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A SIMPLIFIED METHOD FOR ELASTIC-PLASTIC-CREEP STRUCTURAL ANALYSIS

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ABSTRACT

A simplified inelastic analysis computer program (ANSYMP) was developed for predicting the stress-strain history at the critical location of a thermomechanically cycled structure from an elastic solution. The program uses an iterative and incremental procedure to estimate the plastic strains from the material stress-strain properties and a plasticity hardening model. Creep effects can be calculated on the basis of stress relaxation at constant strain, creep at constant stress or a combination of stress relaxation and creep accumulation. The simplified method was exercised on a number of problems involving uniaxial and multiaxial loading, isothermal and nonisothermal conditions, dwell times at various points in the cycles, different materials and kinematic hardening. Good agreement was found between these analytical results and nonlinear finite-element solutions for these problems. The simplified analysis program used less than 1 percent of the CPU time required for a nonlinear finite-element analysis.

INTRODUCTION

The drive toward better performance and fuel economy for aircraft gas turbine engines has resulted in higher turbine inlet temperatures, pressure ratios, and rotor speeds. These more severe operating conditions have subjected the hot section components to thermomechanical load cycles that induce significant inelastic strains and eventual fatigue cracking. It has become increasingly difficult to design reliable components to meet both the engine life and performance requirements. Improvements in the durability of these components depend on accurate structural analysis and life prediction. Life prediction methods have been under development by the NASA Lewis Research Center and other organizations (1-4). Application of these methods requires knowledge of the temperature-stress-strain history at the critical crack initiation location of the structure.

The primary structural parameters of interest for life prediction purposes are generally the total strain range and the mean cyclic stress. For most practical cases, the critical location and the total strain range can be satisfactorily obtained from an elastic analysis (3-5). However, in cases involving purely mechanical load cycling, creep or large plastic strains, an elastic analysis may not be adequate to determine the total strain range. Mean stresses for hot section components, as well as multiaxial and thermomechanical fatigue specimens, must be calculated from some type of nonlinear analysis. The accuracy of the solutions is largely dependent on the adequacy of the material properties and the creep-plasticity models used in the analysis.

Nonlinear finite-element analysis is being increasingly used for calculating inelastic structural response. However, nonlinear methods are not feasible for use as a component design tool because of the high computing costs associated with the iterative and incremental nature of the inelastic solutions. Computing costs are further increased by the presence of high thermal gradients and geometrical irregularities, such as cooling holes, which necessitate three-dimensional analyses. Three-dimensional, nonlinear finite-element analyses are prohibitively time consuming and expensive to conduct in the early design stages for combustor and turbine structures.

To improve the design of hot section components, it is necessary to develop simpler and more economical methods for representing structural behavior under cyclic loading. Development of life prediction methods would also benefit from a simplified analysis method for determining the structural behavior of multiaxial and thermomechanical fatigue specimens.

This study was conducted to develop a fully automated simplified analytical procedure for estimating the stress-strain history of a thermomechanically loaded structure subject to cyclic inelasticity. A computer program (ANSYMP) was created to predict the cyclic stress-strain history at the critical location of a structure from a calculated elastic solution or one constructed from strain measurements at the critical location. An incremental and iterative procedure estimates the plastic strains from the material stress-strain properties and a plasticity hardening model. Creep options are incorporated into the program on the basis of stress relaxation at constant strain, creep at constant stress or a combination of stress relaxation and creep accumulation.

Analytical predictions from the simplified method were compared with nonlinear finite-element solutions from the MARC computer program (6) for a number of problems. Initial development of the program was based on a strain-controlled uniaxial problem with cyclic stress-strain and creep properties for Inconel 718 alloy. Verification of the simplified procedure was conducted using test problems involving an Inconel 718 benchmark notch specimen that was load cycled in an experiment designed to verify structural analysis methodologies (7) and a double-edge wedge specimen that had been thermally cycled in fluidized beds (8). A kinematic hardening model was used for all of these problems. In (5) a MARC elastic-plastic analysis using a kinematic hardening model gave excellent agreement with the experimental results for the benchmark notch specimen under continuous mechanical load cycling. Cyclic stress-strain and creep properties for IN 100 alloy (5) were used in the analyses for the wedge specimen problem. Variations of these three problems, including imposed creep dwell times at different points in the cycles, were exercised with both the ANSYMP simplified analysis program and the MARC program. Verification of the simplified analysis procedure was made on the basis of how well it was able to duplicate the stress-strain hysteresis loops from MARC elastic-plastic-creep analyses of these problems.

NOMENCLATURE

A,B,C	temperature-dependent constants in creep power law, Eq. (5)
E	modulus of elasticity
K,n	temperature-dependent constants in stress-strain equation, Eq. (1)
m	kinematic work hardening slope (Fig. 1)
t	time
$\Delta\sigma_y$	yield stress shift due to load reversal
ϵ_c	creep strain
ϵ_p	plastic strain
ϵ_p'	maximum plastic strain in cyle (Fig. 1)
ϵ_t	total strain
σ	stress
σ_y	current yield stress
σ_{yi}	initial yield stress
σ'	maximum stress in cycle (Fig. 1)
ν	Poisson's ratio

ANALYTICAL PROCEDURE

A simplified inelastic procedure was developed for calculating the stress-strain history at the critical fatigue location of a structure subjected to cyclic thermomechanical loading. The fundamental assumption in this procedure is that the inelastic region is local and is constrained from redistribution by the surrounding elastic material. It follows from this assumption that the total strain history at the critical location can be defined by an elastic solution. Justification for the assumption of elastic constraint of local inelasticity can be found in (3-5), where structural analyses of combustor liners, air-cooled turbine blades, and wedge fatigue specimens have shown that the total strain ranges from elastic and nonlinear solutions are in close agreement. A corollary to this assumption is that the elastic loading and unloading segments of the effective stress-equivalent total strain hysteresis loops constructed from an elastic-plastic analysis will be parallel to the elastic hysteresis loop. This is demonstrated by comparing the nonlinear and elastic hysteresis loops in (5).

The basic problem in developing the simplified analytical procedure was to characterize the yield surface in terms of the total strain obtained from an elastic analysis or strain measurements. Classical plasticity theory characterizes the yield surface by a yield condition to describe yielding under multiaxial stress states and by a hardening model to establish the location of the yield surface during cycling. The simplified procedure was set up to accommodate itself to any yield criterion or hardening model. The only requirements are that the elastic input data be consistent with the yield criterion and that the appropriate material properties be used in conjunction with the hardening model.

