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SUMMARY

This paper presents a closed-form expression for the weight function for a strip with a single edge crack. The expression is valid for relative crack lengths from zero to unity. It is based on the assumption that the shape of an opened edge crack can be approximated by a conic section. The results agree well with published values for weight functions, stress intensity factors, and crack mouth opening displacements.

INTRODUCTION

Bueckner's weight function concept (refs. 1 and 2) has proven to be a useful tool for elastic crack analysis. The weight function is a function of specimen (or structure) and crack configurations only and is independent of the load system. If an expression for the weight function is available, the stress intensity factor for any load system may be computed from the stress distribution in the uncracked body by simple integration. In presenting this concept Bueckner used the example of a strip with a single edge crack, which is a common test specimen configuration and which also has application to real-world structural problems. He presented (ref. 2) a simple expression for the weight function for this configuration which is useful for relative crack lengths from zero to one-half.

Rice (ref. 3) showed that weight functions can be calculated from crack surface displacements and stress intensity factors. Thus, if the stress intensity factor and the complete crack displacement solution are available for a given cracked-body configuration under one loading condition (say, simple tension), the weight function can be computed. This configuration and loading is usually called the reference solution.

At about the same time, this author (ref. 4) proposed that the opening shape of an edge crack could be approximated by a conic section. In the present paper, the conic section approximation is used to develop a closed-form expression for the weight function which is valid for all values of relative crack length, i.e., from zero to unity.

ANALYSIS

The generalized equation for the conic section (ref. 4) and the notation used are shown in figure 1. The conic section coefficients which are shown $(m = -1, 0, 10, \infty)$ correspond respectively to an ellipse, a parabola, a hyperbola, and a pair of straight lines. Using relations given in reference 4, the opening displacement (η) may be written as

$$\eta = \frac{2}{E'} \left(\frac{2}{\pi}\right)^{1/2} K_I \, x^{1/2} \left(1 + \frac{m}{2}\frac{x}{a}\right)^{1/2} \tag{1}$$

where E' is the effective elastic modulus, K_I is the opening-mode stress intensity factor, and m is the conic section coefficient. Rice (ref. 3) showed that the weight function (h) may be written as

$$h = \frac{E'}{2K_I} \frac{\partial \eta}{\partial a} \tag{2}$$

In comparing weight function formulations, note that Bueckner's form M(x) and Rice's form h(x) are related by

$$M(x) = \sqrt{2\pi}\, h(x)$$

Now a general expression for the weight function may be determined by differentiating equation (1). In doing so, note that the coordinate system in figure 1 is located at the crack tip. If the crack extends an amount da, the coordinate system moves a like amount (i.e., dx = da). After performing the required differentiation, Bueckner's reduced form of the weight function (N(x)) may be written as

$$N\left(\frac{x}{a}\right) = x^{1/2} M\left(\frac{x}{a}\right) = \left[2\frac{x}{a}\left(\frac{\lambda}{Y}\frac{dY}{d\lambda} + \frac{1}{2}\right) + 1\right]\left(1 + \frac{m}{2}\frac{x}{a}\right)^{1/2}$$
$$+ \frac{x}{a}\left[\frac{x}{a}\left(\frac{\lambda}{2}\frac{dm}{d\lambda} - \frac{m}{2}\right) + \frac{m}{2}\right]\left(1 + \frac{m}{2}\frac{x}{a}\right)^{-1/2} \tag{3}$$

where $\lambda = a/W$ is the relative crack length and $Y = K_I/\sigma\sqrt{a}$ is the dimensionless stress intensity factor coefficient. Thus the weight function can be computed if we know how the conic section and stress intensity factor coefficients vary with relative crack length for one form of loading, say, simple tension.

