NASA CONTRACTOR REPORT 166533

Analytical Model of Rotor Wake Aerodynamics in Ground Effect

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CHAPTER 1

INTRODUCTION

Since helicopters and other VTOL aircraft are designed to take-off and land vertically, to hover for rescue attempts and other purposes, and also to be able to fly close to the ground for a long time, study of their performance in ground effect is very important.

Calculation of stability and control derivatives of helicopters and aircraft requires knowledge of the behavior of aerodynamic forces and moments for different flight conditions. Since the wake of fixed-wing aircraft is left behind in forward flight, there is no interaction between the newly generated vortices and old vortices. This simplifies the wake study of fixed-wing aircraft compared with helicopters. There are three major differences associated with helicopter wakes.

First, the wake of a helicopter does not move away from the rotor as it does in conventional airplanes. In slow forward flight, the vortices shed from the blade tips move downward below the rotor as a helix. Secondly, the problem becomes more complicated as the helicopter approaches the ground. In the presence of the ground the wake contacts the ground, rebounds and in certain flight conditions is drawn through the rotor again. Thirdly, the wake can roll up in front and around a helicopter like a horseshoe. This vortex (which is close to the ground) is called a "ground vortex". As speed increases a helicopter overruns the ground vortex ahead of it causing a transient disturbance well known to pilots.

All these problems complicate the study of helicopter aerodynamic forces and moments. Most of the investigations of the helicopter wake in ground effect have been experimental and qualitative, although there has been some theoretical study of hover in ground effect by modeling the wake as flat vortex rings. So far, there has not been a successful study of the wake in ground effect which covers all flight conditions because of difficulties mentioned previously. The current study has been made to address these problems.

Precise mathematical description of the flow field for a rotor is made difficult by the interdependence of the velocity and the wake position. That is, the boundary condition is known but the location of the boundary is not known. This situation is characteristic of problems that can be generally classified as "free-boundary problems". The method of solution that can sometimes be used on this class of problems is to guess the position of the boundary, compute the solution, and determine if the computed solution is consistent with the assumed boundary location. This approach is based on the use of space coordinates as independent variables; computation is cast as some sort of feedback scheme in which the differences between the assumed boundary location and the resulting solution are used to determine a new estimated boundary position. Advantage is taken of the azimuthal periodicity of the rotor position and independent variable is chosen to be time (or azimuth angle of a reference blade). The point of view, then, is that new elements of wake vorticity are generated as the blades rotate and translate. These elements are onvected with velocities determined by their self-induced velocities as well as velocities induced by existing wake structure, the bound vorticity of the blades, and the image vortices introduced to satisfy the ground-plane boundary condition. It is assumed that eventually a stabilized periodic wake array solution can be obtained, since the condition of a fixed periodic blade loading was imposed.

The continuous distorted helix was selected as the model for the current effort. The ground boundary condition was enforced by influence of an image wake.

In the present study it has been found that the distorted helix model, in the absence of ground yields average velocities that agree well with measurements made in and about a helicopter rotor wake (reference [2]). There is a good qualitative check on the wake of a helicopter in ground effect, but no check could be made quantitatively because these measurements are not available.

The digital computer program is based on the assumption that the rotor is in steady level flight (or hovering) with a specified tip-path-plane angle. Shaft rotational speed, rotor force, initial vortex core radius, number of blades and the ratio of the rotor radius to height above the ground are also inputs. Information relative to the computing program is given in Appendix B.

It is the purpose of this study to provide the numerical methods from which to calculate aerodynamics (stability derivatives) necessary for the modeling of helicopter dynamics needed for design of stability and control systems.

CHAPTER 2

IDEALIZED MODEL

Computations of aerodynamic forces, moments, and stability derivative of helicopter directly depend upon induced velocity at the rotor disc. As induced velocity is a consequence of a wake structure, it is necessary to have a wake structure which gives relatively accurate velocity on the rotor disc.

In addition it is important to make the model as simple as possible to reduce expensive computation time. In the following sections a wake model will be presented and studied. Section 2-1 describes and compares the prescribed and free wake analysis. Sections 2-2 through 2-7 explain the free wake structure and discuss the reason for keeping important parts of the wake and ignoring unimportant parts of the wake. In section 2-8 the basic formula to calculate induced velocity of a rectilinear vortex segment at a point in the space will be formulated. Section 2-9 describes the structure of the computational model, and in section 2-10 the induced velocity formula will be modified for special cases. Section 2-11 considers the inboard wakes and finally discusses the reasons for considering only one of the inner wakes.

2-1 FREE WAKE ANALYSIS AND PRESCRIBED WAKE ANALYSIS.

Wake analysis of helicopters has been the topic of many researchers for many years. It is one of the most important aspects of helicopter study, because the majority of the characteristics of a helicopter depends on its wake structure.

Among the many methods of analyzing helicopter wakes, two methods currently are employed by the majority of helicopter researchers: prescribed wake analysis and free wake analysis.

In the former method, for each flight condition the location and intensity of the vortices of a wake are measured. Having these locations and intensities, aerodynamic forces and moments are computed. As previously mentioned, for each flight condition a number of experiments for different points in the space must be carried out. An increase in accuracy can be obtained by increasing the number of points on the wake to be examined.

In the free wake analysis, an initial wake with some initial properties is assumed. Then it is possible to compute the induced velocity of any point in the space, including the points on the wake. The new location of the wake solution can be obtained after an increment of time by using velocities of the points on the wake with simple forward integration. Also the properties of the wake can be modified to be consistent with the new wake. This procedure can be repeated until the wake location stabilizes and reaches its periodic steady state.

Similar to the prescribed wake analysis, free wake analysis accuracy can be increased by increasing the number of the segments. Because the computation time will grow exponentially with increase in number of segments, there is a practical limit on the number of segments.

To study the wake in ground effect, the detailed analysis of the prescribed wake is prohibitively expensive in terms of manpower required and difficulties in measuring the location and circulation of the wake segments close to the ground, especially in presence of a ground vortex. Free wake analysis is considered to be a better alternate for the wake analysis, because it can cover all the points in the wake including the points close to the ground in almost all flight conditions, and it is more efficient and less expensive. For these reasons the free wake method was chosen to study the ground effect.

2-2 WAKE STRUCTURE.

In a given flight condition, the wake of a helicopter contains several different vortices. Considering potential flow around an airfoil, each blade may be replaced by a vortex line having the same lift and approximately the same flow field. This vortex line is called a bound vortex. A complete model of a bound vortex for each blade consists of radial variation of circulation, as well as tangential variation with azimuth angle.

Due to steep change in circulation at the tip of a blade, tip vortices are generated; and because of the variation of vorticity along each blade, inboard vortex filaments parallel to tip vortices are released. Blade pitch angle variation or velocity change of a blade results in vortex shedding by the rotating rotor blades (figures 2-1 and 2-2).

Near the ground, a mirror image of the whole wake must be added to satisfy the boundary conditions on the ground.

Consideration of the ground effect will result in large increases in computation time for the following reasons:

i)Computation of the induced velocity of the mirror image wake doubles the computation time.

ii) The number of points for wake analysis in-ground-effect should be more than out of ground effect wake study. The reason for this increase is that during flight out of the ground effect, as time passes the vortices move away from the rotor and their effects can be neglected; whereas in ground effect, old vortices may hit the ground, bounce back and in some flight conditions interact with the wake and cause major variations in forces on the rotor. Therefore it is very important to keep the effect of the old wake which can have a significant effect on the velocity distribution of the rotor.

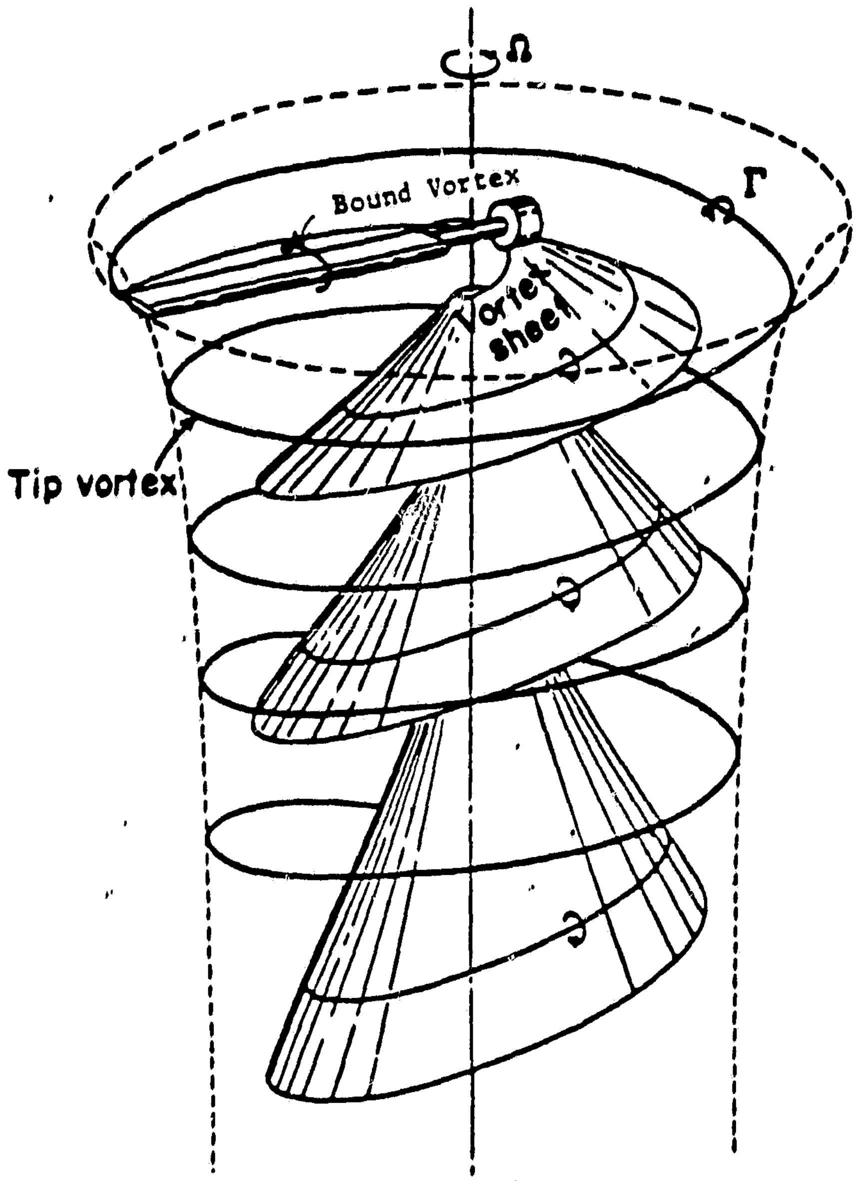

Figure 2-1. Schematic of Rotor Wake Structure.

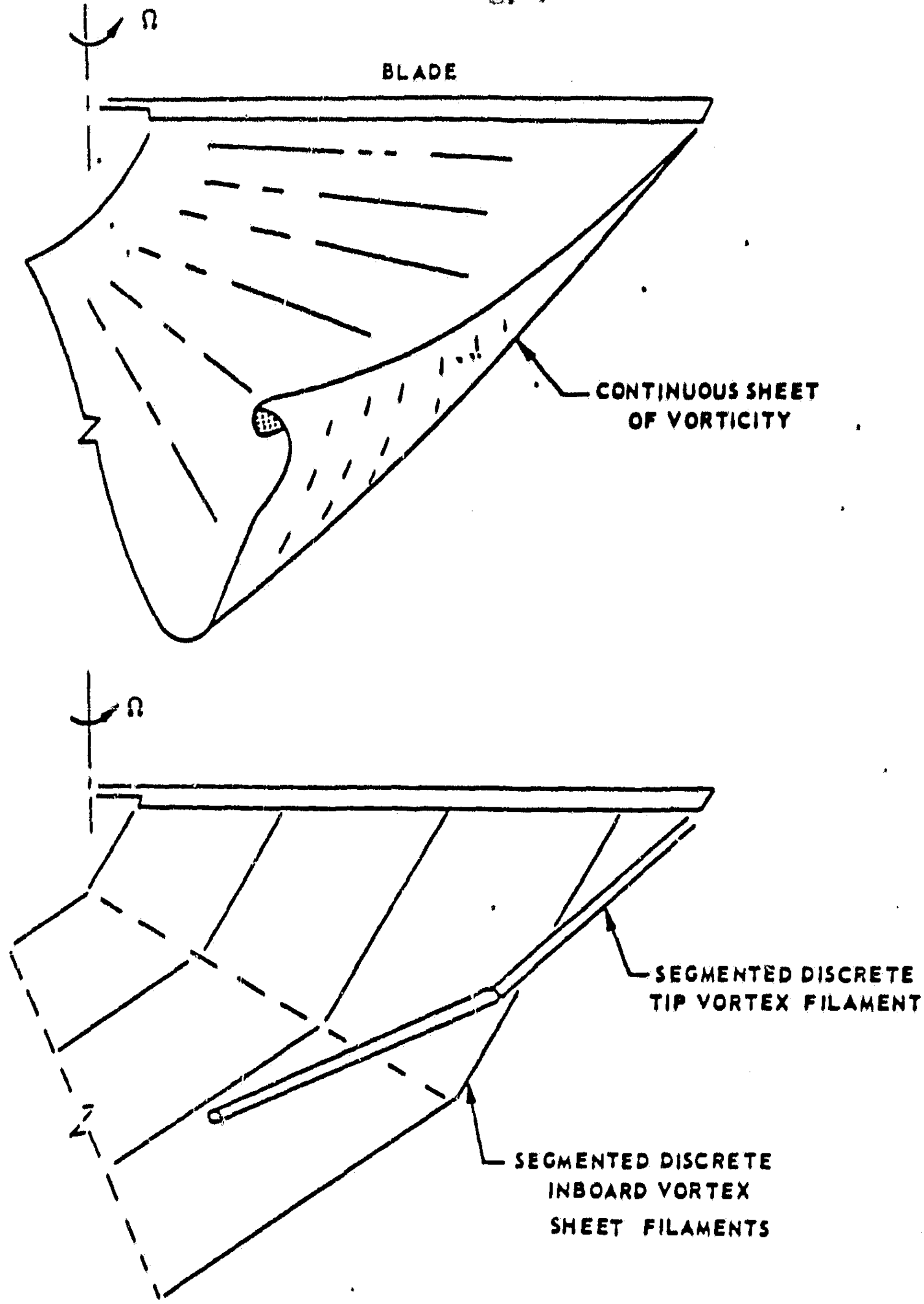

Figure 2-2. Segmented Discrete Vortex Representation of the Wake.

A detailed analysis of the whole wake is practically impossible especially in the presence of the ground and ground vortex. Therefore, it is important to make the model as simple and as accurate as possible utilizing the parts which describe the wake efficiently for velocity computations.

2-3 TIP AND ROOT VORTICES

As was mentioned in the previous section, a quick change in circulation close to the tip of the blade causes a steep continuous sheet of vorticity close to the tip to be released (see figure 2-1 to 2-3 for continuous sheet vortices and discrete filaments). Flow visualization studies indicate that this sheet of vorticity rolls up within a few chord lengths of the blade and forms a single concentrated vortex line. To avoid complexity it will be assumed that tip vortices are fully rolled up from beginning as they are released. Tip vortices are the most important part of the wake, as they carry a considerable amount of energy. The velocity distribution of the rotor disc greatly depends on the locations and strengths of the tip vortices. Therefore the primary goal is to compute the locations and strengths of the tip vortices.

Similar but weaker vortices are created near the blade root, however analytical and experimental studies have shown that they rapidly dissipate. Even if root vortices are considered, their contribution to the induced velocity is negligible. For this reason, as well as the savings in computation time, they will not be considered.

2-4 BOUND VORTICES

Among many models available for bound vortices, the simplest one has been chosen to replace the blades and their images. Each blade is replaced by a single radial vortex line with constant vorticity along its length. The strength of these vortices is computed by assuming the total load on all the blades approximately equals the helicopter weight. The reason for choosing such a simple model is to avoid large increase in computation time.

2-5 INBOARD VORTICES

As previously mentioned, the accuracy will increase with utilization of more realistic models. The mathematical model will be more realistic if large number of inboard vortex filaments are included. Consideration of even a few inner vortices will result in an increase in computation time by a large factor. Therefore it is desired to include as few inner wake filaments as possible. It will be shown that if only one inside vortex filament at r/R=0.7 is included, the accuracy of the velocity close to rotor plane will improve considerably. Also computation time is only increased slightly.

2-6 SHED VORTICES

In forward flight, the pitch angle of each blade will vary periodically (once per revolution). This variation will result in variation of circulation of each blade with azimuth angle of the rotor. Consequently, periodic variation in blade circulation results in release of a continuous vortex sheet parallel to the blade which may be modeled as vortex filaments. Fortunately, during flight conditions in which ground effect is important, the vortex filaments are not very strong and they can be neglected.

2-7 MIRROR IMAGE VORTICES

To satisfy boundary conditions on the ground, it can be assumed that corresponding to each blade bound vortex and each wake segment, there is an image vortex with opposite circulation below the ground at the same distance. The induced velocity of the image segments at points close to the rotor disc is small and for the points close to the ground, the induced

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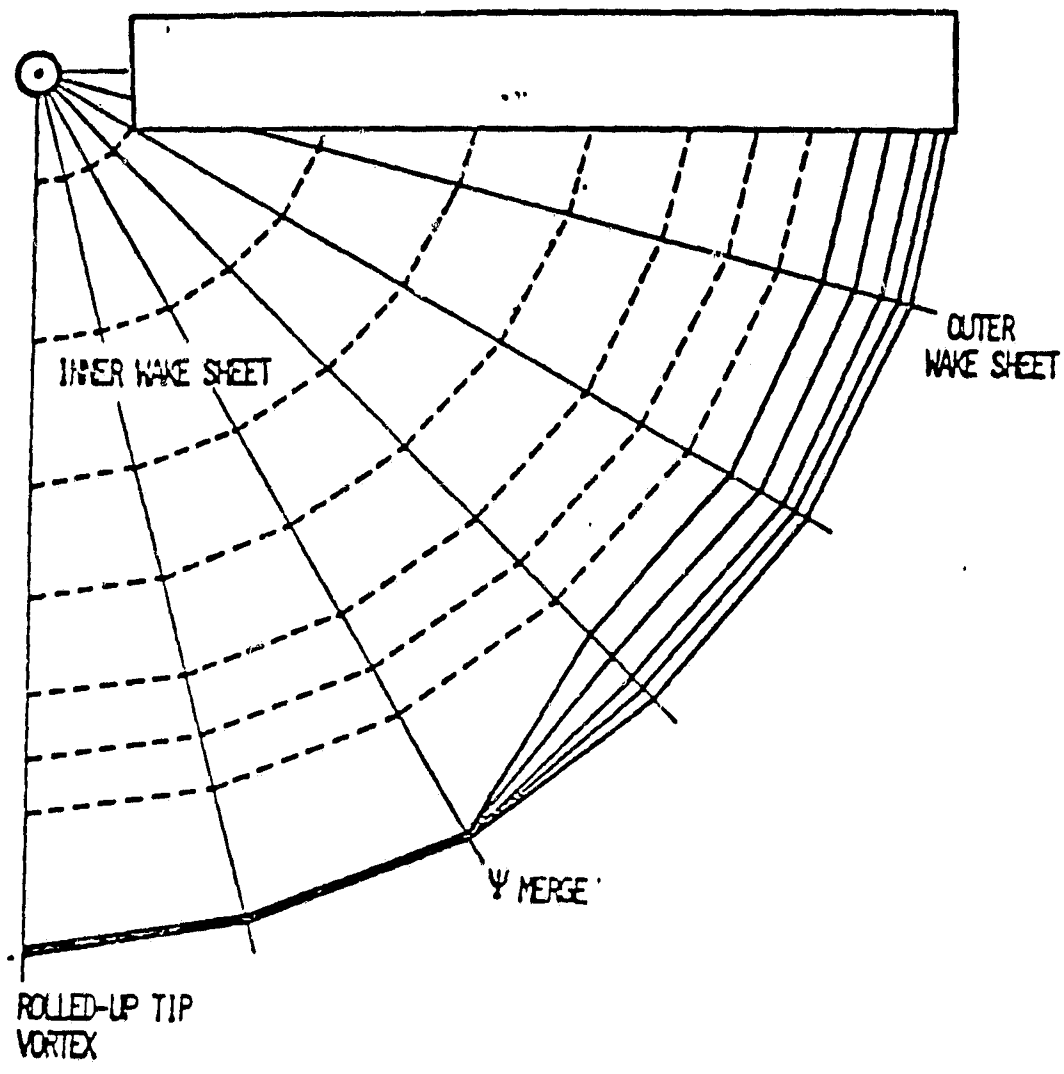

Figure 2-3. Rotor Wake Geometry (Top View).

velocity is large. This effect forces the wake to satisfy the boundary condition on the ground and consequently forms the wake close to the ground. In some flight conditions the wake close to the ground rolls up, passes through the rotor and forms a new configuration for the wake (ground vortex). Therefore, it is necessary to take into account the image wake as an important part of the whole wake.

2-8 FORMULATION OF THE MODEL

Formulation of the flow corresponding to the simplified model can be accomplished as follows: A coordinate system fixed in the tip path plane, a plane which passes through tip of the blades, is introduced (figure 2-4).

The vector $\bar{V}_f$ in Figure 2-4 is the free-stream velocity (the negative of the translational velocity of the aircraft), inclined at an angle α_T to the X-axis and parallel to the X-Z plane. The angle ψ denotes the azimuthal positioning of a given point with respect to the origin.

