

NASA Contractor Report 3732

NASA  
CR  
3732  
c.1

# Ultrasonic Wave Propagation in Two-Phase Media: Spherical Inclusions

L. S. Fu and Y. C. Sheu

LOAN COPY: RETURN TO  
AFWL TECHNICAL LIBRARY  
KIRTLAND AFB, N.M. 87117

GRANT NSG3-340  
OCTOBER 1983



25th Anniversary  
1958-1983

**NASA**



NASA Contractor Report 3732

# Ultrasonic Wave Propagation in Two-Phase Media: Spherical Inclusions

L. S. Fu and Y. C. Sheu  
*The Ohio State University*  
*Columbus, Ohio*

Prepared for  
Lewis Research Center  
under Grant NSG3-340



National Aeronautics  
and Space Administration

Scientific and Technical  
Information Branch

1983

## INTRODUCTION

Data analysis in ultrasonic interrogation often requires quantitative methods that provide solution in analytic form. The scattering theory recently developed within the scheme of an extended version of the equivalent inclusion method [1,2] gives such a solution form. Of particular interest is that this solution reduces to that of the static solution of the Eshelby problem [3]. This suggests that many of the research techniques developed in the static theories of composite materials and constitutive equations can be extended to include the frequency spectra of the ultrasonic waves such that the signals received at the probe(s) can be interpreted in terms of the microstructural parameters. The far field solutions needed in data analysis of ultrasonic evaluation therefore are often complemented by the near field solutions obtained in fracture mechanics or theories of composites.

The determination of the static effective properties of composites or poly-crystalline alloys is an area that constitutes extensive body of literature. Basically, there are two major theoretical approaches that can be used to describe the global material properties which include the causal effect introduced by the constituents, i.e. the statistical or deterministic approach. Both approaches depend upon the availability of the solution of a single inhomogeneity, be it a crack or an ellipsoidal inclusion material, and an averaging theorem. A critique of the approaches and a review can be found in [4] and [5], respectively. Recent textbooks on composites are also available for references, [6,7].

Solution to the scattering of a single embedded inhomogeneity is available by different methods that are appropriate at different frequency ranges [8]. The methods that offer a solution in an analytic form are:

the Born approximation [9], the longwave approximation [10-12], the extended method of equivalent inclusion [1], and the extended theory of geometric diffraction theory [13,14].

There are several averaging schemes or theorems that exist in the literature for finding the dynamic effective moduli and mass density. Kuster and Toksoz [15] considered a sphere of radius "a" consisting of small spheres of inclusion material in an elastic matrix. At Rayleigh limit and at low concentrations of inclusion material they showed that the scattered displacements are equivalent to those obtained by a homogeneous sphere with effective properties. Berryman [16,17] formulated a self-consistent dynamic theory of composites by requiring the scattered displacements to vanish on the average. His theory is appropriate for the longwavelength regime. Gaunaurd and Uberall [18] studied the resonant scattering from spherical cavities in elastic and viscoelastic media.

Employing the concept of volume averaging for such physical quantities as stress, strain and displacements, the moduli and mass density of the effective medium are derived by matching the strain energy and the kinetic energy of the effective and the physical media. The approach taken is similar to those of Christensen and Lo [19], Chou, Nomura and Taya [20] and Kohn [21] except the schemes developed in References [19,20] apply to the static case, and that in Reference [21] applies to the Rayleigh limit.

## THEORETICAL CONSIDERATIONS

Consider the problem of a matrix material, with elastic moduli  $C_{jkrS}$  and mass density  $\rho$ , that contains a concentration of second-phase particles of moduli  $C'_{jkrS}$  and mass density  $\rho'$ , randomly distributed over the material system. A schematic diagram of the system is as shown in Fig. 1. The true composite thus occupies the whole region and possesses effective properties  $C^*_{jkrS}$  and  $\rho^*$ . To determine the effective moduli and mass density, the following definitions are used:

$$\langle \sigma \rangle = C^* \langle \epsilon \rangle \quad (1)$$

$$\frac{1}{2} \langle \sigma \epsilon \rangle = \frac{1}{2} C^* \langle \epsilon \rangle \langle \epsilon \rangle \quad (2)$$

$$\frac{1}{2} \langle \rho \underline{v} \underline{v} \rangle = \frac{1}{2} \rho^* \langle \underline{v} \rangle \langle \underline{v} \rangle \quad (3)$$

where  $\sigma$ ,  $\epsilon$  and  $\underline{v}$  are the stress, strain and velocity fields, and the notation  $\langle \rangle$  denotes the volume average of a field quantity, e.g.

$$\langle \underline{F} \rangle = (V)^{-1} \int_V \underline{F}(\underline{x}_1) dV \quad (4)$$

in which  $V$  represents volume. The left-hand-side and the right-hand-side in each of the Equations (1-3) can be shown to be equivalent in a self-consistent manner [22,23].

Using the elastodynamic solution for a single ellipsoidal inhomogeneity in a linear elastic medium developed in [1], the displacement and strain fields inside an inhomogeneity are found, for incident time-harmonic plane wave, to be:

$$[\underline{u}^{(m)}(\underline{r})]^< = \underline{D}(\underline{r}) \underline{u}^{(a)} \quad (5)$$

$$[\underline{\epsilon}^{(m)}(\underline{r})]^< = \underline{A}(\underline{r}) \underline{\epsilon}^{(a)} \quad (6)$$

in which the superscripts (m), (a) and < denote "mis-match," "applied," and "inside the inhomogeneity," respectively. Employing the volume averaging process as described by Eq. (4) and substituting in Eqs. (1-3), the effective properties are easily defined as follows:

$$\rho^* = \rho + f \Delta\rho \underline{D} \underline{D} \quad (7)$$

$$\underline{C}^* = \underline{C} + f \Delta\underline{C} \underline{A} \quad (8)$$

where  $\Delta\rho = \rho' - \rho$ ,  $\Delta\underline{C} = \underline{C}' - \underline{C}$  and  $f$  is volume fraction of inclusion material. The tensor fields  $\underline{D}$  and  $\underline{A}$  are of ranks two and four, respectively, and they are functions of wavenumber, geometric properties, and effective and inclusion material properties:

$$\underline{D}_{mj} = \delta_{mj} - \langle f_{mj}^<(\bar{r}) \rangle / \{ f_{mj}[0] + 4\pi(\rho' - \rho^*)\omega^2 \} \quad (9)$$

$$\underline{A}_{mnpq} = \langle F_{mjk,n}^<(\bar{r}) + F_{nj,k,m}^<(\bar{r}) \rangle S_{jkpq} / 2\rho^*\omega^2 + \delta_{mp}\delta_{nq} \quad (10)$$

in which the  $\underline{f}$  and  $\underline{F}$  tensors are defined in [1] and  $S_{jkpq}$  is the connecting tensor between the eigenstrains and the applied strains, i.e.

$$\epsilon_{jk}^{*(1)} = S_{jkpq} \epsilon_{pq}^{(a)} \quad (11)$$

for the case of uniform eigenstrains and eigenforces. In developing these expressions the volume average of the  $\Phi$ -integrals, as defined in References [1,2], must be evaluated. Finally, it should be noted that  $\rho^*$  and  $\underline{C}^*$  are physical quantities and hence only the real parts of  $\underline{D} \underline{D}$  and  $\underline{A}$  should enter into consideration in Eqs. (7,8).

