\underline{\gamma}}(\underline{x})\cdot\underline{\underline{G}}_{11}\cdot\underline{\underline{\gamma}}(\underline{x}')\cdot\underline{\underline{G}}_{11}\cdot\underline{\underline{\gamma}}(\underline{x}'') & 0 \\ 0 & 0 \end{pmatrix} + \dots \Bigg] \begin{bmatrix} \underline{E}(\underline{x}') \\ \underline{H}(\underline{x}') \end{bmatrix} \tag{A.5}$$

Now, we can conclude by comparing both sides of eq. (A.5) that for dielectric scatterer

$$\underline{\underline{t}}_{12} = \underline{\underline{t}}_{22} = \underline{\underline{t}}_{21} = 0 \tag{A.6}$$

and

$$\underline{\underline{t}}_{11}(\underline{x},\underline{x}') \neq 0 \tag{A.7}$$

with

$$\underline{\underline{t}}_{11} = \frac{-1}{i\omega\mu_0}\, \underline{\underline{t}}(\underline{x},\underline{x}')$$

To find the relationship between the transition operator and the scattering amplitude, we write out eq. (2.17) explicitly:

$$\Psi_s(\underline{x}) = L^{-1}T\Psi_0 = \int d\underline{x}' G(\underline{x},\underline{x}') \int d\underline{x}'' t(\underline{x}',\underline{x}'')\Psi_0(\underline{x}'') \tag{A.8}$$

Here the free space dyadic Green's function $G(\underline{x},\underline{x}')$ is obtained from Felsen and Marcuvitz, Chapter I, Sec. 1.1, it is given by

$$G(\underline{x},\underline{x}') = \begin{bmatrix} -i\omega\mu_0(\underline{\underline{I}}+\frac{\nabla\nabla}{k_0^2}) & \nabla x\underline{\underline{I}} \\ -\nabla x\underline{\underline{I}} & -i\omega\varepsilon_0(\underline{\underline{I}}+\frac{\nabla\nabla}{k_0^2}) \end{bmatrix} \frac{e^{ik_0|\underline{x}-\underline{x}'|}}{4\pi|\underline{x}-\underline{x}'|} \tag{A.9}$$

To obtain $\Psi_s(\underline{x})$ in the radiation zone, the far field expression for $G(\underline{x},\underline{x}')$ will be required [Twersky, 1967]:

$$G(\underline{x},\underline{x}') \sim H\, e^{-ik_0\,\underline{0}\cdot\underline{x}'} \frac{e^{ik_0|\underline{x}|}}{4\pi|\underline{x}|} \qquad |\underline{x}|\to\infty \tag{A.10}$$

where

$$H = \begin{bmatrix} -i\omega\mu_0(\underline{\underline{I}}-\underline{0}\,\underline{0}) & ik_0\,\underline{0}\,x(\underline{\underline{I}}) \\ -ik_0\,\underline{0}\,x(\underline{\underline{I}}) & -i\omega\varepsilon_0(\underline{\underline{I}}-\underline{0}\,\underline{0}) \end{bmatrix} \tag{A.11}$$

Substituting eqs. (A.10) and (A.11) in (A.8) and using for a incident wave

$$\Psi_0(\underline{x}) = \Psi_0(0)\, e^{ik_0\underline{i}\cdot\underline{x}''} \tag{A.12}$$

and

$$\tilde{t}(k_0\underline{0},k_0\underline{i}) = \frac{1}{(2\pi)^3}\int d\underline{x}'d\underline{x}''\,t(\underline{x}',\underline{x}'')\, e^{ik_0(\underline{i}\cdot\underline{x}''-\underline{0}\cdot\underline{x}')} \tag{A.13}$$

we obtain

$$\Psi_s(\underline{x}) = 2\pi^2 H\,\tilde{t}(k_0\underline{0},k_0\underline{i})\Psi_0(0)\,\frac{e^{ik_0|\underline{x}|}}{|\underline{x}|} \tag{A.14}$$

where

$$\Psi_s = \begin{bmatrix} \underline{\underline{e}}_s \\ \underline{\underline{h}}_s \end{bmatrix} \tag{A.15}$$

Using eqs. (A.15) and (A.7) and substituting H into the eq. (A.14), we obtain for dyadic scattered wave

$$\underline{\underline{e}}_s(\underline{x}) = 2\pi^2[(\underline{\underline{I}} - \underline{0}\,\underline{0})\cdot\tilde{\underline{\underline{t}}}(k_0\underline{0},k_0\underline{i})\cdot\underline{\underline{e}}_i(0)]\,\frac{e^{ik_0|\underline{x}|}}{|\underline{x}|} \tag{A.16}$$