The air velocity $\bar{V}_p$ at a given point located by the vector $\bar{r}_p$ may be expressed in the form

$$V(\bar{r}_p) = -\frac{1}{4\pi}\int_{C_v} \frac{\Gamma(r)\,\bar{r}_1 \times d\bar{r}}{r_1^3} \tag{2-1}$$

where $\bar{r}_1 = \bar{r} - \bar{r}_p$ and $\Gamma(r)$ is the circulation about the vortex element at r. The line integral is to be taken over all vortices in the flow and the image system whose paths are collectively denoted by C_v. It is necessary to modify this expression when $r_p = r$ and this point is discussed later. The

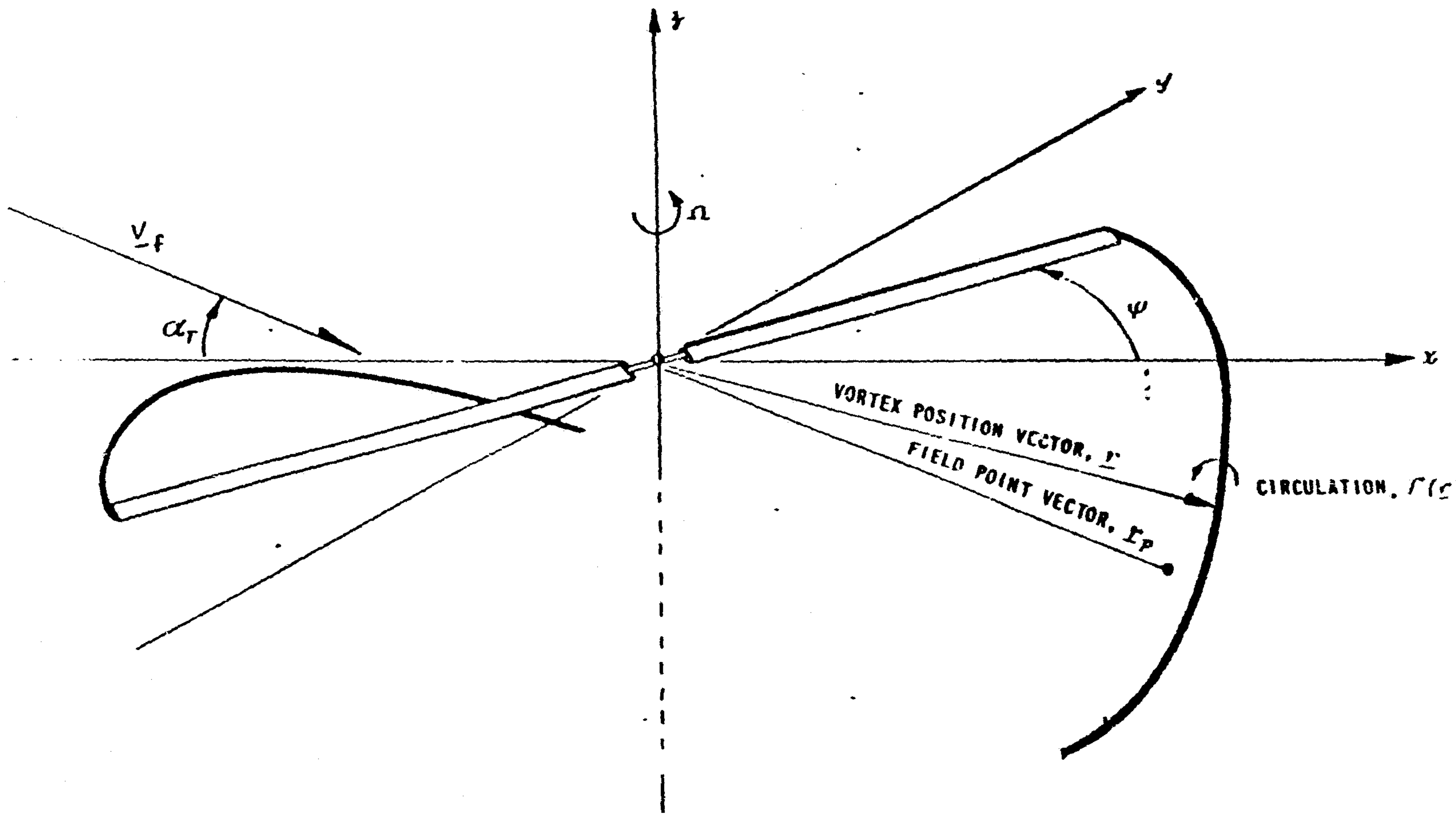

Figure 2-4. The Coordinate System

circulation, Γ , about the blade vortices is estimated directly in terms of flight parameters. The circulation about a wake vortex at a given point is simply that value assigned to the blade vortex when it was generating that segment of the wake.

The only information lacking for the complete specification of the flow at a given instant, then, is the location of the wake vortices at that instant. The position of a given point on a wake vortex located by the vector r is the time integral of the velocity experienced by that fluid particle

$$r(t)=r(t_\circ)+\int_{t_\circ}^{t} V(r(\tau))\ d\tau \qquad (2\text{-}2)$$

Thus, even after being simplified, the flow can only be obtained as the solution of the nonlinear integral formed by the substitution of Equation (2-1) into Equation (2-2). A direct analytical solution is not feasible, but the problem is amenable to solution by numerical methods using a high speed computer. The manner in which the formulations of Equations (2-1) and (2-2) were implemented for digital computation will be discussed in the following sections.

2-9 DESCRIPTION OF COMPUTATION MODEL

For the purpose of numerical analysis, the wakes produced by each blade are divided into small segments (figure 2-5). These segments are chosen to be sufficiently short so that, they can be considered rectilinear vortices having constant circulation along their lengths for purpose of computation of the induced velocities.

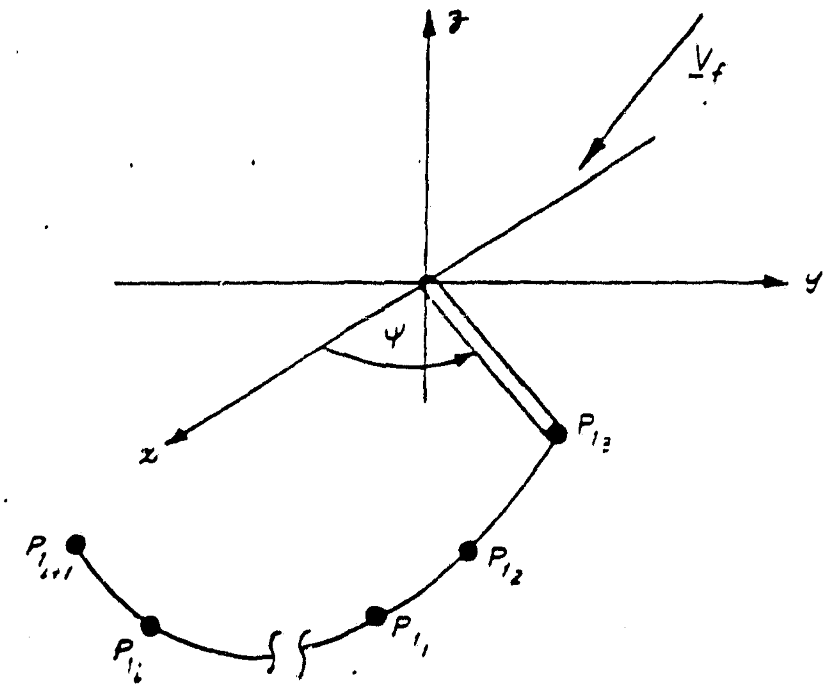

Figure 2-5. Wake Reference Point Identification

Each segment is defined by its two ends. Segment S_i is between point i and point i+1. The wake configuration at any instant is defined by the location of these end points.

Computation is initiated by specifying that each of the wake vortices lies on a prescribed curve. The curve is chosen to be a single vertical helix, or a previous solution for a flight condition close to the required one. The numerical version of equations (2-1) and (2-2) are then performed by first summing the velocity contribution of all the vortex elements in the flow at each reference point (Equation 2-1), and then using these velocities to compute the new location for each point for a time interval Δt (Equation 2-2). The time interval Δt is chosen to correspond to a small finite change in the azimuth position ψ of the blades.

$$\Delta \psi = \Omega \, \Delta t \qquad (2\text{-}3)$$

where Ω is rotor angular speed. Once the new coordinates of each reference point is computed, the azimuth of the blade vortices is increased by $\Delta \psi$ and the velocity computation is performed again. As the blade vortices are repositioned, a new wake vortex element is added to the flow at the tip of each blade vortex, the added vortex having a length of approximately $R \Delta \psi$, where R is rotor radius. A corresponding element is dropped from the computations at the downstream end of each wake vortex to maintain a wake of constant size. The computations are continued in this manner for a specified time and the results are inspected. If a nearly periodic solution is established in the space volume of interest, the calculation is terminated.

The total number of the wake vortices taken into account and the magnitude of $\Delta \psi$ determine the accuracy of the flow representation at a given point. It is believed that, for a two-blade rotor, a value for $\Delta \psi$ of thirty degrees is sufficiently small to furnish an acceptable estimate of the time variations of the flow consistent with the other approximations introduced. The number of wake elements to be considered depends on the region of interest, forward speed and height of the rotor above the ground. If the free stream does not clear the wake under the rotor, the number of wake elements must be sufficiently large to include all the wake elements close to the rotor. This phenomenon greatly depends on forward speed and height of the rotor above the ground.

2-10 VELOCITY INDUCED AT POINT P BY THE WAKE AND THE BLADES.

The velocity induced at an arbitrary point P by the vortices representing the wake and the blades is simply the sum of the effects of an array of rectilinear vortex segments. If V denotes the velocity induced at P by the elements between points P_1 and P_2, it is found from equation (2-4) (Ref. [1] Page 152) that

$$V = \Gamma (\cos\theta_1 - \cos\theta_2)/(4\pi h) \qquad (2\text{-}4)$$

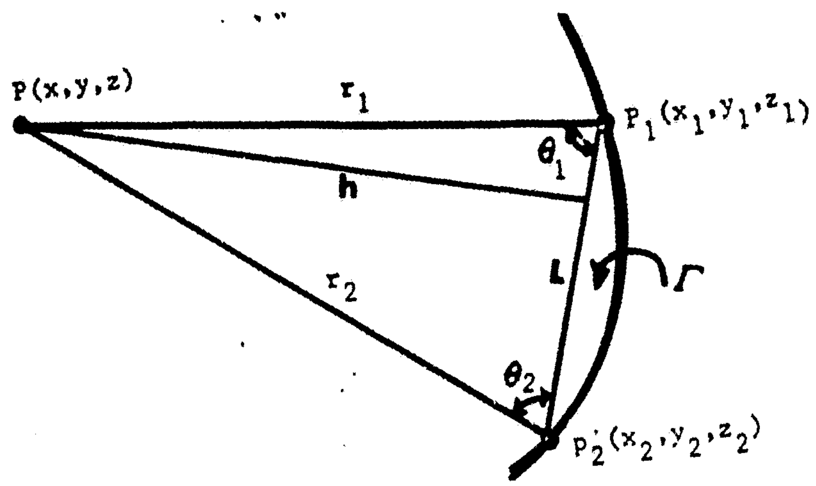

Figure 2-6. GEOMETRIC RELATIONSHIPS DEFINING THE FLOW INDUCED BY A RECTILINEAR VORTEX ELEMENT

where Γ is the strength of the element and θ_1, θ_2 and h are defined in figure 2-6. The velocity is directed normal to the plane containing P_1, P_2 and P.

As the field point P is made to approach any point on the line joining P_1 and P_2 the induced velocity increases without limit, because h tends to zero (Equation 2-4); the velocity becomes indeterminate for h=0. Because the velocity of air can not reach infinity, another model is employed for small h.

Experimental studies [2] of the structure of trailing vortices show that for sufficiently small h, the flow rotates as a rigid body. The region where vortex filaments have rigid rotation is called the core. Among many models suggested for Γ of the core, Scully's model [3] is believed to give the best results.

$$\Gamma_c = \frac{(h/a)^2}{1 + (h/a)^2} \tag{2-5}$$

Here a is defined as the radius of the core. The Scully model approaches a potential flow just a few core radii away from the filament. Because of favorable comparison with experimental data and smoothness for small h [4], the Scully model was used in all calculations. Figure 2-7 compares the normalized vorticity and normalized velocity of the Scully model with wind tunnel experimental data [2]. One of the advantages of this model over other models is that there is not a sharp change in velocity profile at the boundary of the core. This smooth behavior helps to avoid large changes in

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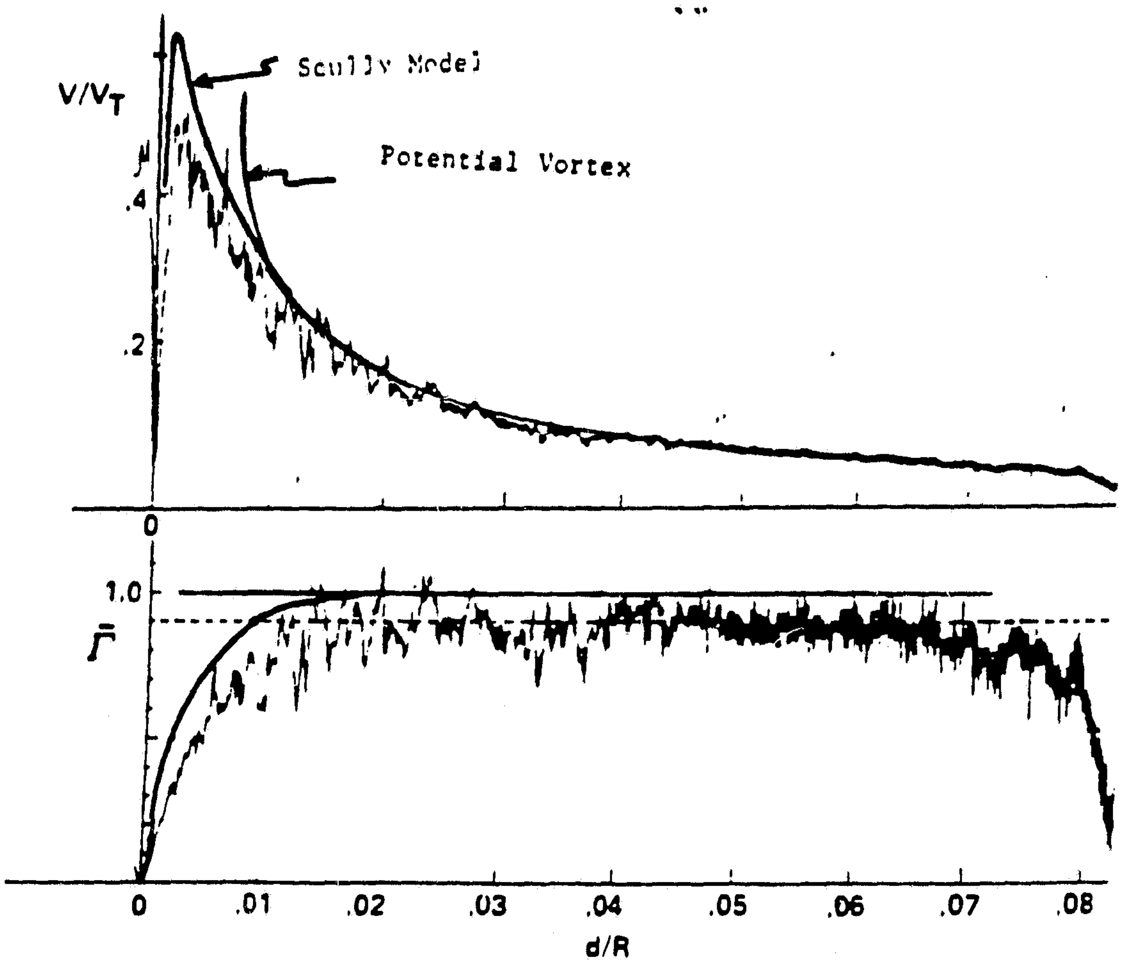

Figure 2-7. Comparision Of Experimental Data And The Scully Core Model

computation of the induced velocity at a point close to a vortex filament which may result in numerical instability.

In previous formulations, point P was not assumed to be one of the end points of vortex segments. If point P is one of the end points, then it should be assumed to be a point on an arc segment containing the two segments which P is one of their ends and undergoes self-induced velocity.

The self-induced velocity of the two segments is formulated in appedix A. The velocity of the point P_2 is considered to be the same as self-induced velocity of an arc consisting half of S_1 and half of S_2.

$$V = \Gamma_1 [Ln((8R/a_1)*Tan(\phi_1/4))-1.]/(8\pi R) +$$

$$\Gamma_2 [Ln((8R/a_2)*Tan(\phi_2/4))-1.]/(8\pi R) \qquad (2\text{-}6)$$

where ϕ_1 and ϕ_2 are defined in figure 2-8. The self-induced velocity is directed normal to the plane of the arc of the two segments. The approximate core radius of a given element may be assigned on a rotational basis using energy considerations.

2-11 INNER WAKE

For computation of the stability derivatives the induced velocity on the rotor disc should be known more accurately than at the points far from it. To improve on simple model of the blade with constant vorticity along its length, the blades are discretize into tiny segments and all small inboard wake filaments parallel to the tip vortices are considered. Since computation time limits the total number of the segments only one inside wake at r=0.7R with $\Delta\Gamma$ =0.5 was assumed. Since the slope of the circulation along the blade is steeper around r=0.7R (Figure 2-9) consideration of an inside wake at this point is a better representation of the wake. It is expected that this model will have better results than any other single inside wake.

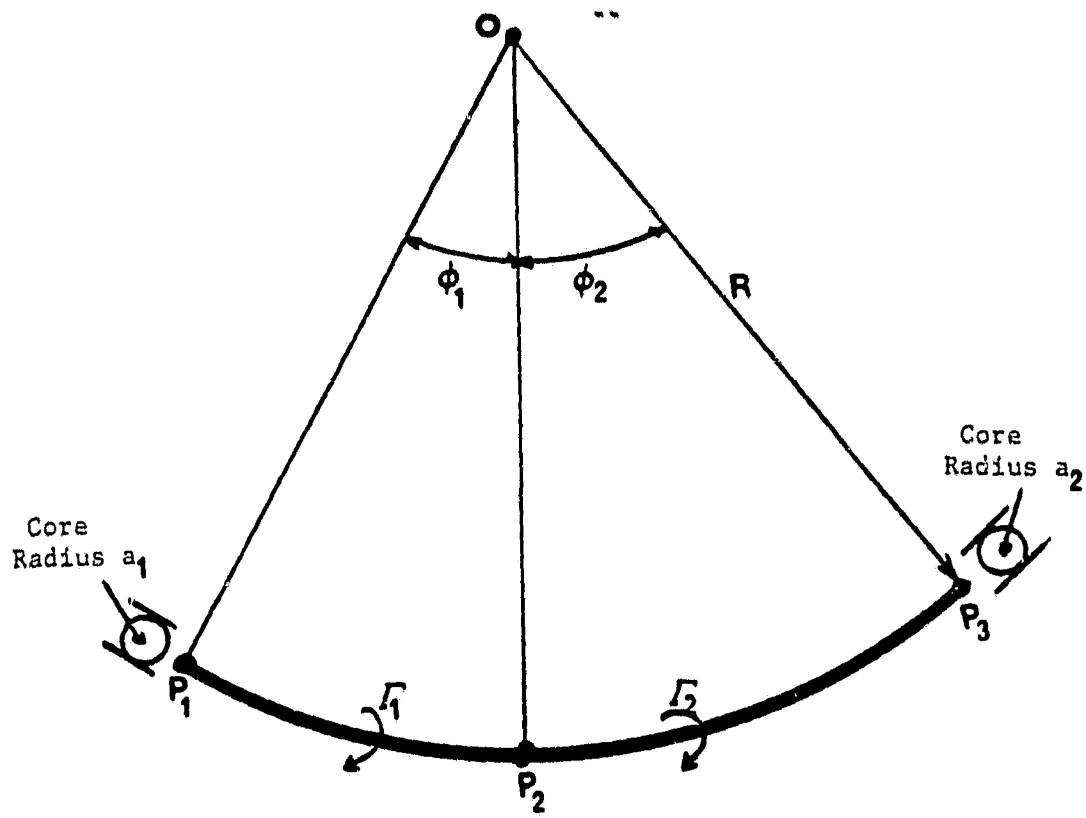

Figure 2-8. GEOMETRIC RELATIONSHIPS DEFINING THE SELF-INDUCED VELOCITY AT WAKE POINT P_2

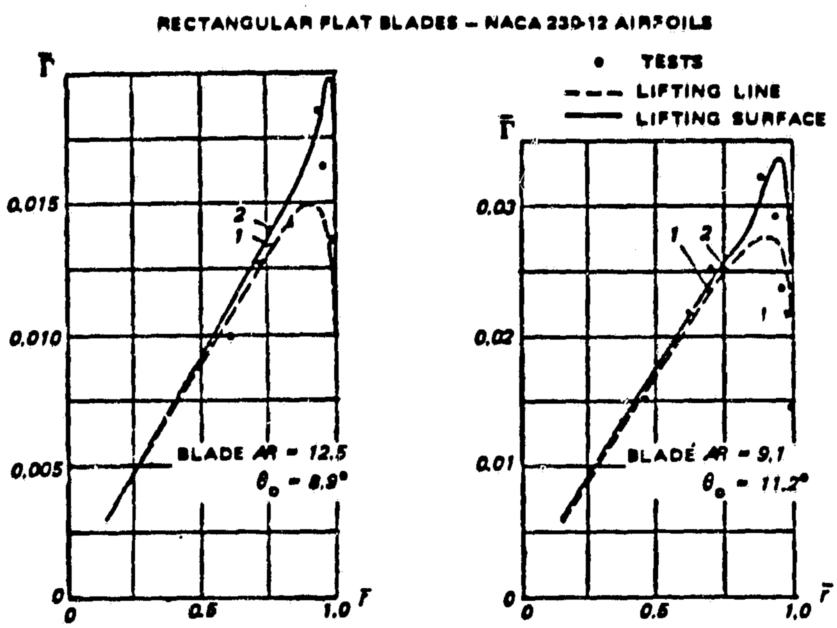

Figure 2-9. Normalized Vorticity Distribution Along A Blade

CHAPTER 3

NUMERICAL FORMULATION

The models of the rotor, the wake and image wake described in the previous chapter were formulated as countinuous functions of time. A digital computer can not integrate continous function exactly, therefore step-wise and interpolative approximations have to be made.

A rectangular integration scheme is used in performing integrations in time. That is, when integrating velocity to compute displacement, the velocity is assumed to remain constant over the interval of time corresponding to a small finite change in the azimuth position of the blades.

Spacial integrations over the wake vortices are performed by assuming that these vortices are made up of small rectilinear vortex segments whose circulation is constant form one point to the next. The position of the wake is then defined by the location of the end points of these segments. Consistent with the approximation made in the time integration, the initial length of each wake segments the length of the arc swept out by the blade tip over the interval used for time integration. Self-induced effects at a given wake point are computed by taking, as the local curvature, the reciprocal of the radius of the circle passing through the wake point in question and the two wake points adjacent to it.

The basic equation for velocity computation of the main wake (tip vortices), inner wake, bound vortices and image wake, at an arbitrary point P was taken as

$$V=\Gamma\,(\cos\theta_1-\cos\theta_2)/(4\pi h)$$

and for self-induced velocity to be

$$V = \Gamma_1 [Ln((8R/a_1)*Tan(\phi_1/4)-1.]/(8\pi R)+$$

$$\Gamma_2 [Ln((8R/a_2)*Tan(\phi_2/4)-1.]/(8\pi R)$$

These two equations will be formulated in three dimensional space in terms of coordinates of the segments and their vorticities, so the velocity components induced by different segments at a point can be summed.

In this chapter all necessary formulas required by the computer code will be derived and all the flight parameters will be calculated.

3-1 INDUCED VELOCITY OF A VORTEX SEGMENT AT POINT P IN THREE DIMENSIONS.

In equation (2-4) the velocity vector induced by segment S at the point P is perpendicular to a plane containing the segment and point P. As we go from one segment to another the direction of the plane changes, consequently the velocity vector will change direction too. In order to calculate the total velocity at point p, it is necessary to formulate the components of velocity vector. The components of the total velocity is the sum of the components of induced velocities of all the segments at p.

In three dimensional space, point P is defined as P(x,y,z). The two ends of segment S, are points P_1 and P_2 with coordinates (x_1,y_1,z_1) and (x_2,y_2,z_2) respectively.