Values of the conic section coefficient are compuated as follows. Equation (5) of reference 4 gives the relationship between stress intensity factor, crack mouth displacement, and conic section coefficient as

$$E'\eta_0/\sigma a = 2Y[(2 + m)/\pi]^{1/2}$$

Thus if stress intensity and mouth displacement are known, the conic section coefficient can readily be determined. Since the tip radius of an opened crack is proportional to the square of the stress intensity factor, this approach is equivalent to fitting a conic section to the opened crack shape at the mouth and at the tip.

In order to determine values of the conic section coefficient, values for the stress intensity and crack mouth displacement were taken from the literature. Stallybrass (ref. 5) gives the stress intensity factor and crack opening displacement for an edge crack in a half-plane as

$$Y = 0.7930\sqrt{2\pi} = 1.988$$

$$E'\eta_0/\sigma a = 0.7930 \times 2\pi\sqrt{2/(\pi^2 - 4)} = 2.908$$

from which the conic section coefficient at $\lambda = 0$ is

$$m = (8 - \pi^2)/(\pi^2 - 4) = -0.3185$$

Keer and Freedman (ref. 6) give stress intensity and opening displacement values for various edge cracks in finite plates. However, their results at $\lambda = 0.85$ appear to be inaccurate and were ignored (their value for the stress intensity coefficient is $Y = 42.9$, whereas Benthem and Koiter (ref. 7) give $Y = 33.0$). Benthem and Koiter (ref. 7) treat the edge-dam in a half-plane, which corresponds to the major limit of the single edge crack. From their work, the limit values ($\lambda = 1$) of the conic section coefficient and its slope are

$$\lim_{\lambda \to 1}[m(1 - \lambda)] = 3.104$$

$$\lim_{\lambda \to 1}d[m(1 - \lambda)]/d\lambda = 8.034$$

Values of the conic section coefficient were computed from the results of references 5 to 7 using equation (5) of reference 4, which was given earlier.

A polynomial in terms of relative crack length was derived from these coefficients as follows. First, the conic section coefficients were put into finite form and a polynomial fit by the method of least squares. Then the polynomial coefficients were adjusted by trial and error to provide the correct magnitude and slope at each extreme. The resulting polynomial is

$$m(1 - \lambda) = -0.3185 + 0.3185\lambda + 1.3765\lambda^2 + 6.2475\lambda^3 - 8.82\lambda^4 + 4.3\lambda^5 \qquad (4)$$

The polynomial is everywhere finite, has the correct magnitudes and slopes at the extremities, is within 1 percent of the reference values, and is shown in figure 2.

Finally a wide-range expression for the stress intensity coefficient for tension is needed, and Benthem and Koiter's (ref. 6) should suffice. Since only tabular values are given in reference 6, the equation was rederived as

$$Y = \sqrt{\pi}(1 + 2\lambda)(1 - \lambda)^{-3/2}\left[21.006 + 87.065\lambda + \right.$$
$$\left. -(395.38 + 3618.5\lambda + 7584.88\lambda^2)^{1/2}\right] \qquad (5)$$

The term in brackets is the coefficient tabulated in reference 7 and agrees with the tabulations within 0.001. Now having wide-range expressions for the conic section and stress intensity coefficients, the calculation of weight functions is straightforward.

Once the weight function has been determined, the stress intensity factor for any loading can be determined by simple integration, as

$$K_I = \sqrt{2/\pi} \int_0^a p(x/a) \; N(x/a) \; x^{-1/2} \; dx \tag{6}$$

where $p(x/a)$ is the stress distribution over the crack location in the uncracked body.

RESULTS AND DISCUSSION

Weight function values computed from equations (3) to (5) and those given by Bueckner (refs. 1 and 2) are presented in table I. The conic-section weight functions are within 4 percent of Bueckner's for relative crack lengths of one-half and less. As a further check, stress intensity coefficients for tension and pure bending were computed by integrating the conic-section weight functions. These results are presented in table II, and are within 1.8 percent (tension) and 2.7 percent (bending) of those given by Benthem and Koiter (ref. 7) over the entire range of relative crack lengths from zero to unity. Crack mouth displacement coefficients are presented in table III and will be discussed later.