In this study, all of the analyses were performed with a kinematic hardening model. A representation of a cyclic stress-strain curve by a bilinear kinematic hardening model is illustrated in Fig. 1. The loci of the tips of the cyclic curves is described by the equation

$$\sigma = K(\epsilon_p)^n \tag{1}$$

The work hardening slope for the kinematic hardening model was determined from energy considerations to give the same strain energy, as indicated by the enclosed area in Fig. 1, as the actual stress-strain curve. This work hardening slope will be defined by

$$m = (\sigma'/\epsilon_p')(2n/(1 + n)) \tag{2}$$

and the initial yield point, $\Delta\sigma_{yi}$, by

$$\Delta\sigma_{yi} = \sigma' - m\,(\epsilon_p') \tag{3}$$

The yield stress shift ($\Delta\sigma_y$) due to load reversal under kinematic hardening is

$$\Delta\sigma_y = 2\,(\sigma_y - m\,(\epsilon_p)) = 2\,(\sigma_{yi}) \tag{4}$$

Creep characteristics of the material were incorporated into the program in the power law

$$\epsilon_c = (\sigma/A)^B t^C \tag{5}$$

A strain-hardening law (9) was used to accumulate creep strain under changing stress. Any of three creep options can be selected; (1) stress relaxation at constant strain, (2) cumulative creep at constant stress, and (3) a combination of (1) and (2).

Most nonlinear computer programs use the von Mises yield criterion and incremental plasticity theory. Implicit in the von Mises yield criterion is the conversion of the total strain from a uniaxial stress-strain curve to modified equivalent total strain, as discussed in (9). The modified elastic equivalent total strain corresponds to the uniaxial total elastic strain multiplied by $2(1 + \nu)/3$. This relationship must be taken into account for multiaxial problems in applying strain results from elastic finite-element analyses or strain measurements as input for the simplified inelastic analysis. Both elastic and nonlinear finite-element analyses for this study were conducted with the MARC computer program. The elastic solutions computed from MARC for input into the simplified analysis method were automatically obtained in terms of von Mises effective stresses and modified equivalent total strains.

The elastic input data are subdivided into a sufficient number of increments to define the stress-strain cycle. Dwell times are specified for increments which require creep analysis. The increments are analyzed sequentially to obtain the cumulative plastic and creep strains and to track the yield surface. An iterative procedure is used to calculate the yield stresses for increments undergoing plastic straining. First, an estimated plastic strain is assumed for calculating an initial yield stress from the stress-strain properties and the simulated hardening model. Second, a new plastic strain is calculated as

$$\epsilon_p = \epsilon_t - \epsilon_c - \sigma_y/E \qquad (6)$$

The yield stress is then recalculated using the new plastic strain. This iterative procedure is repeated until the new and previous plastic strains agree within a a tolerance 1 percent.

A FORTRAN IV computer program (ANSYMP) was created to automatically implement the simplified analytical procedure. The program consists of the main executive routine, ANSYMP, and four subroutines, ELAS, YIELD, CREEP, and SHIFT. The incremental elastic data and temperatures are read into subroutine ELAS. Material stress-strain properties as a function of temperature and a simulated hardening model are incorporated in subroutine YIELD and the creep characteristics are incorporated in subroutine CREEP. Subroutine SHIFT is required to update the temperature effects on the yield stress shift. SHIFT also serves the function of deciding the future direction of the yield surface under nonisothermal conditions by determining the relation of future to past thermal loading.

The ANSYMP program is available from the Computer Software Management Information Center (COSMIC), University of Georgia, Athens, Ga. 30602 under LEW 14011. A flow chart of the program and sample input and output data are presented in (10).

The calculational scheme initially follows the effective stress-equivalent strain input data from subroutine ELAS until the occurrence of initial yielding. The stress-strain solution then proceeds along the yield surface as determined from the stress-strain properties in subroutine YIELD. At each increment during yielding the stress shift (difference between new yield stress and stress predicted from elastic analysis) from the original input data is calculated. Elastic load reversal is signaled when the input stress is less than the yield stress from the previous increment. During elastic unloading, the stresses are translated from the original elastic analysis solution by the amount of the calculated stress shift. Reverse yielding occurs when the stress reaches the reverse yield surface as determined from the hardening model incorporated in subroutine YIELD. Again, the solution follows the yield surface until another load reversal is indicated when the stress based on the shifted elastic solution is less than the yield stress. The elastic response during load reversal is obtained by translating the original elastic solution according to the new stress shift calculated during reversed yielding. The stress-strain response for subsequent cycles is computed by repeating this procedure of identifying load reversals, tracking reverse yield surfaces and translating the original elastic solution during elastic loading and unloading. Creep computations are performed for increments involving dwell times using the creep equation and strain hardening rule incorporated in subroutine CREEP. Depending on the nature of the problem, the creep effects are determined on the basis of one of the three options provided in the subroutine.

The computer program was verified by conducting simplified analyses for a series of three problems and comparing the results to those from MARC nonlinear analyses. The first of these problems was a simulation of a uniaxial specimen subjected to strain cycling under isothermal conditions. Variations of this problem were run with no creep dwell times and with dwell times at minimum and intermediate total strain levels. A kinematic hardening model was used with cyclic stress-strain and creep properties for Inconel 718 alloy obtained from (7). Nonlinear and elastic MARC analyses of this problem were performed by using a single 20 node, three-dimensional element. The MARC solutions for the uniaxial problem were computed for the centroid of the single solid element model. The second problem considered was a mechanically load-cycled benchmark notch specimen shown in Fig. 2. This specimen was tested under isothermal conditions as part of a program to provide controlled strain data for constitutive model verification (7). A MARC analysis of this problem using kinematic hardening demonstrated excellent agreement with experimental data in (5). A number of variations were run with both the MARC and ANSYMP programs. These variations included dwell times at maximum, minimum and intermediate total strains and dwell times at increments where tensile yielding occurred. The simplified analysis of the benchmark notch problem used the kinematic hardening model and cyclic stress-strain and data for Inconel 718 alloy. The third problem was an IN 100 double-edge wedge specimen that was thermally cycled in the fluidized bed facility described in (8). This problem provides a nonisothermal case for evaluating the computer program. Both the MARC and ANSYMP analyses used the kinematic hardening model and the IN 100 cyclic stress-strain and creep properties reported in (5). The geometry of the double-edge wedge specimen is illustrated in Fig. 3. The wedge problem was analyzed without dwell times and with dwell times at maximum and minumum total strain levels. The MARC solutions shown for the benchmark notch and wedge specimens were computed at the closest Gaussian integration point to the critical crack initiation location.

