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# Luminosity Enhancement in Relativistic Jets and Altered Luminosity Functions for Beamed Objects

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## Abstract

Due to relativistic effects, the observed emission from relativistic jets is quite different from the rest frame emission. We discuss systematic differences between the observed and intrinsic intensities of sources in which jet phenomena are occurring. Assuming that jets have a power law luminosity function of slope  $B$ , we calculate the observed luminosity distribution as a function of the velocity of the jet, the spectral index of the rest frame emission, and the range of angles of the jets relative to our line of sight. The result is well-approximated by two power laws, the higher luminosity end having the original power law index  $B$  and the lower luminosity end having a flattened exponent independent of  $B$  and only slightly greater than 1. We then investigate a model consisting of beamed emission from a jet and unbeamed emission from a stationary central component. The luminosity functions for these two-component sources are calculated for two ranges of angles. For sources in which beaming is important, the luminosity function is much flatter. Because of this, the relative numbers of "beamed" and "unbeamed" sources detected on the sky depend strongly on the luminosity at which the comparison is made.

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## I. Introduction

Relativistic jets emanating from the cores of compact extragalactic objects have been inferred from high resolution radio maps, superluminal expansion, and brightness temperature limits. Roughly 70% of the VLA- mapped radio sources have jets, some of which may be relativistic (Bridle 1982). If these relativistic jet phenomena are common, the relativistic motion of the emitting plasma has a tremendous effect on the observed emission and the consequences for observational work are striking. Specifically, the observed luminosity of a relativistically moving source may be considerably brighter or considerably dimmer than the luminosity emitted in the rest frame, depending on the direction of the motion with respect to the observer. Taking this effect into account is important in the derivation of luminosity functions, in the calculation of energy conversion efficiencies, and in any discussion of the physics of sources with relativistic jets.

This paper investigates the difference between observed and intrinsic luminosity functions for sources in which relativistic jets are important. A two-component model, consisting of a jet plus a stationary component, is considered. When the jet points within a certain angle,  $\theta_c$ , of the line-of-sight, its relativistically enhanced emission will dominate; outside of this angle, the jet is decreasingly important. We calculate luminosity functions for these two-component sources for two ranges of jet angles:  $0^\circ < \theta < \theta_c$ , and  $\theta_c < \theta < 90^\circ$ . The results are applicable to any frequency band, assuming only a power law continuum with energy index  $\alpha$ . (For a more complicated spectral shape, the power law approximation will still be valid for a sufficiently narrow bandwidth. The implications for sources with complex spectra are discussed in §III.) A forthcoming paper, Urry and Mushotzky (1984), discusses the relevance of the quite general result presented here to BL Lacertae objects, in which the jet may be the distinguishing characteristic. In §II of the present work

we describe our assumptions and outline the calculation. In §III, results for representative parameters are shown and the important features are enumerated. In §IV, we summarize our findings and discuss the implications.

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## II. Luminosity Enhancement

The observed luminosity,  $L$ , of a relativistic jet is related to its emitted luminosity,  $\mathcal{L}$ , via

$$L = \delta^p \mathcal{L}, \quad (1)$$

where  $\delta$ , the kinematic Doppler factor for the jet, is defined by  $\delta = (\gamma(1 - \beta \cos \theta))^{-1}$ ,  $\beta$ =velocity in units of the speed of light,  $\gamma = (1 - \beta^2)^{-1/2}$  is the Lorentz factor, and  $\theta$  is the angle between the velocity vector and the line-of-sight.

The exponent  $p$  depends on assumptions about the spectrum of the jet emission (Ryle and Longair 1967), the structure of the jet (Scheuer and Readhead 1979), and the frequencies at which the comparisons are made. For the spectrum we assume a power law of the form  $F_\nu \sim \nu^{-\alpha}$ . Then if  $L$  refers to observed monochromatic luminosity ( $\text{ergs s}^{-1} \text{ Hz}^{-1}$ ) at frequency  $\nu$  compared to  $\mathcal{L}(\nu)$ ,  $p=3+\alpha$ . Equivalently,  $L(\nu_0) = \delta^3 \mathcal{L}(\nu_e)$ , where the frequency of observation  $\nu_0$  is  $\delta$  times the emitted frequency  $\nu_e$ . Two powers of the enhancement are due to aberration: the emission is beamed in the forward direction due to the relativistic motion of the jet. Another power comes from contraction of the time interval by a factor  $\delta$  so that the number of photons

per unit time is enhanced by  $\delta$ . The remaining power of  $\alpha$  present in the comparison at a fixed frequency is due to the actual blue-shifting of the photons. (The photons observed at frequency  $\nu$  were emitted at frequency  $\delta^{-1}\nu$ , so that there were a factor  $\delta^\alpha$  more than were emitted at frequency  $\nu$ .) If instead  $L$  refers to integrated luminosity ( $\text{ergs s}^{-1}$ ) over corresponding bands, then  $p=4$ , since the blue-shifting of the photons changes the observed bandwidth by a factor  $\delta$ . Scheuer and Readhead (1979) noted that if discrete components are ejected, in pairs, the observed lifetimes of approaching and receding components differ, with more of the receding blobs being visible at any one time. Under these conditions, and with reference to flux densities or monochromatic luminosities at a fixed frequency,  $p=2+\alpha$ . In practice, the receding jets are so underluminous due to relativistic effects that in the context of a two-component model (jet plus stationary source), they may be ignored. Calculations for  $90^\circ \leq \theta \leq 180^\circ$  show that even if the beamed and unbeamed components are comparable in intrinsic luminosity, the receding beamed component will be less than one percent of the total luminosity for likely values of  $\gamma$ .

