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NONLINEAR TRANSFORMAT

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ABSTRACT

A technique for designing automatic flight controllers for aircraft which utilizes the transformation theory of nonlinear systems to linear systems is presently being developed at NASA Ames Research Center. We mention a method for taking controllable linear systems to Brunovsky canonical form, and introduce a linear approximation to the nonlinear system called the modified tangent model. We show how this model is easily computed. Constructing the transformation for this model enables us to find an approximate transformation for the nonlinear system.

I. INTRODUCTION

We are interested in designing an automatic flight control system for aircraft that have complex characteristics and operational requirements such as powered-lift STOL and V/STOL configurations. The technique developed is effective for a large class of dynamic systems that require multi-input control and that have highly coupled nonlinearities and complex multidimensional flight envelopes. This work is driven by George Meyer's research at NASA Ames Research Center. His current application is to the UH-1H helicopter.

The main idea in our approach is to simplify the representation of the plant dynamics by means of a change of coordinates of the state and control. First, the given nonlinear system is transformed to a controllable linear system in Brunovsky [1] canonical form. Second, standard linear and nonlinear design techniques, such as Bode plots, pole placement, LQC, and phase plane, are used to design a control law for this simple representation. Last, the resulting control law is transformed back out into the original coordinates to obtain a control law in terms of available controls.

Meyer's approach, first outlined in [2], has been applied to several aircraft of increasing complexity. The completely automatic flight control system was successfully tested on a DHC-6 [3], and the reference trajectory used exercised a substantial part of the operational envelope of the aircraft. Next, the technique was applied to the Augmentor Wing Jet STOL Research aircraft, the convincing flight test results being provided in [4]. Methods for providing pilot inputs to the scheme were examined in [5], and application on an A-7 aircraft for carrier landing and testing in manned simulation is reported in [6] and [7]. Other recent results are contained in [8], [9], and [10].

For this paper we concentrate on the transformation aspect of our design method. Necessary and sufficient conditions for a nonlinear system to be transformable to a controllable linear system in Brunovsky form are presented in [11], [12], [13], [14], [15], [16].

In his early work Meyer considered systems that were in block triangular form, and transformations for such systems can be constructed as in [2] and [4]. The transformation theory in [12] applies to systems which are much more general than block triangular. As is indicated there, finding a transformation depends on solving a system of partial differential equations, which can be reduced to ordinary differential equations. It is not always possible to solve these equations in closed form, but cases where this can be done are presented in the Ph.D. thesis of the first author [17]. Numerical techniques are also introduced in that thesis. In addition there is a remarkable technique for changing a controllable linear system to Brunovsky form which involves taking no inverses of matrices and introduces a block triangular form as an intermediate step.

For cases where exact transformations cannot be found, George Meyer considered constructing approximate transformations by using his tangent model [18]. Recently, we have introduced the modified tangent model and have indicated how to construct the linear part of a transformation about a point x_0 in state space without knowing the actual transformation. This modified tangent model was first derived in [19] by examining the partial differential equations from [12]. In addition, we showed that the same model could be found by using the theory of canonical expansions of nonlinear systems [20].

In this paper we mention the method of [17] for taking a linear system to Brunovsky form. Next we define the modified tangent model (without going into the partial differential equations or canonical expansions), and indicate how Ford's technique can be adapted to change the modified tangent model to canonical form. We also show how to easily compute the modified tangent model, despite the fact that it is defined through

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nonlinear equations.

II. NONLINEAR TRANSFORMATIONS

For our nonlinear system we take

$$\dot{x}(t)=f(x(t))+\sum_{i=1}^{m} u_i(t)g_i(x(t)), \tag{1}$$

where $f,g_1,\ldots,g_m$ are C^∞ vector fields on R^n, and $g_1,g_2,\ldots,g_m$ are linearly independent (this is assumed for convenience). The transformation results from [12] are local, and global theorems are given in [13]. For the sake of notation we assume that this system is transformable on all of R^n to the Brunovsky form

$$\dot{y}(t)=A_0y+B_0v \tag{2}$$

with Kronecker indices $\kappa_1\geq\kappa_2\geq\cdots\geq\kappa_m$ and A_0 is nxn, B_0 is nxm. By

$$\dot{x}(t)=Ax+Bu \tag{3}$$

we denote a controllable linear system with the same matrix dimensions as (2).

