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A SIMPLE APPROACH TO DETERMINATION OF STIFFNESS CHARACTERISTICS OF UNIDIRECTIONAL COMPOSITES

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16. Abstract

The test data on a carbon fiber reinforced epoxy composite material was processed using a simplified method. The results show that this simplified approach is feasible, practical, and economical. Basic details of the method are given.

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A SIMPLE APPROACH TO DETERMINATION OF STIFFNESS CHARACTERISTICS OF UNIDIRECTIONAL COMPOSITES* **/420

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1. INTRODUCTION

In the design of composite material laminated sheets, the stiffness characteristics for a single layer sheet are indispensible design materials. Usually a unidirectional laminated plate sample is used to test for E_1, E_2, ν_{12} (or ν_{21}). One test is to stretch uniformly along the direction of the grain. Another test is to stretch uniformly along a direction perpendicular to the direction of the grain [1] (Figure 1a,b). There are many methods to test G_{12}. For example, it is measured by the off-axial stretching of a unidirectional laminated plate along an angle $\theta°$ with respect to the grain [1]-[3] (Figure 1c). Some other methods frequently used to measure G_{12} include trajectory shearing, unidirectional stretching of ±45° laminated plate, and twisting experiment of a thin circular tube [1], [3], [4]. However, the aforementioned methods are either relatively more complicated to realize technically or too costly in fabrication of the specimens, or require too many specimens in type and quantity. Large numbers of characteristic tests require a simple, economical and effective method. The simplified approach provided in this paper only requires two types of specimens and tests to determine the entire stiffness characteristics E_1, E_2, ν_{12} and G_{12}. One of them is the homogeneous stretching of a unidirectional laminated plate specimen along its grain direction (Figure 2a). The other is the unidirectional stretching /421
of the laminated plate specimen which is laid down symmetrically at

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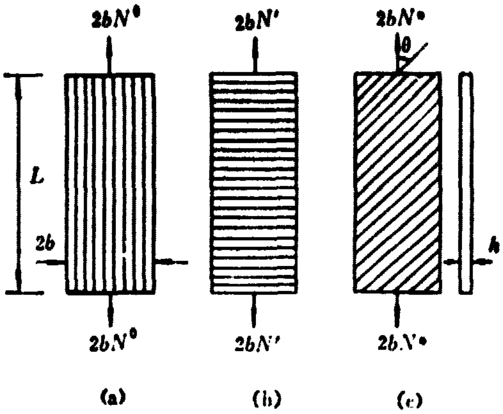

Figure 1

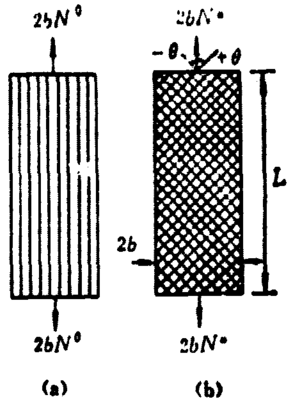

Figure 2

an equal angle of inclination (Figure 2b). The longitudinal and transverse strains at the center of the specimen are obtained through a certain measuring technique (such as strain slices adhered to both sides of the specimen in the center part). The first type of specimen and experiment determine E_1 and ν_{12} directly. With regard to the second specimen test, based on the longitudinal and transverse strain values measured, as well as the E_1 and ν_{12} values already measured, it is possible to determine E_2 and G_{12} using the classical theory of laminated plates.

The test data of a carbon fiber reinforced epoxy composite material was processed using the simplified approach presented. The result shows that this simplified approach is feasible. Moreover, it is simple, practical and economical.

2. BASIC THEORY

The composite material to be studied is limited to linearly elastic materials. Let us consider a laminated plate laid down symmetrically at an equal angle of inclination $(\pm\theta)_{ns}$. The thickness of each layer of plate is identical. According to classical theory of laminated plates [1], the relation between the planar internal force N_x, N_y, N_{xy} of the laminated plate and the strain on the plane ε_x^0, ε_y^0, γ_{xy}^0 is

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \tag{1}$$

or

$$\frac{1}{h}\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 \\ 0 & 0 & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} \tag{2}$$

when h is the thickness of the laminated plate, and

$$\begin{aligned} \bar{Q}_{11} &= Q_{11}\cos^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta + \cos^4\theta) \\ \bar{Q}_{22} &= Q_{11}\sin^4\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta \end{aligned} \tag{3}$$

while

$$Q_{11} = \frac{E_1^2}{E_1 - E_2\nu_{12}^2} \qquad Q_{12} = \frac{E_1E_2\nu_{12}}{E_1 - E_2\nu_{12}^2} \qquad Q_{22} = \frac{E_1E_2}{E_1 - E_2\nu_{12}^2} \qquad Q_{66} = G_{12} \tag{4}$$

Now, let us consider two kinds of plate strip specimens. One is a unidirectional composite material laminated plate strip specimen and the other is a symmetrically laid equal angle of inclination laminated plate strip specimen. The shapes of the specimens and the directions of the stretching loads 26N are shown in Figure 2. Through a certain measuring technique, the longitudinal (along the stretching direction) and transverse (perpendicular to the stretching direction) strains on both sides in the center of the specimens under the stretching load ε_{10}^0, ε_{20}^0 (for the first specimens) and ε_{x0}^0, ε_{y0}^0 (for the second specimen) are measured.