The definition of scattering amplitude given by [Ishimaru, 1978]

$$\underline{\underline{e}}_s(\underline{x}) \sim \underline{\underline{f}}^{(E)}(\underline{0},\underline{i}) \frac{e^{ik_0|\underline{x}|}}{|\underline{x}|} \quad , \quad |\underline{x}| \to \infty \quad , \quad \underline{\underline{f}} \equiv \underline{\underline{f}}^{(E)} \tag{A.17}$$

Assuming dyadic incident wave, $\underline{\underline{e}}_i(0) = (\underline{\underline{I}} - \underline{i}\ \underline{i})$, finally we obtain the required result

$$\underline{\underline{f}}^{(E)}(\underline{0},\underline{i}) = 2\pi^2(\underline{\underline{I}} - \underline{0}\ \underline{0})\cdot\tilde{\underline{\underline{t}}}(k_0\underline{0},k_0\underline{i})\cdot(\underline{\underline{I}} - \underline{i}\ \underline{i}) \tag{A.18}$$

Equation (A.18) is the same as Lang [1981] obtained with different method. From eq. (A.14) we can also obtain

$$\underline{\underline{h}}_s(\underline{x}) = 2\pi^2\eta[(\underline{0}\times\underline{\underline{I}})\cdot\tilde{\underline{\underline{t}}}(k_0\underline{0},k_0\underline{i})\cdot\underline{\underline{e}}_i(0)] \frac{e^{ik_0|\underline{x}|}}{|\underline{x}|} . \tag{A.19}$$

The definition of scattering amplitude for the H field given by

$$\underline{\underline{h}}_s(\underline{x}) \sim \underline{\underline{f}}^{(H)}(\underline{0},\underline{i}) \frac{e^{ik_0|\underline{x}|}}{|\underline{x}|} \quad , \quad |\underline{x}| \to \infty \tag{A.20}$$

For a dyadic incident wave $\underline{\underline{e}}_i(0) = (\underline{\underline{I}} - \underline{i}\ \underline{i})$ we have

$$\underline{\underline{f}}^{(H)}(\underline{0},\underline{i}) = 2\pi^2\eta\,(\underline{0}\times\underline{\underline{I}})\cdot\tilde{\underline{\underline{t}}}(k_0\underline{0},k_0\underline{i})\cdot(\underline{\underline{I}} - \underline{i}\ \underline{i}) \tag{A.21}$$

where

$$\eta = \frac{k_0}{\omega\mu_0} = \sqrt{\frac{\varepsilon_0}{\mu_0}} \tag{A.22}$$

In order to obtain the relationship between two different scattering amplitudes, we take the cross product of eq. (A.21) with respect to $\underline{0}$ direction we find

$$\underline{0}\times\underline{\underline{f}}^{(H)} = 2\pi^2\eta\,\underline{0}\times(\underline{0}\times\underline{\underline{I}})\cdot\tilde{\underline{\underline{t}}}\cdot(\underline{\underline{I}} - \underline{i}\ \underline{i}) \tag{A.23}$$

but

$$\underline{0}\times(\underline{0}\times\underline{\underline{I}}) = -(\underline{\underline{I}} - \underline{0}\ \underline{0}) \tag{A.24}$$

then eq. (A.23) becomes

$$\underline{0}\times\underline{\underline{f}}^{(H)} = \eta 2\pi^2(\underline{\underline{I}} - \underline{0}\ \underline{0})\cdot\tilde{\underline{\underline{t}}}\cdot(\underline{\underline{I}} - \underline{i}\ \underline{i}) \tag{A.25}$$

Using eq. (A.18) in eq. (A.25) we obtain the following relationship for scattering amplitudes for E and H fields

$$\underline{\underline{f}}^{(E)}(\underline{0},\underline{i}) = \sqrt{\frac{\mu_0}{\varepsilon_0}}\,\underline{0} \times \underline{\underline{f}}^{(H)}(\underline{0},\underline{i}) \qquad (A.26)$$

ORIGINAL PAGE IS
OF POOR QUALITY

APPENDIX B

EVALUATION OF ZEROTH ORDER SOLUTION

OF SLOWLY VARYING COEFFICIENTS

In this section, we will evaluate eq. (3.59), which is given by

$$\lim_{z\to\infty} \frac{1}{z}\, \phi^{(1)}(z,\bar{z}) = 0 \tag{B.1}$$

where

$$\phi^{(1)}(z,\bar{z}) = -\frac{d\phi^{(0)}(\bar{z})}{d\bar{z}}\, z + \bar{\rho}(\bar{z})\left\{\int_0^z dz' \int ds \int dz''\, y^{-1}(z')\tilde{\bar{X}} y(z'')\phi^{(0)}(\bar{z})\right\} \mathrm{rect}_d(z) \tag{B.2}$$