The larger the value of  $p$ , the greater the effect on the observed luminosity. Thus, sources with intrinsically steep spectrum emission are more affected than those with flat spectra. If the source spectrum is more complex than a single power law, its shape remains unchanged but (on a plot of flux density versus frequency, and with  $\delta > 1$ ) it is simply shifted up by a factor of  $\delta^3$  and to the right by a factor of  $\delta$ . Observations in a given band therefore depend crucially on what is emitted at frequencies a factor of  $\delta$  lower, so that complex spectra can give rise to serious complications. A case in point involves X-ray observations of BL Lac objects, which may be dominated by relativistically beamed emission (Weistrop et al. 1981; Urry and Mushotzky 1982; Worrall and Bruhweiler 1982; Urry et al. 1982a,b; Worrall et al. 1984). The few BL Lac X-ray spectra that have reasonable statistics suggest there are two components to the spectrum, a steep one at low energies and a flat one at

higher energies (Mushotzky et al. 1978; Riegler, Agrawal, and Mushotzky 1979; Worrall et al. 1981; Urry and Mushotzky 1982; Urry et al. 1982b). The location of this steep-to-flat break seems to occur anywhere from the extreme ultraviolet to the medium X-ray ( $\sim 10$  keV) range. Thus, if the X-rays originate in a relativistic beam, the degree of enhancement in the X-ray band can be determined only with sufficient spectral information. In this paper we have ignored the spectral complications and simply assumed the emission is described by a simple power law.

We begin with a large number of sources, each having a pair of oppositely directed jets so that each source has an approaching jet. Our basic assumption is that these jets are randomly oriented on the sky, i.e. the probability distribution in  $\theta$  is uniform ( $\propto d(\cos\theta)$ ). For simplicity we assume that all jets have the same fixed Lorentz factor. While not exact, this is a good approximation if the range in  $\gamma$  is small. The results presented below used a  $\gamma$  of 5, as suggested by the results of Orr and Browne (1982), and likely values are in the range 2 to 10 (Blandford, McKee, and Rees 1977, and references therein; Cohen et al. 1977; Marscher and Broderick 1981; Sherwood et al. 1983; Unwin et al. 1983; Readhead et al. 1983; Madejski and Schwartz 1983; Henrikson, Marshall, and Mushotzky 1983). For  $2 < \gamma < 10$  the shape of the observed luminosity function is qualitatively the same: the normalization is proportional to  $\gamma^{-1}$  and the luminosity at which the two power laws join is proportional to  $\gamma^p$  (see Eqn. 7 below). Thus, introducing a distribution in  $\gamma$  would tend to blur the sharp two-power law form of the resulting observed luminosity function, smoothing the break between the two power laws and spreading it out over a range of luminosities. The true distribution of Lorentz factors in extragalactic jets is unknown, but our results for the extreme values of  $\gamma$  will describe the envelope of a more realistic calculation.

Since the Doppler factor,  $\delta$ , is a function of  $\theta$  and  $\gamma$  only and the latter is fixed, the probability distribution in  $\delta$  is easily derived from  $P(\theta)$ :  $P(\delta) =$

$(\beta\gamma\delta^2)^{-1}$ . This is twice the probability calculated by Schwartz, Madejski, and Ku (1981), corresponding to the picture that jets come in pairs. Given a particular emitted luminosity  $\mathcal{L}$ , we then know the probability of observing luminosity  $L = \delta\mathcal{L}$ . That is,

$$P(L|\mathcal{L}) = P(\delta) \frac{d\delta}{dL} = \frac{1}{\beta\gamma p} \mathcal{L}^{1/p} L^{-(p+1)/p} . \quad (2)$$

For any form of intrinsic differential luminosity function  $\phi_e(\mathcal{L})$ , the observed differential luminosity function will be given by

$$\phi_o(L) = \int d\mathcal{L} \phi_e(\mathcal{L}) P(L|\mathcal{L}) . \quad (3)$$

We have restricted ourselves to simple luminosity functions of the form

$$\phi_e(\mathcal{L}) = \begin{cases} K \mathcal{L}^{-B} , & \mathcal{L}_1 < \mathcal{L} < \mathcal{L}_2 \\ 0 , & \mathcal{L} < \mathcal{L}_1 \text{ or } \mathcal{L} > \mathcal{L}_2 . \end{cases} \quad (4)$$

The sharp cutoffs are almost certainly unrealistic but they conform to our practice of using the simplest picture that is still consistent with observation, in this case that luminosity functions must turn over at low luminosities (Piccinotti et al. 1982; Elvis, Soltan, and Keel 1983; Gaston 1983).