The material properties and kinematic hardening models were coded into subroutines YIELD, SHIFT and CREEP. The program input initially involves specification of the number of increments of elastic input data, number of increments with dwell times, number of subincrements the dwell times are to be subdivided for creep calculations, the selected creep option and a pointer that refers to the type of problem to be solved and the set of material properties to be used in the analysis. The temperature, stress and total strain for the elastic solution is then listed for each increment. The elastic input data were repeated a second time to conduct the simplified analyses for two cycles for all the problems considered in this study. Finally, the dwell times are specified for those increments where creep calculations are to be performed. The output includes an echo of the parameters and the increment dwell times. For each increment, the temperature, stress, and the total, plastic and creep strains are listed. The output lists double the increments that were specified for the input because the stress-strain solution is printed for the beginning and end of each dwell increment.

DISCUSSION OF ANALYTICAL RESULTS

The results of the simplified elastic-plastic-creep analyses of the uniaxial, benchmark notch, and wedge specimen cases are discussed herein. Comparisons are made with MARC inelastic solutions. Stress-strain cycles used for comparison purposes are in terms of effective stresses and equivalent total strains based on the von Mises yield criterion. The discussion is based on the critical location in the specimen where fatigue cracking would start.

Uniaxial Problem

The uniaxial problem was used for the basic development of the simplified approach and computer program. Since the loading was strain-controlled, the maximum and minimum total strains were identical for the MARC elastic and nonlinear finite-element solutions. Also the effect of creep dwell time at any increment was to cause stress relaxation under constant total strain.

Four variations of the uniaxial problem were considered in this study. These were initial compressive loading without creep dwell times (Fig. 4(a)), dwell time at maximum strain (Fig. 4(b)), dwell time at minimum strain (Fig. 4(c)), and dwell times at minimum and intermediate strains (Fig. 4(d)). A constant temperature of 649° C was assumed during the strain cycling. Creep option 1 (stress relaxation at constant strain) was used for all the creep computations.

A comparison of the stress-strain cycles obtained from the simplified and MARC elastic-plastic-creep analyses is shown in Fig. 4. Agreement between the ANSYMP and MARC nonlinear solutions is seen to be excellent for all the uniaxial cases.

Benchmark Notch Problem

The benchmark notch test was conducted by mechanical load cycling at a constant temperature of 649° C. A mechanically loaded structure, especially where the peak strain occurs at a discontinuity, is most likely to violate the basic assumption of the simplified approach that strain redistribution is prevented by containment of the local plastic region by the surrounding elastic material. The computed total strain range from the MARC elastic-plastic analysis was 20 percent greater than that obtained from the MARC elastic analysis. This foreshortening of the elastic strain range would cause the simplified procedure to truncate the stress-strain hysteresis loop. Therefore, the elastic solution was constructed from strain measurements obtained at the notch root in (7). When the elastic solution was extended to be consistent with the measured notch root strain, the agreement between the simplified and MARC elastic-plastic stress-strain hysteresis loops was excellent as demonstrated in Fig. 5(a). Both the ANSYMP and MARC elastic-plastic analyses gave stable stress-strain hysteresis loops for the second cycle. Further study is required to develop rules or guidelines for adjusting the elastic solution in this type of problem. The extended elastic strain range was used for all analyses of the benchmark notch problem in this study.

In Fig. 5 comparisons are shown of simplified and nonlinear finite-element analytical results for benchmark notch cases invoving dwell times at maximum strain (Fig. 5(b)), at minimum strain (Fig. 5(c)), at intermediate strain in compressive yield (Fig. 5(d)) and at all increments involving tensile yielding (Fig. 5(e)). ANSYMP analyses were performed using all three creep options for each of benchmark cases. The creep analyses using option 3 (combined stress relaxation and creep accumulation) gave the most consistent agreement with the MARC nonlinear finite-element solutions. This would indicate that creep option 3 should be used in most cases other than strain controlled problems.

In terms of cycle mean stresses, the simplified procedure gave results more compatible with MARC elastic-plastic analyses than were possible from an elastic solution. The mean stresses from the simplified and MARC elastic solutions were 68 and 223 megapascals, respectively, compared to 77 megapascals for the MARC elastic-plastic solution. The application of creep dwell times did not significantly alter the cycle mean stresses. The ANSYMP analyses of the benchmark notch problem used less than 1 percent of the central processor unit (CPU) time required by the MARC nonlinear analyses.

Wedge Specimen Problem

The double-edge wedge specimen provided a nonisothermal case for evaluation of the simplified procedure and the operation of the ANSYMP program. Because of the incremental temperature changes, the elastic solution was no longer linear as for the isothermal uniaxial and benchmark notch cases.

In Fig. 6(a), the stress-strain hysteresis loops calculated from the ANSYMP simplified procedure and MARC elastic-plastic analyses are compared for two thermal cycles without dwell times. Reasonably good agreement is shown between the ANSYMP and MARC stress-strain hysteresis loops in Fig. 6(a). The mean stress for the second MARC stress-strain cycle was 55 megapascals. The simplified procedure predicted a mean stress of 20 megapascals compared to -201 megapascals for the elastic solution.

These analyses were repeated with dwell times imposed at the maximum strain level. As shown in Fig. 6(b), the predicted ANSYMP solution for this case was not in good agreement with the MARC nonlinear stress-strain cycles. This was due to the extreme sensitivity of creep computations to small variations in stress. The maximum tensile stresses predicted from ANSYMP for the elastic-plastic case (Fig. 6(a)) were not accurate enough to use for creep calculations. Better agreement between ANSYMP and MARC elastic-plastic-creep solutions is shown in Fig. 6(c) for dwell times applied at the minimum strain level of the cycle. This is due to the better agreement between ANSYMP and MARC stress predictions in compressive yield shown in Fig. 6(a).