Performing eq.(B.1), we obtain

$$\frac{d}{d\bar{z}}\phi^{(0)}(\bar{z}) = \bar{\rho}(\bar{z}) M \phi^{(0)}(\bar{z})\,\mathrm{rect}_d(z) \tag{B.3}$$

where

$$M = \lim_{z\to\infty}\frac{1}{z}\int_0^z dz' \int_{-\infty}^{\infty} ds \int_{-\infty}^{\infty} dz''\, y^{-1}(z')\tilde{\bar{X}}(\underline{k}_{t_0}, z'-s, z''-s)\, y(z'') \tag{B.4}$$

Now, using eq. (3.51) we have

$$M = \lim_{z\to\infty}\frac{1}{z}\int_0^z dz' \int ds dz'' \begin{bmatrix} \tilde{\underline{\underline{X}}}_\phi^{(-,-)} & \tilde{\underline{\underline{X}}}_\phi^{(-,+)} \\ \tilde{\underline{\underline{X}}}_\phi^{(+,-)} & \tilde{\underline{\underline{X}}}_\phi^{(+,+)} \end{bmatrix} \tag{B.5}$$

where

$$\tilde{\underline{\underline{X}}}_\phi^{(\alpha,\beta)}(z',z'',s) = \frac{(\alpha)}{2N_T}\underline{\underline{I}}^\alpha \cdot \tilde{\underline{\underline{X}}}_{11}(\underline{k}_{t_0}, z'-s, z''-s)\cdot \underline{\underline{I}}^\beta e^{-i\kappa(\alpha z' - \beta z'')} \tag{B.6}$$

$$\alpha,\beta \in \{+,-\}$$

The integrals over ds and dz´´ will now be evaluated. First define

$$\underline{\underline{P}}^{(\alpha,\beta)} = \int ds dz'' \tilde{\overline{\underline{\underline{X}}}}_{\phi}^{(\alpha,\beta)}(z',z'',s) \tag{B.7}$$

By using eq. (B.6) in eq. (B.7), we obtain

$$\underline{\underline{P}}^{(\alpha,\beta)} = \frac{(\alpha)}{2N_T} \underline{\underline{I}}^{\alpha} \cdot \underline{\underline{Q}}^{(\alpha,\beta)} \cdot \underline{\underline{I}}^{\beta} \tag{B.8}$$

where

$$\underline{\underline{Q}}^{(\alpha,\beta)} = \int ds dz'' \tilde{\overline{\underline{\underline{X}}}}_{11}(\underline{k}_{t_0}, z'-s, z''-s) e^{-i\kappa(\alpha z' - \beta z'')} \tag{B.9}$$

The quantity $\tilde{\overline{\underline{\underline{X}}}}_{11}$ will be related to $\tilde{\underline{\underline{t}}}_{11}$. We note that

$$\tilde{\overline{\underline{\underline{X}}}}_{11}(\underline{k}_{t_0}, z'-s, z''-s) = \int d\underline{x}_t \overline{\underline{\underline{X}}}_{11}(\underline{x}_t, z'-s, z''-s) e^{-i\underline{k}_{t_0} \cdot \underline{x}_t} d\underline{x}_t \tag{B.10}$$

and from eq. (3.86) we have

$$\overline{\underline{\underline{X}}}_{11}(\underline{x}_t, z'-s, z''-s) = \int d\hat{\underline{s}}_t \, \overline{\underline{\underline{t}}}_{11}(-\hat{\underline{s}}_t, -\underline{x}_t - \hat{\underline{s}}_t, z'-s, z''-s) \tag{B.11}$$

We also have from eq. (2.22)

$$\overline{\underline{\underline{t}}}_{11}(\underline{x}_t, \underline{x}_t', z, z') = \frac{1}{(2\pi)^3} \int d\underline{\kappa}_t d\kappa_z d\underline{\kappa}_t' d\kappa_z' \tilde{\underline{\underline{t}}}_{11}(\underline{\kappa}_t, \kappa_z; \underline{\kappa}_t', \kappa_z') \, e^{i[\underline{k}_t \cdot \underline{x}_t - \underline{k}_t' \cdot \underline{x}_t' + \kappa_z z - \kappa_z' z']} \tag{B.12}$$

By using eqs. (B.10), (B.11) and (B.12) in eq. (B.9), evaluating the resulting integrals over $d\underline{x}_t$, $d\hat{\underline{s}}_t$, ds and dz´´, and using the resulting Dirac delta functions to eliminate the remaining integrals, we obtain

$$\underline{\underline{Q}}^{(\alpha,\beta)} = (2\pi)^3 \tilde{\underline{\underline{t}}}_{11}(\underline{k}_{t_0}, \beta\kappa; \underline{k}_{t_0}, \beta\kappa) e^{i\kappa(\beta-\alpha)z'} \tag{B.13}$$