Combining equations (2) and (4) with equation (3), we can perform the integral over  $\mathcal{L}$  to obtain the transformed luminosity function,  $\phi_o(L)$ . It will be useful to consider the luminosity function for various fixed ranges of  $\theta$ . Let  $\theta_{\min}$  and  $\theta_{\max}$  represent the limits of the range under consideration. Then  $\delta$  has minimum and



maximum values corresponding to  $\theta_{\max}$  and  $\theta_{\min}$  respectively. Clearly  $\delta_{\max} = (\gamma(1 - \beta \cos \theta_{\min}))^{-1}$  and  $\delta_{\min} = (\gamma(1 - \beta \cos \theta_{\max}))^{-1}$ . (At the limits  $0^\circ$  and  $90^\circ$ ,  $\delta_{\max} = 2\gamma$  and  $\delta_{\min} = \gamma^{-1}$ .) Thus, only part of the  $L$ - $\mathcal{L}$  plane is accessible, as illustrated in Figure 1. The limits of the integral depend on the value of  $L$ , for which there are three regions. These are indicated in Fig. 1 by the dashed lines at  $L_{\min} = \delta_{\min}^p \mathcal{L}_1$ ,  $L_3 = \delta_{\min}^p \mathcal{L}_2$ ,  $L_4 = \delta_{\max}^p \mathcal{L}_1$ , and  $L_{\max} = \delta_{\max}^p \mathcal{L}_2$ . The formal result for the observed luminosity function has two forms, depending on whether  $L_3$  is less than or greater than  $L_4$ . With the abbreviation  $C = B - 1/p - 1$ , the results are:

when  $L_3 < L_4$ ,

$$\phi_0(L) = \frac{K}{\beta \gamma p C} \begin{cases} 0, & L < L_{\min} \\ \mathcal{L}_1^{-C} L^{-(p+1)/p} - \delta_{\min}^{pC} L^{-B}, & L_{\min} < L < L_3 \\ L^{-(p+1)/p} (\mathcal{L}_1^{-C} - \mathcal{L}_2^{-C}), & L_3 < L < L_4 \\ \delta_{\max}^{pC} L^{-B} - \mathcal{L}_2^{-C} L^{-(p+1)/p}, & L_4 < L < L_{\max} \\ 0, & L > L_{\max} \end{cases} \quad (5)$$

and when  $L_3 > L_4$ ,

$$\phi_0(L) = \frac{K}{\beta \gamma p C} \begin{cases} 0, & L < L_{\min} \\ \mathcal{L}_1^{-C} L^{-(p+1)/p} - \delta_{\min}^{pC} L^{-B}, & L_{\min} < L < L_4 \\ L^{-B} (\delta_{\max}^{pC} - \delta_{\min}^{pC}), & L_4 < L < L_3 \\ \delta_{\max}^{pC} L^{-B} - \mathcal{L}_2^{-C} L^{-(p+1)/p}, & L_3 < L < L_{\max} \\ 0, & L > L_{\max} \end{cases} \quad (6)$$

In each line, the first term dominates the second, except at the limits. Neglecting the second terms, both cases reduce to the same approximation:

$$\phi_0(L) = \frac{K}{\beta \gamma p C} \begin{cases} 0, & L < L_{\min} \\ \mathcal{L}_1^{-C} L^{-(p+1)/p}, & L_{\min} < L < L_4 \\ \delta_{\max}^{pC} L^{-B}, & L_4 < L < L_{\max} \\ 0, & L > L_{\max} \end{cases} \quad (7)$$

Thus the beamed object luminosity function is a broken power law. At the low luminosity end the index is  $(p+1)/p$ , which is between 1 and 1.5 for reasonable values of  $p$ . All of the objects in this range have the lowest intrinsic luminosity  $\mathcal{L}_1$  and are Doppler boosted by the full range of Doppler factors, so that the observed luminosity function  $\phi_0(L)$  simply follows the probability distribution of Eqn. (2). Toward the high luminosity end there is a break at  $L = L_4 = \delta_{\max}^2 \mathcal{L}_1$  to a (steeper)

power law with the original index  $B$ . This occurs because the large number of sources at the low luminosity cutoff cannot be boosted any higher than  $L_4$ , so the number of sources observed above that luminosity falls off exactly as the number of sources existing above  $L_1$ , although the normalization is different. If the lower cutoff is not sharp but a gradual rollover, the break at  $L_4$  will be similarly gradual. The strong dependence of the result on the low luminosity cutoff is explicit in Eqn. (7); the upper cutoff plays no role at all, except for determining  $L_{\max}$ . In principle, it is possible that the moderate luminosity beamed sources from the flat part of the observed luminosity function may arise from a population of sources intrinsically too faint to have been seen when unbeamed.