# SPHERICAL INCLUSION MATERIALS

Let the spherical inclusion materials of radius "a" be randomly distributed over the whole volume of the matrix. If the matrix and the inhomogeneities are isotropy, the effective medium is also isotropic.

It is straight forward to show that

$$D_{mj} = \delta_{mj} \{1 - \langle f_{33}(\bar{r}) \rangle / f_{33}[0] + 4\pi(\rho' - \rho^*)\omega^2\} = \delta_{mj} D \quad (12)$$

and

$$S_{j k p q} = S_{k j p q} = S_{j k q p} = S_{p q j k} \quad (13)$$

$$S_{111} = S_{222} = S_{333} = C_1$$

$$S_{2323} = S_{1313} = S_{1212} = C_3$$

$$S_{1122} = S_{1133} = S_{2233} = C_2$$

where

$$C_1 = (C_1^* + C_2^* - 2GC_2^*) / [(C_1^*)^2 + C_1^* C_2^* - 2(C_2^*)^2]$$

$$C_2 = (C_1^* G - C_2^*) / [(C_1^*)^2 + C_1^* C_2^* - 2(C_2^*)^2]$$

$$C_3 = \{2 F_{122,1}[0] + \mu^* / (\mu' - \mu^*)\}$$

$$C_1^* = G F_{111,1}[0] + (G+1)D_{122,1}[0] + H$$

$$F = -(\lambda^* + 2\mu^*)/G$$

$$G = (\lambda' - \lambda^*) / [(\lambda' - \lambda^*) + 2(\mu' - \mu^*)]$$

$$H = \lambda/G$$

Following the theory developed in the previous section, the effective moduli and mass density are found to be

$$\rho^* = \rho + f \Delta \rho D^2 \quad (14)$$

$$\lambda^* = \lambda + f[\Delta \lambda(A_{1111} + 2A_{1122}) + 2\Delta \mu A_{1122}] \quad (15)$$

$$\mu^* = \mu + f \Delta \mu(A_{1212} + A_{1221}) \quad (16)$$

$$K^* = K + f[(A_{1111} + 2A_{1122})\Delta \lambda + (2/3)(3A_{1122} + A_{1212} + A_{1221})\Delta \mu] \quad (17)$$

It is clearly seen that the velocities are dispersive. At frequency range above that of the Rayleigh limit, this phenomenon is pronounced. From Figs. 2-7, the bulk moduli, shear moduli and longitudinal velocities are shown as functions of volume concentration of spherical inclusion materials for the cases of (1) aluminum spheres in germanium and (2) voids in silicon nitride, HS 130.

As an example of application to detect localized damage by void nucleation, let all voids be locally nucleated within a region  $\Omega$  of radius  $R$ ,  $R \gg a$ , Fig. 8. The effective moduli of this composite can therefore be obtained as before. If void nucleation outside the region  $\Omega$  can be ignored, then the scattering of the composite sphere can be easily obtained. Using the program developed in [24], the scattering cross section for a composite sphere consisted of small voids in titanium is displayed as a function of dimensionless wavenumber for different concentration of voids, Fig. 9. The scattering cross section, which is essentially proportional to the attenuation [25], increases with increasing concentration  $f$ . It appears that these curves can be used to locate and calibrate porosity in a structural component. Effective properties for this material system are presented in [26].



## CLOSING REMARKS

The velocity and attenuation of ultrasonic waves in two-phase media are studied by using a self-consistent averaging scheme, that require the effective medium to possess the same potential and kinetic energy as the physical medium. The concept of volume averaging for physical quantities is employed and the solution depend upon the scattering of a single inhomogeneity. The theory is general in nature and can be applied to any two-phase material system. Since the scattering of an ellipsoidal inhomogeneity is known, the average theorem presented in this report can be used to study the velocity and attenuation of distributed inhomogeneities of shapes such as disks, short fibres, etc. The introduction of the orientation of these inhomogeneities besides only their sizes as in the spherical geometry will necessarily induce anisotropy in the effective medium.

Results for randomly distributed spherical inclusions of radius "a" are presented. Effective moduli and mass density are found to be dispersive. When the inhomogeneities or voids are nonuniformly distributed, attenuation occurs. The case of some localized damage is studied. Since it is well known that porosity is directly related to the strength of bone [27] and ceramics [28] it appears that the theoretical study of velocity and attenuation in two-phase media presents a valuable means for data analysis in ultrasonic evaluation of material properties.

1. L.S. Fu and T. Mura, "The Determination of the Elastodynamic Fields of an Ellipsoidal Inhomogeneity," Trans. ASME, J. Appl. Mech., 50, 390-396 , (June, 1983).
2. L.S. Fu and T. Mura, "Volume Integrals of Ellipsoids Associated with the Inhomogeneous Helmholtz Equation," Wave Motion, 4, 141-149, (1983).
3. J.D. Eshelby, "Elastic Inclusions and Inhomogeneities," in Progress in Solid Mechanics, Vol. 2, ed. Sneddon and Hill, North-Holland, Amsterdam, 89-140, (1961).
4. C.C. Chamis and G.P. Sendeckyj, "Critique on Theories Predicting Thermoelastic Properties of Fibrous Composites," J. Composite Materials, 2(3), 332-358 (July, 1968).
5. T. Mura, Micromechanics of Defects in Solids, Nijhoff Pub., 1982.
6. S.W. Tsai and M. Thomas Hahn, Int. to Composite Materials, Technomic Pub., Conn., 1980.
7. R.M. Christensen, Mechanics of Composite Materials, Wiley, N.Y., 1979.
8. L.S. Fu, "Mechanics Aspects of NDE by Sound and Ultrasound," Applied Mechanics Reviews, 35(8), 1047-1057, (August, 1982).
9. J.E. Gubernatis, E. Domany, J.A. Krumhansl, and M. Huberman, "The Born Approximation in the Theory of the Scattering of Elastic Waves by Flaws," J. Appl. Physics, 48(7), 2812-2819 (1977).
10. A.K. Mal and L. Knopoff, "Elastic Wave Velocities in Two-Component Systems," J. Inst. Math. Appl., 3, 376-387 (1967).
11. J.R. Willis, "A Polarization Approach to the Scattering of Elastic Waves - I. Scattering by a Single Inclusion," J. Mech. Phys. Solids, 28, 287-305 (1980).
12. B. Budiansky and J.R. Rice, "On the Estimation of a Crack Fracture Parameter by Long Wavelength Scattering," J. Appl. Mech., Trans. ASME, 45, 453-454 (1978).
13. E. Resende, Propagation, Reflection and Diffraction of Elastic Waves, Ph.D. Thesis, New York University, 1963.
14. J.D. Achenbach and A.K. Gautesen, "Geometrical Theory of Diffraction for 3-D Elastodynamics," J. Acoust. Soc. Am., 61, 413-421 (1977).
15. G.T. Kuster and N. Toksöz, "Velocity and Attenuation of Seismic Waves in Two-Phase Media: Part I. Theoretical Formulations," Geophysics, 39(5), 587-606, (1974).