Putting eq. (B.13) in eq. (B.8), we have

$$\underline{\underline{P}}^{(\alpha,\beta)} = \frac{(\alpha)(2\pi)^3}{2N_T} \underline{\underline{I}}^{\alpha} \cdot \tilde{\underline{\underline{t}}}_{11}(\underline{k}_{t_0}, \beta\kappa; \underline{k}_{t_0}, \beta\kappa) \cdot \underline{\underline{I}}^{\beta} \cdot e^{i\kappa(\beta-\alpha)z'} \tag{B.14}$$

The final expression for M can now be obtained. We put eq. (B.14) into eq. (B.5) and perform the integration over z′ and also take the limit as z goes to infinity. We obtain

$$M = \begin{bmatrix} \underline{\underline{m}}^- & 0 \\ 0 & \underline{\underline{m}}^+ \end{bmatrix} \tag{B.15}$$

where

$$\underline{\underline{m}}^\beta = \frac{\beta(2\pi)^3}{2N_T} \underline{\underline{I}}^\beta \cdot \tilde{\underline{\underline{t}}}_{11}(\underline{k}_{t_0}, \beta\kappa; \underline{k}_{t_0}, \beta\kappa) \cdot \underline{\underline{I}}^\beta \tag{B.16}$$

$$\beta = \{+,-\}$$

Now employing eqs. (2.20), (2.23), (3.27b) and (3.37a), we have

$$\underline{\underline{m}}^\beta = \frac{\beta i 4\pi^3}{\kappa} \underline{\underline{I}}^\beta \cdot \tilde{\underline{\underline{t}}}(k_0 \underline{i}^0_\beta, k_0 \underline{i}^0_\beta) \cdot \underline{\underline{I}}^\beta \tag{B.17}$$

$$\beta \in \{+,-\}$$

APPENDIX C

EVALUATION OF $\underline{\underline{T}}$ AND $\underline{\underline{\Gamma}}$

To find an expression for $\underline{\underline{T}}$ we use eq. (3.83) in eq. (3.81). We find

$$\underline{A}(-\bar{d}) = -\frac{1}{2N_T}(y^{-T}\Gamma y_g^-)\cdot\underline{\underline{T}}\cdot\underline{A}(-\bar{d})e^{i(\kappa_g-\kappa)d} \tag{C.1}$$

where $(y^{-T}\Gamma y_g^-)$ is a dyadic and $N_T=\eta\kappa/k_0$. By taking the inverse of this dyadic an expression for $\underline{\underline{T}}$ can be obtained. It is:

$$\underline{\underline{T}} = -2N_T(y^{-T}\Gamma y_g^-)^{-1}\cdot\underline{\underline{I}}^-\ e^{i(\kappa-\kappa_g)d} \tag{C.2}$$

where we have used the identities $\underline{\underline{I}}^-\cdot\underline{A} = \underline{A}$ and $(y^{-T}\Gamma y_g^-)^{-1}\cdot(y^{-T}\Gamma y_g^-) = \underline{\underline{I}}_g^-$. The expression for $\underline{\underline{\Gamma}}$ can now be obtained by employing eq. (3.84) and eq. (C.2). We have

$$\underline{\underline{\Gamma}} = -(y^{+T}\Gamma y_g^-)\cdot(y^{-T}\Gamma y_g^-)^{-1}\cdot\underline{\underline{I}}^-\ e^{2i\kappa d} \tag{C.3}$$

The quantities $y^{+T}\Gamma y_g^-$ and $y^{-T}\Gamma y_g^-$ need to be evaluated. From eq. (3.31), we write

$$y^- = \Psi_{-1}\,\underline{v}_-^0 + \Psi_{-2}\,\underline{h}^0 \quad\text{and}\quad y^+ = \Psi_{+1}\,\underline{v}_+^0 + \Psi_{+2}\,\underline{h}^0 \quad,\quad z>-d$$

$$y_g^- = \Psi_{1g}\,\underline{v}_g^0 + \Psi_{2g}\,\underline{h}^0 \quad,\quad z<-d \tag{C.4}$$

where $\Psi_{\pm1}$, $\Psi_{\pm2}$, Ψ_{1g} and Ψ_{2g} are given by eq. (3.28). Using these expressions, we find

$$y^{+T}\Gamma y_g^- = c\underline{v}_+^0\underline{v}_g^0 + \hat{c}\underline{v}_+^0\underline{h}^0 + \hat{d}\,\underline{h}^0\underline{v}_g^0 + d\,\underline{h}^0\underline{h}^0 \tag{C.5a}$$

and

$$y^{-T}\Gamma y_g = a\underline{v}_-^0\underline{v}_g^0 + \hat{a}\underline{v}_-^0\underline{h}^0 + \hat{b}\,\underline{h}^0\underline{v}_g^0 + b\,\underline{h}^0\underline{h}^0 \tag{C.5b}$$

