Temp # - 23337

NASA Contractor Report 172212

MINIVER UPGRADE FOR THE AVID SYSTEM

VOLUME I: LANMIN USER'S MANUAL

Carl D. Engel
Sarat C. Praharaj

REMTECH, Inc.
Huntsville, AL 35805

Contract NAS1-16983
August 1983

(NASA-CR-172212) MINIVER UPGRADE FOR THE AVID SYSTEM. VOLUME 1: LANMIN USER'S MANUAL (Remtech, Inc., Huntsville, Ala.) 127 p HC A07/MF A01 CSCL 09B

N84-10779

Unclas
G3/61 42337

NASA

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

FOREWORD

This final report presents work which was conducted for Langley Research Center (LaRC) in response to requirements of Contract NAS1-16983. The work presented was performed by REMTECH Inc., Huntsville, Alabama and is entitled ''MINIVER Upgrade for the AVID System.'' The final report consists of three volumes.

Volume 1: LANMIN User's Manual

Volume 2: LANMIN Input Guide

Volume 3: EXITS User's and Input Guide

The NASA technical coordination for this study was provided by Ms. Kathryn E. Wurster of the Vehicle Analysis Branch of the Space Systems Division.

TABLE OF CONTENTS
VOLUME I
LANMIN USER'S MANUAL

Section 1.0

INTRODUCTION

The successful design of thermal protection systems for vehicles operating in atmosphere and near-space environments requires accurate analyses of heating rate and temperature histories encountered along a trajectory. For preliminary design calculations, however, the requirement for accuracy must be tempered by the need for speed and versatility in computational tools used to determine such ''thermal environments.''

The MINIVER program (Ref. 1) over the last decade has been found to provide the proper balance between versatility, speed and accuracy for an aerothermal prediction tool. The advancement in computer aided design concepts at Langley Research Center (LaRC) in the past few years has made it desirable to incorporate the MINIVER program into the LaRC AVID system (Ref. 2). The purpose of the AVID system is to provide the preliminary design engineer with a useful tool for multi-discipline interactions to perform partial or complete vehicle synthesis.

In order to effectively incorporate MINIVER into the AVID system, several changes to MINIVER were made. The thermal conduction options in MINIVER were removed and a new Explicit Interactive Thermal Structures (EXITS) code was developed. Many upgrades to the MINIVER code were made and a new Langley version of MINIVER called LANMIN was created.

This report is divided into three volumes. Volume I describes the theoretical methods and subroutine functions used in LANMIN. Volume II provides a user input guide for LANMIN. Volume III describes the EXITS code and provides an input guide.

The documentation presented in Volume I utilized prior documentation where applicable and added new material for upgrade areas. The primary sources of prior documentation are Ref. 1 for the original MINIVER code, Ref. 3 for subroutine descriptions and Ref. 4 for rarefied flow updates.

Section 2.0

GENERAL UTILITY ROUTINES

The program uses several subroutines of general utility such as specification of printing of input and output data, interpolation techniques and the managing of all other subroutines. This section gives a description of these utility routines with a discussion of their capabilities.

2.1 MAIN

MAIN performs the task of managing the rest of LANMIN. This routine is best described by the flow diagram in Figure 2-1. The MAIN routine sets up the input, calls the calculation subroutines in proper sequence, saves and controls the output, and controls the case to case input. Most of the input is accomplished through an array W(700). Many of the input array elements are equivalent to other variable names. A complete description of the input is given in Volume II of this report.

2.2 SUBROUTINE WRINP

Subroutine WRINP is called by MAIN to print out all of the input data for each separate case. This routine prints out the pertinent input information.

2.3 SUBROUTINE VANOUT

Subroutine VANOUT is called by MAIN and is used to print the output in a specified format. The parameters in the CALL statement of VANOUT are as follows:

D - (input) This matrix dimensioned (100, 13) contains 13 variables as a function of time. The 13 variables are: time, altitude, velocity, Mach number, angle of attack, Reynolds number/ft, heat transfer coefficient, recovery enthalpy, equilibrium wall temperature, convective heating rate, heating load, local pressure, and flow type.

IF - (input) Number of time points.

TITLE - (input) This array dimensioned (18) contains an input title for the case.

AMET - (input) This variable corresponding to W(646) determines the output units. AMET = 0.0 corresponds to English units and AMET > 0.0 corresponds to metric units for output.

BPN - (input) Body point number corresponding to W(611).

This subroutine writes to paper and to a file. Each case on the file is ended with a flag having a numerical value of -100. for the variable TIME.

2.4 SUBROUTINE TBLIN

Subroutine TBLIN is a utility linear interpolation routine which is used extensively throughout LANMIN by other routines. The following discussion considers the use of this routine in conjunction with an atmospheric table which does not exist as a routine in the program, but rather comes into the program via input data.

The argument of the routine is TBLIN (X, XX, Y, YY, Z, ZZ, N). The parameter X acts as an independent variable and the parameters Y and Z act as functions of X. When a value X is input to TBLIN, the values Y and Z are returned at the corresponding X. The array XX contains all of the table values of the independent variables X in a one-dimensional array. The one-dimensional arrays YY and ZZ contain all of the table values of the dependent variables. Since each of these arrays (XX, YY, ZZ) contains the same number of elements, the parameter N is used to indicate how many values are contained in each array and thus how large each array is to be dimensioned in TBLIN.

It should be noted that this routine can only be used when the values in array XX are input in nondecreasing order. The results of this linear interpolation routine are not valid if the values in the independent variable array XX are not listed in increasing order. An example of the use of this routine occurs when a value of altitude (X) is input. The routine will return values of ambient temperature (Y) and pressure (Z) via linear interpolation from the temperature and pressure arrays, YY and ZZ.

Upon entering the routine, a check is made to determine if the independent variable X is less than the first (smallest) value in the array XX. If it is, then the first element of array YY and the first element of array ZZ are returned as Y and Z, respectively, to the calling routine. If the independent variable X is greater than the last (largest) element in array XX, then the last element of array YY and the last element of array ZZ are returned as Y and Z, respectively, to the calling routine. If the value of the independent variable X is not out of the range of the array XX, then linear interpolation is employed to determine the corresponding values of Y and Z.

2.5 SUBROUTINE TINT6

Subroutine TINT6 is a linear interpolation routine. This routine interpolates six or less dependent variables simultaneously corresponding to an input independent variable. The argument of TINT6 contains 16 elements. The first two elements in the argument are values of the independent variable and the array of stored values of the independent variable. The next 12 locations of the subroutine argument occur in six pairs corresponding to the six dependent variables. The first element of the pair is the value of the dependent variable returned by the program, and the second element of the pair is the array of input values corresponding to the values in the independent variable array. The independent and dependent variable arrays may each contain a maximum of 10 values. The last element of the subroutine argument corresponds to the number of values contained in the independent variable input array (10 or less).

The next-to-last element of the subroutine argument contains an integer value. If the value of this integer is greater than 0, then TINT6 performs interpolation assuming the independent variable is nonincreasing. If the value of this integer is less than zero, then TINT6 performs linear interpolation assuming the independent variable to be nondecreasing.

Now consider the case where the independent variable is nondecreasing. Before interpolation is attempted, the input value of the independent variable is checked to determine if it is less than the first element in the independent variable array. If the input value is less than this first element, then the values of the dependent variables returned to the calling routine are set equal to the first element of each array. If the independent variable is larger than the first element, then another check occurs to determine if the independent variable is greater than the last element in the independent variable array. If it is, then the values of the dependent variables returned to the calling routine are the last values in each dependent variable array. If the independent variable passes both tests, then TINT6 inspects to resolve whether or not the independent variable is equal to one of the values in the independent variable input array. If equality occurs, then the value returned for each dependent variable is that value in each dependent array corresponding to the independent variable.

If the value of the independent variable is not found to be less than the first element, greater than the last element, or equal to any of the elements in the input array, then linear interpolation occurs using the two elements of each dependent array which corresponds to the two input independent variables bracketing the input independent variable.

An analogous procedure is performed by TINT6 if the independent variable is nonincreasing instead of nondecreasing.

This routine is written for either a nondecreasing independent variable or a nonincreasing independent variable, such as time increasing during a trajectory.

2.6 FUNCTIONS INTP1 AND INTP2

These functions are used to perform linear interpolations in semi $\log_{10}$ space. The arguments of INTP1 are:

I - (input) Pressure index
J2 - (input) Highest temperature index
J1 - (input) Lowest temperature index
K - (input) Thermal property index
F - (input) Delta value of independent variable is (X-X1)/(X2-X1)

Values are interpolated from the G(I, J, K) matrix.

The arguments of INTP2 are:

X - (input) Lowest value of dependent variable
Y - (input) Highest value of dependent variable
F - (input) Delta value of independent variable

Both functions linearly interpolate in log10 space of the dependent variable.

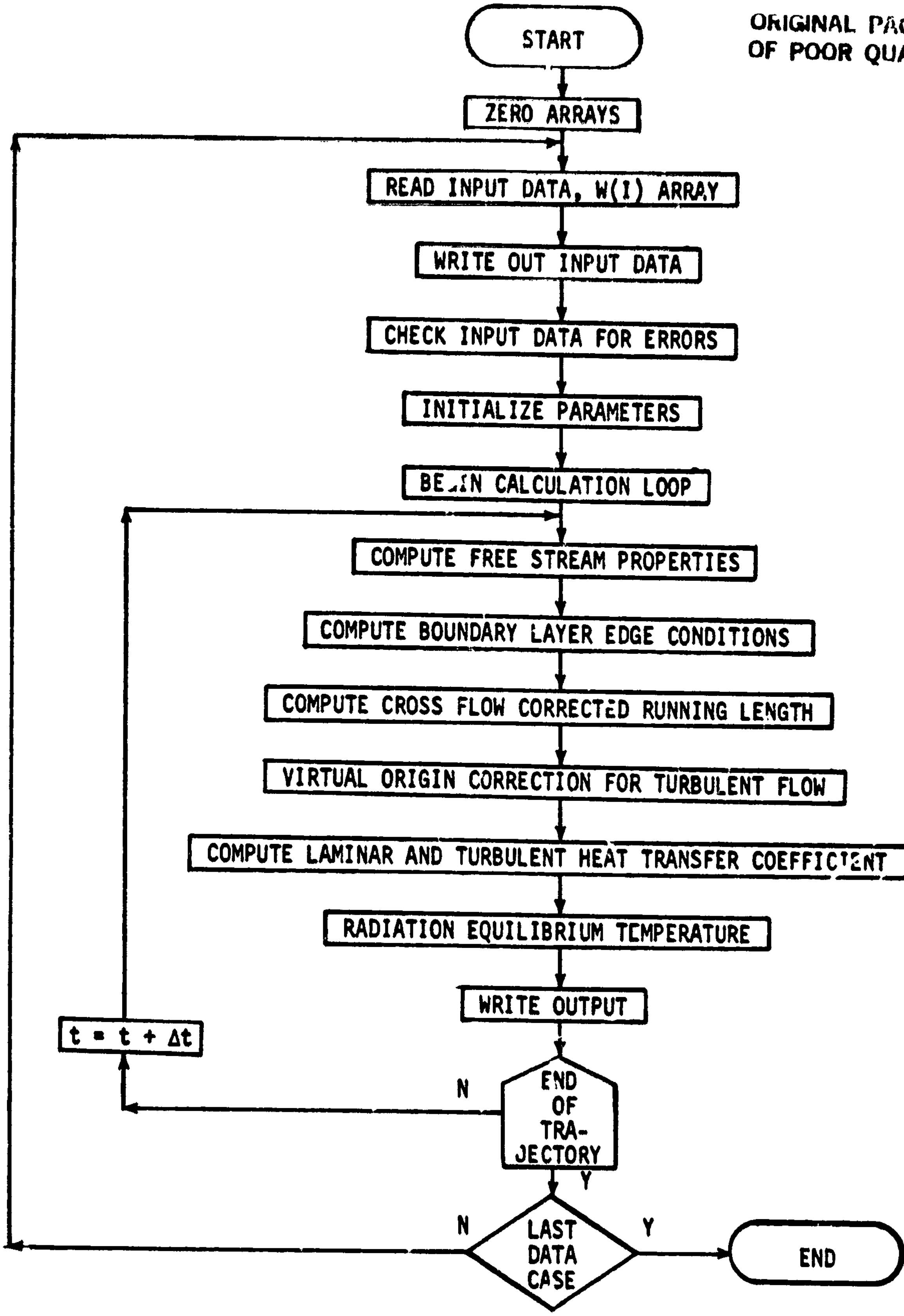


Fig. 2.1 Routine MAIN

Section 3.0

FLUID PROPERTY ROUTINES

In this section, the options of computing the free-stream air properties are set forth. These include ideal and real gas effects.

3.1 SUBROUTINE AIR62

This is one of the atmosphere routines in MINIVER. The routine describes the U. S. Standard Atmosphere, 1962 (ref. 2). Given altitude (ft), the routine yields ambient temperature (degrees R), density (slug/ft^3), pressure (lbf/ft^2), and speed of sound (ft/sec). The altitude range of the table is from 0 to 2,068,776 feet above sea level. The thermodynamic properties, temperature and density, are exponential curve-fits to the U. S. Standard Atmosphere, 1962. The ambient pressure and speed of sound are computed from the ideal gas equations. This routine does not call any other routines and returns only to the calling routine.

3.2 SUBROUTINE ATMS4

This is the 1963 Patrick atmosphere routine. The input into the routine is altitude (ft) and the routine returns ambient temperature (degrees R), density (slug/ft^3), pressure (lbf/ft^2), and speed of sound (ft/sec). The range of altitudes is from 0 to 2,300,000 feet with errors of less than 3 percent below 400,000 feet. This routine is entire within itself and does not call any other subroutines. The routine breaks up the altitude span into several ranges. Each altitude range then has the atmospheric properties calculated via a combination of polynomials and exponential curve-fits.

3.3 SUBROUTINE VRA71

This is the Vandenberg 1971 reference atmosphere. The input into the routine is altitude (ft) and the routine returns ambient temperature (degrees

R), density (slug/ft³), pressure (lbf/ft²), and speed of sound (ft/sec). The range of altitude is from 0 to 2,300,000 ft. This routine does not call any other routines. The subroutine calculates ambient viscosity and molecular weight, but these values are not used.

3.4 ROUTINES MOLIER, BLOCK, BLOCKA AND SLOPE

These are the routines constituting the air Mollier diagram and act as an equation of state to account for real gas effects. The data are contained in the G(I, J, K) matrix.

G(I, J, 1)	- Temperature	BLOCK
G(I, J, 2)	- Enthalpy	
G(I, J, 3)	- Entropy	
G(I, J, 4)	- Viscosity	
G(I, J, 5)	- Prandtl Number	BLOCKA
G(I, J, 6)	- Speed of Sound	
G(I, J, 7)	- Density	

The block data BLOCK contains the pressure array PT(14) and the variables G(I, J, K), K = 1,4. The block data BLOCKA contains the variables G(I, J, K), K = 5,7.

Four options may be used to look up properties based on two independent variables. These options are:

NOPT =	0	P,H	(Pressure, Enthalpy)
	1	P,S	(Pressure, Entropy)
	2	P,T	(Pressure, Temperature)
	3	H,S	(Enthalpy, Entropy)

Based on the options selected, the limits of the program are checked

$$10^{-10} \leq P \leq 10^{3} \text{ Atm.}$$
$$48. \leq H \leq 75580 \text{ Btu/lbm}$$
$$200. \leq T \leq 30000 \text{ R}$$
$$.925 \leq S/R \leq 12.368$$

If the independent variables are outside of the preceding range of validity, the routine sets all thermodynamic variables to zero and returns with IDEAL = 1.

Given the independent variables, the dependent variables are obtained by interpolation in the G matrix. For NOPT = 0, 1, and 2, the interpolation procedure is the same. The pressure index is found corresponding to the table pressures below and above the input pressure level. At the lower pressure level, the dependent variables are determined by two calls. First, the function SLOPE is called to calculate

$$FT1 = \frac{X - X1}{X2 - X1}$$

where X is the log10 of the other independent variable and the X1 and X2 values are the log10 values of the nearest points in the G matrix for the lower pressure. Given the value of FT1, all of the dependent variables are calculated by linear interpolation in log10 space using FT1 in a call to INTP1. Likewise, the dependent variables are determined at the higher pressure level in the tables. Finally, the dependent variables are determined for the input pressure level by linear interpolation in log10 (pressure) space by calling INTP2.

For NOPT = 3 the interpolation procedure is similar but uses different logic. The G matrix is sorted to obtain the surrounding points of enthalpy and entropy. The nondimensional entropy is used in a call to SLOPE to obtain the parameter

$$FT1 = \frac{S - S1}{S2 - S1}$$

at the lower pressure level where S is the log10 of the entropy. A call is made to INTP1 to obtain the dependent variables at this pressure level. This process is repeated for the higher pressure level. Finally, the dependent variables are determined at the given enthalpy level by a call to INTP2.

After the calculations and interpolations are made, some variables are converted in units and the compressibility and specific heat ratio are calculated for output.

The arguments of the call for MOLIER are

H	-	(input/output)	Enthalpy
P	-	(input/output)	Pressure
NOPT	-	(input)	Independent Variable Option
T	-	(input/output)	Temperature
Z1	-	(output)	Compressibility Factor
S	-	(output)	Entropy
R	-	(output)	Density
G1	-	(output)	Specific Heat Ratio

In addition the viscosity is output via the common statement /HANK/.

3.5 SUBROUTINE BINTRP

Subroutine BINTRP can be used to calculate the Prandtl number Pr, Lewis number Le, and dissociation enthalpy Hd for air. The Prandtl number is computed as a function of temperature (degrees R) and as a function of the ratio of the pressure at the stagnation point to standard pressure. The Lewis number and dissociation enthalpy are calculated as a function of temperature (degrees R) and as a function of the ratio of the density at the stagnation point to the density at standard conditions (2.37×10^{-3} slug/ft^3). Routine BINTRP is composed of three tables. Table I corresponds to dissociation enthalpy, Table II corresponds to Lewis number, and Table III corresponds to Prandtl number. The first two elements of the routine argument are the temperature (degrees R) and the pressure ratio, or the temperature (degrees R) and the density ratio. The third element of the routine argument is the table number (I, II, III). The last (fourth) element of the routine argument is either Hd, Le, or Pr, depending upon the input. Upon entering this routine, various checks occur to insure that the input values are not less than those values for which a calculation can be performed. If the temperature is negative, the pressure ratio is less than 10^{-6}

or the density ratio is less than 10^{-8}, then an error message is printed and NERROR = 1 is returned to the calling routine. If the tests are passed, then BINTRP proceeds to use double interpolation to determine the appropriate output. If Hd < 0, then Hd = 0 is returned. If Hd > 12450 Btu/lbm, then Hd = 12450 Btu/lbm is returned. If Le < 0.5, then Le = 0.5 is returned to the calling routine. If Pr < 0.678, then Pr = 0.678 is returned. If Pr > 0.734 and T > 24000 degrees R, then Pr = 0.734 is returned. Presently this routine is used only to determine Hd and Pr. Thus, the other one-third of this routine is not used.

3.6 SUBROUTINE HANSEN

Subroutine HANSEN is called by routine FLOW to calculate viscosity as a function of temperature and pressure. This subroutine calls MOLIER and returns viscosity via common HANK. If the viscosity is greater than the switch value of 0.29531E-08 slug/ft sec, the viscosity from MOLIER is returned. If the viscosity from MOLIER is less than the switch value, the viscosity is calculated based on the relations of Hansen and Helms (NACA TN 4359).

$$\mu = \mu_s \, B/D$$

where

$$PL = \log_{10} P/2116.$$

$$A = ((T/1800)(1-0.125PL) - 6.5)/(1.5 + 0.125PL)$$

$$B = 1. + 0.023(T/1800.)(1 + \mathrm{Tanh}(A))$$

$$C = (T/1800 - 14.5 - 1.5PL)/(0.9 + 0.1PL)$$

$$D = e^{C} + 1.0$$

$$\mu_s = 2.27 \times 10^{-8}\, T^{1.5}/(T + 198.6) \qquad \text{(Sutherlands)}$$

T = Temperature (Degrees R)

P = Pressure (lbf/ft^2)

If the value of B/D < 0.04 then B/D is set equal to 0.04.

Section 4.0

FLOWFIELD ROUTINES

This section of the report deals with those subroutines that are used to compute the local flowfield and boundary-layer edge conditions around the body under consideration. The methods used to evaluate shock angles, flowfields after shocks, and Prandtl-Meyer expansions are presented.

4.1 SUBROUTINE FLOW

Subroutine FLOW is called by MAIN. The subroutine argument contains two parameters, FF and ALPHA. The element FF is a one-dimensional array containing a possibility of nine consecutive flowfield and pressure flags (always occurring in pairs with the exception of the swept cylinder flowfield FF = 39 and the Prandtl-Meyer expansion flag FF = 29). The element ALPHA is a one-dimensional array containing a possibility of nine consecutive flowfield angles corresponding to the flowfield flags. The transfer of other information, such as upstream conditions, downstream conditions, etc., is performed through common blocks. The free-stream temperature and pressure are first needed to compute free-stream viscosity via subroutine HANSEN. The first check performed by FLOW determines if the Mach number upstream M_u of the first flowfield flag is greater than unity. If $M_u < 1$ and this is the first or second flag in the array FF, then the program ignores the flag and proceeds to calculate stagnation conditions based on the isentropic ideal gas relations. If the flag following the $M_u < 1$ verification calls for a Prandtl-Meyer expansion (FF = 29), then a deceleration of the flow occurs. The flow decelerates to a higher pressure calculated by modified Newtonian theory.

Knowing the local stagnation conditions and a local subsonic modified

Newtonian pressure, all of the other local properties are determined by the ideal gas isentropic relations. If the deceleration angle is greater than or equal to 90 degrees, then the local conditions are set equal to stagnation conditions. For subsonic flow, the local conditions determined above are also set equal to the local conditions along the stagnation line of an infinite swept cylinder. Having the local flowfield conditions, the Eckert reference properties are next calculated. If the heat transfer flag is equal to 5 or 7, then the $\rho_r\mu_r$ properties are also calculated. The routine then returns to MAIN.

If $M_n < 1$ and this is not the first flowfield or pressure flag, then the local stagnation conditions are taken to be the same as those corresponding to the conditions downstream of the previous pressure or Prandtl-Meyer flag. If the flowfield flag following $M_n < 1$ calls for a Prandtl-Meyer expansion (FF = 29), then a deceleration of the flow occurs. The flow decelerates to a higher pressure again computed via modified Newtonian theory. Knowing the pressure and also having the entropy from the (isentropic) upstream conditions, the other local thermodynamic properties are determined by using the Mollier diagram (subroutine MOLIER). If MOLIER returns IDEAL > 0, then the program switches to the ideal gas calculation previously discussed for $M_n < 1.0$. If MOLIER returns IDEAL = 0, then FLOW checks to determine if the deceleration angle is greater than or equal to 90 degrees. If the deceleration angle is greater than 90 degrees, then the local conditions are set equal to the local stagnation conditions. For subsonic flow, the final computed local conditions are also set equal to local conditions along the stagnation line of an infinite swept cylinder. Knowing the local flow properties, the Eckert reference properties are next calculated. If the heat transfer flag is equal to either 5 or 7, then the $\rho_r\mu_r$ properties are also calculated. The FLOW returns to MAIN. The difference between M_u < 1 and FF = 1 or 2, and M_u < 1 and FF > 2 is that for FF = 1 or 2 the fluid is treated as an ideal gas, and for FF > 2 the fluid is treated as a real gas. Only the equations of state are different.

Now if M_u > 1, FLOW checks to determine if FF = 35, FF = 36, FF = 38 (there is no FF = 37) or FF = 39. Each of these flowfield flags corresponds to a different technique for determining the shock angle β. For FF = 35 or 36, the value of ALFA is taken as the flow deflection angle for a sharp wedge or sharp cone, respectively. For FF = 38, the shock angle is read into the program as ALPHA. And FF = 39 corresponds to the swept cylinder flowfield and the shock angle is equal to ALFA. Subroutine FLOW (for FF = 35 or 36) calls routine PCSW to determine the attached shock angle as a function of M_u and the wedge or cone deflection angle. If NERROR ≠ 0 is returned from PCSW, then the input deflection angle must have been either negative or equal to zero. FLOW then immediately returns to MAIN and transfers to the next data case. The next check is to determine the value of OFT. If OFT = 1, then the flow upstream of the wedge must have been subsonic. If OFT = 2, then the shock associated with the wedge is detached and the shock angle is, therefore, taken to be 90 degrees. (If the shock is detached, FLOW performs a normal shock expansion (modified Newtonian) to the pressure flag angle associated with the flowfield angle.) If OFT = 0, then PCSW has successfully determined the shock angle as a function of upstream Mach number and deflection angle. If OFT = 1, then the routine proceeds to make a subsonic calculation as discussed above.

Having obtained a shock angle β (either input for FF = 38, β = ALFA for FF = 39, or for a sharp wedge or cone from PCSW) (90 degrees for a detached wedge or cone shock), FLOW next calls DWNSTM to determine the properties downstream of the shock. Subroutine DWNSTM uses β along with an air Mollier diagram (subroutine MOLIER) as an equation of state and the upstream conditions to compute the conditions downstream of the shock. If the properties required from the Mollier diagram cannot be obtained, then IDEAL = 1 or 2 may be returned to FLOW. If IDEAL = 1, then the routine proceeds by employing ideal gas relationships (discussed later). If IDEAL = 0 or 2, then the program proceeds by employing the Mollier diagram as the equation of state.

Once the conditions downstream of the shock are known and assuming IDEAL = 0, FLOW then proceeds to determine the local wall pressure based on the pressure flag (14, 15, 16, 17, 18) following the flowfield flag (35, 36, 38). If FF = 14, then Subroutine TBLIN is called to interpolate the pressure coefficient from an input set of tabular values with coefficient of pressure given as a function of free-stream Mach number. If FF = 15 or 16, then subroutine PCSW is called by FLOW to determine the pressure coefficient for a sharp wedge or cone as a function of M_u and deflection angle. Again if NERROR $\neq$ 0 is returned from PCSW, then FLOW returns to MAIN and skips to the next data case. If OFT = 1 is returned, then the program switches to perform a subsonic calculation. If OFT = 2 is returned (detached shock), then FLOW computes the local modified Newtonian pressure. If OFT = 0 is returned, then the local edge of the boundary-layer pressure is determined based on the pressure coefficient returned from PCSW. If FF = 18, then the local pressure is determined from modified Newtonian theory. The other local thermodynamic properties are obtained from the Mollier diagram routine (MOLIER) as a function of local edge pressure and the downstream entropy. Knowing the local enthalpy and the stagnation enthalpy, the velocity is computed from the energy equation.

If FF = 17, the pressure angle is the surface angle-of-attack or flow deflection angle. This pressure flag provides the pressure solution for a surface whose shock angle is slightly greater than the surface angle-of-attack or flow deflection angle. The shock angle must be known and input under FF = 38. The edge of the boundary-layer velocity is computed from the surface angle-of-attack, the shock angle and the upstream velocity. Then the local enthalpy is computed from the energy equation. Knowing the local enthalpy and the entropy behind the shock from DWNSTM (FF = 38), the other local thermodynamic properties are obtained via the air Mollier diagram routine. The wall pressure is an output of the Mollier diagram, rather than being obtained from the pressure coefficient (from PCSW) as was done for FF = 15 or 16. If the surface angle-of-attack is larger than the shock angle, then the program proceeds to compute the edge conditions via the parallel shock technique (with β as the shock angle). If this occurs, then any following flowfield flags are ignored, and the program proceeds to compute $\rho_r\mu_r$ and/or Eckert reference properties before returning to MAIN.

If FF = 14, 15, 16, 17, 18 and no abnormality occurs such as $\alpha > \beta$ (FF = 18), then before proceeding to process the next flowfield flag a check occurs to determine if a Prandtl-Meyer expansion is desired. If a Prandtl-Meyer expansion is desired, then FLOW calls PMEXPN to calculate conditions downstream of the expansion. If no Prandtl-Meyer expansion is desired, then FLOW bypasses PMEXPN. Next a check occurs to determine if additional flowfield flags (with or without pressure flags) are present. If there are, then the present available local conditions are taken as upstream conditions for the next flowfield. If no more flowfield flags are present, then FLOW proceeds to calculate $\rho_r\mu_r$ and/or Eckert reference properties.

If FF = 39 is used, then FLOW uses the downstream entropy and the stagnation line enthalpy to compute the other local thermodynamic properties. Having the local thermodynamic properties, the local stagnation and static flow properties are computed. Then the $\rho_r \mu_r$ and/or Eckert reference properties are computed. The swept cylinder flowfield option may be used alone or as a last flowfield flag in a series. Any flowfield option used after FF = 39 will be ignored by FLOW.

If in the use of the Mollier diagram routine the error message IDEAL = 1 or 2 is returned to FLOW, then FLOW skips the real gas calculation and performs an ideal gas calculation. The Mollier diagram routine (MOLIER) is capable of handling both ideal and real gas calculations. The ideal gas relations used in the program are primarily used when nothing else works. This is to keep the program running rather than stop the program because the Mollier diagram routine cannot extrapolate to a value off the table. Once FLOW switches to the ideal gas calculation, all the flowfield and pressure options are executed using the ideal gas relationships. The ideal gas equation of state is implicitly used instead of MOLIER. The shock angles and pressure coefficients are determined exactly the same way as for a real gas calculation. The downstream conditions are now determined by the ideal gas routine DOWNID instead of DWNSTM and the conditions downstream of the Prandtl-Meyer expansion are computed in the ideal gas routine PMID instead of PMEXPN. All logic and error checks are precisely the same as for the real gas calculation including the ideal gas determination of the $\rho_r \mu_r$ and/or Eckert reference properties.

4.2 SUBROUTINE DWNSTM

Subroutine DWNSTM is called by FLOW to calculate the conditions downstream of a shockwave (normal, oblique or parallel). This routine is used to calculate the downstream conditions for both real and ideal gases. However, the calculation procedure is based upon the assumption that a real gas calculation will take place. If subroutine MOLIER determines that an ideal gas calculation is sufficient, then the ideal gas equation of state is used instead of the Mollier diagram values.