SUMMARY OF RESULTS

A simplified analysis procedure was developed for calculating the stress-strain history at the critical location of a thermomechanically cycled structure. A FORTRAN IV computer program, ANSYMP, was created to implement this procedure. The general conclusions and observations that were drawn from the evaluation of the method are as follows:

1. The predicted stress-strain response showed good to excellent agreement with elastic-plastic finite-element analysis solutions using the MARC program.

2. The predicted creep response showed generally good agreement with comparable MARC analytical results. However, the accuracy of the creep calculations was very sensitive to variations in the calculated effective stresses from the MARC solution for the elastic-plastic case without creep. The creep option averaging the effects of stress relaxation at constant stress

and cumulative creep at constant total strain demonstrated the most consistent agreement with MARC creep computations except for strain-controlled problems.

3. Mean cyclic stress predictions were in considerably better agreement with MARC nonlinear analysis results than mean stresses obtained from elastic solutions.

4. Nonlinear stress-strain histories were computed from the ANSYMP program with less than 1 percent of the CPU time required by the MARC program.

5. The accuracy of the simplification process is problem dependent. Strain redistribution adversely affects the solution accuracy. This is most likely to occur with mechanical load cycling at regions of high strain gradients.

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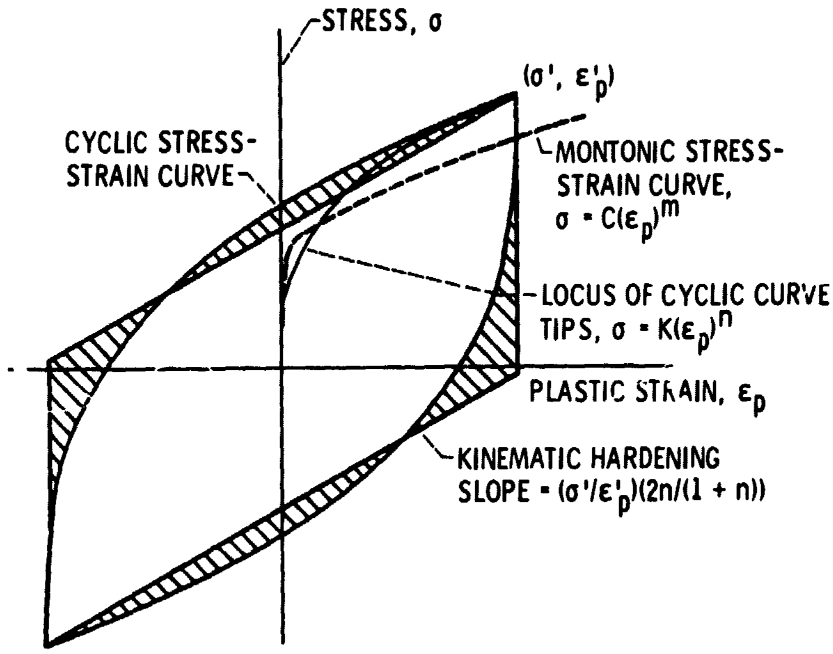

Figure 1. - Representation of stress-strain curves.

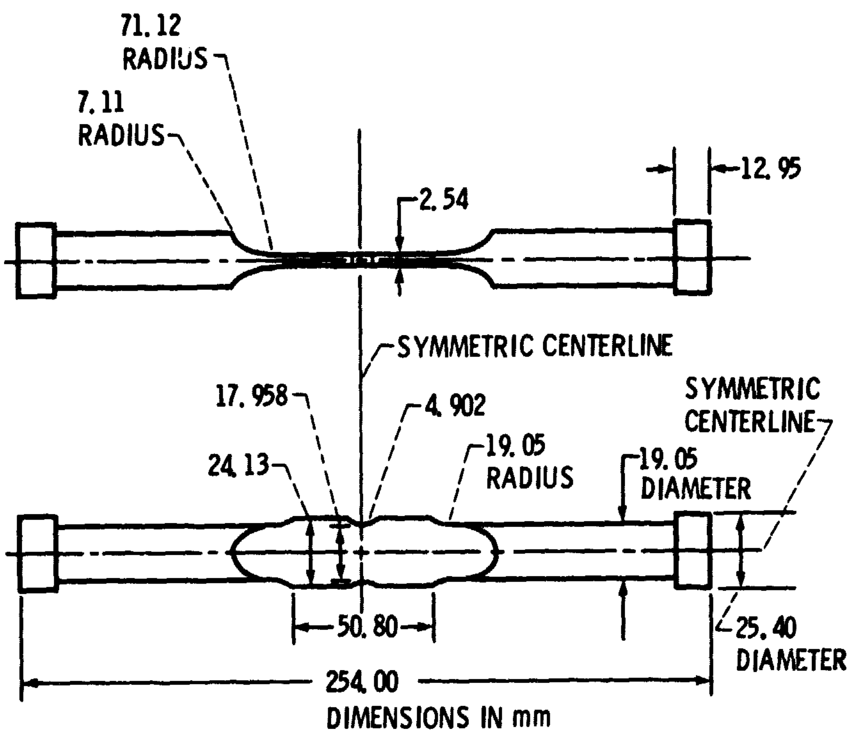

Figure 2. - Benchmark notch specimen (K_t = 1.9).