where

$$a = (\Psi_{-1},\Gamma\Psi_{1g}) \qquad \hat{a} = (\Psi_{-1},\Gamma\Psi_{2g}) \tag{C.6a}$$

$$b = (\Psi_{-2},\Gamma\Psi_{2g}) \qquad \hat{b} = (\Psi_{-2},\Gamma\Psi_{1g}) \tag{C.6b}$$

$$c = (\Psi_{+1},\Gamma\Psi_{1g}) \qquad \hat{c} = (\Psi_{+1},\Gamma\Psi_{2g}) \tag{C.6c}$$

$$d = (\Psi_{+2},\Gamma\Psi_{2g}) \qquad \hat{d} = (\Psi_{+2},\Gamma\Psi_{1g}) \tag{C.6d}$$

Substituting the expressions for $\Psi_{\pm i}$, Ψ_{ig}, i=1,2 in eqs. (C.6) we have

$$a = \eta_g \underline{v}^0_- \cdot \underline{z}^0 x \underline{h}^0 - \eta \underline{h}^0 \cdot \underline{z}^0 x \underline{v}^0_g \qquad \hat{a} = 0 \tag{C.7a}$$

$$b = -\eta_g \underline{h}^0 \cdot \underline{z}^0 x \underline{v}^0_g + \eta \underline{v}^0_- \cdot \underline{z}^0 x \underline{h}^0 \qquad \hat{b} = 0 \tag{C.7b}$$

$$c = \eta_g \underline{v}^0_+ \cdot \underline{z}^0 x \underline{h}^0 - \eta \underline{h}^0 \cdot \underline{z}^0 x \underline{v}^0_g \qquad \hat{c} = 0 \tag{C.7c}$$

$$d = -\eta_g \underline{h}^0 \cdot \underline{z}^0 x \underline{v}^0_g + \eta \underline{v}^0_+ \cdot \underline{z}^0 x \underline{h}^0 \qquad \hat{d} = 0 \tag{C.7d}$$

Now using eqs. (3.24) and (3.25) in the expressions for a-d in eqs. (C.7), we have

$$a = -\eta(\varepsilon_g \kappa + \kappa_g)/k_g \tag{C.8a}$$

$$b = -\eta(\kappa + \kappa_g)/k_0 \tag{C.8b}$$

$$c = \eta(\varepsilon_g \kappa - \kappa_g)/k_g \tag{C.8c}$$

$$d = \eta(\kappa - \kappa_g)/k_0 \tag{C.8d}$$

Proceeding, we use the orthogonality results of eqs. (C.7) which yield

$$y^{+T} \Gamma y^-_g = c\, \underline{v}^0_+ \underline{v}^0_g + d\, \underline{h}^0 \underline{h}^0 \tag{C.9a}$$

$$y^{-T} \Gamma y^-_g = a\, \underline{v}^0_- \underline{v}^0_g + b\, \underline{h}^0 \underline{h}^0 \tag{C.9b}$$

To complete the calculation, the inverse of $y^{-T}\Gamma y_g$ is required. It requires that

$$(y^{-T}\Gamma y_g)^{-1}\cdot(y^{-T}\Gamma y_g) = \underline{\underline{I}}_g^- \tag{C.10}$$

or

$$(y^{-T}\Gamma y_g)^{-1}\cdot(a\underline{v}^0_-\underline{v}^0_g + b\underline{h}^0\underline{h}^0) = \underline{v}^0_g\underline{v}^0_g + \underline{h}^0\underline{h}^0 \tag{C.11}$$

If

$$(y^{-T}\Gamma y_g)^{-1} = \frac{1}{a}\,\underline{v}^0_g\underline{v}^0_- + \frac{1}{b}\,\underline{h}^0\underline{h}^0 \tag{C.12}$$

then eq.(C.11) is satisfied. Putting eqs. (C.9a) and (C.12) into eqs. (C.2) and C.3) we have

$$\underline{\underline{T}} = \underline{\underline{T}}_g\, e^{i(\kappa-\kappa_g)d} \tag{C.13}$$

where

$$\underline{\underline{T}}_g = T_{gh}\,\underline{h}^0\underline{h}^0 + T_{gv}\,\underline{v}^0_g\underline{v}^0_- \tag{C.14}$$

and

$$\underline{\underline{\Gamma}} = \underline{\underline{\Gamma}}_g\, e^{2i\kappa d} \tag{C.15}$$

where

$$\underline{\underline{\Gamma}}_g = \Gamma_{gh}\,\underline{h}^0\underline{h}^0 + \Gamma_{gv}\,\underline{v}^0_-\underline{v}^0_+ \tag{C.16}$$