16. J.G. Berryman, "Long-Wavelength Propagation in Composite Elastic Media, I. Spherical Inclusions," J. Acoust. Soc. Am., 68(6), 1809-1819 (Dec. 1980).
17. J.G. Berryman, "Long-Wavelength Propagation in Composite Elastic Media, II. Ellipsoidal Inclusions," J. Acoust. Soc. Am., 68(6), 1820-1831 (Dec. 1980).
18. G. Gaunaurd and H. Uberall, "Resonance Theory of the Effective Properties of Perforated Solids," J. Acoust. Soc. Am., 71(2), 282-295 (1982).
19. R.M. Christensen and K.H. Lo, "Solutions for Effective Shear Properties in Three Phase Sphere and Cylindrical Models," J. Mech. Phys. Solids, 27(4), (1979).
20. T.W. Chou, S. Nomura, and M. Taya, "A Self-Consistent Approach to the Elastic Stiffness of Short-Fiber Composites," J. Comp. Materials, 14, 178-186 (July, 1980).
21. W. Kohn, "Propagation of Low Frequency Elastic Disturbances in a Three-Dimensional Composite Material," J. Appl. Mech., Trans. ASME, 42, 159(1975).
22. R. Hill, "A Self-Consistent Mechanics of Composite Materials," J. Mech. Phys. Solids, 13, 213-222 (1965).
23. B. Budiansky, "On the Elastic Moduli of Some Heterogeneous Materials," J. Mech. Phys. Solids, 13, 223-227 (1965).
24. Y.C. Sheu and L.S. Fu, "The Transmission or Scattering of Elastic Waves by an Inhomogeneity of Simple Geometry: A Comparison of Theories," NASA Contractor Report #3659, Jan. 1983.
25. R. Truell, C. Elbaum and B.B. Chick, Ultrasonic Methods in Solid State Physics, Academic Press, N.Y., 1969.
26. L.S. Fu and Y. Sheu, "Velocity and Attenuation in Two-Phase Media: Spherical Inclusions," 14th Symposium of NDE, April, 1983.
27. S.H. Advani, R.B. Martin and W.R. Powell, "Mechanical Properties and Constitutive Equations of Anatomical Materials," in Applied Physiological Mechanics, ed. D.N. Ghista, Bioengineering and Computational Series, Vol. 1, 31-103, Harwood Acad. Pub. (1979).
28. A.G. Evans, D.R. Biswas and Fulrath, "Some Effects of Cavities on the Fracture of Ceramics: I and II," J. Am. Ceram. Soc., 62(1), 95-100, and 101-106, (1979).

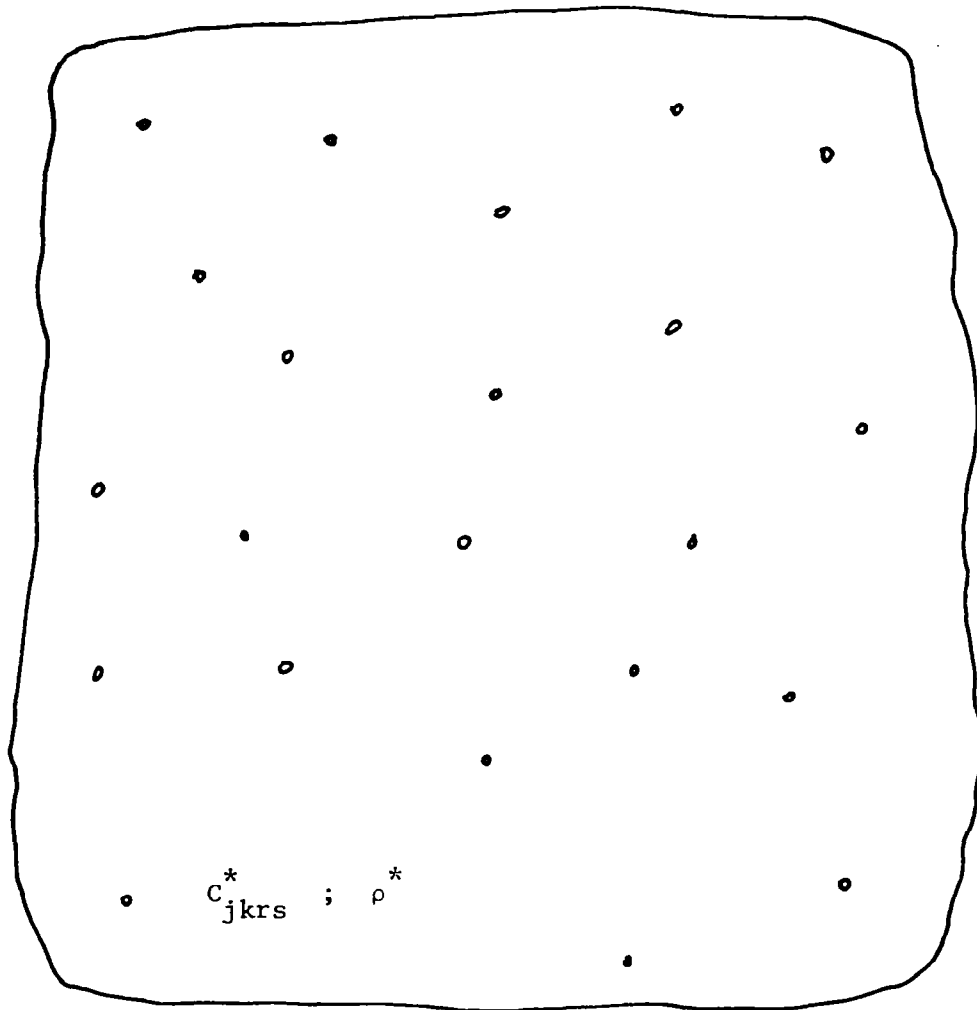


Fig. 1 A schematic diagram of an inhomogeneity matrix system with effective properties  $C_{jkr}^*$  and  $\rho^*$