### III. Results

A representative calculation is shown in the two panels of Figure 2. We assumed that the jet was a continuous stream of outward-moving particles (Konigl 1981, Marscher 1980), and we considered luminosity comparisons made at a fixed frequency, such that  $p=3+\alpha$ . The functions plotted in Fig. 2 used the value  $p=4$ , corresponding to a spectral index  $\alpha=1$ . Radio spectra are often flatter (Scheuer and Williams 1968, Kellerman and Pauliny-Toth 1969, Condon et al. 1978); optical, ultraviolet, and X-ray spectra have a wide range of spectral indices (Richstone and Schmidt 1980, Mushotzky et al. 1980, Worrall et al. 1981, Urry et al. 1982a, Bregman et al. 1982) but  $\alpha=1$  is a good representative value. We assumed all of the luminosity was in the jet, and the value used for the Lorentz factor was  $\gamma=5$ , as suggested by the results of Orr and Browne (1982). Values for  $\theta$  were allowed to range between  $0^\circ$  and  $90^\circ$

since we are neglecting radiation from jets which point away. We defined a critical angle  $\theta_c$  such that  $\delta(\gamma, \theta_c) = 1$ . With  $\gamma = 5$ ,  $\theta_c$  is  $\sim 35^\circ 3$ .

In Figure 2 we have plotted  $\log [L\phi_0(L)]$  versus  $\log L$ . The solid line in the top panel represents the intrinsic luminosity function used (Eqn. 4), with slope  $B=2.75$  and cutoffs  $\mathcal{L}_1=1$  and  $\mathcal{L}_2=10^4$ . (The units are arbitrary but a tic mark on either axis represents a decade.) The dash-dot line in Fig. 2a shows the observed luminosity function as it has been altered by the effects of relativistic beaming. All angles are included,  $0^\circ < \theta < 90^\circ$ , corresponding to Eqn. 7 with  $\delta_{\min} = \delta(90^\circ)$  and  $\delta_{\max} = \delta(0^\circ)$ . In Fig. 2b the objects have been separated according to their orientation angle, with the dotted curve representing the luminosity function for jets with angles  $35^\circ 3 < \theta < 90^\circ$  and the dashed curve representing jets with angles  $0^\circ < \theta < 35^\circ 3$ . As expected, small orientation angles correspond to high observed luminosities.

As discussed above, we do not expect all of the luminosity of a source to arise in a jet. Instead, let the total luminosity of a source  $L_T$  be composed of an unbeamed part  $\mathcal{L}_u$  and the jet luminosity  $L_j$ . The simplest assumption we can make is that the intrinsic luminosity of the jet is some fixed fraction  $f$  of the unbeamed luminosity, i.e.,  $\mathcal{L}_j = f\mathcal{L}_u$ . Then

$$\begin{aligned} L_T &= \mathcal{L}_u + L_j \\ &= (1 + f\delta^p) \mathcal{L}_u. \end{aligned} \quad (8)$$

The probability  $P(L|\mathcal{L})$ , where  $L=L_T$  refers to the total luminosity and  $\mathcal{L}=\mathcal{L}_u$  refers to the unbeamed luminosity, is again derived from  $P(\delta)$  according to Eqn. (2), with the result

$$P(L|\mathcal{L}) = \frac{1}{B\gamma^p} f^{1/p} \mathcal{L}^{-1} \left( \frac{L}{\mathcal{L}} - 1 \right)^{-(p+1)/p}. \quad (9)$$

The integral in Eqn. (3) is now done numerically and the results are shown in Figure 3. Once again the luminosity functions have been separated according to orientation angle. In this case  $\theta_c$  was defined as the angle at which the unbeamed luminosity

was equal to the beamed luminosity of the jet. That is,  $f\delta^p(\theta_c)$  is 1, or  $\theta_c = \arccos(\beta^{-1} (1 - \gamma^{-1}f^{1/p}))$ . Thus, the critical angle changes as the fraction  $f$  is varied.

Figure 3 illustrates the results for this two-component model, using the same intrinsic luminosity function as in Figure 2. For each value of  $f$  the sources are divided into two classes: the jet-dominated sources with  $0^\circ < \theta < \theta_c$  and the non-jet-dominated objects with  $\theta_c < \theta < 90^\circ$ . The dashed curves in Fig. 3 represent the luminosity functions for jet-dominated sources. The values of  $f$  shown are 0.001, 0.01, 0.1 and 1.0, for which the corresponding critical angles are  $10^\circ 2'$ ,  $17^\circ 0'$ ,  $25^\circ 1'$ , and  $35^\circ 3'$ , respectively. The solid curves represent the luminosity functions for the non-jet-dominated sources. In these sources the jets point outside  $\theta_c$ , so that relativistic effects actually diminish the observed jet intensities. The four curves are nearly identical, lying one on top of the other, and are effectively the same as the unbeamed, intrinsic luminosity function assumed. This is the case both because the jet luminosity is insignificant when  $f\delta^p$  is less than unity, and because only a small number of objects ( $\sim \pi\theta_c^2/4\pi$ ) are jet-dominated and are subtracted from the initial population.

#### IV. Discussion and Conclusions

We have described the impact of relativistic enhancement on measurements of the luminosity function for objects with relativistic jets. We divide such sources into two classes: those which are jet-dominated and those in which the jet contributes negligibly to the total luminosity of the source. Let the latter objects be called

parents, in reference to the parent population which gives rise to the class of jet-dominated objects. The main results of our study are illustrated in Figure 3. First, the jet-dominated objects tend to be more luminous than the parents, although there are fewer of them overall.