The Fresnel transmission and reflection coefficients are given by

$$T_{gh} = \frac{2\kappa}{\kappa+\kappa_g}\ , \qquad T_{gv} = \frac{2\kappa\sqrt{\varepsilon_g}}{\varepsilon_g\kappa+\kappa_g} \tag{C.17}$$

and

$$\Gamma_{gh} = \frac{\kappa-\kappa_g}{\kappa+\kappa_g}\ , \qquad \Gamma_{gv} = \frac{\varepsilon_g\kappa-\kappa_g}{\varepsilon_g\kappa+\kappa_g} \tag{C.18}$$

APPENDIX D

CALCULATION OF $\overline{f}_{vh}(\underline{i}^0,\underline{i}^0)$

In this Appendix, we will show that $\overline{f}_{vh}=\overline{f}_{vh}=0$. We start by writting scattering amplitude that is given by LeVine, et al [1982].

$$\underline{\underline{f}}(\underline{i}^0,\underline{0}^0) = \left(\frac{k_0^2}{4\pi}\right) T\Delta\tilde{S}(\nu_t)(\underline{h}^0\underline{h}^0 e_h^+ + \underline{v}^0_{\varepsilon+}\underline{v}^0_- e_v^-)\,\text{sinc}\theta^-$$
$$+ (\underline{h}^0\underline{h}^0 e_h^- + \underline{v}^0_{\varepsilon-}\underline{v}^0_- e_v^-)\ \text{sinc}\theta^+ \qquad \text{(D.1)}$$

where $\underline{i}^0$ and $\underline{0}^0$ are the directions of incident and scattering waves respectively. The expressions appearing in eq. (D.1) are defined in chapter V.

We need f_{vh} in forward direction. Using $\underline{i}^0=\underline{0}^0$ and dotting eq. (D.1) by h_i from the right and by $\underline{v}_i^0$ from the left, we find:

$$f_{vh} = \underline{v}_i^0\cdot\underline{\underline{f}}(\underline{i}^0,\underline{i}^0)\cdot\underline{h}_i^0 = \beta\{[ae_h^+ + b_+ce_v^+]\text{sinc}\theta^- + [ae_h^- + b_-ce_v^-]\text{sinc}\theta^+\} \qquad \text{(D.2)}$$

where

$$\beta = \left(\frac{k_0^2}{4\pi}\right)T\Delta\tilde{S}_0$$
$$a = (\underline{v}_i^0\cdot\underline{h}^0)(\underline{h}^0\cdot\underline{h}_i^0)$$
$$b_\pm = \underline{v}_i^0\cdot\underline{v}_{\varepsilon\pm} \qquad \text{(D.3)}$$
$$c = \underline{v}_-^0\cdot\underline{h}_i^-$$

and $\underline{h}_i^0$, $\underline{v}_i^0$ are the unit horizontal and vertical vectors of the incident wave.

Next by expanding the unit vectors in cartesian coordinates, we determine that f_{vh} has the following dependence on $\overline{\phi} = \phi - \phi_i$

$$f_{vh} = h(\sin\overline{\phi})\cos\overline{\phi} \tag{D.4}$$

where $h(\zeta)$ is a single valued function of ζ. Using eq. (D.4), and assuming ϕ is uniformily distributed, we have:

$$\overline{f}_{hv} = \frac{1}{2\pi}\int_{-\pi}^{\pi} f_{vh}d\phi = \frac{1}{2\pi}\int_{-\pi}^{\pi} f_{vh}d\overline{\phi} = \frac{1}{2\pi}\int_{-\pi}^{\pi} h(\sin\overline{\phi})\cos\overline{\phi}d\overline{\phi} = 0 \tag{D.5}$$

The last integral is zero because of the even and odd character of $\cos\overline{\phi}$ and $\sin\overline{\phi}$; thus $\overline{f}_{vh}=0$. We also find that $\overline{f}_{vh}=0$ using the same method.

Using a similar procedure one can show that

$$\kappa_p^+ = \kappa_p^- = \kappa_p \tag{D.6}$$

REFERENCES

Barber, P., and C. Yeh (1975), "Scattering of Electromagnetic Waves by Arbitrarily Shaped Dielectric Bodies," Applied Optics, 14(12), 2864-2872.

Bellman, R. (1960), Introduction to Matrix Analysis, McGraw-Hill, New York.

Born, M., and E. Wolf (1964), Principles of Optics, Macmillan, New York.

Broadhurst, M.G. (1970), "Complex Dielectric Constant and Dissipation Factor of Foliage," U.S. National Bureau of Standards Report 9592.

Cole, R. (1968), Theory of Ordinary Differential Equations, Appleton-Century-Crofts.

Cole, J.D. (1968), Perturbation Methods in Applied Mathematics, Balisdell Publ. Co., Waltham, Mass.

Collin, R. and F. Zucker (1969), Antenna Theory Part I, 62-69, McGraw Hill, New York.

