CORONAL HEATING BY WAVES

Joseph V. Hollweg

Space Science Center, Dept. of Physics, Univ. of New Hampshire, Durham, NH 03824

Abstract. We show that Alfven waves or Alfvenic surface waves can carry enough energy into the corona to provide the coronal energy requirements. Coronal loop resonances are an appealing means by which large energy fluxes can enter active region loops. The wave dissipation mechanism still needs to be elucidated, but a Kolmogoroff turbulent cascade is fully consistent with the heating requirements in coronal holes and active region loops.

Introduction

The solar chromosphere and corona are heated mechanically. The energy requirements of the chromosphere and corona are roughly comparable but one usually speaks of 'the coronal heating problem', presumably because of the spectacularly high temperatures there; it is probably a mistake to conceptually separate the chromospheric and coronal heating problems, but space constraints require us to do so here. Spicules present the dual problems of heating and accelerating the chromospheric gas. Locally, in a spicule, the energy requirements are comparable to the chromospheric and coronal heating requirements; again, it is probably a mistake to separate the spicule problem from the overall energy balance of the solar atmosphere.

In this review we discuss theories which invoke waves to heat the corona. If we are willing to interpret the word 'waves' broadly enough, there are good reasons for invoking waves. First, any mechanical process requires that the convection zone do work, followed by the mechanical transfer of energy upwards into the corona. The solar atmosphere must move if work is to be done, and it is a fact of life that virtually all motions in the solar atmosphere obey hyperbolic equations which yield wave or wave-like solutions. For example, the linearized versions of the twisting motions invoked by Parker in the next paper obey the Alfvenic wave equation. Second, the corona is observed to contain ubiquitous non-thermal motions of the order of 10-30 km s^{-1} (rms) (e.g. Bonnet, 1978; Cheng et al., 1979; Doschek and Feldman, 1977; Feldman et al., 1975). These motions are unresolved in space and time. For the reasons given above, it is likely that these motions can be thought of as waves. We will argue below that the observed motions may contain sufficient energy to heat the corona, and a wave theory of coronal heating seems possible. Third, the solar wind may serve as a prototype. Alfven waves (e.g. Belcher and Davis, 1971) and/or surface waves (Hollweg, 1982a) appear copiously in the solar wind beyond 0.3AU and there is some radio evidence for the presence of significant wave fluxes in the acceleration region of the wind (Hollweg et al., 1982a). Successful Alfven-wave-driven solar wind models have been constructed (e.g. Hollweg, 1978a). And the behavior of heavy ions in the solar wind suggests the operation of wave-particle interactions (see Isenberg's review in this volume). The solar wind may be telling us that the sun radiates energetically significant wave fluxes, and that waves can heat (and accelerate) at least part of the solar atmosphere.

In short, it seems that it should be possible to construct a successful wave theory for coronal heating. And in view of the first point in the previous paragraph, it is possible that virtually every theory can at some level

be thought of as a wave theory. Nonetheless, a successful wave theory has not yet emerged. In the following we will summarize some current thinking on the subject. We will point out where wave theories succeed and where they fail. And we will suggest some possible routes to be followed in the future.

Some other recent reviews are Hollweg (1981a), Kuperus et al. (1981), Priest (1982a), and Wentzel (1978a, 1981).

Energies Required and Available

We begin with a brief definition of the problem in terms of the energies required to heat the corona. It is useful to split the corona into three types of regions: i. coronal holes, out of which high-speed solar wind streams flow (e.g. Hundhausen, 1977; Zirker, 1977); ii. quiet corona, consisting of large-scale closed field regions, such as the helmet streamers; iii. active region loops, consisting of small-scale coronal regions of enhanced pressure with a loop-like morphology, presumably tracing out closed magnetic field lines (e.g. Rosner et al., 1978; Webb, 1981; Withbroe, 1981). Convenient summaries of the energy requirements of these regions have been given by Withbroe (1976, 1981).

Coronal holes and quiet corona lose energy via radiation and heat conduction back down into the chromosphere. In addition, coronal holes lose energy by heat conduction out into the solar wind; this latter energy loss could be very large if the high-speed streams are driven thermally, as discussed by Olbert in this volume. Holes and quiet corona require an energy flux density of a few times 10^5 erg cm^{-2} s^{-1} entering from below. And if the high-speed streams are thermally driven, the holes may require as much as 10^6 erg cm^{-2} s^{-1}. The volumetric heating rate can be estimated by dividing the energy flux density by the distance over which the heating occurs. If we take 3×10^5 erg cm^{-2} s^{-1} for the flux density, then the heating rate lies between 10^{-4} and 10^{-5} erg cm^{-3} s^{-1} for heating distances in the range $(0.05 - 0.5)r_s$.

The active region loops lose energy via radiation and via heat conduction along the magnetic field lines back down into the chromosphere. In this case a useful rule-of-thumb can be obtained by paraphrasing Rosner et al. (1978) and Withbroe (1981). Let E_H be the volumetric heating rate of the loop plasma. The radiation loss out of the (optically thin) plasma is $n_e^2\Phi$ erg cm^{-3} s^{-1}, where n_e is electron concentration and Φ is a function of T_e (electron temperature) only. If we neglect flows, the energy equation is

$$\nabla \cdot \underset{\sim}{q} = E_H - n_e^2 \Phi \tag{1}$$

where $\underset{\sim}{q}$ is heat conduction. Assuming that $\underset{\sim}{q}$ is classical electron heat conduction along the magnetic field, we can rewrite (1) in the form

$$-\frac{1}{2} q_e^2 = K_o \int E_H T_e^{5/2}\, dT_e - \frac{K_o}{\kappa^2} \int p_e^2\, \Phi\, T_e^{1/2}\, dT_e \tag{2}$$

where it has been assumed that the loop's cross-sectional area is constant (κ is Boltzmann's constant, p is pressure, and $K_o T_e^{5/2}$ is the heat conductivity). We now take E_H, Φ and p_e to be constants. The loop is assumed to have a maximum temperature T_{max} where $q_e = 0$, and it is assumed that $q_e \to 0$ as $T_e \to 0$; the latter constraint requires

$$E_H = \frac{7}{3} \frac{p_e^2\, \Phi}{\kappa^2 T_{max}^2} \tag{3}$$

and we obtain

$$q_e^2 = \frac{4}{3}\frac{K_o p_e^2 \Phi}{\kappa^2} T_e^{3/2}\left[1 - \left(\frac{T_e}{T_{max}}\right)^2\right] \tag{4}$$