The velocity vector V at point p is perpendicular to the plane of P, P_1 and P_2. The unit vector in velocity direction may be calculated as:

$$\hat{\rho} = (\bar{r}_1 X \bar{L}) / \left|\bar{r}_1 X \bar{L}\right| \qquad (3\text{-}1)$$

also

$$\left|\bar{r}_1 X \bar{L}\right| = r_1 L \sin\theta_1 = h\, r_1 \qquad (3\text{-}2)$$

assuming

$$\bar{r}_1 \times \bar{L} = \nu_1 \hat{i} + \nu_2 \hat{j} + \nu_3 \hat{k}$$

The unit vector can be decomposed as

$$\rho_k = \nu_k / (L\,h) \qquad k=1,2,3 \qquad (3\text{-}3)$$

ν_1, ν_2 and ν_3 are the components of vector $(\bar{r}_1 \times \bar{L})$ in X, Y and Z directions. Using equation (3-3) in (2-4), components of induced-velocity of a segment at point p may be written as

$$V_k = \Gamma (\mathrm{Cos}\,\theta_1 + \mathrm{Cos}\,\theta_2)\, \nu_k / (4 \pi L h^2) \qquad k=1,2,3 \qquad (3\text{-}4)$$

where $\mathrm{Cos}\,\theta_1 + \mathrm{Cos}\,\theta_2$ and h may be calculated in terms of r_1, r_2 and L:

$$\mathrm{Cos}\,\theta_1 + \mathrm{Cos}\,\theta_2 = (r_1 + r_2)[L^2 - (r_1 - r_2)^2]/(2\, r_1 r_2 L) \qquad (3\text{-}5)$$

$$h^2 = [(r_1 + r_2)^2 - L^2][L^2 - (r_1 - r_2)^2]/(4L^2) \qquad (3\text{-}6)$$

By substituting (3-5) and (3-6) in (3-4) components of induced velocity can be computed in terms of coordinates of points p, p_1, p_2 and Γ.

$$V_k = \Gamma (r_1 + r_2)/[2 \pi r_1 r_2 ((r_1 + r_2)^2 - L^2)]\, \nu_k \qquad k=1,2,3 \qquad (3\text{-}7)$$

where

$$r_1^2 = (x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2$$

$$r_2^2 = (x - x_2)^2 + (y - y_2)^2 + (z - z_2)^2$$

$$L^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

and

$$\nu_1 = (y-y_1)(z_1-z_2)-(z-z_1)(y_1-y_2)$$

$$\nu_2 = (z-z_1)(x_1-x_2)-(x-x_1)(z_1-z_2)$$

$$\nu_3 = (x-x_1)(y_1-y_2)-(y-y_1)(x_1-x_2)$$

3-2 SELF-INDUCED VELOCITY IN THREE DIMENSION

In equation (2-6) the velocity vector $\ddot{V}$ is perpendicular to the plane containing segment S_1 and S_2. The unit vector in velocity direction can be computed similar to the method given in the previous section.

$$\hat{\rho} = (\bar{L}_1 \times \bar{L}_2)/|\bar{L}_1 \times \bar{L}_2| \qquad (3\text{-}8)$$

If

$$\nu_1 = (y_1-y_2)(z_2-z_3)-(z_1-z_2)(y_2-y_3)$$

$$\nu_2 = (z_1-z_2)(x_2-x_3)-(x_1-x_2)(z_2-z_3)$$

$$\nu_3 = (x_1-x_2)(y_2-y_3)-(y_1-y_2)(x_2-x_3)$$

then

$$\bar{L}_1 \times \bar{L}_2 = \nu_1 \hat{i} + \nu_2 \hat{j} + \nu_3 \hat{k}$$

and

$$\rho_k = {}_k/[\nu_1^2 + \nu_2^2 + \nu_3^2]^{1/2} \qquad (3\text{-}9)$$

the radius of the circle formed by S_1 and S_2 may be calculated from following formula.

$$R^2 = (L_1^2 * L_2^2 * L_3^2)/[(L_1 + L_2 - L_3)(L_1 + L_2 + L_3)(-L_1 + L_2 + L_3)$$

$$(L_1 - L_2 + L_3)] \tag{3-10}$$

where

$$L_1^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

$$L_2^2 = (x_2 - x_3)^2 + (y_2 - y_3)^2 + (z_2 - z_3)^2$$

$$L_3^2 = (x_1 - x_3)^2 + (y_1 - y_3)^2 + (z_1 - z_3)^2$$

ϕ_1 and ϕ_2 can be computed from

$$\phi_1 = 2 \sin^{-1} [L_1/(2R)] \tag{3-11}$$

$$\phi_2 = 2 \sin^{-1} [L_2/(2R)] \tag{3-12}$$

Substitution of equations (3-10) to (3-12) in (2-6) provides the components of the self-induced velocity of an arc containg S_1 and S_2 in terms of Γ_1, Γ_2, and the coordinates of p_1, p_2 and p_3.

3-3 VARIATION OF CIRCULATION WITH AZIMUTH ANGLE.

In hover with no wind, the velocity of the air with respect to a point on a blade remains constant with azimuth angle:

$$V(\psi, r) = \Omega r$$

Assuming that the induced velocity at the rotor disc is small in comparison with the velocity of the blade, Kutta-Joukowsky law may be used to calculate elementary thrust and elementary rolling moment:

$$dT = \rho \Gamma V dr$$

$$dM_x = \rho \Gamma V r \sin\psi dr$$

Integration of these two equations over the entire disc (with the assumption of constant vorticity along the blades and the fact that the thrust appoximately equals the weight of the aircraft) yeilds;

$$\Gamma = (2W)/(b \rho V_{t.} R) \qquad (3\text{-}13)$$

and

$$M_{x\ average} = 0$$

Here b is the number of the blades,

V_t is the velocity of the tip of the blades and

R is the rotor radius.

In the absence of the wind or in hover equation (3-13) may be used for computation of circulation Γ. But in the presence of the wind or in forward flight it may not be used, because the velocity of the air relative to a point on the blades is not constant and varies with azimuth angle.

$$V(\psi, r) = \Omega r + V_f \Omega \sin\psi \qquad (3\text{-}14)$$

Here V_f is the forward speed or wind velocity. There will be some average rolling moment per revolution if it is assumed that the vorticity does not vary with azimuth angle. Asymmetry in velocity profile with constant Γ is the cause of the appearance of the rolling moment. The rolling moment can be found by integrating the infinitesimal moment over the entire disc.

$$dT(\psi, r) = \rho \Gamma (\Omega r + V_f \sin\psi) dr$$

$$dM_x(\psi, r) = \rho \Gamma (\Omega r + V \sin\psi) \sin\psi \, r dr$$

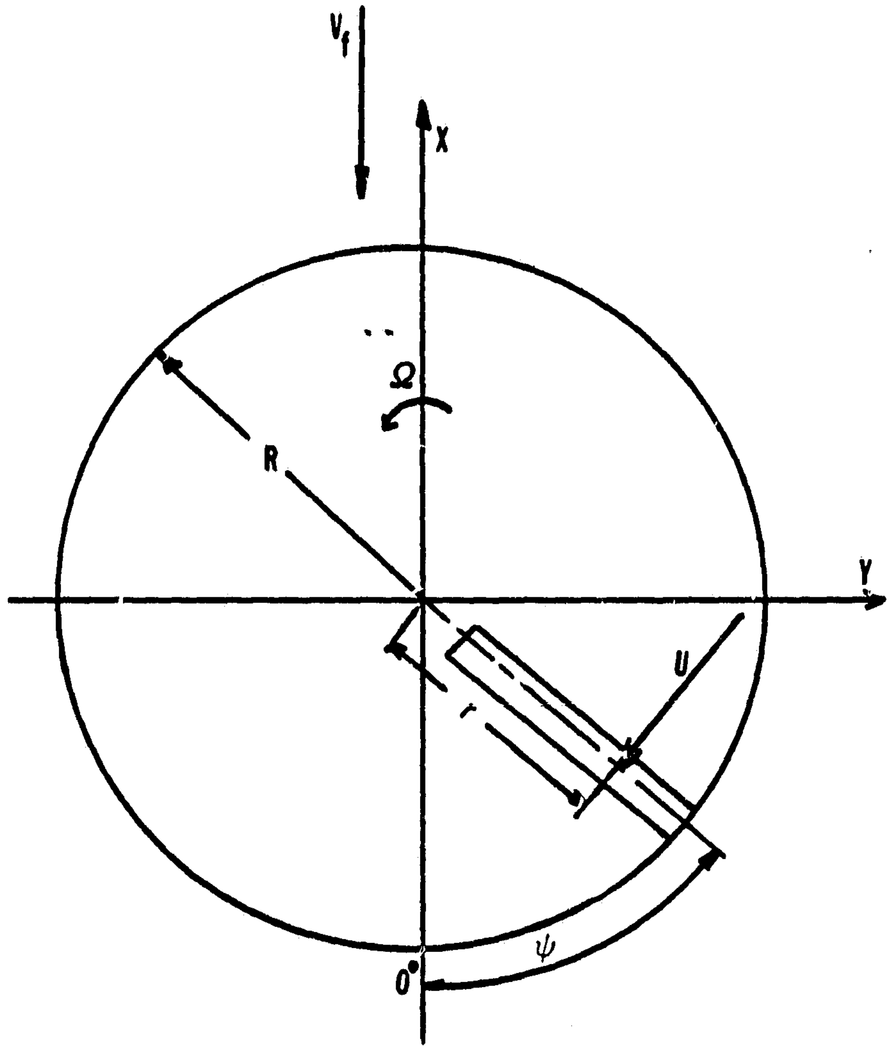

Figure 3-1. Rotor Disc In Forward Flight

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Integration along the blade at a constant azimuth angle provides:

$$T(\psi) = (\rho \Gamma R^2)(1+2\mu \sin\psi)/2 \qquad (3\text{-}15)$$

$$M_x(\psi) = (\rho \Gamma R^3)(1+1.5\mu \sin\psi)\sin\psi/3 \qquad (3\text{-}16)$$

where

$$\mu = V_f/(\Omega R)$$

Finally the average rolling moment per revoltion

$$M_{x\ average} = -b\rho\Gamma R^2 V_t/4 \qquad (3\text{-}17)$$

Thrust offset can be found by eliminating Γ between the thrust and the average rolling moment.

$$y_o = 0.5\mu$$

As forward speed or velocity of the wind increases the thrust offset increases too. Such large thrust offsets are not realistic for conventional helicopters, although they might be tolorated in some other configurations like side-by-side helicopters.

Elimination of average rolling moment can be achieved by postulating that the blade thrust moment with respect to its flapping axis remains constant.

$$M_f = (\rho\Omega R^3/3)\,\Gamma(1+1.5\mu \sin\psi) = \text{const.}$$

This requires that circulation to be

$$\Gamma = (\text{const.})/(1+1.5\mu \sin\psi) \qquad (3\text{-}18)$$

Equation (3-18) have been employed in the computer code for blade circulation at different ψ.

3-4 CORE SIZE

As a point approaches the center of a potential vortex, the velocity induced at that point by the vortex tends to go to infinity. Because the velocity of the air is finite and can not reach infinity, this tendency is not realistic. Therefore some modifications are needed to overcome this problem. In chapter 2 the Scully model [3]

$$\Gamma_c = \Gamma (h/a)^2 / [1. + (h/a)^2]$$

was assumed to have proper characteristics. This model required the knowledge of core radius and intensity of the vortices.

In reference [2] an average value for radius of fully rolled up tip vortices has been suggested as

$$a=0.003\ R \tag{3-19}$$

The results were obtained by experimental data and verified to be quite accurate for high aspect ratio rotor blades. In the present study, rotors have been assumed to have high aspect ratio blades, the tip vortices are assumed fully rolled up from the time they are generated, and the ground has no effect on the core size. With these assumptions the average value of a=.003 R was used as starting value for the vortex segments when they are at the tip of the blades. The core radii of the vortex segments at other places are calculated by assuming that the volume of a vortex segment remains constant as time passes. Therefore

$$a_2^2 = a_1^2\ L_1 / L_2 \tag{3-20}$$

where L is the length of the segments.

Considering the fact that there is no air where the blades are and the bound vortices are blade replacements, the core radius of the bound vortices was chosen to be one-half of the chord length.

Unlike the tip vortices or bound vortices, the inner wake is not a single concentrated vortex line but a continous sheet of vortices which was replaced by a vortex filament. The velocity induced by this filament at neighboring points is quite high considering the smooth velocity profile inside the wake. To overcome the unrealistic behavior around the center of this filament, the radius for core of these segments was chosen to be a=0.03R. A few other core sizes for inner wake were examined. The results have indicated the choice of the smaller radius will result in numerical instability which is one of the main difficulties of this approach.

3-5 INCREASE IN ACCURACY AND FASTER ALGORITHM

Another major difficulty of free wake approach is the consumption of a large amount of computation time. If N is the total number of the segments constructing the wake, the computation of velocity at only one point including the image wake requires 2N times calculation of equations (2-4) or (2-6) which themselves require number of operations. There are N points on the wake whose velocities are computed at each time slice. Therefore at each iteration $2N^2$ times equations (2-4) or (2-6) are computed. Considering time for integration of velocities and other procedures, computation time becomes proportional to N^3.

On the other hand, the accuracy of the calculations greatly depend on the number of the segments per revolution and number of the revolution in the wake. The shorter the segment length the better the results. Therefore there is a trade-off between accuracy and computation time.

Most of the contribution of induced velocity at a point comes from the segments in the vicinity of that point. To increase the accuracy of these velocity contributions, each segment close to the point of interest has been broken into smaller segments and velocity induced by each smaller segment has been completed and added together.

The segment division was done by passing a circle through two neighboring segments and dividing the arc passing through the end of each segment into two equal parts.

Also if $h/R > 2.5$, the contribution of induced velocity of this segment in comparison with the contribution of another segment with $h/R < 0.1$ is negligible. Therefore it is not necessary to compute the velocity contributions of segments with distances beyound $h/R=2.5$.

3-6 COORDINATES OF THE IMAGE WAKE

As previously mentioned, to include the effect of the ground on helicopter rotor aerodynamics a mirror image for the wake has to be assumed. The velocity induced by this wake is computed the same way as for the main wake with the exception of the self-induced velocity. The image wake was broken into segments exactly like the main wake, so that segments of the image wake are the mirror image of the segments of the main wake.

Assuming the rotor can only tilt forward, the coordinates of the mirror image of point P(x,y,z) for the rotor with α_t (Tip Path Plane angle) and H (the height above the ground) can be obtained from the following equations.

$$x_{mi} = x \, \mathrm{Cos}(2\alpha_t) - z \, \mathrm{Sin}(2\alpha_t) - 2 H \, \mathrm{Sin}\,\alpha_t \qquad (3\text{-}21)$$

$$y_{mi} = y \qquad (3\text{-}22)$$

$$z_{mi} = - x \, \mathrm{Sin}(2\alpha_t) - z \, \mathrm{Cos}(2\alpha_t) - 2 H \, \mathrm{Cos}\,\alpha_t \qquad (3\text{-}23)$$

x_{mi}, y_{mi} and z_{mi} are the coordinates of the mirror image of point p.

3-7 Γ ASSIGNMENT

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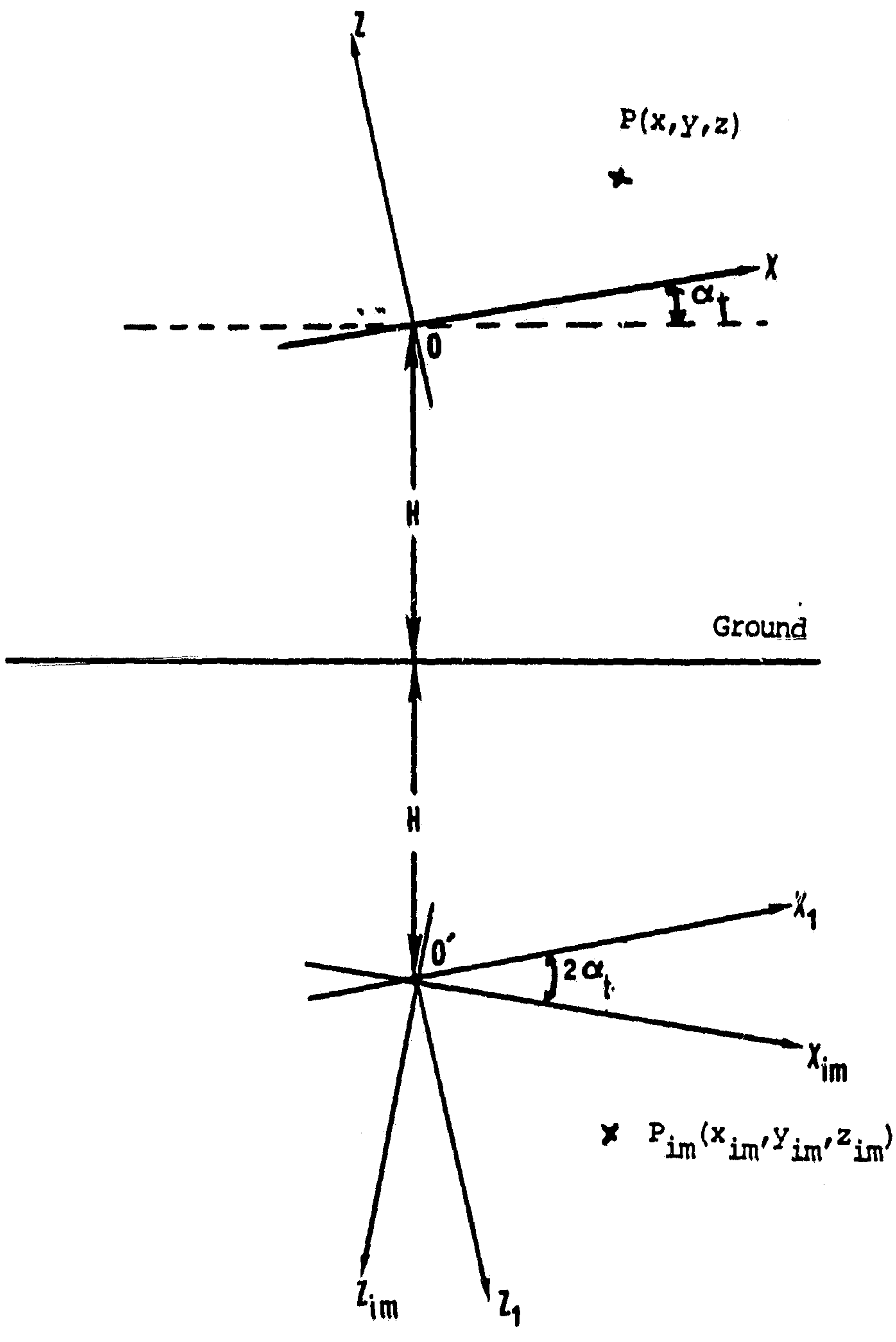

Figure 3-2. Coordinate System And Its Mirror Image

Theoretical studies of helicopter blades have indicated that there is a direct correlation between load of the rotor and the circulation of the blades. If for simplisity circulation is assumed to be constant along the blade, equations (3-13) or (3-15) can be used to calculate average circulation. For a two-blade rotor

$$\Gamma_m/(\Omega R^2) = \pi C_T \ldots \qquad (3\text{-}24)$$

where C_T is defined from

$$C_T = W/[(\pi R)^2 \rho (\pi R^2)]$$

Somewhat better results are obtained if elliptic profile is assumed for spanwise blade circulation.

$$\Gamma_m/(\Omega R^2) = 4 C_T \qquad (3\text{-}25)$$

However even more realistic and better results can be obtained using experimental data. Figure 3-3 was plotted using data extracted from reference [2]. The slope of a nearest straight line to the points extracted from experiments is assumed to be a better relation between C_T and Γ_m.

$$\Gamma_m/(\Omega R^2) = 5.075 C_T \qquad (3\text{-}26)$$

Considering the variation with azimuth angle, we obtain

$$\Gamma_m/(\Omega R^2) = 5.075 C_T/(1 + 1.5 \mu \sin\Psi) \qquad (3\text{-}27)$$

Equation (3-26) has been implemented to assign circulation to the vortices released at the tip of the blade at the time of generation.

To be able to compute the vorticity of each segment at a later time, the vorticity dissipation should be known. Since the total time which a vortex segment is in the domain of velocity computation is less than two seconds, the dissipation of vorticity can be neglected. The assumption of

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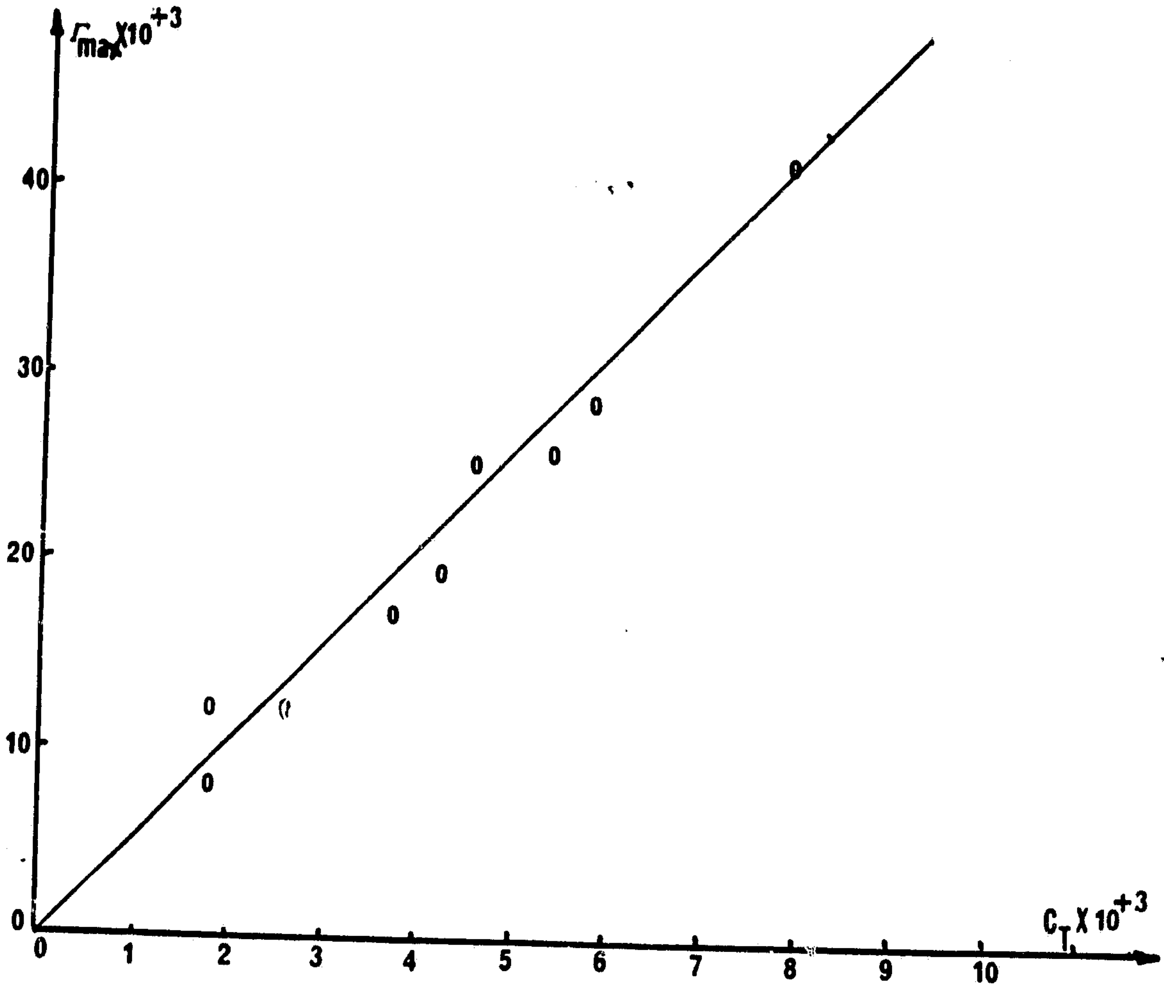

Figure 3-3. Nondimentional Vorticity Vs. Thrust Coefficient
(Extrated From Reference [2])

constant vorticity provides a simple formula for vorticity computation of the segments as they shrink or elongate.

$$\Gamma_1 = \Gamma_2 L_2 / L_1 \qquad (3\text{-}28)$$

3-8 COORDINATE DESCRIPTION AND NOMENCLATURE

A coordinate system fixed in the tip path plane of the rotor is used. The model for a two blade rotor and its wake is shown in figure 3-4. As noted on the figure a free stream of dimensionless magnitude μ is directed at an angle α_t to the tip path plane. The azimuth angle position ψ of the rotor is defined to be the angle between blade vortex number one and X-axis, as shown in the figure. The point P_{ij} is the wake reference point; the first subscript, i, increases successively proceeding down the wake vortex for a given blade, and the second subscript, j, denotes the blade number of the blade which generates that wake vortex. Each wake segment is associated with that end point having lower first subscript. In the coordinate system described above the location of point P_{ij} is then defined by X_{ij}, Y_{ij} and Z_{ij}. The total velocities associated with the X, Y and Z directions are defined U_{ij}, V_{ij} and W_{ij} respectively. And finally each element (i,j) is assigned a dimensionless core radius a_{ij} and strength Γ_{ij}.

