STELLAR WINDS DRIVEN BY MULTI-LINE SCATTERING

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ABSTRACT

This paper presents a model of a radiation-driven stellar wind with overlapping spectral lines. It is based on the Castor, Abbott, and Klein (CAK) theory. The presence of overlapping lines allows a photon to be scattered many times in different lines. I assume a random separation between strong lines, which makes it possible to find the angular distribution of the wavelength-averaged intensity of radiation from the star. The properties of the wind at any point depend on this intensity, which in turn depends on the structure of the wind. A self-consistent wind model is found. The mass loss rate does not saturate as line overlap becomes more pronounced, but continues to increase. The terminal velocity is much larger than in the CAK model, while the velocity law is shallower. This model might help explain the massive winds from Wolf-Rayet stars.

Introduction

Previous models for radiation-driven stellar winds [e.g., Castor, Abbott, and Klein, 1975; hereafter CAK] have not considered the fact that if the spectral lines that drive the wind are packed densely enough, a photon can scatter in more than one line. Multiple scatterings of photons in overlapping lines may explain some aspects of the winds from early-type stars. "Overlap" in this context means that the lines are separated by less than twice the wind velocity Doppler shift:

$$\frac{\Delta\lambda}{\lambda} = \frac{2v}{c} , \qquad (1)$$

where v is the velocity of the wind. How overlap causes multiple scatterings, and how we include this effect in the radiation force, is discussed in the next section.

Another shortcoming of the earliest stellar wind models is that they do not include the correct angle integration of the core radiation in the calculation of the radiation force, but instead assume radial streaming. This angle integration is important for the multi-scattering process because, as we shall see, line overlap affects the wind through the angular distribution of radiation.

The combination of the angle integration and overlapping lines may be able to explain the observed terminal velocities of early-type stellar winds. The CAK theory predicts that the terminal velocity is proportional to the escape velocity:

$$v_\infty = \left(\frac{\alpha}{1-\alpha}\right)^{1/2} v_{esc} , \qquad (2)$$

where α is essentially independent of spectral type, and has a value between 0.5 and 0.7 [Abbott, 1982a]. Thus, the CAK model would predict $v_\infty = (1.0-1.5)v_{esc}$. This relation may be correct for late B and early A stars [Abbott, 1982a], but is incorrect for O stars, which have a constant of proportionality of about 3 [Abbott, 1978; Garmany et al., 1981]. In section IV of this paper we will see that a self-consistent multi-scattering model does indeed predict that the terminal velocity is higher than in CAK, and increases with increasing line overlap.

The quantity $\varepsilon = \dot{M}v_\infty c/L$, which is the ratio of the momentum carried to infinity by the wind to the radiative momentum of the star, can be thought of as an efficiency of ejecting matter in the wind. ($\dot{M}$ is the mass loss rate and L is the stellar luminosity.) In the "single scattering limit," in which a photon can give its momentum to the wind only once, ε cannot exceed unity. ε is observed to be greater than one in some Of stars [Ferrari-Toniolo, Persi, and Grasdalen, 1981] and much greater than one in Wolf-Rayet stars [Barlow, Smith and Willis, 1981; Abbott, 1982b]. It is often claimed that this is evidence that these winds are not radiatively driven. We will see that the multi-scattering model presented here has ε values substantially larger than unity, and it appears that ε could be arbitrarily large if there are enough lines.

Method

If we consider only monotonically increasing velocity laws, a photon is continually redshifted, in the reference frame of the expanding gas, as it travels outward through the wind. After scattering in a line, if there is no other line with a rest wavelength within the wind-velocity Doppler shift of the first line (as described by Eq. (1)), the photon can escape from the atmosphere. If the lines overlap, as is the case for the UV resonance lines in OB star atmospheres [Panagia and Macchetto, 1982], the photon can now scatter in another line. It will continue scattering in lines with longer wavelengths until it escapes from the atmosphere or collides with the photosphere. Figure 1 illustrates the process schematically.

This is a formidable problem to treat accurately. One feasible, but costly, approach is to do a Monte-Carlo simulation of the multiple scattering process using a complete tabulation of scattering lines. A more approximate but much less costly method is to make a statistical assumption. The real wavelength distribution of lines is markedly clumpy, since lines are often present as fairly closely spaced multiplets. When the richness of the spectrum increases, however, the chance groupings of lines of different elements and ions may become more prominent than the multiplet structure. This is the regime in which a statistical treatment may succeed.