Second, there are luminosity ranges within which the numbers of jet-dominated objects are comparable to or even greater than the numbers of parents. For example, in Figure 3 the numbers of jet-dominated objects and parent objects for  $f=1.0$ ,  $0.1$ , and  $0.01$  are roughly equal at  $L=8.6 \mathcal{L}_1$ ,  $L=13 \mathcal{L}_1$ , and  $L=20 \mathcal{L}_1$ , respectively; for  $f=0.001$ , there are more parent objects at all luminosities below  $\mathcal{L}_2$ . This is in marked contrast to the naive expectation that because  $\theta < \theta_c$  only infrequently, we should see far fewer beamed objects than parents irrespective of luminosity (or  $f$  for that matter).

Third, over a large range in luminosity, the jet-dominated objects will have a flat luminosity function, generally much flatter than is typical for suggested parent populations. The flat slope results from the many objects at the lowest luminosity being boosted by a range of Doppler factors. Thus we are making a specific prediction: if a given class of object consists of sources with observed emission primarily from relativistic jets pointed at the Earth, that class should have a flatter luminosity function than the class of similar sources with jets pointed in other directions. This may explain the apparently flat slope of the X-ray luminosity function for BL Lac objects (Schwartz and Ku 1983) compared to the slope for any other class of extragalactic object (Piccinotti et al. 1982). Because of the flattening of the luminosity function for jet-dominated objects, the ratio of the volume density of the jet-dominated objects to the volume density of the parents varies with observed luminosity. Estimates of the required volume density of the parent population giving rise to BL Lacertae objects have not taken this effect into account (Schwartz and Ku 1983, Browne 1983).

Jets are present in extragalactic objects. If the jets are relativistic, the proper treatment of the luminosity enhancement effect is important. If 3C273, for example, exhibits superluminal expansion because of a directed jet, (Cohen et al. 1971, 1977, 1979; Sherwood et al. 1983) it may be much less luminous intrinsically than observed, at least in the radio, depending on the value of  $f$  for the source. If the jet contribution to the total luminosity is significant, one should not expect to see a few dozen sources as bright as 3C273 in the same co-moving volume as their jets will likely be pointing away. Finally, if this beaming picture is correct, the highest luminosity objects observed should be those which are dominated by a relativistic jet. These should tend to have strong nonthermal (jet-dominated) continua, and to be highly variable, highly polarized, and possibly superluminal.

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## Figure Captions

Figure 1. Observed luminosity  $L$  versus emitted luminosity  $\mathcal{L}$ .

$\mathcal{L}_1$  and  $\mathcal{L}_2$  are the cutoffs for the rest frame luminosity function and  $L_{\min}$  and  $L_{\max}$  are the resulting cutoffs in the observed luminosity function. The dashed lines indicate the ranges for  $L$ , which are delimited by the values  $L_{\min} = \delta_{\min}^p \mathcal{L}_1$ ,  $L_3 = \delta_{\min}^p \mathcal{L}_2$ ,  $L_4 = \delta_{\max}^p \mathcal{L}_1$ , and  $L_{\max} = \delta_{\max}^p \mathcal{L}_2$ . The case  $L_3 < L_4$  is illustrated. In the two-component model, where the intrinsic jet luminosity is  $f\mathcal{L}$  if the unbeamed luminosity is  $\mathcal{L}$ , the expression  $(1+f\delta_i^p)$  must be substituted for  $\delta_i^p$ , with  $i=\min$  or  $\max$ .

Figure 2. Luminosity Functions for Jet Component.

(a) The solid line represents  $\mathcal{L}\phi_e(\mathcal{L})$ , where  $\phi_e(\mathcal{L})$ , the intrinsic differential luminosity function assumed, is a power law given by Eqn. (4) with index  $B=2.75$  and with luminosity cutoffs  $\mathcal{L}_1=1$  and  $\mathcal{L}_2=10^4$ . The units are arbitrary. The dash-dot line is the observed luminosity function due to relativistic beaming with  $\gamma=5$  and  $p=4$ , for sources at all angles  $0^\circ < \theta < 90^\circ$ .

(b) The observed luminosity functions for beamed sources are separated into two groups according to jet orientation angle. Dotted line: jets with angles  $35.3^\circ < \theta < 90^\circ$ . Dashed line: jets with angles  $0^\circ < \theta < 35.3^\circ$ . The sum of these two lines is given by the dash-dot line in panel (a).

Figure 3. Total Luminosity Functions for Two-Component Model with Beamed and Unbeamed Emission.

The observed luminosity functions for sources with total luminosity comprised of the



unbeamed luminosity  $\mathcal{L}$  and the beamed (jet) luminosity  $f\delta^p\mathcal{L}$ . These sources have been divided into two classes: those dominated by unbeamed emission as defined by  $\theta_c < \theta < 90^\circ$  (solid lines), and those dominated by beamed emission from a jet with  $0^\circ < \theta < \theta_c$  (dashed lines). Four values of  $f$  were used, as indicated in the figure. The curves for non-jet-dominated sources are virtually unaffected by the value of  $f$ , and depart from a single line only at the endpoints.