An alternate approach to the approximation of weight functions was presented by Petroski and Achenbach (ref. 8). They prescribe the crack opening displacement in the form

$$\eta = \frac{\sigma}{\sqrt{2} \; E'} \left[4 \frac{K_I}{\sigma \sqrt{\pi a}} a^{1/2} x^{1/2} + G(\lambda) \; a^{-1/2} x^{3/2} \right] \tag{7}$$

in which the term $G(\lambda)$ is determined by requiring that the stress intensity factor, when determined by integration as in equation (6), be identical to the value used in equation (7). In other words, equations (6) and (7) must be self-similar. The approximate weight function is then determined using equation (2). This approach is advantageous if crack mouth displacements are not available for the reference problem. However, if such displacements are available, the conic section approximation yields more accurate displacement coefficients, as might be expected. This can be seen in table III. The coefficients computed from equation (1) are within 0.7 percent of Keer and Freedman's (ref. 6) up to $\lambda = 0.65$ and within 2.1 percent at $\lambda = 0.80$. As mentioned earlier, the results in (ref. 6) for $\lambda = 0.85$ are probably inaccurate. For comparison, the interpolation equations of Srawley and Gross (ref. 9) and Tada (ref. 10) give crack mouth displacement coefficients of 150.1 and 149.8, respectively, at $\lambda = 0.85$.

Even though these results may be satisfactory, some further attention should be paid to the matter of crack opening shape. Consider Stallybrass's solution (ref. 5) for the edge crack in a half-plane, which was mentioned earlier. Note that the conic section coefficient is an exact value, independent of his numerical computations. This might lead one to expect that the crack opening shape is an exact conic section. However, this is not the case, as can be seen in figure 3. Here the displacements from the conic section

model are compared with Bueckner's (ref. 1) and the earlier results of Wigglesworth (ref. 11). Although they seem small in figure 3(a), the differences are significant, as can be seen in figure 3(b).* The conic-section displacements agree with Wigglesworth's at the crack tip and mouth, but are larger in between. They are smaller than Bueckner's at the tip and mouth but, again, are larger in between. Similar small but consistent differences can also be found in the weight functions themselves (see table I). The conic-section weight functions are almost everywhere larger than Bueckner's for $\lambda \leq 0.40$ and smaller for $\lambda \leq 0.45$. These small unidirectional errors do not average out when the weight function is integrated to determine a stress intensity factor. For example, the stress intensity coefficient at $\lambda = 0$, when calculated by integration, is 1.1413 (table II). This compares with the value 1.122 from which the conic section coefficient was calculated. The conic section model is obviously not self-similar.

In figure 3(b), the conic-section model and Petroski and Achenbach's model provide at best only a rough approximation of the crack profile. Both models are essentially two-parameter representations. It seems doubtful that any two-parameter representation could closely match the Bueckner or Wigglesworth crack profiles. Although more elaborate models could be constructed, the effort may not be warranted. Either method may be adequate for most engineering applications. The choice of method will most likely depend on the information available for the reference problem.

SUMMARY OF RESULTS

A closed-form expression is given for the weight function for a strip with a single edge crack. The expression is valid for relative crack lenghts from zero to unity. Computed weight functions are within 4 percent of reference values for relative crack lengths of one-half and less. Computed stress intensity coefficients are within 1.8 percent (tension) and 2.7 percent (bending) of reference values for relative crack lengths from zero to unity. Crack mouth opening displacement coefficients are within 2.1 percent of reference values for relative crack lengths of 0.8 and less.

This paper and (ref. 8) each present methods for developing approximate weight functions. Each assumes a form for the crack opening displacements. Neither method appears to have an overwhelming advantage, and the choice may depend on the problem to be solved and the available information.

