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EIGENMODE ANALYSIS OF UNSTEADY ONE-DIMENSIONAL EULER EQUATIONS

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EIGENMODE ANALYSIS OF UNSTEADY ONE-DIMENSIONAL EULER EQUATIONS

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Abstract

The initial boundary value problem describing the evolution of unsteady linearized perturbations of a steady, uniform subsonic flow is analyzed. The eigenmodes and eigenfrequenices of the system are derived and several examples are presented to illustrate the effect of different boundary conditions on the exponential decay rate of the eigenmodes. The resultant implications for the stability and convergence rates of finite difference computations are discussed.

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INTRODUCTION

In finite difference calculations of steady-state subsonic solutions of quasi-one-dimensional and two-dimensional Euler equations using time marching methods, it is often observed that when the solution has almost converged to steady-state the remaining residual is due to the propagation of low frequency waves up and down the domain. These waves are largely unaffected by numerical viscosity and are dissipated through the interaction with the inflow and outflow boundary conditions. The purpose of this paper is to examine this process by analyzing the unsteady linearized perturbations of a one-dimensional, steady, uniform, subsonic flow. For this linear problem with constant coefficients it is possible to derive the exact eigenmodes and eigenfrequencies of the initial boundary value problem. This is the classical technique used to analyze physical and acoustical vibrations in a finite domain [5] and more recently used in numerical analysis to examine the P-stability of finite difference approximations to scalar equations [2,3]. The exponential decay rate of the physical eigenmodes is computed for several different sets of boundary conditions commonly used in finite difference calculations and the implications for the stability and convergence rates of these calculations are discussed.

The wellposedness of both the initial boundary value problem (i.b.v.p.) and the steady-state boundary value problem (b.v.p.) is discussed briefly. The definitive analysis of the i.b.v.p. for multi-dimensional hyperbolic systems is given by Kreiss in [4]. Oliger and Sundström [7], use an energy method to establish sufficient conditions for the wellposedness of the Euler i.b.v.p. Finally, the wellposedness of the steady-state solution to the nonlinear quasi-one-dimensional and two-dimensional Euler equations will be discussed in a forthcoming paper by Wornom and Hafez [8].

2. ANALYSIS

The equation for the unsteady linearized perturbation of a steady, uniform one-dimensional flow is,

$$\begin{pmatrix} \tilde{\rho} \\ \tilde{u} \\ \tilde{p} \end{pmatrix}_T + \begin{pmatrix} \bar{u} & \bar{\rho} & 0 \\ 0 & \bar{u} & \bar{\rho}^{-1} \\ 0 & \gamma\bar{p} & \bar{u} \end{pmatrix} \begin{pmatrix} \tilde{\rho} \\ \tilde{u} \\ \tilde{p} \end{pmatrix}_X = 0 , \qquad (1)$$

where $\tilde{\rho}$, $\tilde{u}$, $\tilde{p}$ are the perturbation density, velocity and pressure and $\bar{\rho}$, $\bar{u}$, $\bar{p}$ are the steady, uniform values.

The analysis is greatly simplified by defining the following non-dimensional variables

$$\rho = \tilde{\rho}/\bar{\rho} \qquad (2)$$

$$u = \tilde{u}/\bar{c} \qquad (3)$$

$$p = \tilde{p}/\bar{\rho}\,\bar{c}^2 \qquad (4)$$

$$x = X/L \qquad (5)$$

$$t = T\bar{c}/L, \qquad (6)$$

where $\bar{c} = [\gamma\bar{p}/\bar{\rho}]^{1/2}$ is the speed of sound. L is the physical length of the domain considered, so in the non-dimensional domain the subsonic inflow is at $x = 0$ and the outflow is at $x = 1$.

The resultant non-dimensional equation is

$$\begin{pmatrix} \rho \\ u \\ p \end{pmatrix}_t + A \begin{pmatrix} \rho \\ u \\ p \end{pmatrix}_x = 0, \qquad (7)$$

where

$$A = \begin{pmatrix} M & 1 & 0 \\ 0 & M & 1 \\ 0 & 1 & M \end{pmatrix}, \tag{8}$$

and M is the Mach number of the unperturbed flow.