Felsen, L.B. and N. Marcuvitz (1973), Radiation and Scattering of Waves, Prentice-Hall, Englewood Cliffs, N.J.

Debye, P. (1929), Polar Molecules, Chemical Catalog, New York.

de Loor, G.P. (1968), "Dielectric Properties of Heterogeneous Mixtures Containing Water," J. Microwave Power, Vol. 3, No. 2, pp. 67-73.

Du, L., and W. H. Peake (1969), "Rayleigh Scattering From Leaves," Proc. IEEE, 57(6), 1227-1229.

Foldy, L. (1945), "The Multiple Scattering of Waves," Phys. Rev., 67(3), 107-119.

Frisch, U. (1968), "Wave Propagation in a Random Medium," in Probabilistic Methods in Applied Math. Vol. I, edited by A.T. Bharucha-Reid, 75-198, Academic Press, New York.

Fung, A. and H. Fung (1977), "Application of First-Order Renormalization Method to Scattering From a Vegetated-like Half-space," IEEE Trans. Geosci. Electron., 15(4), 189-195.

Fung, A. and F. Ulaby (1978), "A Scatter Model for Leafy Vegetation," IEEE Trans. Geosci. Electron., 16(4), 281-286.

Fung, A. (1979), "Scattering From a Vegetation Layer," IEEE Trans. Geosci. Electron, 17(1), 1-6.

Harrington, R.F. (1968), Field Computation by Moment Methods, The Macmillan Company, New York.

Hevenor, R. (1976), "Backscattering of Radar Waves by Vegetated Terrain," Tech. Rep. ETL-0105, U.S. Army Engineer Topographical Laboratories, Fort Belvoir, VA.

Ishimaru, A. (1978a), Wave Propagation and Scattering in Random Media, Vols. 1 and 2, pp. 572, Academic Press, New York, NY.

Ishimaru, A. (1978b), "Diffusion of a Pulse in Density Distributed Scatterers," J. Opt. Soc. Am., 68(8), 1045-1050.

Ito, S. and S. Adachi (1977), "Multiple Scattering Effect on Backscattering from a Random Medium," IEEE Trans. Antennas Propagation, AP-25(2), 205-208.

Keller, J.B. (1962), "Wave Propagation in Random Media," in Proc. Symp. Appl. Math., 13, 145-170, Amer. Math. Soc., Providence, RI.

Keller, J.B. (1964), "Stochastic Equations and Wave Propagation in Random Media," Proc. Symp. Appl. Math., 16, 145-170, Amer. Math. Soc., Providence, RI.

Keller, J.B. and F.C. Karal, Jr. (1966), "Effective Dielectric Constant, Permeability, and Conductivity of a Random Medium and the Velocity and Attenuation Coefficient of Coherent Waves," J. Math. Phys., 7(4), 661-670.

Kohler, W., and G.C. Papanicolaou (1973), "Power Statistics for Wave Propagation in One-dimension and Comparison with Radiative Transport Theory," J. Math Phys., 14(12), 1733-1745.

Lang, R.H., and S. Sokolakis (1979), "Radar Backscattering From a Vegetated Terrain: A Discrete Scattering Approach," Tech. Rept. ETL-0159, U.S. Army Engineer Topographic Laboratories, Fort Belvoir, VA.

Lang, R.H. (1981), "Electromagnetic Backscattering From a Sparse Distribution of Lossy Dielectric Scatterers," Radio Sci., 16(1), 15-30.

Lang, R.H., and J. Sidhu (1983), "Electromagnetic Backscattering from a Layer of Vegetation: A Discrete Approach" to be published in IEEE Trans. Geosci. Electron.

Lax, M. (1951), "Multiple Scattering of Waves," Rev. Mod. Phys., 23(4), 287-310.

Lax, M. (1952), "Multiple Scattering of Waves, II. The Effective Field in Dense Systems," Phys. Rev., 85(4), 621-629.

LeVine, D.M., R. Meneghini, R. Lang and S. Se[illegible] (1982), "High Frequency Scattering from Arbitrarily Oriented Diele[illegible] Disks," NASA Technical Memo. 83910.

Papanicolaou, G.C. and R. Burridge (1975), "Transport Equations for the Stokes Parameters from Maxwell's Equations in a Random Medium," J. Math Phys., 16(10), 2074-2085.

Papanicoloau, G.C. (1969), "On Stochastic Differential Equations and Applications," Ph.D. dissertation, New York University, NY.

Rosenbaum, S. (1971), "The Mean Green's Function: A Non-linear Approximation," Radio Sci., 4(4), 379-386.

Rosenbaum, S. and W. Bowles (1974), "Clutter Return From Vegetated Areas," IEEE Trans. Antennas Propaga., AP-22(2), 227-236.

Shin, R.T. and J.A. Kong (1981), "Radiative Transfer Theory for Active Remote Sensing of a Half-space Random Medium with Three-Dimensional Variations, Radio Sci., 14(3), 359-369.