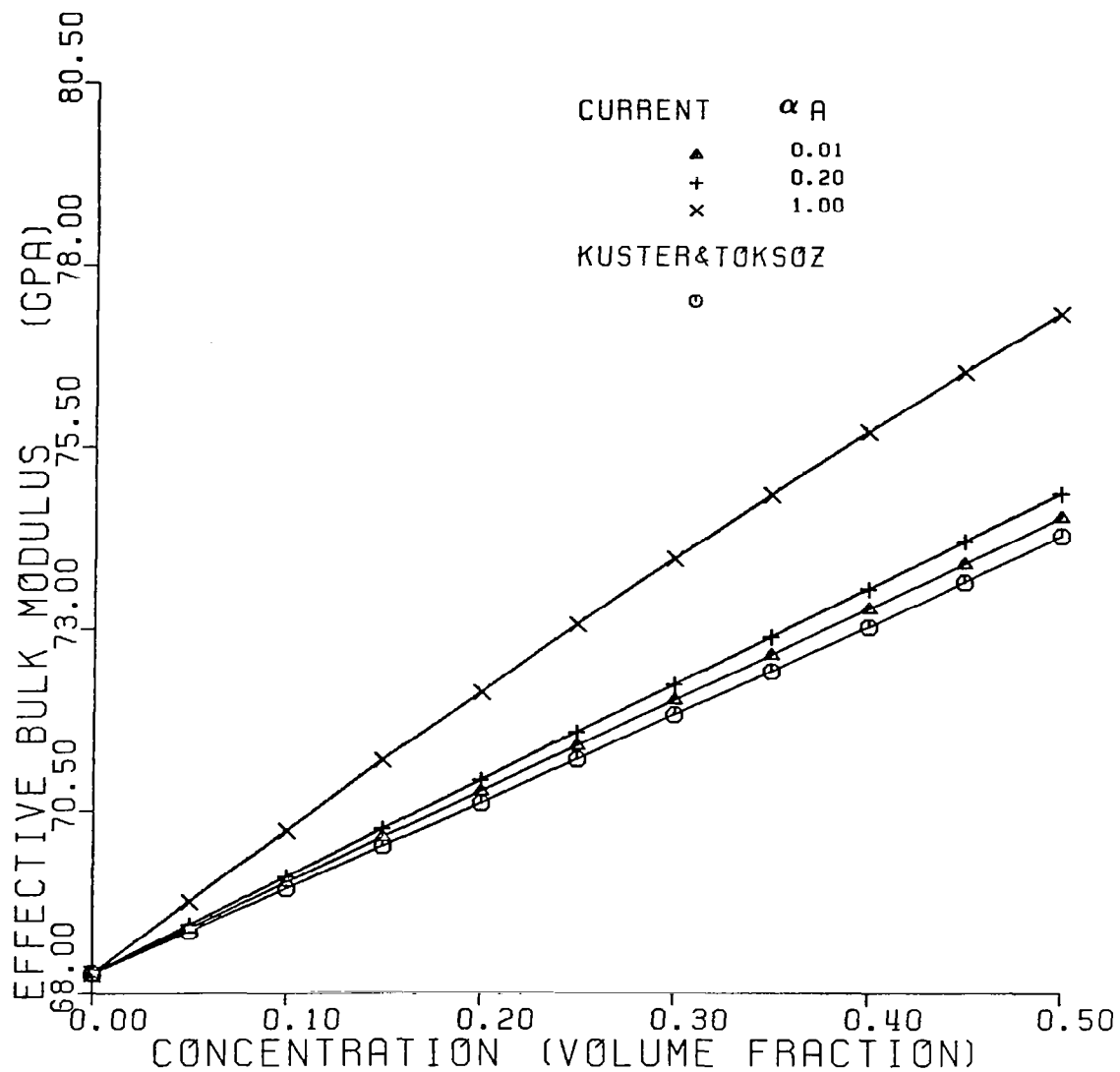


Fig. 2 Effective bulk modulus vs. concentration:  
aluminum spheres in germanium.