Equation (7) has wave-like solutions

$$\begin{pmatrix} \rho \\ u \\ p \end{pmatrix} = \exp[i(kx - \omega t)]U \tag{9}$$

provided

$$(kA - \omega I)U = 0, \tag{10}$$

so ω/k is an eigenvalue of A and U is the corresponding eigenvector. The three eigenvalues of A and their corresponding eigenvectors are

$$\lambda_1 = M \qquad U_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \tag{11a,b}$$

$$\lambda_2 = M + 1 \qquad U_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \tag{12a,b}$$

$$\lambda_3 = M - 1 \qquad U_3 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}. \tag{13a,b}$$

A general eigenmode of the initial boundary value problem can be written as a sum of the three eigenwaves,

$$U = e^{-i\omega t}[\alpha_1 e^{i(\omega/\lambda_1)x} U_1 + \alpha_2 e^{i(\omega/\lambda_2)x} U_2 + \alpha_3 e^{i(\omega/\lambda_3)x} U_3]. \tag{14}$$

The eigenfrequency ω and the values of the constants $\alpha_1, \alpha_2, \alpha_3$ are determined by the three boundary conditions.

At the inflow boundary at $x = 0$ there are two boundary conditions which when linearized and non-dimensionalized have the form,

$$C_{in} U = 0, \tag{15}$$

where C_{in} is a 2×3 matrix. Substitution of (14) into (15) yields the equation,

$$\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = 0 \tag{16}$$

where

$$\begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{pmatrix} = C_{in}\begin{pmatrix} U_1 & U_2 & U_3 \end{pmatrix}. \tag{17}$$

A necessary condition for the initial boundary value problem to be wellposed is that the 2×2 matrix

$$\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$$

is nonsingular and so can be inverted to obtain α_1 and α_2, the values of the incoming characteristics, as a function of α_3, the value of the outgoing characteristic.

Similarly the outflow boundary condition yields one equation of the form

$$C_{out} U = 0 \tag{18}$$

and substitution of (14) produces

$$\begin{pmatrix} b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = 0, \tag{19}$$

where

$$\begin{pmatrix} b_{31} & b_{32} & b_{33} \end{pmatrix} = C_{out}\begin{pmatrix} e^{i(\omega/\lambda_1)} U_1 & e^{i(\omega/\lambda_2)} U_2 & e^{i(\omega/\lambda_3)} U_3 \end{pmatrix}. \tag{20}$$

The second necessary condition for the wellposedness of the initial boundary value problem is that b_{33} is nonzero so that α_3 the value of the incoming characteristic can be determined as a function of α_1 and α_2 the values of the outgoing characteristics.

Equations (16) and (19) can be written jointly as

$$B(\omega) \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = 0. \tag{21}$$

To obtain a nontrivial eigenmode $B(\omega)$ must be singular and the vector $\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \end{pmatrix}^T$ must be a corresponding null vector. Thus the eigenfrequenices can be calculated from the following determinant equation

$$\det B(\omega) = 0. \tag{22}$$

The matrix B can also be used to examine whether the steady-state boundary value problem is wellposed. The three requirements for wellposedness are that a solution exists, is unique, and small perturbations in the boundary data produce small perturbations in the solution.

The linearized steady-state boundary value problem has a zero solution and this solution is unique provided there are no nonzero solutions to

$$B(0)\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix} = 0, \tag{23}$$

i.e., provided that $B(0)$ is non-singular.

A perturbation of the boundary data leads to an equation of the form

$$B(0)\begin{pmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\end{pmatrix} = \begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\end{pmatrix}, \tag{24}$$

which, provided $B(0)$ is nonsingular, can be solved to obtain $(\alpha_1 \quad \alpha_2 \quad \alpha_3)^T$ which define the characteristic perturbations of the steady-state solution.

Thus the linearized steady-state boundary value problem is wellposed if, and only if, $\det B(0)$ is nonzero, or alternatively the initial boundary value problem does not have a zero eigenfrequency.