3-9 NUMERICAL DAMPING

The goal in the present approach is to compute the location of the tip vortices by iteratively computing the velocities and the new locations of the tip vortices. The computation is terminated when a periodically steady state solution for the locations of tip vortices is achieved.

Succesive computation of velocities and locations does not always result in a steady state solution. If flight conditions are such that in

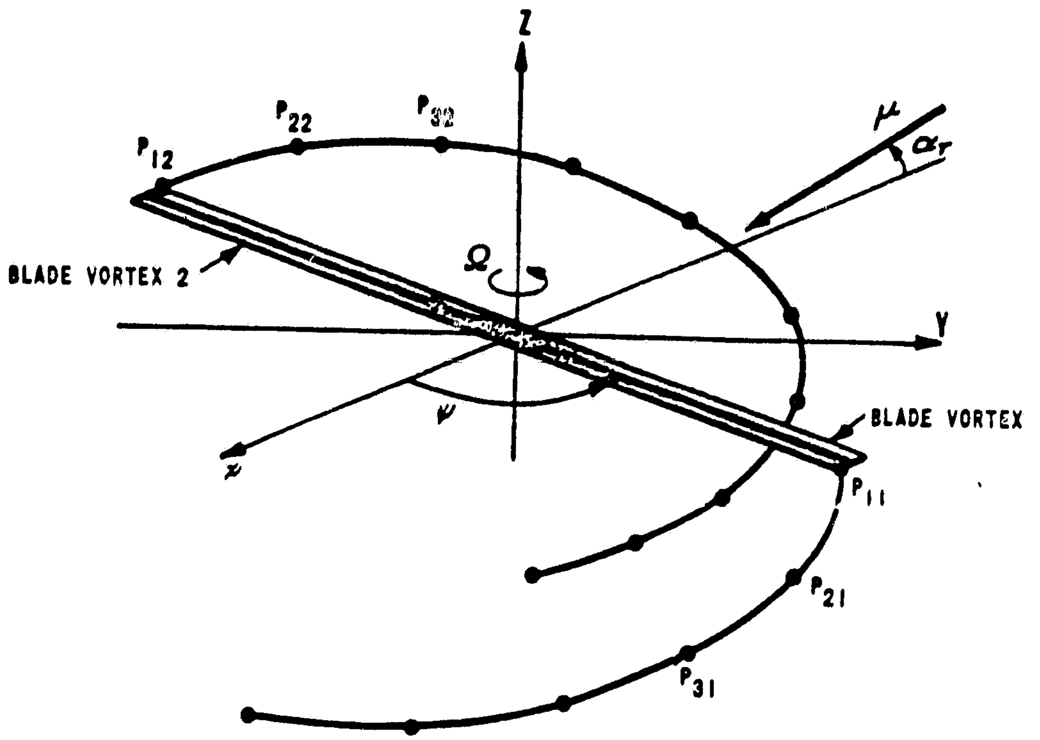

Figure 3-4. Coordinate System For Two-Blade Rotor

reality the wake is unsteady or if interaction of the old vortices and newly generated are present, then the computations will result in an unstable solution which does not necessarily represent the true answer. It is also possible that the wake is stable in reality, but its numerical iteration is unstable due to either method of integration, or poorly chosen initial conditions.

To overcome these problems, and also to be able to obtain an average location for the case in which the wake is unstable in reality, numerical damping was introduced.

Detailed study of the main wake indicated that the computation instability was started and magnified when a vortex segment of the wake of the blade number one and a vortex segment of the wake of the blade number two moved very close to each other. Because the velocity induced by a vortex is inversely proportional to the distance from the vortex, the two vortices induce large velocities on each other. In the next step of integration they unrealistically move far from each other. This behavior may deform the wake such that the continuation of the computation only worsens the results. It is possible to introduce an upper limit for the velocity to prevent large wake deformation. This can stabilize the iteration but convergence becomes very slow, therefore this concept was rejected.

In steady state there should be no difference between each successive blade. If at a particular time when blade number one is at some azimuth angle, point p_{1j} on the wake blade number one is at point (x_0, y_0, z_0), then when blade number two is at the same azimuth angle, point p_{2j} should be at the same point (x_0, y_0, z_0). Based on this criteria numerical damping can be introduced. In attempts to reach faster steady state solutions, if the location of a point on wake of blade number one was too far from the location of the corresponding point for the wake of blade number two, a point between these two locations was used as a better estimate of the new location of the two points.

This method was employed in our computer program. Use of a point from 65% to 75% of the distance between the old location and the newlocation gave the best convergence and most stable solution.

3-10 STRUCTURE OF THE COMPUTER PROGRAM

The computer code documented in appendix B has been constructed to simulate the physical flow by means of the modeling the main rotor, the tip vortices and the ground. Given an initial wake geometry and aircraft flight condition, it proceeds to compute the velocity and integrate in time succesively until a periodically steady solution is obtained.

The program has been written so that it can be used for out-of-ground-effect (OGE), as well as in-ground-effect (IGE). By proper usage of flags, the wake can be shortened or enlarged; the iteration can be applied to a portion of the wake or to the whole wake. The flow of information as computation proceeds, is presented schematically in figure 3-5.

At the beginning the program reads the flags and flight conditions. If an initial wake is available then the program reads the wake and its properties; otherwise it creates a simple helix for use as an initial wake. Subroutine "VELOC" computes the total velocity at the vortex segment's ends; subroutine "NEWLOC" computes the new location of the end points and subroutine "SMOOTH" (which is called by "NEWLOC" subroutine) applies the numrical damping. If an acceptable solution is reached the program is terminated, otherwise it continues until the number of iterations exceeds a given maximum.

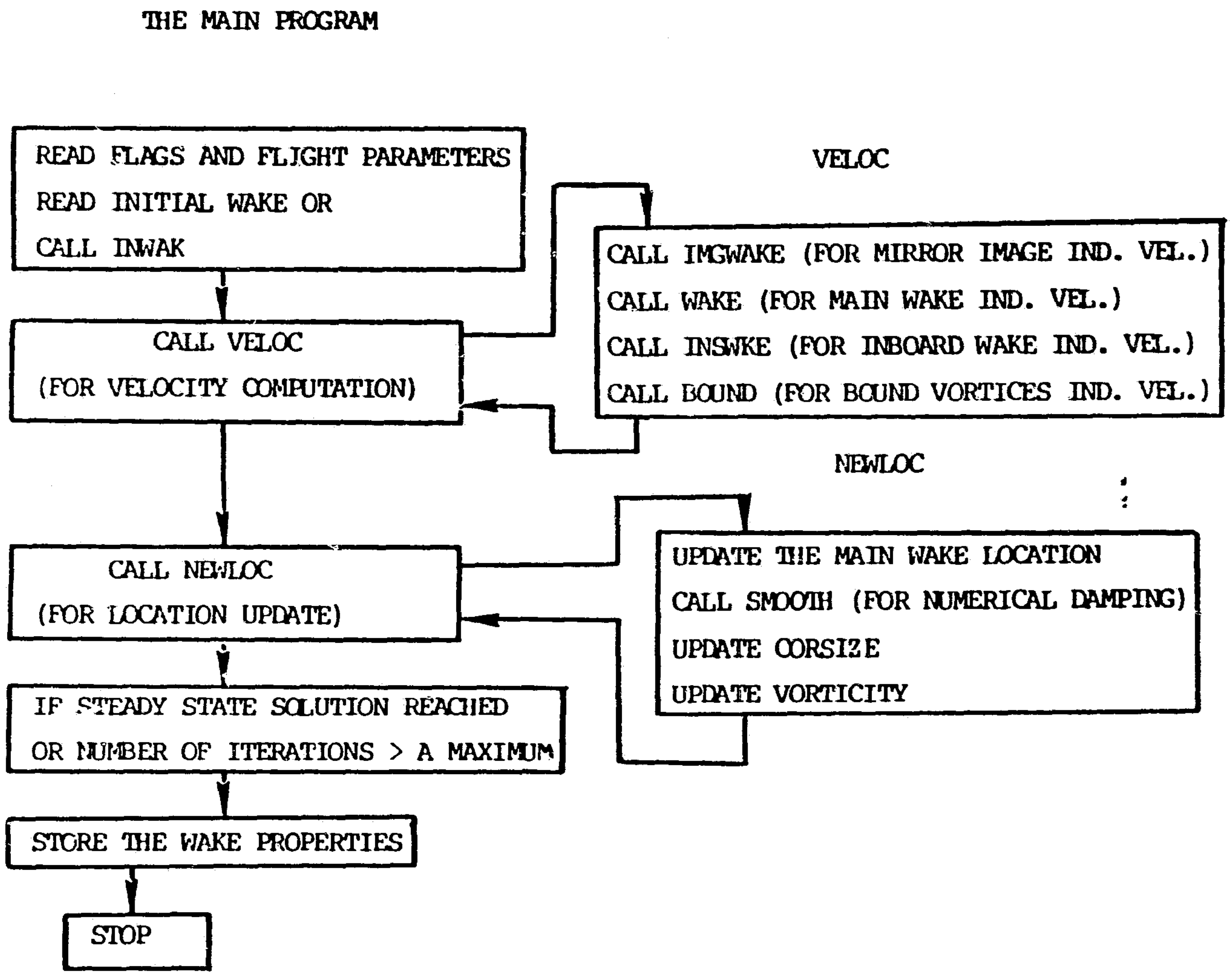

FIGURE 3-5. Flow-Chart Of The Computer Program

The description of what the subroutines do as well as their inputs and their outputs are given at the beginning of each subroutine as comment statements. The main program and all subroutines are documented in appendix B.

3-11 FLIGHT CONDITION PARAMETERS

The formulation of the model was nondimensionalized for the purpose of coding, with lengths made dimensionless by rotor radius "R" and velocities by rotor tip speed "V_t". The flight conditions of the aircraft being represented relate to computer program through the following parameters:

$\mu = V_f / V_t$	advance ratio
$\bar{\Gamma} = \Gamma / (\Omega R^2)$	vorticity
α_t	tip-path-plane angle
a/r	core radii
$C_T = W / (\rho \Omega^2 \pi R^4)$	thrust coefficient
H/R	height above the ground and
R/C	aspect ratio.

CHAPTER 4

AERODYNAMIC RESULTS

The computing program given in appendix B, computes the location, velocity, core radius and circulation of the fully rolled up tip vortices as they are generated and moved in space. Later modifications to the program allowed the computation of the velocity of any particle in space, and as a consequence its new location after a short period of time. This enabled the plotting of streamlines or streaklines. Some of the results of this chapter were obtained using the modified version of the program.

It should be remembered that this program was originally developed for stability derivative computation in ground effect and not for detailed study of the aerodynamics of the rotor. Therefore, a number of assumptions consistent our purpose was made. The assumptions were:

a. potential flow,

b. effect of fuselage and tail rotor were neglected,

c. tip vortices are fully rolled up as they are generated,

d. circulation along the blades changes in two steps,

1. at the tip $r / R = 1.0$ and
2. at the location $r / R = 0.7$

e. blades were replaced by line vortices,

f. no decay in circulation and no merging of vortices,

g. vortex segments are assumed to be straight lines and

h. the wake is stable.

4-1 UNSTABLE WAKE (NO NUMERICAL DAMPING)

As was mentioned in the previous chapter, with no numerical damping an unstable wake might result from successive computation of the velocities and locations. In figure 4-1 the cross-section of the location of the main wake

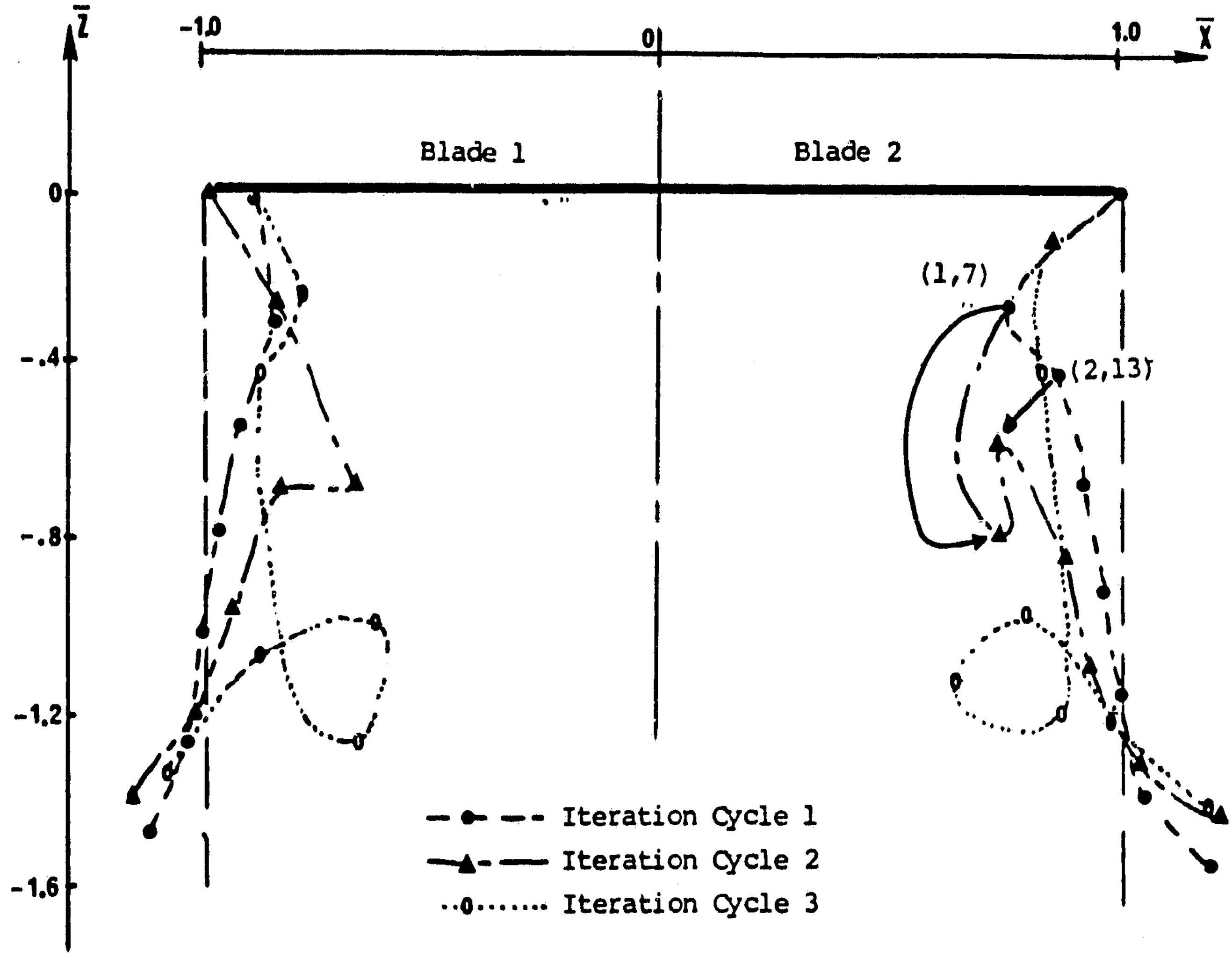

Figure 4-1. Numerically Unstable Wake For Cycle 1, 2 and 3
(No Numerical Damping)

in hover and out of ground effect for cycles 1, 2, and 3 of iteration with no numerical damping are shown. As can be seen, in iteration number 2 the end points of two segments of the wakes of the two blades have approached each other closely. Because point (2,13) is close to segment (1,7), the total induced velocity at this point has wrong magnitude and is in the wrong direction. In the next cycle it circles the segment (1,7) (see figrue 4-1). As a consequence the whole wake is distorted and if the computations continue similar instabilities are repeated. In ground effect the results were worse because of additional interaction of the old and new vortices.

4-2 STABLE WAKE

The use of numerical damping resulted in a stable wake for a rotor in the same conditions as the previous section. Figure 4-2 and 4-3 show the wake cross-section and three-dimensional wake geometry after reaching a periodic steady state solution. Figure 4-4 is the same rotor in ground effect and in hover. for this case again a periodic steady state solution was obtained.

4-3 COMPARISON OF THEORY AND MEASUREMENT

To check the results obtained from the computer code, they were compared to the results obtained from wind tunnel tests performed in reference [1].

As can be seen in figure 4-5 the computational results are very close to the measured ones, especially in the first cycle which is the area of interest. The error between the new wake and the measured wake increases as one goes downstream. The reason for this discrepancy is the error in forward integration and number of the assumptions made. However this error is not very important, because the effect of the far vortices on the rotor disc is small compared to that of the newest cycle.

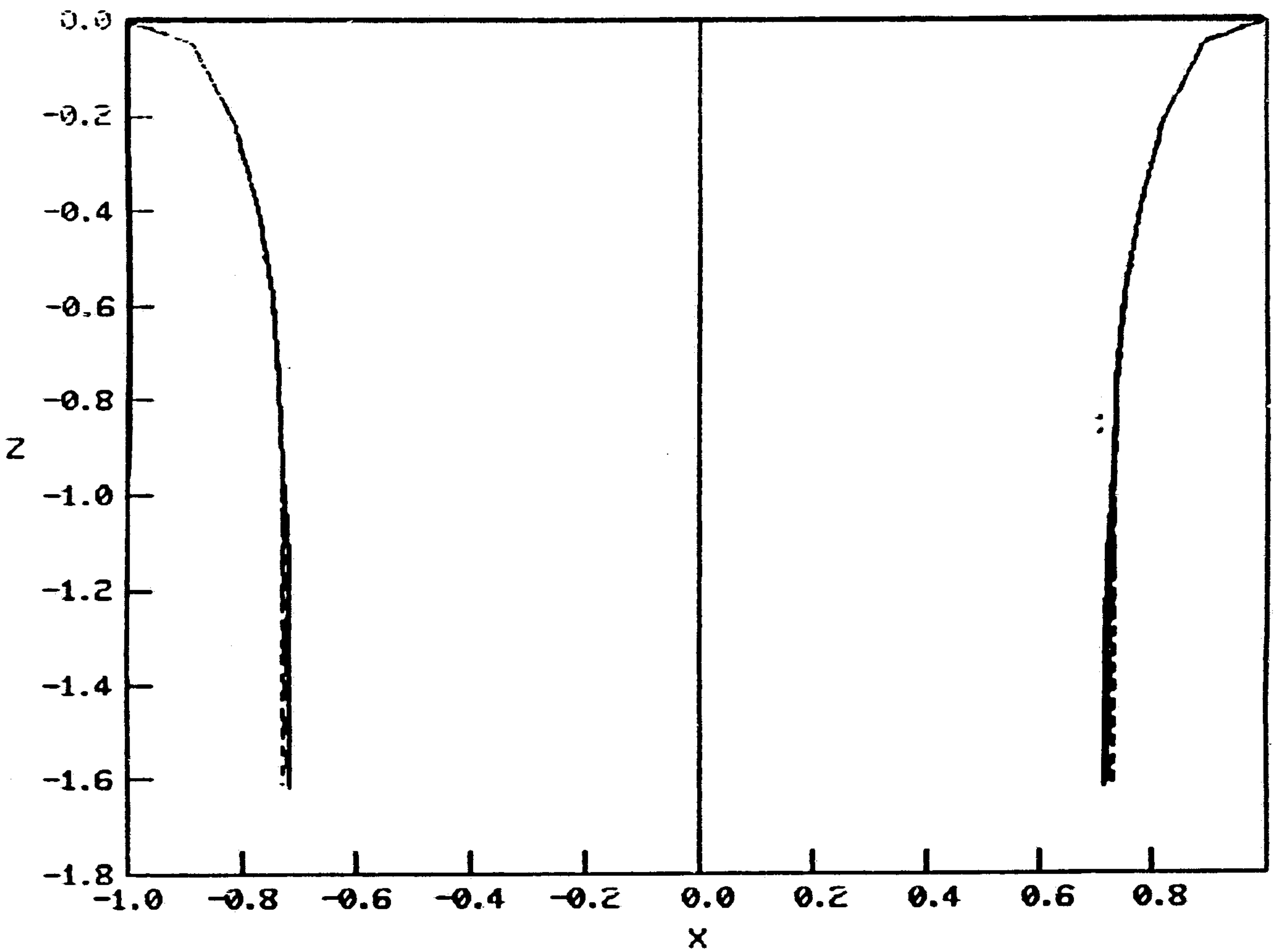

Figure 4-2. The Cross-Section Of The Stable Wake Out Of Ground Effect (C_T=0.0037, α_t=0.0, Numerical Damping=70%, And Aspect Ratio=13.7)