The shock angle β is input into DWNSTM through the subroutine argument. For an attached wedge or cone shock, β is determined in subroutine PCSW. If the shock is determined to be detached by PCSW, then β is taken as 90 degrees for input into DWNSTM. If the parallel shock option is chosen, then the shock angle is set equal to the effective angle-of-attack.

Having an input shock angle β DWNSTM next requires the input of conditions upstream of the shock. The upstream conditions are input into DWNSTM via a common block with FLOW. Once the upstream conditions and shockwave angle are available, DWNSTM proceeds to calculate the downstream conditions. First, the normal components of the upstream velocity* and Mach number are calculated. Then an initial guess is made for ρ_2/ρ_1. If $V_1 \leq 4000$ ft/sec, the ideal gas equations are used to make the initial guess. If $V_1 > 4000$ ft/sec, the initial guess for ρ_2 is made using various empirical equations depending on whether 4000 ft/sec $< V_1 \leq 8000$ ft/sec, 8000 ft/sec $< V_1 \leq 14000$ ft/sec, 14000 ft/sec $< V_1 \leq 26000$ ft/sec, or 26000 ft/sec $< V_1$. Then a check occurs to insure that $\rho_2/\rho_1 > 1.0$. Subroutine DWNSTM then begins the first of a maximum of 50 iterations to determine the downstream properties.

---

*This discussion employs the following subscripts: 1, 2, N, T. The subscripts 1 and 2 correspond to conditions upstream and downstream of the shock, respectively. Subscripts N and T correspond to velocity components normal and tangent to the shock, respectively.

Having an initial guess for the downstream density $\rho_2^{(1)}$, the downstream normal component of velocity, V_{2_N}, is calculated from Equation (4-1).

$$V_{2_N} = V_{1_N}\left(\frac{\rho_1}{\rho_2^{(1)}}\right) \tag{4-1}$$

and then the downstream enthalpy and pressure are computed based on $\rho_2^{(1)}$

$$h_2 = h_1 + \left[\frac{V_{1_N}^{\ 2}}{2} - \frac{V_{2_N}^{\ 2}}{2}\right] \tag{4-2}$$

$$P_2 = P_1 + \left[\rho_1 V_{1_N}^{\ 2} - \rho_2^{(1)} V_{2_N}^{\ 2}\right] \tag{4-3}$$

Now $\rho_2^{(2)}$ is determined from subroutine MOLIER via Equation (4-4).

$$\rho_2^{(2)} = \rho(h_2, P_2) \tag{4-4}$$

If $\dfrac{\rho_2^{(2)} - \rho_2^{(1)}}{\rho_2^{(1)}} < 0.01$, then convergence is considered to have been achieved. If convergence is not achieved, then $\rho_2^{(1)}$ is replaced by $\rho_2^{(2)}$ and the calculations, beginning with Equation (4-1) are repeated. This process is continued until two consecutive iterations are within 1 percent of one another, or until a maximum of 25 iterations is attained. After 25 iterations alternate damping coefficients are chosen to obtain a guessed density

$$\rho_2^{(1)} = c\rho^{(2)} + (1 - c)\rho^{(1)}$$

where $c = 0.7$ between 26 and 35 iterations and $c = 0.4$ between 36 and 50 iterations. Once convergence is obtained, all of the static conditions behind the shock are known via Equations (4-1), (4-2), (4-3) and subroutine MOLIER. The velocity behind the shock is given by

$$V_2 = \sqrt{\left(\frac{\rho_1}{\rho_2}\right)^2 V_{1_N}^{\ 2} + (V_1\cos\beta)^2} \tag{4-5}$$

since $V_{1T} = V_1 \cos\beta = V_{2T}$.

Having the total enthalpy and S_2, all of the total conditions behind the shock are computed. Also this routine calculates the stagnation line enthalpy for the parallel shock. The stagnation line enthalpy is given by

$$h_{SL} = h_2 + \frac{V_{2N}^2}{2} \tag{4-6}$$

Subroutine DWNSTM does not calculate the other stagnation line properties for the parallel shock (swept cylinder stagnation line flowfield). The other properties are calculated after returning to FLOW. Subroutine FLOW uses S_2 and h_{SL} as inputs to MOLIER in determining the other parallel shock properties.

4.3 SUBROUTINE PCSW

Subroutine PCSW is called by routine FLOW and has the purpose of providing the shock angles and pressure coefficients for sharp wedges and cones. This routine contains four tables: one for the sharp-wedge shock angle as a function of upstream Mach number and deflection angle, one for the sharp-cone shock angle as a function of upstream Mach number and deflection angle, one for the sharp-wedge pressure coefficient as a function of upstream Mach number and deflection angle, and the last one for the sharp-cone pressure coefficient as a function of upstream Mach number and deflection angle. The argument of PCSW contains five parameters. The first parameter corresponds to the upstream Mach number, and the second parameter corresponds to the deflection angle. The third parameter ITABLE, is an integer value:

ITABLE = 1 indicates shock angle for a sharp wedge is desired

ITABLE = 2 indicates shock angle for a sharp cone is desired

ITABLE = 3 indicates pressure coefficient for a sharp wedge is desired

ITABLE = 4 indicates pressure coefficient for a sharp cone is desired

The fourth parameter, OFFTBL, is calculated in PCSW and may have one of three possible values. If OFFTBL = 1.0 is returned, then the upstream Mach number and the deflection angle correspond to conditions producing a detached shock. If OFFTBL = 0.0, then PCSW has successfully completed its function. The last parameter in the subroutine argument is the value of the shock angle or pressure coefficient for either the wedge or cone, depending upon which is requested from the subroutine.

Having successfully entered the routine, several checks occur. If the upstream Mach number is greater than 26, the M_u (upstream Mach number) is set equal to 26. If $M_u < 1$, then the routine returns OFFTBL = 1.If δ (deflection angle) ≤ 0, then PCSW prints out an error message and returns NERROR = 1. This causes PCSW to return to the MAIN program. The MAIN program then will skip to read input data for the next case if W(642) = 0.0, or will print the stored output in routine VANOUT and then read input data for the next case if W(642) $\neq$ 0.0. If $\delta > 60$ degrees, then the shock is taken to be detached for the cone and OFFTBL = 2.0 is returned to the calling program. If $\delta > 55$ degrees, then the shock is considered to be detached for the wedge and OFFTBL = 2.0 is returned to the calling routine.

Next, the value of ITABLE (1, 2, 3, 4) is checked to determine which table is to be used. For each value of ITABLE several checks occur to determine if the shock is detached. If these tests are passed, then PCSW proceeds to calculate either a shock angle or a pressure coefficient.

The wedge and cone tables for shock angle and pressure coefficient are represented by curve-fits. The pressure coefficient and shock angle are curve-fitted as a function of Mach number and deflection angle. Each of the four tables consists of several patches, each corresponding to various regions for each table.

After the value of shock angle or pressure coefficient is calculated, a final check is made to determine if $\beta \geq 90$ degrees or $C_p > 1.8$. This implies a detached shock and OFFTBL = 2.0 is returned to the calling program.

4.4 SUBROUTINE DOWNID

Subroutine DOWNID is called by subroutine FLOW to calculate the conditions downstream of a shockwave (normal, oblique, and parallel). This routine only performs calculations applicable for an ideal gas. The shock angle β is input into DOWNID through the subroutine argument. The upstream conditions are input into this routine via a common block with FLOW. Since an ideal gas calculation is being undertaken, there is no need for an iteration process as in DWNSTM. The calculation procedure begins with the calculation of the normal component* of free-stream Mach number

$$M_{1_N} = M_1 \sin \alpha \qquad (4-7)$$

---

*This discussion employs the following subscripts: 1, 2, N, T. The subscripts 1 and 2 correspond to conditions upstream and downstream of the shock, respectively. Subscripts N and T correspond to velocity components normal and tangent to the shock, respectively.

ORIGINAL PAGE IS
OF POOR QUALITY

The downstream pressure, temperature and Mach number are given by

$$\frac{P_2}{P_1} = \frac{2\gamma M_{1_N}^{\;2} - (\gamma - 1)}{(\gamma + 1)} \tag{4-8}$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} \frac{(\gamma - 1) M_{1_N}^{\;2} + 2}{(\gamma + 1) M_{1_N}^{\;2}} \tag{4-9}$$

$$M_2 = \frac{T_1}{T_2} \sqrt{\left[M_1^{\;2} - \frac{2}{(\gamma + 1)} \left\{\frac{P_2}{P_1} - \frac{T_2 P_1}{T_1 P_2}\right\}\right]} \tag{4-10}$$

The total pressure behind the shock, the static density, local speed of sound, downstream velocity and static enthalpy and given by

$$P_{o2} = P_1 \left[1 + \frac{(\gamma - 1)}{2} M_2^{\;2}\right]^{\gamma/\gamma-1} \tag{4-11}$$

$$\rho_2 = \frac{P_2}{RT_2} \tag{4-12}$$

$$a_2 = \sqrt{\gamma RT_2} \tag{4-13}$$

$$V_2 = M_2 a_2 \tag{4-14}$$

$$h_2 = .24T_2 \tag{4-15}$$

and then DOWNID returns to FLOW.

4.5 SUBROUTINE PMEXPN

Subroutine PMEXPN is the real gas Prandtl-Meyer expansion routine for air. The input conditions to the routine are upstream static properties and stagnation enthalpy. If the Prandtl-Meyer expansion angle θ_{pm} is greater than 103.2 degrees, then the program conveys an error message to the user.

The differential equation describing the real gas expansion is

$$\frac{dF}{d_{\theta pm}} = \frac{2F}{\sqrt{\frac{2F}{a^2} - 1}} \tag{4-16}$$

with $F = h_o - h = u^2/2$

and $a = a(h,s)$, $s = s_1$ = const.

where $a = a(h,s)$ is a functional representation of the Mollier chart (Subroutine MOLIER).

Knowing the value of θ_{pm}, one desires to determine enthalpy, h, at the end of the expansion fan. Once h is determined, all other properties are immediately available. To determine h, the above equation must be solved for F. This is achieved by employing the fourth-order Runge-Kutta method.

If 86 degrees < θ_{pm} < 103.2 degrees, then θ_{pm} is divided into twelve equal segments for numerical integration. If 68.8 degrees < $\theta_{pm} \leq$ 86 degrees, then θ_{pm} is separated into 10 equal divisions. If 51.6 < $\theta_{pm} \leq$ 68.8 degrees, then θ_{pm} is split into eight equal intervals. If 34.4 degrees < $\theta_{pm} \leq$ 51.6 degrees, then θ_{pm} is separated into six equal segments. If 17.2 degrees < $\theta_{pm} \leq$ 34.4 degrees, then θ_{pm} is divided into four equal intervals. And, if θ_{pm} < 17.2 degrees, then θ_{pm} is divided into two equal divisions. Having successfully determined F at the end of the expansion fan, all of the properties downstream of the fan are returned to the main program.

4.6 SUBROUTINE PMID

This is the ideal gas Prandtl-Meyer expansion routine. The input is the Prandtl-Meyer expansion angle, the upstream temperature, pressure and Mach number. The output consists of local Mach number, temperature, pressure, density, enthalpy, speed of sound and velocity after the expansion. The Mach number after the expansion fan is determined by using the Newton-Raphson iteration method. A maximum of 25 iterations is allowed.

Section 5.0

HEATING ROUTINES

The subroutines used to calculate heat transfer coefficients to various geometries are presented in this section.

5.1 SUBROUTINE FAYRID

Subroutine FAYRID is called by MAIN and is used to compute the heating at the stagnation point of a sphere. The arguments of the CALL statement are RN and ENCL. The nose radius, RN, is input and the laminar value of the heat transfer coefficient, ENCL, is output if a boundary layer calculation is made. The FAYRID subroutine has been modified to compute free molecular flow heating and rarefied flow heating in addition to boundary layer flow heating by the method of Fay and Riddell. The boundary layer flow method has been modified to calculate subsonic flow heating by a modification of the velocity gradient.

The first subroutine called by FAYRID is REGIME. Based on the value of IRE, the output from REGIME, a boundary layer, rarefied or free molecular flow calculation is made.

<u>If IRE = 1 Boundary Layer Flow</u>

A boundary layer flow stagnation point heating calculation is made based on the method of Fay and Riddell as described in Table 5.1. The specific dissociation enthalpy H_d is obtained from BINTRP as a function of the stagnation temperature behind the normal shock and the ratio of the stagnation density behind the normal shock to the density at standard atmospheric conditions. If NERROR $\neq$ 0 is returned by BINTRP, then FAYRID returns to MAIN and MAIN reads the next data case. Having obtained the heat transfer coefficient ENCL, FAYRID returns to the calling routine. PCT is set equal to 0.0 for output description purposes.

If IRE = 2 Rarefied Flow

A rarefied flow stagnation point heating calculation is made by calling STHEAT. PCT is set equal to 2.0 for output description purposes. The heat transfer coefficient is output through the labeled common /FREMO/ by the variable HTIL.

If IRE = 3 Free Molecular Flow

A free molecular flow heating calculation is made for a plate perpendicular to the flow direction by calling FMHEAT. PCT is set equal to 10.0 for output description purposes.

One labeled COMMON has been added to FAYRID and it is called /FREMO/. The variables transferred through this statement are:

IRE	- (output)	1 Boundary Layer 2 Rarefied Flow 3 Free Molecular Flow
PCT	- (output)	A variable specifying the calculation method used
HTIL	- (output)	Heat transfer film coefficient based on enthalpy
H	- (output)	Heat transfer film coefficient based on temperature
ATPBA	- (output)	Angle of attack plus local body angle
NONCON	- (output)	Integer value of W(646) If NONCON > 0 a rarefied flow calculation can be made if IRE > 1 If NONCON ≤ 0 only boundary layer calculations can be made (i.e. the program operates identically to the unmodified MINIVER program)
AK2	- (output)	Rarefaction parameter ZeK^2 from STHEAT
CHT	- (output)	Stanton number from STHEAT
CHI ZETA CHMI AK2CYL XI	}	Not derived from this subroutine or associated calls.

Since SWCYL calls FAYRID, this subroutine should be discussed here. No modifications have been made to SWCYL. However, if W(646) = NONCON is input as greater than zero and SWCYL is called, then a rarefied calculation could be performed if IRE > 1. This method is not recommended.

5.2 SUBROUTINE REGIME

This routine determines which flow regime is appropriate for heating calculations. The Mach number (AM), velocity (UI), density (RI), temperature (T), pressure (P), and characteristic length (X) are input through the argument. The subroutine calls DWNSTM to determine the post-shock compressibility factor. The flow regime is determined by the relations given in Table 5.2 and returns the integer IRE.

IRE = 1 Boundary layer (continuum)
    = 2 Rarefied (transitional)
    = 3 Free Molecular

Subroutine REGIME is called by FAYRID, SPCHI and SWCYL2.

5.3 SUBROUTINE STHEAT

This subroutine calculates the heat transfer coefficient, HTIL, to the stagnation point of a sphere in the rarefied flow regime. The equational basis for this calculation is given in Table 5.3. This subroutine calls DWNSTM to get the post normal shock temperature, TD, and compressibility factor, Z. Subroutine HANSEN is called to obtain the viscosity, XMU, at post-shock pressure and reference temperature levels.

Subroutine STHEAT is called by FAYRID. The arguments of the CALL statement are:

UI	- (input)	Free stream velocity
RI	- (input)	Free stream density
T	- (input)	Free stream temperature
P	- (input)	Free stream pressure
AM	- (input)	Free stream Mach number
GAMMA	- (input)	Free stream specific heat ratio
HI	- (input)	Free stream static enthalpy
TW	- (input)	Wall temperature
HW	- (input)	Wall enthalpy
R	- (input)	Body radius
QD	- (output)	Convective heating rate
H	- (output)	Heat transfer film coefficient based on temperature
HTIL	- (output)	Heat transfer film coefficient based on enthalpy
HAW	- (input)	Laminar adiabatic wall enthalpy
TAW	- (output)	Laminar adiabatic wall temperature

AK2 - (output) Rarefaction parameter
CHT - (output) Stanton number

Labeled commons /FRSTM/ and /DNSTRM/ are used to transfer information for the shock calculation.

5.4 SUBROUTINES FMHEAT AND ERF

Subroutine FMHEAT is used to compute free molecular flow heating to any windward facing surface. The function program ERF is the error function call in subroutine FMHEAT. The equations programmed in subroutine FMHEAT and function ERF are given in Table 5.4.

Subroutine FMHEAT is called by FAYRID, SPCHI and SWCYL2. The arguments of the call statement are:

UI - (input) Free stream velocity
RI - (input) Free stream density
T - (input) Free stream temperature
P - (input) Free stream pressure
AM - (input) Free stream Mach number
GAMMA - (input) Free stream specific heat ratio
HI - (input) Free stream static enthalpy
TW - (input) Wall temperature
HW - (input) Wall enthalpy
QC - (output) Convective heating rate
H - (output) Heat transfer film coefficient based on temperature
HTIL - (output) Heat transfer film coefficient based on enthalpy
HAW - (input) Laminar adiabatic wall enthalpy
TAW - (output) Laminar adiabatic wall temperature
TH - (input) Theta, θ, surface angle

No common block statements are used to transfer information.

5.5 SUBROUTINE SWCYL

Subroutine SWCYL is called by MAIN to compute the heat transfer coefficient along the stagnation line of a swept cylinder for laminar or turbulent flow. This is done using empirical relations based on the work of Cato for laminar flow and Johnson for turbulent flow as given in Table 5.5. Both techniques are based on empirical adjustments to the stagnation point heat transfer coefficient on a sphere obtained from FAYRID. The sphere is taken to have a radius equal to the radius of the cylinder. If the upstream Mach number is less than 1.12 the velocity gradient is adjusted per the relations given in Table 5.1 to yield the appropriate stagnation point heat transfer coefficient, ENF, from FAYRID.

After computing h_L and h, the Reynolds analogy factor $S = Pr^{2/3}$ is calculated and SWCYL returns to MAIN. Presently SWCYL contains four parameters: RN the cylinder radius, PHI the sweep angle, ENCL the laminar heat transfer coefficient, and ENCT the turbulent swept cylinder heat transfer coefficient. The parameters RN and PHI are inputs with ENCL and ENCT as outputs.

5.6 SUBROUTINE SWCYL2

Subroutine SWCYL2 is used to compute the heat transfer coefficient along the stagnation line of an infinite swept cylinder. This subroutine has been modified to compute rarefied flow heat transfer coefficients.

An initial check is made on NONCON = W(646) to determine if rarefied flow is to be considered. Subsequently, if NONCON > 0, the subroutine REGIME is called by SWCYL2. Based on the value of IRE, the output from REGIME, a boundary layer, rarefied or free molecular flow calculation is made. There are three parameters in the argument of the CALL statement for subroutine SWCYL2:

RN	–	(input)	Cylinder radius (ft)
ENCL	–	(output)	Swept-cylinder stagnation line laminar heat transfer coefficient
ENCT	–	(output)	Swept-cylinder stagnation line turbulent heat transfer coefficient

<u>If IRE = 1 Boundary Layer Flow</u>

A boundary layer flow calculation is made as described in Table 5.6a and 5.6b both for supersonic and subsonic flow conditions. PCT is set equal to 0.0 for output purposes. Subroutine MOLIER is called to obtain wall properties.

HANSEN is called to obtain wall viscosity. FAYRID is called to obtain the sphere stagnation point heat transfer coefficient.

If IRE = 2 Rarefied Flow

A rarefied flow calculation is made by calling CYT. PCT is set equal to 3.0 for output purposes.

If IRE = 3 Free Molecular Flow

A free molecular flow calculation is made for a plate at the local sweep angle. PCT is set equal to 10.0 for output purposes.

The labeled COMMON has been added to SWCYL2 and it is called /FREMO/. The variables from this subroutine transferred through the COMMON/FREMO/ are:

AK2CYL - Rarefaction parameter, $\tilde{K}^2{}_\Lambda$

XI - Stanton Number parameter, ξ

plus the variables defined in section 5.1.

5.7 SUBROUTINE CYT

Subroutine CYT is used to compute rarefied flow stagnation line heating rates for a right circular cylinder at arbitrary yaw angles. The equations programmed in this subroutine are given in Table 5.7.

Subroutine CYT is called by SWCYL2. The arguments of the CALL statement are:

UI	- (input)	Free stream velocity
RI	- (input)	Free stream density
T	- (input)	Free stream temperature
P	- (input)	Free stream pressure
AM	- (input)	Free stream Mach number
GAMMA	- (input)	Free stream specific heat ratio
HI	- (input)	Free stream static enthalpy
TW	- (input)	Wall temperature
HW	- (input)	Wall enthalpy
X	- (input)	Distance along the surface
QC	- (output)	Convective heating rate
H	- (output)	Heat transfer film coefficient based on temperature
HTIL	- (output)	Heat transfer film coefficient based on enthalpy

HAW - (input) Laminar adiabatic wall enthalpy
TAW - (output) Laminar adiabatic wall temperature
DLAMB - (input) Sweep angle
AK2CYL- (output) Rarefaction Parameter, K^2_Λ
XI - (output) Stanton Number Parameter, ξ

Subroutine CYT calls DWNSTM to calculate post normal shock temperatures and pressures. Subsequently subroutine HANSEN is called to compute viscosity.

Labeled commons /FRSTM/ and /DNSTRM/ are used to transfer information for the shock calculations.

5.8 SUBROUTINE SWCYL3

Subroutine SWCYL3 is also called by MAIN and employs the $\rho_r\mu_r$ technique for computing the heat transfer coefficient along the stagnation line of an infinite swept cylinder as described in Table 5.8. This routine calls subroutines MOLIER and BINTRP. The argument of the CALL statement for SWCYL3 contains the three following parameters:

RN - (input) Cylinder radius
ENCL - (output) Laminar heat transfer coefficient along the stagnation line
ENCT - (output) Turbulent heat transfer coefficient along the stagnation line of an infinite swept cylinder.

All other input information necessary for computation is brought into SWCYL3 through common blocks.

5.9 SUBROUTINE DETRAL

Subroutine DETRAL is called by MAIN and is used in calculating the laminar and turbulent heat transfer coefficients about hemispherical nose shapes. This routine enables the evaluation of circumferential as well as stagnation point heat transfer coefficients. The laminar equations of Lees, given in Table 5.9a, are used to compute the laminar heat transfer coefficient. The turbulent equations of Detra and Hidalgo, given in Table 5.9b, are used to compute the turbulent heat transfer coefficient.

The arguments of the CALL statement for DETRAL contain the following parameters:

RN	–	(input)	Sphere radius
EL	–	(input)	Running length from stagnation point
PHI	–	(input)	Body angle (90 degrees at the stagnation point)
ENCL	–	(output)	Laminar heat transfer coefficient
ENCT	–	(output)	Turbulent heat transfer coefficient

DETRAL calls FAYRID to obtain the laminar stagnation point heat transfer coefficient and calls BINTRP to obtain the dissociation enthalpy, HD.

5.10 SUBROUTINE ECKERT

Subroutine ECKERT is called by MAIN and is used to calculate the laminar and turbulent heat transfer coefficients for flow over a flat plate with negligible pressure gradient. The CALL statement for subroutine ECKERT contains 6 parameters:

ELL	–	(input)	Laminar running length (already corrected for crossflow if crossflow option was chosen)
ELT	–	(input)	Turbulent running length (already corrected for crossflow and virtual origin adjustment if these options were chosen)
ENL	–	(input)	Laminar Mangler transformation factor to account for axisymmetric flow correction to flat-plate heating
ENT	–	(input)	Turbulent Mangler transformation factor to account for axisymmetric flow correction to flat-plate heating
ENCL	–	(output)	Laminar heat transfer coefficient based on enthalpy
ENCT	–	(output)	Turbulent heat transfer coefficient based on enthalpy

Subroutine BINTRP is called to determine the Prandtl number Pr_L as a function of the Eckert reference temperature for laminar flow (recovery factor is 0.85) and the ratio of the boundary-layer edge pressure to standard atmospheric pressure. The Eckert heat transfer coefficient for laminar flow is calculated using the relations in Table 5.10a. Next subroutine BINTRP is called to determine the Prandtl number Pr_T as a function of the Eckert reference temperature for turbulent flow (recovery factor is 0.88) and the ratio of the boundary-layer edge

edge pressure to standard atmospheric pressure. The turbulent flow heat transfer coefficient is calculated using the relations given in Table 5.10b.

If the parameter NERROR $\neq$ 0 is returned from BINTRP, then ECKERT returns to MAIN and proceeds to read in the next data case. The Eckert reference properties (laminar and turbulent) are calculated in subroutine FLOW based on a constant Prandtl number (laminar recovery factor is 0.85 and the turbulent recovery factor is 0.88). Using this constant Prandtl number the Eckert reference temperature is determined. This reference temperature is then used in conjunction with BINTRP to determine the Prandtl number (now considered as a function of temperature and pressure) which was originally taken to be a constant. Strictly speaking, the Prandtl number should be determined by an iterative procedure. However, this probably does not cause any appreciable error.

5.11 SUBROUTINES SPCHI AND FSUBC

Subroutine SPCHI is called by MAIN and is used to calculate the turbulent heat transfer coefficient for turbulent flow, and also the Eckert reference technique is used to compute the laminar heat transfer coefficient. Both correspond to flow over a flat plate with a negligible pressure gradient. The CALL statement for subroutine SPCHI contains seven parameters:

ELL	-	(input)	Laminar running length (already corrected for crossflow if crossflow option was chosen)
ELT	-	(input)	Turbulent running length (already corrected for crossflow and virtual origin adjustment if these options were chosen)
ENL	-	(input)	Laminar Mangler transformation factor to account for axisymmetric flow correction to flat-plate heating
ENT	-	(input)	Turbulent Mangler transformation factor to account for axisymmetric flow correction to flat-plate heating
ENCL	-	(output)	Laminar heat transfer coefficient based on enthalpy
ENCT	-	(output)	Turbulent heat transfer coefficient based on enthalpy
RANFLG	-	(input)	If a value greater than zero is input into location 919, then the Von Karman form of the Reynolds analogy factor is used.

An initial check is made on NONCON = W(646) to determine if boundary layer flow is to be considered (yes if NONCON > 0). Subsequently, if NONCON > 0, the subroutine REGIME is called by SPCHI. Based on the value of IRE, the output from REGIME, a boundary layer, rarefied or free molecular calculation is made.

If IRE = 1 Boundary Layer Flow

A boundary layer flow calculation is made as described in Table 5.11 for turbulent flow and Table 5.10a for laminar flow. PCT is set equal to 0.0 for output purposes.

SPCHI calls subroutine FSUBC to compute F_c for a real gas using MOLIER. The argument of FSUBC contains five parameters:

HE	- (input)	Enthalpy at the edge of the boundary layer
HW	- (input)	Wall enthalpy
HRECT	- (input)	Turbulent Eckert reference enthalpy
PE	- (input)	Boundary-layer edge pressure
FCINV	- (output)	$1/F_c$

If no problem is encountered with the Mollier diagram, then F_c is returned to SPCHI. If the thermodynamic properties cannot be determined by MOLIER, then IDEAL = 1 is returned to SPCHI and an ideal gas calculation of F_c is undertaken. Having obtained F_c, the turbulent heat transfer coefficient is then determined from the Spalding-Chi equations which have been checked and found to be correct. If a value greater than zero is stored in location 319, then the Von Karman Reynolds analogy factor is used in determining the turbulent heat transfer coefficient. However, if zero is stored in location 319, then the Reynolds analogy factor is used as $Pr^{2/3}$.

After the turbulent heat transfer coefficient has been determined, SPCHI next proceeds to compute the laminar heat transfer coefficient by the Eckert reference technique. Both the laminar and turbulent heat transfer coefficients are corrected for crossflow and for axisymmetric flow effects (via the laminar and turbulent Mangler transformation factors) before being used in the heat transfer coefficient equations.

<u>If IRE = 2 Rarefied Flow</u>

NFCS > 0.0, NFCS = W(314)

If NFCS > 0.0, then SPCHI calls SPFP.

The rarefied flow heat transfer coefficient for a sharp flat plate at zero angle of attack is computed. PCT is set equal to 4.0 for output purposes.

NFCS $\leq$ 0.0

If NFCS $\leq$ 0.0 then SPCHI calls CONE. The rarefied flow heat transfer coefficient for a sharp nose cone is computed. PCT is set equal to 5.0 for output purposes.

<u>If IRE = 3 Free Molecular Flow</u>

A free molecular flow heating calculation is made for a plate at the local body angle by calling FMHEAT. PCT is set equal to 10.0 for output purposes.

5.12 SUBROUTINE SPFP

Subroutine SPFP is used to compute flat plate heating rates in the rarefied flow regime. The equations programmed in this subroutine are given in Table 5.12.

Subroutine SPFP is called by SPCHI. The arguments of the CALL statement are:

UI	- (input)	Free stream velocity
RI	- (input)	Free stream density
T	- (input)	Free stream temperature
P	- (input)	Free stream pressure
AM	- (input)	Free stream Mach number
GAMMA	- (input)	Free stream specific heat ratio
HI	- (input)	Free stream static enthalpy
TW	- (input)	Wall temperature
HW	- (input)	Wall enthalpy

X - (input) Distance along the surface
QC - (output) Convective heating rate
H - (output) Heat transfer film coefficient based on temperature
HTIL - (output) Heat transfer film coefficient based on enthalpy
HAW - (input) Laminar adiabatic wall enthalpy
TAW - (output) Laminar adiabatic wall temperature
CHMI - (output) Rarefaction parameter, $M_\infty \sqrt{C_*/Re}$

Subroutine SPFP calls HANSEN to compute viscosity for use in calculating the Chapman-Rubesin constant.

No common statements are used.

5.13 SUBROUTINE CONE

Subroutine CONE is used to compute rarefied flow heating rates for a sharp cone. The equations programmed in this subroutine are given in Table 5.13.