3. EXAMPLES

(a) <u>Entropy, Enthalpy Specified at Inflow, Pressure at Outflow</u>

The physical boundary conditions are

$$X = 0 \quad \begin{cases} p'/\rho'^{\gamma} = \overline{p}/\overline{\rho}^{\,\gamma} & (25a) \\ \dfrac{\gamma-1}{2}\, u'^{2} + \dfrac{\gamma p'}{\rho'} = \dfrac{\gamma-1}{2}\, \overline{u}^{2} + \dfrac{\gamma \overline{p}}{\overline{\rho}} & (25b) \end{cases}$$

$$X = L \qquad p' = \overline{p}, \tag{25c}$$

where ρ', u', p' are the unsteady physical variables which are a sum of the steady-state and unsteady perturbation variables. The corresponding linearized non-dimensionalized equations are

$$x = 0 \qquad \begin{pmatrix} -1 & 0 & 1 \\ -1 & (\gamma-1)M & \gamma \end{pmatrix} \begin{pmatrix} \rho \\ u \\ p \end{pmatrix} = 0, \tag{26a}$$

$$x = 1 \qquad \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \rho \\ u \\ p \end{pmatrix} = 0. \tag{26b}$$

At $x = 0$ substitution of the eigenvector definitions (11b), (12b), and (13b) into the eigenmode definition (14) yields

$$\begin{pmatrix} \rho \\ u \\ p \end{pmatrix} = e^{-i\omega t} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}. \tag{27}$$

Substitution of this equation into (26a) produces the characteristic inflow boundary condition

$$\begin{pmatrix} -1 & 0 & 1 \\ -1 & (\gamma-1)M & \gamma \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 & 0 \\ -1 & (\gamma-1)(1+M) & (\gamma-1)(1-M) \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = 0. \tag{28}$$

Similarly at $x = 1$

$$\begin{pmatrix} \rho \\ u \\ p \end{pmatrix} = e^{-i\omega t} \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha_1 \exp(i\omega/\lambda_1) \\ \alpha_2 \exp(i\omega/\lambda_2) \\ \alpha_3 \exp(i\omega/\lambda_3) \end{pmatrix}, \tag{29}$$

and substitution into (26b) produces the characteristic outflow boundary condition

$$(0 \quad 0 \quad 1) \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha_1 \exp(i\omega/\lambda_1) \\ \alpha_2 \exp(i\omega/\lambda_2) \\ \alpha_3 \exp(i\omega/\lambda_3) \end{pmatrix}$$

$$= \left(0 \quad \exp(i\omega/\lambda_2) \quad \exp(i\omega/\lambda_3)\right) \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = 0 . \tag{30}$$

Together equations (28) and (30) define the matrix B

$$B(\omega) = \begin{pmatrix} -1 & 0 & 0 \\ -1 & (\gamma-1)(1+M) & (\gamma-1)(1-M) \\ 0 & \exp(i\omega/\lambda_2) & \exp(i\omega/\lambda_3) \end{pmatrix} . \tag{31}$$

The eigenfrequencies are given by

$$\det B = (\gamma-1)\left[(1-M)\exp(i\omega/\lambda_2) - (1+M)\exp(i\omega/\lambda_3)\right] = 0, \tag{32}$$

$$\Longrightarrow \exp\left(\frac{2i\omega}{1-M^2}\right) = \frac{1+M}{1-M}, \tag{33}$$

$$\Longrightarrow \omega = \frac{1-M^2}{2}\left[-i\log\left(\frac{1+M}{1-M}\right) + 2n\pi\right], \tag{34}$$

where n is an integer.

Thus there is an infinite set of discrete eigenfrequencies. It is useful to define a decay rate σ_n

$$\sigma_n \stackrel{\text{def}}{=} -\operatorname{Im}(\omega_n). \tag{35}$$

For this example

$$\sigma_n = \frac{1-M^2}{2}\log\left(\frac{1+M}{1-M}\right). \tag{36}$$

The amplitude of the eigenmode grows, or decays, as $\exp(-\sigma t)$, so the requirement for all eigenmodes to decay is $\sigma_n > 0$ for every n. In this example the requirement is satisfied and so any initial disturbance at $t = 0$ will decay exponentially.