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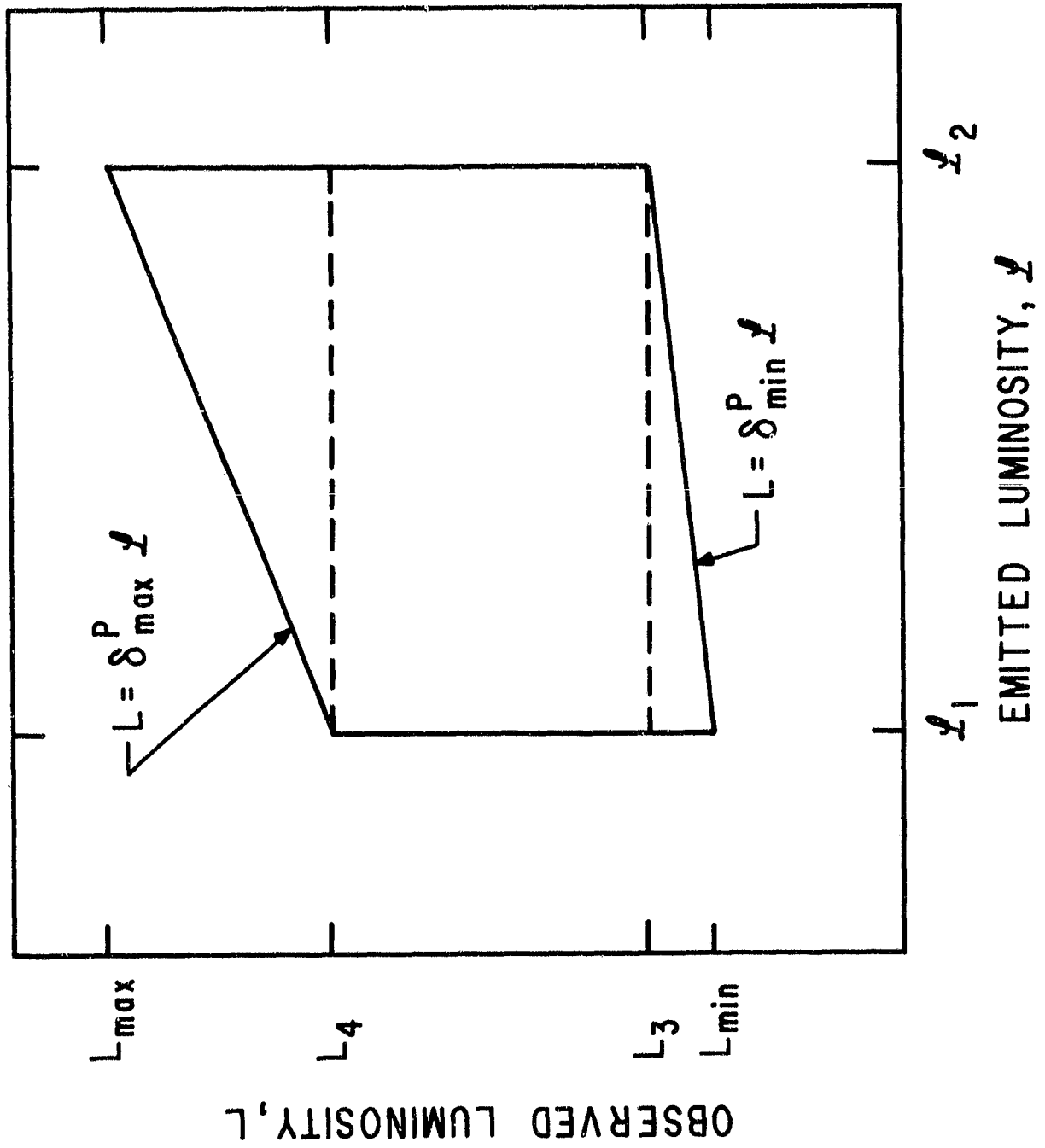
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