The quantity Φ can be eliminated in favor of the loop length as follows: From the usual expression for q_e we have

$$ds = \frac{K_o T_e^{5/2}\, dT_e}{|q_e|} \tag{5}$$

where s is distance along the loop. Inserting (4) into (5) and integrating from $T_e = 0$ to $T_e = T_{max}$ gives

$$p_e L \approx \left(\frac{3K_o}{\Phi}\right)^{\frac{1}{2}} \kappa \left(T_{max}\right)^{11/4} \tag{6}$$

where L is twice the distance from $T_e = 0$ to T_{max}. Inserting (6) into (3) gives finally

$$E_H L \approx \frac{7K_o}{L} T_{max}^{7/2} \tag{7}$$

Since Φ does not appear in (7), it is probable that errors associated with restricting Φ to be a constant will not be very significant. Equation (7) gives the required energy flux density if all the energy comes up along the loop from one of its footpoints; half that value is required if equal fluxes come up both footpoints. For a short (long) loop we take $L \approx 6\times10^9$ (6×10^{10}) cm and $T_{max} = 2 \times 10^6$ (2.5×10^6) K, and with $K_o = 8.4 \times 10^{-7}$ (c.g.s.) we obtain $E_H L \sim 1.1 \times 10^7$ (2.4×10^6) erg cm^{-2} s^{-1}; the corresponding volumetric heating rates are 1.8×10^{-3} (4×10^{-5}) erg cm^{-3} s^{-1}. It is interesting to note that both the energy flux density and the volumetric heating rate are smaller on longer loops; it is the task of theory to explain this.

Can the observed nonthermal velocities in the corona supply the required energy flux densities? It is easy to show that slow (sound) waves cannot do the job, and we will henceforth ignore them. For the fast or Alfven modes, we calculate the energy flux density to be $2\rho_o\, \delta v_{rms}^2\, v_A$ where v_A is the Alfven speed and ρ_o is the coronal density (the factor '2' allows for 2 polarization states). For coronal holes we take $\rho_o = 3.3 \times 10^{-16}$ gm cm^{-3}, B = 8 Gauss, $\delta v_{rms} = 30$ km s^{-1}, and the energy flux density is 7.4×10^5 erg cm^{-2} s^{-1}. For active region loops we take $\rho_o = 5.2 \times 10^{-15}$ gm cm^{-3} (for a mean molecular weight of ½, this density yields a total pressure of 2 dyne cm^{-2} if $T = 2.3 \times 10^6$ K), B = 100 Gauss, $\delta v_{rms} = 30$ km s^{-1}, and we obtain 3.7×10^7 erg cm^{-2} s^{-1} for the energy flux density. These energy fluxes are adequate to supply the required energies, if the 30 km s^{-1} nonthermal motions are fast or Alfven waves.

Can the nonthermal velocities supply the required volumetric heating rates? Write

$$E_H = 2\omega_i\, (2\rho_o \delta v_{rms}^2) \tag{8}$$

where ω_i is the imaginary part of the wave (angular) frequency (the second factor '2' again allows for 2 polarization states). Coronal holes require $E_H \lesssim 10^{-4}$ erg cm^{-3} s^{-1}, and (8) implies $\omega_i/\omega \lesssim 0.4$ if the wave period is 300s (the

latter figure is a guess). Active region loops require $E_H \lesssim 2 \times 10^{-3}$ erg cm^{-3} s^{-1}, and (8) implies $\omega_i/\omega \lesssim 0.17$ if the period is 100s (the reason for this choice of period will be given below). Our estimates of ω_i/ω are not larger than 1, and it can be meaningful to talk about propagating, but damped, waves.

Finally, we must ask whether the convection zone can do enough work on the system. We will assume that the work is done on the intense photospheric magnetic flux tubes (for a review see Spruit, 1981a). If the energy propagates as an Alfven wave, the convective motions can supply an energy flux density to the corona of

$$2\rho_c \; \delta v_{c,rms}^2 v_{Ac} \; (B_{cor}/B_c) \tag{9}$$

Here the subscript 'c' refers to the top of the convection zone, the factor '2' allows for two polarizations, and the factor (B_{cor}/B_c) represents the area expansion of the flux tube. Taking $\rho_c = 3 \times 10^{-7}$ gm cm^{-3} and $\delta v_{c,rms} = 1$ km s^{-1}, we obtain energy flux densities of 2.5×10^7 erg cm^{-2} s^{-1} in a coronal hole ($B_{cor} = 8$ Gauss) and 3×10^8 erg cm^{-2} s^{-1} in an active region loop ($B_{cor} = 100$ Gauss). These values exceed the requirements by more than an order-of-magnitude. As we shall see, this is fortunate, since most of the wave energy is reflected before reaching the corona. (Strictly speaking, equation 9 ignores the details of the wave generation process at the top of the convection zone. A detailed analysis of the coupling between convection and the photospheric flux tubes is really needed, but not available.)

Reflection and Transmission

Fast waves

The coronal pressure is small compared to the magnetic pressure. The fast mode dispersion relation is then approximately $\omega^2 = k^2 v_A^2$, where k is the wavenumber. Upon splitting k into horizontal (h) and vertical (v) components, we obtain

$$k_v^2 = \frac{\omega^2}{v_A^2} - k_h^2 \tag{10}$$

Taking the $\omega - k_h$ structure for the known solar motions in the photosphere and chromosphere, Hollweg (1978b) found that the sun yields $k_v^2 < 0$ in the corona. Fast waves can be expected to be evanescent in the corona, i.e. they suffer total internal reflection somewhere below the corona. Fast waves can not be expected to supply the required energies to the corona. Moreover, the situation is worst in the active regions, which have the greatest energy requirements; the reason is that the active regions are observed to be highly structured (implying large k_h) and to have strong magnetic fields (implying large v_A). Leroy and Schwartz (1982) and Schwartz and Leroy (1982) concur with this conclusion. (See also Osterbrock, 1961.)

However, we note that interesting results have been obtained by Habbal et al. (1979). They postulate the presence of coronal fast waves with a period of 3s. They use ray-tracing techniques to follow the propagation of the wave energy, and emphasize the tendency of the fast waves to refract into regions with smaller v_A; some waves can even be trapped inside a dense loop, in analogy with the trapping of light in an optical fiber. They consider the Landau/transit-time damping of the waves (e.g. Barnes, 1966). An interesting feature of the damping is that it increases with $\beta_p = 8\pi p_p/B^2$, if β_p is small

(the subscript 'p' refers to the protons). In the isothermal model of Habbal et al., β_p is largest where v_A is smallest. Thus the waves refract toward regions of larger damping. This means that the fast waves can heat some coronal regions more strongly than others. The effect is enhanced by a 'positive feedback', whereby local heating increases β_p along a given field line (since heat conduction spreads the heat along $\underset{\sim}{B}$), which in turn increases the heating, and so on. A 'catastrophic' situation can occur in which once fast waves begin heating the plasma, they dump all their energy in a small flux tube. Habbal et al. suggest that such a scenario can account for the highly structured nature of the corona in a natural way. But it has to be shown that the waves exist, and that equation (10) can be overcome.