Subroutine CONE is called by SPCHI. The arguments of the CALL statement are:

UI - (input) Free stream velocity
RI - (input) Free stream density
T - (input) Free stream temperature
P - (input) Free stream pressure
AM - (input) Free stream Mach number
GAMMA - (input) Free stream specific heat ratio
HI - (input) Free stream static enthalpy
TW - (input) Wall temperature
HW - (input) Wall enthalpy
X - (input) Distance along the surface
QC - (output) Convective heating rate
H - (output) Heat transfer film coefficient based on temperature
HTIL - (output) Heat transfer film coefficient based on enthalpy
HAW - (input) Laminar adiabatic wall enthalpy
TAW - (output) Laminar adiabatic wall temperature
TCD - (input) Semivertex angle of the cone
CHI - (output) Rarefaction parameter, $\overline{\chi}_C$
ZETA - (output) Stanton Number parameter, ξ
NSB - (input) NSB = W(650)
NSB = 0 Sharp cone curve fit
NSB = 1 Blunt cone curve fit

Subroutine CONE calls DWNSTM to calculate post normal shock temperature and pressure. Subsequently subroutine HANSEN is called to compute viscosity based on a reference temperature.

Labeled commons /FRSTM/ and /DNSTRM/ are used to transfer information for the shock calculation.

5.14 SUBROUTINE RHOMUR

Subroutine RHOMUR is called by MAIN and is used to compute laminar and turbulent heat transfer coefficients over a flat plate using the relations in Table 5.14. Subroutine BINTRP is called by RHOMUR to compute the Prandtl number as a function of reference temperature and as a function of the ratio of the boundary-layer edge pressure to standard atmospheric pressure. Also BINTRP is called to compute the specific dissociation enthalpy as a function of reference temperature and as a function of the ratio of the reference density to standard atmospheric density. The expressions for the laminar and turbulent heat transfer coefficient have been somewhat simplified from the original $\rho_r\mu_r$ expressions. The equivalent running length is determined from CRSFLW and the original $\rho_r\mu_r$ crossflow expressions are not used. Also there is no adjustment to the heat transfer coefficient to account for the increased heating due to axisymmetric flow (Mangler transformation factor). The argument of the CALL statement for RHOMUR contains the four following parameters:

ELL	- (input)	Laminar running length which may have been corrected for crossflow effects
ELT	- (input)	Turbulent running length which may have been corrected for crossflow effects and virtual origin adjustment
ENCL	- (output)	Laminar heat transfer coefficient
ENCT	- (output)	Turbulent heat transfer coefficient

All other information necessary for computation is brought into RHOMUR through common blocks.

5.15 SUBROUTINE LESIDE

Subroutine LESIDE is called from MAIN after a call to FAYRID, to calculate the average heat transfer coefficient to the leeside of an orbiter. The average heat transfer coefficient is calculated using the relations in Table 5.15. This subroutine is called if NHFLAG = 9. The arguments of the call are:

```
RN   - (input)  Radius of scale, full scale RN = 1.0 ft.
HWD  - (input)  W(21), Windward Wall enthalpy (Btu/lbm)
ENCL - (output) Average turbulent or laminar leeward side heat
                transfer coefficient
ENCT - (output) = ENCL
```

The common FLWFLD is used to transfer post normal shock values obtained by the call to FAYRID.

5.16 SUBROUTINES FLAPH, DEFL, DSTML, CPIAF AND BTHICK

Subroutine FLAPH is calle from MAIN when NHFLAG = 10 to calculate the peak reattachment heating to a flap. The equations used for this calculation are given in Table 5.16. The arguments of the call are as follows:

```
EL     - (input)  Running length
FLENG  - (input)  Flap length
FANG   - (input)  Flap angle
TW     - (input)  Wall temperature
HT     - (input)  Total enthalpy
ENCLO  - (output) Laminar heat transfer coefficient
ENCTO  - (output) Turbulent heat transfer coefficient
IQUIT  - (output) Flag IQUIT = 0 Separation occurs
                       IQUIT = 1 No separation occurs
PCT    - (output) Transition percentage
```

A common statement, FLPETG, is used to transfer the following edge properties from MAIN into the subroutine:

```
EDGEP(1,I)  -  Pressure
EDGEP(2,I)  -  Temperature
EDGEP(3,I)  -  Density
EDGEP(4,I)  -  Speed of sound
EDGEP(5,I)  -  Velocity
EDGEP(6,I)  -  Mach number
EDGEP(7,I)  -  Viscosity
EDGEP(8,I)  -  Specific heat ratio
```

where I = 1 - Before the flap shock edge conditions
      I = 2 - After the flap shock edge conditions

Subroutine FLAPH calls BTHICK to obtain a laminar and turbulent boundary layer thickness. This calculation is made using Eckert reference conditions in the following equations:

$\delta = 5.2X/\sqrt{Re_x^*}$: Laminar boundary layer thickness

$\delta = 0.154X/(Re_x^*)^{1/7}$: Turbulent boundary layer thickness

Next subroutine CPIAF is called to calculate the incipient separation pressure coefficient. This is followed by a call to DEFL to determine if the input flap angle is sufficiently large enough to produce separation. If the input angle is insufficient to cause separation, IQUIT is set equal to 1 and the subroutine returns to the MAIN. If the subroutine returns to the MAIN with IQUIT = 1, then the heating is calculated by a call to SPCHI.

If separation is found to occur, the plateau pressure is calculated and DEFL is called to calculate the dividing streamline angle. Next the separation length is calculated by a call to DSTML. If the separation geometry indicates that impingement will not occur for the input flap length, IQUIT = 1 and the subroutine returns to the MAIN.

If impingement occurs, the shear layer thickness is calculated. Subroutine HANSEN is called to calculate the wall viscosity. Finally, the heat transfer coefficient is calculated for either laminar or turbulent flow with PCT set equal to 0.0 or 1.0. The laminar and turbulent heat transfer coefficients are equated since this routine can calculate only one type for a given flowfield. Note that in MAIN if NHFLAG = 10 and IQUIT = 0 then the call to TRANS is skipped.

5.17 SUBROUTINES FINH, FINPKH AND FINPKP

Subroutine FINH is called by MAIN when NHFLAG = 11 to calculate the peak interference heating produced by a fin on the adjacent surface. The equations used for this calculation are given in Table 5.17 The arguments of the call are as follows:

ALP	– (input)	Effective fin angle of attack
XP	– (input)	Length along fin surface
YP	– (output)	Normal distance from fin to peak heating
EL	– (input)	Running length to fin leading edge
TW	– (input)	Wall temperature
HO	– (input)	Total enthalpy
AMPPT	– (output)	Turbulent pressure amplification
AMPPL	– (output)	Laminar pressure amplification
AMPHL	– (output)	Laminar heating amplification
AMPHT	– (output)	Turbulent heating amplification
ENCLO	– (output/input)	Laminar heat transfer coefficient
ENCTO	– (output/input)	Turbulent heat transfer coefficient

The common statement, FLPHTG, is used to transfer edge properties upstream of the fin shock from MAIN into the subroutine. The variables transferred are defined in subsection 5.16.

Subroutine FINH calls BTHICK to obtain the laminar and turbulent boundary layer thickness using the relations given in subsection 5.16. Next the shock angle produced by the fin is computed using a call to PCSW. If the shock is attached, the peak pressure amplification is calculated for laminar and turbulent flow using two calls to FINPKP. The peak heating amplification for laminar and turbulent is then computed using two calls to FINPKH. The input heat transfer coefficients are modified using the heating amplification factors.

Subroutine SPCHI is always called before the call to FINH to obtain the undisturbed heat transfer coefficient. Note that in MAIN, if NHFLAG = 11, the heating amplification factor and edge pressure are calculated based on PCT after the call to TRANS.

5.18 SUBROUTINE RADEQT

Subroutine RADEQT is called by MAIN and is used to compute the radiation equilibrium temperature. The radiation equilibrium temperature is determined via the Newton-Raphson iteration technique. The wall temperature is used as an initial guess and convergence to the radiation equilibrium temperature is considered to be achieved after 50 iterations, or after two successive iterations are within 0.5 percent of one another. The first of these two conditions to occur defines convergence.

The argument of the CALL statement for subroutine RADEQT contains the following five parameters:

ENC	-	(input)	Heat transfer coefficient
HR	-	(input)	Recovery enthalpy
EMIS	-	(input)	Effective emissivity - product of shape factor and emissivity
TW	-	(input)	Conduction or thin-skin wall temperature which is used as an initial guess to the radiation equilibrium temperature in the iteration scheme
TRE	-	(output)	Radiation equilibrium temperature

Table 5.1

STAGNATION POINT HEATING

Stagnation point heating is based on the theory of Fay and Riddell.

$$q = \frac{N_u}{\sqrt{R_e}} \sqrt{g_c \rho_w \mu_w \left(\frac{dU_e}{dx}\right)} \frac{(H_t - H_w)}{P_{r_w}}$$

where

$$\frac{N_u}{\sqrt{R_e}} = 0.76\, P_{r_w}^{0.4} \left(\frac{\rho_t \mu_t}{\rho_w \mu_w}\right)^{0.4} \left\{1 + (L_e^{.52} - 1)\frac{H_d}{H_t}\right\}$$

$$\left(\frac{dU_e}{dx}\right) = \frac{1}{R}\sqrt{2g_c \frac{(P_t - P_\infty)}{\rho_t}} \text{ for a sphere and } M_\infty > 1.22$$

$$= \frac{1}{2R}\sqrt{2g_c \frac{(P_t - P_\infty)}{\rho_t}} \text{ for a cylinder and } M_\infty > 1.12$$

$$= \frac{U_\infty}{R}(1.5 - 0.378M_\infty^2 - 0.02625\, M_\infty^4) \text{ for a sphere and } M_\infty \leq 1.22$$

$$= \frac{U_\infty}{R}(2.0 - 0.872M_\infty^2 - 0.328M_\infty^4) \text{ for a cylinder and } M_\infty \leq 1.12$$

and the dissociation enthalpy

$$H_d = C_o h_o^\circ + C_N h_N^\circ$$

$$h_o^\circ = 6636.26 \text{ Btu/Lbm}$$

$$h_N^\circ = 14456.53 \text{ Btu/Lbm}$$

Nomenclature

q = Heating Rate (Btu/ft^{2}sec.)
g_c = 1 (slug ft./lb$_f$sec.^{2})
g = 32.174 (Lbm/slug)
ρ = Density (slug/ft.^{3})
μ = Viscosity (Lb$_f$sec/ft^2)
U = Velocity (ft./sec.)
P = Pressure (Lb$_f$/ft.^{2})
H = Enthalpy (Btu/lbm)

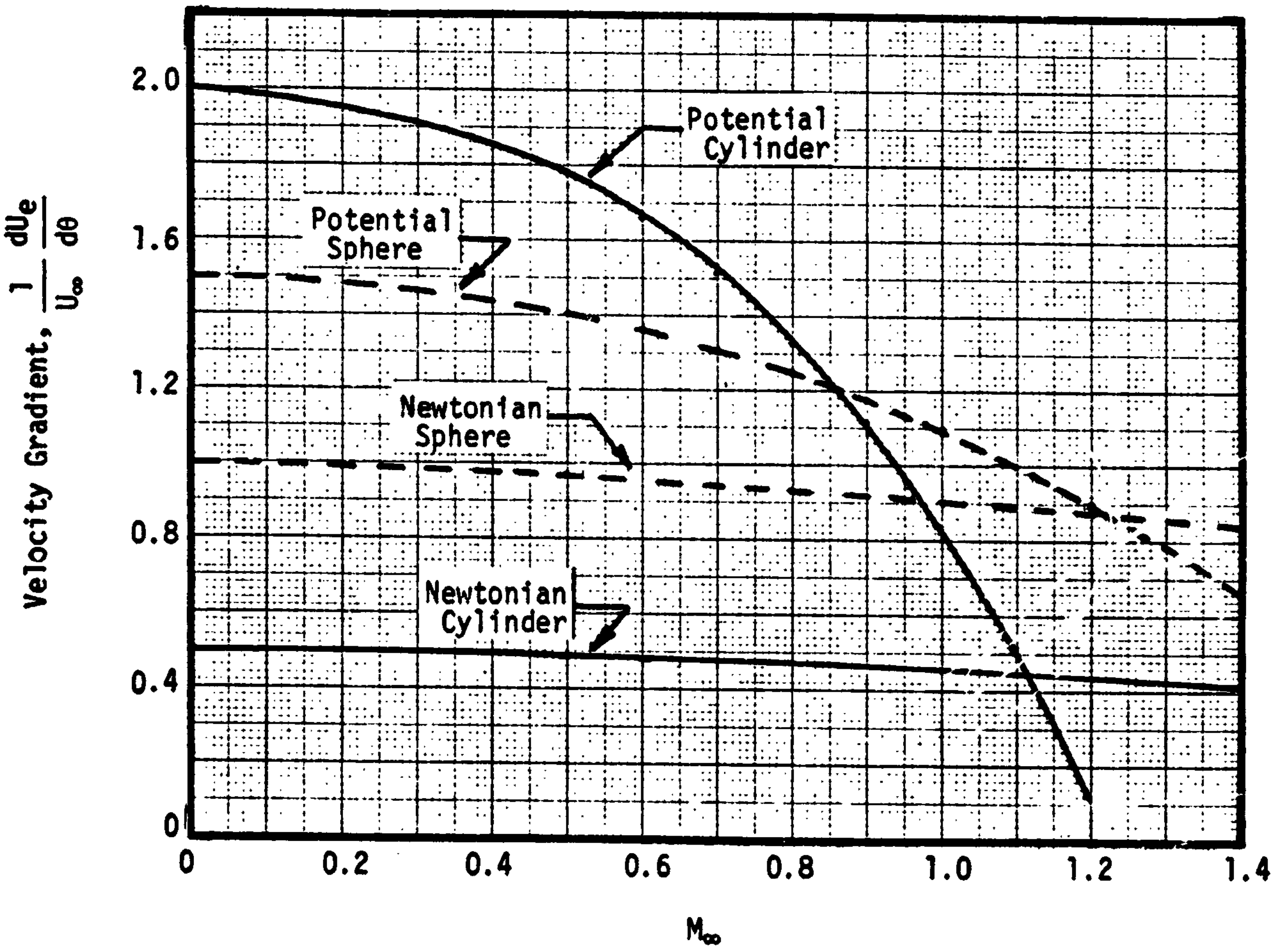
Table 5.1 (Cont. 1)

ORIGINAL PAGE IS
OF POOR QUALITY

L_e = Lewis Number
H_d = Dissociation enthalpy of air (Btu/Lbm)
C_i = Mass fraction of species i (i = O, oxygen and i = N, nitrogen)
R = Nose radius (ft.)
P_r = Prandtl Number
O = Sphere Body Angle

Subscripts

e = Edge
t = Stagnation (post shock)
w = Wall
∞ = Free stream



References

Fay, J. A. and Riddell, F. R., "Theory of Stagnation Point Heat Transfer in Dissociated Air", J. Aeronaut. Sci., Vol. 25, No. 2, Feb. 1958, pp. 78-85, 121.

Table 5.2

BOUNDARY LAYER, RAREFIED AND
FREE MOLECULAR FLIGHT REGIME CRITERION

$A = \dfrac{M_\infty}{Ze(Re_{\infty X})^{0.5}}$	Flight Regime Selection Parameter
If $A \leq 0.05$	Boundary Layer
If $0.05 < A < 3.0$	Rarefied
If $A > 3.0$	Free Molecular

where

M_∞ = Free Stream Mach Number

$Re_{\infty X}$ = Free Stream Reynolds Number Based on Running Length or Radius

Ze = Post Normal Shock Compressibility

Table 5.3

RAREFIED FLOW STAGNATION POINT HEAT TRANSFER EQUATIONS

The rarefied flow heating to the sphere stagnation point based on the work of Engel and Praharaj is as follows.

(1) $T_r = \left(T_\delta + T_W\right)/2$ (Reference temperature)

(2) $T_0 = T_\infty\left(1 + \frac{\gamma-1}{2}M_\infty^2\right)$ (Free stream stagnation temperature)

(3) $K^2 = \epsilon\left(\frac{\rho_\infty U_\infty R}{\mu_r}\right)\left(\frac{T_r}{T_0}\right)$ (Rarefaction parameter)

where $\epsilon = \frac{\gamma-1}{2\gamma}$

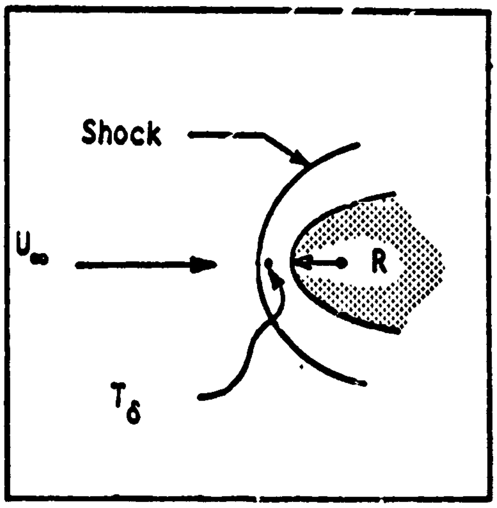
(4) Heat transfer coefficient

$$\log_{10}(C'_H) = \sum_{i=0}^{2} a_i(\log_{10} ZeK^2)^i$$

$a_0 = -0.235256$

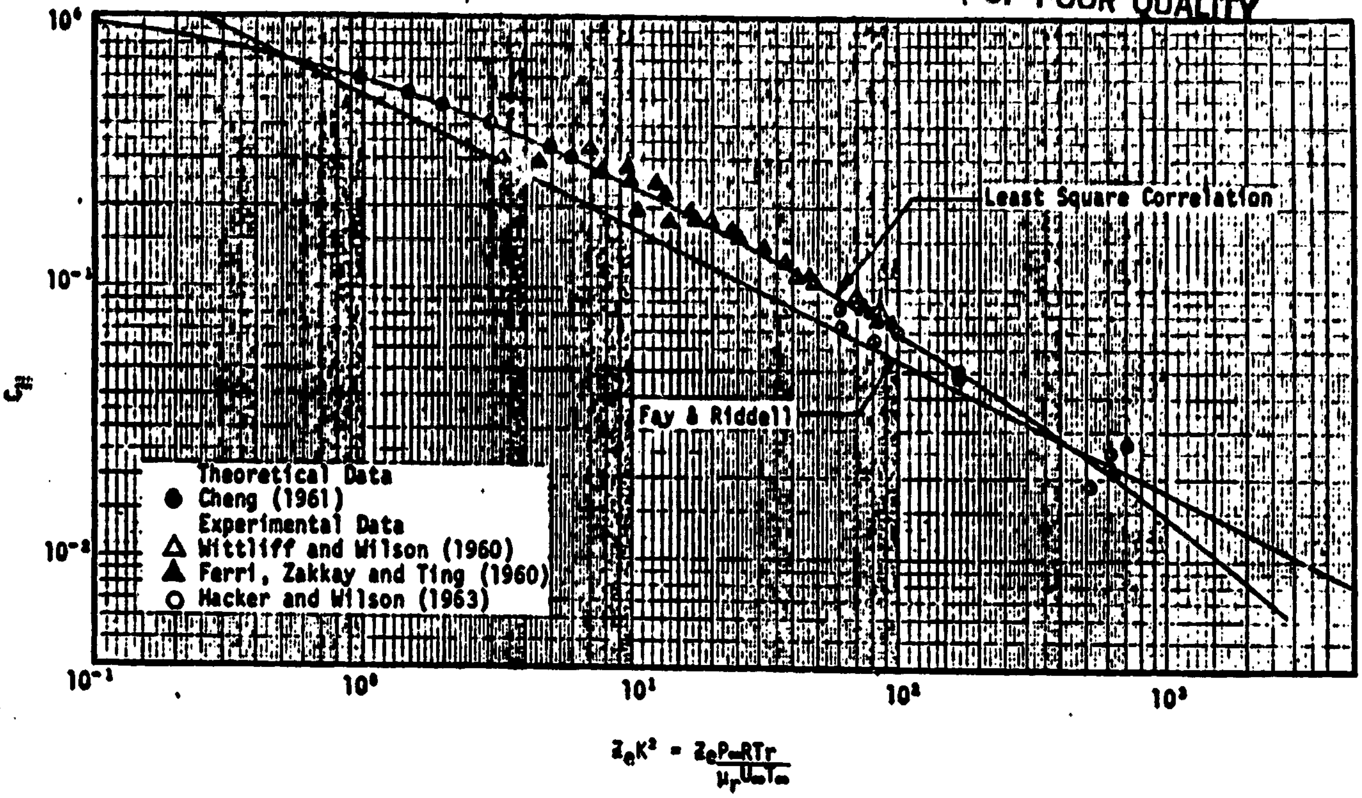
$a_1 = -0.303095$

$a_2 = -0.0779538$



(5) Heat transfer

$$q = \rho_\infty U_\infty C_H(H_0 - H_W)$$



Stagnation Point Heat Transfer in Low Reynolds Number Flow

Nomenclature

C_H	Stanton Number
H	Enthalpy
M_∞	Free Stream Mach Number
T	Temperature
U	Velocity
Z	Post Normal Shock Compressibility
ρ	Density
γ	Specific Heat Ratio
μ	Viscosity

Subscripts

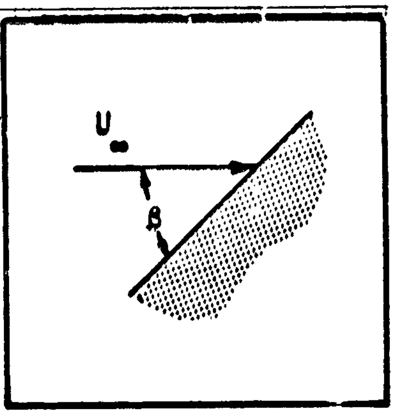
∞	Free Stream
W	Wall
$\delta = e$	Post Normal Shock
o	Total

Reference

Engel, C.D. and Praharaj, S.C., "External Tank Rarefied Aerothermodynamics," REMTECH Inc. RTR 022-1, January 1978.

Table 5.4

FREE MOLECULAR FLOW HEATING EQUATIONS



(1) $N = \frac{P}{kT_\infty}$ (particles/unit volume)

(2) $S = \sqrt{\frac{\gamma}{2}}\, M_\infty$ (molecular speed ratio)

(3) $\psi = \frac{NU_\infty}{2\sqrt{\pi}\, S}$

(4) $\eta = S \sin \beta$

(5) $n = \psi \left[e^{-\eta^2} + \sqrt{\pi}\,\eta(1 + \text{erf}\ \eta)\right]$

= number of molecules striking a unit area per unit time

where

$$\text{erf}\ \eta = \frac{2}{\sqrt{\pi}} \int_0^\eta e^{-x^2} dx \quad \text{(error function)}$$

Rational approximation ($o \leq \eta < \infty$) from Abramowitz and Stegun

$$\text{erf}\ \eta = 1. -(a_1 t + a_2 t^2 + a_3 t^3) e^{-\eta^2} + \varepsilon(\eta)$$

$$t = \frac{1}{1+a_0 \eta}$$

$$a_0 = 0.47047$$

$$a_1 = 0.3480242$$

$$a_2 = -0.0958798$$

$$a_3 = 0.7478556$$

$$|\varepsilon(\eta)| \leq 2.5 \times 10^{-5}$$

Table 5.4 (Cont. 1)

(6) $\phi = \frac{\psi}{2} e^{-\eta^2}$

(7) $q = \alpha \left\{ \frac{\gamma+1}{2(\gamma-1)} nkT_W - \left[\left(S^2 + \frac{\gamma}{\gamma-1} \right) \eta - \phi \right] kT_\infty \right\}$

These equations are based on the work of Oppenheim.

Nomenclature

k - Boltzman's Constant
M_∞ - Freestream Mach Number
P - Freestream Pressure
q - Heating Rate
T_W - Wall Temperature
T_∞ - Free Stream Temperature
U_∞ - Free Stream Velocity Heat Ratio
γ - Free Stream Specific Heat Ratio
α - Accommodation Coefficient

References

Abramowitz, M., and Stegun, I. A., eds, Handbook of Mathematical Functions, Dover Publications, New York, 1965.

Oppenheim, A. K., "Generalized Theory of Convective Heat Transfer In A Free-Molecule Flow", J. Aeron. Sci., Jan. 1953, p. 49.

Table 5.5

CATO/JOHNSON SWEPT CYLINDER HEATING

The empirical correlations of Cato for laminar flow and of Johnson for turbulent flow are given below along with a comparison with experimental data.

For laminar flow,

$$h_L = 0.75\, h_o \left[1 - 1.857 \left(\frac{\Lambda}{90^\circ}\right)^2 + 1.097 \left(\frac{\Lambda}{90^\circ}\right)^3\right]$$

For turbulent flow,

$$h_t = 0.75 \frac{K_T}{K_L} h_o \left[\frac{2\rho_u V_u R_N}{\mu_u}\right]^{0.3} \left[0.01714 + 0.01235 \sin\left[3.53\,(\Lambda - 10^\circ)\right]\right]$$

The laminar multiplication factor, K_L, is included in the definition of h_o.

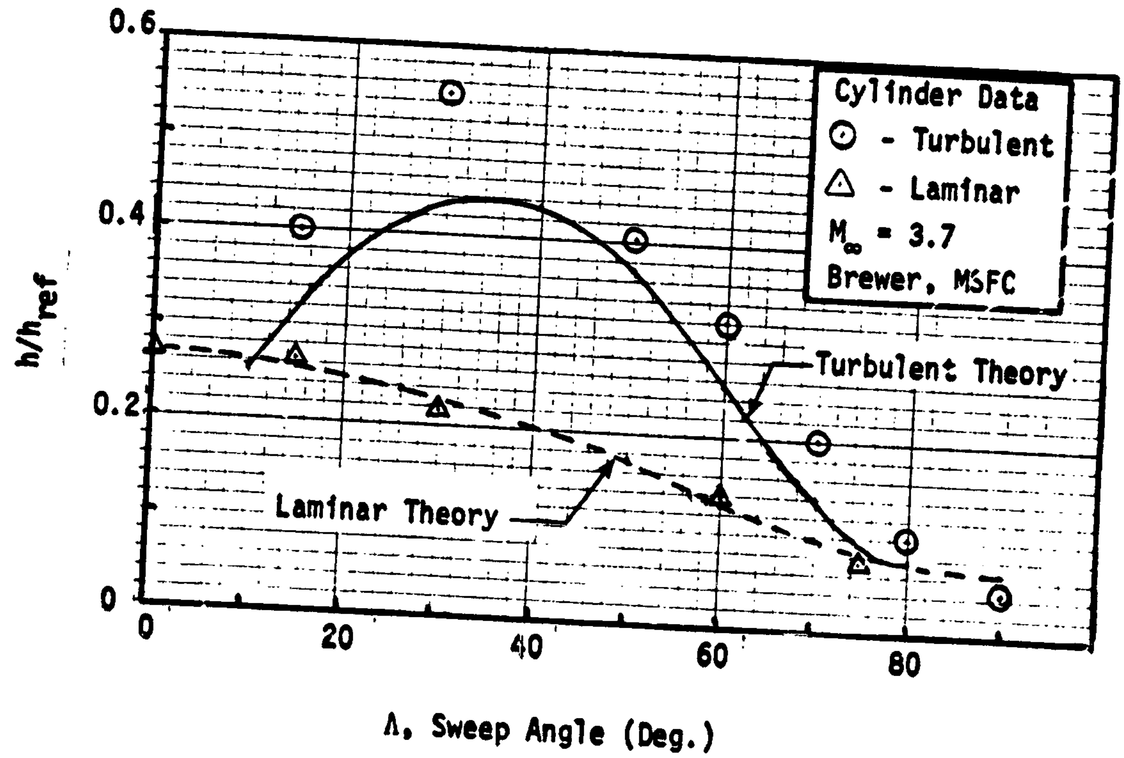


Table 5.5 (Cont. 1)

Nomenclature

h_o	Stagnation Point Heat Transfer Coefficient (Sphere)
h_L	Laminar Heat Transfer Coefficient
h_t	Turbulent Heat Transfer Coefficient
K	Multiplier Factor
R_N	Cylinder Radius
V_u	Velocity (Upstream)
ρ_u	Density (Upstream)
Λ	Sweep Angle
μ_u	Viscosity (Upstream)

References

Cato, G.C., "Heat Transfer to the Leading Edge of a Yawed Wing", Memorandum A-260-TH-57-115.

Johnson, W.A., "Turbulent Heat Transfer to a Yawed Circular Cylinder", Memorandum A2-260-TH-59-218, September 1959.

Table 5.6a

BECKWITH AND GALLAGHER TURBULENT REAL GAS YAWED CYLINDER STAGNATION LINE HEATING

$$h_{SL} = \frac{0.0323}{P_r^{0.6667}} \left(\frac{u_\infty \sin\Lambda}{\mu_o}\right)^{0.6} (g\rho^*\mu^*)^{0.8} \left(\frac{du_e}{d_x}\right)^{0.2}_{x\,=\,0} \qquad (1)$$

$$\left(\frac{du_e}{d_x}\right)_{x\,=\,0} = \frac{1.414}{R} \left(\frac{P_e - P_\infty}{\rho_e}\right)^{0.5} \qquad (2)$$

Nomenclature

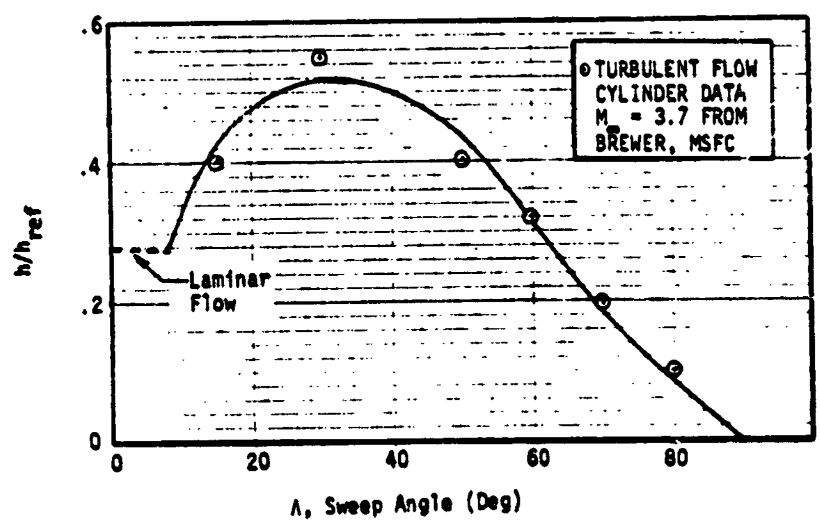
g	32.174	Lbm/slug
h	Heat Transfer Coefficient	Lbm/ft^{2}sec
P	Pressure	Lbf/ft^2
P_r	Prandtl Number @ Edge Conditions	Dimensionless
R	Radius of Cylinder	Feet
u	Velocity	Ft/sec
x	Distance from Stagnation Line	Feet
Λ	Sweep Angle	Degrees
μ	Viscosity	Lbm/ft-sec
ρ	Density	Slugs/ft^3

Subscripts:

SL	Stagnation Line
e	Edge Conditions
o	Total

Superscript:

*	Evaluate at Eckert's Reference Enthalpy & Edge Pressure



Reference:

Beckwith, I. E., and Gallagher, J. J., "Local Heat Transfer and Recovery Temperatures on a Yawed Cylinder at a Mach Number of 4.15 and High Reynolds Number, NASA TR R-104, 1961.