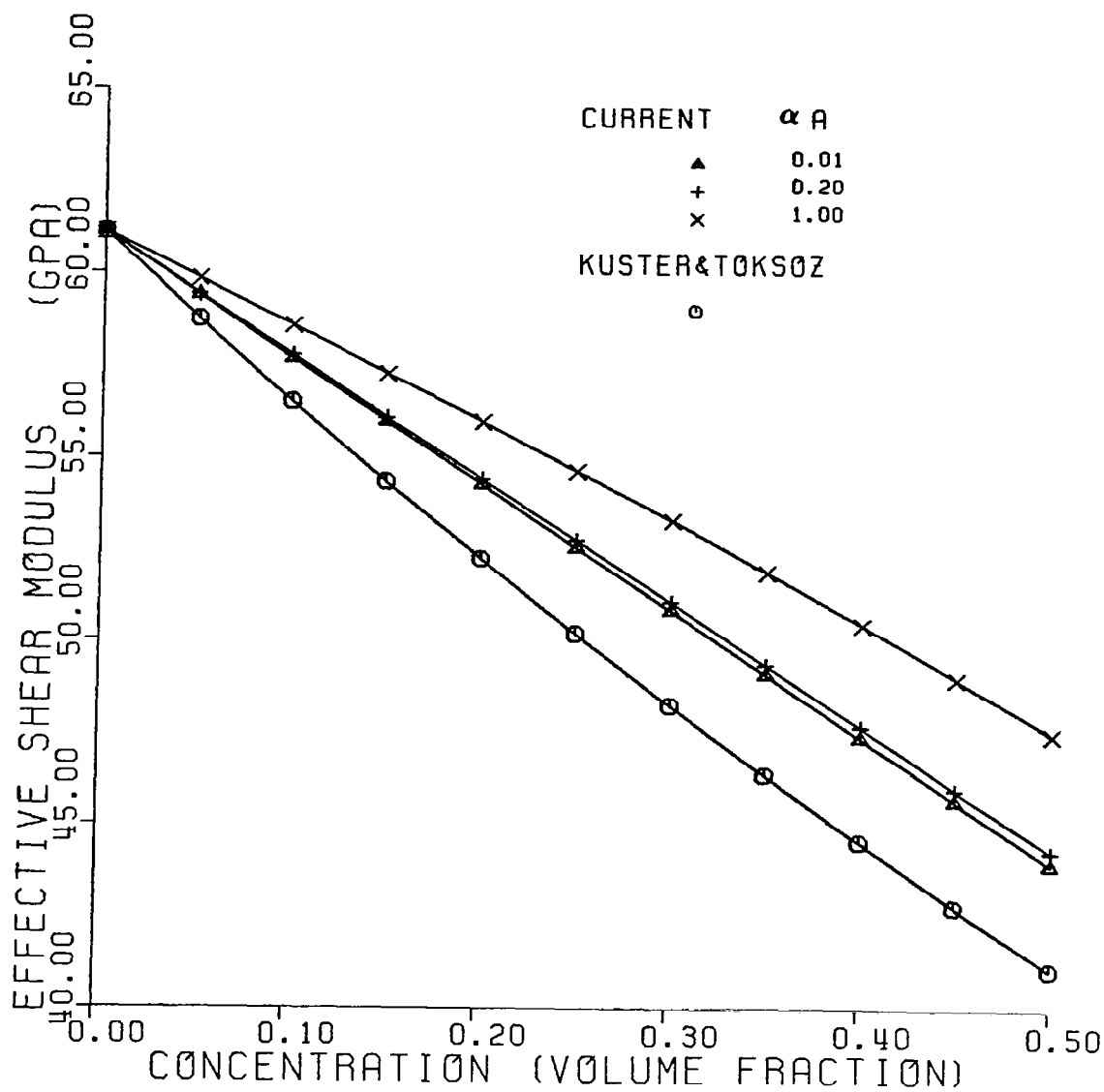


Fig. 3 Effective shear modulus vs. concentration aluminum spheres in germanium.

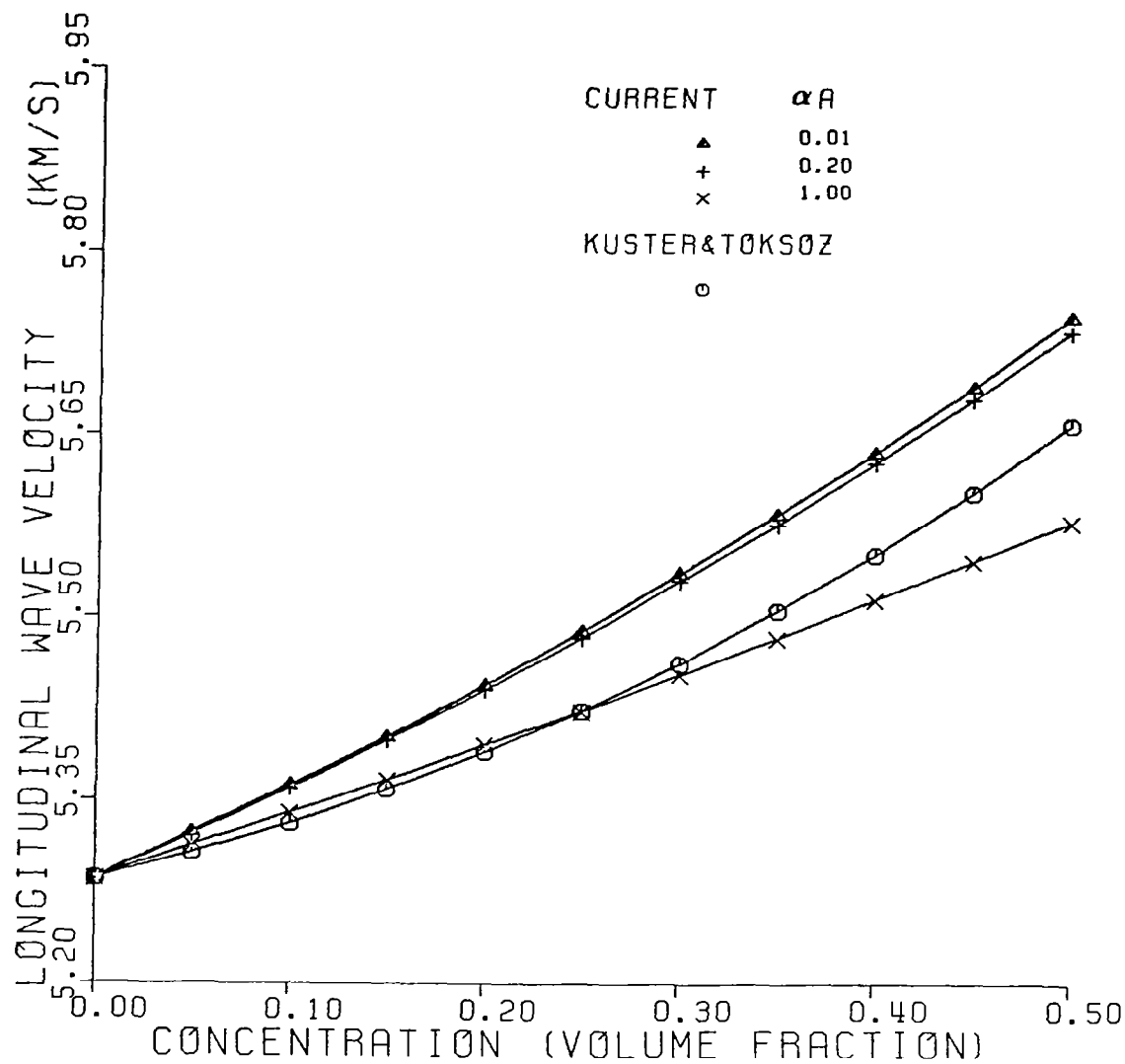


Fig. 4 Longitudinal wave velocity vs. concentration:  
aluminum spheres in germanium.