Through the experimental data of the first specimen ε_{10}^0, ε_{20}^0 and N^0, we can immediately determine

$$E_1 = N^0/h\varepsilon_{10}^0 \qquad \nu_{12} = -\varepsilon_{20}^0/\varepsilon_{10}^0 \tag{5}$$

For the second specimen test, if we let $E_x = N^0/h\varepsilon_{x0}^0$, $\nu_{xy} = -\varepsilon_{y0}^0/\varepsilon_{x0}^0$, then because $\gamma_{xy0}^0 = 0$, it is possible to obtain the following two equations by using equation (2)

$$\begin{aligned} \bar{Q}_{11} - \nu_{xy}\bar{Q}_{12} &= E_x \\ \bar{Q}_{12} - \nu_{xy}\bar{Q}_{22} &= 0 \end{aligned} \tag{6}$$

Substituting the Q_{ij} in (3) into (6), after rearranging the order, we have

$$\begin{aligned} &Q_{11}[\cos^4\theta - \nu_{xy}\cos^2\theta\sin^2\theta] + Q_{12}[2\cos^2\theta\sin^2\theta - \nu_{xy}(\cos^4\theta + \sin^4\theta)] + \\ &+ Q_{22}[\sin^4\theta - \nu_{xy}\sin^2\theta\cos^2\theta] + Q_{66}(1 + \nu_{xy})4\sin^2\theta\cos^2\theta = E_x \end{aligned} \tag{7}$$

$$Q_{11}[\sin^2\theta\cos^2\theta - \nu_{xy}\sin^4\theta] + Q_{12}[(\cos^4\theta + \sin^4\theta) - 2\nu_{xy}\sin^2\theta\cos^2\theta] + $$
$$+ Q_{22}[\sin^2\theta\cos^2\theta - \nu_{xy}\sin^4\theta] - Q_{66}(1+\nu_{xy})4\sin^2\theta\cos^2\theta = 0 \quad (8)$$ /422

(7) + (8), after rearranging, we have

$$k_{11}Q_{11} + + k_{12}Q_{12} + k_{22}Q_{22} = E_x$$

Here

$$k_{11} = \cos^4\theta - \nu_{xy}\sin^4\theta + (1-\nu_{xy})\sin^2\theta\cos^2\theta$$
$$k_{12} = (1-\nu_{xy})(\cos^4\theta + \sin^4\theta) + 2(1-\nu_{xy})\sin^2\theta\cos^2\theta$$
$$k_{22} = \sin^4\theta - \nu_{xy}\cos^4\theta + (1-\nu_{xy})\sin^2\theta\cos^2\theta$$

Noticing equation (4), finally, we can get

$$E_2 = \frac{E_1(E_x - k_{11}E_1)}{E_1(k_{12}\nu_{12} + k_{22}) + E_x\nu_{12}^2} \quad (9)$$

Q_{66} is G_{12}, which can be calculated from equations (7) and (8).

Up to this moment, we can use the data obtained in the two aforementioned two specimen tests and equations (5) and (9), as well as equations (7) and (8), to determine the entire stiffness characteristic parameters E_1, E_2, ν_{12} and G_{12} of a unidirectional composite material.

3. EXPERIMENTAL RESULTS AND COMPARISON

The application and effectiveness of this simplified measuring method are explained through the processing of the experimental data of a carbon fiber reinforced epoxy composite material.

According to the usual measuring method, the specimens are as shown in Figure 1. Here we used a ±45° laminated plate specimen to replace the off-axial stretching or other shear specimen as shown in Figure 1c. The four stiffness characteristic parameters are calculated from the following equations

$$E_1 = \frac{N^*}{h\varepsilon_{10}^0} \qquad \nu_{12} = -\frac{\varepsilon_{20}^0}{\varepsilon_{10}^0}$$

$$E_2 = \frac{N'}{h\varepsilon_{20}'} \qquad G_{12} = \frac{N^*}{2h(\varepsilon_{x0}^0 - \varepsilon_{y0}^0)}$$

where N' and ε_{20}' are the load and strain of the specimen in Figure 1b along the direction of the added load.