Table 5.6b
LAMINAR CYLINDER STAGNATION LINE HEATING

ORIGINAL PAGE IS
OF POOR QUALITY

$$h = 0.707\, h_s \cos^{1.1} \Lambda$$

h_s = Sphere Stagnation Point Heat Transfer Coefficient
h = Cylinder Stagnation Line Heat Transfer Coefficient
Λ = Sweep Angle

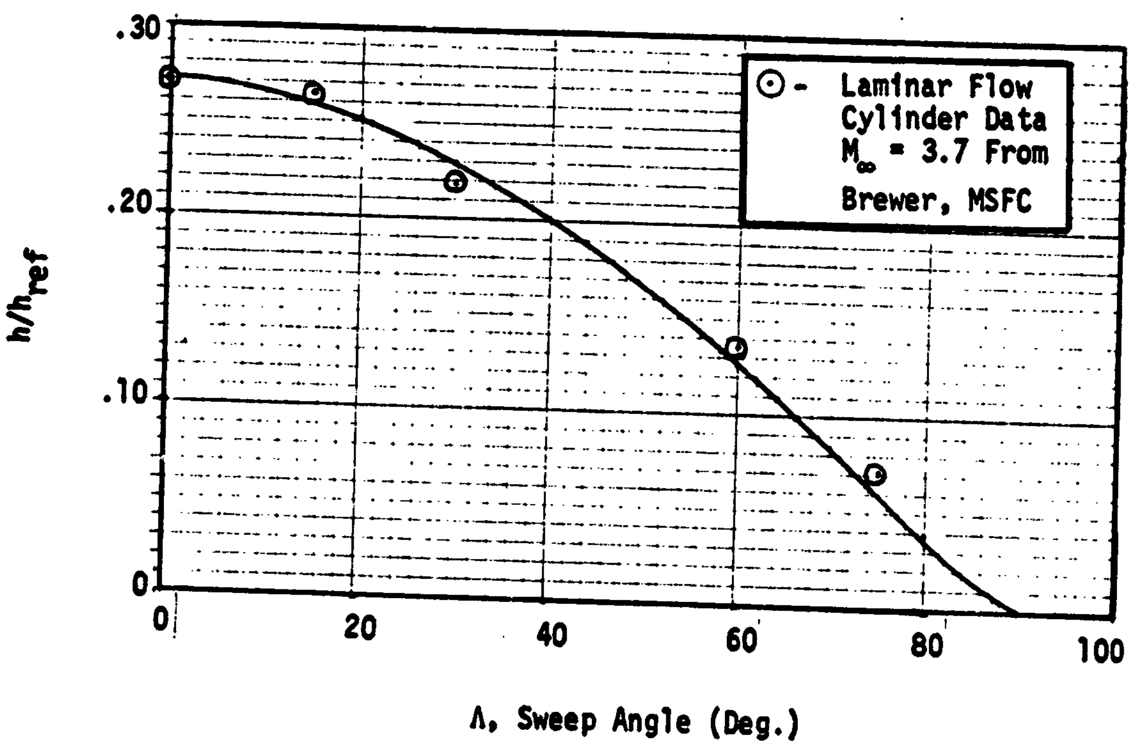


Table 5.7

RAREFIED FLOW YAWED CYLINDER
STAGNATION LINE HEAT TRANSFER EQUATIONS

The rarefied flow heating to a cylinder based on the work of Engel and Praharaj is as follows:

(1) $T_r = \frac{1}{2} (T_W + T_\delta \cos^2\Lambda)$

(2) $Re_\infty = \frac{\rho_\infty U_\infty R}{\mu_\infty}$

(3) $C_* = \frac{\mu_r T_\infty}{\mu_\infty T_r}$

(4) $\tilde{K}^2_\Lambda = \frac{Re_\infty \ Ze}{\gamma_\infty M_\infty^2 C_* (\cos^2\Lambda + P_\infty \sin^2\Lambda/\rho_\infty U_\infty^2)^{.5}}$

(5) $\xi = C_H/(\cos^2\Lambda + P_\infty \sin^2\Lambda/\rho_\infty U_\infty^2)^{.5}$

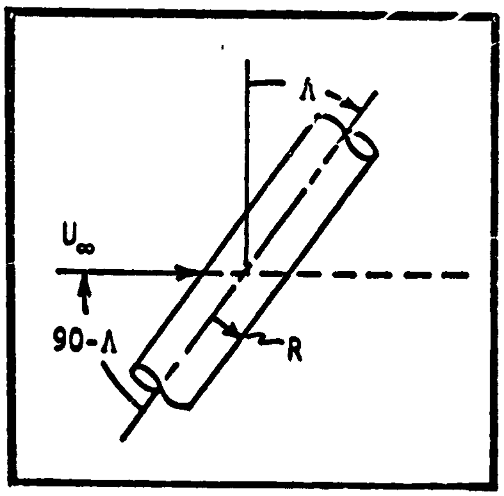
(6) Heat Transfer Correlation

$$\log_{10} \xi = \sum_{i=0}^{2} a_i (\log_{10} \tilde{K}^2_\Lambda)^i$$

$a_0 = -0.377656$

$a_1 = -0.368580$

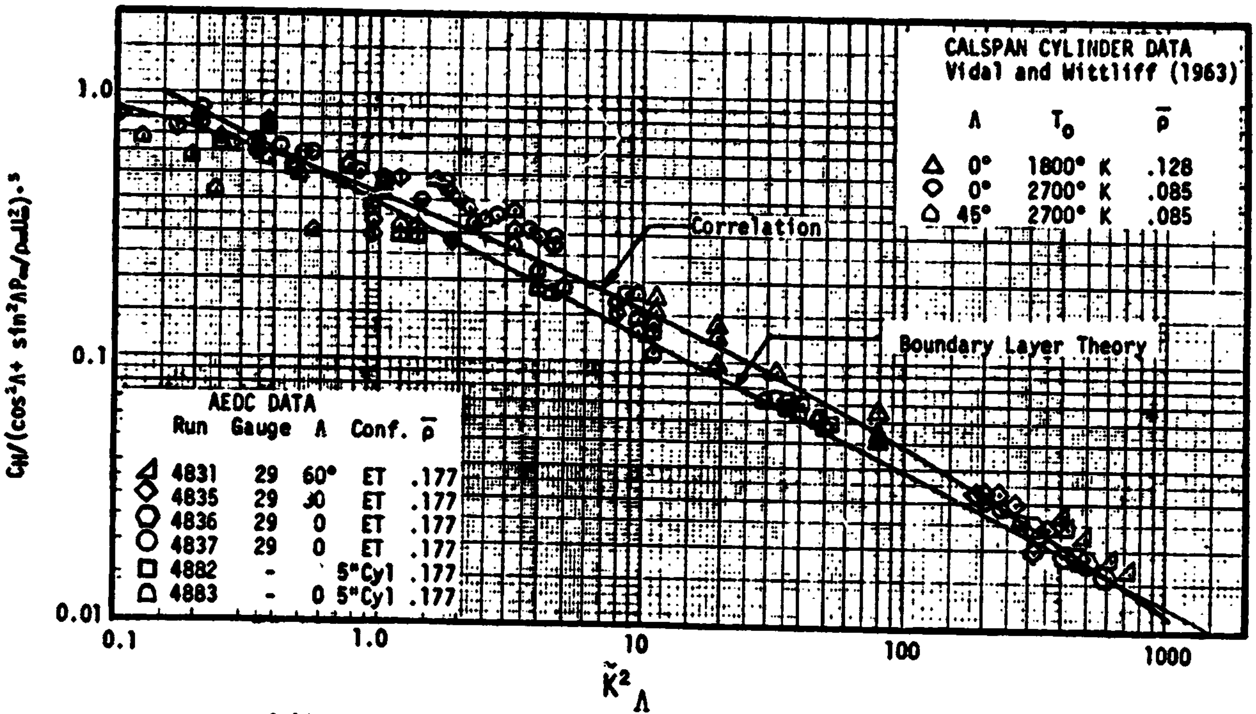
$a_2 = -0.0461064$



(7) Heat Transfer

$q = \rho_\infty U_\infty C_H (H_0 - H_W)$

Table 5.7 (Cont. 1)



Cylinder Stagnation Line Heat Transfer Data with Sweep Data Included

Nomenclature

C_H	Stanton Number
H	Enthalpy
M_∞	Free Stream Mach Number
T	Temperature
U	Velocity
Z	Post Normal Shock Compressibility
ρ	Density
γ	Specific Heat Ratio
μ	Viscosity

Subscripts

∞	Free Stream	e = edge
W	Wall	
δ	Post Normal Shock	
o	Total	

Reference

Engel, C.D. and Praharaj, S.C., "External Tank Rarefied Aerothermodynamics," REMTECH Inc. RTR 022-1, January 1978.

Table 5.8
"$\rho\mu$" SWEPT CYLINDER HEATING

The "$\rho_r\mu_r$" method for swept cylinders is defined as follows:

$$H_{mc,o} = 0.5(H_{\mathcal{L}} + H_w) \quad , \quad \rho_{mc,o} = f(H_{mc,o}, P_{\mathcal{L}})$$

$$\Sigma_o = \frac{\rho_{\mathcal{L}}}{\rho_{mc,o}} \quad , \quad \Gamma_o = 0.96\left(\Sigma_o\right)^{.55} - 0.5$$

$$\alpha_r = 0.90\left(\frac{\rho_{stag}\mu_{stag}}{\rho_w\mu_w}\right)^{0.5}$$

$$P_{r_r} = f(P_{\mathcal{L}}, T_r)$$

$$\overline{E}_{L,o} = \left[1 + \Gamma_o\right] P_r^{\alpha_r} \quad , \quad \overline{E}_{T,o} = \left[1 + 0.77\ \Gamma_o\right] P_r^{\alpha_r}$$

$$K_r = \frac{1}{V_{\mathcal{L}}R_N}\sqrt{\frac{2(P_{\mathcal{L}} - P_u)}{\rho_{\mathcal{L}}}}$$

$$Xeq_{L,o} = \frac{1}{2K_r\overline{E}_{L,o}} \quad , \quad Xeq_{T,o} = \frac{4}{5K_r\overline{E}_{T,o}}$$

$$F_x = \left[\frac{Xeq_{T,o}}{Xeq_{L,o}}\right]^{0.333}$$

$$n_r = \sqrt{\frac{\rho_{stag}\mu_{stag}}{\rho_{\mathcal{L}}\mu_{\mathcal{L}}}}$$

Table 5.8 (Cont. 1)

$$H_{mc} = H_{mc,o} + 0.206(H_T - H_\ell)P_r^{n_r}$$

$$\rho_{mc} = f(H_{mc}, P_\ell)$$

$$\Sigma_c = \frac{\rho_\ell}{\rho_{mc}} \quad , \quad \Gamma_c = \left[0.96\left(\Sigma_c\right)^{.55} - 0.5\right]$$

$$\bar{E}_L = (1 + \Gamma_c)P_r^{\alpha_r}$$

$$\mu_{T''} = \mu_r\left(\frac{H_T}{H_r}\right)^{1.5} \frac{(T_r + 2000)}{\left[T_r\left(\frac{H_T}{H_r}\right) + 200\right]}$$

$$Re_{r,cyl} = \frac{\rho_r \mu_r V_\ell (Xeq_{L,o})^2}{F_x^2 \, \mu_{T''}^2 \, (Xeq_L)}$$

$$Cf_{r,L} = \frac{0.664}{\sqrt{Re_{r,cyl}}} \quad , \quad Cf_{r,t} = \frac{0.370}{\left[\log_{10}(Re_{r,cyl} + 3000)\right]^{2.584}}$$

$$Le = 1.4 \quad , \quad H_{D,r} = f(\rho_r, T_r)$$

$$\mathcal{L} = 1 + (Le^{0.52} - 1)\frac{H_{D,r}}{H_\ell}$$

Table 5.8 (Cont. 2)

For laminar flow,

$$h_L = 0.5g \frac{K_L}{P_{r_r}^{0.645}} \mathcal{L}\left(\frac{\mu_{T''} F_x}{Xeq_{L,o}}\right) Re_{r,cyl} Cf_{r,L}$$

For turbulent flow,

$$h_t = 0.5g \frac{K_T}{P_{r_r}^{0.645}} \mathcal{L}\left(\frac{\mu_{T''} F_x}{Xeq_{L,o}}\right) Re_{r,cyl} Cf_{r,t}$$

Nomenclature

Cf	Skin Friction Coefficient
h	Heat Transfer Coefficient
H	Enthalpy
Le	Lewis Number
P	Pressure
Pr	Prandtl Number
Re	Reynolds Number
R_N	Cylinder Radius
T	Temperature
V	Velocity
ρ	Density
μ	Viscosity

Subscripts

L	Laminar
r	Eckerts Reference
stag	Post Normal Shock Stagnation
S	Stagnation Line
T	Total
t	Turbulent

References

Nagel et al., "Analysis of Hypersonic Pressure and Heat Transfer Tests on Delta Wings with Laminar and Turbulent Boundary Layers," NASA CR-535, August 1966.

Thomas et al., "Advanced Re-entry Systems Heat Transfer Manual for Hypersonic Flight," Technical Report AFFDL-TR-65-195, October 1966.

Table 5.9a

LEES' HEMISPHERICAL DISTRIBUTION METHOD FOR LAMINAR FLOW

Lees' heating distribution over a sphere for an ideal gas is expressed as:

$$h = h_{sp} \frac{2\Theta \sin\Theta \left[\left(1 - \frac{1}{\gamma M_\infty^2}\right) \cos^2\theta + \frac{1}{\gamma M_\infty^2} \right]}{\sqrt{D(\Theta)}}$$

$$D(\Theta) = \left(1 - \frac{1}{\gamma M_\infty^2}\right) \left[\Theta^2 - \frac{\Theta \sin 4\Theta}{2} + \frac{1 - \cos 4\Theta}{8} \right] + \frac{4}{\gamma M_\infty^2} \left[\Theta^2 - \Theta \sin 2\Theta + \frac{1 - \cos 2\Theta}{2} \right]$$

Nomenclature

- h = Heat Transfer Coefficient
- h_{sp} = Stagnation Point Heat Transfer Coefficient
- M_∞ = Free Stream Mach Number
- γ = Free Stream Specific Heat Ratio
- Θ = Local Body Angle

Reference

Lees, Lester, "Laminar Heat Transfer Over Blunt-Nose Bodies at Hypersonic Flight Speeds", Jet Propulsion, April 1956.

Table 5.9b

DETRA-HIDALGO TURBULENT HEATING DISTRIBUTION

Detra and Hidalgo developed a method of calculating the turbulent distribution over a hemisphere.

$$h_t = 0.029\, g \frac{K_T}{P_r^{0.667}} \frac{\mu_e}{L} \left[\frac{\rho_e V_e L}{\mu_e}\right]^{0.8} \left[1.037\, f(\delta)^{0.2}\right] \left[1 + 0.58 \frac{H_{D,e}}{H_T}\right]$$

where

$$f(\delta)^{0.2} = \sum_{i=o}^{N} A_i \delta^i$$

i	A_i ($\delta < 25^\circ$)	A_i ($\delta \geq 25^\circ$)
1	1.03754	0.96451
2	0.0043776	0.01107
3	-0.6187×10^{-4}	0.842558×10^{-4}

Nomenclature

g	Gravitational Constant
h_t	Turbulent Heat Transfer Coefficient
H	Enthalpy
K_T	Turbulent Multiplier Factor
L	Running Length
P_r	Prandtl Number
V	Velocity
ρ	Density
μ	Viscosity
δ	Sphere Tangency Angle (90 Degrees at the Stagnation Point)

Subscripts

D	Dissociation
e	Edge
T	Total

Reference

Detra, H.W. and Hidalgo, H., "Generalized Heat Transfer Formulae and Graphs", AVCO Research Report 72, March 1960.

Table 5.10a

ECKERT's LAMINAR FLAT PLATE HEATING METHOD

Eckert's laminar flat plate heating relation can be expressed as:

$$h = 0.332 \frac{g_c}{(P_r^*)^{2/3}} \frac{\rho^* u_e}{\sqrt{R_e^*/\tau_m}} \qquad (\text{lbm/ft}^2\text{sec})$$

where

$$R_e^* = \frac{\rho^* u_e x}{\mu^*}$$ Reynolds no. evaluated at reference conditions

The reference conditions are obtained from the reference enthalpy

$$H^* = H_e + 0.5\,(H_w - H_e) + 0.22(H_{aw} - H_e)$$

and edge pressure. Other reference properties are evaluated as:

$$\rho^* = f\,(H^*, P_e)$$

$$\mu^* = f\,(H^*, P_e)$$

For geometries other than a flat plate, the Mangler transformation may be evaluated as:

τ_m = 1 Flat Plate

= 3 Cone

<u>Nomenclature</u>

g_c = 32.174 lbm/slug
h = Heat Transfer Coefficient (lbm/ft^{2}sec)
H_e = Edge Enthalpy (Btu/lbm)
H_{aw} = Adiabatic Wall Enthalpy (Btu/lbm)
H_w = Wall Enthalpy (Btu/lbm)
P_e = Edge Pressure (atm.)
u_e = Edge Velocity (ft/sec)
x = Surface Distance From Origin to Point of Interest
ρ^* = Reference Density
μ^* = Reference Viscosity
τ_m = Mangler Transformation

Table 5.10b

SCHULTZ-GRUNOW TURBULENT FLAT PLATE METHOD

Turbulent heating relations for a plate using the Schultz-Grunow skin function law can be expressed as:

$$h = \frac{0.185g}{Pr_*^{0.667}} \frac{\rho^* u_e}{\left[\log_{10} (R_{e_*}/\tau_m)\right]^{2.584}}$$

where

$$Re_* = \frac{\rho^* UeX}{\mu^*}$$ Reynolds number evaluated at Eckert reference conditions

For geometries other than a flat plate, the Mangler transformation may be evaluated as

τ_m = 1 Flat Plate

2 Cone

Nomenclature

g_c	=	32.174 lbm/slug
h	=	Heat Transfer Coefficient (lbm/ft^{2}sec)
Pr_*	=	Prandtl Number at Reference Conditions
u_e	=	Edge Velocity (ft/sec)
x	=	Surface Distance From Origin to Point of Interest
ρ^*	=	Reference Density
μ^*	=	Reference Viscosity
τ_m	=	Mangler Transformation

Reference

Schultz-Grunow, F., "A New Resistance Law for Smooth Plates," Luftfahrt Forsch, Vol. 17 (1940), pp. 239-246: (translation) NACA TM 986, 1941.

Table 5.11

SPALDING-CHI METHOD MODIFIED FOR REAL GAS HEAT TRANSFER FOR TURBULENT BOUNDARY LAYER FLOW

(1) Spalding-Chi define F_c, F_{R_θ}, F_{R_s} (Spalding and Chi) which are functions of Mach Number and Temperature alone such that

$$1/2\, C_f F_c = \psi_\theta (F_{R_\theta} Re_\theta) = \psi_s (F_{R_s} Re_s)$$

(2) $$F_c = \frac{1}{Z_e T_e} \left[\int_0^1 \left(\frac{1}{ZT}\right)^{1/2} d\left(\frac{u}{u_e}\right) \right]^{-2}$$

Where $ZT = f\ (H, P_e)$ & $H = H_w + (H_{aw} - H_w)\left(\frac{u}{u_e}\right) - (H_{aw} - H_e)\left(\frac{u}{u_e}\right)^2$

(3) $$F_{R_\theta} = \left(\frac{H_{aw}}{H_w}\right)^{0.772} \left(\frac{H_e}{H_w}\right)^{0.702} \qquad \text{(Wallace)}$$

(4) $$F_{R_s} = F_{R_\theta}/F_c$$

(5) $$C_{f_i} = 1/2 \exp \left\{ \sum_{i=1}^{10} g(i) \left[\ln\left(F_{R_s} \frac{Re_s}{\tau_m}\right) \right]^{i-1} \right\} = 1/2\, C_f F_c$$

Where
$g(1) = 9.2808635$
$g(2) = -4.7340248$
$g(3) = 6.6858663\ 10^{-1}$
$g(4) = -4.1876614\ 10^{-2}$
$g(5) = -5.5054577\ 10^{-4}$
$g(6) = 2.8367291\ 10^{-4}$
$g(7) = -2.1249608\ 10^{-5}$
$g(8) = 8.0162000\ 10^{-7}$
$g(9) = -1.5900985\ 10^{-8}$
$g(10) = 1.3236350\ 10^{-10}$

$$Re_s = \frac{\rho_e u_e s}{\mu_e}$$

τ_m = Mangler Transformation (Komar)

Table 5.11 (Cont. 1)

Nomenclature

C_f	Skin Friction Coefficient
h_{TURB}	Heat Transfer Coefficient For Turbulent Flow
H	Enthalpy
P	Pressure
P_r	Prandtl Number
Re_{Θ}, Re_s	Local Reynolds Number Based on Momentum Thickness Θ and Characteristic Length s Respectively
S	Inverse of Von Karman Reynolds Analogy Factor
S_t	Stanton Number
T	Temperature
u	Velocity
Z	Compressibility
ρ	Density

Subscripts

aw	Recovery
e	Local
i	Incompressible
w	Wall

References

Spalding, D.B., and Chi, S.W., "The Drag of A Compressible Turbulent Boundary Layer on a Smooth Flat Plate With and Without Heat Transfer", Journal of Fluid Mechanics, Vol. 18, Part I, pp. 117-143, Jan. 1964.

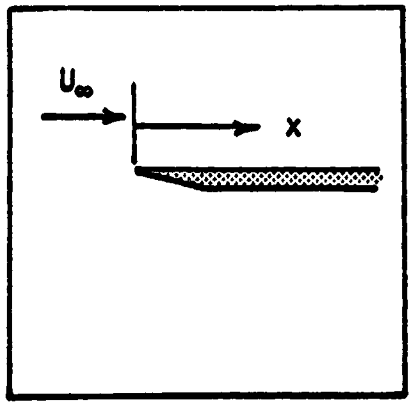
Wallace, J.E., "Hypersonic Turbulent Boundary Layer Studies at Cold Wall Conditions", 1967 Heat Transfer and Fluid Mechanics Institute, La Jolla, CA June 1967.

Komar, J.J., "Improved Turbulent Skin-Friction Coefficient Predictions Utilizing the Spalding-Chi Method", Douglas Aircraft Company, Douglas Report DAC-59801, Nov. 1966.

Table 5.12

RAREFIED FLOW SHARP FLAT PLATE HEAT TRANSFER EQUATIONS

Rarefied flat plate heating correlations based on the work of Shorenstein and Probstein are given below.



(1) $T_o = T_\infty \left(1 + \frac{\gamma-1}{2} M_\infty^2\right)$

(2) $Re_\infty = \frac{\rho_\infty U_\infty X}{\mu_\infty}$

(3) $C_* = \frac{\mu_W T_\infty}{\mu_\infty T_W}$

(4) $\beta = (T_W/T_o)^{1/2} M_\infty^2 C_*/Re_\infty$

(5) $C_{H_{si}} = (0.368 T_W/T_o + 0.0684) \left[M_\infty (C_*/Re_\infty)^{1/2}\right]^{3/2}$

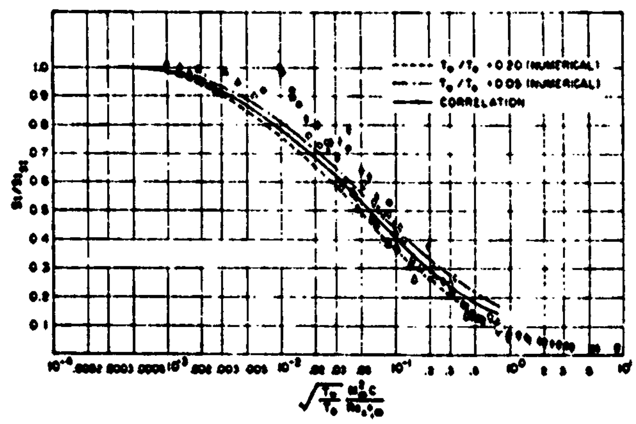
(6) $\frac{C_H}{C_{H_{si}}} = \frac{1}{2}\left[1 - \tanh(0.91 \log_{10}\beta + 1.10)\right]$

for $\beta < 0.1$

(7) Heat Transfer

$q = \rho_\infty U_\infty C_H (H_o - H_W)$

From Shorenstein and Probstein



Heat-transfer rate correlation.

Nomenclature

CH	Stanton Number
H	Enthalpy
M	Mach Number
T	Temperature
U	Velocity
X	Running Length
ρ	Density
γ	Specific Heat Ratio
μ	Viscosity

Subscripts

∞	Free Stream
w	Wall
o	Total

Reference

Shorenstein, M.L. and Probstein, R.F., "The Hypersonic Leading-Edge Problem," AIAA J., Vol. 6, No. 10, October 1968.

Table 5.13

RAREFIED FLOW SHARP AND BLUNT CONE
HEAT TRANSFER EQUATIONS

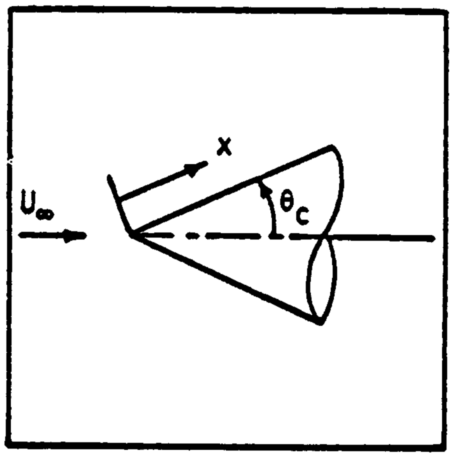
The rarefied flow heating to sharp and blunt cones based on the work of Engel and Praharaj is as follows:

(1) $T_r = T_w + (T_\delta + T_w)/2 - T_\delta \cos^2\theta_c/3$

(2) $Re_\infty = \dfrac{\rho_\infty U_\infty x}{\mu_\infty}$

(3) $C^* = \dfrac{\mu_r \; T_\delta}{\mu_\delta \; T_r}$

(4) $\xi = \dfrac{.9\, C_H}{(\sin^2\theta_c + P_\infty \cos^2\theta_c/\rho_\infty U_\infty^2)^{.5}}$



(5) $\bar{x}_c = \dfrac{Re_\infty \; Ze}{M_\infty^2 \gamma_\infty C_* \cos\theta_c}$

(6) Correlation Equation

$$\log_{10}(\xi) = \sum_{i=0}^{3} a_i \, (\log_{10}\bar{x}_c)^i$$

Sharp:

$a_0 = -0.344074$

$a_1 = -0.349130$

$a_2 = -0.104455$

$a_3 = +0.022766463$

Blunt:

$a_0 = -0.647813$

$a_1 = -0.365587$

$a_2 = -0.0143793$

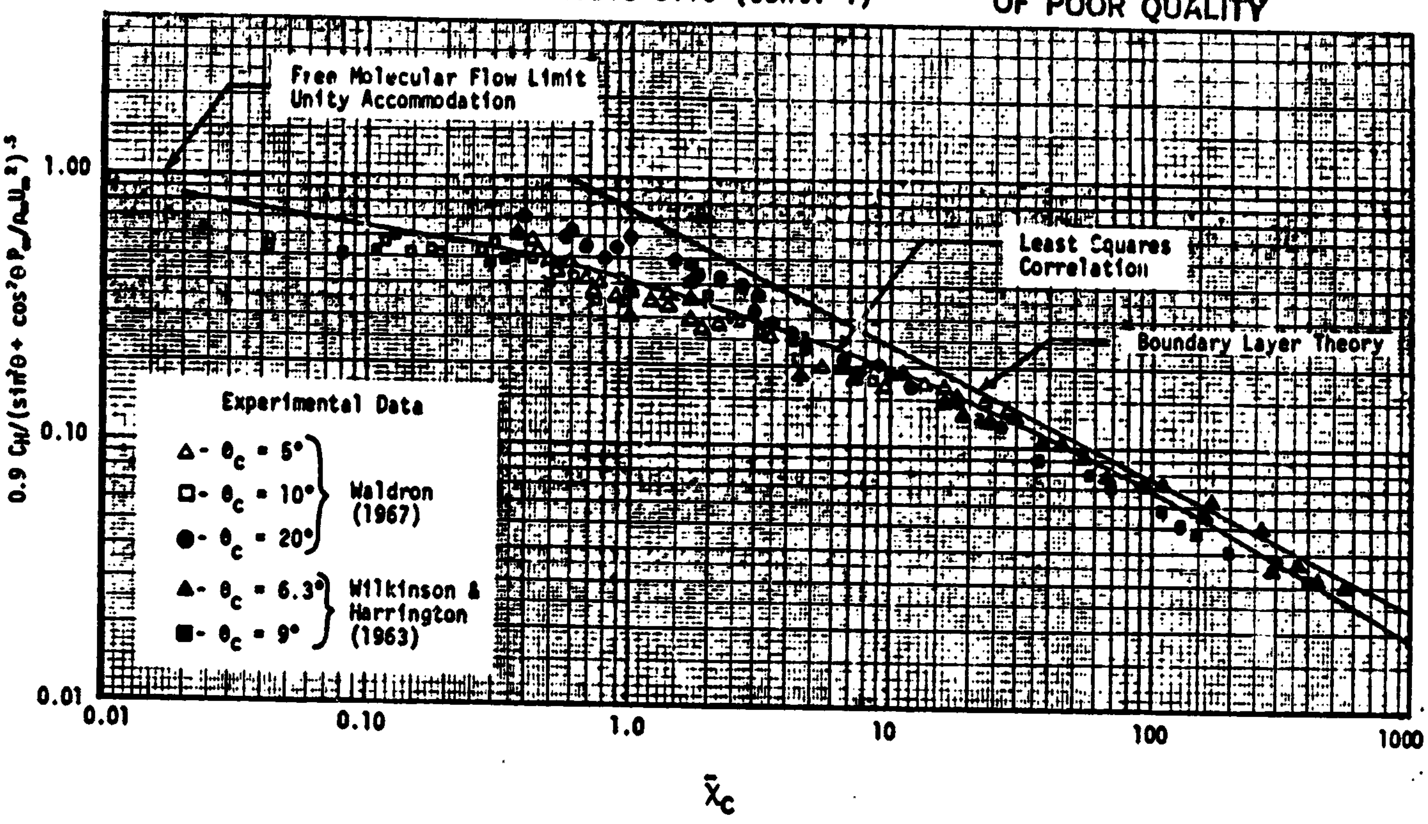
$a_3 = +0.003281793$

(7) Heat Transfer

$$q = \rho_\infty \, U_\infty \, C_H \, (H_0 - H_W)$$

Table 5.13 (Cont. 1)

ORIGINAL PAGE IS OF POOR QUALITY



Sharp Cone Heat Transfer Data

Nomenclature

C_H	Stanton Number
H	Enthalpy
M_∞	Free Stream Mach Number
T	Temperature
U	Velocity
Z	Post Normal Shock Compressibility
ρ	Density
γ	Specific Heat Ratio
μ	Viscosity

Subscripts

∞	Free Stream
W	Wall
δ	Post Normal Shock
o	Total

Reference

Engel, C.D. and Praharaj, S.C., "External Tank Rarefied Aerothermodynamics," REMTECH Inc. RTR 022-1, January 1978.