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*The special form of the displacement coefficient used in figure 3(b) deserves comment. It can be shown that

$$(x/a)^{-1/2} \ (E'\eta/\sigma a)\Big|_{x/a=0} \ = \sqrt{8} \ K_I/\sigma\sqrt{\pi a}$$

Thus the form used in figure 3(b) is particularly useful since the intercept at $x/a = 0$ is proportional to the stress intensity factor and the intercept at $x/a = 1$ is the crack mouth displacement. In this way, two significant features of the opening shape may be clearly seen on the same plot.

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TABLE I. - WEIGHT FUNCTIONS FOR THE EDGE-CRACKED STRIP

(a) Bueckner (taken from second table in [2] except as noted).

a/W	x/a									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0[a]
	N(x/a)									
0.00[b]	1.0736	1.1407	1.2079	1.2797	1.3590	1.4468	1.5524	1.6433	1.7453	1.8424
.05	1.0659	1.1361	1.2115	1.2928	1.3806	1.4751	1.5756	1.6806	1.7874	1.9161
.10	1.0818	1.1688	1.2614	1.3601	1.4652	1.5768	1.6942	1.8161	1.9398	2.0697
.15	1.1061	1.2176	1.3348	1.4581	1.5881	1.7248	1.8679	2.0158	2.1656	2.3264
20	1.1390	1.2841	1.4358	1.5945	1.7606	1.9343	2.1151	2.3020	2.4934	2.6889
.25	1.1771	1.3623	1.5554	1.7569	1.9670	2.1859	2.4132	2.6477	2.8874	3.1637
.30	1.2252	1.4610	1.7077	1.9653	2.2338	2.5132	2.8033	3.1032	3.4107	3.7650
35	1.2830	1.5814	1.8949	2.2233	2.5665	2.9424	3.2962	3.6811	4.0772	4.5178
.40	1.3501	1.7224	2 1162	2.5306	2.9650	3.4190	3.8918	4.3820	4.8867	5.4635
.45	1.4348	1.9024	2.4012	2.9296	3.4864	4.0705	4.6809	5.3160	5.9737	6.6653
.50	1.5376	2.1237	2.7550	3.4288	4.1430	4.8956	5.6848	6.5086	7.3649	8.2149

[a] This column calculated from eqn. (2.5) of [1] except for $\lambda = 0$.
[b] This row calculated from eqns. (4.32) and (4.35) of [2].

(b) This report, Eqs. (3) to (5).

a/W	x/a									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
	N(x/a)									
0.00	1.0768	1.1548	1.2343	1.3152	1.3975	1.4815	1.5670	1.6542	1.7431	1.8338
.05	1.0836	1.1686	1.2549	1.3426[c]	1.4318	1.5225	1.6147	1.7087	1.8043	1.9018
.10	1.1004	1.2022	1.3056	1.4104	1.5169	1.6250	1.7349	1.8465	1.9600	2.0755
.15	1.1242	1.2505	1.3787	1.5090	1.6415	1.7762	1.9132	2.0526	2.1943	2.3387
.20	1.1452	1.2924	1.4417	1.5932	1.7470	1.9033	2.0620	2.2233	2.3873	2.6949
.25	1.1911	1.3875	1.5894	1.7967	2.0095	2.2279	2.4519	2.6815	2.9169	3.1581
.30	1.2352	1.4793	1.7322	1.9440	2.2646	2.5440	2.8323	3.1293	3.4352	3.7499
.35	1.2882	1.5906	1.9069	2.2372	2.5811	2.9385	3.3093	3.6932	4.0903	4.5002
.40	1.3521	1.7262	2.1218	2.5382	2.9752	3.4320	3.9085	4.4040	4.9183	5.4509
.45	1.4300	1.8933	2.3885	2.9144	3.4701	4.0545	4.6667	5.3058	5.9712	6.6620
.50	1.5263	2.1019	2.7242	3.3910	4.1001	4.8497	5.6382	6.4641	7.3261	8.2230

[c] Largest difference, +3.85 percent.