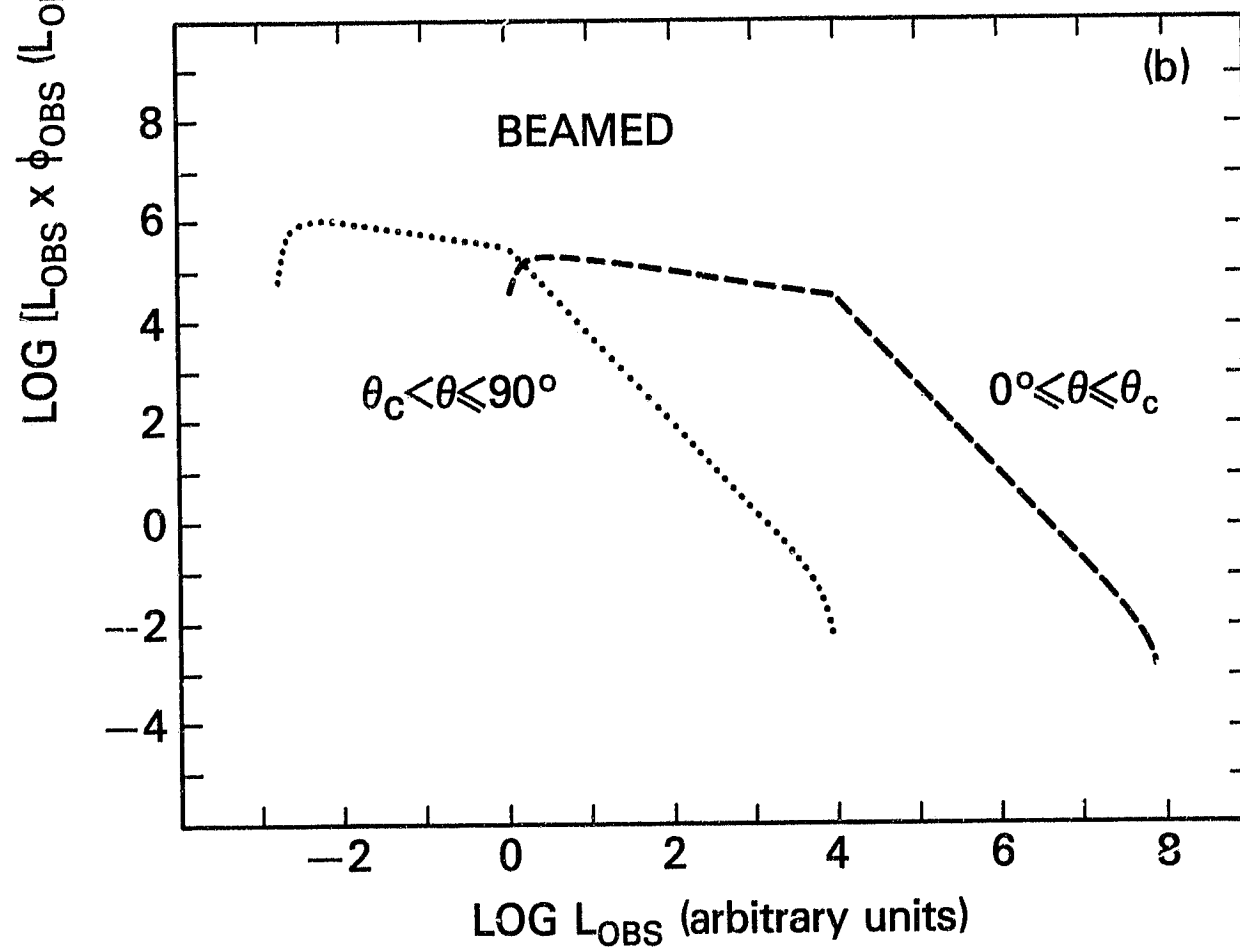
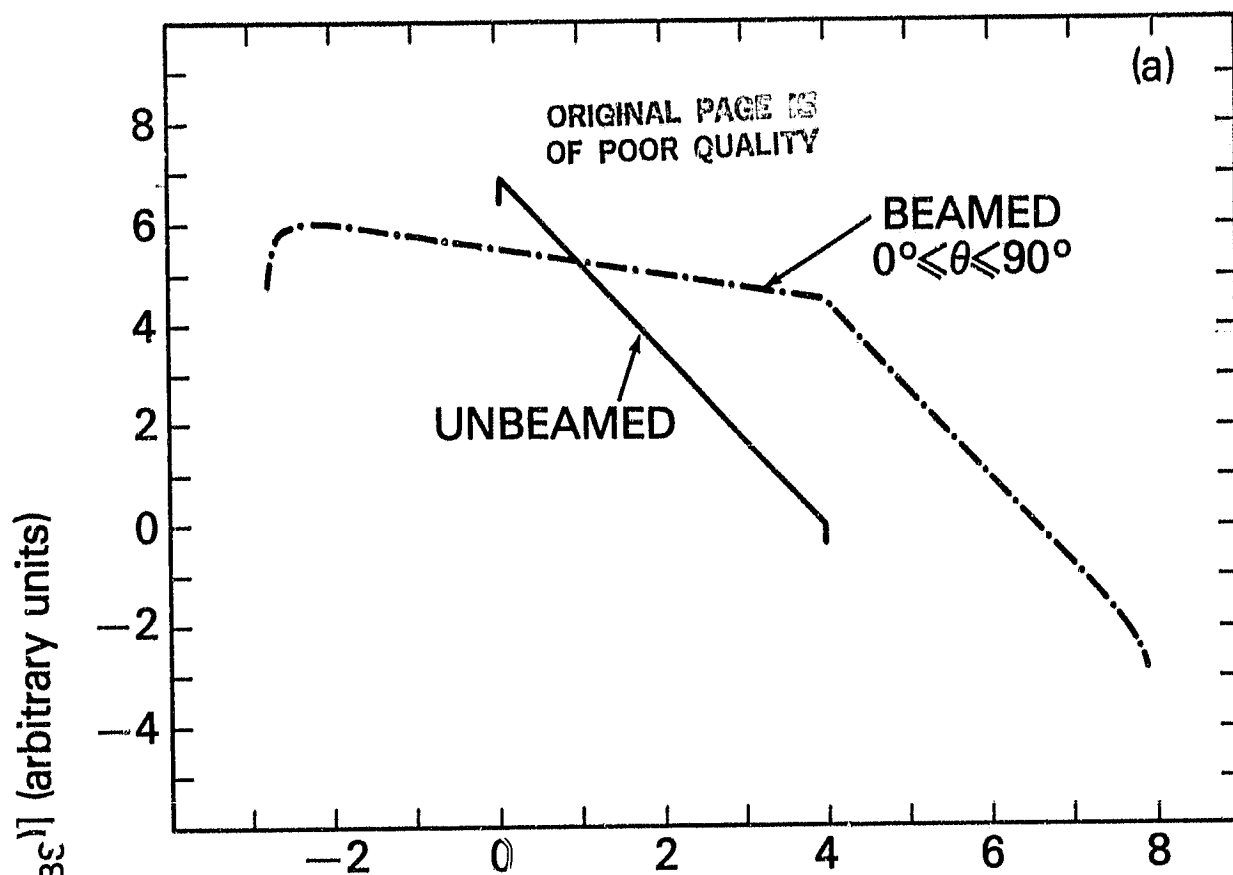