Table 5.14

"$\rho\mu$" FLAT PLATE HEATING

ORIGINAL PAGE IS
OF POOR QUALITY

The "$\rho_r\mu_r$" method was developed by Hanks from Boeing and is documented by Nagel and Thomas.

For laminar flow,

$$h_L = 0.332g \frac{K_L}{P_{r_r}^{0.645}} \mathcal{L} \sqrt{\frac{\rho_r \mu_r V_e}{L}}$$

For turbulent flow,

$$h_t = 0.185g \frac{K_T}{P_{r_r}^{0.645}} \frac{\mathcal{L}}{\mu_{T''}} \frac{\rho_r \mu_r V_e}{\left[\log_{10} (Re_r + 3000)\right]^{2.584}}$$

where

$$\mathcal{L} = 1 + (Le^{0.52} - 1) \frac{H_{D,r}}{H_e}$$

$$H_{D,r} = f(\rho_r, T_r) \quad \text{and} \quad L_e = 1.4$$

$$\mu_{T''} = \mu_r \left(\frac{H_T}{H_r}\right)^{3/2} \frac{(T_r + 200)}{\left[T_r\left(\frac{H_T}{H_r}\right) + 200\right]}$$

Nomenclature

g	Gravitational Constant
h	Heat Transfer Coefficient
H	Enthalpy
L	Running Length
Le	Lewis Number
K_L	Laminar Multiplier Factor
K_T	Turbulent Multiplier Factor
P_r	Prandtl Number

Table 5.14 (Cont. 1)

Re	Reynolds Number
T	Temperature
V	Velocity
ρ	Density
μ	Viscosity

Subscripts

e	Edge
D	Dissociation
L	Laminar
r	Eckert Reference Condition
t	Turbulent
T	Total

References

Nagel et al., "Analysis of Hypersonic Pressure and Heat Transfer Tests on Delta Wings With Laminar and Turbulent Boundary Layers," NASA, CR-535, August 1966.

Thomas et al., "Advanced Re-entry Systems Heat Transfer Manual for Hypersonic Flight," Technical Report AFFDL-TR-65-195, October 1966.

Table 5.15

AVERAGE SEPARATED LEESIDE ORBITER HEATING

The average separated leeside heating relations are based on correlations of wind tunnel data for the Space Shuttle Orbiter configuration by Bertin and Goodrich. The average leeward heating (turbulent or windward flow) is given by:

$$\bar{S}_t = \left(1.067\left(\frac{H_{wwd}}{H_t}\right) + 0.7905\right)\left(0.00282(Re_{ns})^{-0.37}\right)$$

$$Re_{ns} = \rho_\infty U_\infty R_{ref}/\mu_{ns}$$

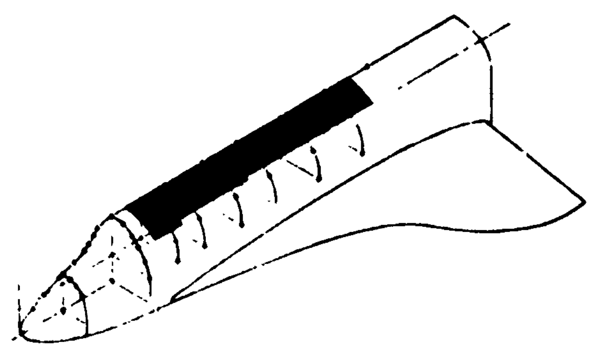
$$R_{ref} = 1.0 \text{ foot for full scale}$$

$$\bar{q} = \rho_\infty U_\infty (H_t - H_w)\, \bar{S}_t$$

where the windward to total enthalpy ratio replaces the temperature ratio in the original paper.

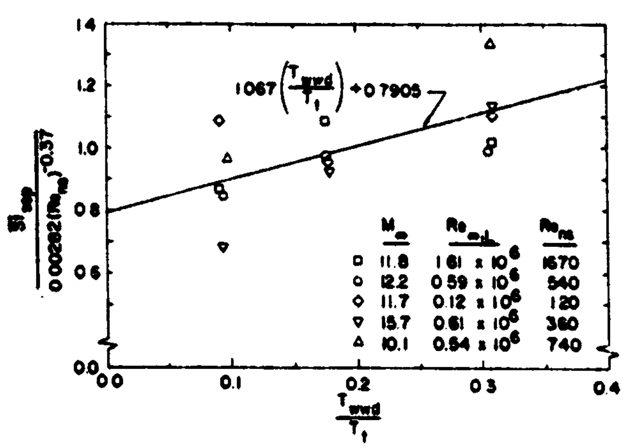
Nomenclature

H_{wwd}	Windward Wall Enthalpy
H_t	Total Enthalpy
H_w	Local Wall Enthalpy
Re	Reynolds Number
R	Radius
ρ_∞	Freestream Density
U_∞	Freestream Velocity
$\bar{q}$	Average Leeside Heating Rate
μ_{ns}	Post Normal Shock Viscosity

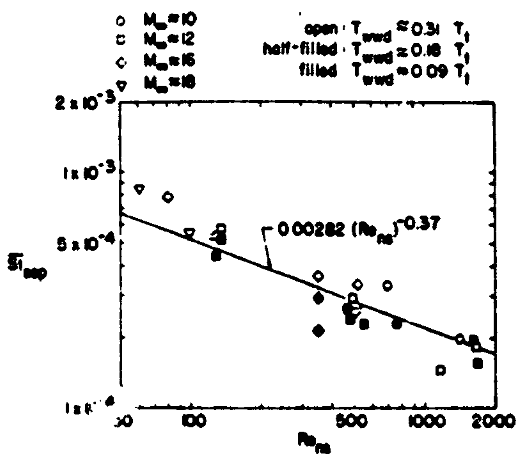


Leeward surface area over which the heat-transfer measurements were averaged to obtain $\bar{St}_{sep}$.

A comparison of the correlation and data from the original paper are given below.



Windward-surface-temperature effect on the average heating in the leeward "separated" region.



Average Stanton number for the leeward "separated" region.

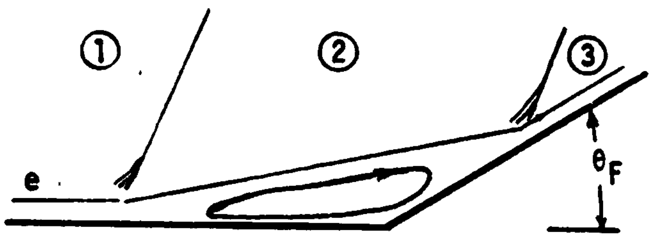
Reference

Bertin, J.J. and Goodrich, W.D., "Effects of Surface Temperature and Reynolds Number on Leeward Shuttle Heating," Journal of Spacecraft, Vol. 13, No. 8, August 1976, pp. 473-480.

Table 5.16

PEAK FLAP INTERFERENCE HEATING

The procedure for calculating peak flap interference heating follows that of Fivel except for the separation length calculations.



Flap
Flowfield

TRANSITION

Re_δ/M_e^3 < 400 Laminar
$\geq$ 400 Turbulent

where Re_δ = Undisturbed B.L. edge Reynolds number based on boundary layer thickness

δ = Boundary Layer (B.L.) thickness

Me = Edge Mach Number

INCIPIENT SEPARATION (Kessler, Reilly and Mockapetris)

C_{p_i} = Incipient separation pressure coefficient

Laminar

$$\log_{10} C_{p_i} = -0.361397 \log_{10} (Re_\delta/M_e^3) - 0.662427$$

Turbulent

If $2.6 \leq \log_{10} Re_\delta/M_e^3 \leq 3.8$

$$C_{p_i} = 0.2615 (|\log_{10} Re_\delta/M_e^3 - 3.8|)^{3.5} + 0.405$$

If $\log_{10} Re_\delta / M_e^3 > 3.8$

$$C_{p_i} = 0.0354\,(\log_{10} Re_\delta / M_e^3 - 3.8)^{1.6} + 0.405$$

DEFLECTION ANGLE

$$\frac{P_3}{P_1} = 1 + \frac{\gamma}{2}\, C_p\, M_e^2$$

$$\tan^2\theta_D = \left(\frac{\frac{P_3}{P_1} - 1}{\gamma M_e^2 - \frac{P_3}{P_1} + 1}\right)^2 \frac{2\gamma M_e^2 - (\gamma - 1) - (\gamma + 1)\frac{P_3}{P_1}}{(\gamma + 1)\frac{P_3}{P_1} + \gamma - 1}$$

C_p = Separation Region Pressure Coefficient

θ_D = Deflection Angle of Separated Streamline

Note if $C_p = C_{p_i}$ = Incipient Separation Coefficient

then $\theta_D = \theta_i$ = Wedge Angle for Incipient Separation

PLATEAU PRESSURE (Wuerer and Clayton)

$$\left(C_{p_{PLAT}}\right)_L = 1.60\left[Re_x(M_e^2 - 1)\right]^{-1/4}$$

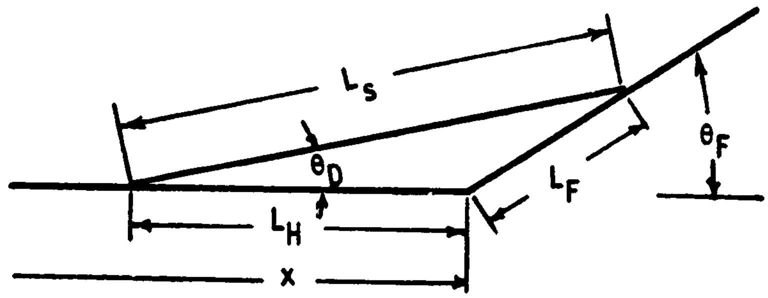
$$\left(C_{p_{PLAT}}\right)_T = 1.70(Re_x)^{-1/10}\,(M_e^2 - 1)^{-1/4}$$

$C_{p_{PLAT}}$ = Plateau Pressure Coefficient

Re_x = Reynolds Number Based on Distance along a Streamline to the Hinge Line

Table 5.16 (Cont. 2)

SEPARATION GEOMETRY



Laminar (Giles and Thomas)

$$L_S/\delta = \frac{35}{M_e^3}\left[\left(\frac{P_F - P_{PL}}{Pe}\right) Re_{ex}^{1/4}\right]^{0.98}$$

Turbulent (Popinski and Ehrlich)

$$L_H = \frac{\frac{x}{4} Re_{ex}^{.2}\left(\frac{P_F}{Pe}\right)^{1.025}}{\left(M_e^2 - 1\right)^{1.17}\left(\log_{10} Re_{ex}\right)^{2.58}}$$

where

Pe	=	Upstream Edge Pressure
P_{PL}	=	Plateau Pressure
P_F	=	Post Shock Flap Pressure
x	=	B.L. Running Length to Hinge Line
δ	=	B.L. Thickness Upstream of Separation

Geometry

$$L_H = L_S \sin(\theta_F - \theta_D)/\sin(180 - \theta_F)$$

$$L_F = L_S \sin\theta_D/\sin(180 - \theta_F)$$

Table 5.16 (Cont. 3)

PEAK REATTACHMENT HEATING (Bushnell & Weinstein)

$$Re_{shear} = \frac{\rho_W U_F \delta_s}{\mu_W \sin(\theta_F - \theta_D)}$$

where

U_F = Velocity at Reattachment Region

ρ_W = Density at Reattachment Region at Wall

μ_W = Viscosity at Reattachment Region at Wall

$$(\delta_s)_L = \delta + 5\left(\frac{L_s \mu_s}{\rho_s U_s}\right)$$

$$(\delta_s)_T = \delta + 1.6\ L_s/13$$

where

U_s = Shear Layer Edge Velocity

ρ_s = Shear Layer Edge Density

μ_s = Shear Layer Edge Viscosity

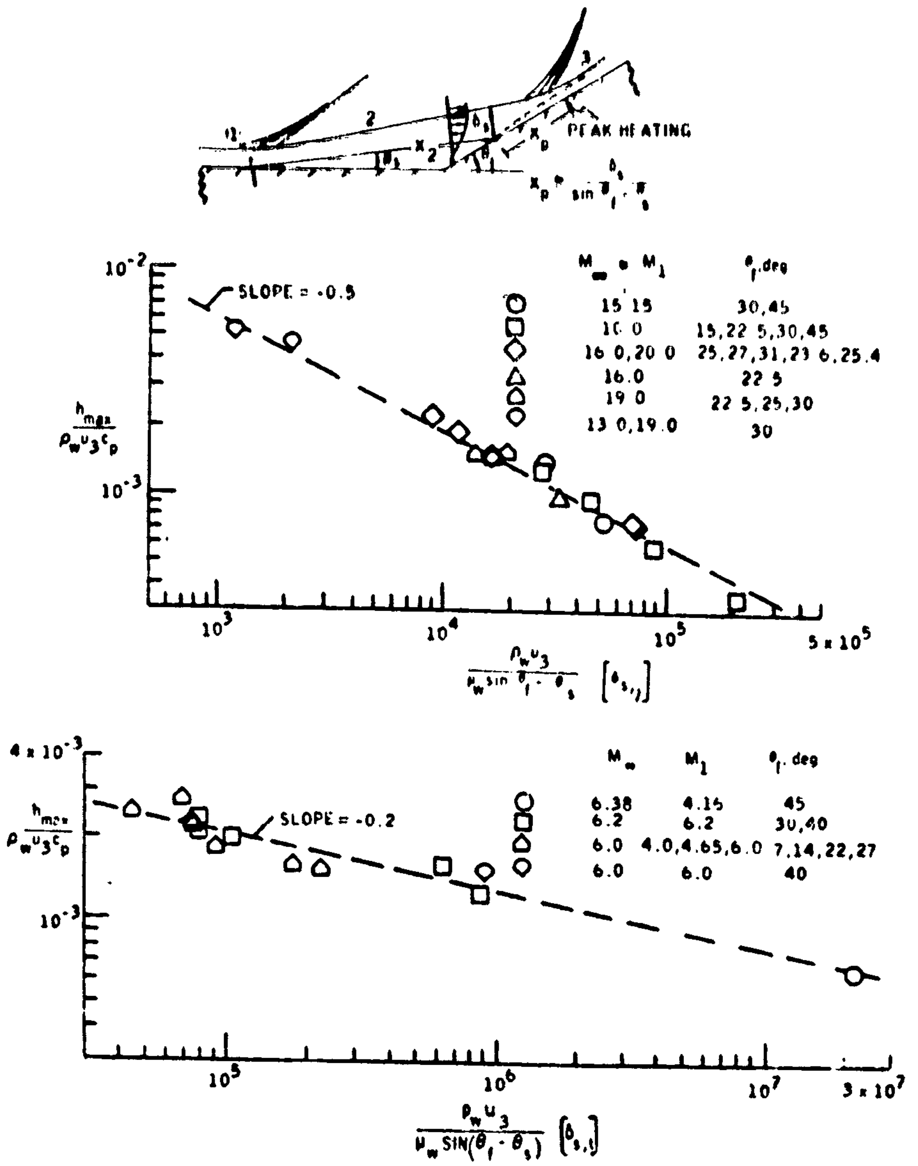
PEAK STANTON NUMBER AT REATTACHMENT

$$(St)_{PK} = C_{St}(Re_{shear})^{n_{St}}$$

where

	Lam	Turb
C_{St}	0.199	0.0204
n_{St}	-0.5	-0.2

Table 5.16 (Cont. 4)



Correlation of Peak Heating at Reattachment for Laminar and Turbulent Separated Flows (Bushnell and Weinstein)

Table 5.16 (Cont. 5)

REFERENCES

Fivel, H.J., "Numerical Flow Field Program for Aerodynamic Heating Analysis," AFFDL-TR-79-3128, Vol. 1, December 1979.

Kessler, W.C., Reilly, J.F., and Mockapetris, L.J., "Supersonic Turbulent Boundary Layer Interaction With An Expansion Ramp and Compression Corner," McDonnell Douglas Astronautics Company, St. Louis Report MDC E0264, 17 December 1970.

Wuerer, J.E. and Clayton, F.I., "Flow Separation in High Speed Flight. A Review of the State-of-the-Art," Douglas Report SM-46429, April 1965.

Giles, H.L. and Thomas, J.W. "Analysis of Hypersonic Pressure and Heat Transfer Tests on a Flat Plate With Flap and a Delta Wing With Body, Elevons, Fins, and Rudders," National Aeronautics and Space Administration, Washington, D.C., (NASA CR-536), August 1966.

Popinski, Z. and Ehrlich, C.F., "Development Design Methods for Predicting Hypersonic Aerodynamic Control Characteristics," AFFDL-TR-66-85, September 1966.

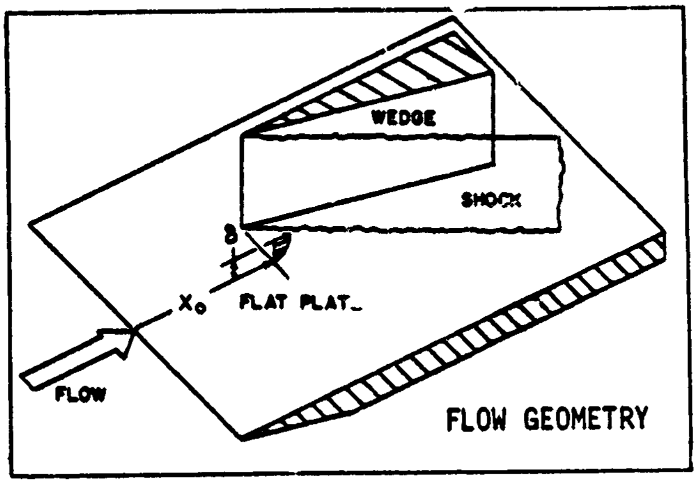
Bushnell, D.M. and Weinstein, L.M., "Correlation of Peak Heating for Reattachment of Separated Flows," Journal of Spacecraft and Rockets, Vol. 5, No. 9, September 1963, pp. 1111-1112.

Table 5.17

FIN-PLATE PEAK INTERFERENCE HEATING

ORIGINAL PAGE IS OF POOR QUALITY

The peak fin-plate interference heating can be calculated using the method presented by Fivel, which basically came from Hayes. The peak interference heating occurs along a line near the fin on the plate. The peak heating angle is correlated using:

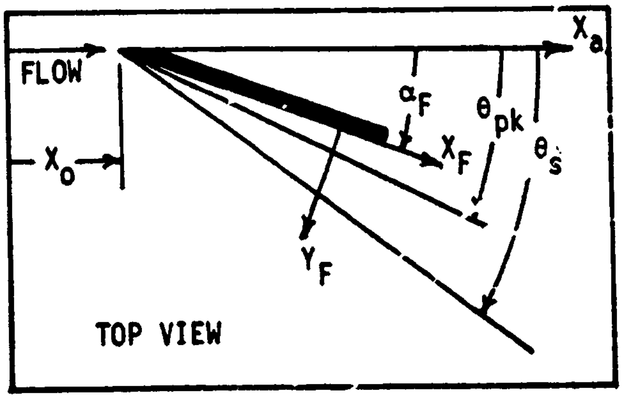


$$\theta_{pk} = 0.24\,(\theta_s - \alpha_F) + \alpha_F$$

The location of the peak heating line is given by:

$$Y_{FP} = X_F \,\mathrm{Tan}(\theta_{pk} - \alpha_F)$$

The boundary layer thickness at the leading edge of the fin based on the running length to the fin leading edge, X_o, is required.



The peak pressure ratio is based on correlated test data

$$\frac{P_{pk}}{P_u} = (Me\ \mathrm{Sin}\theta_s)^{n_{pk}}$$

Table 5.17 (Cont. 1)

Likewise, the peak heating ratio is based on a correlation of test data

$$\frac{h_{pk}}{h_u} = (Me\ Sin\theta_s - 1.0)n_{st} + 0.75$$

The exponent n_{pk} and coefficient n_{st} are obtained from interpolation of a faired curve through data from Hayes.

X_a/δ	n_{pk}
0.0	1.25
1.25	1.50
2.50	1.71
3.75	1.87
5.00	1.98
6.25	2.03
7.50	2.09
8.75	2.14
10.00	2.18
11.25	2.20
12.50	2.23
13.75	2.25
15.00	2.26
17.50	2.29
20.00	2.32
22.50	2.34
25.00	2.36
27.50	2.38
30.00	2.40

If $X_a/\delta > 30$, $n_{pk} = 2.4$

n_{pk} is obtained by linear interpolation given X_a/δ.

X_a/δ	n_{st}
1.0	1.62
2.0	2.20
3.0	2.98
4.0	3.57
5.0	3.87
6.0	4.00
7.0	4.05

If $X_a/\delta \leqslant 0.183$, $n_{st} = 1.0$

If $0.183 < X_a/\delta < 1.0$,
$n_{st} = 0.35612 \ln(X_a/\delta) + 1.62$

If $1.0 \leq X_a/\delta \leq 7.0$
n_{st} determined by linear interpolation given X_a/δ

If $X_a/\delta > 7.0$
$n_{st} = 0.35612 \ln(X_a/\delta) + 3.357$

Nomenclature

h	Heat Transfer Coefficent
P	Pressure
Me	Edge Mach Number Upstream of Fin
X_a	Distance Aft of Fin Leading Edge in Streamwise Direction
X_o	Running Length to Start of Fin in Streamwise Direction
X_F	Running Length Along Fin
Y_{FP}	Peak Heating Location Normal to Fin
α_F	Effective Angle of Attack of Fin
θ_{pk}	Angular Location of Peak Heating
θ_s	Shock Angle
δ	Upstream Boundary Layer Thickness

Subscripts

pk	Peak
u	Undisturbed

References

Fivel, H.J., "Numerical Flow Field Program for Aerodynamic Heating Analysis," AFFDL-TR-79-3128, Vol. 1, December 1979.

Hayes, R., "Prediction Techniques for the Characteristics of Fin Generated Three Dimensional Shock Wave Turbulent Boundary Layer Interactions," AFFDL-TR-77-10, May 1977

Section 6.0

LENGTH AND TRANSITION ROUTINES

This section describes the subroutines which determine boundary layer transition, running length corrections and cross flow adjustments.

6.1 SUBROUTINE TRANS AND EDPRAM

The purpose of routine TRANS is to determine if the flow is laminar, transitional or turbulent. The equations for each of the transition criteria are presented in Table 6.1. If a transition option greater than 8.0 is used, then an error message is printed from TRANS and the routine returns NERROR = 1 to MAIN. This causes MAIN to void the remainder of this problem and read in data for the next case. If the flow is determined to be laminar, then the heat transfer coefficient returned from TRANS is equal to the laminar heat transfer coefficient. If the flow is fully turbulent, then the heat transfer coefficient returned from TRANS is the turbulent heat transfer coefficient. If the flow is determined to be transitional, then the heat transfer coefficient returned from TRANS is determined by linear interpolation between the laminar and turbulent values.

The interpolation parameter for TRANS = 1 and 2 is time (TIME). The interpolation parameter for TRANS = 3 is the Reynolds number based on running length (corrected for crossflow) or body diameter and free-stream properties. However, no correction is made to account for axisymmetric flow. For TRANS = 4, the interpolation parameter is compressible momentum thickness Reynolds number. Transition option 5 uses

$$PARA = \psi$$

as the interpolation parameter. For transition option 6, the interpolation parameter is the laminar running length corrected for crossflow and corrected for axisymmetric flow (Mangler transformation factor). Transition option 7 uses the input running length as the interpolation parameter. Transition option 8 uses Re^*_θ /Me as the interpolation parameter. Whenever the flow is laminar, then PCT = 0.0, turbulent PCT = 1.0, and transitional 0.0 < PCT < 1.0. The parameter PCT represents how close the flow is to being fully turbulent based on the above interpolation parameters. The parameter PCT is also used in the output routine (VANOUT) to identify the flow type. It should also be noted that the recovery enthalpy for transitional flow is also determined by linear interpolation between the laminar and turbulent recovery enthalpies.

The following is a list of the parameters used in the CALL TRANS statement:

ENCL	- (input)	Laminar heat transfer coefficient
ENCT	- (input)	Turbulent heat transfer coefficient
PARA1	- (input)	Onset of transitional flow parameter
PARA2	- (input)	Onset of fully turbulent flow parameter
PARA	- (output)	For transition flag 4, this is equal to the compressible momentum thickness Reynolds number. For transition flag 5, this is equal to the MDAC-EAST transition parameter. For transition flag 6, this is equal to the corrected (crossflow and axisymmetric flow) running length to the onset of transition. This is not used for the other transition options.
TIME	- (input)	Trajectory time
ENC	- (output)	Heat transfer coefficient
HRECOV	- (output)	Recovery enthalpy
NTR	- (output)	NTR = 1 corresponds to laminar flow NTR = 2 corresponds to transitional flow NTR = 3 corresponds to turbulent flow
PCT	- (output)	Fraction of turbulent flow PCT = 0.0 corresponds to laminar flow PCT = 1.0 corresponds to fully turbulent flow 0.0 < PCT < 1.0 corresponds to transitional flow
TRFLAG	- (input)	Transition option
EL	- (input)	Geometric running length
ELFAC	- (input)	Multiplication factor which, when multiplied times the running length to the onset of transition, gives the running length to the onset of fully turbulent flow
ENL	- (input)	Laminar Mangler transformation factor to account for axisymmetric flow corrections
ELL	- (input)	Laminar running length corrected for crossflow effects

Another routine which is closely related to TRANS is EDPARM. Routine EDPARM is used to determine the MDAC-E onset of transition parameter solely as a function of angle-of-attack. The angle-of-attack range is from 0 to 70 degrees. There are six data points in the data table (α = 0, 30, 40, 50, 60, and 70 degrees) along with the corresponding values of the transition parameter. Interpolation at angles-of-attack not in the table is performed with the aid of routine TBLIN.

6.2 SUBROUTINE VRUNL

Subroutine VRUNL is called by MAIN and is not called unless heat transfer technique 3, 4, or 5 (flat-plate laminar and turbulent methods) is being used. Subroutine VRUNL insures that the length parameter used in the turbulent heat transfer equations (when using transition options 3, 4, 5, 6, or 7) is corrected for the assumption that the running length should be measured from the onset of transition. The argument of subroutine VRUNL contains the following parameters:

TRFLAG	- (input)	May have values 3.0, 4.0, 5.0, 6.0, or 7.0 corresponding to transition options 3.0, 4.0, 5.0, 6.0, or 7.0. (It should be noted that VRUNL will not be called unless a positive value is stored in location 29.)
ELTRAN	- (output)	Running length at onset of transition with or without crossflow correction
ELL	- (input)	Equivalent running length with or without crossflow correction for laminar flow
ELT	- (input) and (output)	Equivalent running length with or without crossflow for turbulent flow and is corrected in VRUNL to account for virtual origin from the onset of transition
ELTP	- (output)	Equal to input value of ELT
PARA1	- (input)	This parameter denotes the onset of fully transitional flow
PARA2	- (input)	This parameter denotes the onset of fully turbulent flow
ENL	- (input)	Laminar multiplication factor

If PARA2 < PARA1, then PARA = PARA2. If PARA2 > PARA1, then PARA = PARA1. If TRFLAG < 3.0, then MAIN does not call VRUNL. Therefore, VRUNL is really only meaningful for TRFLAG = 3.0, 4.0, 5.0, 6.0, or 7.0. The values TRFLAG = 1.0 or 2.0 correspond to time dependent transition and are handled through the main routine. If TRFLAG = 6.0, then VRUNL calls EDPARM as a function of angle-of-attack to determine PARA. The value of ELTRAN is given by

$$\text{ELTRAN} = L'_{TR}$$

If TRFLAG = 7.0, then the logarithm (base 10) of the transition Reynolds number RELG, based on edge conditions, is determined as a function of edge Mach number M_e from a built-in table (North American Rockwell technique). The value of ELTRAN is given by

$$\text{ELTRAN} = (10^{Re_{TR}}\,\mu_e)/\rho_e u_e$$

The interpolation to determine $\log_{10} Re_{TR}$ is performed with the aid of routine TBLIN. If TRFLAG = 5, then ELTRAN is the same as for option 5. However, the value of PARA = PARA1 is input through the routine argument to define transition onset instead of being determined by EDPARM. If TRFLAG = 4, then the running length at the onset of transition is determined based on the compressible momentum thickness Reynolds number:

$$\text{ELTRAN} = \left[\frac{Re_\theta^* \,\mu_e}{.664}\right]^2 \frac{1}{\rho^* \mu^* u_e}$$

where PARA = Re_θ^* is input to define the onset of transition. If TRFLAG = 3, then the running length at the onset of transition is determined based on the Reynolds number computed using boundary-layer edge conditions:

$$\text{ELTRAN} = \frac{Re_L \mu_e}{\rho_e u_e}$$

where Re_L = PARA.