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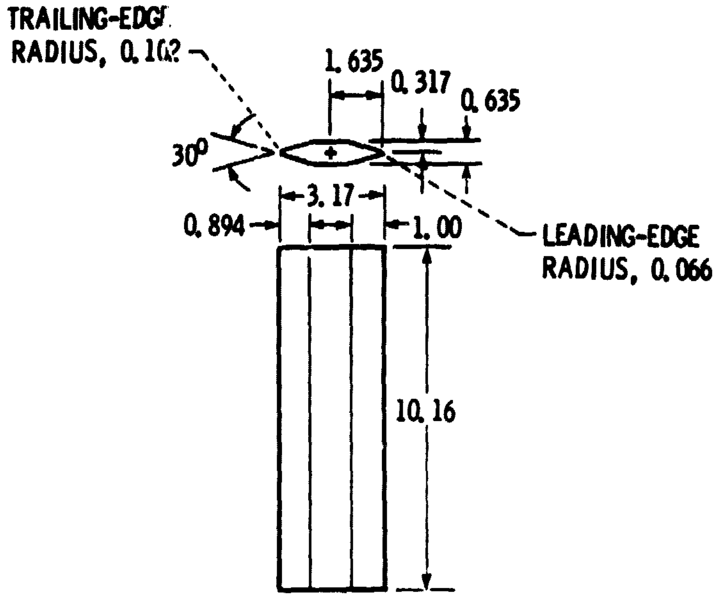
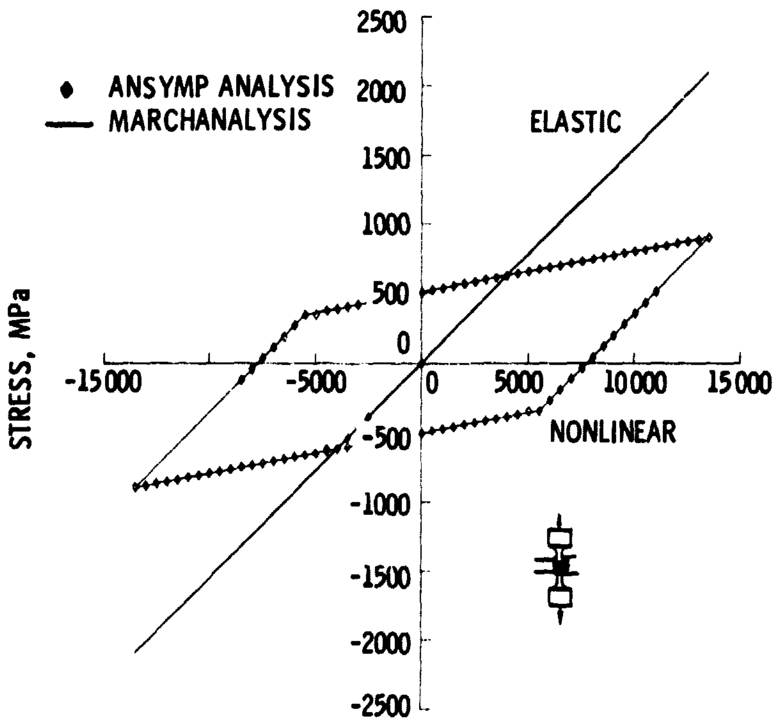


Figure 3. - Double-edge wedge. (All linear dimensions in centimeters.)



(a) No creep dwell time.

Figure 4. - Comparison of ANSYMP and MARC analysis results for uniaxial problem.

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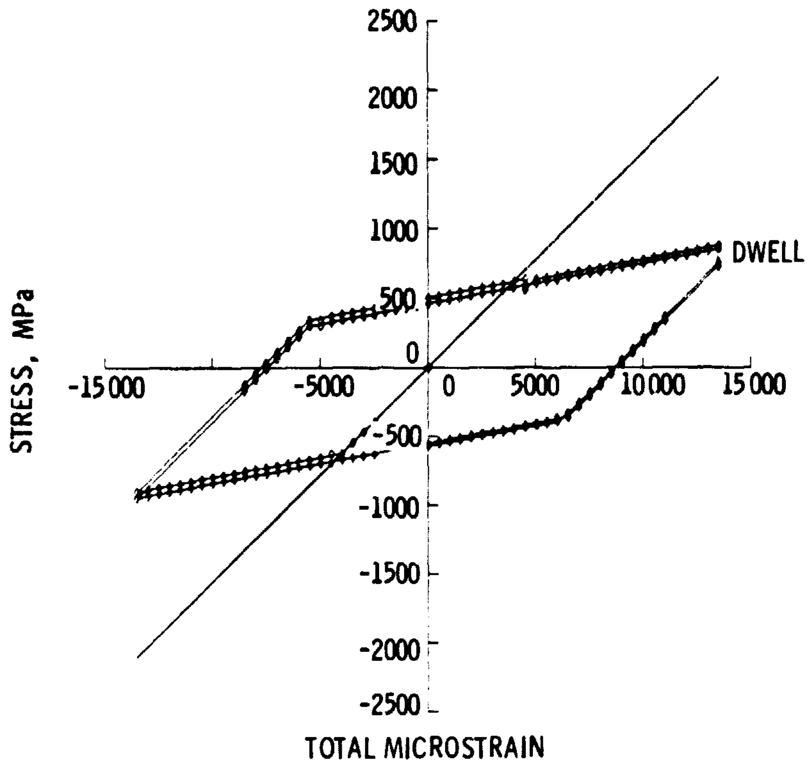

(b) Dwell time at maximum strain.

Figure 4. - Continued.

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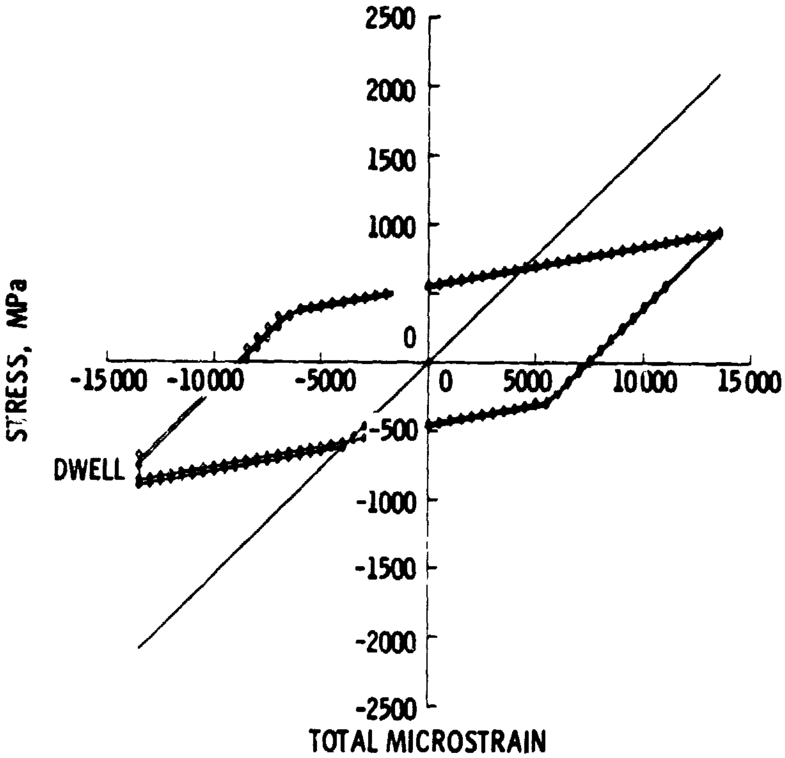
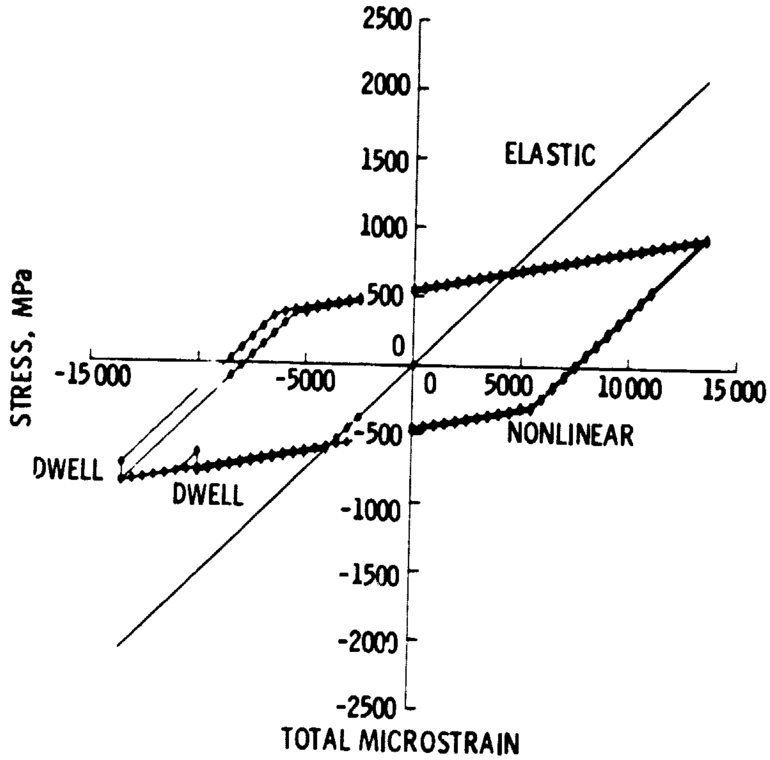