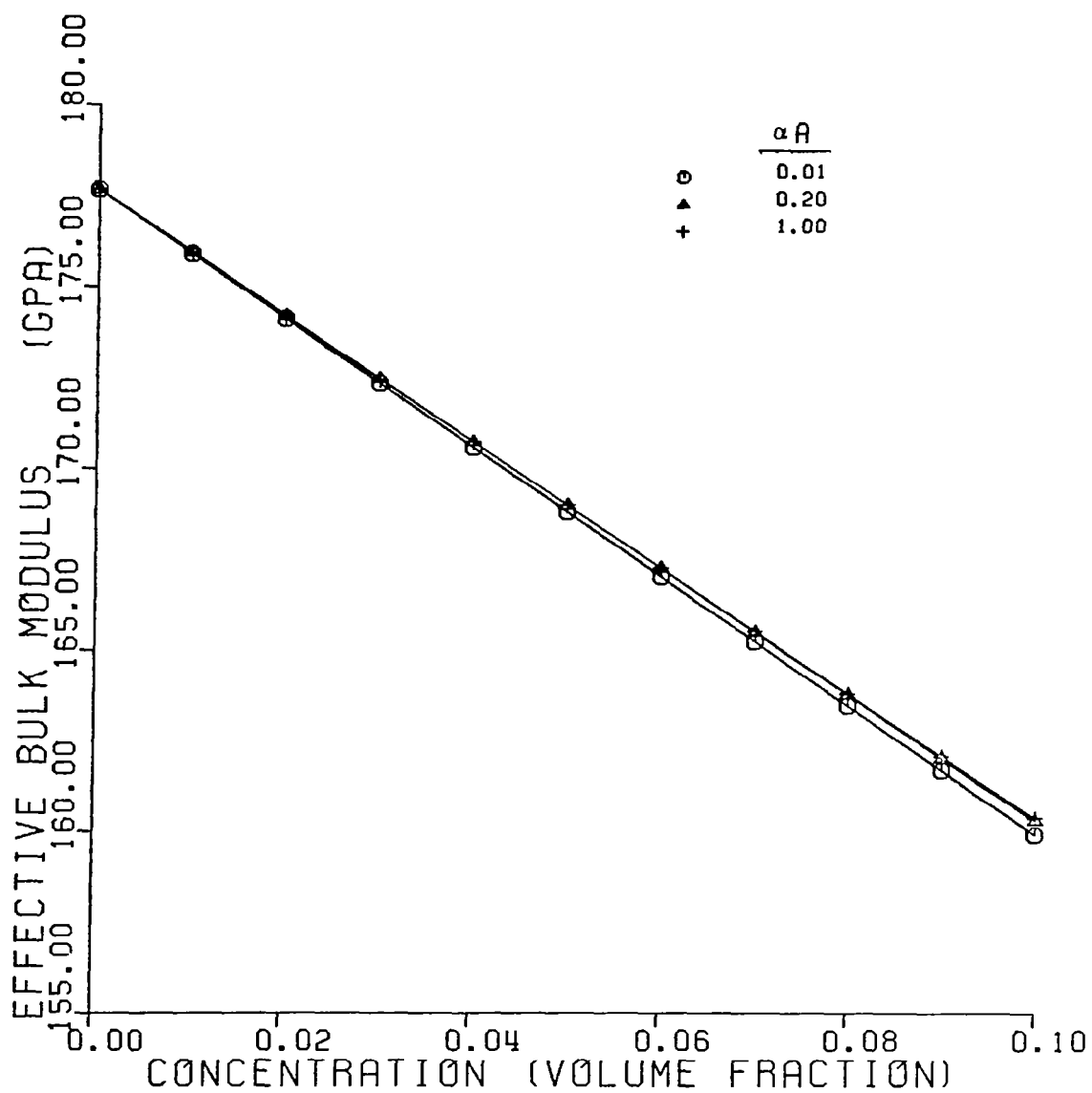


Fig. 5 Effective bulk modulus vs. concentration: voids in silicon nitride



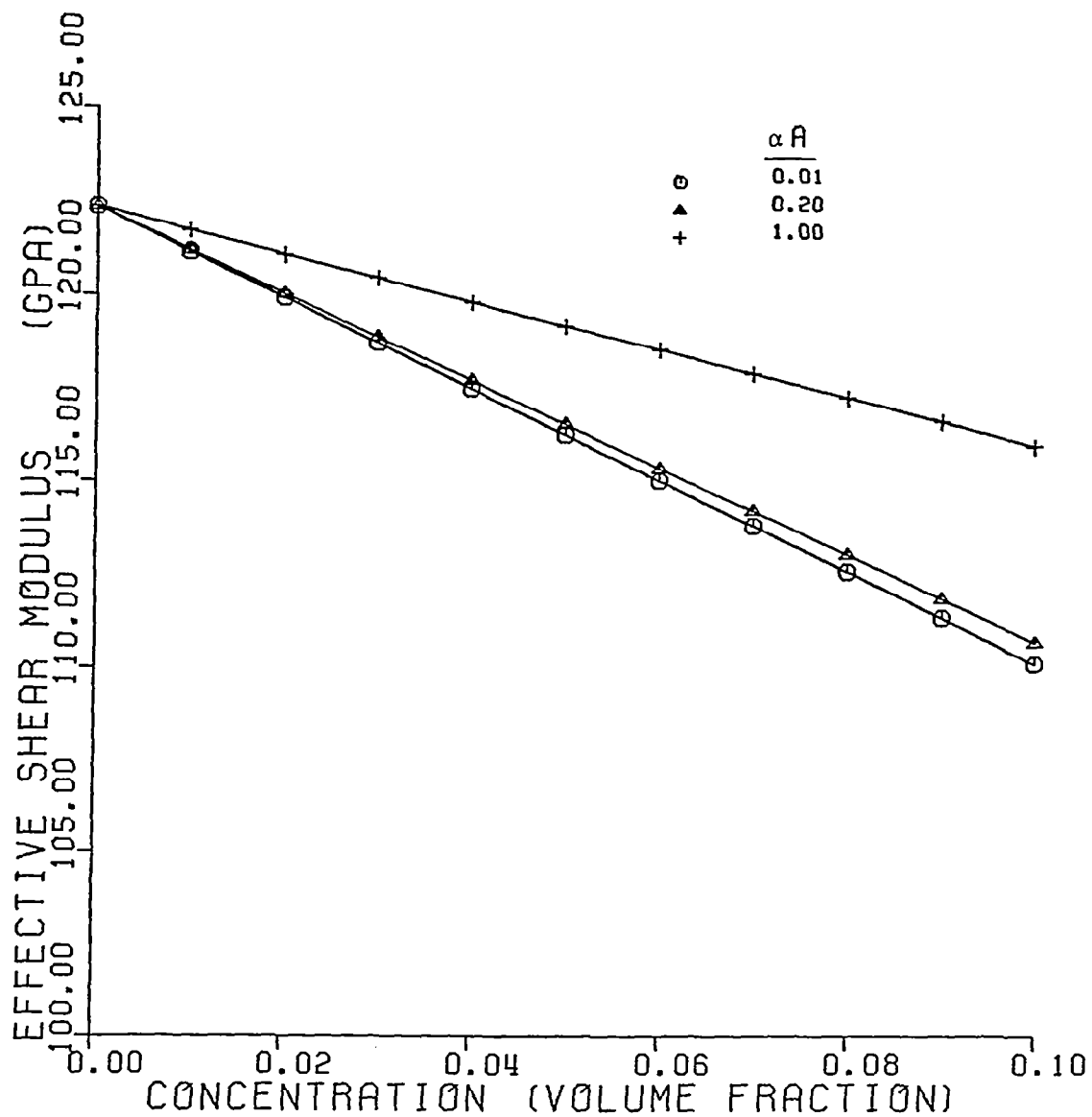


Fig. 6 Effective shear modulus vs. concentration:  
voids in silicon nitride.