(b) <u>Mass Flux, Enthalpy Specified at Inflow, Pressure at Outflow</u>

The physical boundary conditions are

$$X = 0 \quad \begin{cases} \rho' u' = \bar{\rho}\,\bar{u} & (37a) \\[2ex] \dfrac{\gamma-1}{2} u'^2 + \dfrac{\gamma p'}{\rho'} = \dfrac{\gamma-1}{2}\bar{u}^2 + \dfrac{\gamma \bar{p}}{\bar{\rho}} & (37b) \end{cases}$$

$$X = L \qquad p' = \bar{p}. \tag{37c}$$

Omitting the algebraic details the resultant matrix B is

$$B = \begin{pmatrix} M & 1 + M & M - 1 \\ -1 & (\gamma-1)(1+M) & (\gamma-1)(1-M) \\ 0 & \exp(i\omega/\lambda_2) & \exp(i\omega/\lambda_3) \end{pmatrix} . \tag{38}$$

The eigenfrequencies are

$$\omega_n = \frac{1 - M^2}{2} \left[- i \log\left(\frac{(1 + M)[1 + M(\gamma - 1)]}{(1 - M)[1 - M(\gamma - 1)]}\right) + 2(n + 1/2)\pi \right] . \tag{39}$$

The decay rates are

$$\sigma_n = \frac{1 - M^2}{2} \log\left(\frac{(1 + M)[1 + M(\gamma - 1)]}{(1 - M)[1 - M(\gamma - 1)]}\right) . \tag{40}$$

(c) <u>Density, Pressure Specified at Inflow, Pressure at Outflow</u>

The physical boundary conditions are

$$X = 0 \quad \begin{cases} \rho' = \bar{\rho} & (41a) \\ p' = \bar{p} & (41b) \end{cases}$$

$$X = L \qquad p' = \bar{p} . \tag{41c}$$

The matrix B is

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & \exp(i\omega/\lambda_2) & \exp(i\omega/\lambda_3) \end{pmatrix} . \tag{42}$$

The eigenfrequenices are

$$\omega_n = (1 - M^2)n\pi \ , \tag{43}$$

and the decay rates are zero.

Since one of the eigenfrequenices is zero the steady-state boundary value problem is ill-posed, as discussed earlier.

(d) Density, Velocity Specified at Inflow, Pressure at Outflow

The physical boundary conditions are

$$X = 0 \quad \begin{cases} \rho' = \bar{\rho} & (44a) \\ u' = \bar{u} & (44b) \end{cases}$$

$$X = L \qquad p' = \bar{p}. \tag{44c}$$

The matrix B is

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & \exp(i\omega/\lambda_2) & \exp(i\omega/\lambda_3) \end{pmatrix} . \tag{45}$$

The eigenfrequenices are

$$\omega_n = (1 - M^2)(n + 1/2)\pi \tag{46}$$

and the decay rates are zero.

In this example the steady-state boundary value problem is wellposed but because of the zero decay rates unsteady oscillations will continue indefinitely without exponential growth or decay.

(e) Non-reflecting Boundary Conditions

The full nonlinear non-reflecting boundary conditions specify entropy and the appropriate Riemann invariant at the inflow, and the other Riemann invariant at the outflow [4]

$$X = 0 \quad \begin{cases} p'/\rho'^{\gamma} = \overline{p}/\overline{\rho}^{\gamma} & (47a) \\ u' + \frac{2}{\gamma-1} c' = \overline{u} + \frac{2}{\gamma-1} \overline{c} & (47b) \end{cases}$$

$$X = L \qquad u' - \frac{2}{\gamma-1} c' = \overline{u} - \frac{2}{\gamma-1} \overline{c} \, . \tag{47c}$$

The matrix B is

$$B = \begin{pmatrix} -1 & 0 & 0 \\ -\frac{1}{\gamma-1} & 2 & 0 \\ \frac{1}{\gamma-1} \exp(i\omega/\lambda_1) & 0 & -2 \exp(i\omega/\lambda_3) \end{pmatrix} . \tag{48}$$

Det B = 0 leads to $\sigma = +\infty$ which reflects the fact that with these boundary conditions the unsteady perturbations become zero after the finite time it takes for all three characteristic waves to cross the domain once.