(c) Dwell time at minimum strain.

Figure 4. - Continued.



(d) Dwell times at minimum and intermediate strains.

Figure 4. - Concluded.

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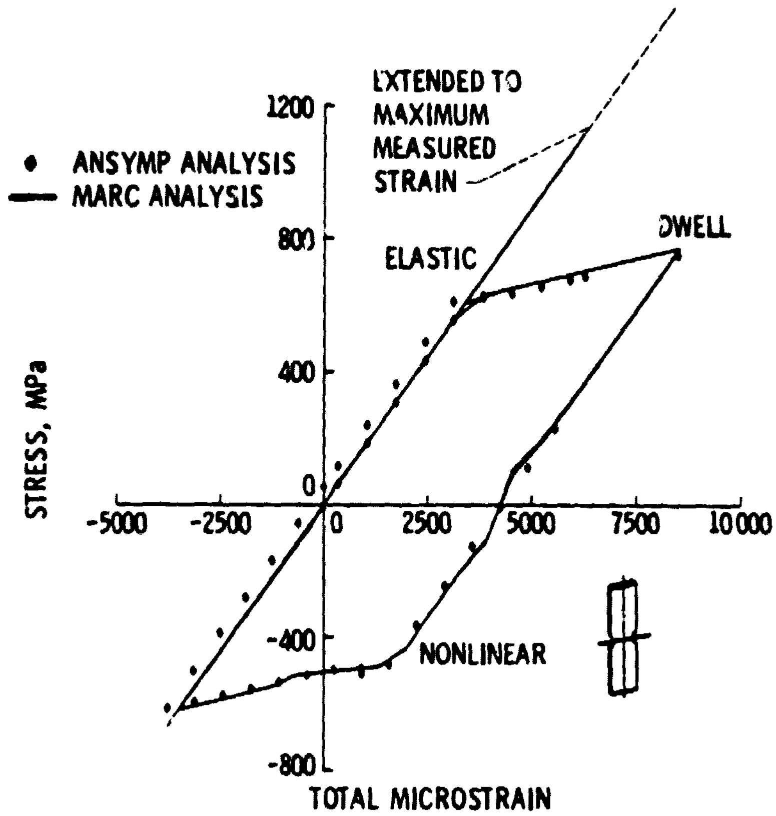

(a) No creep dwell time.

Figure 5. - Comparison of ANSYMP and MARC analysis results for benchmark problem

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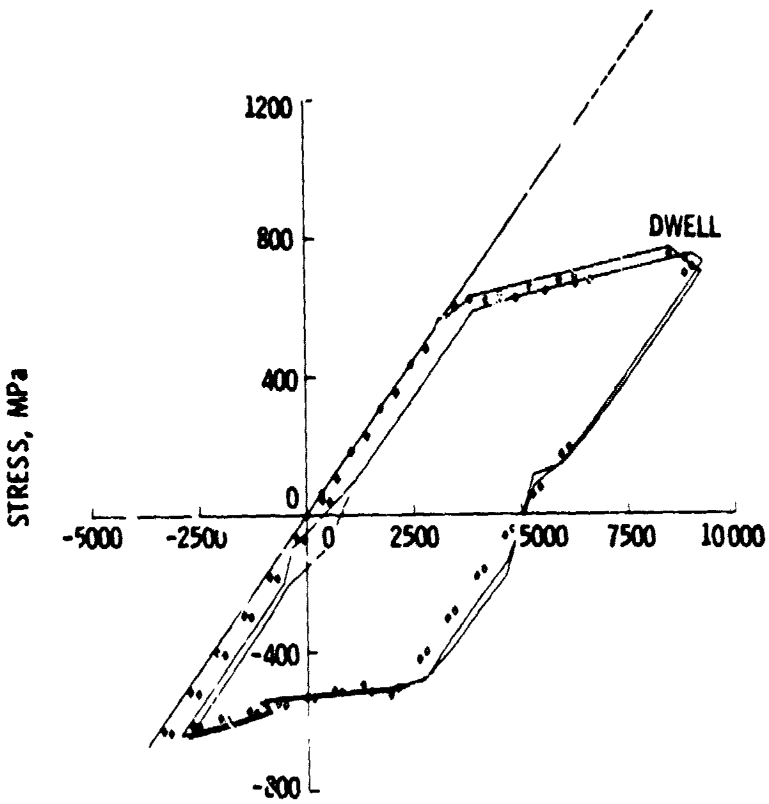

(b) Dwell time at maximum strain - creep option 3.

Figure 5. - Continued.