Alfven waves

Unlike fast waves, these waves never totally internally reflect, and we consider them in some detail. Hollweg (1981b) has considered the propagation of small-amplitude (linearized) axisymmetric twists on a background potential magnetic field which has an axisymmetric untwisted fleur-de-lis structure. If the axis of symmetry is vertical, the twisting motions do not couple to gravity or to the radiation field, and they are non-compressive. If the motions vary as exp(iωt), they obey

$$-\omega^2 r^2 \chi = \frac{B_o}{4\pi\rho_o} \frac{\partial}{\partial s} \left[r^2 B_o \frac{\partial \chi}{\partial s} \right] \tag{11}$$

$$-\omega^2 y = r^2 B_o \frac{\partial}{\partial s} \left[\frac{B_o}{4\pi\rho_o r^2} \frac{\partial y}{\partial s} \right] \tag{12}$$

$$i\omega y = r^2 B_{os} \frac{\partial \chi}{\partial s} \tag{13}$$

where $\chi \equiv \delta v_\theta / r$, $y \equiv r\, \delta B_\theta$, the subscript 'o' refers to the background, the prefix 'δ' refers to the wave, θ is the angle about the axis of symmetry, s is distance along any field line, and r(s) is the distance from the axis to the field line. For field lines near the axis we expect $r^2 B_o \approx$ constant, and (11) becomes

$$\left(v_A^2 \frac{\partial^2}{\partial s^2} + \omega^2 \right) \frac{\delta v_\theta}{r} = 0 \tag{14}$$

Equation (14) is the Alfvenic wave equation; we deal with it in what follows.

If $v_A \sim e^{s/2h}$, then (14) has the solution

$$\frac{\delta v_\theta}{r} = \left[a H_o^{(1)}(\xi) + b\, H_o^{(2)}(\xi) \right] e^{i\omega t} \tag{15}$$

where $\xi = 2h\omega / v_A(s)$, and a and b are complex constants. From (13)

$$\delta B_\theta = \frac{-ir\, B_{os}}{v_A} [aH_1^{(1)}(\xi) + bH_1^{(2)}(\xi)]\, e^{i\omega t} \tag{16}$$

The time-averaged Poynting flux, $\langle \underset{\sim}{S} \rangle$, is along the magnetic field:

$$\langle \underset{\sim}{S} \rangle = - \frac{\underset{\sim}{B}_o}{4\pi} \langle \delta v_\theta\ \delta B_\theta \rangle \tag{17}$$

From (15) and (16) we obtain

$$\langle S_s \rangle = \frac{B_o^{\,2} r^2}{8\pi^2 h\omega} (|a|^2 - |b|^2) \tag{18}$$

From the form of (18), we interpret the $H_o^{(1)}$ part of (15) as the upward-going wave, and the $H_o^{(2)}$ part of (15) as the downward-going wave (Hollweg, 1972).

Now the exponential behavior of v_A used in (15) - (18) roughly represents the probable behavior in the chromosphere and transition region of a flux tube, but v_A is much more nearly constant in the coronal part of a flux tube (Figs. 3 and 10 of Hollweg, 1981b). If the corona extends to infinity, as in a coronal hole, it is possible to obtain some useful analytical results by considering a two-layer model, in which

$$v_A = v_{A,cor} = \text{constant},\ s > 0 \tag{19a}$$

$$v_A = v_{A,cor}\, e^{s/2h},\ s < 0 \tag{19b}$$

Here $s > 0$ represents the corona, and $s<0$ represents the chromosphere and transition region. Equations (15) - (18) apply in $s < 0$, while the usual harmonic solutions apply in $s > 0$. At $s = 0$ we apply the matching conditions that δv_θ and δB_θ be continuous (this ensures continuity of $\langle S_s \rangle$). The matching conditions give

$$\frac{b}{a} = - \frac{H_o^{(1)} - i\, H_1^{(1)}}{H_o^{(2)} - i\, H_1^{(2)}} \tag{20}$$

where the argument of the Hankel functions is $2h\omega/v_{A,cor} \equiv \alpha$. From (20) we can calculate the reflection coefficient, R, i.e. the ratio of downward-going energy flux to upward-going:

$$R = |b|^2/|a|^2 \tag{21}$$

In the limit of small α, we obtain

$$R \underset{\sim}{\sim} 1-2\pi\alpha \tag{22}$$

(Equations 19-22 have been given by Leer et al. (1982). They are a special case of Hollweg (1972).) Since equations (20) - (22) are obtained from the matching conditions at $s = 0$ it seems natural to think of the reflection as

occurring at the discontinuity in scale height there (Hollweg, 1972). However, Wentzel (1978b), following Tolstoy (1973), and Leroy (1980) have argued that R is really the accumulated reflection off of the entire chromosphere and transition region. The latter interpretation is probably correct, but we are then perplexed as to why R can be calculated solely from the matching conditions at s = 0.

For a flux tube entering a coronal hole we might have h = 250 km, and $v_{A,cor}$ = 1200 km s^{-1}. If the wave period is 300s, the energy transmission coefficient is 1-R = 0.055. In the previous section we estimated that the convective work can supply 2.5×10^7 erg cm^{-2} s^{-1}. Multiplying by 1-R gives 1.4×10^6 erg cm^{-2} s^{-1} available to the corona. This is enough to supply the energy requirements.

The situation is different on an active region loop. We imagine the loop to be stretched out so that (possibly important) complications arising from the curvature are ignored (e.g. Wentzel, 1978b). But we must now at the very least consider a 3-layer model, since there is a chromosphere and transition region at each end of the loop (an 18-layer model was considered by Hollweg, 1981b). We take

$$v_A = v_{A,cor} \exp(s/2h), \quad s < 0 \tag{23a}$$

$$v_A = v_{A,cor}, \quad 0 < s < L \tag{23b}$$

$$v_A = v_{A,cor} \exp[-(s-L)/2h'], \quad s > L \tag{23c}$$

We imagine there to be a source at the far end of the loop, in s > L. We impose the boundary condition that there be only an outgoing (from the source) wave in the region s < 0. The other two regions have both upward-and downward-going solutions. Applying matching conditions at s = 0 and s = L allows us to calculate the ratio of downward-going energy flux to upward-going flux in the region s > L. Again calling this ratio R, we obtain

$$R = |N/D|^2 \tag{24}$$

where

$$\begin{aligned} N = &\left[H_1^{(1)}(\beta)H_o^{(2)}(\alpha) + H_o^{(1)}(\beta)H_1^{(2)}(\alpha)\right]\cos kL \\ &+ \left[H_1^{(1)}(\beta)H_1^{(2)}(\alpha) - H_o^{(1)}(\beta)H_o^{(2)}(\alpha)\right]\sin kL \end{aligned} \tag{25}$$

and

$$\begin{aligned} D = &\left[H_1^{(2)}(\beta)H_o^{(2)}(\alpha) + H_o^{(2)}(\beta)H_1^{(2)}(\alpha)\right]\cos kL \\ &+ \left[H_1^{(2)}(\beta)H_1^{(2)}(\alpha) - H_o^{(2)}(\beta)H_o^{(2)}(\alpha)\right]\sin kL \end{aligned} \tag{26}$$

We have defined $\alpha = 2h\omega/v_{A,cor}$, $\beta = 2h'\,\omega/v_{A,cor}$, and $k = \omega/v_{A,cor}$.