After the running length at the onset of transition is calculated, the turbulent running length is stored (ELTP = ELT) and a check is made to determine if ELL < ELTRAN. If ELL < ELTRAN, then ELT = ELTP. If ELL > ELTRAN, then ELT = ELT - ELTRAN. Thus when VRUNL returns to the calling routine, ELTP contains the input value of ELT and ELT may or may not contain the input value of ELT depending on whether ELL < ELTRAN or ELL > ELTRAN. If ELL > ELTRAN, then ELT contains the turbulent running length measured from the onset of transition.

6.3 SUBROUTINE CRSFLW

Subroutine CRSFLW is called by MAIN and the purpose of this routine is to account for the effects of streamline divergence on basic geometries at angle-of-attack. Two types of geometry are considered in this routine: a constant width shape (rectangle) and a sharp-edge triangle (delta wing). For each of the two geometries it is possible to get an ideal or real gas calculation for both laminar and turbulent flow. In each circumstance, a correction factor is applied to the input geomtric length, and this corrected running length is used in the flat-plate heat transfer equations in place of the input geometric length. The parameters in the argument of the CALL CRSFLW statement are listed and defined below:

CFFLG	- (input)	Type of crossflow correction selected • Constant width rectangle assuming ideal gas chordwise velocity gradient • Constant width rectangle using a real gas chordwise velocity gradient • Sharp-edged delta configuration assuming ideal gas chordwise velocity gradient • Sharp-edge delta configuration using a real gas chordwise velocity gradient
ELL	- (output)	Laminar running length corrected for crossflow
ELT	- (output)	Turbulent running length corrected for crossflow
EL	- (input)	Physical geometric running length input into MINIVER
ELMBDA	- (input)	Delta sweep angle
DSUBO	- (input)	Rectangle width
CORNR	- (input)	Rectangle corner radius
UE	- (input)	Velocity at the edge of the boundary layer
PE	- (input)	Pressure at the edge of the boundary layer

RHOE	- (input)	Density at the edge of the boundary layer
PU	- (input)	Pressure upstream of the previous shock (if no shock, then freestream pressure)
ALPHA	- (input)	Surface effective angle-of-attack
UDOT	- (output)	Designates the nondimensional crossflow, stagnation-point, velocity gradient at a point on the centerline (of the wing) as a ratio to the velocity gradient at the stagnation point of a sphere with a diameter equal to the planform width
VU	- (input)	Velocity upstream of the previous shock (if no shock then free-stream Mach number)
XMACHU	- (input)	Mach number upstream of the previous shock (if no shock then freestream Mach number)

Having entered CRSFLW, the first check that occurs is to determine if ALPHA ≤ 0, if so then ELL = EL. If ALPHA > 0, then the decision is made as to which crossflow option should be used. If CFFLG = 1, the necessary program input is CORNR and DSUBO. The routine then proceeds to compute the parameter $\frac{X\bar{V}}{D_0}$. If $\frac{X\bar{V}}{D_0} < 10^{-5}$, then no crossflow is used. This indicates the velocity gradient is so small that crossflow is insignificant. If $\frac{X\bar{V}}{D_0} > 6$, then the routine shifts to a simplified calculation of ELL and ELT (this situation corresponding to the case of a very large velocity gradient). If $10^{-5} < \frac{X\bar{V}}{D_0} < 6$ then the standard crossflow correction expressions are used to compute ELL and ELT and then CRSFLW returns to MAIN. If CFFLG = 3, then the ideal gas crossflow correction for a sharp-edged delta is used to compute ELL and ELT. When CFFLG = 3, the only necessary input is the delta sweep angle, ELMBDA.

If CFFLG = 2, then the crossflow correction for a constant width rectangle using a real gas chordwise velocity gradient is used to compute ELL and ELT. The required input are DSUBO and UDOT. If CFFLG = 4, then the crossflow correction for a sharp delta configuration using a real gas chordwise velocity gradient is employed to compute ELL and ELT the required inputs are the delta sweep normal component of upstream Mach number XMACHU is less than one. If XMACHU $<$ 1, then a subsonic technique is used to compute the crossflow correction. If XMACHU $\geq$ 1, then a supersonic technique is used for crossflow correction. However for CFFLG = 1 or 3, no distinction is made for XMACHU $<$ 1 and XMACHU $\geq$ 1.

Table 6.1

TRANSITION OPTIONS

There are currently eight transition options in the LANMIN code consisting of the following:

1. <u>Time Dependence: Laminar to Turbulent</u>

$t \le t_I$ Laminar

$t \ge t_{II}$ Turbulent

$t_I < t < t_{II}$ Transitional

$\eta = \dfrac{t-t_I}{t_{II}-t_I}$ Percentage of Fully Turbulent Flow

2. <u>Time Dependence: Turbulent to Laminar</u>

$t \le t_I$ Turbulent

$t \ge t_{II}$ Laminar

$t_I < t < t_{II}$ Transitional

$\eta = \dfrac{t_{II}-t}{t_{II}-t_I}$ Percentage of Fully Turbulent Flow

3. <u>Reynolds Number Dependence</u>

$Re_{\infty D} = 2\rho_\infty U_\infty R/\mu_\infty$ for Swept Cylinders

$Re_X = \rho_e U_e X/\mu_e$ for Plate Options

$Re \le Re_I$ Laminar

$Re \ge Re_{II}$ Turbulent

$Re_I < Re < Re_{II}$ Transitional

$\eta = \dfrac{Re-Re_I}{Re_{II}-Re_I}$ Percentage of Fully Turbulent Flow

4. <u>Compressible Momentum Reynolds Number Dependency</u>

$Re_\theta^* = 0.664 \sqrt{\dfrac{\rho^*\mu^* Re_X}{\rho_e \mu_e \tau_L}}$ Compressible Momentum Thickness Reynolds Number

Table 6.1 (Cont. 1)

$Re_\theta^* \leq Re_{\theta I}$ Laminar

$Re_\theta^* \geq Re_{\theta II}$ Turbulent

$Re_{\theta I} < Re_\theta^* < Re_{\theta II}$ Transitional

$$\eta = \frac{Re_\theta^* - Re_{\theta I}}{Re_{\theta II} - Re_{\theta I}}$$ Percentage of Fully Turbulent Flow

5. <u>MDAC-E Transition Parameter</u>

This transition option was developed by Masek and Kipp

$$\psi = Re_\theta^* / [Me(\rho_e U_e/\mu_e)^{0.2}]$$

$\psi \leq \psi_I$ Laminar

$\psi \geq \psi_{II}$ Turbulent

$\psi_I < \psi < \psi_{II}$ Transitional

$$\eta = \frac{\psi - \psi_I}{\psi_{II} - \psi_I}$$ Percentage of Fully Turbulent Flow

6. <u>MDAC-E Transition Table Look-up Dependency</u>

The built in tabular function of angle of attack from Masek and Kipp was developed for conical flows. Transition is based on the parameter ψ from option 5.

α	$\log_{10} \psi_{TR}$	
0	1.0	Linear Interpolation is used
30	1.0	
40	1.0412	
50	1.1383	
60	1.30103	
70	1.69897	

Once the transition onset parameter, ψ_{TR}, is calculated, the transition length is determined:

$$L_{TR}' = 2.26 \frac{(\psi_{TR} M_e)^2 \rho_e^{.4} \mu_e^{1.6}}{\rho_* \mu_* U_e^{.6}}$$

Table 6.1 (Cont. 2)

If a crossflow option has been flagged, the following procedure is used: Since the transition parameter ψ was derived for complex shapes using the laminar crossflow length in the calculation of Re_θ^*, it is necessary to remove this correction factor so that the resulting transition length can be compared with the geometric length.

For crossflow options for rectangular shapes the transition length is

$$L_{TR} = -0.5\,\bar{V}\,\ln\left[1 - \frac{2L_{TR'}}{\bar{V}}\right]$$

For delta wings options the transition length is

$$L_{TR} = BL_{TR'}$$

When the Mangler transformation, τ_L, is used, the transition length is

$$L_{TR} = \tau_L\, L_{TR'}$$

Two methods for treating the extent of transition have been provided. The first is rather simple in that the fully turbulent length, L_{FT}, is a factor times the onset length

$$L_{FT} = K_{TE} \cdot L_{TR}$$

The second method defines the factor K_{TE} in terms of the boundary layer edge Reynolds number at the onset length, which is defined as

$$Re_{TE} = \frac{\rho_e U_e L_{TR}}{\mu_e}$$

The table, which is built in, consists of

$\log_{10} Re_{TE}$	K_{TE}	
5.	5.64	Linear Interpolation is used
6.518	2.	
6.778	1.702	
7.	1.605	
7.301	1.535	
10.	1.535	

Table 6.1 (Cont. 3)

The state of flow is determined by

$L_{LCF} \le L_{TR}$	Laminar
$L_{LCF} \ge L_{FT}$	Turbulent
$L_{TR} < L_{LCF} < L_{FT}$	Transitional

$$\eta = \frac{L_{FT} - L_{LCF}}{L_{FT} - L_{TR}} \qquad \text{Percentage of Fully Turbulent Flow}$$

Since the transition criteria was developed for a conical shock flowfield, some judgement must be used in applying it to other situations.

7. NAR Transition Parameter

A criteria developed by North American Rockwell is based on edge Reynolds number as a function of edge Mach number. The built in table is

Me	$\log_{10} Re_{TR}$
0.5	5.30103
1.0	5.54407
1.5	5.81291
2.0	6.00
3.5	6.00
4.5	6.07918
5.5	6.25527

Linear Interpolation is used

$L_{TR} = Re_{TR}\, \mu_e/(\rho_e U_e)$ Transition length

The transition extent is the same as for option 6. The state of flow determination is based on L_{Geo} rather than L_{LCF}.

8. Compressible Re^*_θ/Me Dependency

$$Re^*_\theta = 0.664 \sqrt{\frac{\rho^* \mu^*}{\rho_e \mu_e} \frac{Re_x}{\tau_L}}$$

Table 6.1 (Cont. 4)

$$R_f = Re_\theta^* / Me$$

$R_f \leq R_{fI}$	Laminar
$R_f \geq R_{fII}$	Turbulent
$R_{fI} < R_f < R_{fII}$	Transitional

$$n = \frac{R_f - R_{fI}}{R_{fII} - R_{fI}}$$ Percentage of Fully Turbulent Flow

Nominal Values

$R_{fI} = 150$

$R_{fII} = \sqrt{2}\ R_{fI}$

Nomenclature

B	Cross Flow Parameter
L	Length
M	Mach Number
t	Time
U	Velocity
$\bar{V}$	Chordwise Velocity Gradient
X	Cross Flow Corrected Running Length
ρ	Density
μ	Viscosity
τ	Mangler Factor

Subscripts

D	Diameter
e	Edge
Geo	Geometric
I	Initial
II	Final
L	Laminar
LCF	Laminar Cross Flow
TE	Transition Extent
TR	Transition
θ	Momentum Thickness

References

Masek, R.V., Boundary Layer Transition at High Angles of Attack, Space Shuttle Technology Symposium Proceedings, NASA/LeRC, July 15, 1970.

Kipp, H.W. and Masek, R.V., Aerodynamic Heating Constraints on Space Shuttle Vehicle Design, ASME paper 70-HT/SPT-45, June 1970.

 C-2

Section 7.0

APPLICATION TECHNIQUES

The purpose of this section is to provide some guidance to the engineer in applying the LANMIN code in the prediction of heating conditions. In order to help achieve this objective, several sample cases are shown. These cases contain experimental wind tunnel data compared with different options available in LANMIN. Based on these results and past experience with the code, a set of recommended options are given.

7.1 GENERAL APPLICATION

In order to achieve the best prediction method using the LANMIN code several key parameters need to be examined.

1. Determine the shock option which establishes the boundary layer edge entropy level.
2. Determine the geometric Mangler transformation which gives the appropriate running length adjustment for the given geometry.
3. Determine the pressure option which best matches the surface pressure.
4. Determine the heating method and associated Reynolds analogy factor which best fits the experimental heating data.

The shock option is usually selected by determining which option produces the approximate shock angle which processes the boundary layer flow. This is usually established by using wind tunnel schlieren.

The Mangler transformation used in LANMIN is the geometric transformation of the running length for a body into the equivalent length for a flat plate. The mathematical relations which can be used for a compound geometry body which is axisymmetric are as follows:

$$\tau_m = \frac{rs}{\int_0^s r\,ds} \qquad \text{Laminar}$$

$$\tau_m = \frac{r^2 s}{\int_0^s r^2\,ds} \qquad \text{Turbulent}$$

where r is the radius of the body normal to the axis of symmetry and s is the surface distance from the nose to the point of interest on the body. The preceding equations can be numerically integrated for compound body shapes to obtain the laminar and turbulent Mangler factor as a function of running length.

Selection of the pressure option and Reynolds analogy factor for a problem is based primarily on experience. In order to document some of this experience, several sample cases will be examined. Figure 7.1 shows a comparison of wind tunnel pressure data with the LANMIN tangent cone option and a method of characteristic (MOC) calculation. The data and theories are for the cylinder section of the Space Shuttle external tank at zero angle of attack. For this situation, the tangent cone option does not predict the data well. This option would also correspond to the modified Newtonian pressure option. The MOC results do compare well with data. Thus, results from a more exact theoretical method should be used as input to LANMIN, where available, to establish the edge pressure.

Heating results from LANMIN using both pressure options are shown in Figure 7.2 using the best heating prediction method. The overprediction trend of the tangent cone option, shown in Figure 7.1a, is translated into an over prediction in heating as shown in Figure 7.2. Likewise, when the pressures from the more accurate MOC solution are used, the heating is more accurately predicted.

The Reynolds analogy factor effect is examined in Figure 7.3. The Colburn analogy factor is shown to overpredict the data and a unity Reynolds analogy factor underpredicts the data. The von Karman analogy factor when used with the Spalding-Chi heating method with accurate pressures provides the best comparison with data. This trend is substantiated further by the results given in Figure

7.4. The pressure at small to moderate angles of attack along the windward streamline is predicted well by both the tangent cone option and MOC solutions. Thus the heating data in Figure 7.4a compares well with calculations using both pressure options. The theory and data comparisons in Figure 7.4b and c demonstrate that the von Karman Reynolds analogy factor gives the best prediction.

Results from the Spalding-Chi option using the von Karman Reynolds analogy are shown in Figure 7.5 along with an independent heating theory for a different geometry. Heating data for a sphere-cone-cylinder (SRB) body are shown. The DIRLIN results are from an integral momentum solution based on the work of Dirling (Ref. 7). The two theories are in good agreement and compare well with the wind tunnel data.

Another application of the LANMIN code is shown in Figure 7.6. This case is a sharp wedge at three angles of attack. Three-dimensional effects on the wedge produced pressures higher than the two-dimensional wedge pressure option. By using the wedge shock entropy and inputting the measured pressures, the LANMIN results compare well with data and results from an independent theory of Dirling (Ref. 7).

All preceding results and comparisons have been for turbulent flow. The laminar method of Eckert in LANMIN is compared with wind tunnel data in Figure 7.7. The tangent cone pressure option was used for this calculation. Using more precise pressures would have slightly lowered the predicted values in the range $0.08 < X/L < 0.22$ and slightly increased the values in the range $0.22 < X/L < 0.60$.

Other comparisons of data and theory have been presented in the tables of Section 5.0. These results along with other experience were used to determine a recommended set of heating options. These options are given in Table 7.1.

7.2 GENERIC ORBITER APPLICATIONS

Heating to a generic orbiter geometry can be calculated by a numerous combination of options for a specific location on the vehicle. The success in accurately predicting the heating to any given location is determined in large part by the proper selection of options and the engineering skill of the user. A general approach is given herein for orbiter application. Specific geometries and flow situations may require modification of the guidelines given.

The approach taken follows the selection of the four key parameters given in Section 7.1 as specifically applied to a generic orbiter shape. Figure 7.8 gives the selected generic orbiter design methods in summary form. The approach taken was to select methods which would best represent measured data. If a design conservatism is to be introduced, it should be introduced as a known constant multiplier. In order to demonstrate and help explain the options selected, a discussion is given of three of the principal areas on an orbiter.

<u>Nose Region</u>

The heating to the bottom centerline of an orbiter nose can be calculated using the options in LANMIN by determining an effective sphere value. The effective sphere value is calculated with normal shock entropy. The heating methods of Lees and Detra-Hidalgo are used for laminar and turbulent flow respectively. Consider the laminar case as an example. The heat transfer coefficient is determined by

$$h = \left(h_o\sqrt{\frac{R_o}{r}}\right)\left(\frac{h}{h_o}\right)_{Lees}$$

where

ho is the Fay and Riddell value for radius Ro

r is the local radius of the body

h/ho is the Lees distribution ratio at the local body angle plus angle of attack

An example of this method being applied to a sphere-cone geometry is given in Fig. 7.9. The Lees distribution value on the cone is evaluated at 90 - 20 - 15 = 55 degrees, and the local radius, r, is a function of axial location. This rather simple method provides quite good agreement with the data shown.

This method was also applied to the nose region of an orbiter configuration as shown in Fig. 7.10. The area of application was $0 \leq X/L < 0.05$ on the orbiter bottom centerline where theory and data agree for all three angles of attack.

Thus by inputting the body angle and local radius an effective sphere distribution value is calculated which represents the nose region bottom centerline. The effect of angle of attack is accounted for by the program calculating an effective body angle.

<u>Bottom Centerline Heating</u>

In order to select a usable method for LANMIN for the bottom centerline of an orbiter, each component of the calculation must be considered. The comparisons given in Fig. 7.10 for $X/L > 0.05$ are sample trials of different options. At one angle of attack, a given set of options may agree with data but not agree with data at other angles of attack. This approach confounds the effects of the different options in a set. Thus, final agreement for one case does not assure applicability over a wide range of conditions.

Let us consider the shock shape which controls the boundary layer edge conditions. Figure 7.11 presents a correlation of shock shapes first published for delta wing data by Dunavant (Ref. 14). The generic orbiter data from four sources were added in the present study. Virtually all of the data agrees with tangent cone shock theory for angles of attack from 10 to 45 degrees. Most all orbiters fly in this angle of attack range during the highest heating period. Thus the shock option to be used is the tangent cone option.

Determining the correct running length to use is the most complicated and most critical decision to be made. The LANMIN code has cross-flow correction options and Mangler transformations to assist in correcting the actual running length to an equivalent running length.* For some geometries these running length corrections can be quite accurate and easy to select. This is not the case for generic orbiter geometries.

Another approach is selected here based on the modification of earlier work by Pond et al (Ref. 15). aminar heat transfer data are used to calculate the effective running length to the body location of interest. This procedure finds the actual flat plate equivalent running length which is required for input into LANMIN. This effective running length is then used for flight prediction calculations. A sample case is described to provide details of the approach.

* Other analyses have developed complex streamline tracing methods which requires an accurate and smooth pressure distribution input. These methods are out beyond the scope of the LANMIN approach.

Pond et al (Ref. 15) tested the geometry shown in Fig. 7.12 to obtain heating data. The heating data were used with Eckerts heating method to calculate the equivalent running lengths shown in Fig. 7.13. For this geometry it is evident that the equivalent running lengths are a strong function of angle of attack and axial location.

Work by Dunavant (Ref. 14) which was later amplified by Newmann and Renfroe (Ref. 16) dealt with obtaining an outflow correction for the running length. For hypersonic flow, this correction can simply be written as

$$Xe = X(\tan s/\tan \alpha) \qquad (7.1)$$

where

Xe = equivalent running length

X = actual running length

α = angle of attack

s = 90 - sweep angle

By looking at the geometry in Fig. 7.12 a single value to use for s is not evident. However, by taking the data in Fig. 7.13 and using it in the preceding equation, s can be calculated. The results are shown in Fig. 7.14. All of the 20, 30 and 40 degree angle of attack data collapse to give an effective sweep angle complement as a function of axial location. The α = 10 data is for an inflow condition and is thus not expected to correlate. The data for α = 55 exhibit a trend which is not completely understood and may be due to lack of quality paint data for the original heating data. Since the preceding relation does collapse the data in the primary range of interest, it can be used along with limited wind tunnel data to develop equivalent running lengths for input.

To complete the explanation of the procedure, the equivalent running length data in Fig. 7.13 was used in LANMIN for a completely different orbiter shape. The results from LANMIN are compared in Fig. 7.15 with data from a phase B orbiter geometry. The comparison is quite good and significantly improved over the comparisons made in Fig. 7.10. Note that the tangent wedge pressure option was used. Unfortunately, pressure data are not available for comparison. However, Mach number data on orbiter shapes are usually overpredicted when wedge pressures are used. Thus, wedge pressures are most likely too low. This might explain why the theoretical results are slightly lower than measured in Fig. 7.15.

Thus far only laminar equivalent lengths and results have been discussed. The laminar heating rates are more sensitive to the equivalent length ($h \sim Xe^{1/2}$) than turbulent heating rates ($h \sim Xe^{.2}$). The laminar and turbulent equivalent lengths are different. For example on a cone the differences are:

$$Xe_L = X/3 \qquad \text{from Mangler relations}$$
$$Xe_t = X/2$$

Thus an adjustment must be made to the laminar equivalent length to obtain a turbulent equivalent length. Since no data were available for turbulent equivalent lengths, the cone ratio was used

$$Xe_t = 3Xe_L/2$$

to obtain input to LANMIN for the case shown in Fig. 7.16. The comparison of data and theory is quite good. Note that a 50% increase in the running length when used in turbulent heating equations only reduces the heating by 8 percent.

The procedure to use the equivalent running length approach is as follows:

(1) Obtain laminar wind tunnel data for the generic orbiter of interest.

(2) Use LANMIN parametrically to generate laminar equivalent running lengths.

(3) If data are available, repeat step (1) and (2) to verify the laminar to turbulent transformation.

(4) If limited data are available, use equation 7.1 along with an effective s based on the limited data to expand the angle of attack range.

(5) If no wind tunnel data are available use Fig. 7.13 or closest geometry equivalent length data (Note a set of plots/tables of equivalent lengths for a set of geometries should be developed as a data base).

(6) Use the equivalent running length in LANMIN for flight applications along with the same shock, pressure and heating rate options used to generate the equivalent running lengths.

This method has been discussed in terms of the bottom centerline. However, it has a much broader application. This method may be applied to any body location which is not influenced by separation or shock interference.

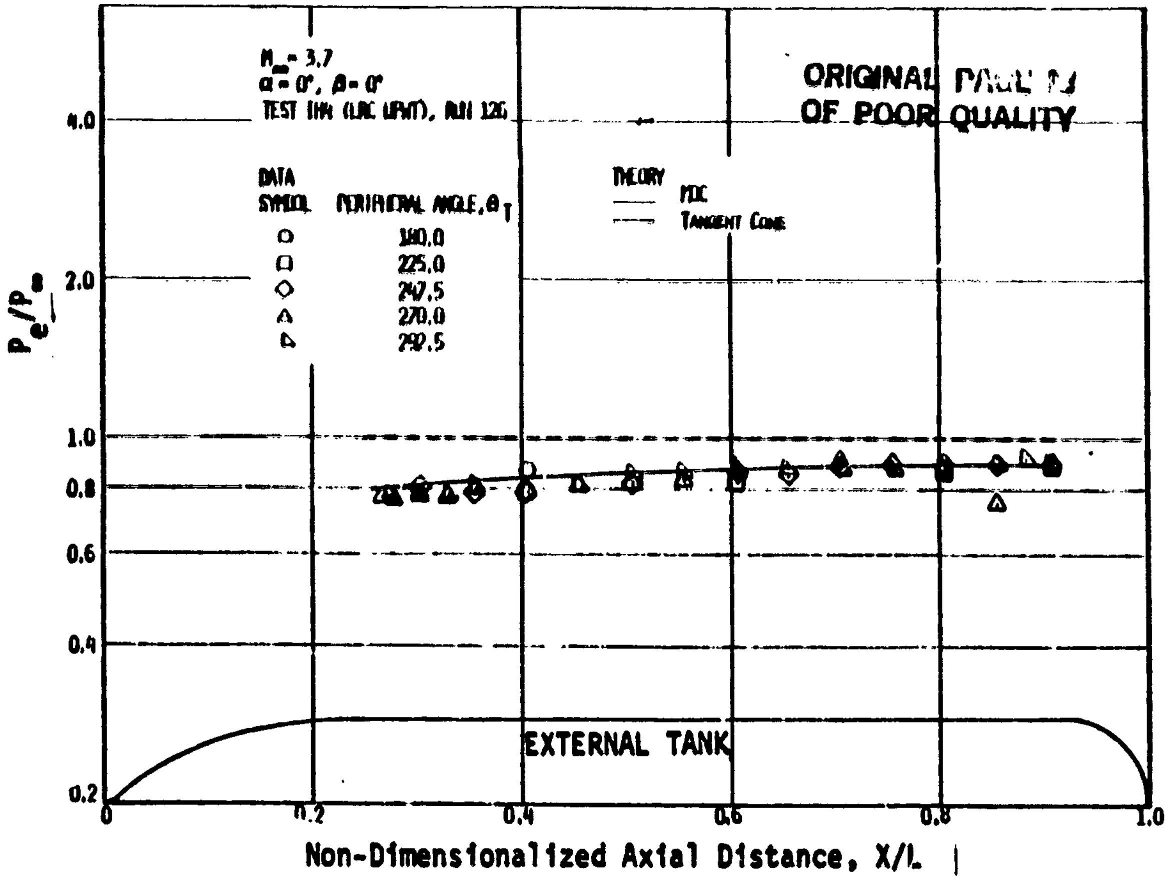
Wing Leading Edge

The stagnation line of a wing is approximated well by swept cylinder theory. The shock lies parallel to the wing over a considerable portion of the wing as shown in the schlieren of Fig. 7.17. The laminar flow data agree well with the correlation in LANMIN. A comparison for turbulent flow of Beckwith-Gallagher with data is given in Table 5.6a.