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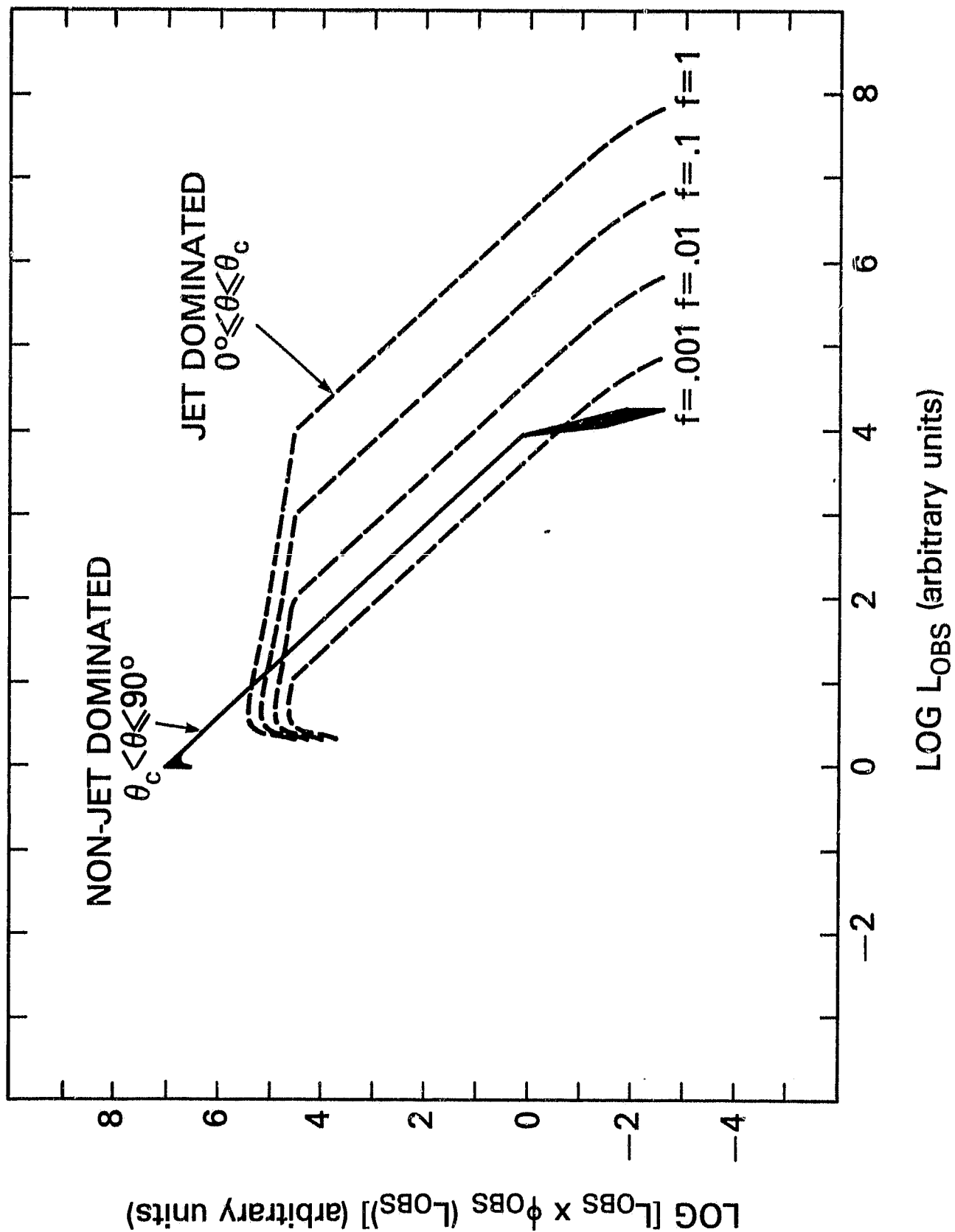
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