Expressions (24) - (26) have not yet been analyzed in detail. But they take a simpler form if $\alpha = \beta$, and if $\alpha, \beta << 1$. Then

$$N \approx Y_1^2 \sin kL + 2\, Y_o Y_1 \cos kL \tag{27}$$

and

$$D \approx N + 2iY_1 \cos kL \tag{28}$$

It is interesting to note that N can be exactly zero at certain resonant frequencies which are approxmately given by

$$\omega_{res} \approx m\pi v_{A,cor}/L \tag{29}$$

where m = 1, 2, 3... (For example, the resonant period is 40s if m = 1, $v_{A,cor}$ = 3000 km s^{-1}, and L = 6 x 10^4 km.) At these frequencies the reflections vanish, and a large energy flux can pass through the corona and out the other end of the loop. In the vicinity of one of the resonances it is possible to use (24), (27) and (28) to obtain the following expression for the energy transmission coefficient:

$$1-R = \frac{1}{1 + (Y_1 L/2v_{A,cor})^2(\omega-\omega_{res})^2} \tag{30}$$

Denoting the full-width-at-half-maximum by $\Delta\omega$, we find

$$\frac{\omega_{res}}{\Delta\omega} \equiv Q \approx \frac{L}{4\pi h} \tag{31}$$

(Q denotes the quality). If L = 6 x 10^4 km and h = 150 km, then Q = 32; the resonance is moderately high quality.

We have already estimated that the convection zone work can supply 3 x 10^8 erg cm^{-2} s^{-1}. If this power is in a band width B_ω, then the energy flux density passing through the corona in one resonant peak is

$$F_{res} = (3 \times 10^8) \frac{\Delta\omega}{B_\omega} \frac{\pi}{2} \tag{32}$$

where the factor ($\pi/2$) comes from integrating the area under (30). From Fig. 3 of Ionson (1982), we estimate $B_\omega \approx 3 \times 10^{-2} s^{-1}$. Then from (29) and (31)-(32) we find

$$F_{res} = 6.2 \times 10^{11} mh\, v_{A,cor}/L^2 \tag{33}$$

Note that longer loops receive less power, as (qualitatively) observed. If we take $h = 150$ km, $v_{A,cor} = 3000$ km s^{-1}, $L = 6 \times 10^4$ km and $m = 1$, then $F_{res} = 7.8 \times 10^7$ erg cm^{-2} s^{-1}. This is more than enough energy to supply the loop, but this energy goes right through the loop, since we have put no damping into the calculation. The calculation with damping still needs to be done, but work in progress shows that adequate energies can reach and stay in the loop, since damping tends to broaden the resonance peak, compensating for its reduction in height.

Equations (25)-(33) are new, but some of these ideas have been considered previously. Ionson (1978) was the first to show that standing waves can be excited on active region loops. The first paper to show that resonances can eliminate the reflections and allow Alfven wave energy to enter the corona was by Hollweg (1981b). The same point was made independently by Zugzda and Locans (1982), but their Alfven wave equation differs from (11) and (12) above. Ionson (1982) has discussed the resonances using RLC circuits as an analogy. This approach omits some physics, however. For example, in the absence of dissipation Ionson gets $Q = \infty$, and he uses $Q = 10^3 - 10^4$ in his paper; by contrast, we find $Q = 30$ or so, even in the absence of dissipation.

An interesting point made by Ionson (1982) is that there appear to be two classes of loops (active region loops and 'large scale structures'). There may also be two peaks in the photospheric power spectrum. Ionson suggests that a narrow-band resonance is a natural means by which the details of the power spectrum could be mapped into the corona, and thus a double-peaked spectrum can give rise to two classes of loops.

Alfvenic Surface Waves

The corona is highly structured. In the limit that the structuring takes the form of discontinuities, it is possible to find new wave modes supported by the surfaces. These surface waves have recently been of considerable interest. See Edwin and Roberts (1982), Gordon and Hollweg (1983), Hollweg (1982a), Ionson (1978), Roberts (1981a, b), Wentzel (1979).

Consider a cold, stationary background plasma which varies only in the χ - direction. The background magnetic field vector varies in χ, but its magnitude is constant. The system supports small-amplitude fluctuations obeying

$$\varepsilon \delta v_\chi = \frac{\partial}{\partial \chi} \left[\frac{\varepsilon}{q^2} \frac{\partial \delta v_\chi}{\partial \chi} \right] \tag{34}$$

where $q^2 \equiv k^2 - \omega^2/v_A^2$ and $\varepsilon \equiv 4\pi\rho_o\omega^2 - (\underset{\sim}{k} \cdot \underset{\sim}{B}_o)^2$; the fluctuations vary as $\exp[ik_y y + ik_z z - i\omega t]$. Now assume that everything is uniform except at a discontinuity at $\chi = 0$. If $q^2 > 0$ everywhere, it is possible to find solutions which evanesce away from the discontinuity as $\exp[\pm qx]$. The dispersion relation for these surface waves follows by requiring that δv_x and $(\varepsilon/q^2)\partial\delta v_\chi/\partial\chi$ be continuous at $\chi = 0$.

For example, suppose $\underset{\sim}{B}_o$ is constant and in the z-direction. Suppose also that k_y is large, so that $q^2 \cong k_y^2$. The dispersion relation turns out to be

$$\frac{\omega^2}{k_z^2} = \frac{B_o^2}{4\pi\rho_{av}} \tag{35}$$

where ρ_{av} is the average of the densities on the two sides of the discontinuity. Equation (35) is similar to the dispersion relation for Alfven waves. These surface waves generally turn out to be rather similar to Alfven waves.

In particular, they propagate energy along $\underset{\sim}{B}_o$. Much of what was said about Alfven waves in the previous section applies to surface waves as well. They may be suitable candidates for coronal heating, although many details, such as their ability to propagate energy through the chromosphere and transition region, remain to be worked out.

It is interesting to note that surface waves may be present in the solar wind (Hollweg, 1982a).