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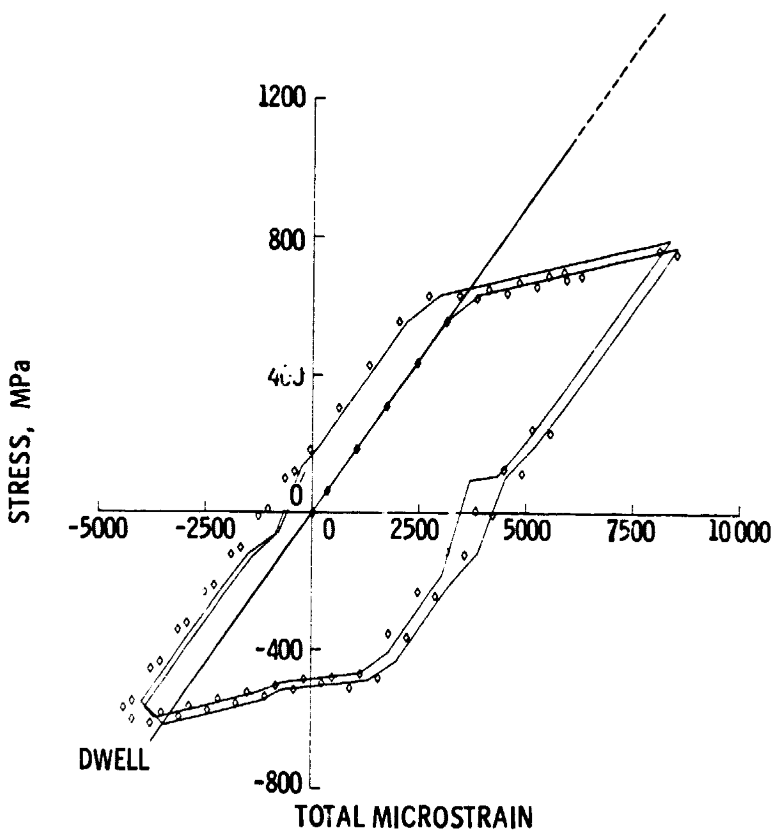

(c) Dwell time at minimum strain - creep option 3.

Figure 5. - Continued.

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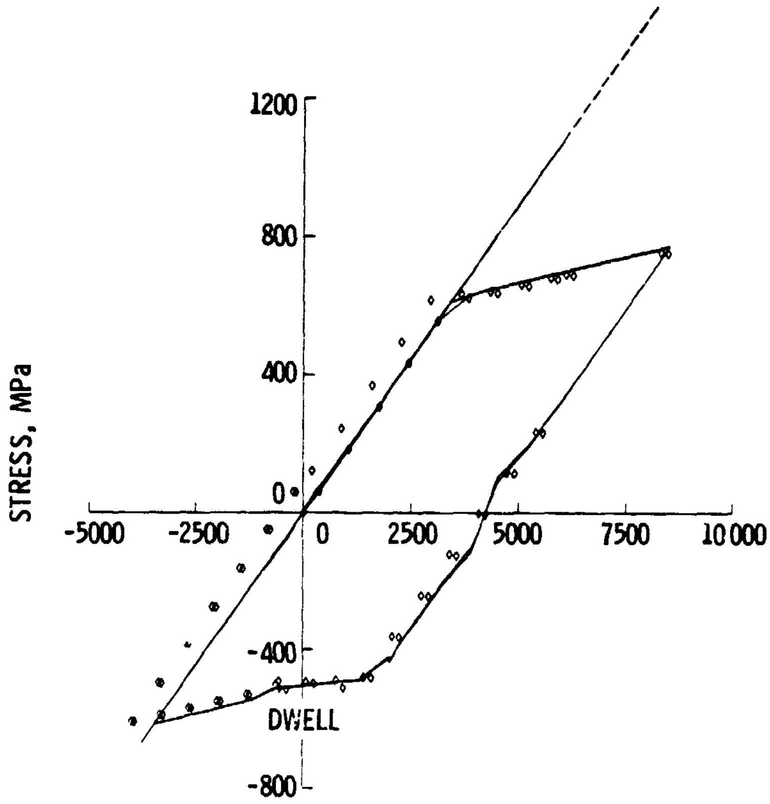

(d) Dwell time at intermediate strain - creep option 3.

Figure 5. - Continued.

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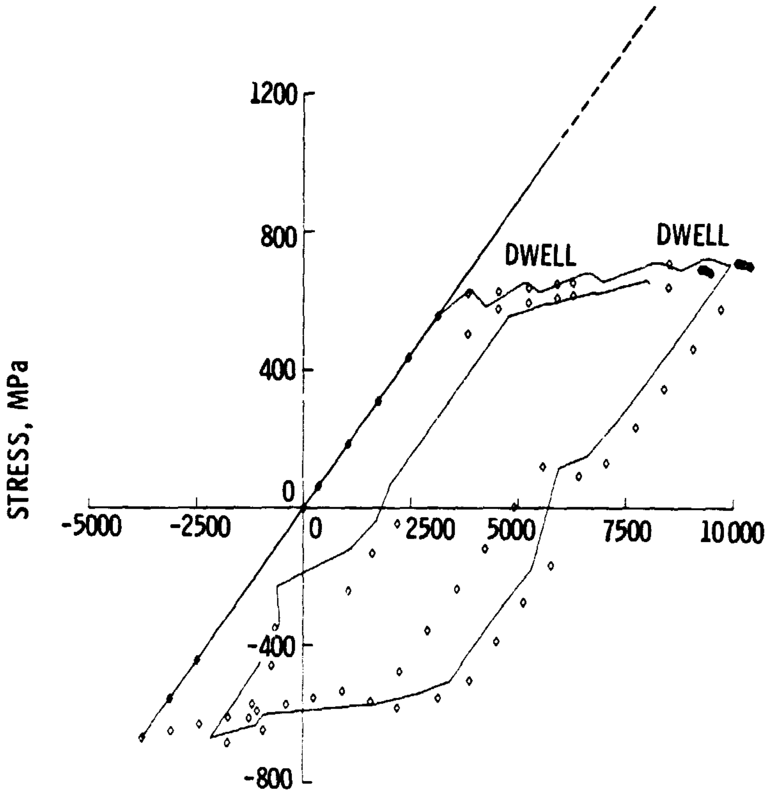

(e) Dwell times at tensile yield strains - creep option 3.