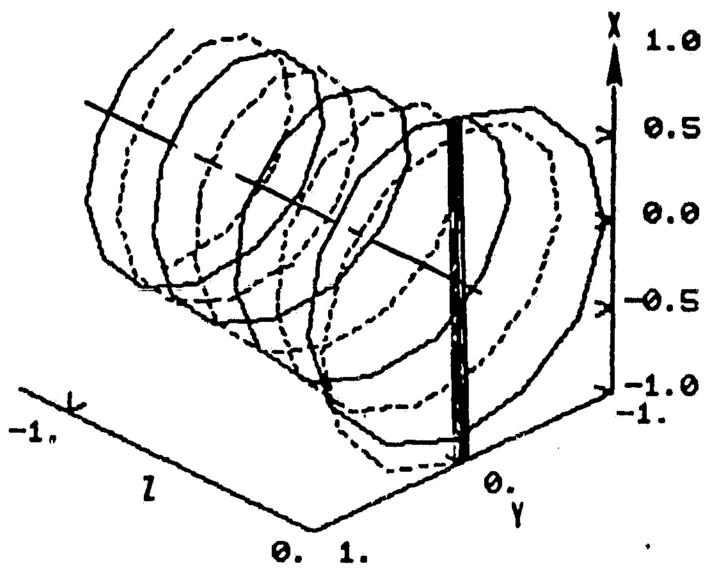

Figure 4-3. Three Dimentional Wake Out Of Ground Effect (C_T=0.0037, α_t=0.0, Numerical Damping=70%, And Aspect Ratio=13.7)

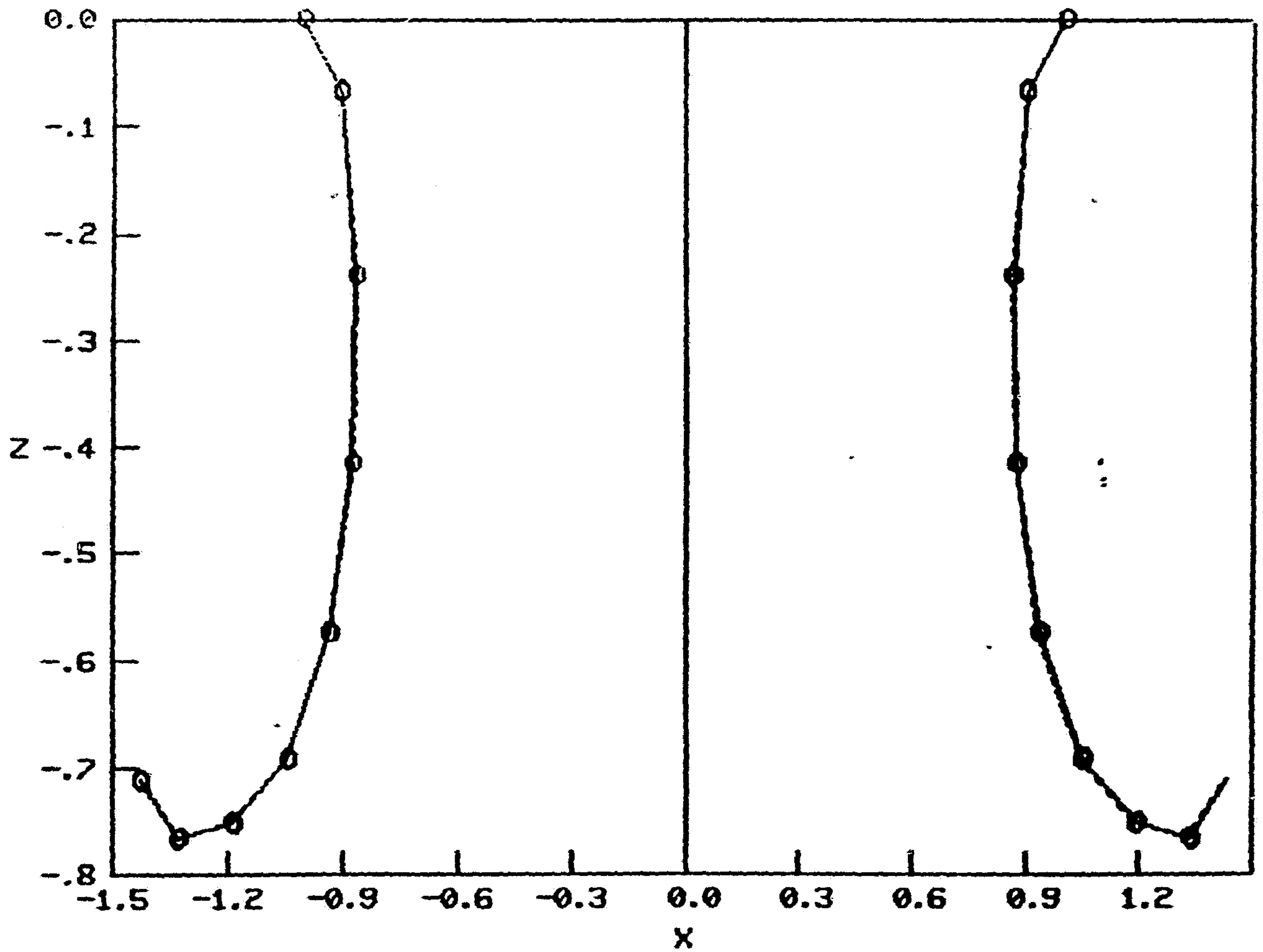

Figure 4-4. The Cross-Section Of The Stable Wake In Ground Effect (C_T=0.0037, α_t=0.0, Numerical Damping=70%, And Aspect Ratio=13.7)

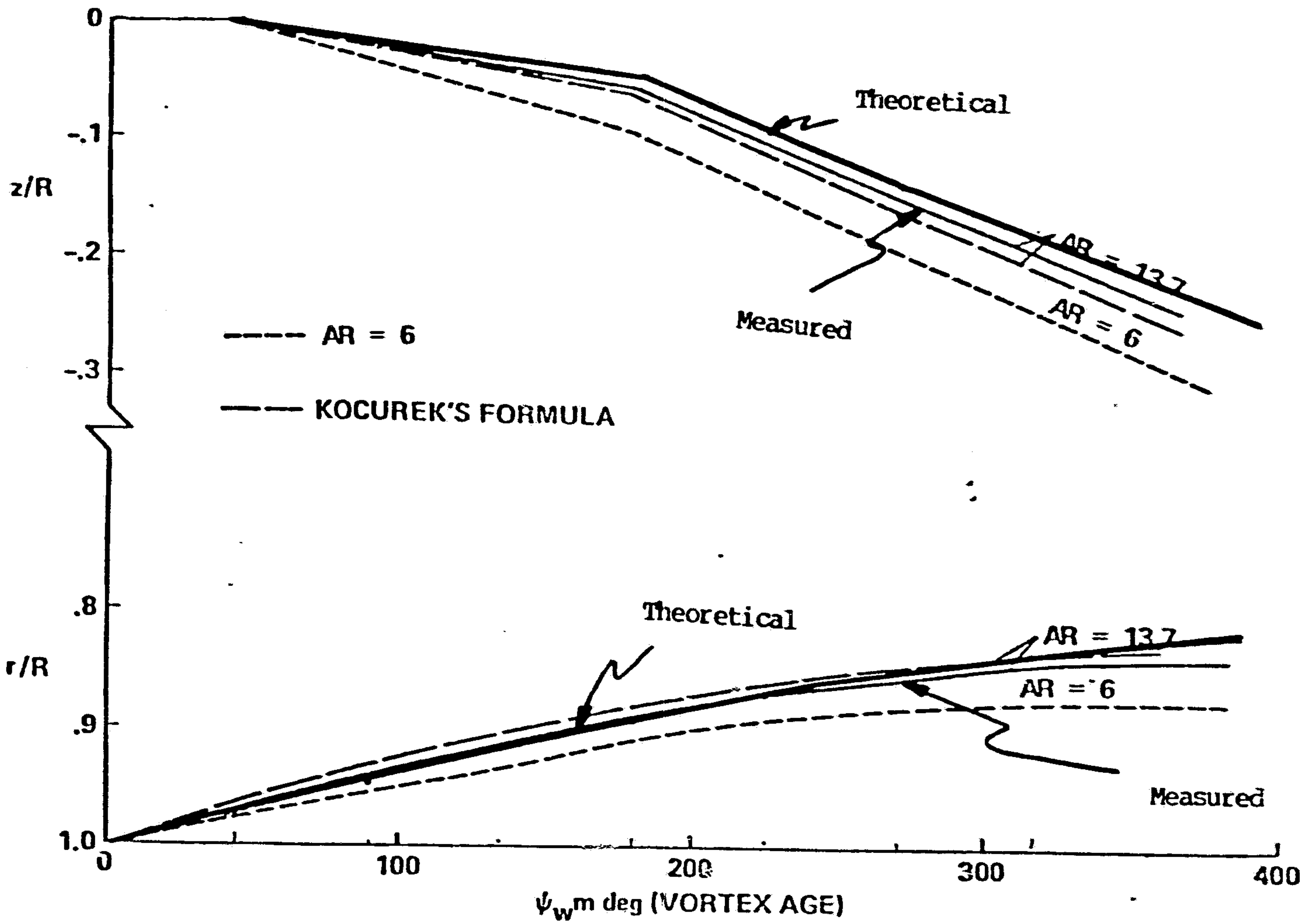

Figure 4-5. Comparison Of Theoretical And Experimental Wake Geometry For C_T=0.0037, $_t$=0.0, Numerical Damping=70%, Aspect Ratio=13.7, And Out Of Ground Effect

Since experimental data for the main wake in ground effect were not available, quantitative comparison in ground effect was not possible. But the following results indicate there is a good comparison of the theory and the actual flow patterns.

4-4 FLOW FIELD IN GROUND EFFECT AND HOVER

Experiments with a radio controlled helicopter showed the smoke coming from the engine exhaust would follow different patterns in different flight conditions.

For hover and very close to the ground, the smoke flowed upward near the center of the the rotor and down ward elsewhere inside the disc area. As the rotor moved further away from the ground, the amount of smoke going upward around the center was reduced. For $h / R > 1.0$ no smoke flowed out of the center of the rotor, as can be seen in figures 4-6 and 4-7. Theoretical results show the same behavior,as can be seen in figures 4-8 and 4-9.

4-5 GROUND VORTEX

Experimental studies of a rotor in ground effect and in hover have shown that the vortices generated by the tip of the blades close to the ground will move away from the rotor, and there is no vortex interaction and no air circulation through the rotor. But in the presence of wind or in slow forward flight, the vortices close to the ground upstream of the rotor will roll up and form a horseshoe vortex around the rotor. This is called a ground vortex and can create problems if there is gusty wind.

For slow forward speed or light winds, the ground vortex will stay far upstream. As the wind velocity increases, it moves closer to the rotor. Interaction between old vortices and new vortices increases as forward speed increases. At a certain speed (depending on the height above the

Figure 4-6. Flow Visualization In Ground Effect H/R=0.4

Figure 4-7. Flow Visualization In Ground Effect H/R>1.0

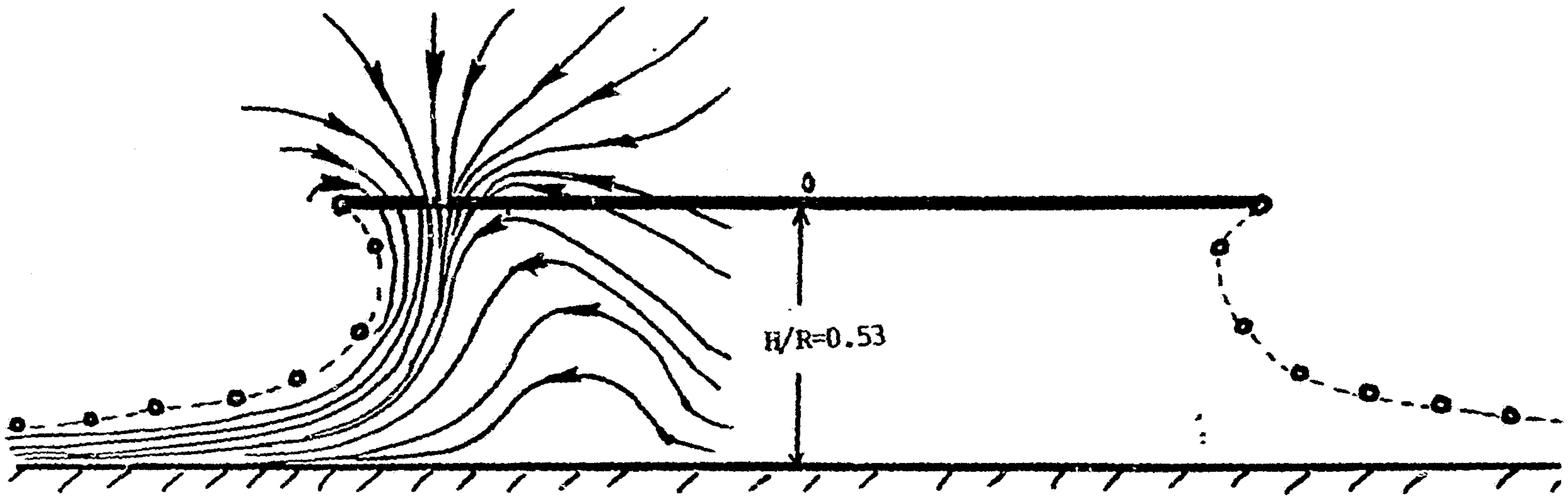

Figure 4-8. Theoretical Flow Pattern In Ground Effect H/R=0.53

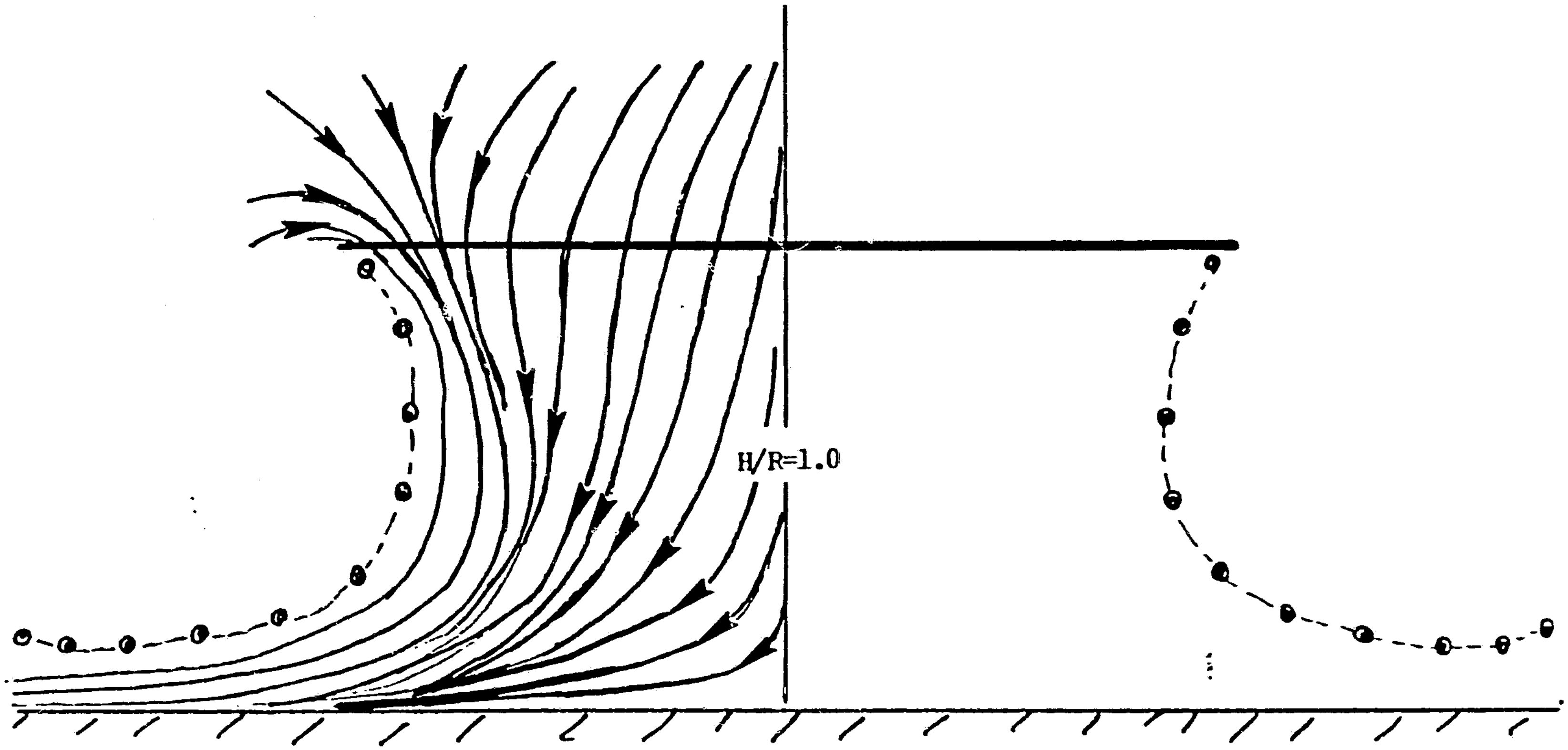

Figure 4-9. Theoretical Flow Pattern In Ground Effect H/R=1.0

ground and tip-path-plane angle), when the ground vortex is nearly under the leading edge of the rotor, the interaction reaches its maximum. If the forward speed increases further, the ground vortex will be washed away downstream. Figures 4-10 and 4-11 show the actual flow at different speeds (reference [5]). Similar results were obtained by the present approach. These results are shown in figures 4-12 and 4-13.

4-6 CONVERGENCE

Detailed study of the locations of the points on the main wake with no numerical damping showed two interesting results which resulted in a method of applying numerical damping which was described in the previous chapter.

First, the points closer to the rotor disc converge to their steady value faster than the point far from the disc. Secondly, the convergence to the final values were oscillatory. Figure 4-14 shows the convergence of the wake after applying numerical damping. As can be seen, the points closer to the rotor converge faster to their final values. It was also concluded that recomputation of the points which have reached their final location is a waste of time. Therefore, another check was added to the program to skip the points whose position changes were very small. This made the program to run faster for each flight condition.

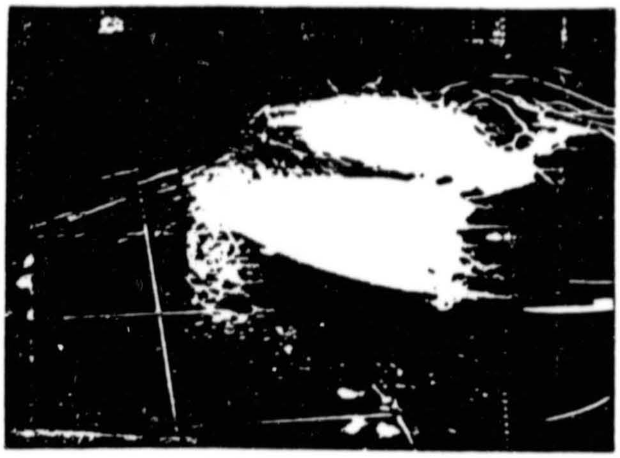

Figure 4-10. Ground Vortex Visualization (H/R=0.53, AR=9.8 And μ=0.05 Reference [5])

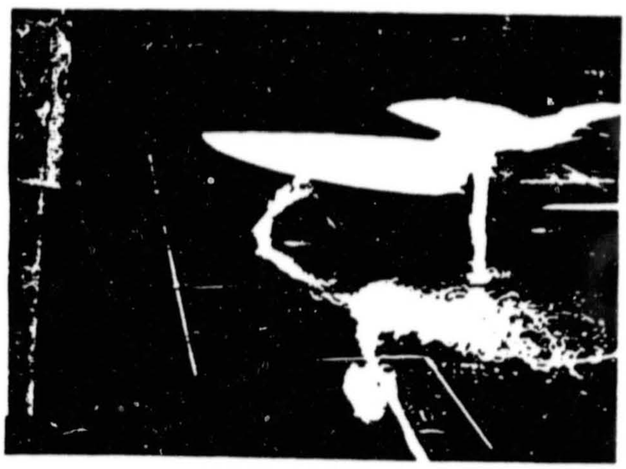

Figure 4-11. Ground Vortex Visualization (H/R=0.53, AR=9.8 And μ=0.055 Reference [5])

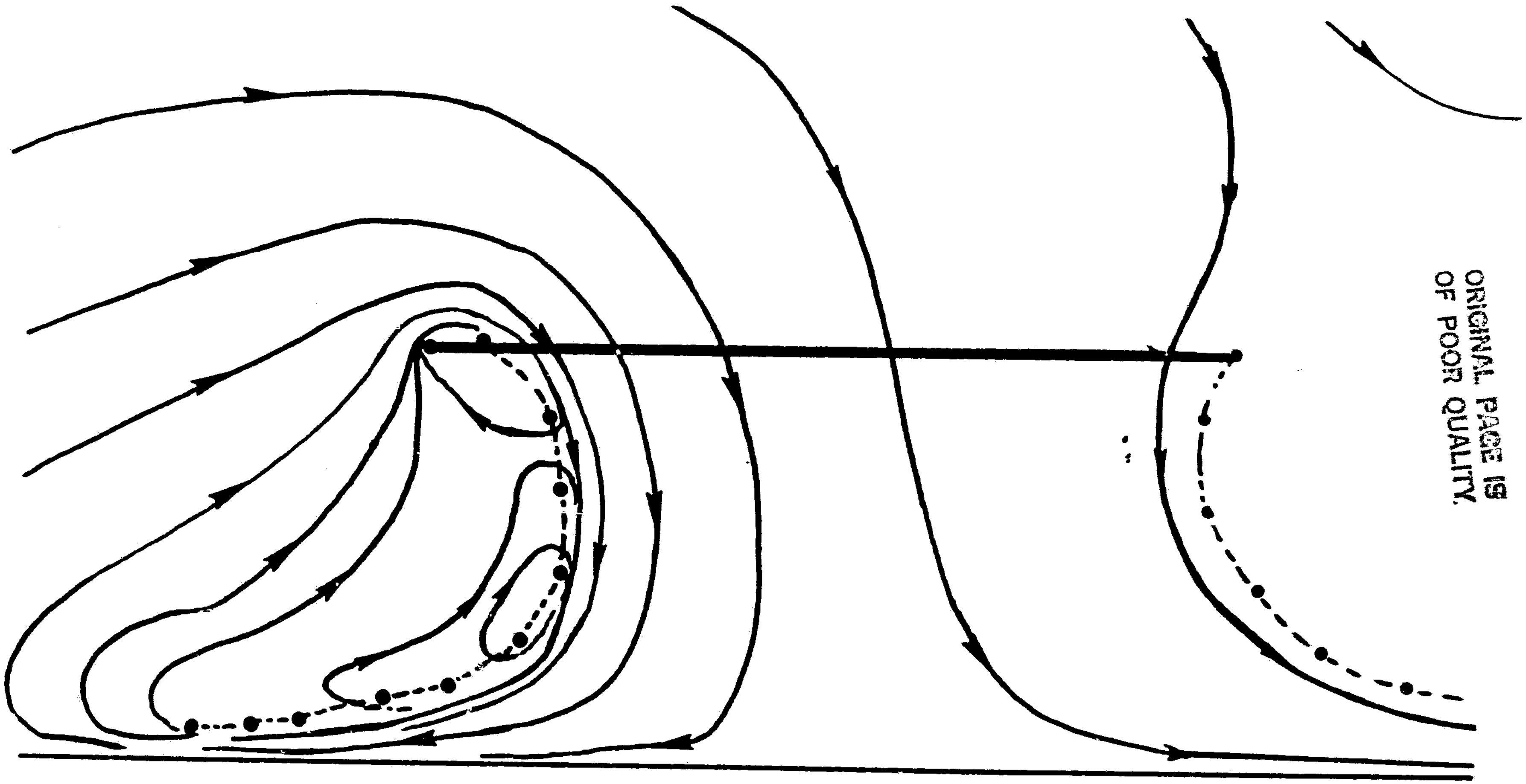

Figure 4-12. Theoretical Ground Vortex (H/R=1.0, AR=9.8 And μ=0.02)