Wiggles of Thin Flux Tubes.

Spruit (1981a, b) has considered transversal oscillations of thin vertical magnetic flux tubes imbedded in a field-free gas. If the tube is in pressure and temperature equilibrium with its surroundings, then the (small) horizontal displacements of the tube obey:

$$(2\beta_t+1)\frac{\partial^2\xi}{\partial t^2} = -g\frac{\partial\xi}{\partial z} + 2\frac{p}{\rho}\frac{\partial^2\xi}{\partial z^2} \tag{36}$$

where g is the gravitational acceleration, and β_t = constant is the ratio of gas to magnetic pressures inside the flux tube. If the atmosphere is isothermal, we can take $\xi \sim \exp[ikz-i\omega t]$ and (36) yields:

$$k = \frac{g\rho}{4p}\left[-i \pm \left(\frac{\omega^2}{\omega_c^2} - 1\right)^{\frac{1}{2}}\right] \tag{37}$$

where

$$\omega_c^2 = \frac{g^2\rho}{8p(2\beta_t+1)} \tag{38}$$

The waves have a low-frequency cutoff. If the temperature is 10^4K, if the molecular weight is 1.3, and if $\beta_t = 1$, we find from (38) that periods longer than 900s are evanescent; this is not a severe restriction.

These waves carry energy along the magnetic field. If $\omega >> \omega_c$, the phase and group velocities are comparable to the sound speed. They could carry energy into the low chromosphere. However, the tubes cease to be thin above the low chromosphere, and a more detailed analysis is necessary. They could couple some energy into the fast and Alfven modes, which could carry the energy to greater heights.

Dissipation

The general conclusion of the previous section is that Alfven or Alfvenic surface waves, and to some extent transversal tube waves, can in principle supply the corona with its energy requirements. This is the first requirement of a coronal heating theory. The second requirement is that the waves deposit energy as heat. Wave theories of coronal heating have so far failed in this regard, but we will summarize some of the possibilities.

One point to note at the outset is that the observed coronal motions, if interpreted as magnetic waves, are quite linear. In a coronal hole we might have $v_A \sim 1200$ km s^{-1} (if B = 8 Gauss and $\rho = 3.3 \times 10^{-16}$ gm cm^{-3}), while in an active region loop we might have $v_A \sim 4000$ km s^{-1} (if B = 100 Gauss and ρ =

5.2×10^{-15} gm cm^{-3}). In all cases $\delta v_{rms}/v_A \ll 1$, in contrast to the solar wind.

Viscous Heating.

Coronal viscosity is mainly due to the protons. Following Braginskii (1965), the proton collision time in an electron-proton plasma is

$$\tau_p = 0.75 T_p^{3/2} n_p^{-1} \text{ s} \tag{39}$$

for a Coulomb logarithm of 22. In the corona $\omega_{cp}\tau_p \gg 1$ (ω_{cp} is the proton cyclotron frequency) and the protons are well-tied to the field lines. Of the five viscosity coefficients given by Braginskii, $\eta_o \sim 10^{-16} T_p^{5/2}$ (cgs) is by far the largest; shear viscosity is smaller by $(\omega_{cp}\tau_p)^{-2}$ (roughly). If only η_o is considered, the viscous heating rate is

$$Q_p = \eta_o \left[\frac{1}{3}(\nabla \cdot \underset{\sim}{v})^2 - 2\frac{\partial v_z}{\partial z} \nabla \cdot \underset{\sim}{v} + 3\left(\frac{\partial v_z}{\partial z}\right)^2\right] \tag{40}$$

where z is along the magnetic field. For purely parallel motion $Q_p = (4/3)\,\eta_o (\nabla \cdot \underset{\sim}{v})^2$ while for purely transverse motions $Q_p = \eta_o (\nabla \cdot \underset{\sim}{v})^2/3$. As a rule-of-thumb we will take $Q_p = \eta_o (\nabla \cdot \underset{\sim}{v})^2$.

In an active region loop we observe $Q_p \lesssim 2 \times 10^{-3}$ erg cm^{-3}s^{-1}. Using the equation of continuity, this requires $\delta\rho_{rms}/\rho_o \lesssim 0.8$ if $T_p = 2.3 \times 10^6$ K and if the wave period is 100s. In a coronal hole we observe $Q_p \lesssim 10^{-4}$ erg cm^{-3}s^{-1}. This requires $\delta\rho_{rms}/\rho_o \lesssim 0.9$ if $T_p = 1.5 \times 10^6$ K and if the wave period is 300s. Such large density fluctuations may occur occasionally in small regions in the corona, but they are not compatible with the requirements of heating the entire corona by any of the waves discussed so far.

Heat Conduction

Coronal heat conduction is due mainly to the electrons. They too are tied to the field lines and the heat conduction is along the magnetic field. The plasma heating rate due to heat conduction damping of waves is

$$Q_e = \kappa_{\parallel,e} \left(\frac{\partial \delta T_e}{\partial z}\right)^2 T_{oe}^{-1} \tag{41}$$

where the heat conductivity is $\kappa_{\parallel,e} = 8.4 \times 10^{-7} T_e^{5/2}$ (cgs) for a Coulomb logarithm of 22.

Low-frequency waves induce nearly adiabatic temperature fluctuations, i.e. $|\delta T_e| \sim (\gamma - 1)\, T_{eo}\, |\nabla \cdot \delta\underset{\sim}{v}|/\omega$, where $\gamma = 5/3$ is the ratio of specific heats. We then find (taking $T_e = T_p$)

$$\frac{Q_e}{Q_p} = 8.4 \times 10^9\, (\gamma-1)^2 T_e k_z^2/\omega^2 \tag{42}$$

If $T_e = 2 \times 10^6$ K, we find that viscous heating dominates heat conduction if $\omega/k_z > 860$ km s^{-1}. Coronal Alfven speeds generally exceed this value, and we deduce that heat conduction will be even less effective than viscosity in damping the waves.

For completeness we should mention that heat conduction smooths out the temperature fluctuations in high-frequency waves. This reduces the

effectiveness of heat-conduction damping even further. See Gordon and Hollweg (1983).

Shocks.

We have already mentioned that Alfven, fast, or Alfvenic surface waves are to a good approximation linear in the corona. Shocks probably do not form in the corona. However, Hollweg et al. (1982b) have suggested the possibility that shocks can form in the chromosphere, on their way to the corona. Hollweg et al. considered Alfven waves, which steepen into a train of shocks which are nearly switch-on shocks (see, for example, Boyd and Sanderson, 1969). These shocks enter the corona, and can carry substantial energy fluxes. Hollweg et al. suggested that some of the impulsive events observed in the transition region (see the article by Dere in this volume) could in fact be the shocks.