Let $[f,g]$ be the usual Lie bracket for vector fields f and g; i.e.

$$[f,g]=\frac{\partial g}{\partial x}f-\frac{\partial f}{\partial x}g,$$

where $\frac{\partial g}{\partial x}$ and $\frac{\partial f}{\partial x}$ are Jacobian matrices. We set

$$(ad^0f,g)=g$$
$$(ad^1f,g)=[f,g]$$
$$(ad^2f,g)=[f,[f,g]]$$
$$\vdots$$
$$(ad^kf,g)=[f,(ad^{k-1}f,g)].$$

Let

$$C=\{g_1,[f,g_1],\ldots,(ad^{\kappa_1-1}f,g_1),g_2,[f,g_2],\ldots,(ad^{\kappa_2-1}f,g_2),\ldots,g_m,[f,g_m],\ldots,(ad^{\kappa_m-1}f,g_m)\}$$

$$C_j=\{g_1,[f,g_1],\ldots,(ad^{\kappa_j-2}f,g_1),g_2,[f,g_2],\ldots,(ad^{\kappa_j-2}f,g_2),\ldots,g_m,[f,g_m],\ldots,(ad^{\kappa_j-2}f,g_m)\}$$

for $j=1,2,\ldots,m$.

We want a nonsingular one-one transformation taking system (1) to system (2) so that the new states (in y space) are functions of the old states (in x space), and the new controls v are functions of both x and u. The following local theorem is proved in [12].

<u>Theorem</u>. The nonlinear system (1) is transformable to the linear system (2) if and only if (with possible reordering of the vector fields $g_1,g_2,\ldots,g_m$)

i) the n vector fields in C are linearly independent,
ii) the sets C_j are involutive for $j=1,2,\ldots,m$, and
iii) the span of C_j equals the span of $C_j\cap C$ for $j=1,2,\ldots,m$.

We assume that our system (1) satisfies these three conditions. Before we consider the modified tangent model, we mention the technique from [17] for moving from a controllable linear system (3) to Brunovsky canonical form (2).

We begin with (3) and form the (n+m)x(n+m) matrix

$$\left[\begin{array}{c|c} A & B \\ \hline 0 & 0 \end{array}\right]. \tag{4}$$

An orthogonal coordinate change on R^n is computed (we actually have a program) to take this matrix to generalized lower Hessenberg form. In this form all elements above the first m superdiagonals are zero, and for our special case, the two zeros in (4) are retained. We remark that the Kronecker indices, if unknown, can be found in this manner (see [21] for a similar method).

Now the system (3) after this orthogonal coordinate change is in block triangular form [4]. It is easy to construct a transformation to Brunovsky canonical form. For example, in a single input system, matrix (4) is

$$\begin{bmatrix} a_{11} & a_{12} & 0 & \cdots & 0 & 0 \\ a_{21} & a_{22} & a_{23} & & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & 0 & 0 \\ & & & & a_{n-1n} & 0 \\ a_{n1} & a_{n2} & a_{n3} & \cdots & a_{nn} & b_{1n} \\ 0 & 0 & 0 & & 0 & 0 \end{bmatrix}. \tag{5}$$

Then the map

$$T_1=x_1$$
$$T_2=\dot{T}_1=\dot{x}_1=a_{11}x_1+a_{12}x_2$$
$$T_3=\dot{T}_2=a_{11}\dot{x}_1+a_{12}\dot{x}_2=a_{11}(a_{11}x_1+a_{12}x_2)$$
$$\qquad +a_{12}(a_{21}x_1+a_{22}x_2+a_{23}x_3)$$
$$T_4=\dot{T}_3=\sum_{i=1}^{3}\frac{\partial T_3}{\partial x_i}\dot{x}_i$$
$$\vdots$$
$$T_n=\dot{T}_{n-1}=\sum_{i=1}^{n-1}\frac{\partial T_{n-1}}{\partial x_i}\dot{x}_i$$
$$T_{n+1}=\dot{T}_n=\sum_{i=1}^{n}\frac{\partial T_n}{\partial x_i}\dot{x}_i$$

carries system (3) to system (2) with $(T_1,T_2,\ldots,T_n,T_{n+1})=(y_1,y_2,\ldots,y_n v)$.

We return to our nonlinear system (1). Given a point x_0 in x-state space, we wish to approximate this system by a more general linear system than (3) and construct an approximate transformation for the nonlinear system by taking a system like (3) to (2). The usual approach is to linearize (1) about x_0 using the Taylor formula. This is fine if x_0 is an equilibrium of $f(x)$ or in certain cases (see [19] for an example). In general, the Taylor approach did not prove advantageous if we wish to find the linear part of an actual transformation at x_0. For this reason we introduced the modified tangent model in [19].

Definition. The modified tangent model for the system (1) at x_0 is the linear system

$$\dot{x}(t)=f(x_0)-Ax_0+Ax+Bu \tag{5}$$

where A is nxn and B is nxm and satisfy

$$A^k b_1=\pm(ad^k f,g_1)(x_0),\ k=0,1,\ldots,\kappa_1$$
$$A^k b_2=\pm(ad^k f,g_2)(x_0),\ k=0,1,\ldots,\kappa_2$$
$$\vdots \tag{6}$$
$$A^k b_m=\pm(ad^k f,g_m)(x_0),\ k=0,1,\ldots,\kappa_m.$$

Here we take + for k even and - for k odd.