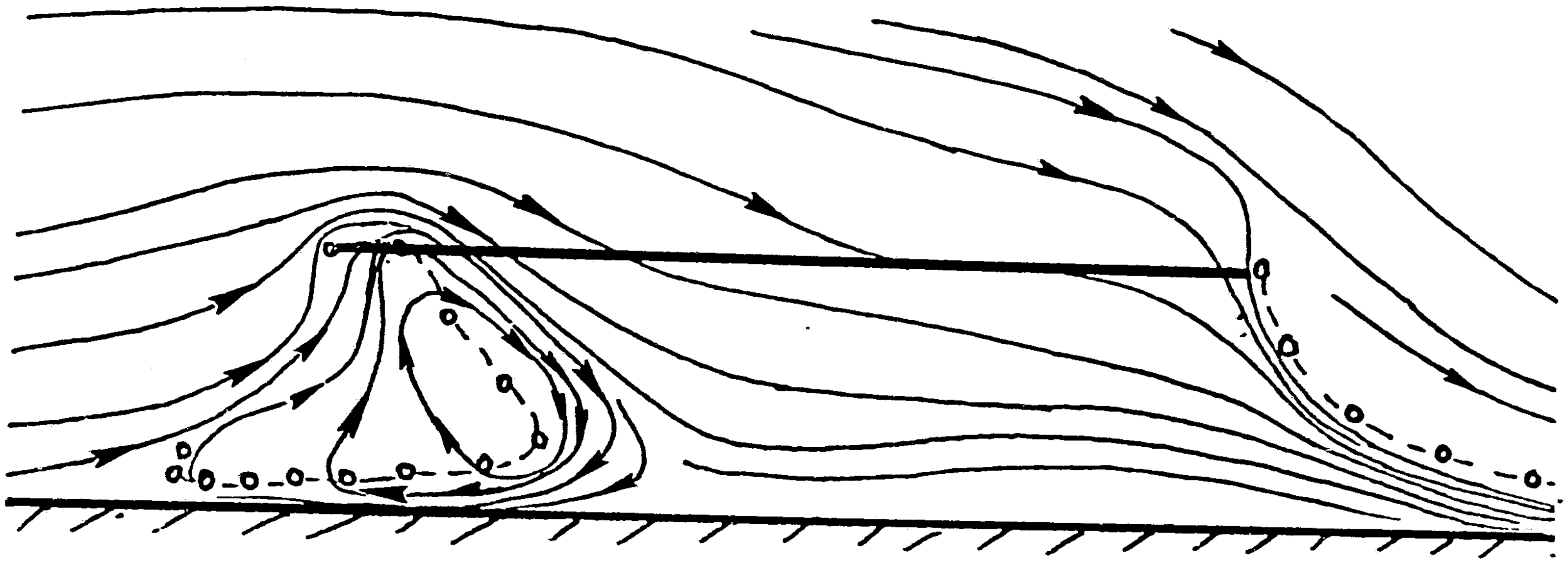

Figure 4-13. Theoretical Ground Vortex (H/R=0.53, AR=9.8 And μ=0.055)

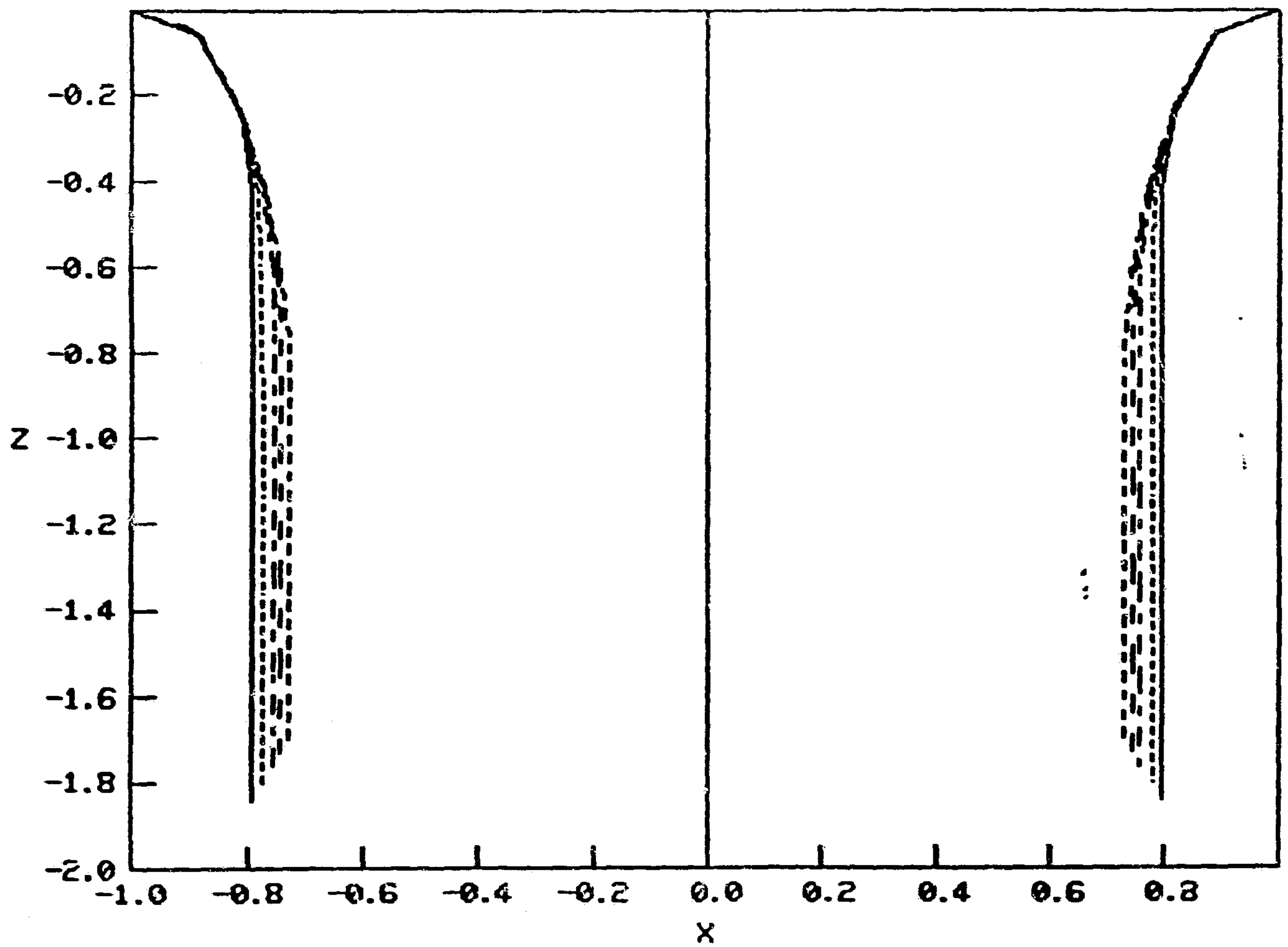

Figure 4-14. Wake Convergence Out Of Ground Effect With 70% Numerical Damping

CHAPTER 5

CONCLUSIONS

The model and the computer program developed in this study provides the velocity, location, and circulation of the tip vortices of a two-blade helicopter in and out of the ground effect. Comparison of the theoretical results with some experimental measurements for the location of the wake indicate that there is excellent accuracy in the vicinity of the rotor and fair amount of accuracy far from it. Having the location of the wake at all times enables us to compute the history of the velocity and the location of any point in the flow. The main goal of our study, induced velocity at the rotor, can also be calculated in addition to stream lines and streak lines. Since the wake location close to the rotor is known more accurately than at other places, the calculated induced velocity over the disc should be a good estimate of the real induced velocity, with the exception of the blade location, because each blade was replaced only by a vortex line.

Because no experimental measurements of the wake close to the ground were available to us, quantitative evaluation of the theoretical wake was not possible. But qualitatively we have been able to show excellent agreement. Comparison of flow visualization with our results has indicated the location of the ground vortex is estimated excellently. Also, the flow field in hover is well represented. The addition of numerical damping provided the three important following results:

1. faster convergence,
2. stable numerical solutions for steady wake and
3. computation a time average location for an unstable wake.

These results for stable and unstable wakes in steady flight conditions should be accurate enough for stability derivative computations.

The computation of stability derivatives requires the knowledge of the aerodynamic forces and moments around a trim condition. As a consequence induced velocity around the trim condition should be known. Therefore, the computation of stability derivatives requires the execution of the present computer code for a variety of different flight parameters around the trim condition. As a result, that portion of study requires a considerable amount of computer time, the lack of which has delayed progress on computation of stability derivatives.

As mentioned in previous chapters, because of the importance of the old vortices in low altitude and low advance ratio, and also because of the consideration of the mirror image wake the number of points in the wake should be enlarged. The time required for each iteration was proportional to the cube of the number of points in the wake. Although the program has been made to have faster convergence, it still requires a large amount of computing time for near hover conditions. However for high advance ratios, or for high altitude the vortices do not interact with each other or with the rotor; therefore a smaller wake is sufficient for stability computations.

This program can also be employed for two-blade propeller study in or out of the ground effect. With some modification, it can also be used for rotors and propellers with more than two blades. A fair amount of modifications is required to employ this program for the interaction of two rotors or two propellers or more. The modifications are only in usage of different subprograms and memories required, not the formulation or method of approach.

The preliminary study of a cylinder vortex sheet in ground effect has indicated that when a rotor in hover is close to the ground, the ground behaves like a spring with a damper. The spring constant increases, as the rotor approaches the ground. A better understanding of the ground effect in hover and forward flight can be made with the help of the present program.

A quantitative validation of the present study could have been carried out if more experimental data of measured wake in ground effect were available. However for qualitative study, the present work is a very good tool for prediction of the ground vortex, and computation of the induced velocity.

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APPENDIX A

A-1 SELF-INDUCED VELOCITY

The self-induced velocity of segment APC in figure a-1 may be computed by subtraction of the induced velocity of segment ABC at point P from the self-induced velocity of a whole vortex ring with the same curature as segment APC.

The Biot-Savart Law for the induced velocity of segment ABC at point p can be caculated as follows:

$$V_P = \Gamma/(4\pi)\int_{\phi_0}^{2\pi-\phi_0} (\overline{d} \times \overline{ds}) / d^3 \qquad \text{(a-1)}$$

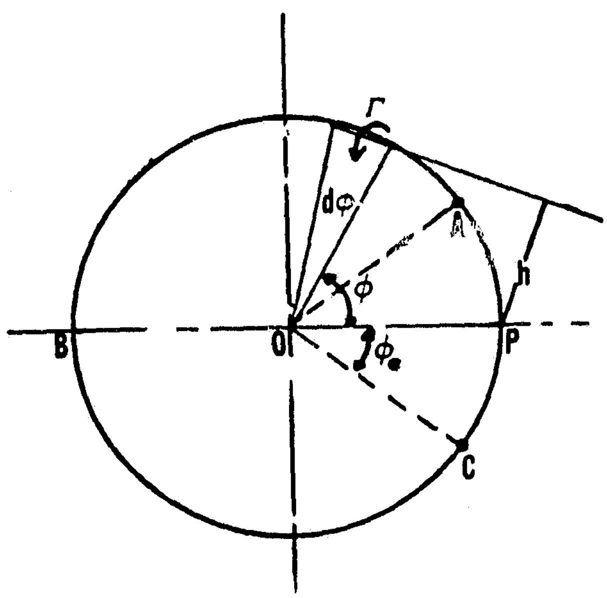

Figure A-1. Vortex Ring

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Assuming the point P and the whole ring is in X-Y plane, then

$$\bar{d} = R\,(\cos\phi - 1.)\,\hat{i} + R\,\sin\phi\,\hat{j} \qquad \text{(a-2)}$$

$$\bar{ds} = R\,[-\sin\phi\,\hat{i} + \cos\phi\,\hat{j}] \qquad \text{(a-3)}$$

And

$$d^3 = R^3\,[\,2 - 2\cos\phi\,]^{3/2} \qquad \text{(a-4)}$$

Using equations (a-2) through (a-4) in the induced velocity equation (a-1) can be reformulated as an elliptic integral.

$$V_p = \Gamma/(2\sqrt{8}\,\pi\,R)\int_{\phi_o}^{\pi} d\phi\;/[1. - \cos\phi]^{1/2} \qquad \text{(a-5)}$$

Let $\phi = 2\theta$ then

$$V_p = \Gamma/(4\,\pi\,R)\int_{\phi_o/2}^{\pi/2} d\phi/[1. - \cos^2\phi\,]^{1/2} \qquad \text{(a-6)}$$

or

$$V_p = -\,\Gamma\,/(4\,\pi\,R)\;\mathrm{Ln}(\,\mathrm{Tan}(\,\phi_o\,/4)) \qquad \text{(a-7)}$$

Subtraction of V_p from self-induced velocity of the whole vortex ring

$$V_s = \Gamma/(4\,\pi\,R)\;\;[\mathrm{Ln}(8R/a) - 1.) \qquad \text{(a-8)}$$

leads to calculation of the self-induced velocity of segments APC.

$$V_p = \Gamma/(4\,\pi\,R)\,[\mathrm{Ln}(8R/a\;\mathrm{Tan}(\,\phi_o/4)) - 1.] \qquad \text{(a-9)}$$

APPENDIX B

B-1 OPERATIONAL INFORMATION FOR THE COMPUTER PROGRAM

The program presented in this appendix was written in Fortran IV and executed on DEC-20, IBM-370, and VAX-11.

The inputs to the program are as follows:

NCOMP	Number of point to be iterated on
NB	Number of blades
NW	Number of segments for each blade wake
NCONS	Number of points that does not need the computations
IADD	Set to 0 if wake should not be enlarged
MIRIMG	Set to 0 for in ground effect ans 1 for out of ground effect
PSI0	Initial azimuth angle
REV	Number of revolution to be iterated
CT	Thrust Coefficient
XMU	Advance ratio
ALPHAT	Tip-path-plane angle
RB	Aspect ratio
H	Height ABove the Ground
ALFR	Damping percentage in r direction
ALFZ	Damping percentage in z direction

```
C        THIS IS THE LAST VESION OF HELLICAL WAKE TILL JUNE 83.
C        FOR39 IS INPUT FILE READING FILE (INPUT.DAT).
C        FOR40 IS STORING DATA FOR NEXT STEP (INTER.DAT).
C        FOR50 IS FOR BLADE VORTICES LOCATOINS (OUTPUT).
C        ****** MAIN PROGRAM *****
C       -----------------------------------------------------------
C       THIS PROGRAM CALCULATES THE INDUCED VELOCITY OF A HELICOPTER
C       ROTOR IN GROUND EFFECT.
C       --------------------------------------------------------
        COMMON/ALLSUB/ALFO,ALPHAT,CAT,CAT2,CA2T,CT,DALFO,DPSI,EPS,
     1        H,IADD,IAV,MIRIMG,NA,NCONS,NDPSI,NPR,NRW,NW,NW1,PI,
     2        PSI,PSIF,PSIO,RAD,RC,REV,SAT,SAT2,SA2T,THCAT,
     3        TFRC,THSAT,XMU,XX,YY,ZZ,ALFR,BETR,ALFZ,BETZ,
     4        A(180,2),GAMA(180,2),GAMB(30),SEG(180,2),U(180,2),
     5        V(180,2),W(180,2),X(180,2),Y(180,2),Z(180,2)
       COMMON/MIS/ SX(180,18),SY(180,18),SZ(180,18)
       COMMON/NTURN/NCOMP
       PI=4.*ATAN(1.)
       RAD=PI/180.
       NCT=0
C
C      ---- READS ALL THE FLAGS ,FLIGHT CONDITIONS ,NUMERICAL ----
C      ---- DAMPING PROPERTIES,COR OF EACH SEGMENT OF THE     ----
C      ----   CIRCULATION OF EACH SEGMENT AND WAKE LOCATION   ----
C      ---- FROM PREVIOUS RUN.                                ----
C
       READ(39,*) NCOMP
       READ(39,*)NA,NW,NCONS,IADD,MIRIMG,NDPSI,NPR
       READ(39,*)PSIO,REV,CT,XMU,ALPHAT,RB,H
       READ(39,*)ALFR,BETR,ALFZ,BETZ,EPS
       WRITE(50,1000)NA,NW,NCONS,IADD,MIRIMG,NDPSI,NPR
       WRITE(50,1004) PSIO,REV,CT,XMU,ALPHAT,RB,H
       WRITE(50,1001)ALFR,BETR,ALFZ,BETZ,EPS
       NW1=NW+1
       NRW=NW/NA
       RC=2.*RB
   101 IF(PSIO.LT.1) GO TO 301
       READ(39,*)((A(I,J),I=1,NW1),J=1,2)
       READ(39,*)((GAMA(I,J),I=1,NW),J=1,2)
       READ(39,*)((X(I,J),Y(I,J),Z(I,J),I=1,NW1),J=1,2)
       READ(39,*)((SX(I,J),SY(I,J),SZ(I,J)),I=1,NW1),J=1,NA)
       NAHP=NA/2+1
       IF(IADD.EQ.0) GO TO 301
       CALL ADDP(NAHP)
       NW=NW1-1
       A(NW1,1)=A(NW,1)
       A(NW1,2)=A(NW,2)
       GAMA(NW,1)=GAMA(NW-1,1)
       GAMA(NW,2)=GAMA(NW-1,2)
   301 CONTINUE
       DPSI=2.*PI/NA
C
C      ---- VORTICITY OF THE FIRST SEGMENT VERSUS AZIMUTH OF THE ROTOR ----
C      ---- BLADE.                                                     ----

       SIE=0.
       DO 299 I=1,NA
```

```
           S1=SIN(SIE)
           S2=SIN(SIE+DPSI)
           GAM=.5/(1.+1.5*XMU*S1)+.5/(1.+1.5*XMU*S2)
           GAMB(I)=5.1*GAM/(2.*PI)
           SIE=SIE+DPSI
       299 CONTINUE
           SAT=SIN(ALPHAT*RAD)
           CAT=COS(ALPHAT*RAD)
           SAT2=SAT*SAT
           CAT2=CAT*CAT
           SA2T=SIN(2.*ALPHAT*RAD)
           CA2T=COS(2.*ALPHAT*RAD)
           THSAT=2.*H*SAT
           THCAT=2.*H*CAT
           TMP=RC*DPSI
           TMP1=SQRT(TMP*(TMP+2.))
           TFRC=(TMP-TMP1+ALOG(1.+TMP+TMP1))/DPSI
           PSIFF=PSIO+360.*REV
           PSI=PSIO*RAD
           PSIO=PSI
      C
      C    ---- THE FIRST TIME OF A FLIGHT CONDITION INITIAL WAKE IS ----
      C    ---- CALLED TO PRODUCE AN INITIAL CONDITION AN INITIAL    ----
      C    ---- CONDITION FOR THE WAKE.                              ----
      C
           PSIF=PSIFF*RAD+.05
        15 IF(PSIO.LT.0.01) CALL INWAKE
      C
      C    ---- COMPUTES LENGHTH OF EACH SEGMENT.                    ----
      C
           DO 250 J=1,2
           DO 250 I=1,NW
           I1=I+1
           DX=X(I1,J)-X(I,J)
           DY=Y(I1,J)-Y(I,J)
           DZ=Z(I1,J)-Z(I,J)
           SEG(I,J)=SQRT(DX*DX+DY*DY+DZ*DZ)
       250 CONTINUE
      C
      C    ---- START OF THE MAIN LOOP OF THE MAIN PROGRAM.           ----
      C    ---- SUBROUTINE VELOCITY COMPUTES THE TOTAL INDUCED        ----
      C    ---- VELOCITY AT ALL THE POINTS OF THE WAKE                ----
      C
        20 CALL VELOC
        40 IF(NCT.NE.0) GO TO 50
           TN=IFIX((PSI+.05)/DPSI)/(1.*NA)
           WRITE(50,1002)TN
           NAH=NA/2
      C
      C    ---- WRITES THE CROSS SECTION OF THE WAKE ----
      C    ---- P L A N E    X-Z                     ----
      C
           NWE=NW1-NAH
           DO 10 I=1,NWE,NA
           WRITE(50,1005)I,X(I,1),Y(I,1),Z(I,1),W(I,1)
           K=I+NAH
        10 WRITE(50,1005)K,X(K,2),Y(K,2),Z(K,2),W(K,2)
```

```
        WRITE(50,1005)NW1,X(NW1,1),Y(NW1,1),Z(NW1,1)
        WRITE(50,1000)
        DO 30 I=1,NWE,NA
        WRITE(50,1005)I,X(I,2),Y(I,2),Z(I,2),W(I,2)
        K=I+NAH
     30 WRITE(50,1005)K,X(K,1),Y(K,1),Z(K,1),W(K,1)
        WRITE(50,1005)NW1,X(NW1,2),Y(NW1,2),Z(NW1,2)
        WRITE(50,1000)
     50 NCT=NCT+1
        IF(NCT.GE.NDPSI)NCT=0
        PSI=PSI+DPSI
        IF(PSI.LT.PSIF)GO TO 80
C
C       ---- STORE PROPERTIES OF THE WAKE ON A FILE TO BE ----
C       ---- TO BE READ FOR THE NEXT RUN.                 ----
C
        WRITE(40,1000) NCOMP
        WRITE(40,1000)NA,NW,NCONS,IADD,MIRIMG,NDPSI,NPR
        WRITE(40,1004) PSIFF,REV,CT,XMU,ALPHAT,RB,H
        WRITE(40,1001)ALFR,BETR,ALFZ,BETZ,EPS
        WRITE(40,1001)((A(I,J),I=1,NW1),J=1,2)
        WRITE(40,1002)((GAMA(I,J),I=1,NW),J=1,2)
        WRITE(40,1001)((X(I,J),Y(I,J),Z(I,J),I=1,NW1),J=1,2)
        WRITE(40,1001)((SX(I,J),SY(I,J),SZ(I,J),I=1,NW1),J=1,NA)
        STOP
C
C       ---- THIS SUBROUTINE USES THE INDUCED VELOCITIES AND  ----
C       ---- OLD LOCATION OF EACH POINT AND COMPUTES NEW      ----
C       ---- LOCATION OF  THE POINTS ON THE WAKE USING SIMPLE ----
C       ---- FORWARD EULER METHOD.                            ----
C
     80 CALL NEWLOC
        GO TO 20
C
C       ---- END OF THE MAIN LOOP                             ----
   1000 FORMAT(12I5)
   1001 FORMAT(8F9.4)
   1002 FORMAT(7F10.6)
   1003 FORMAT(3F10.5,I10)
   1004 FORMAT(F12.2,2X,F6.3,2F12.5,3F7.2)
   1005 FORMAT(I8,E18.5,3E15.5)
        END
```

```
C     ***** SUB. INWAKE *****
C     --------------------------------------------------
C     THIS SUBROUTINE COMPUTES THE INITIAL WAKE LOCATION.
C     --------------------------------------------------
      SUBROUTINE INWAKE
C
C     ---- THIS SUBROUTINE COMPUTES THE COORDINATES OF INITIAL ----
C     ---- WAKE AND ITS PROPERTIES . INITIAL WAKE IS A HELICAL ----
C     ---- WITH CONSTANT RADIUS.                               ----
C
      COMMON/ALLSUB/ALFO,ALFHAT,CAT,CAT2,CA2T,CT,DALFO,DPSI,EPS,
     1      H,IADD,IAV,MIRIMG,NA,NCONS,NDPSI,NFR,NRW,NW,NW1,PI,
     2      PSI,PSIF,PSIO,RAD,RC,REV,SAT,SAT2,SA2T,THCAT,
     3      TFRC,THSAT,XMU,XX,YY,ZZ,ALFR,BETR,ALFZ,BETZ,
     4      A(180,2),GAMA(180,2),GAMB(30),SEG(180,2),U(180,2),
     5      V(180,2),W(180,2),X(180,2),Y(180,2),Z(180,2)
      COR=.003
      IF(MIRIMG.NE.0) H=2.5
      SIE=0.
      DO 10 J=1,2
      IF(J.EQ.2) SIE=PI
      DO 10 I=1,NW1
      A(I,J)=COR
      X(I,J)=COS(SIE)
      Y(I,J)=SIN(SIE)
      Z(I,J)=-(H-COR)*(I-1)/NW1
      SIE=SIE-DPSI
   10 CONTINUE
      CALL SMOOTH
      DO 30 J=1,2
      K=NA
      IF(J.EQ.2)K=NA/2
      DO 30 I=1,NW
      GAMA(I,J)=GAMB(K)
      K=K-1
      IF(K.EQ.0) K=NA
   30 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE ADDP(NAHP)
C
C  ---- THIS SUBROUTINE ADDS HALF A TURN TO THE WAKE WHEN MORE---
C  ---- ACCURACY FOR THE WAKE IS REQUIRED. THIS SUBROUTINE IS ---
C  ---- CALLED WHEN IADD=1                                     ---
C
      COMMON/ALLSUB/ALFO,ALFHAT,CAT,CAT2,CA2T,CT,DALFO,DPSI,EPS,
     1      H,IADD,IAV,MIRIMG,NA,NCCHG,NDPSI,NPR,NRW,NW,NW1,PI,
     2      PSI,PSIF,PSIO,RAD,RC,REV,SAT,SAT2,SA2T,THCAT,
     3      TFRC,THSAT,XMU,XX,YY,ZZ,ALFR,BETR,ALFZ,BETZ,
     4      A(180,2),GAMA(180,2),GAMB(30),SEG(180,2),U(180,2),
     5      V(180,2),W(180,2),X(180,2),Y(180,2),Z(180,2)
      COMMON/MIS/ SX(180,18),SY(180,18),SZ(180,18)
      NAH=NAHP-1
      DO 10 KK=1,NAH
      NW=NW1
      NW1=NW1+1
      A(NW1,1)=A(NW,1)
      A(NW1,2)=A(NW,2)
      GAMA(NW,1)=GAMA(NW-1,1)
      GAMA(NW,2)=GAMA(NW-1,2)
      NWA=NW1-NAH
      NWB=NW1-NA
      X(NW1,1)=2.*X(NWA,2)-X(NWB,1)
      Y(NW1,1)=2.*Y(NWA,2)-Y(NWB,1)
      Z(NW1,1)=2.*Z(NWA,2)-Z(NWB,1)
      X(NW1,2)=2.*X(NWA,1)-X(NWB,2)
      Y(NW1,2)=2.*Y(NWA,1)-Y(NWB,2)
      Z(NW1,2)=2.*Z(NWA,1)-Z(NWB,2)
      DO 10 J=1,NA
      K=J+NAH
      IF(K.GT.NA) K=K-NA
      SX(NW1,J)=2.*SX(NWA,K)-SX(NWB,J)
      SY(NW1,J)=2.*SY(NWA,K)-SY(NWB,J)
      SZ(NW1,J)=2.*SZ(NWA,K)-SZ(NWB,J)
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE VELOC
C       ------------------------------------------------------
C       FOR A GIVEN WAKE LOCATION THIS SUBROUTINE COMPUTES THE
C       TOTAL INDUCED VELOCITY FOR ALL THE POINTS THE WAKE.
C       ------------------------------------------------------
      COMMON/ALLSUB/ALFO,ALPHAT,CAT,CAT2,CA2T,CT,DALFO,DPSI,EPS,
     1        H,IADD,IAV,MIRIMG,NA,NCONS,NDPSI,NFR,NRW,NW,NW1,PI,
     2        PSI,PSIF,PSIO,RAD,RC,REV,SAT,SAT2,SA2T,THCAT,
     3        TFRC,THSAT,XMU,XX,YY,ZZ,ALFR,BETR,ALFZ,BETZ,
     4        A(180,2),GAMA(180,2),GAMB(30),SEG(180,2),U(180,2),
     5        V(180,2),W(180,2),X(180,2),Y(180,2),Z(180,2)
      COMMON /NTURN/ NCOMP
C
C     ---- TRANSITION VELOCITY                              ----
C
      XMUX=XMU*CAT/CT
      XMUZ=-XMU*SAT/CT
      DO 10 I=1,NCONS
      DO 10 J=1,2
      U(I,J)=0.
      V(I,J)=0.
   10 W(I,J)=0.
      NSTOP=NCONS+NCOMP
      IF(NSTOP .GT. NW) NSTOP=NW
C
C     ---- START OF THE MAIN LOOP                         ----
C
      DO 28 I=NCONS,NSTOP
   13 DO 29 J=1,2
C
C     ---- CHECK FOR HOVER CONDITION                      ----
C
      IF(XMU.GT.0.00001.OR.J.NE.2) GO TO 150
      U(I,2)=-U(I,1)
      V(I,2)=-V(I,1)
      W(I,2)= W(I,1)
      GO TO 29
  150 CONTINUE
      XX=X(I,J)
      YY=Y(I,J)
      ZZ=Z(I,J)
      U(I,J)=0.
      V(I,J)=0.
      W(I,J)=0.
      IDUM=I
      JDUM=J
C
C     ---- INDUCED VELOCITY BY THE MIRROR IMAGE OF THE WAKE AT ----
C     ---- POINT (I,J)                                         ----
C
      IF(MIRIMG.EQ.0) CALL IMGWAK(IDUM,JDUM)
C
C     ---- INDUCED VELOCITY BY THE MAIN WAKE AT POINT (I,J)    ----
C
      CALL WAKE(IDUM,JDUM)
      IF(I.GT.2*NA+4) GO TO 140
C
```

```
C     ---- INDUCED VELOCITY BY THE INSIDE WAKE AT POINT (I,J)  ----
C
      CALL INSWKE(IDUM,JDUM)
C
C     ---- INDUCED VELOCITY BY THE BOUND VORTICES AT POINT (I,J)----
C
  140 CALL BOUNDE(IDUM,JDUM)
      U(I,J)=U(I,J)+XMUX
      W(I,J)=W(I,J)+XMUZ
   29 CONTINUE
   28 CONTINUE
C
C     ---- END OF THE MAIN LOOP                              ----
C
      RETURN
      END
```