The volumetric heating rate due to a periodic train of weak switch-on shocks with period τ is

$$Q_{sos} = \frac{B_o^2}{32\pi\tau} \left(\frac{\Delta v_t}{v_{Ao}} \right)^4 \tag{43}$$

if the coronal pressure is small compared to the magnetic pressure (Hollweg, 1982b); Δv_t is the jump in the velocity component transverse to the shock normal. Since $Q \sim B_o^{-2}$, this mechanism yields negligibly small heating rates in active regions. However, it could conceivably work in coronal holes or quiet corona if B_o and τ are small enough, and if Δv_t is large enough. For example, if $B_o = 5$ Gauss, $\rho = 3.3 \times 10^{-16}$ gm cm^{-3}, $\Delta v_t = 200$ km s^{-1}, and $\tau = 100$s, we obtain $Q_{sos} = 10^{-5}$ erg cm^{-3} s^{-1}. The corona can be heated over an extended distance by this mechanism.

Switch-on shocks are probably a worst case. A best case for shocks is a train of fast shocks propagating across the magnetic field. If the shocks are weak and if the coronal pressure is small compared to the magnetic pressure, the volumetric heating rate is

$$Q_{fs} = \frac{B_o^2}{16\pi\tau} \left(\frac{\Delta v}{v_{Ao}} \right)^3 \tag{44}$$

where Δv is the velocity jump across the shock. In an active region loop we might have B_o=100 Gauss and v_{Ao} = 3000 km s^{-1}. If we take Δv = 200 km s^{-1} and τ = 100s we find $Q_{fs} = 6 \times 10^{-4}$ erg cm^{-3} s^{-1}. This is comparable to the heating requirements of some moderate length loops. In a coronal hole we might have B_o = 5 Gauss and v_{Ao} = 800 km s^{-1}, and we find $Q_{fs} = 7.8 \times 10^{-5}$ erg cm^{-3} s^{-1}. The heating of the coronal hole is substantial in this case.

Thus fast shocks propagating across $\underset{\sim}{B}_o$ can yield substantial heating. The same is presumably true for fast shocks propagating at some not-too-small angle to $\underset{\sim}{B}_o$. But where would such shocks come from? One guess is that they could form in the chromosphere in the manner investigated by Hollweg et al. (1982b) for switch-on- shocks. In fact, the study of Hollweg et al. is probably unrealistically restricted, since there is no reason to expect that the sun will yield shock normals which are nearly aligned along the magnetic field; in view of the strong cross-field structuring observed on the sun, the opposite is probably the case, and further studies must be done. In any event, we will ultimately have to rely on observations to tell us whether shocks with the required frequency and amplitude do in fact form in the chromosphere and enter the corona.

Phase Mixing and Turbulence.

Consider two neighboring magnetic field lines along which energy is propagating in the form of an Alfven wave. If the phase velocities are different on the two field lines, then the motions on those field lines will not always maintain the same phase relationship. The motions will move in and out of phase, and (in the latter case) extremely large cross-field velocity gradients can develop. Heyvaerts and Priest (1982) (see also Priest, 1982b) have pointed out that this phase mixing is ripe for viscous dissipation. They consider the special case where $\underset{\sim}{B}_o$ is in the z- direction, $\delta\underset{\sim}{v}$ is in the y-direction, and $v_A = v_A(\chi)$. Unfortunately, since the motions in this case are shears, the viscous damping will be weak, because $\omega_{cp}\tau_p >> 1$; this limitation of the efficacy of viscosity was not considered by Heyvaerts and Priest.

Heyvaerts and Priest consider also the possibility that large velocity shears between neighboring field lines can result in Kelvin-Helmholz instabilities. (They consider in particular the situation on coronal active region loops, where the resonances discussed in section IIIB lead to nearly-standing waves on the loops. A similar idea was mentioned earlier by Hollweg (1981b) with regard to Alfven waves in the chromosphere. Since the solar atmosphere is not homogeneous, and since there is no reason to expect phase coherence between motions on different field lines, phase mixing may be a ubiquitous and important effect.) Priest (1982b) estimates the Kelvin-Helmholz growth rate to be $\omega_i \approx k_\perp |\delta\underset{\sim}{v}|$, where $k_\perp$ is the transverse wave number associated with the velocity shears. But it must be kept in mind that the driving velocity shears in this problem are time-dependent. For the analysis (which assumes that the driving shears are steady) to hold, it is necessary that $\omega_i >> \omega$. If we take $\omega = 2\pi/(100s)$, and $|\delta\underset{\sim}{v}| = 40$ km s^{-1} (corresponding to $\delta v_{rms} \approx 30$ km s^{-1}), we require $\lambda_\perp = 2\pi/k_\perp << 4000$ km. This is not a strong constraint (the active region loops are only a few thousand km in diameter), and Kelvin-Helmholz instabilities may occur. The instabilities may initiate a turbulent cascade to higher wavenumbers where viscosity (or some other process) can convert the energy into heat.

Heyvaerts and Priest (1982) and Priest (1982b) consider also the possibility that the phase relationship between Alfven waves on neighboring field lines is such as to produce large magnetic shears. Tearing-mode instabilities are then possible. (This idea was also discussed earlier by Hollweg, 1981b.) Priest (1982b) estimates the tearing growth time to be

$$\tau_{tmi} \approx |\delta\underset{\sim}{v}|^{-4/7} \eta^{-3/7} k_\perp^{-10/7} \qquad (45)$$

where $\eta \approx 2 \times 10^{13}\ T_e^{-3/2}$ (cgs) is the magnetic diffusivity; in computing η we have taken the perpendicular electrical conductivity (Braginskii, 1965) and a Coulomb logarithm of 22. (Note that η is so small in the corona that classical electrical resistivity fails miserably as a dissipation mechanism.) For the analysis (which assumes that the magnetic shears are steady) to be valid, it is necessary that $\omega\tau_{tmi} << 1$. If the wave period is 100s, this requires $\lambda_\perp << 10$ km, where we have taken $T_e = 2.3 \times 10^6$ K, and $|\delta\underset{\sim}{v}| = 40$ km s^{-1}. It is not known whether the coronal waves are structured on these scales. But tearing instabilities, if they occur, can initiate a turbulent cascade to higher wavenumbers.