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According to the simplified method, all the specimens are as shown in Figure 2. Here, $\theta = \pm 45°$. The four stiffness characteristic parameters are calculated according to the following formula:

$$E_1 = \frac{N^0}{h\varepsilon_{10}^0} \qquad \nu_{12} = \frac{-\varepsilon_{20}^0}{\varepsilon_{10}^0}$$

$$E_2 = \frac{E_1(2E_x - E_1(1-\nu_{xy}))}{(2\nu_{12}+1)(1-\nu_{xy})E_1 + 2E_x\nu_{12}^2} \qquad G_{12} = \frac{E_x}{2(1+\nu_{xy})}$$

Specimen material: Carbon fiber reinforced epoxy. The composition materials are carbon fiber (Liao Yuan) and 648 epoxy (Shanghai Resin Factory). Fiber volume ratio $\nu_f = 0.60$. Thermal press molding technology (155° ±5°C, 1 hour, pressure 5 kg/cm^2).

specimen dimensions: 220 mm x 25 mm x 4 mm

number of layers : 24 layers (n = 12)

experimental data and its processing:

I. Uniform stretching specimen along the fiber direction (cross-sectional area 2.482 x 0.472 cm^2)

| N^0/h (kg/cm^2) | ε_{10}^0 (10^{-6}) | ε_{20}^0 (10^{-6}) | E_1 (10^5 kg/cm^2) | ν_{12} | 423
|---|---|---|---|---|
| 428 | 456 | −135 | 9.39 | 0.298 |
| 855 | 925 | −270 | 9.21 | 0.292 |
| 1280 | 1380 | −405 | 9.28 | 0.291 |
| 1710 | 1870 | −551 | 9.11 | 0.295 |
| average | | | 9.26 | 0.295 |

II. Uniform stretching specimen perpendicular to the fiber direction (cross-sectional area 2.50 x 0.416 cm^2)

N'/h kg/cm^2)	ε_{20} (10^{-6})	E_2 (10^5 kg/cm^2)
24.1	266	0.904
48.1	532	0.901
72.2	798	0.905
96.2	1090	0.882
average		0.899

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III. Unidirectional stretching ±45° laminated plate strip specimen (cross-sectional area 2.496 x 0.438 cm^2)

N°/h kg/cm^2)	$\varepsilon^0_{x0}(10^{-6})$	$\varepsilon^0_{y0}(10^{-6})$	$E_x(10^5$ kg/cm^2)	ν_{xy}
45.9	319	−253	1.1[illegible]	0.793
91.8	638	−472	1.13	0.721
138	957	−695	1.13	0.726
184	1276	−902	1.13	0.707
average			1.13	0.737

Comparison of stiffness characteristic parameters measured by the usual and simplified methods

	E_1 (kg/cm^2)	E_2 (kg/cm^2)	G_{12} (kg/cm^2	ν_{12}
usual method	9.26×10^5	0.899×10^5	0.41[illegible] 10^5	0.[illegible]
simplified method	9.26 × 10[illegible]	0.911×10^5	0.415 10[illegible]	0.[illegible]

4. CONCLUSIONS AND DISCUSSION 424

(1) The simplified method was used to measure the entire stiffness characteristic parameters of a carbon fiber reinforced epoxy unidirectional composite material. Comparing the results measured using the commonly used methods indicates that this simplified test method is feasible.

(2) The specimen material and the testing work load of the simplified method are one-third less than those of the commonly used methods. Furthermore, all tests involve the unidirectional stretching of plate strip specimens. It is relatively easier to realize technologically in the experiment. Therefore, it is a simple, practical and economical test method.

(3) The theoretical basis of this simplified method is the classical theory of laminated plates. This theory considers that $\varepsilon_z=\gamma_{xz}=\gamma_{yz}=0$, It is equivalent to neglecting the stress between

layers. However, for a symmetrically laid laminated plate at an arbitrary angle of inclination, usually its distortion does not obey the above assumption even when the plate thickness is relatively thin and under a unidirectional uniform stretch. That means there will be a normal stress σ_z and shear stress τ_{xz}, τ_{yz} between the layers. Hence, when applying the classical theory for laminated plates, we must notice this point. References [1], [5], [6] and [7] have discussed stresses between layers. Both three-dimensional elastic approximation analysis and experiment proved that stress between layers mainly occurs in the free boundary region. Moreover, the maximum is reached at the free boundary. This type of edge effect occurs within the dimensional range approximately equal to the thickness h of the laminated plate. In regions far away from the free boundary, it still can be considered that stress between layers is zero. That means the classical theory of laminated plates still applies. When measuring the stiffness characteristic parameters using the simplified method, it is required that the width 2b of the $[\pm\theta°]_{ns}$ laminated plate strip must satisfy $2b/h \geq 4$.

(4) In the description of the basic principle, it is required that the stress state at the center of the specimen should be $\sigma_x = \sigma$, $\sigma_y = \tau_{xy} = 0$. In order to ensure that such a stress state is realized, the length to width ratio of the specimen should be selected to be slightly larger.

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