The statistical model that is advanced is this: lines have random wavelengths and strengths such that different strengths form independent processes. This assumption enables us to use the crucial simplification that the intensity of the radiation field depends only on the wavelengths and strengths of lines that have already scattered photons, and hence the presence or absence of a line at a given location, and its strength if there is one, are statistically independent of the intensity. The radiative transfer problem is then tractable, and we may solve for the angle-dependent intensity of radiation (averaged over a suitable wavelength range). For the details of this calculation, see Friend and Castor [1983].

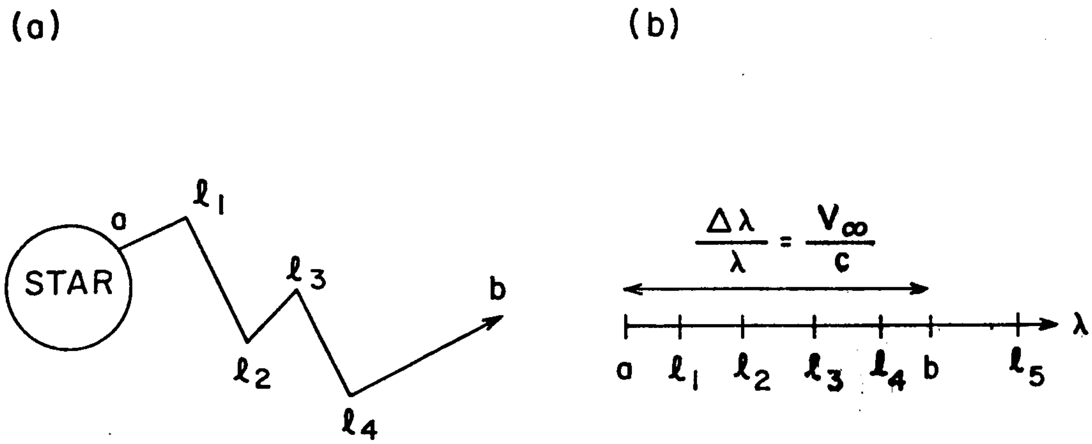

Figure 1. (a) Diagram of the path of a photon from the stellar surface (point a) to its escape from the atmosphere (point b). It is scattered by four lines in the process. (b) Wavelengths of spectral lines ℓ_1 through ℓ_5 are shown, along with the wavelength of the photon at points a and b and the wind terminal velocity Doppler shift $\Delta\lambda/\lambda = v_\infty/c$, in the reference frame of the expanding wind. Lines ℓ_1 through ℓ_4 "overlap" and each scatters the photon, while line ℓ_5 is at too long a wavelength.

The solution gives the blocking factor, B, which is the fraction of the outward flux that is reflected back into the photosphere, and the correction factor to the line force in the CAK model of radially-streaming radiation, which we call F_a.

The radiation force in our model is obtained by multiplying the force multiplier function M(t) of CAK by the correction factor F_a. Recall that in CAK M(t) is approximated by $kt^{-\alpha}$, where t is the optical depth parameter [see CAK equation (5)]. We will add an additional complication here, which takes into account the variation of ionization state with density. Abbott [1982a] showed that the dependence of the radiation force on density is approximately given by $(\rho/W)^{0.1}$ (see his eq. (12)), where W is the dilution factor. Since ρ is proportional to $\dot{M}/vr^2$ and W is approximately $(1/4)(R_*/r)^2$, we write the new force multiplier as

$$M(t) = A \left(\frac{\dot{M}}{R_*^2 v}\right)^{0.1} \left(\frac{v_{th}}{t}\right)^{\alpha} F_a \quad . \tag{3}$$

The $v_{th}{}^{\alpha}$ term is included since the CAK force constant k is actually proportional to it (see CAK, eq. (15)). For ease of comparison with CAK, we will use $\alpha = 0.7$ as they did. The constant A is a measure of the number of strong lines.