Table 7.1

RECOMMENDED METHODS

GEOMETRY	HEATING OPTION	HEAT TRANSFER OPTION NUMBER
SPHERE	Fay and Riddell	1
SPHERE DISTRIBUTION	Laminar: Lees Turbulent: Detra and Hidalgo	8
SWEPT CYLINDER	Laminar: Correlation Turbulent: Beckwith-Gallagher	6
CONE, WEDGE, OGIVE & ORBITER BOTTOM CENTERLINE	Laminar: Eckert Turbulent: Spalding-Chi with Von Karman Reynolds Analogy	4
ORBITER LEESIDE	Bertin and Goodrich	9
FLAP	Bushnell and Weinstein	10
FIN INTERFERENCE ON PLATE	Fivel	11



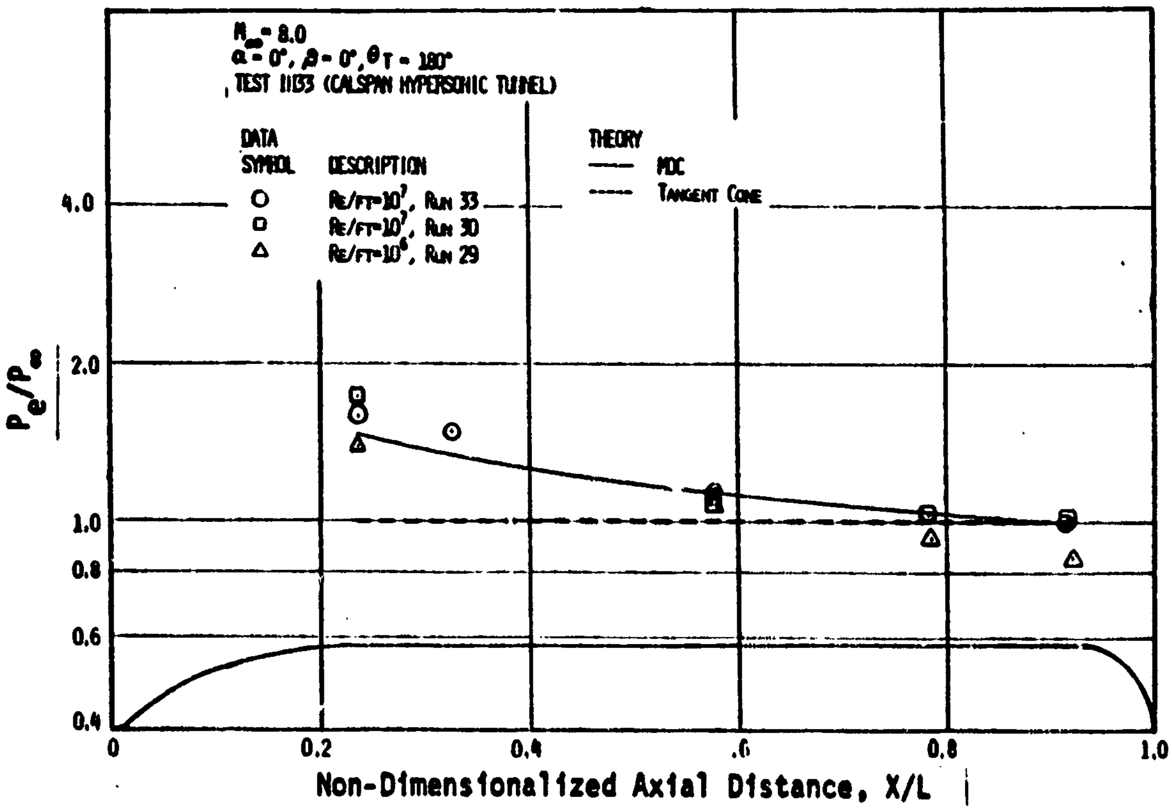
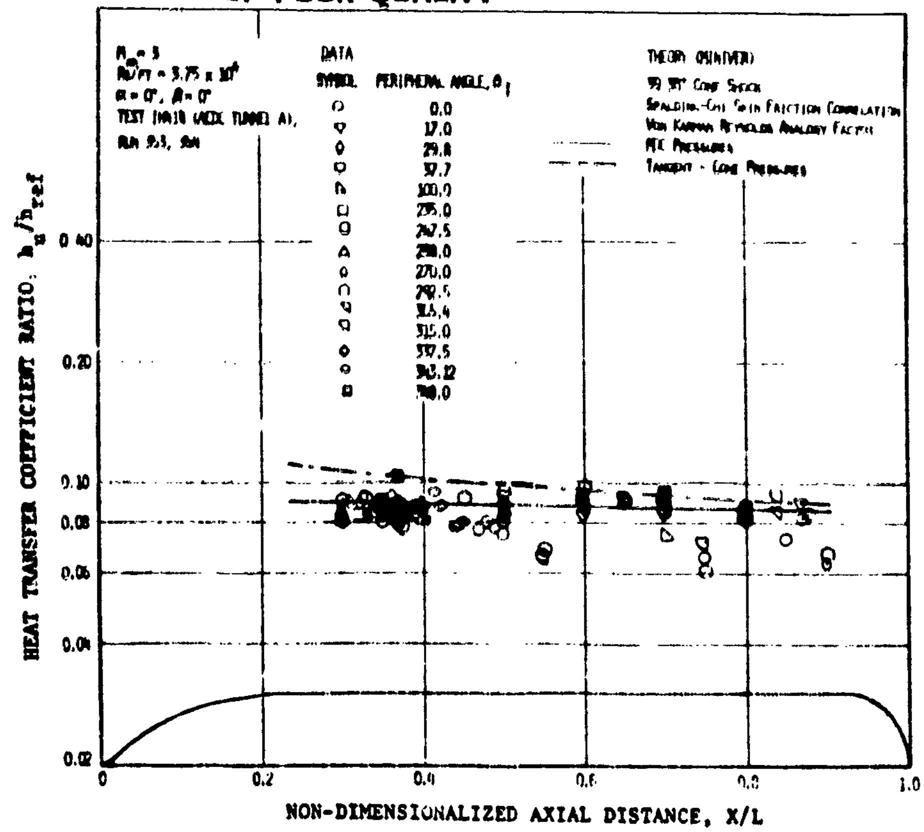


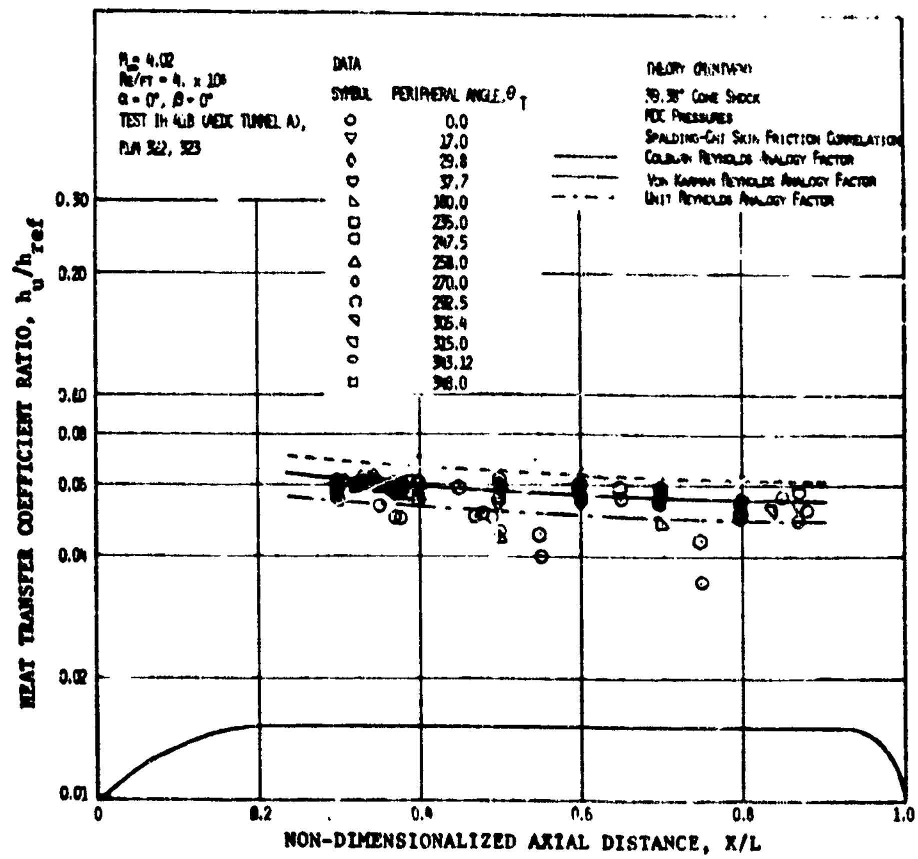
Fig. 7.1 Pressure Ratio on the Space Shuttle External Tank Barrel (From Ref. 5)

ORIGINAL PAGE IS
OF POOR QUALITY



From
Ref. 5

FIG. 7.2 HEAT TRANSFER DISTRIBUTION ON THE SPACE SHUTTLE EXTERNA .



From
Ref. 5

FIG. 7.3 HEAT TRANSFER DISTRIBUTION ON THE SPACE SHUTTLE EXTERNAL TANK

(a)

(b)

(c)

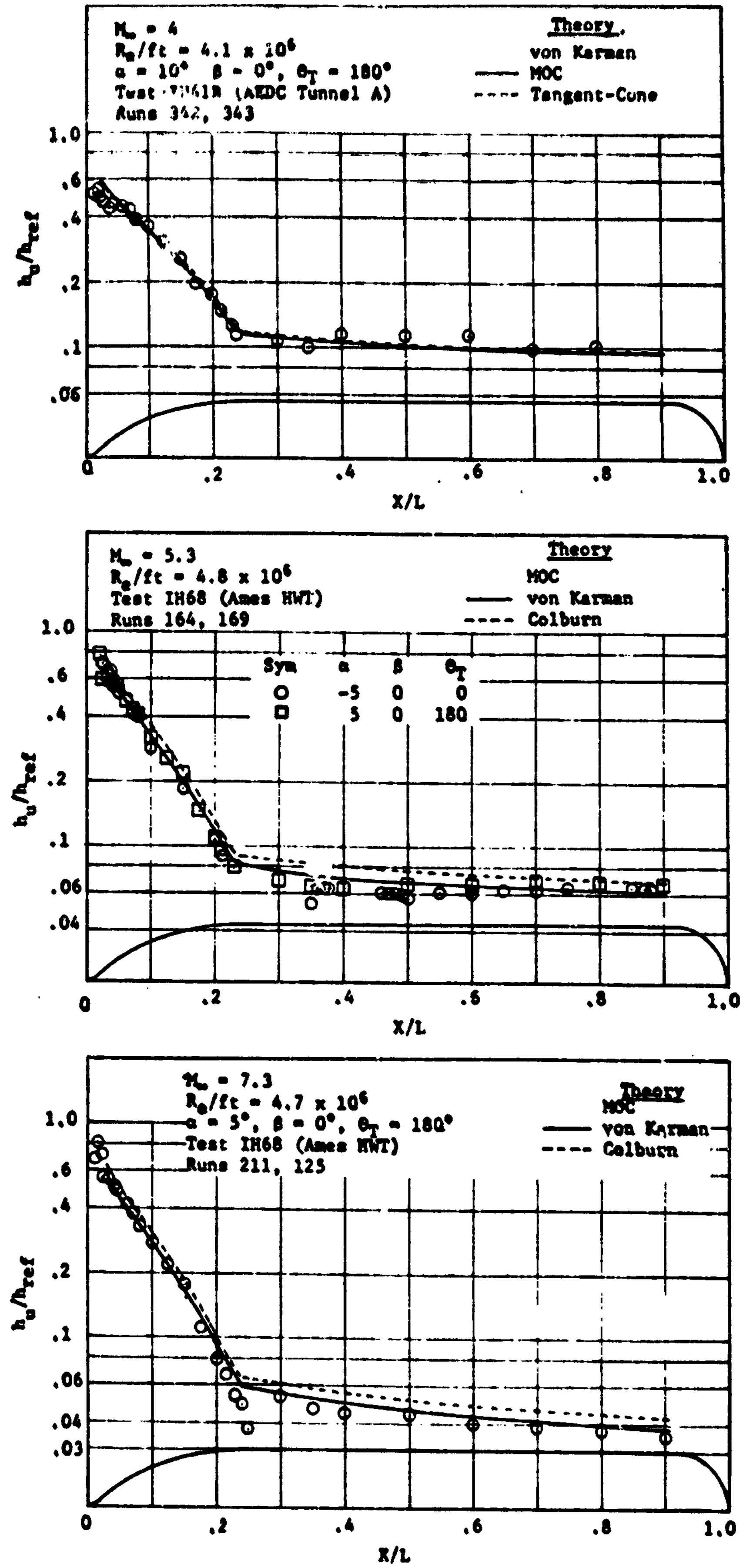


Fig. 7.4 Heat-transfer Distribution on the Space Shuttle External Tank for Various Tunnel Conditions (From Ref. 6)

ORIGINAL PAGE IS
OF POOR QUALITY

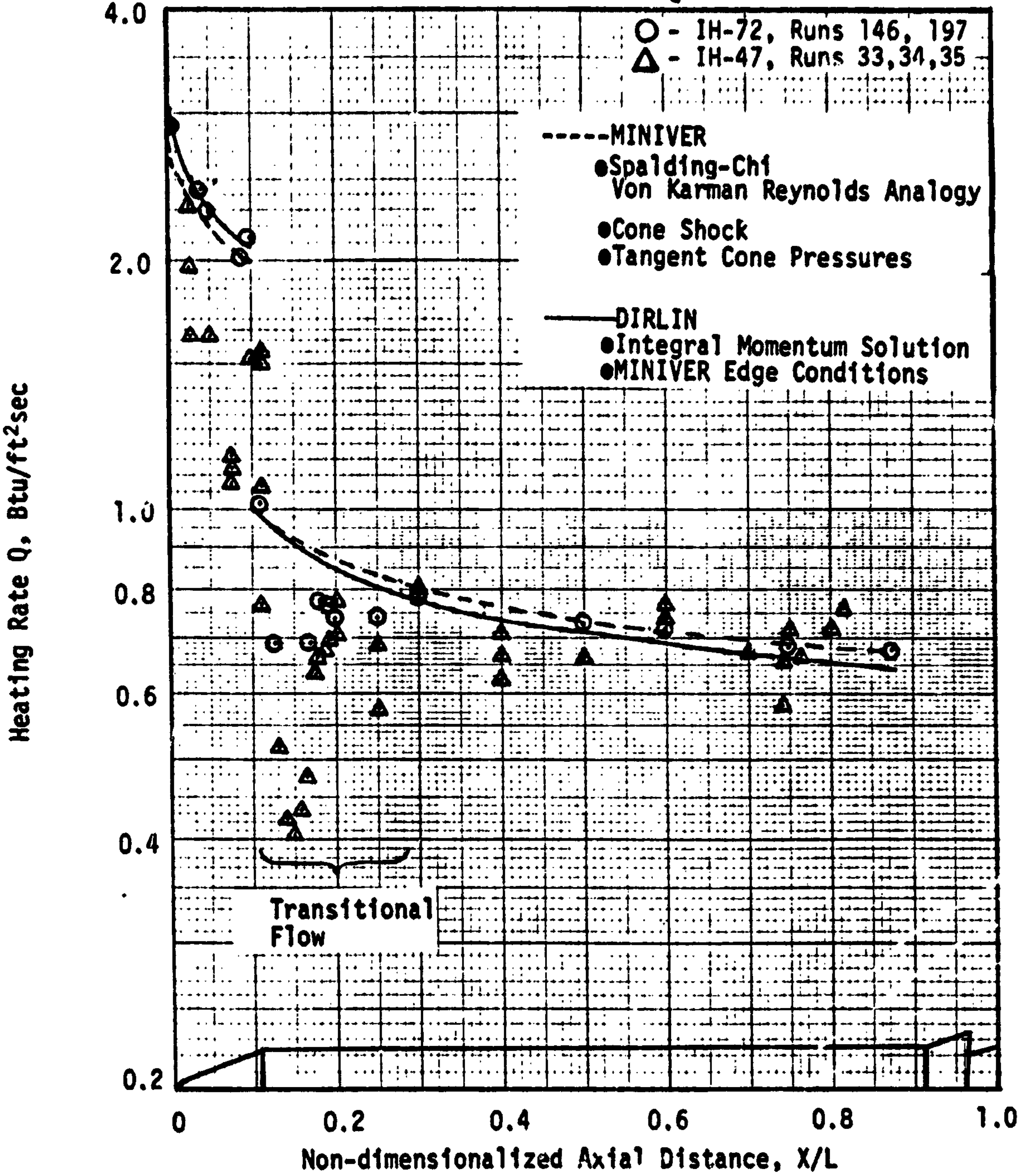


Fig. 7.5 Comparison of Experimental Data and MINIVER and DIRLIN Results for Right SRB Unmated Configuration at $M_\infty = 3$ ($\alpha = 0°$, $\beta = 0°$)

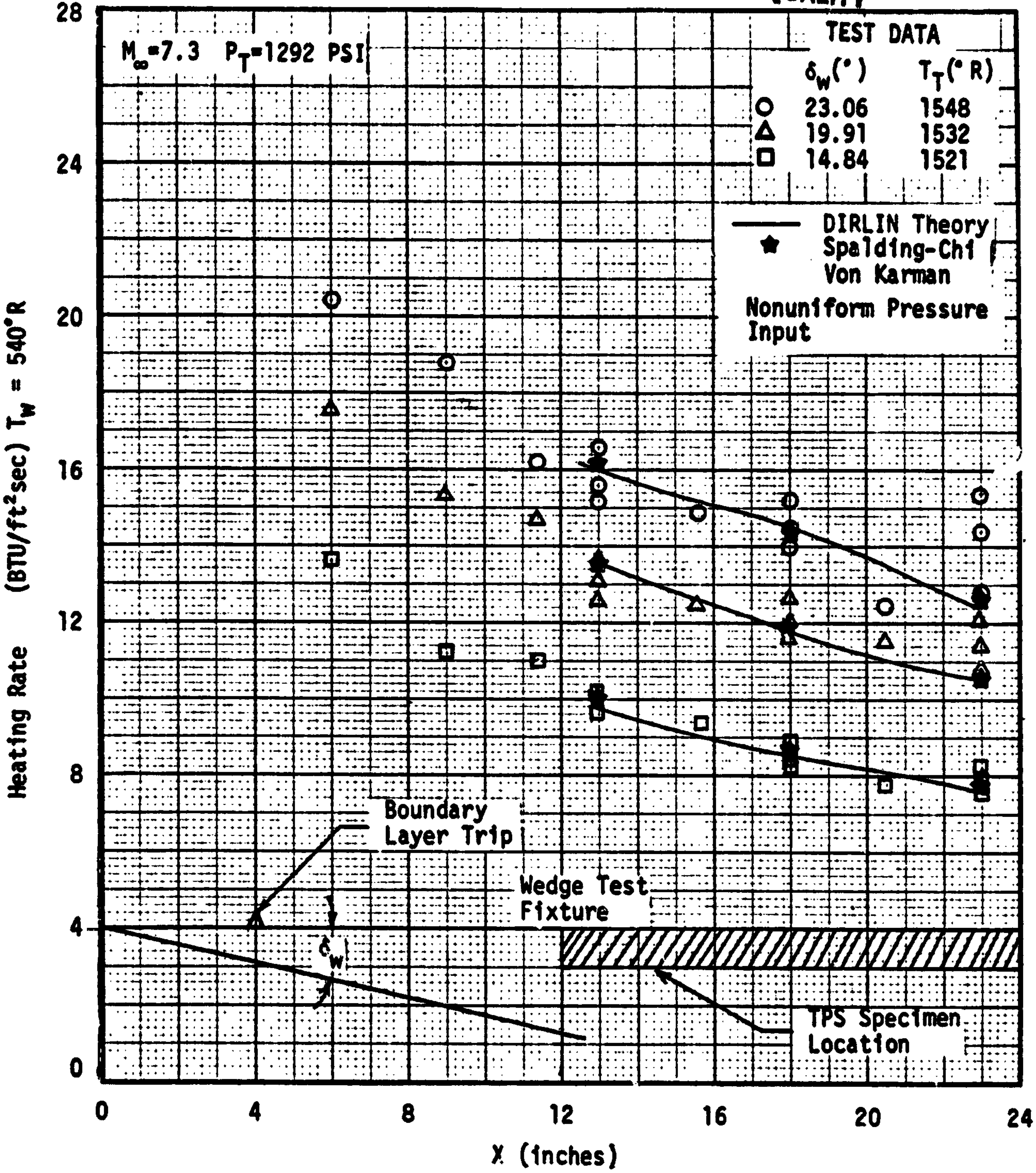


Fig. 7.6 Heating Rate Versus Length for Three Test Conditions at the Ames TPS Calibration Test

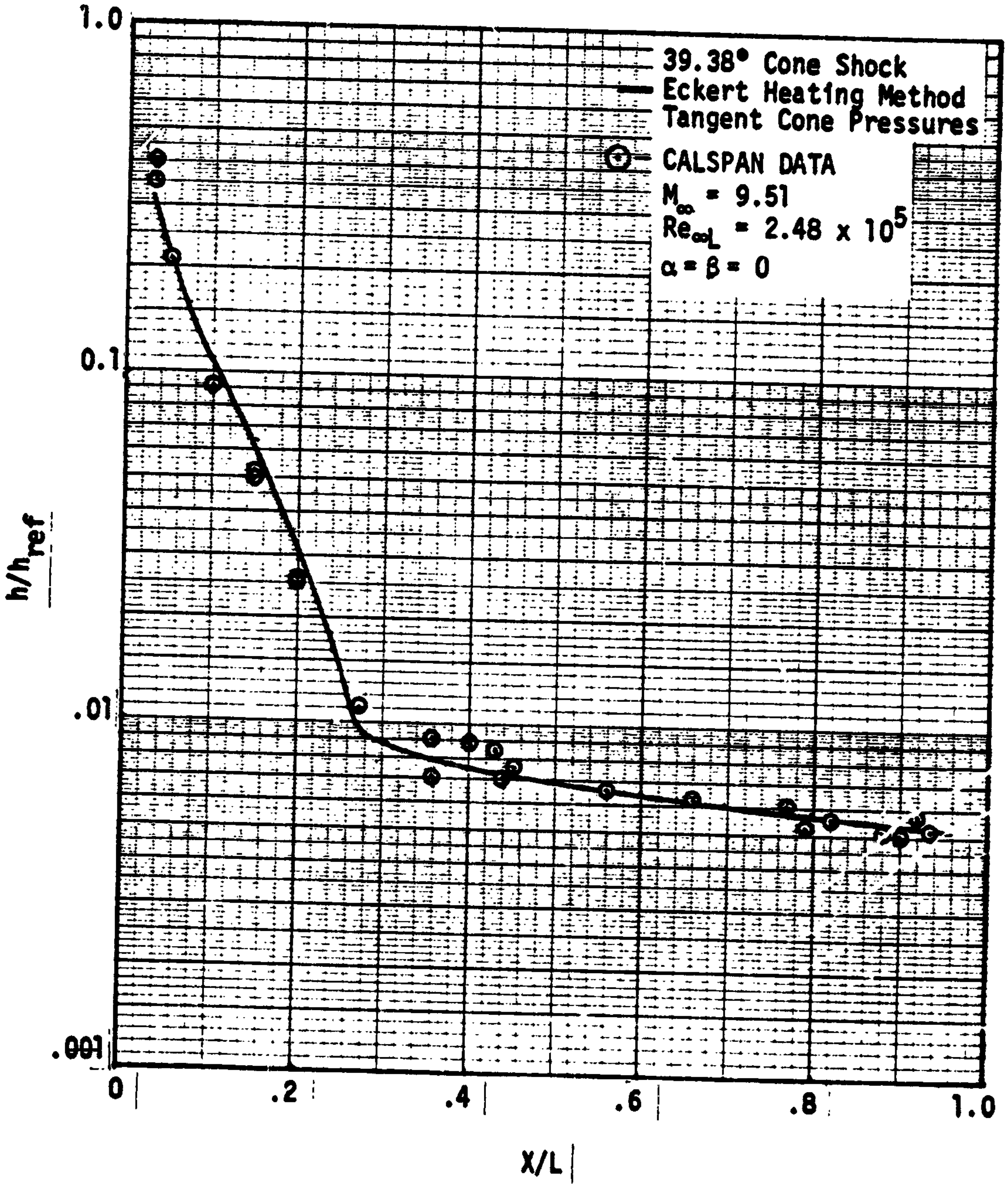
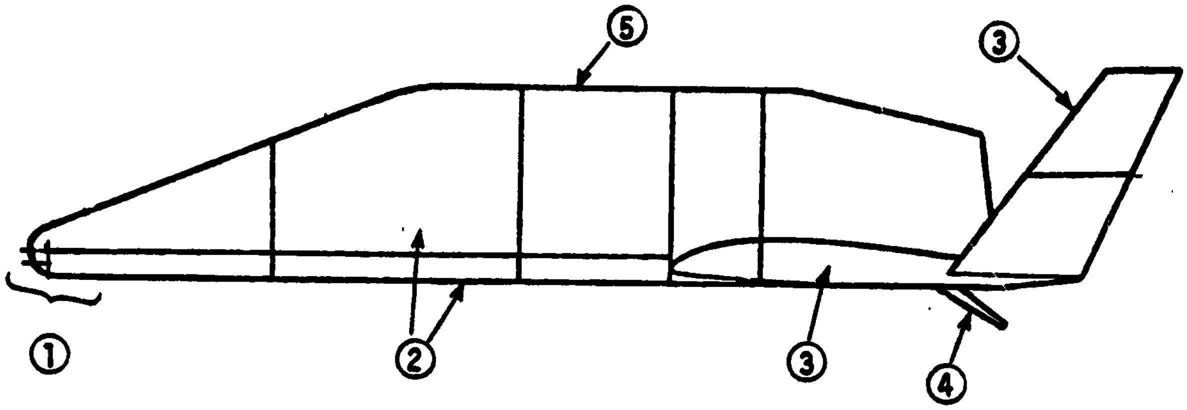


Fig. 7.7 Laminar Heat Transfer Distribution on the Space Shuttle External Tank

ORIGINAL PAGE IS
OF POOR QUALITY



1
- Normal Shock
- Stagnation Point: Fay & Riddell
- Distribution : Laminar - Lees
Turbulent - Detra & Hidalgo

With Effective Radius

2
Tangent Cone Shock
Tangent Wedge Pressures
Laminar: Eckert With Tunnel Derived Equivalent Running Lengths
Turbulent: Spalding-Chi With Turbulent $\tau_m = 2/3$
Von Karman

3
Swept Cylinder
Parallel Shock
Laminar: Correlation
Turbulent: Beckwith-Gallagher

4
Tangent Cone Shock
Tangent Wedge Pressures
Tangent Wedge Shock
Tangent Wedge Pressures
Bushnell-Weinstein

5
Normal Shock
Bertin-Goodrich

Fig. 7.8 LANMIN Generic Orbiter Design Methods

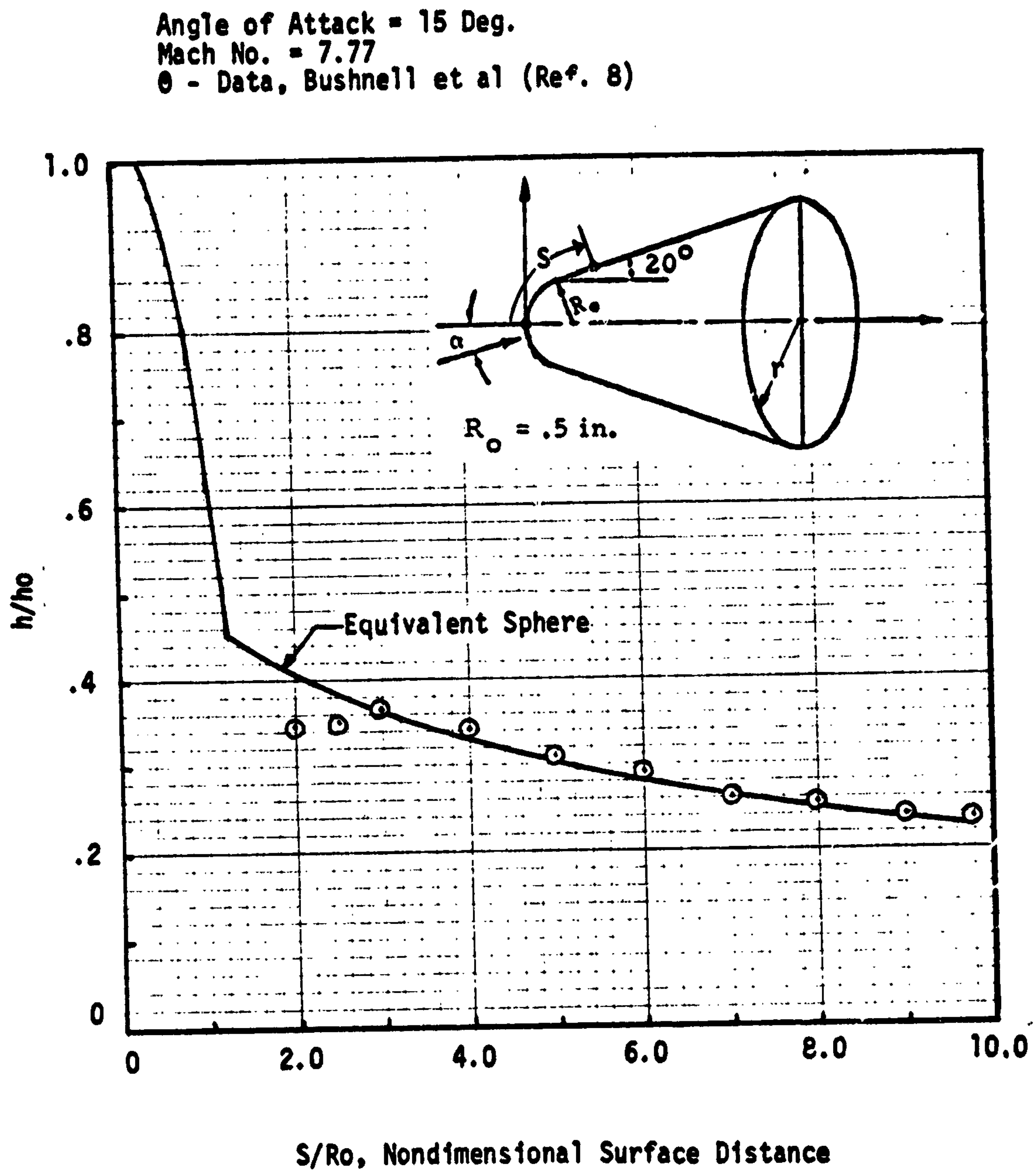


Fig. 7.9 Windward Centerline Heat Transfer Over a 20-Degree Half Angle Blunt Cone at Angle of Attack