Stogryn, A. (1974), "Electromagnetic Scattering by a Random Dielectric Constant Fluctuations in a Bounded Medium, Radio Sci., 9(5), 509-518.

Tan, H.S., A.K. Fung and H. Eom (1980), "A Second-order Renormalization Theory for Cross-polarized Backscatterer from a Half-space Random Medium," Radio Sci., 15(6), 1059-1065.

Tartarskii, V.I., and M.E. Gertsenshtein (1963), "Propagation of Waves in a Medium With Strong Fluctuation of the Refractive Index," Soviet Physics JETP, 17(2), 458-463.

Tsang, L. and J.A. Kong (1976), "Microwave Remote Sensing of Two-layer Random Medium," IEEE Trans. Antennas Propagation, AP-24(3), 283-288.

Tsang, L. and J.A. Kong (1978), "Radiative Transfer Theory for Active Remote Sensing of Half-space Random Media," Radio Sci., 13(5), 763-773.

Tsang, L. and J.A. Kong (1979), "Wave Theory for Microwave Remote Sensing of a Half-space Random Medium with Three-dimensional Variations," Radio Sci., 14(3), 359-369.

Tsang, L. and J.A. Kong (1980), "Multiple Scattering of Electromagnetic Waves by Random Distributions of Discrete Scatterers With Coherent Potential and Quantum Mechanical Formulism," J. Appl. Phys., 51(7), 3465-3485.

Tsang, L., M.C. Kubacsi and J.A. Kong (1981), "Radiative Transfer Theory for Active Remote Sensing of a Layer of Small Ellipsoidal Scatterers," Radio Sci., 16(3), 321-329.

Twersky, V. (1962), "On Scattering of Waves by Random Distributions. I. Free Space Scatter Formalism," J. Math. Phys., 3(4), 700-715.

Twersky, V. (1964), "On Propagation in Random Media of Discrete Scatterers," in Proc. Symp. Appl. Math., 16, 84-116, Amer. Math. Soc., Providence, RI.

Twersky, V. (1970), "Interference Effects in Multiple Scattering by Large, Low-refracting, Absorbing Particles," J. Opt. Soc. Am., 60(7), 908-914.

Twersky, V. (1967), "Multiple Scattering of Electromagnetic Waves by Arbitrary Configurations," J. Math. Phys., 8(3), 589-610.

Twersky, V. (1978), "Coherent, Electromagnetic Waves in Pair-Correlated Random Distribution of Aligned Scatterers," J. Math. Phys., 19(1), 215-230.

Wang, J.R., T.J. Schmugge (1980), "An Empirical Model for the Complex Dielectric Permittivity of Soils as a Function of Water Content," IEEE Trans. Geosci. Electron., 18(4), 288-295.

Zuniga, M., T. Habashy and J. Kong (1979), "Active Remote Sensing of Layered Random Media," IEEE Trans. Geosci. Electron., 17(4), 296-302.

Zuniga, M., J.A. Kong (1981), "Active Remote Sensing of Random Media," J. Appl. Phys., 51(1), 74-79.

Zuniga, M., J.A. Kong (1981), "Mean Dyadic Green's Function for a Two Layer Random Medium," Radio Sci., 16(6), 1255-1270.

GRANT PRESENTATIONS AND PUBLICATIONS

"Electromagnetic Backscattering From a Layer of Vegetation: A Discrete Approach," with J.S. Sidhu. IEEE GE-21. pp. 62-71. Jan. 1983.

"High Frequency Scattering From Aribtrarily Oriented Dielectric Disks" with D.M. LeVine, R. Meneghini and S.S. Seker, NASA Technical Mem. 83910, June, 1982.

"Scattering From a Random Layer of Leaves in the Physical Optics Limit," with S.S. Seker and D.M. LeVine, presented at IGARSS Meeting, Munich, Germany, Conf. Proc. TA-1, 3.1-3.6, June, 1982.

"High Frequency Scattering From Arbitrarily Oriented Dielectric Disks," with D.M. LeVine, R. Meneghini and S.S. Seker, presented at IEEE/APS-URSI Meeting, Albuquerque, N.M., May, 1982.

"Resonant Backscattering From a Layer of Vegetation," with S.S. Seker, presented at XXth URSI General Assembly, Washington, D.C., August, 1981.

"Electromagnetic Backscattering From a Layer of Vegetation," with J. Sidhu, presented at the URSI Commission F Symposium on Signature Problems in Microwave Remote Sensing of the Surface of the Earth. University of Kansas, Lawrence, KA, January, 1981.

"Electromagnetic Scattering by Sparsely Distributed Discrete Random Medium," (invited talk), presented at Naval Research Laboratory, Washington, D.C., December, 1980.