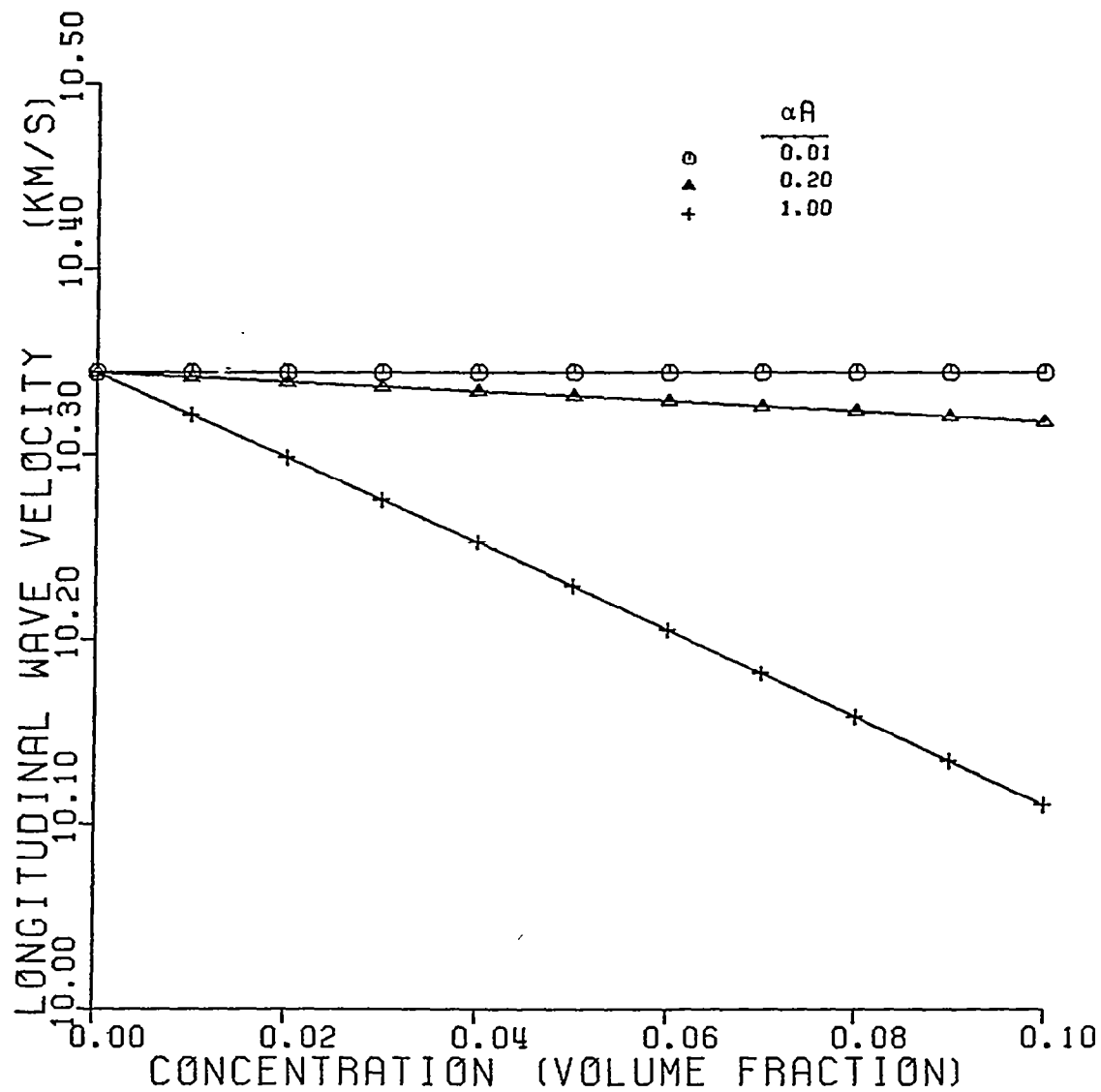


Fig. 7 Longitudinal wave velocity vs. concentration:  
voids in silicon nitride.

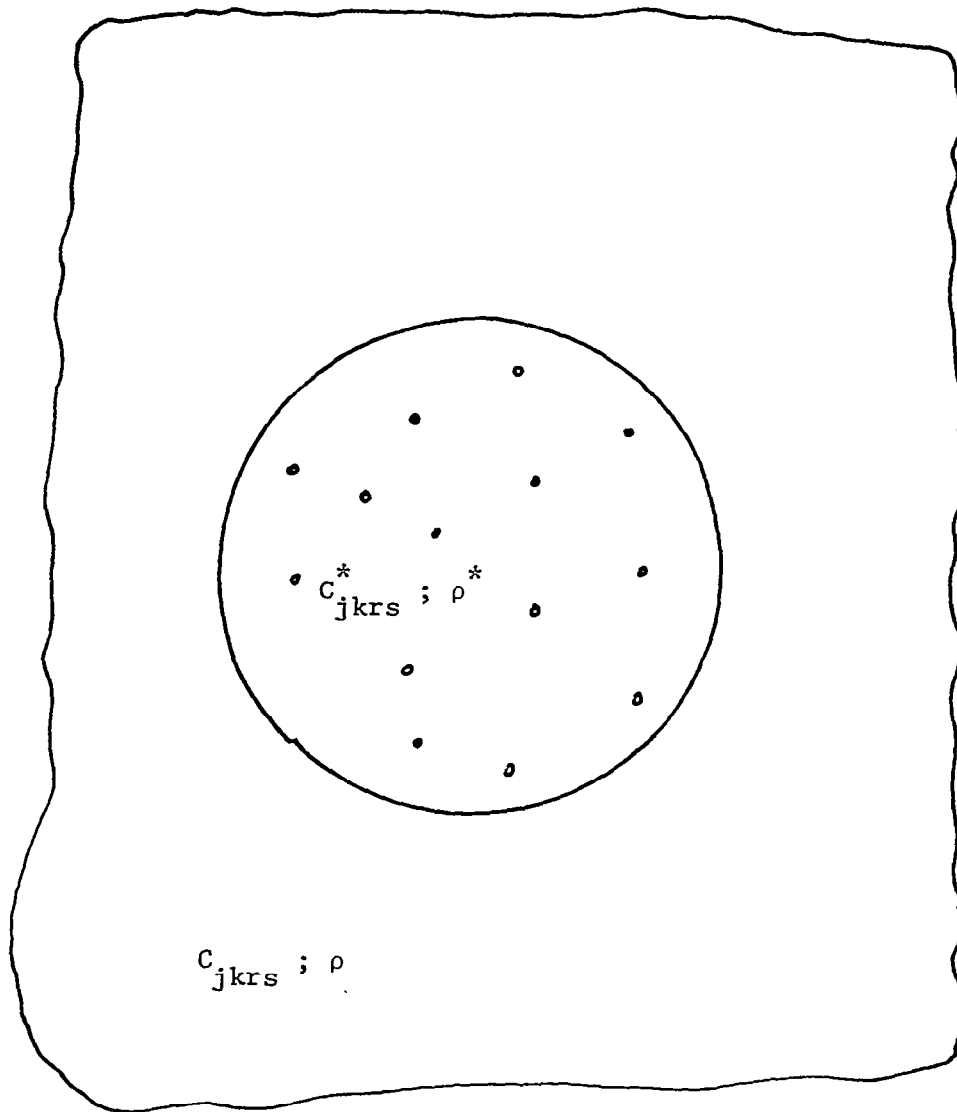


Fig. 8 A schematic diagram of localized damage.