ORIGINAL PAGE IS
OF POOR QUALITY

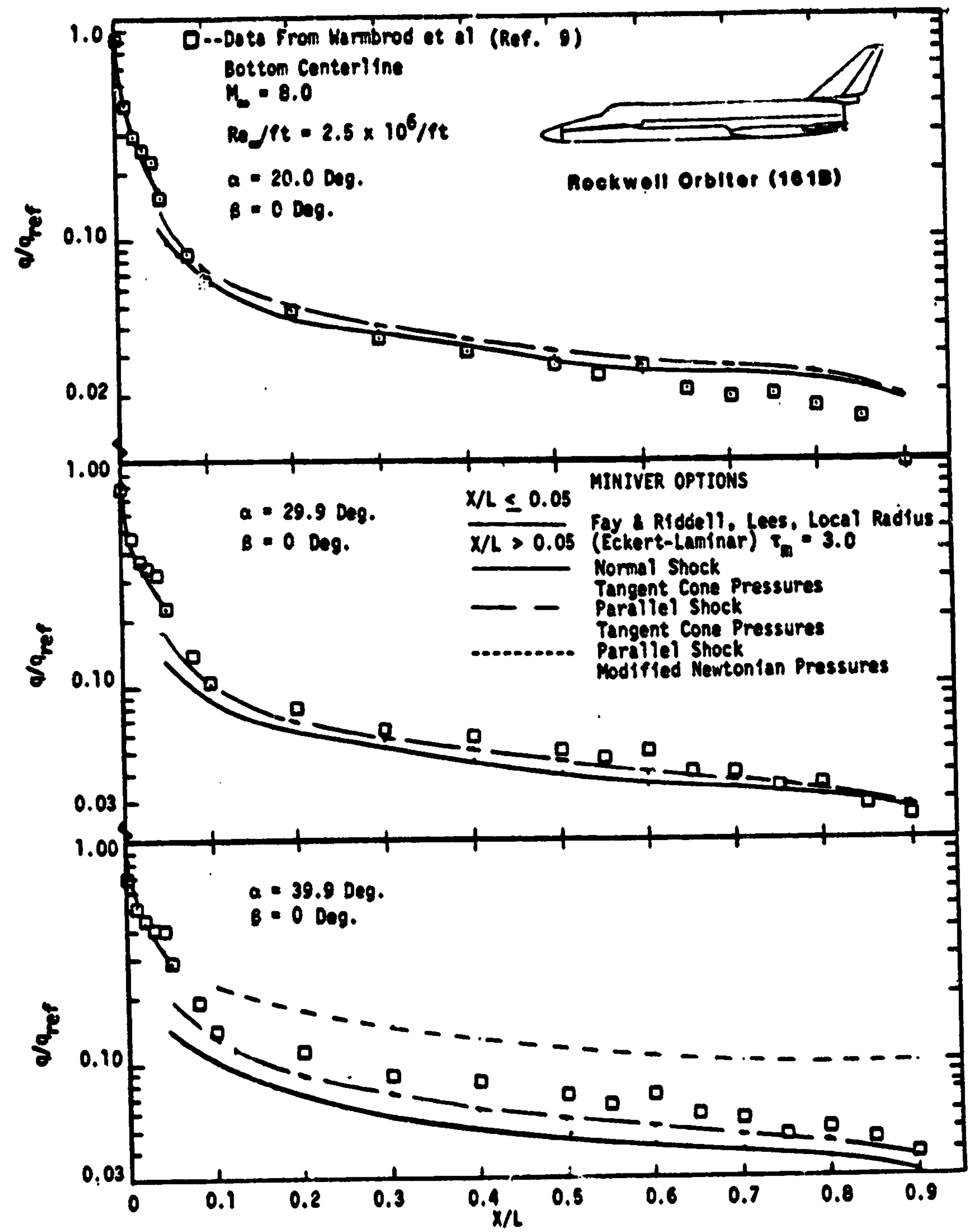


Fig. 7.10 MINIVER Theory Compared With Phase B Orbiter Configuration Data

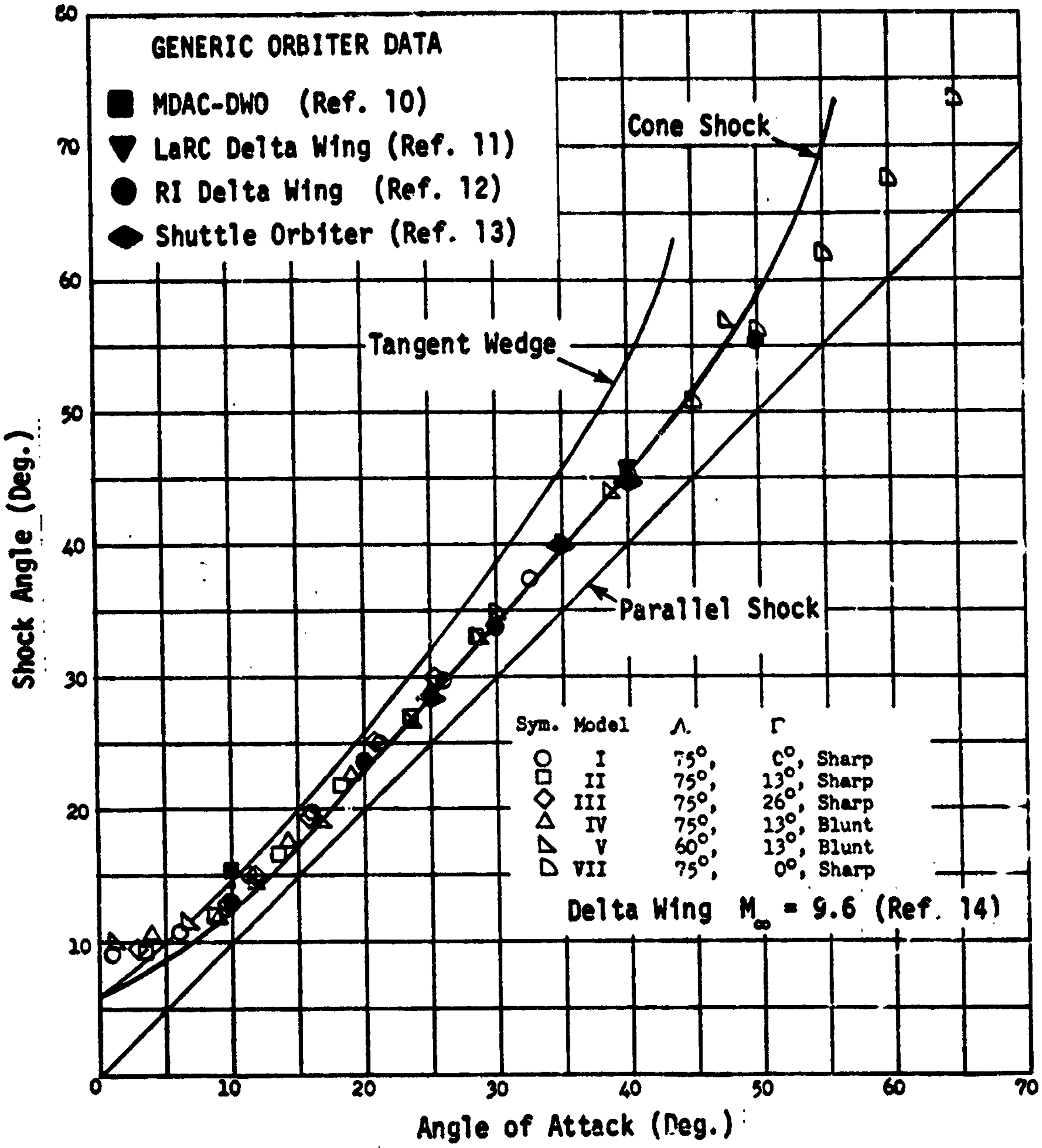


Fig. 7.11 Shock-Wave-Angle Correlation

ORIGINAL PAGE IS
OF POOR QUALITY

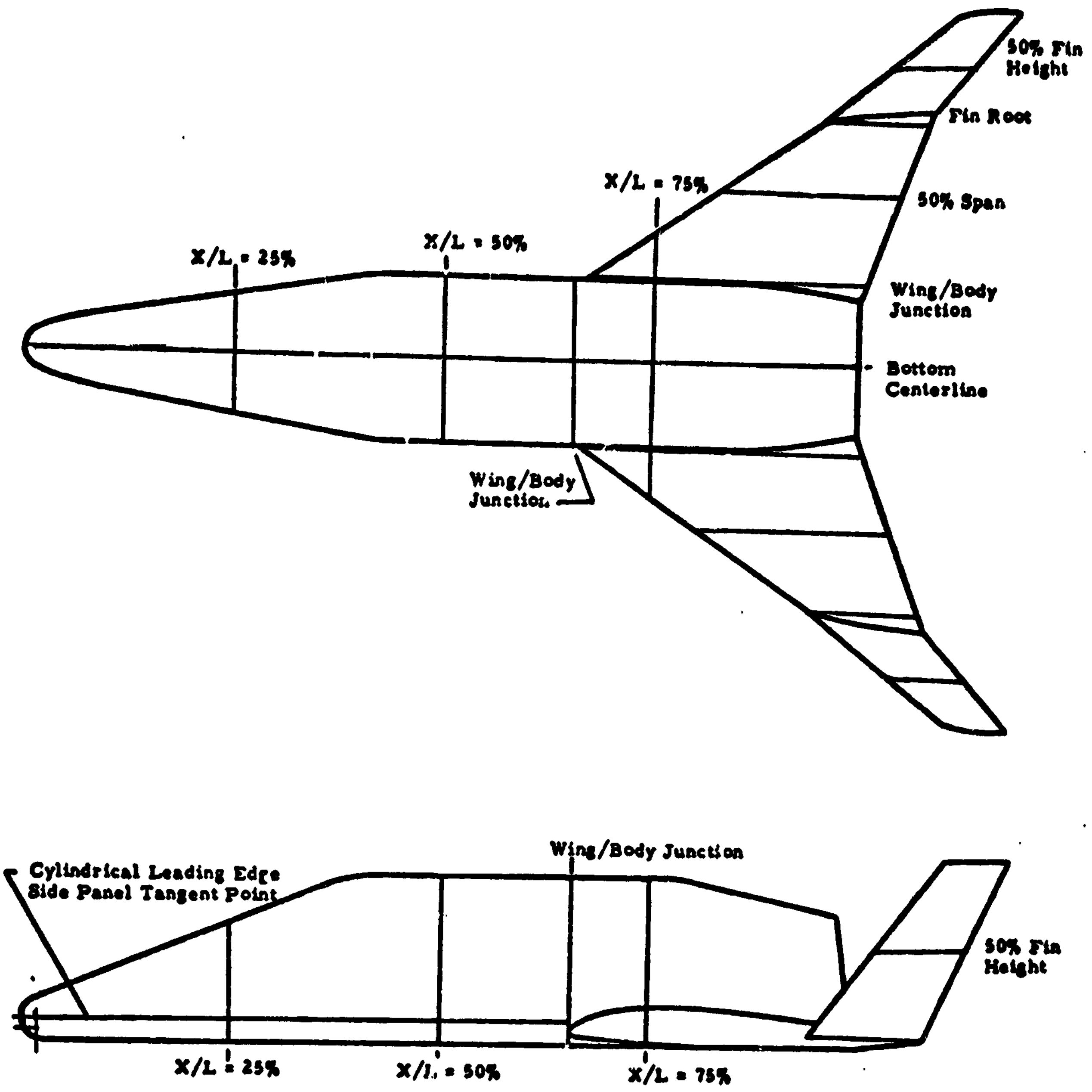


Fig. 7.12 - Geometry of the MSFC 437 Aerodynamic Heating Data Model

ORIGINAL PAGE IS
OF POOR QUALITY

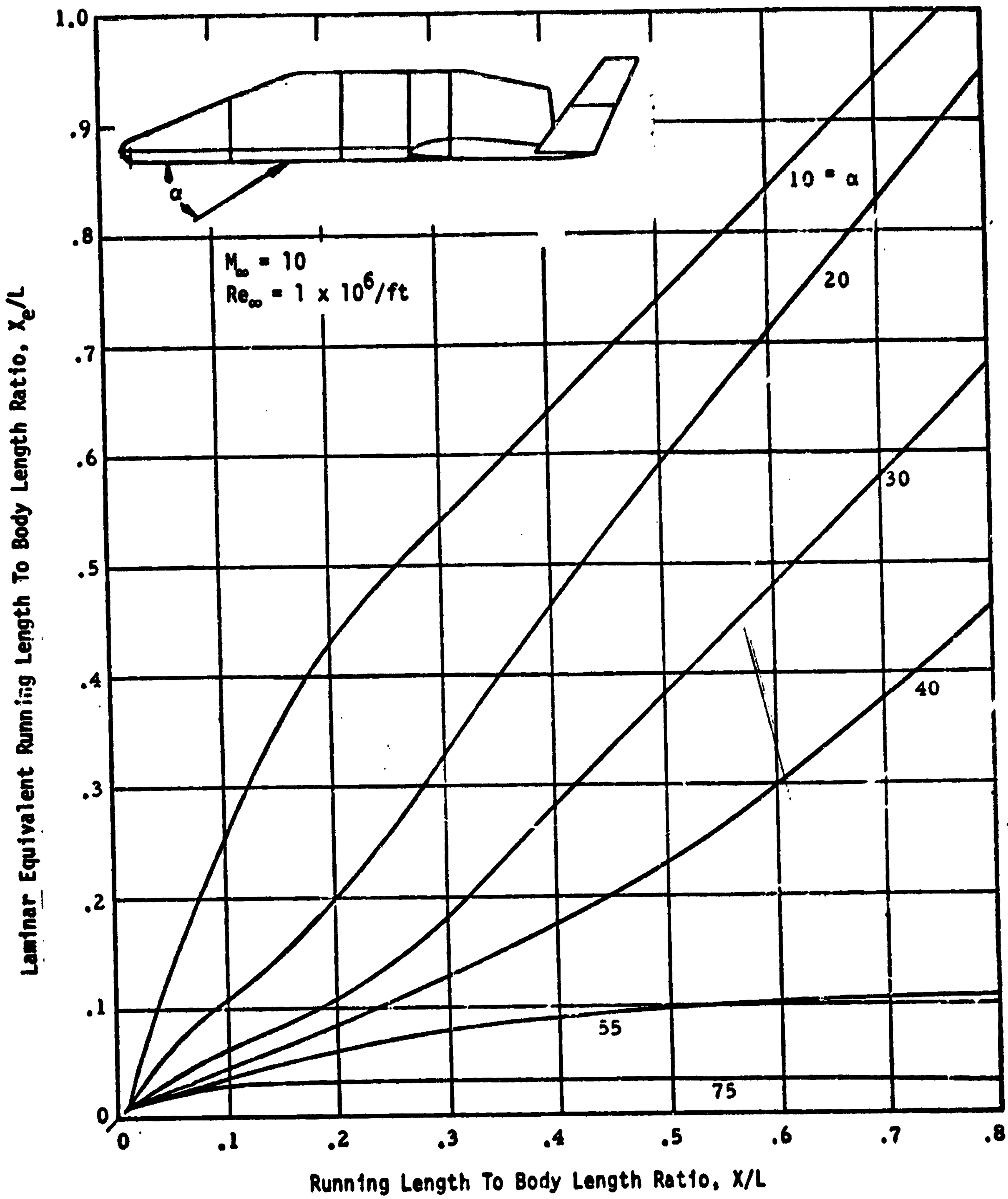


Fig. 7.13 Equivalent Running Length For The Bottom Centerline Of The MSFC 437 Orbiter

ORIGINAL PAGE IS
OF POOR QUALITY

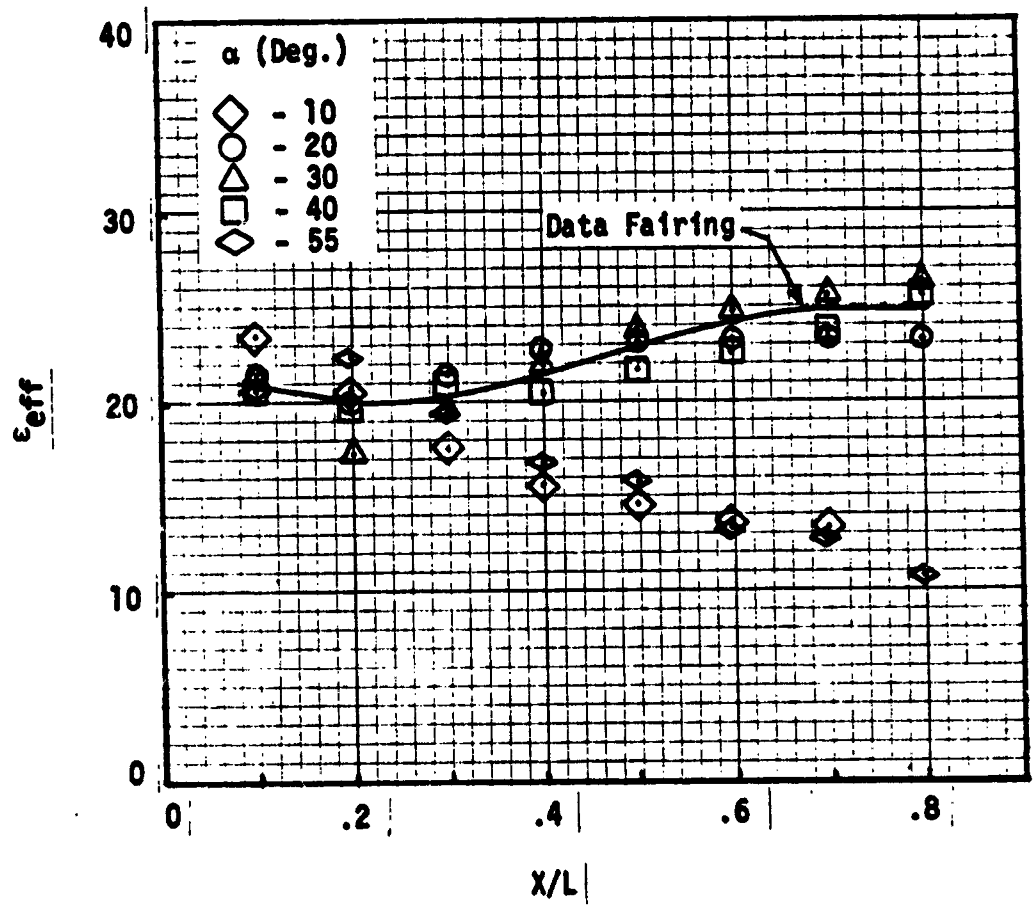
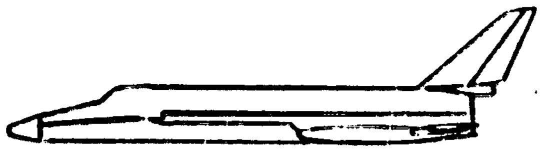


Fig. 7.14 Effective Sweep Angle Compliment For MSFC 437 Orbiter



Rockwell Orbiter (161B)

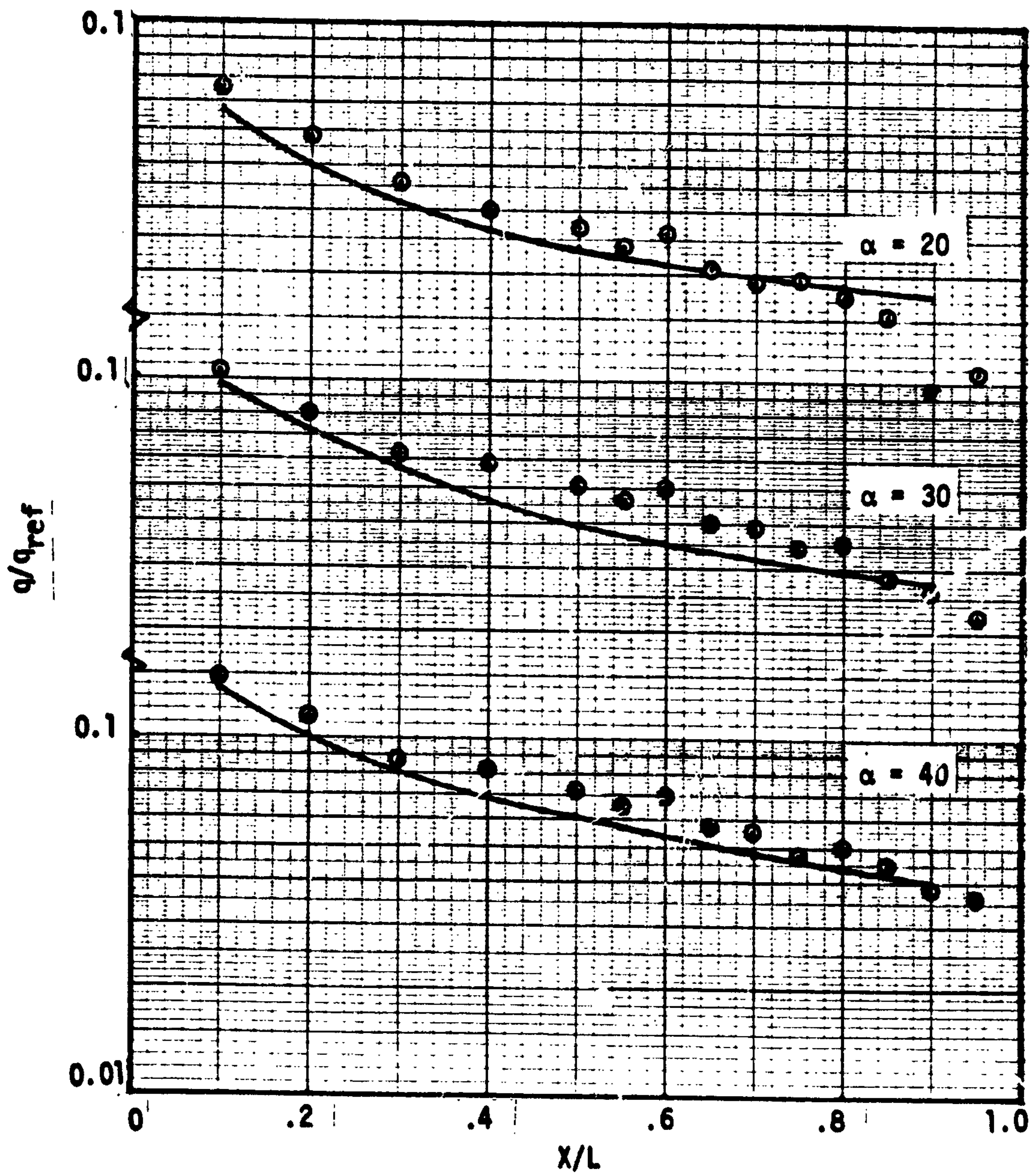
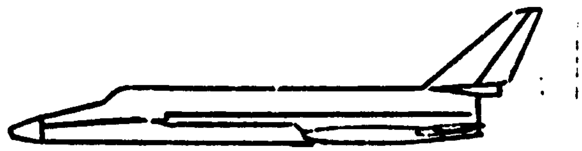


Fig. 7.15 LANMIN Theory Compared With Phase B Orbiter Configuration Bottom Centerline Laminar Data

ORIGINAL PAGE IS
OF POOR QUALITY

⊕ Data From Warmbrod et al (Ref. 9)
$M_\infty = 8.0$
$Re_\infty/ft = 3.75 \times 10^6/ft$
Grit on Nose

—— Theory
Tangent Cone Shock
Tangent Wedge Pressure
Spaulding-Chi
Von Karman
$\tau_m = .6667$
Equivalent Laminar Running Lengths



Rockwell Orbiter (161B)

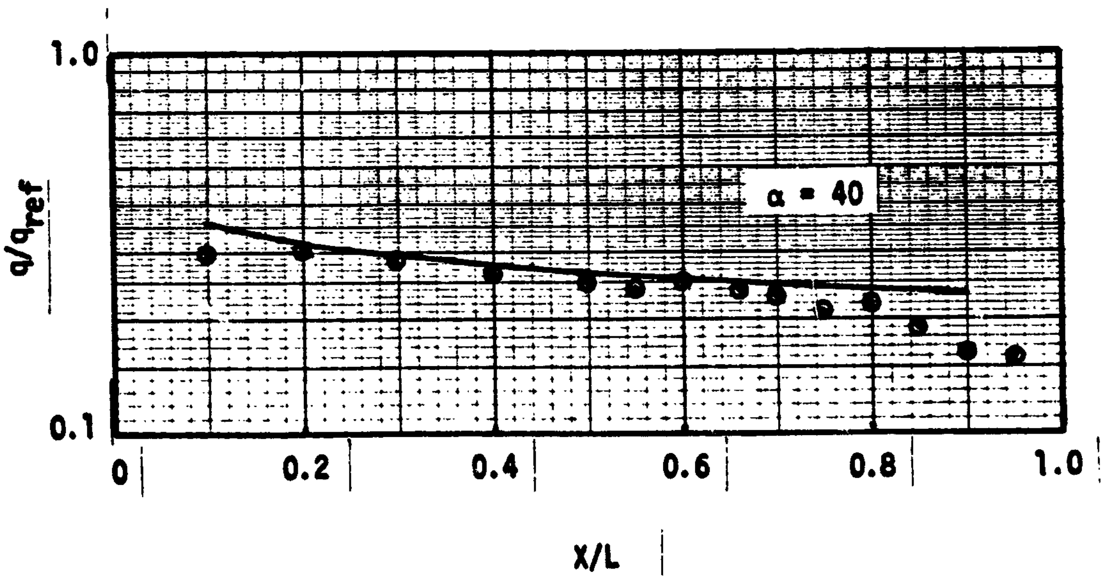


Fig. 7.16 LANMIN Theory Compared With Phase B Orbiter Configuration Bottom Centerline Turbulent Data

ORIGINAL PAGE IS
OF POOR QUALITY

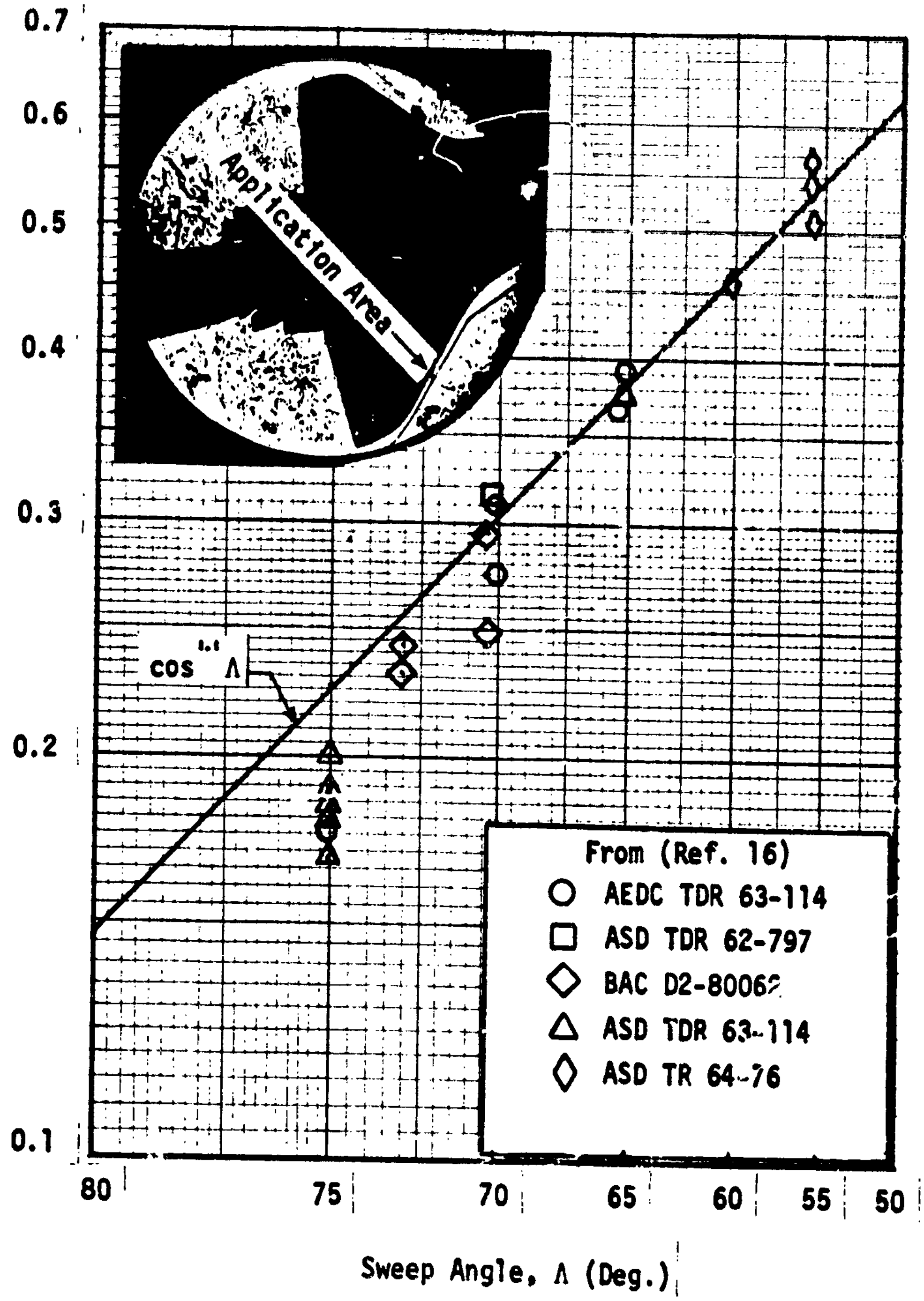


Fig. 7.17 Laminar Leading Edge Stagnation Line Heating Compared With the Heat Transfer Option 6 Method

Section VIII

REFERENCES

1. Hender, D.R., ''A Miniature Version of the JA70 Aerodynamic Heating Computer Program, H800 (MINIVER),'' McDonnell Douglas Astronautics Co., Huntington Beach, Calif., MDC Report G-0462, June 1970 (revised Jan. 1972).

2. Wilhite, A.W. and J.J. Rehder, ''AVID: A Design System For Technology Studies of Advanced Transportation Concepts,'' AIAA Paper 79-0872, AIAA/NASA Conference on Advanced Technology for Future Space Systems, Hampton, VA., May 8-11, 1979.

3. Roux, J.A. and D. Somogyi, ''A Review of NS1-MSFC Generalized Aerodynamic Heating Program (MINEQ),'' Northrop Services, Inc. TR-230-1202, June 1973.

4. Engel, C.D., ''MINIVER Computer Program Modifications for Rarefied Flow Aeroheating,'' RM 008-05, REMTECH, Inc., March 1973.

5. Praharaj, S.C., ''Undisturbed Heat Transfer Data Analysis on the Space Shuttle External Tank Barrel Section,'' AIAA Paper 78-41, AIAA 16th Aerospace Sciences Meeting, Huntsville, AL, January 16-18, 1978.

6. Praharaj, S.C., ''Undisturbed Flowfield and Turbulent Heat-transfer Analysis on the Space Shuttle External Tank,'' AIAA Paper 79-0378, AIAA 17th Aerospace Sciences Meeting, New Orleans, La., January 15-17, 1979.

7. Engel, C.D. and R.C. Lewis, ''Rough Wall Aerodynamic Heating Computer Program,'' RM 017-6, REMTECH, Inc., August 1977.

8. Bushnell, D.M., R.A. Jones, and J.K. Huffman, ''Heat Transfer and Pressure Distributions on Spherically Blunted 25° Half-Angle Cone at Mach 8 and Angles of Attack up to 90°,'' NASA TN D-4792, October 1968.

9. Warmbrod, J.D., W.R. Martindale, and R.K. Matthews, ''Heat Transfer Rate Measurements on North American Rockwell Orbiter (161B) at Nominal Mach Number of 8,'' DMS-DR-1177, Vol. III, AEDC, December 1971.

10. Eaves, R.H., T.D. Buchanan, and J.D. Warmbrod, ''Heat Transfer Investigation of the McDonnell-Douglas Delta Wing Orbiter at a Nominal Mach Number of 10.5,'' DMS-DR-1206, CR-120,024, Vol. 1, AEDC, May 1972.

11. Eaves, R.H., T.D. Buchanan, J.D. Warmbrod, and C.B. Johnson, ''Heat Transfer Investigation of Two Langley Research Center Delta Wing Configurations at a Mach Number of 10.5,'' DMS-DR-1224, CR-120, 036, Vol. 1, AEDC, March 1972.

12. Matthews, R.K., W.R. Martindale, and J.D. Warmbrod, ''Heat Transfer Rate Distribution on North American Rockwell Delta Wing Orbiter Determined by Phase Change Point Technique at Mach Number of 8,'' DMS-DR-1231, CR-120,048, Vol. 1, AEDC, March 1972

13. Nutt, K.W., G.L. Dommerman, and A.C. Mansfield, ''Test Results from NASA/Rockwell International Space Shuttle Orbiter Tests (OH-84B, IH-102, OH-105),'' AEDC-TSR-79-V42, August 1979.

14. Dunavant, J.C., ''Investigation of Heat Transfer and Pressures on Highly Swept Flat and Dihedraled Delta Wings at Mach Numbers of 6.8 and 9.6 and Angle of Attack to 90°,'' LaRC NASA TMX-688, June 1962.

15. Pond, J.E., and P.O. McCormick, ''Shuttle Reentry Aerodynamic Heating Test,'' Lockheed Missile and Space Company, Huntsville, AL, LMSC-HREC D225535, December 1971.

16. Newmann, R.D. and P.G. Renfroe, ''Evaluation of the Heat Transfer and Flow Fields About Highly Swept Delta Wings in Hypersonic Flow,'' Wright-Patterson Air Force Base, ASD TDR 64-76, April 1964.