Solution of Equation of Motion

The equation of motion for the wind, from CAK, is

$$v\frac{dv}{dr} = -\frac{GM}{r^2} + \frac{\sigma_e L}{4\pi c r^2}[1+M(t)] - \frac{1}{\rho}\frac{dP_g}{dr} \quad . \tag{4}$$

(σ_e is the electron scattering opacity.) The gas pressure P_g will be written as ρa^2, where a is the isothermal sound speed (assumed constant). Using Eq. (3) for M(t), substituting ρa^2 for P_g, and using the continuity equation $\dot{M} = 4\pi\rho v r^2$ to eliminate ρ, yields

$$\left(v - \frac{a^2}{v}\right)\frac{dv}{dr} = \frac{-GM(1-\Gamma)}{r^2} + \frac{2a^2}{r} + \frac{\Gamma GMA}{r^2}\left(\frac{\dot{M}}{R_*^2 v}\right)^{0.1} F_a\left(\frac{4\pi}{\sigma_e \dot{M}} r^2 v \frac{dv}{dr}\right)^{\alpha} \tag{5}$$

for the equation of motion. Γ is the ratio of the luminosity to the Eddington luminosity:

$$\Gamma = \frac{L}{L_{EDD}} = \frac{\sigma_e L}{4\pi GMc} \quad . \tag{6}$$

The topology of solutions is exactly the same as in CAK, and the only acceptable wind solution passes through a unique critical point. The critical point conditions give us the velocity and velocity gradient at the critical point, and the mass loss rate, given the critical radius.

We must find a wind model whose velocity law and mass loss rate are consistent with the angle-dependent radiation force. We may begin by specifying the factor F_a as a function of radius, and then solving the wind equation of motion and critical point conditions to get the wind structure. We must make an intitial guess for the location of the critical point, and then adjust the critical radius until the electron scattering optical depth is unity at the specified photospheric radius. The wind parameters thus found can then be used to calculate the radiation field and $F_a(r)$. The process is repeated until convergence is obtained. About 10 to 20 iterations were required for $F_a(r)$ to converge.

Results

The procedure described in the previous section was performed for stellar data representing an O5f star and five values of the radiation force constant A (see section II). The stellar parameters chosen were the ones used in section IV of CAK: $L = 9.66 \times 10^5\ L_\odot$, $M = 60\ M_\odot$, and $R_* = 13.8\ R_\odot$. Figure 2 shows $F_a(r)$ for three different values of log A (in cgs units). For small r there is not much variation in F_a for different A, but at large radii the force is much larger for larger A. The radiation force is smaller than in CAK at small r ($r < 2R_*$) and larger than in CAK for large r. This has the effect of making the velocity law more gradual and the terminal velocity larger.

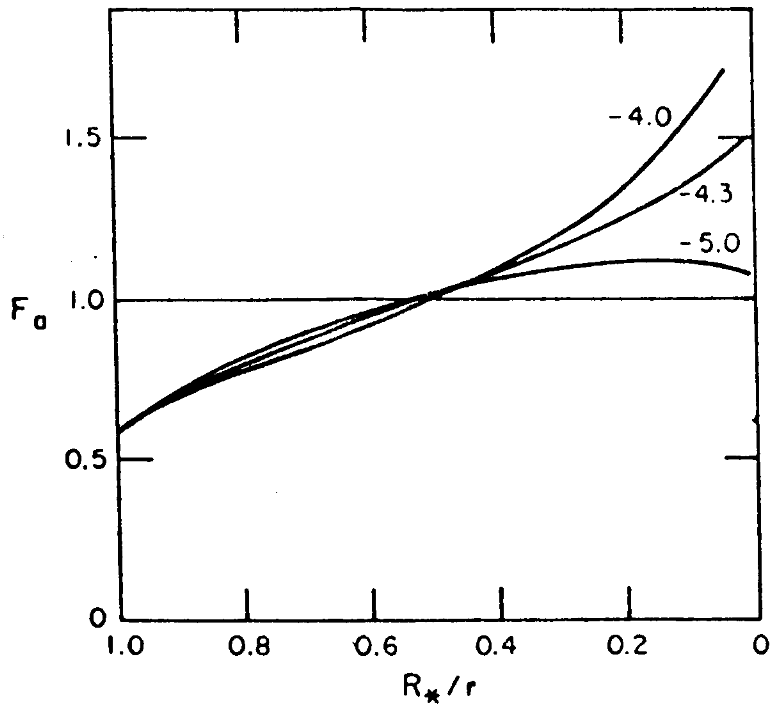

Figure 2. Correction factor F_a as a function of radius (plotted as R_*/r). The three curves are labelled with the value of log A (in cgs units).

The increase of F_a at large radii with increasing A is due to a broadening of the angular distribution of radiation. Increasing the overlap (larger A) means increasing the number of scatterings, which broadens the angular distribution. (The intensity at large angles goes up as more photons are scattered.) This makes the optical depth parameter t smaller (for large r). The radiation force then increases since it is proportional to $t^{-\alpha}$.