TABLE II. - STRESS INTENSITY FACTOR COEFFICIENTS

a/W	Uniform tension, $(K_I/\sigma\sqrt{\pi a})(1 - \lambda)^{3/2}/(1 + 2\lambda)$		Pure bending, $(K_I/\sigma\sqrt{\pi a})(1 - \lambda)^{3/2}$	
	Ref. [7]	This report	Ref. [7]	This report
0.00	1.122	1.1413	1.122	1.1413
.05	.961	.9780	.990	1.0107
.10	.849	.8631	.870	.9122
.15	.767	.7768	.811	.8328
.20	.703	.7096	.747	.7667
.25	.653	.6558	.693	.7108
.30	.611	.6121	.649	.6633
.35	.577	.5761	.610	.6224
.40	.548	.5457	.577	.5869
.45	.523	.5205	.549	.5562
.50	.501	.4984	.523	.5290
.55	.482	.4790	.501	.5047
.60	.464	.4618	.481	.4830
.65	.449	.4466	.463	.4635
.70	.435	.4329	.446	.4461
.75	.423	.4207	.432	.4306
.80	.411	.4097	.418	.4167
.85	.401	.3999	.406	.4043
.90	.391	.3908	.394	.3932
.95	.382	.3823	.384	.3831
1.00	.374		.374	

TABLE III. - CRACK MOUTH
DISPLACEMENT COEFFICIENTS

a/W	$E'\eta_0/\sigma a$		
	Ref. [6]	Ref. [8]	Eq. (1)
0.05	2.967	2.878	2.965
.10	3.107	3.011	3.116
.15	3.336	3.238	3.354
.20	3.667	3.562	3.689
.30	4.707	4.607	4.739
.40	6.549	6.468	6.589
.50	9.925	9.938	9.972
.60	16.68	16.92	16.70
.65	22.73	23.29	22.69
.80	81.26	82.15	79.55
.85	168.9	184.6	148.7