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```
C       ***** SUB. NEWLOC *****
C       --------------------------------------------------
C       HAVING OLD WAKE LOCATION AND VELOCITIES OF ALL THE
C       POINTS ON THE WAKE COMPUTES THE NEW WAKE LOCATION.
C       THE METHOD OF INTEGRATION IS THE FORWARD EULER
C       METHOD.
C       --------------------------------------------------
        SUBROUTINE NEWLOC
        COMMON/ALLSUB/ALFO,ALPHAT,CAT,CAT2,CA2T,CT,DALFO,DPSI,EPS,
     1         H,IADD,IAV,MIRIMG,NA,NCONS,NDPSI,NFR,NRW,NW,NW1,PI,
     2         PSI,PSIF,PSIO,RAD,RC,REV,SAT,SAT2,SA2T,THCAT,
     3         TFRC,THSAT,XMU,XX,YY,ZZ,ALFR,BETR,ALFZ,BETZ,
     4         A(180,2),GAMA(180,2),GAMB(30),SEG(180,2),U(180,2),
     5         V(180,2),W(180,2),X(180,2),Y(180,2),Z(180,2)
      COMMON /NTURN/ NCOMP
      DATA COR/0.003/
      NSTART=NCONS+1
      NSTOP1 =NSTART + NCOMP
      IF(NSTOP1 .GT. NW1) NSTOP1 =NW1
C
C     ---- COMPUTES THE NEW WAKE LOCATION ----
C
   32 DO 35 J=1,2
      TX1=X(NCONS,J)
      TY1=Y(NCONS,J)
      TZ1=Z(NCONS,J)
      DO 33 I=NSTART,NSTOP1
      TX2=TX1
      TY2=TY1
      TZ2=TZ1
      TX1=X(I,J)
      TY1=Y(I,J)
      TZ1=Z(I,J)
      X(I,J)=TX2+U(I-1,J)*DPSI*CT
      Y(I,J)=TY2+V(I-1,J)*DPSI*CT
      Z(I,J)=TZ2+W(I-1,J)*DPSI*CT
      IF(MIRIMG.NE.0) GO TO 33
C
C     ---- CHECKES IF POINT (I,J) PASSES THROUGH THE GROUND.----

  320 IF(X(I,J)*SAT+Z(I,J)*CAT+H.GT.A(I,J)) GO TO 33
      X(I,J)=(X(I,J)*CAT-Z(I,J)*SAT)*CAT-(H-A(I,J))*SAT
      Z(I,J)=-((H-A(I,J))*CAT+(X(I,J)*CAT-Z(I,J)*SAT)*SAT)
   33 CONTINUE
  351 XJ=FLOAT(J-1)*PI
      X(1,J)=COS(PSI+XJ)
      Y(1,J)=SIN(PSI+XJ)
      Z(1,J)=0.
   35 CONTINUE
C
C     ---- USING NUMERICAL DAMPING TO AVOID NUMERICAL INSTABILITY ----
C
      CALL SMOOTH
C
C     ---- COMPUTATION OF THE NEW WAKE PROPERTIES                ----
C                                                                ----
C
```

```
      DO 38 J=1,2
      SEGOLD=SEG(1,J)
      GAMOLD=GAMA(1,J)
      COROLD=A(1,J)
      DO 37 I=2,NW
      I1=I+1
      DX=X(I1,J)-X(I,J)
      DY=Y(I1,J)-Y(I,J)
      DZ=Z(I1,J)-Z(I,J)
      SEGNEW=SQRT(DX*DX+DY*DY+DZ*DZ)
      CORNEW=COROLD*SQRT(SEGOLD/SEGNEW)
      GAMNEW=GAMOLD*SEGOLD/SEGNEW
      SEGOLD=SEG(I,J)
      COROLD=A(I,J)
      GAMOLD=GAMA(I,J)
      SEG(I,J)=SEGNEW
      A(I,J)=CORNEW
      GAMA(I,J)=GAMNEW
   37 CONTINUE
      DX=X(1,J)-X(2,J)
      DY=Y(1,J)-Y(2,J)
      DZ=Z(1,J)-Z(2,J)
      SEG(1,J)=SQRT(DX*DX+DY*DY+DZ*DZ)
   38 CONTINUE
      NPS=IFIX((PSI+.05)/DPSI)
      LPS=MOD(NPS,NA)+1
      LPD=LPS+NA/2
      IF(LPD.GT.NA) LPD=LPD-NA
      GAMA(1,1)=GAMB(LPS)
      GAMA(1,2)=GAMB(LPD)
      A(1,1)=COR
      A(1,2)=COR
      RETURN
      END
```

```
C       ---------------------------------------------------
C       THIS SUBROUTINE COMPUTES THE SELF INDUCED VELOCITY
C       AT THE POINTS (I,J) FOR SEGMENTS BETWEEN (I-1,J),
C       AND (I+1,J).
C       --
C       ---------------------------------------------------
        SUBROUTINE  SLFIND(IDUM,IRM,IR,IRP,L,GAM,DU,DV,DW)
        COMMON/ALLSUB/ALFO,ALPHAT,CAT,CAT2,CA2T,CT,DALFO,DPSI,EPS,
      1      H,IADD,IAV,MIRIMG,NA,NCONS,NDPSI,NPR,NRW,NW,NW1,PI,
      2      PSI,PSIF,PSIO,RAD,RC,REV,SAT,SAT2,SA2T,THCAT,
      3      TFRC,THSAT,XMU,XX,YY,ZZ,ALFR,BETR,ALFZ,BETZ,
      4      A(180,2),GAMA(180,2),GAMB(30),SEG(180,2),U(180,2),
      5      V(180,2),W(180,2),X(180,2),Y(180,2),Z(180,2)
        IF(IDUM.EQ.1) GO TO 260
        X1=X(IRM,L)
        Y1=Y(IRM,L)
        Z1=Z(IRM,L)
        X2=X(IR,L)
        Y2=Y(IR,L)
        Z2=Z(IR,L)
        X3=X(IRP,L)
        Y3=Y(IRP,L)
        Z3=Z(IRP,L)
        DX1=X1-X2
        DX2=X2-X3
        DX3=X3-X1
        DY1=Y1-Y2
        DY2=Y2-Y3
        DY3=Y3-Y1
        DZ1=Z1-Z2
        DZ2=Z2-Z3
        DZ3=Z3-Z1
        AL2=DX1*DX1+DY1*DY1+DZ1*DZ1
        BL2=DX2*DX2+DY2*DY2+DZ2*DZ2
        AL=SQRT(AL2)
        BL=SQRT(BL2)
        CL2=DX3*DX3+DY3*DY3+DZ3*DZ3
        CL=SQRT(CL2)
        DENR=(AL+BL-CL)*(AL+BL+CL)*(BL+CL-AL)*(AL+CL-BL)
        IF(DENR.GT.0.001) GO TO 10
        DU=0.
        DV=0.
        DW=0.
        RETURN
     10 R2=AL2*BL2*CL2/DENR
        R=SQRT(R2)
        S1=.5*AL/R
        S2=.5*BL/R
        C1=SQRT(1.-S1*S1)
        C2=SQRT(1.-S2*S2)
        T104=S1/(1.+C1)
        T204=S2/(1.+C2)
        GG1=GAMA(IRM,L)*(ALOG(8.*R*T104/A(IRM,L))+.25)
        GG2=GAMA(IR,L)*(ALOG(8.*R*T204/A(IR,L))+.25)
        GGG=(GG1+GG2)/(4.*R)
     20 XMX=DY1*DZ2-DY2*DZ1
        XMY=DZ1*DX2-DZ2*DX1
```

```
      XMZ=DX1*DY2-DX2*DY1
      XML=SQRT(XMX*XMX+XMY*XMY+XMZ*XMZ)
      DU=GGG*XMX/XML
      DV=GGG*XMY/XML
      DW=GGG*XMZ/XML
C1110 WRITE(45,C1111)I,J,DU,DV,DW
C1111 FORMAT(28X,'SELF INDUCED VELOC'/2I10,3F12.6)
      RETURN
  260 DU=0.
      DV=0.
      DW=-GAMA(1,L)*TFRC
C1112 WRITE(45,C1111)I,J,DU,DV,DW
      RETURN
      END
```



```
      SUBROUTINE VEL(GAM,ASQ,DU,DV,DW)
C
C     ---- THIS SUBROUTINE STORES THE TIME HISTORY OF THE WAKE ----
C     ---- FOR HALF OF A REVELOTION IN SX SY SZ. IT ALSO ADDS  ----
C     ---- THE NUMERICAL DAMPING TO AVOID NUMERICAL AND WAKE   ----
C     ---- INSTABILITY.                                        ----
C
      COMMON/VELO/DXA,DYA,DZA,DX,DY,DZ,DXB,DYB,DZB
      R1S=DXA*DXA+DYA*DYA+DZA*DZA
      R1=SQRT(R1S)
      R2S=DX*DX+DY*DY+DZ*DZ
      R2=SQRT(R2S)
      SQ=DXB*DXB+DYB*DYB+DZB*DZB
      SQDUM=(R1+R2)*(R1+R2)-SQ
      HSQ=.25*SQDUM*(SQ-(R1-R2)*(R1-R2))/SQ
      GG=GAM*(R1+R2)/(R1*R2*SQDUM)
      GGG=GG*(HSQ/(ASQ+HSQ))
  161 DU=GGG*(DYA*DZB-DZA*DYB)
      DV=GGG*(DZA*DXB-DXA*DZB)
      DW=GGG*(DXA*DYB-DYA*DXB)
      RETURN
      END
```

```
C     ***** SUB. WAKEF *****
C     -------------------------------
C     VELOCITY INDUCED BY WAKE ITSELF.
C     -------------------------------
      SUBROUTINE WAKE(I,J)
C
C     ---- THIS SUBROUTINE COMPUTES THE INDUCED VELOCITY OF THE ----
C     ---- MAIN WAKE  AT THE POINT (I,J) ON THE WAKE ITSELF.    ----
C     ---- (POINT XX,YY,ZZ OR X(I,J),Y(I,J),Z(I,J))
C
      COMMON/ALLSUB/ALFO,ALPHAT,CAT,CAT2,CA2T,CT,DALFO,DPSI,EPS,
     1     H,IADD,IAV,MIRIMG,NA,NCONS,NDPSI,NPR,NRW,NW,NW1,PI,
     2     PSI,PSIF,PSIO,RAD,RC,REV,SAT,SAT2,SA2T,THCAT,
     3     TFRC,THEAT,XMU,XX,YY,ZZ,ALFR,BETR,ALFZ,BETZ,
     4     A(180,2),GAMA(180,2),GAMB(30),SEG(180,2),U(180,2),
     5     V(180,2),W(180,2),X(180,2),Y(180,2),Z(180,2)
      COMMON/HALFS/RR,X1U,Y1U,Z1U,X2U,Y2U,Z2U,
     1            X1,X2,X3,Y1,Y2,Y3,Z1,Z2,Z3
      COMMON/VELO/DXA,DYA,DZA,DX,DY,DZ,DXB,DYB,DZB
      NH=NA/2
      TH=2.*H
C
C     ---- LOOP FOR THE WAKES PRODUCED BY TWO BLADES. ----
C
      DO 25 L=1,2
      LL=L
      DXA=XX-X(1,L)
      DYA=YY-Y(1,L)
      DZA=ZZ-Z(1,L)
      R1S=DXA*DXA+DYA*DYA+DZA*DZA
      R1=SQRT(R1S)
C
C     ---- LOOP FOR ALL THE POINTS ON THE WAKE ----
C
      DO 80 IR=1,NW
      IRP=IR+1
      DX=XX-X(IRP,L)
      DY=YY-Y(IRP,L)
      DZ=ZZ-Z(IRP,L)
      R2S=DX*DX+DY*DY+DZ*DZ
      R2=SQRT(R2S)
C
C     ---- CHECK IF THE POINT (I,J) IS END OF A SEGMENT BETWEEN ----
C     ---- POINTS (IR,L) AND (IR+2,L)                          ----

      IF(L.NE.J) GO TO 50
      IF(IR.LT.I-1.OR.IR.GT.I) GO TO 50
      IF(I.EQ.1.OR.IR.EQ.I-1)GO TO 20
      IF(IR.EQ.I) GO TO 70
   20 IRM=I-1
      IF(I.EQ.1) IRM=1
      IR1=IRM+1
      IRP1=IR1+1
      IDUM=I
      GAM=(GAMA(IRM,L)+GAMA(IR1,L))/2.
      LDUM=L
C
```

```
C      ---- COMPUES SELF INDUCED VELOCITY FOR THE TWO SEGMENTS ----
C      ---- AROUND POINT (I,J).                                ----

       CALL SLFIND(IDUM,IRM,IR1,IRP1,LDUM,GAM,DU,DV,DW)
    10 U(I,J)=U(I,J)+DU
       V(I,J)=V(I,J)+DV
       W(I,J)=W(I,J)+DW
C1111 WRITE(45,C2221)L,IR,DU,DV,DW
C2221 FORMAT(4X,'WAKE  ',2I8,3F12.6)
       GO TO 70
C
C      ---- CHECK IF THE POINT (I,J) IS VERY CLOSE TO OR VERY ----
C      ---- FAR FROM SEGMENT BETWEEN POINTS (IR,L) AND        ----
C      ---- (IR+1,L)                                           ----
C
    50 IF(R1.GT.TH.AND.R2.GT.TH) GO TO 70
C
C                                                              ----
C
       IF(R1.LT.0.2.OR.R2.LT.0.2)GO TO 67
    68 SQ=SEG(IR,L)*SEG(IR,L)
       SQDUM=(R1+R2)*(R1+R2)-SQ
       IF(SQDUM.LT.0.001) SQDUM=.001
       HSQ=.25*SQDUM*(SQ-(R1-R2)*(R1-R2))/SQ
       ASQ=A(IR,L)*A(IR,L)
       GG=GAMA(IR,L)*(R1+R2)/(R1*R2*SQDUM)
       GGG=GG*(HSQ/(ASQ+HSQ))
       DXB=X(IR,L)-X(IRP,L)
       DYB=Y(IR,L)-Y(IRP,L)
       DZB=Z(IR,L)-Z(IRP,L)
   161 XNU1=DYA*DZB-DZA*DYB
       XNU2=DZA*DXB-DXA*DZB
       XNU3=DXA*DYB-DYA*DXB
       DU=XNU1*GGG
       DV=XNU2*GGG
       DW=XNU3*GGG
       U(I,J)=U(I,J)+DU
       V(I,J)=V(I,J)+DV
       W(I,J)=W(I,J)+DW
C1112 WRITE(45,C2221)L,IR,DU,DV,DW
       GO TO 70
C
C      ---- POINT (I,J) IS CLOSE TO THE SEGMENT.          ----
C
    67 IRM3=IR
       IR3=IR+1
       IRP3=IR3+1
       IF(IRP3.GT.NW1) GO TO 68
       LL=L
C
C      ---- CALL HALFST SUBROUTINE TO REDUCE THE STEP SIZE ----
C
       CALL HALFST(IRM3,IR3,IRP3,LL)
       DXA=XX-X1
       DYA=YY-Y1
       DZA=ZZ-Z1
       DX=XX-X1U
```

```
      DY=YY-Y1U
      DZ=ZZ-Z1U
      DXB=DX-DXA
      DYB=DY-DYA
      DZB=DZ-DZA
      GAM=GAMA(IRM3,LL)
      ASQ=A(IRM3,LL)*A(IRM3,LL)
      CALL VEL(GAM,ASQ,DU1,DV1,DW1)
      DXA=DX
      DYA=DY
      DZA=DZ
      DX=XX-X2
      DY=YY-Y2
      DZ=ZZ-Z2
      DXB=DX-DXA
      DYB=DY-DYA
      DZB=DZ-DZA
      CALL VEL(GAM,ASQ,DU2,DV2,DW2)
      U(I,J)=U(I,J)+DU1+DU2
      V(I,J)=V(I,J)+DV1+DV2
      W(I,J)=W(I,J)+DW1+DW2
C1113 WRITE(45,C2221) L,IR,DU1,DV1,DW1,L,IR,DU2,DV2,DW2
   70 R1S=R2S
      R1=R2
      DXA=DX
      DYA=DY
      DZA=DZ
   80 CONTINUE
C
C     ---- END OF THE MAIN LOOP                               ----
C
   25 CONTINUE
C
C     ---- END OF THE BLADE LOOP                              ----
C
      RETURN
      END
```