Figure 3 shows velocity laws for the three cases of Fig. 2. For all A values, and especially for the intermediate one, v(r) is close to the relation

$$v(r) = v_\infty(1-R_*/r) \quad , \tag{7}$$

which is closer to what observations indicate is the actual velocity law in early-type stars [Lamers and Morton, 1976; Barlow and Cohen, 1977; Van Blerkom, 1978].

Table 1 gives the mass loss rate and terminal velocity, the blocking factor B, and the radius, velocity and F_a function at the critical point, for the five different values of A. The CAK results for the same star are also shown. The terminal velocity is much higher than in CAK, even for the smallest value of A. This means that simply including the angle integration in the radiation force increases the terminal velocity to a value more consistent with observations.

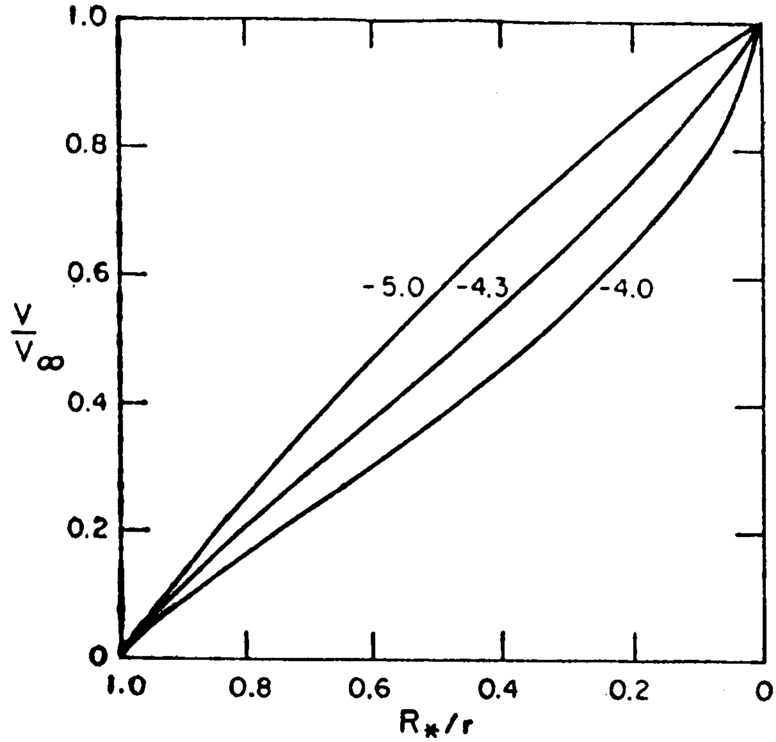

Figure 3. Velocity laws for the three cases shown in Fig. 2.

Table 1. Results of Model

log A	-5.0	-4.6	-4.3	-4.1	-4.0	CAK
$\dot{M}$ (10^{-6} $M_\odot$ yr^{-1})	0.59	2.7	8.6	18	28	6.6
v_∞(km s^{-1})	3350	3500	3900	4300	4600	1515
B	0.03	0.12	0.32	0.49	0.62	-
10B/3(1-B)	0.103	0.455	1.57	3.20	5.44	-
$\dot{M}v_\infty c/L$	0.101	0.483	1.71	3.95	6.58	0.511
r_c/R_*	1.05	1.05	1.06	1.06	1.07	1.50
v_c (km s^{-1})	250	255	274	288	307	950
$F_a(r_c)$	0.65	0.65	0.66	0.66	0.67	-

The mass loss rate for the log A = -4.3 case (which corresponds most closely to CAK) is only slightly higher than the CAK value, so line overlap does not have a significant effect. The variation of $\dot{M}$ with A is due to the increase in the number of lines. What is unexpected is that $\dot{M}$ does not saturate as line line overlap becomes more pronounced. This is probably due to the assumption of random line spacing, as clustering of lines tends to reduce the force [see Olson, 1982]. Another result of Table 1 is that the blocking factor correlates well with the "wind efficiency" $\varepsilon = \dot{M}\, v_\infty c/L$ (see Table 1):

$$\frac{B}{1-B} \approx 0.3\ \varepsilon \quad . \tag{8}$$

For blocking factors approaching one, ε can become very large, so winds with multiple scatterings are much more efficient in ejecting matter than single-scattering winds. This may be especially relevant for Wolf-Rayet stars.

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