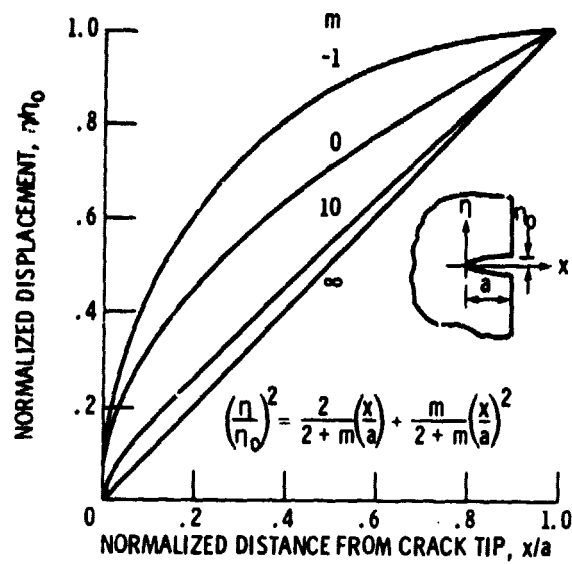$$\left(\frac{\eta}{\eta_0}\right)^2 = \frac{2}{2+m}\left(\frac{x}{a}\right) + \frac{m}{2+m}\left(\frac{x}{a}\right)^2$$
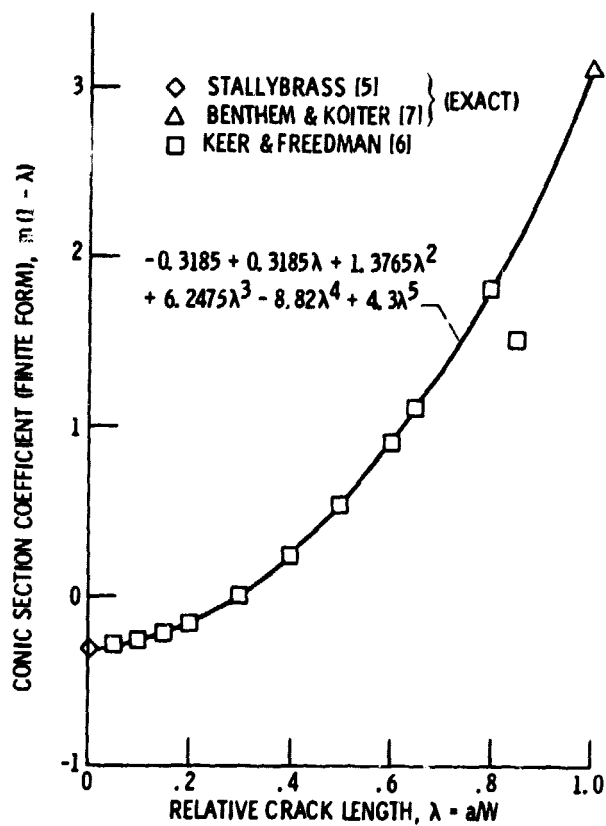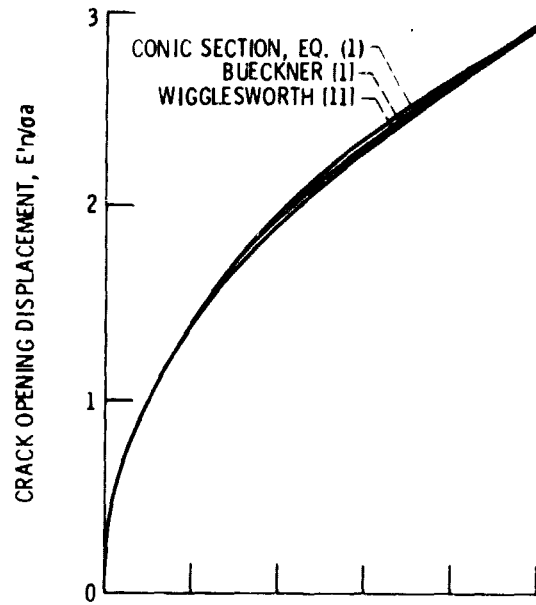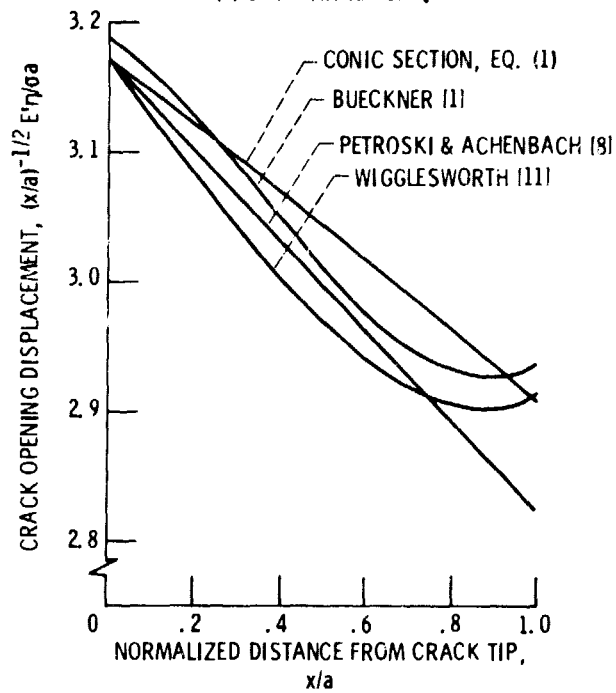
Figure 1. - Conic sections.

Figure 2. - Conic section coefficient for tension load.

(a) Conventional form.

(b) Special form.

Figure 3. - Crack opening displacement for
the edge crack in a half-plane.