```
      U(I,J)=U(I,J)+DU
      V(I,J)=V(I,J)+DV
      W(I,J)=W(I,J)+DW
C1111 WRITE(45,C2221)LL,IRR,DU,DV,DW
C2221 FORMAT(2X,2I7,3F12.6,'   MIRIMG')
      GO TO 1335
 1332 IRM=IRR
      IR=IRR+1
      IRP=IR+1
      IF(IRP.GT.NW1) GO TO 1310
      L=LL
C     ---- REDUCTION OF STEP SIZE IN SEGMENT LENGTH ----
C
      CALL HALFST(IRM,IR,IRP,L)
      DXA=XX-(X1*CA2T-Z1*SA2T-THSAT)
      DYA=YY-Y1
      DZA=ZZ+(X1*SA2T+Z1*CA2T+THCAT)
      DX=XX-(X1U*CA2T-Z1U*SA2T-THSAT)
      DY=YY-Y1U
      DZ=ZZ+(X1U*SA2T+Z1U*CA2T+THCAT)
      DXB=DX-DXA
      DYB=DY-DYA
      DZB=DZ-DZA
      GAM=-GAMA(IRM,LL)
      ASQ=A(IRM,LL)*A(IRM,LL)
      ASQ=.0001
      CALL VEL(GAM,ASQ,DU1,DV1,DW1)
      DXA=DX
      DYA=DY
      DZA=DZ
      DX=XX-(X2*CA2T-Z2*SA2T-THSAT)
      DY=YY-Y2
      DZ=ZZ+(X2*SA2T+Z2*CA2T+THCAT)
      DXB=DX-DXA
      DYB=DY-DYA
      DZB=DZ-DZA
      CALL VEL(GAM,ASQ,DU2,DV2,DW2)
      U(I,J)=U(I,J)+DU1+DU2
      V(I,J)=V(I,J)+DV1+DV2
      W(I,J)=W(I,J)+DW1+DW2
C1112 WRITE(45,C2221) LL,IRR,DU1,DV1,DW1,LL,IRR,DU2,DV2,DW2
 1335 CONTINUE
      R1=R2
      XI=XJ
      YI=YJ
      ZI=ZJ
  134 CONTINUE
  135 CONTINUE
      RETURN
      END
```

```
C     ***** SUB MIRIMGF *****
C     ------------------------------------------
C     VELOCITY INDUCED BY MIRIMG (GROUND EFFECT).
C     ------------------------------------------
      SUBROUTINE IMGWAK(I,J)
C
C     ---- THIS SUBROUTINE COMPUTES THE INDUCED VELOCITY OF ----
C     ---- THE MIRROR IMAGE OF THE THE WAKE AT POINTS (I,J) ----
C
      COMMON/ALLSUB/ALFO,ALPHAT,CAT,CAT2,CA2T,CT,DALFO,DPSI,EPS,
     1      H,IADD,IAV,MIRIMG,NA,NCONS,NDPSI,NPR,NRW,NW,NW1,PI,
     2      PSI,PSIF,PSIO,RAD,RC,REV,SAT,SAT2,SA2T,THCAT,
     3      TFRC,THSAT,XMU,XX,YY,ZZ,ALFR,BETR,ALFZ,BETZ,
     4      A(180,2),GAMA(180,2),GAMB(30),SEG(180,2),U(180,2),
     5      V(180,2),W(180,2),X(180,2),Y(180,2),Z(180,2)
      COMMON/HALFS/RR,X1U,Y1U,Z1U,X2U,Y2U,Z2U,
     1      X1,X2,X3,Y1,Y2,Y3,Z1,Z2,Z3
      COMMON/VELO/DXA,DYA,DZA,DX,DY,DZ,DXB,DYB,DZB
C
C     ---- LOOP FOR WAKE OF BLADE NO.1 AND NO.2            ----
C
  130 DO 135 LL=1,2
      XI=X(1,LL)*CA2T-Z(1,LL)*SA2T-THSAT
      YI=Y(1,LL)
      ZI=-(Z(1,LL)*CA2T+X(1,LL)*SA2T+THCAT)
      R1S=(XX-XI)*(XX-XI)+(YY-YI)*(YY-YI)+(ZZ-ZI)*(ZZ-ZI)
      R1=SQRT(R1S)
C
C     ---- LOOP FOR ALL THE POINTS ON THE WAKE ----
C
  131 DO 134 IRR=1,NW
      IP=IRR+1
      XJ=X(IP,LL)*CA2T-Z(IP,LL)*SA2T-THSAT
      YJ=Y(IP,LL)
      ZJ=-(Z(IP,LL)*CA2T+X(IP,LL)*SA2T+THCAT)
      R2S=(XX-XJ)*(XX-XJ)+(YY-YJ)*(YY-YJ)+(ZZ-ZJ)*(ZZ-ZJ)
      R2=SQRT(R2S)
      IF(R1.GT.THCAT.AND.R2.GT.THCAT) GO TO 1335
      IF(R1.LT.0.2.OR.R2.LT.0.2) GO TO 1332
C
C     ---- CHECK IF POINT (I,J) IS TOO CLOSE TO OR TOO FAR FROM ----
C     ---- MIRROR IMAGE OF SEGMENT BETWEEN POINTS (IRRR-1,L)    ----
C     ---- (IRR,L)
C     ---                                                       ----
C
 1310 SQ=SEG(IRR,LL)*SEG(IRR,LL)
      SQDUM=(R1+R2)*(R1+R2)-SQ
      HSQ=.25*SQDUM*(SQ-(R1-R2)*(R1-R2))/SQ
      ASQ=A(IRR,LL)*A(IRR,LL)
  132 GG=-GAMA(IRR,LL)*(R1+R2)/(R1*R2*SQDUM)
      GGG=GG*(HSQ/(ASQ+HSQ))
  133 XNU1=(YY-YI)*(ZI-ZJ)-(ZZ-ZI)*(YI-YJ)
      XNU2=(ZZ-ZI)*(XI-XJ)-(XX-XI)*(ZI-ZJ)
      XNU3=(XX-XI)*(YI-YJ)-(YY-YI)*(XI-XJ)
      DU=XNU1*GGG
      DV=XNU2*GGG
      DW=XNU3*GGG
```

```
C      ***** SUB. BOUNDE *****
C      -----------------------------------
C      VELOCITY INDUCED BY BOUND VORTICES.
C      -----------------------------------
       SUBROUTINE BOUNDE(I,J)
C
C      ----THIS SUBROUTINE COMPUTES THE INDUCED VELOCITY OF THE ----
C      ----BOUND VORTICES.                                     ----
C
       COMMON/ALLSUB/ALFO,ALPHAT,CAT,CAT2,CA2T,CT,DALFO,DPSI,EPS,
     1      H,IADD,IAV,MIRIMG,NA,NCONS,NDPSI,NPR,NRW,NW,NW1,PI,
     2      PSI,PSIF,PSIO,RAD,RC,REV,SAT,SAT2,SA2T,THCAT,
     3      TFRC,THSAT,XMU,XX,YY,ZZ,ALFR,BETR,ALFZ,BETZ,
     4      A(180,2),GAMA(180,2),GAMB(30),SEG(180,2),U(180,2),
     5      V(180,2),W(180,2),X(180,2),Y(180,2),Z(180,2)
       COMMON/VELO/DXA,DYA,DZA,DX,DY,DZ,DXB,DYB,DZB
       ASQ=1./(RC*RC)
       NPS=IFIX((PSI+.05)/DPSI)
       LPS=MOD(NPS,NA)+1
       LPD=LPS+NA/2
       IF(LPD.GT.NA) LPD=LPD-NA
       IF(MIRIMG.NE.0) GO TO 10
C
C      ---- INDUCED VELOCITY OF THE MIRROR IMAGE OF THE BOUND VORTICES ----
C
       DXA=XX+THSAT
       DYA=YY
       DZA=ZZ+THCAT
       DX=XX-X(1,1)*CA2T+THSAT
       DY=YY-Y(1,1)
       DZ=ZZ+X(1,1)*SA2T+THCAT
       DXB=DX-DXA
       DYB=DY-DYA
       DZB=DZ-DZA
       GAM=-GAMB(LPS)
       CALL VEL(GAM,ASQ,DU1,DV1,DW1)
       DX=XX-X(1,2)*CA2T+THSAT
       DY=YY-Y(1,2)
       DZ=ZZ+X(1,2)*SA2T+THCAT
       DXB=DX-DXA
       DYB=DY-DYA
       DZB=DZ-DZA
       GAM=-GAMB(LPD)
       CALL VEL(GAM,ASQ,DU2,DV2,DW2)
       U(I,J)=U(I,J)+DU1+DU2
       V(I,J)=V(I,J)+DV1+DV2
       W(I,J)=W(I,J)+DW1+DW2
C1111  WRITE(45,C2221)I,J,DU1,DV1,DW1,I,J,DU2,DV2,DW2
C2221  FORMAT(5X,'BOUND ',2I7,3F12.6)
    10 IF(I.NE.1) GO TO 20
       XX=.5*(XX+X(2,J))
       YY=.5*(YY+Y(2,J))
       ZZ=.5*(ZZ+Z(2,J))
C      ----
C      ----INDUCED VELOCITY OF THE BOUND VORTICES ----
C
    20 DXA=XX
```

```
      DYA=YY
      DZA=ZZ
      DX=XX-X(1,1)
      DY=YY-Y(1,1)
      DZ=ZZ-Z(1,1)
      DXB=DX-DXA
      DYB=DY-DYA
      DZB=DZ-DZA
      GAM=GAMB(LPS)
      CALL VEL(GAM,ASQ,DU1,DV1,DW1)
      DX=XX-X(1,2)
      DY=YY-Y(1,2)
      DZ=ZZ-Z(1,2)
      DXB=DX-DXA
      DYB=DY-DYA
      DZB=DZ-DZA
      GAM=GAMB(LPD)
      CALL VEL(GAM,ASQ,DU2,DV2,DW2)
      U(I,J)=U(I,J)+DU1+DU2
      V(I,J)=V(I,J)+DV1+DV2
      W(I,J)=W(I,J)+DW1+DW2
C1112 WRITE(45,C2221)I,J,DU1,DV1,DW1,I,J,DU2,DV2,DW2
      RETURN
      END
```

```
C     --------------------------------
C     VELOCITY INDUCED BY INSIDE WAKE .
C     --------------------------------
      SUBROUTINE INSWKE(I,J)
C
C     ---- THIS SUBROUTINE COMPUTES THE INDUCED VELOCITY   ----
C     ---- THE INSIDE WAKE AT POINTS I,J.(%70 OF THE BLADE ----
C     ---- RADIUS FROM THE ROOT)                           ----
C
      COMMON/ALLSUB/ALFO,ALPHAT,CAT,CAT2,CA2T,CT,DALFO,DPSI,EPS,
     1      H,IADD,IAV,MIRIMG,NA,NCONS,NDPSI,NPR,NRW,NW,NW1,PI,
     2      PSI,PSIF,PSIO,RAD,RC,REV,SAT,SAT2,SA2T,THCAT,
     3      TFRC,THSAT,XMU,XX,YY,ZZ,ALFR,BETR,ALFZ,BETZ,
     4      A(180,2),GAMA(180,2),GAMB(30),SEG(180,2),U(180,2),
     5      V(180,2),W(180,2),X(180,2),Y(180,2),Z(180,2)
      COMMON/VELO/DXA,DYA,DZA,DX,DY,DZ,DXB,DYB,DZB
      NEND=3*NA/2+4
      DO 10 L=1,2
      DXA=XX-.7*X(1,L)
      DYA=YY-.7*Y(1,L)
      DZA=ZZ-Z(1,L)
      DO 10 IR=2,NEND
      IRM=IR-1
      DX=XX-.7*X(IR,L)
      DY=YY-.7*Y(IR,L)
      DZ=ZZ-Z(IR,L)
      DXB=DX-DXA
      DYB=DY-DYA
      DZ=DZ-DZA
      GAM=-.5*GAMA(IRM,L)
      ASQ=A(IRM,L)*A(IRM,L)/.01
      CALL VEL(GAM,ASQ,DU,DV,DW)
      U(I,J)=U(I,J)
      V(I,J)=V(I,J)
      W(I,J)=W(I,J)+DW
C1111 WRITE(45,C2221)L,IR,DU,DV,DW
C2221 FORMAT(2X,' INSWKE ',2I8,3F12.6)
      DXA=DX
      DYA=DY
      DZA=DZ
   10 CONTINUE
      RETURN
      END
```

```
C     ------------------------------------------------
C     ------------------------------------------------
      SUBROUTINE HALFST(IRM,IR,IRP,L)
C
C     ---- THIS SUBROUTINE PASSES A CIRCLE THROUGH THREE POINTS ----
C     ---- AND FINDS THE HALF WAY BETWEEN THREE POINTS AND      ----
C     ---- THE HALF WAY BETWEEN EACH TWO POINTS. IN OTHER WORDS ----
C     ---- IT REDUCES THE STEP SIZE . THIS SUBROUTINE IS CALLED ----
C     ---- WHEN POINT OF INTEREST IS VERY CLOSE TO A SEGMENT OF ----
C     ---- THE WAKE.                                            ----
C
      COMMON/ALLSUB/ALFO,ALPHAT,CAT,CAT2,CA2T,CT,DALFO,DPSI,EPS,
     1      H,IADD,IAV,MIRIMG,NA,NCONS,NDPSI,NPR,NRW,NW,NW1,PI,
     2      PSI,PSIF,PSIO,RAD,RC,REV,SAT,SAT2,SA2T,THCAT,
     3      TFRC,THSAT,XMU,XX,YY,ZZ,ALFR,BETR,ALFZ,BETZ,
     4      A(180,2),GAMA(180,2),GAMB(30),SEG(180,2),U(180,2),
     5      V(180,2),W(180,2),X(180,2),Y(180,2),Z(180,2)
      COMMON/HALFS/R,X1U,Y1U,Z1U,X2U,Y2U,Z2U,
     1      X1,X2,X3,Y1,Y2,Y3,Z1,Z2,Z3
      X1=X(IRM,L)
      Y1=Y(IRM,L)
      Z1=Z(IRM,L)
      X2=X(IR,L)
      Y2=Y(IR,L)
      Z2=Z(IR,L)
      X3=X(IRP,L)
      Y3=Y(IRP,L)
      Z3=Z(IRP,L)
      DX1=X1-X2
      DX2=X2-X3
      DX3=X3-X1
      DY1=Y1-Y2
      DY2=Y2-Y3
      DY3=Y3-Y1
      DZ1=Z1-Z2
      DZ2=Z2-Z3
      DZ3=Z3-Z1
      AL2=DX1*DX1+DY1*DY1+DZ1*DZ1
      BL2=DX2*DX2+DY2*DY2+DZ2*DZ2
      AL=SQRT(AL2)
      BL=SQRT(BL2)
      CL2=DX3*DX3+DY3*DY3+DZ3*DZ3
      CL=SQRT(CL2)
      DENR=(AL+BL-CL)*(AL+BL+CL)*(BL+CL-AL)*(AL+CL-BL)
      IF(DENR.GT.0.001) GO TO 10
      X1U=.5*(X1+X2)
      Y1U=.5*(Y1+Y2)
      Z1U=.5*(Z1+Z2)
      X2U=.5*(X2+X3)
      Y2U=.5*(Y2+Y3)
      Z2U=.5*(Z2+Z3)
      RETURN
   10 R2=AL2*BL2*CL2/DENR
      R=SQRT(R2)
      ANUM=DX3*DX2+DY3*DY2+DZ3*DZ2
      DEN1=DX1*DX2+DY1*DY2+DZ1*DZ2
      DEN =DEN1*DEN1-AL2*BL2
```

```
      CO =.5*ANUM/DEN
      COT=SQRT(R2-.25*AL2)
      T=COT/CO
      IF(CO.GT.0.) COP=R/T-CO
      IF(CO.LE.0.) COP=-R/T-CO
      C1=DEN1*COP
      C2=COP*AL2
      C3=.5-C1
      C4=.5+C1+C2
      X1U=C3*X1+C4*X2-C2*X3
      Y1U=C3*Y1+C4*Y2-C2*Y3
      Z1U=C3*Z1+C4*Z2-C2*Z3
      ANUM=DX1*DX3+DY1*DY3+DZ1*DZ3
      CO=.5*ANUM/DEN
      COT=SQRT(R2-.25*BL2)
      T=COT/CO
      IF(CO.GT.0.) COP=R/T-CO
      IF(CO.LE.0.) COP=-R/T-CO
      C1=COP*DEN1
      C2=COP*BL2
      C3=.5-C1
      C4=.5+C1+C2
      X2U=-C2*X1+C4*X2+C3*X3
      Y2U=-C2*Y1+C4*Y2+C3*Y3
      Z2U=-C2*Z1+C4*Z2+C3*Z3
      RETURN
      END
```

```
      SUBROUTINE SMOOTH
C     ------------------------------------------------------------
C       THIS SUBROUTINE SMOOTHS THE LOCATION OF TIP VORTICES.
C     ------------------------------------------------------------
      COMMON/ALLSUB/ALFO,ALPHAT,CAT,CAT2,CA2T,CT,DALFO,DPSI,EPS,
     1        H,IADD,IAV,MIRIMG,NA,NCONS,NDPSI,NPR,NRW,NW,NW1,PI,
     2        PSI,PSIF,PSIO,RAD,RC,REV,SAT,SAT2,SA2T,THCAT,
     3        TFRC,THSAT,XMU,XX,YY,ZZ,ALFR,BETR,ALFZ,BETZ,
     4        A(180,2),GAMA(180,2),GAMB(30),SEG(180,2),U(180,2),
     5        V(180,2),W(180,2),X(180,2),Y(180,2),Z(180,2)
      COMMON/MIS/ SX(180,18),SY(180,18),SZ(180,18)
      COMMON /NTURN/ NCOMP
      NSTOP = NCONS + NCOMP
      IF(NSTOP .GT. NW) NSTOP =NW
      NSTOP1= NSTOP + 1
      N1=IFIX((PSI+.05)/DPSI)
      PSI=N1*DPSI
      N2=MOD(N1,NA)
      NAH=NA/2
      J=N2+1
      K=J+NAH
      IF(K.GT.NA) K=K-NA
   40 NSTART=NCONS+1
      PSIE=NAH*DPSI-.1*DPSI
      IF(PSI.LT.PSIE)  GO TO 20
      IF(J.EQ.4.OR.J.EQ.10) GO TO 44
      IF(J.NE.1.AND.J.NE.7)GO TO 50
   44 DX1=X(NSTART,1)-SX(NSTART,J)
      DX2=X(NSTART,2)-SX(NSTART,K)
      DY1=Y(NSTART,1)-SY(NSTART,J)
      DY2=Y(NSTART,2)-SY(NSTART,K)
      DZ1=Z(NSTART,1)-SZ(NSTART,J)
      DZ2=Z(NSTART,2)-SZ(NSTART,K)
      ERR1=ALFR*(DX1*DX1+DY1*DY1)+ALFZ*(DZ1*DZ1)
      ERR2=ALFR*(DX2*DX2+DY2*DY2)+ALFZ*(DZ2*DZ2)
      ERR=SQRT((ERR1+ERR2)/(ALFR+ALFZ))
      IF(ERR.GT.EPS) GO TO 50
      IF(NCONS.GE.NW) GO TO 50
      NCONS=NCONS+1
      GO TO 40
   50 CONTINUE
C     PRINT *,' 1 NSTOP1 ,NCOMP,NCONS ',NSTOP1,NCOMP,NCONS
C     PRINT *, ' XE,SXE ', X(NW1,1),SX(NW1,J)
      DO 2 I=1,NCONS
      X(I,1)=SX(I,J)
      X(I,2)=SX(I,K)
      Y(I,1)=SY(I,J)
      Y(I,2)=SY(I,K)
      Z(I,1)=SZ(I,J)
      Z(I,2)=SZ(I,K)
    2 CONTINUE
      IF(NSTOP1 .GE. NW1) GO TO 4
      DO 3 I=NSTOP1,NW1
      X(I,1)=SX(I,J)
      X(I,2)=SX(I,K)
      Y(I,1)=SY(I,J)
      Y(I,2)=SY(I,K)
```

```
      Z(I,1)=SZ(I,J)
      Z(I,2)=SZ(I,K)
    3 CONTINUE
    4 CONTINUE
C      PRINT *,' 2 NSTOP1 ,NCOMP,NCONS ',NSTOP1,NCOMP,NCONS
C      PRINT *, ' XE,SXE ', X(NW1,1),SX(NW1,J)
      DO 10 I=NSTART,NSTOP1
      X(I,1)=ALFR*X(I,1)+BETR*SX(I,J)
      Y(I,1)=ALFR*Y(I,1)+BETR*SY(I,J)
      X(I,2)=ALFR*X(I,2)+BETR*SX(I,K)
      Y(I,2)=ALFR*Y(I,2)+BETR*SY(I,K)
      Z(I,1)=ALFZ*Z(I,1)+BETZ*SZ(I,J)
      Z(I,2)=ALFZ*Z(I,2)+BETZ*SZ(I,K)
   10 CONTINUE
C      PRINT *,' 3 NSTOP1 ,NCOMP,NCONS ',NSTOP1,NCOMP,NCONS
C      PRINT *, ' XE,SXE ', X(NW1,1),SX(NW1,J)
      IF(XMU.GT.0.00001.OR.MIRING.NE.1) GO TO 80
      R1=1.
      DO 60 I=2,NW1
      R2=SQRT(X(I,1)*X(I,1)+Y(I,1)*Y(I,1))
      IF(R1.GT.R2) GO TO 70
      X(I,1)=X(I,1)*R1/R2
      Y(I,1)=Y(I,1)*R1/R2
      R2=R1
   70 X(I,2)=-X(I,1)
      Y(I,2)=-Y(I,1)
      Z(I,2)= Z(I,1)
      R1=R2
   60 CONTINUE
C      PRINT *,' 4 NSTOP1 ,NCOMP,NCONS ',NSTOP1,NCOMP,NCONS
C      PRINT *, ' XE,SXE ', X(NW1,1),SX(NW1,J)
      GO TO 20
   80 CONTINUE
      CALL SMOTH2
   20 CONTINUE
      DO 30 I=1,NW1
      SX(I,J)=X(I,1)
      SX(I,K)=X(I,2)
      SY(I,J)=Y(I,1)
      SY(I,K)=Y(I,2)
      SZ(I,J)=Z(I,1)
      SZ(I,K)=Z(I,2)
   30 CONTINUE
C      PRINT *,' 5 NSTOP1 ,NCOMP,NCONS ',NSTOP1,NCOMP,NCONS
C      PRINT *, ' XE,SXE ', X(NW1,1),SX(NW1,J)
      RETURN
      END
```