A full explanation of the technique of constructing an approximate transformation by applying the modified tangent model is presented in [19]. However, here we only wish to indicate the method of solving equations (6) using the lower Hessenberg form mentioned earlier.

First we check to see if in our Kronecker indices $\kappa_1 \geq \kappa_2$ or $\kappa_1=\kappa_2$, $\kappa_2 \geq \kappa_3$ or $\kappa_2=\kappa_3$, etc. Then we form the following set (assuming $\kappa_1>\kappa_2>\kappa_3\ldots$, with obvious modifications needed if any equalities exist).

$$D=\{(ad^{\kappa_1}f,g_1)(x_0),(ad^{\kappa_1-1}f,g_1)(x_0),$$
$$\ldots,(ad^{\kappa_2}f,g_1)(x_0),(ad^{\kappa_2}f,g_2)(x_0),$$
$$(ad^{\kappa_2-1}f,g_1)(x_0),(ad^{\kappa_2-1}f,g_2)(x_0),$$
$$\ldots,(ad^{\kappa_3}f,g_1)(x_0),(ad^{\kappa_3}f,g_2)(x_0),$$
$$(ad^{\kappa_3}f,g_3)(x_0),(ad^{\kappa_3-1}f,g_1)(x_0),$$
$$(ad^{\kappa_3-1}f,g_2)(x_0),(ad^{\kappa_3-1}f,g_3)(x_0),$$
$$\ldots,g_1(x_0),g_2(x_0),\ldots,g_m(x_0)\}.$$

Now we define an (n+m)x(n+m) matrix whose first column is $(ad^{\kappa_1}f,g_1)(x_0)$ followed by m zeros, second column is the second element of D followed by m zeros, ..., n^{th} column is the n^{th} element of D followed by m zeros, $(n+1)^{th}$ column is $g_1(x_0)$ and m zeros, ..., last column is $g_m(x_0)$ and m zeros. As before, there is an orthogonal coordinate change on R^n which takes our (n+m)x(n+m) matrix to generalized lower Hessenberg form. Thus we retain the zeros in the last m rows and all elements above the m^{th} superdiagonal are zero. No generality is lost in assuming that A and B in (6) start in this form, since we can easily compute the orthogonal transformation and its inverse (transpose).

The equations in the following process are solvable because the set C in our Theorem consists of linearly independent vector fields. We know

$$b_m=\begin{bmatrix}0\\ \vdots\\ 0\\ \vdots\\ *\end{bmatrix},\ b_{m-1}=\begin{bmatrix}0\\ \vdots\\ 0\\ *\\ \vdots\\ *\end{bmatrix},\ldots,b_1=\begin{bmatrix}0\\ \vdots\\ 0\\ *\\ \vdots\\ *\end{bmatrix},$$

where * indicates a possible nonzero entry, and the first * in b_1 is in the $(n-m+1)^{th}$ row. From (6) we take

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$$Ab_m = -[f, g_m](x_0) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ * \\ \vdots \\ * \end{bmatrix},$$

with the first * in the column being at the $(n-m)^{th}$ level. We have

$$\begin{bmatrix} a_{11} & a_{12} \cdots a_{1n} \\ a_{21} & a_{22} \cdots a_{2n} \\ \vdots & \vdots \quad \vdots \\ a_{n1} & a_{n2} \cdots a_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ * \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ * \\ \vdots \\ * \end{bmatrix}$$

and $a_{1n}, a_{2n}, \ldots, a_{nn}$ are easily computed. In the same fashion, $Ab_{m-1} = -[f, g_{m-1}](x_0)$ yields, $a_{1(n-1)}, a_{2(n-1)}, \ldots, a_{n(n-1)}, \ldots,$ and finally $Ab_1 = -[f, g_1](x_0)$ yields $a_{1(n-m+1)}, a_{2(n-m+1)}, \ldots, a_{n\,(n-m+1)}$.

Next we examine

$$A^2 b_m = (ad^2 f, g_m)(x_0)$$

if the vector field on the right hand side is in D. We write this as

$$A^2 b_m = A(Ab_m) = -A((ad^1 f, g_m)(x_0) = (ad^2 f, g_m)(x_0),$$

and compute $a_{1(n-m)}, a_{2(n-m)}, \ldots, a_{n(n-m)}$. Continuing in this way we solve for every entry in A, and the method just described can be implemented on a computer.

Hence the modified tangent model is easily found despite the formidable appearing nonlinear equations defining it. As mentioned earlier, we can construct an approximate transformation from this model.

III. CONCLUSION

We have defined an approximating system to a nonlinear system called the modified tangent model. A technique to solve for this model has been presented. Once such a model is found, an approximating transformation to take the nonlinear system to Brunovsky form can be constructed.

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