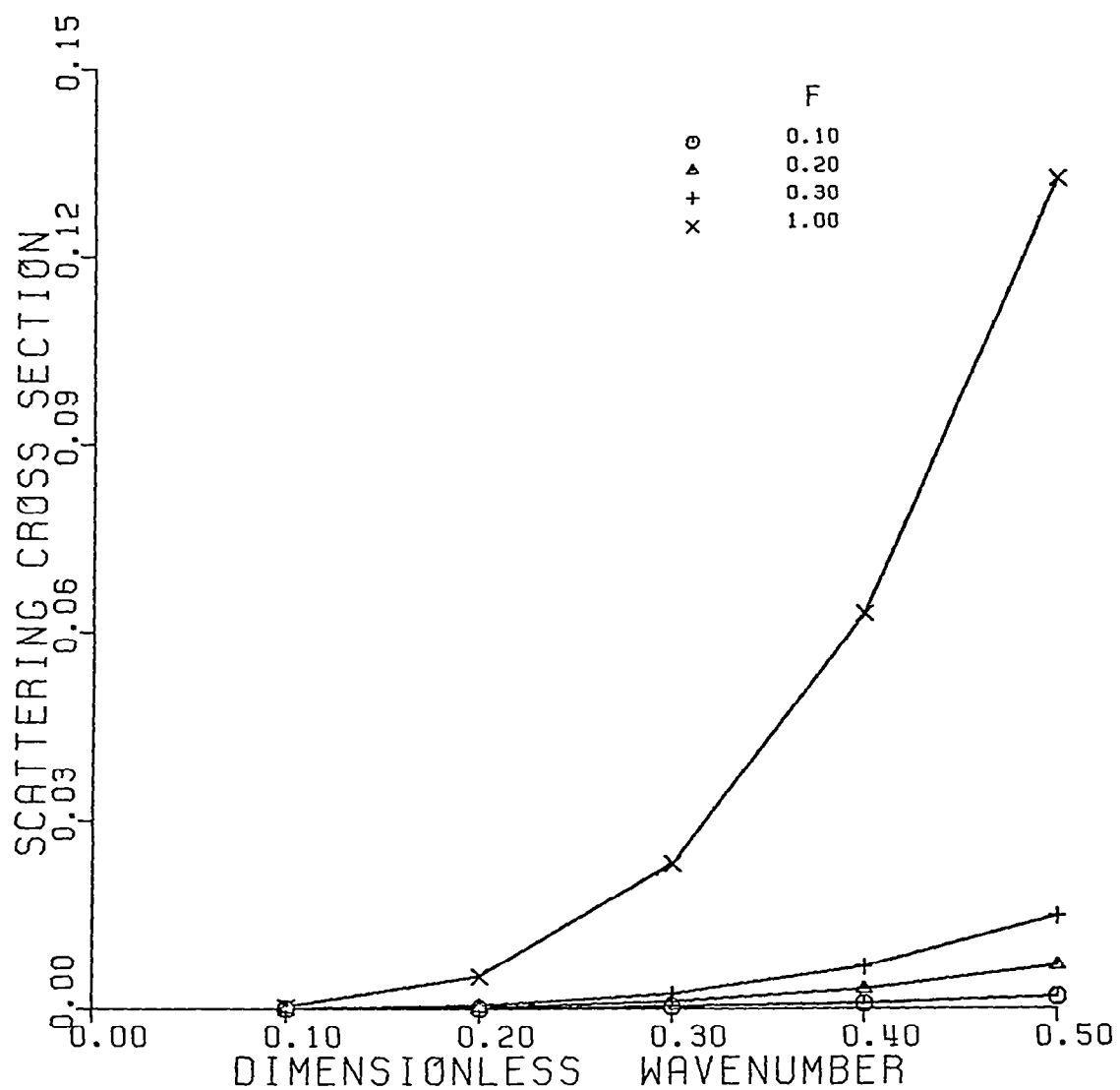


Fig. 9 Scattering cross section vs. dimensionless wavenumber: distributed voids within a spherical domain in titanium matrix,  $a/R=0.01$

1. Report No. NASA CR-3732		2. Government Accession No.		3. Recipient's Catalog No.	
4. Title and Subtitle Ultrasonic Wave Propagation in Two-Phase Media: Spherical Inclusions				5. Report Date October 1983	
				6. Performing Organization Code	
7. Author(s) L.S. Fu and Y.C. Sheu				8. Performing Organization Report No. None	
				10. Work Unit No.	
9. Performing Organization Name and Address The Ohio State University 1314 Kinnear Rd. Columbus, Ohio 43212				11. Contract or Grant No. NSG3-340	
				13. Type of Report and Period Covered Contractor Report	
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546				14. Sponsoring Agency Code 506-53-1A (E-1791)	
15. Supplementary Notes Final report. Project Manager, Alex Vary, Materials Division, NASA Lewis Research Center, Cleveland, Ohio 44135.					
16. Abstract The scattering theory, recently developed via the extended method of equivalent inclusion, is used to study the propagation of time-harmonic waves in two-phase media of elastic matrix with randomly distributed elastic spherical inclusion materials. The elastic moduli and mass density of the composite medium are determined as functions of frequencies when given properties and concentration of the spheres and the matrix. Velocity and attenuation of ultrasonic waves in two-phase media are determined for cases of (1) distributed spheres and (2) localized damage. An averaging theorem that requires the equivalence of the strain energy and the kinetic energy between the effective medium and the original matrix with with spherical inhomogeneities is employed to derive the effective moduli and mass density. The functional dependency of these quantities upon frequencies and concentration provides a method of data analysis in ultrasonic evaluation of material properties. Numerical results for moduli, velocity and/or attenuation as functions of concentration of inclusion material, or porosity, are graphically displayed. In particular, aluminum/germanium and void/silicon nitride (HS130) material systems are given as examples.					
17. Key Words (Suggested by Author(s)) Ultrasonics; Ultrasonic scattering; Ultrasonic wave propagation; Ultrasonic velocity; Ultrasonic attenuation; Two- phase media; Inclusions (spherical)			18. Distribution Statement Unclassified - unlimited STAR Category 38		
19. Security Classif. (of this report) Unclassified		20. Security Classif. (of this page) Unclassified		21. No. of pages 21	
				22. Price* A02	

\*For sale by the National Technical Information Service, Springfield, Virginia 22161

NASA-Langley, 1983