Suppose a turbulent cascade is initiated. How effective will it be? Unfortunately, most turbulence theory has been developed for incompressible fluids with isotropic turbulence. Neither condition can be expected to apply in the corona. However, as Montgomery (this volume) says, "it's the only game

in town", so we'll play anyway. For fully-developed Kolmogoroff turbulence one expects

$$E(k) \cong C_o \varepsilon^{2/3} k^{-5/3} \tag{46}$$

where k is wavenumber, $C_o \sim 1.5$ is a universal (?) constant, $\int E(k)dk = \langle \delta \underset{\sim}{v}^2 \rangle$, and ε (dimensions = velocity^2 time^{-1}) is the rate at which $\langle \delta \underset{\sim}{v}^2 \rangle$ cascades to high wavenumbers. Integrating (46) from k_o to ∞ gives

$$\frac{\varepsilon}{2\langle \delta \underset{\sim}{v}^2 \rangle} = \frac{k_o \langle \delta \underset{\sim}{v}^2 \rangle^{1/2}}{2(1.5\ C_o)^{3/2}} \tag{47}$$

where it has been assumed that all the energy is in $k > k_o$. Now we can regard $\varepsilon/(2\langle \delta \underset{\sim}{v}^2 \rangle)$ as a measure of the damping rate, ω_i, since the cascade pumps energy to arbitrarily high wavenumbers where it is absorbed and converted into heat. Equation (47) then yields (c.f. Section II)

$$E_H = \frac{2^{3/2} k_o \rho_o \delta v_{rms}^3}{(1.5\ C_o)^{3/2}} \tag{48}$$

(we have again allowed for 2 polarization states). The problem is that we are not sure what to put in for k_o. If we take $k_o = 2\pi/\lambda_1$ with $\lambda_1 = 3000$ km (a reasonable guess for an active region loop), $\rho_o \cong 5.2 \times 10^{-15}$ gm cm^{-3}, and $\delta v_{rms} = 30$ km s^{-1}, we find $E_H = 2.5 \times 10^{-3}$ erg cm^{-3} s^{-1}. Comparison with Section II shows that this is adequate to heat the active region loops. In a coronal hole we might have λ_1 to be the mean distance between the photospheric magnetic flux tubes $\sim 10^4$ km, and with $\rho_o = 3.3 \times 10^{-16}$ gm cm^{-3} we obtain $E_H = 4.7 \times 10^{-5}$ erg cm^{-3} s^{-1}. Again, we obtain an adequate heating rate. We tentatively conclude that a turbulent cascade can provide the required heating. But a theory for turbulence which is applicable to the corona must be developed. We suggest that this subject be vigorously pursued in the future. (The author thanks Dr. C. Smith for advice on this paragraph.)

Surface Waves

Ionson (1978) has considered the propagation of surface waves on non-discontinuous 'surfaces'. He considers the problem discussed previously, which has (35) as its dispersion relation in the case of a truly discontinuous surface. He finds, however, that ω is now complex, with

$$\frac{\omega_i}{|\omega_r|} = -\pi \left(\frac{\rho_{o1}}{\rho_{o2}} + \frac{\rho_{o2}}{\rho_{o1}} + 2\right)^{-1} \frac{|k_y| a \Delta v_A (4\pi\rho_{av})^{\frac{1}{2}}}{B_o} \tag{49}$$

where 'a' is the thickness of the 'surface' and Δv_A is the difference between the Alfven speeds on the two sides. Ionson interpreted ω_i as a damping rate, but Lee (1980) pointed out that it is not correct to think of damping of a normal mode, because there are no normal modes in this problem (unless a = 0). Instead, the appearance of ω_i represents a readjustment of the system's energy distribution. (Lee suggests that the situation is analogous to leakage of particles out of a potential well, as in radioactive decay. ω_i represents the leakage out of the well, but there is no net loss of particles.) The energy flows into the surface. Steep gradients develop in the surface, and viscous dissipation will eventually occur in a thin layer. It seems likely that the dissipation in the thin layer will adjust itself to absorb the energy flow

into the surface, as suggested by Ionson (1978). The problem with dissipation still needs detailed study, but it may represent a promising mechanism for converting wave energy into heat in many thin layers in the corona. (See also Lee et al., 1983).

Summary

The corona is observed to move with velocities of 30 km s^{-1} or so, rms. Since most coronal motions obey hyperbolic equations, it is reasonable to think of the motions as being waves.

If the motions are Alfven or Alfvenic surface waves, they can carry the required energies into the corona. Some large fraction of the energy is presumably reflected in the chromosphere and transition region, but theory indicates that sufficient energy can be transmitted into the corona. Loop resonances may play a special role in allowing energy to enter the active region corona.

The problem is with the dissipation. Viscosity is the most promising dissipation mechanism, but special conditions have to be fulfilled. The wave energy must ultimately appear at sufficiently short spatial scales so that viscosity becomes effective. Fast shocks may be suitable, since the shock thickness automatically adjusts itself to yield the required entropy jump across the shock. A turbulent cascade seems even more promising, but an anisotropic compressive turbulence theory still needs to be formulated. Surface waves may be yet another route by which the wave energy finds itself at small enough spatial scales for viscous heating to be effective. But the detailed analyses of these processes still need to be done.

Another area where more work is needed is the following: We have conceptually separated the coronal heating from the chromospheric heating. And we have regarded the chromosphere and transition region as being fixed entities which carry (and reflect) the waves. But the coupling between the chromosphere, corona, and transition region may be an integral part of the entire atmosphere's energy balance. And the coupling may be strong and dynamic. For example, Hollweg (1981b) and Hollweg et al. (1982b) have argued that the chromosphere and transition region may themselves be set into vigorous motion as the waves propagate from the photosphere into the corona. (A possible connection with the spicules has been noted by Hollweg et al. (1982b) and by Hollweg (1982c).) It is not yet clear how this dynamic coupling affects the ideas presented in this review.

Finally, we have been implicitly regarding the waves as non-impulsive. Yet the observations discussed by Dere in this volume suggest the opposite: the transition region moves violently and impulsively. What are the implications of these observations for coronal heating? We have already mentioned that the impulses could be shocks, but the issue is far from being resolved. These observations demand further study.

Acknowledgments. The author thanks J.A. Ionson, M.A. Lee, E.N. Parker, and C. Smith for comments. This work was supported by the NASA Solar-Terrestrial Theory Program under grant NAGW-76.

References

Barnes, A., Collisionless damping of hydromagnetic waves, Phys. Fluids, 9, 1483, 1966.