CONCLUSIONS

The calculation of the exponential decay rates of physical eigenmodes has implications for the stability and convergence rates of time-marching finite difference computations. If the analytic problem has exponentially increasing eigenmodes then for sufficiently fine grid resolution a time-accurate numerical solution will exhibit corresponding exponentially increasing eigenmodes. In a forthcoming paper, Trefethen[1] will prove that for a linear constant coefficient system such as this the three conditions:

(i) Exponentially decaying physical eigenmodes,

(ii) Dissipative interior numerical scheme,

(iii) GKS-stable numerical boundary conditions,

are sufficient to ensure the P-stability of a time-marching method for a sufficiently fine grid. P-stability was defined by Beam, Warming and Yee [2] and corresponds to GKS-stability with the additional requirement that none of the numerical eigenmodes increases exponentially. The precise definition of the theorem and its proof are given by Trefethen[1], but in essence the argument is that condition (i) ensures that low frequency physical waves decay, while conditions (ii) and (iii) ensure the decay of high frequency waves, both physical and non-physical.

The exponential decay rates for the physical eigenmodes also provide a useful lower limit on the spectral radius of the finite difference time-marching procedure. If a physical eigenmode decays as $\exp(-\sigma t)$ with $\sigma > 0$, then for a sufficiently fine grid the corresponding numerical eigenmode decays approximately as $\exp(-\sigma\, n\, \Delta t)$ where n is the iteration number and Δt is the time-step. As the grid is refined with $\Delta t/\Delta x$ held constant, $\Delta t \to 0$ and so the spectral radius is no less than $1 - \sigma\Delta t + O(\Delta t^2)$. If $\sigma = 0$, as in example (d), the physical eigenmodes are neutrally stable and so the numerical convergence rate towards steady-state is due solely to numerical dissipation. If this dissipation is of n^{th} order then the corresponding spectral radius is $1 - O(\Delta t^{n+1})$. Non-reflecting boundary

Trefethen, L. N., 1983, Courant Institute, New York University, NY, personal communication.

conditions as in example (e) clearly give a much faster rate of convergence, but in two or three dimensions perfectly non-reflecting boundary conditions do not exist and in general the best that can be achieved is that there is zero reflection for locally plane waves propagating in a particular chosen direction [1].

It is not clear to what extent the conclusions for this model problem, with linearized perturbations and constant coefficients, are valid for more general flows such as transonic quasi-one-dimensional and two-dimensional flows. Nonlinear mechanisms at sonic lines and shocks are undoubtedly very important. However the decay to steady-state of low frequency waves will still depend on the physical boundary conditions and so this analysis should provide insight into the effect of the boundary conditions.

REFERENCES

[1] A. Bayliss and E. Turkel, "Radiation Boundary Conditions for Wave-Like Equations," Comm. Pure Appl. Math., Vol. 33, 1980, pp. 707-725.

[2] R. M. Beam, R. F. Warming, and H. C. Yee, "Stability Analysis of Numerical Boundary Conditions and Implicit Difference Approximations for Hyperbolic Equations," Numerical Boundary Condition Procedures, NASA CP-2201, 1981, pp. 257-282.

[3] M. Giles and W. T. Thompkins, Jr., "Asymptotic Analysis of Numerical Wave Propagation in Finite Difference Equations," Gas Turbine and Plasma Dynamics Laboratory Report 171, Massachusetts Institute of Technology, 1983.

[4] H.-O. Kreiss, "Initial Boundary Value Problems for Hyperbolic Systems," Comm. Pure Appl. Math., Vol. 23, 1970, pp. 277-298.

[5] J. Lighthill, Waves in Fluids, Cambridge University Press, 1978, pp. 137-152.

[6] P. M. Morse and K. U. Ingard, Theoretical Acoustics, International Series in Pure and Applied Physics, McGraw-Hill, 1968.

[7] J. Oliger and A. Sundström, "Theoretical and Practical Aspects of Some Initial Boundary Value Problems in Fluid Dynamics," SIAM, Vol. 25, No. 3, 1978, pp. 419-446.

[8] S. F. Wornom and M. M. Hafez, "A Rule for Selecting Analytical Boundary Conditions for the Conservative Quasi-One-Dimensional Nozzle Flow Equations," to be presented at AIAA 22nd Aerospace Sciences Meeting, January 1984.

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16. Abstract

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