Figure 5. - Concluded.

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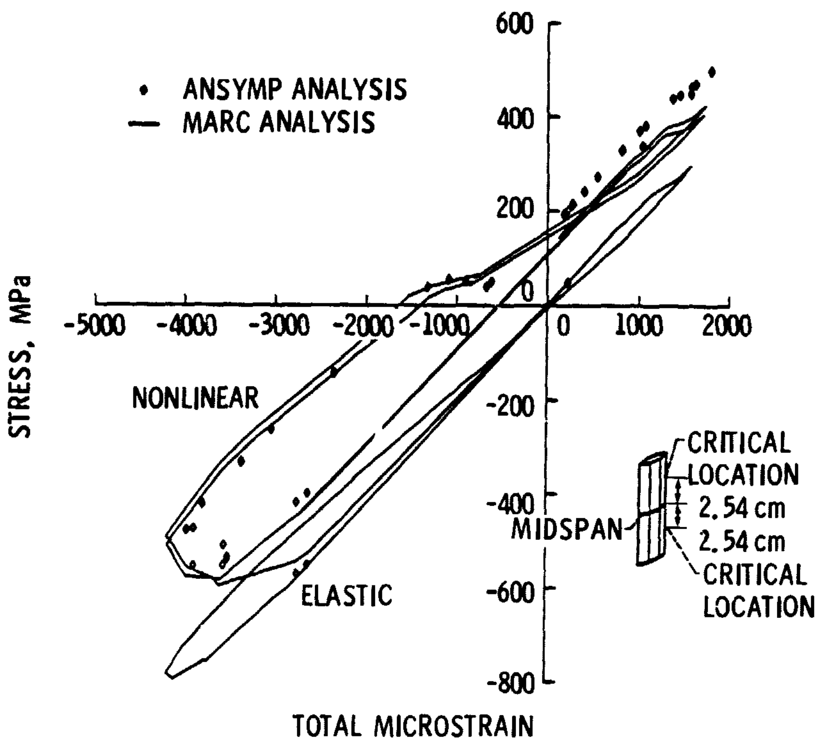
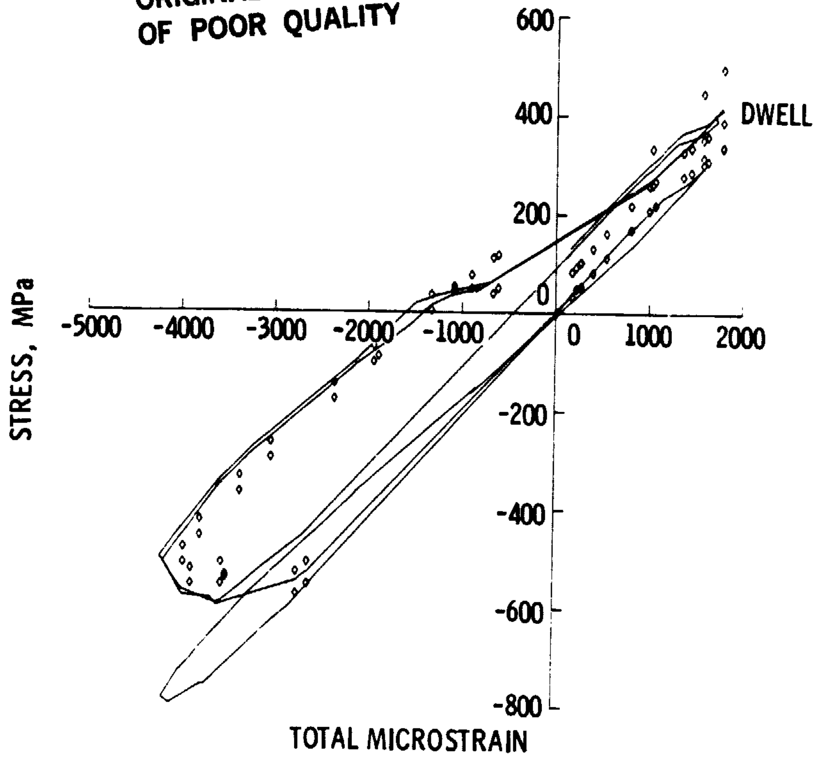


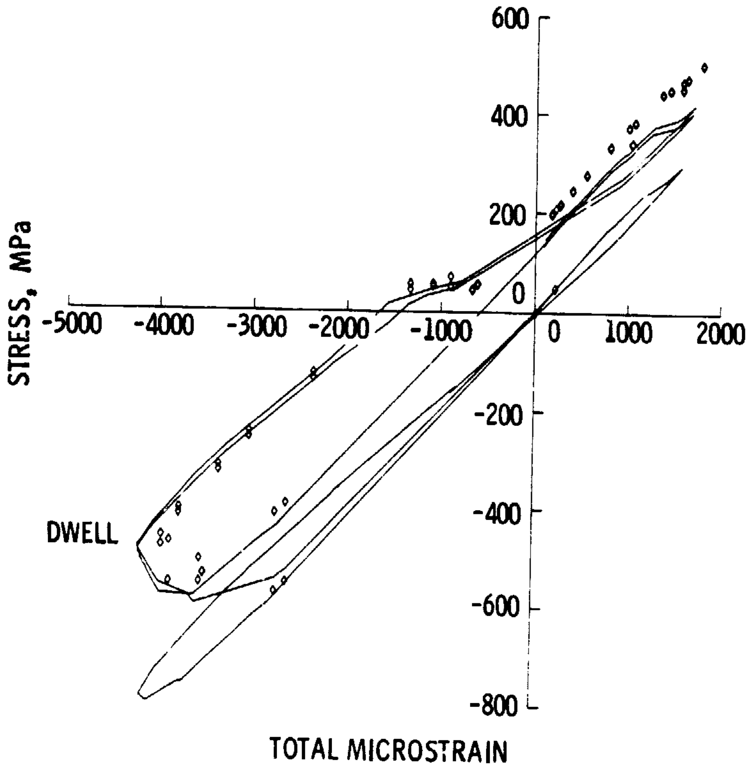
(a) No creep dwell time.

Figure 6. - Comparison of ANSYMP and MARC analysis results for wedge problem.



(b) Dwell time at maximum strain - creep option 3.

Figure 6. - Continued.



(c) Dwell time at minimum strain - creep option 3.

Figure 6. - Concluded.