Belcher, J.W., and L. Davis Jr., Large-amplitude Alfven waves in the

interplanetary medium, 2, J. Geophys. Res., 76, 3534, 1971.
Bonnet, R.M., High resolution UV solar spectroscopy, Space Sci. Rev., 21, 379, 1978.
Boyd, T.J.M., and J.J. Sanderson, Plasma Dynamics, Barnes and Noble, New York, 1969.
Braginskii, S.I., Transport processes in a plasma, Rev. Plasma Phys., 1, 205, 1965.
Cheng, C.-C., G.A. Doschek, and U. Feldman, The dynamical properties of the solar corona from intensities and line widths of EUV forbidden lines of Si VIII, Fe XI, and Fe XII, Astrophys. J., 227, 1037, 1979.
Doschek, G.A., and U. Feldman, The coronal temperature and nonthermal motions in a coronal hole compared with other solar regions, Astrophys. J., 212, L143, 1977.
Edwin, P.M., and B. Roberts, Wave propagation in a magnetically structured atmosphere, 3, the slab in a magnetic environment, Solar Phys., 76, 239, 1982.
Feldman, U., G.A. Doschek, and R. Tousey, The intensities and profiles of XUV transition zone lines in a quiet sun region compared to a polar coronal hole, Astrophys. J., 202, L147, 1975.
Gordon, B., and J.V. Hollweg, Collisional damping of surface waves in the solar corona, Astrophys. J., 266, in press, 1983.
Habbal, S.R., E. Leer, and T.E. Holzer, Heating of coronal loops by fast mode MHD waves, Solar Phys., 64, 287, 1979.
Heyvaerts, J., and E.R. Priest, Coronal heating by phase-mixed shear Alfven waves, Astron. and Astrophys., in press, 1982.
Hollweg, J.V., Supergranulation-driven Alfven waves in the solar chromosphere and related phenomena, Cosmic Electrodyn., 2, 423, 1972.
Hollweg, J.V., Some physical processes in the solar wind, Rev. Geophys. Space Phys., 16, 689, 1978a.
Hollweg, J.V., Fast wave evanescence in the solar corona, Geophys. Res. Lett., 5, 731, 1978b.
Hollweg, J.V., Mechanisms of energy supply, in Solar Active Regions, ed. by F.Q.Orrall, Colorado Associated University Press, 277, 1981a.
Hollweg, J.V., Alfven waves in the solar atmosphere, 2, open and closed magnetic flux tubes, Solar Phys., 70, 25, 1981b.
Hollweg, J.V., Surface waves on solar wind tangential discontinuities, J. Geophys. Res., 87, 8065, 1982a.
Hollweg, J.V., Heating of the corona and solar wind by switch-on shocks, Astrophys. J., 254, 806, 1982b.
Hollweg, J.V., On the origin of solar spicules, Astrophys. J., 257, 345, 1982c.
Hollweg, J.V., M.K. Bird, H. Volland, P. Edenhofer, C.T. Stelzried and B.L. Seidel, Possible evidence for coronal Alfven waves, J. Geophys. Res., 87, 1, 1982a.
Hollweg, J.V., S. Jackson, and D. Galloway, Alfven waves in the solar atmosphere, 3. nonlinear waves on open flux tubes, Solar Phys., 75, 35, 1982b.
Hundhausen, J.R., An interplanetary view of coronal holes, in Coronal Holes and High-Speed Wind Streams, ed. by J. Zirker, Colorado Associated University Press, 225, 1977.
Ionson, J., Resonant absorption of Alfvenic surface waves and the heating of solar coronal loops, Astrophys. J., 226, 650, 1978.
Ionson, J., Resonant electrodynamic heating of stellar coronal loops: an LRC circuit analog, Astrophys. J., 254, 318, 1982.

Kuperus, M., J.A. Ionson, and D.S. Spicer, On the theory of coronal heating mechanisms, Ann. Rev. Astron. Astrophys., 19, 7, 1981.
Lee, M.A., Comments on the dissipation of hydromagnetic surface waves, Astrophys. J., 240, 693, 1980.
Lee, M.A., I. Rae, and B. Roberts, On the dissipation and propagation of hydromagnetic surface waves, Astrophys. J., submitted, 1983.
Leer, E., T.E. Holzer, and T. Fla, Acceleration of the solar wind, preprint, 1982.
Leroy, B., Propagation of waves in an atmosphere in the presence of a magnetic field, 2, the reflection of Alfven waves, Astron. Astrophys., 91, 136, 1980.
Leroy, B., and S.J. Schwartz, Propagation of waves in an atmosphere in the presence of a magnetic field, 5, the theory of magneto-acoustic-gravity oscillations, Astron. Astrophys., 112, 84, 1982.
Osterbrock, D.E., The heating of the solar chromosphere, plages, and corona by magneto-hydrodynamic waves, Astrophys. J., 134, 347, 1961.
Priest, E.R., Solar Magnetohydrodynamics, Reidel, Dordrecht, Holland, 1982a.
Priest, E.R., The magnetohydrodynamcs of solar activity, Plasma Phys., in press, 1982b.
Roberts, B., Wave propagation in a magnetically structured atmosphere, 1, surface waves at a magnetic interface, Solar Phys., 69, 27, 1981a.
Roberts, B., Wave propagation in a magnetically structured atmosphere, 2, waves in a magnetic slab, Solar Phys., 69, 39, 1981b.
Rosner, R., W.H. Tucker, and G.S. Vaiana, Dynamics of the quiescent solar corona, Astrophys. J., 220, 643, 1978.
Schwartz, S.J., and B. Leroy, Propagation of waves in an atmosphere in the presence of a magnetic field, 6, Application of magneto-acoustic-gravity mode theory to the solar atmosphere, Astron. Astrophys., 112, 93, 1982.
Spruit, H.C., Magnetic flux tubes, in The Sun as a Star, ed. by S. Jordan, NASA-SP-450, 1981a.
Spruit, H.C., Motion of magnetic flux tubes in the solar convection zone and chromosphere, Astron. Astrophys., 98, 155, 1981b.
Tolstoy, I., Wave propagation, McGraw-Hill, New York, 1973.
Webb, D.F., Active region structures in the transition region and corona, in Solar Active Regions, ed. by F..Q. Orrall, Colorado Associated University Press, 165, 1981.
Wentzel, D.G., Heating of the solar corona: a new outlook, Rev. Geophys. Space Phys., 16, 757, 1978a.
Wentzel, D.G., Wave reflection and wave disorder in the solar transition zone and corona, Solar Phys., 58, 307, 1978b.
Wentzel, D.G., Hydromagnetic surface waves, Astrophys. J., 227, 319, 1979.
Wentzel, D.G., Coronal heating, in The Sun as a Star, ed. by S. Jordan, NASA-SP-450, 1981.
Withbroe, G., in Energy Balance and Hydro-dynamics of the Solar Chromosphere and Corona, ed. by R.M. Bonnet and Ph. Delache, IAU Colloq. 36, 1976.
Withbroe, G.L., Physics of static loop structures and scaling laws, in Solar Active Regions, ed. by F.Q. Orrall, 199, 1981.
Zirker, J.B., Coronal holes-an overview, in Coronal Holes and High-Speed Wind Streams, ed. by J. Zirker, Colorado Associated University Press, 1, 1977.
Zugzda, Y.D., and V. Locans, Tunnelling and interference of Alfven waves, Solar Phys., 76, 77, 1982.