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THE ELEMENTS OF THE EARTH'S MAGNETISM FOR THE EPOCHS 1600, 1650, 1780, 1842 AND 1885, AND THEIR SECULAR CHANGES DERIVED FROM OBSERVATIONS CALCULATED USING THE COEFFICIENTS OF THE GAUSSIAN "GENERAL THEORY OF EARTH'S MAGNETISM"

Dr. H. Fritsche

Translation of "Die Elemente des Erdmagnetismus fuer die Epochen 1600, 1650, 1700, 1780, 1842 und 1885, und Ihre Saecularen Aenderungen, berechnet mit Huelfe der aus allen brauchbaren Beobachtungen abgeleiteten Coefficiented der Gaussischen "Allgemeinem Theorie Des Erdmagnetismus"," St. Peterburg, A. Yakobson Printer, 1899, pp. 1-9.

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16. Abstract

After a historical review of the theory of the Earth's magnetism, the author indicates how the theory may be improved and gives results of his own calculations for the Epoch 1842.

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THE ELEMENTS OF THE EARTH'S MAGNETISM FOR THE EPOCHS 1600, 1650, 1700, 1780, 1842 AND 1885, AND THEIR SECULAR CHANGES DERIVED FROM OBSERVATIONS CALCULATED USING THE COEFFICIENTS OF THE GAUSSIAN 'GENERAL THEORY OF EARTH'S MAGNETISM'

Dr. H. Fritsche*

CHAPTER 1

Introduction

In the atlas for the Earth's magnetism published by Gauss and Weber in 1840, on page 32 of the text one finds the following citation "this means that the Earth's magnetism has been completely determined for the first time by its elements (just like planetary and cometary trajectories), that is,by the 24 numbers given in Chapter 40. Similar determinations will be repeated in the future and because of the changes which the magnetic state of the Earth experiences, their details will be researched. Data based on experience will be collected to wrtite the history of the Earth's magnetism. This is a goal which would be impossible without any theory by just experience alone. Experience alone does include the changes in individual effects but never effects which act on the whole. The correctness of these observations will become apparent during the course of the following paper, not only for the future but also to a great degree for the past, that is, for the centuries between 1600 and 1900. The Earth magnetic observations which were performed during this interval are often scanty, inaccurate and incomplete (for example, only one element, the declination, has been observed). This means that understanding the overall magnetic state of the Earth (nine elements, the potential, the ideal distribution of the magnetism on the Earth's surface, the forces X, Y, Z, the declination, the inclination I, the horizontal intensity T

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and the total intensity J) for 1600-1900 can only be based on the Gauss theory. Other methods to reach this goal only will be incomplete. For example, the attempt made by Dr. L. A. Bauer* to empirically determine the secular motion of a freely movable magnetic needle at its center of gravity is based on long term observations at a few locations (London, Paris, Rome, etc.) and this covers only a small part of the Earth's surface. As we will see later on, this leads to an incorrect result that everywhere on the Earth's surface the North Pole of the needle rotates in a clockwise direction as seen from its point of suspension.

In addition, I believe that the publication "on the analytical form of the magnetic attraction of the Earth expressed as a function of time" by the Carlheim-Gyllenskoeld Stockholm, 1896, is erroneous because Dr. Carlheim-Gyllenskoeld made almost no comparison of his hypothesis and the observed magnetic elements. For instance, his provisional theory for the Epoch 1600 results in a declination for England which is 16° different from what was observed. Because the author did not use the Gauss theory formulas themselves but differential formulas in order to go back from the 19th Century, because he did not give the numerical details of his calculation (his data are incorrect). The hypothesis of Dr. Carlheim-Gyllenskoeld about the changes in the coefficients in time was examined by me later on in Chapter 3 and was found to be untenable. Also the results of the paper "foundations of the Gauss theory and phenomena of the Earth's magnetism in 1829" written by A. Ermann and H. Peterson at the request of the Royal Admiralty in Berlin in 1874 are not usable because their work is incomplete. They only calculated 24 coefficients of the theory instead of 46 as I have proven are required to satisfactorily describe the observations. The residual average deviation of the calculation from the observations for the forces X, Y, Z are too large in these calculations, that is, ±0.115 GE (see page 24 of the paper mentioned above). My calculations for the same area over which the

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*Contribution to the knowledge of the secular variation of the Earth's magnetism by Louis A. Bauer, doctoral dissertation Berlin, 1895. Dr. Fritsche, Elements of the Earth's Magnetism 1600-1885.

observations extend (longitude from zero to 360°, latitude from about +70 to -70°) and for the Epoch 1842, only amount to ±0.055 GE, that is, barely 1/2 of the above.

The papers of Dr. A. Schmidt (Gotha) about the Earth's magnetism are not yet completely published unfortunately. However, I would here like to discuss a few aspects of publications now available to me with which I do not agree.

The majority of scientists who deal with the Earth's magnetism have tolerated the view for many decades that the Gauss theory of the Earth's magnetism is only an interpolation method and not a theory. This opinion was especially reported by the unsuccessful calculations of Erman, Peterson, Quintus, Julius and Neumeyer. They all determined only 24 coefficients gh and since they did not obtain satisfactory agreement between theory and observations in this way, they expected a future improvement according to a method not yet discovered. I expressed my views in 1897 in a paper called "The Determinations of the Coefficients of the Gauss General Theory of the Earth's Magnetism for 1885 and the Relationship of the Three Earth Magnetic Elements"*. On pages 1 and 2 and I would like to add the following here because Dr. A. Schmidt is a proponent of this improvement method of coefficients gh according to the writings (page 21c) published in the Annals of Hydrography and Maritime Meteorology, January 1898. He only considers this theory as an interpolation method (cf. Dr. A Petermann, Geographic Communications 1898, vol.VII, p.4). The general theory of the Earth's magnetism established by Gauss is, therefore, a theory because Gauss developed the law of the effect of the magnetic elements of the Earth on the points of the Earth's surface (sphere or ellipsoid surface) (according to their position with respect to changes), using the fundamental law of interaction of magnetic masses (proportional to the masses and inversely proportional to the square of the distances). From this there results a very substantial simplification of the problem

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*In the following I will call this paper "1897 Paper".

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when the theory is applied to reality. The theory gives the analytical expressions for the determining equations for the coefficients gh (see paper 1897 page 10). If one were to not use the analytical expressions from the theory and if one were to represent the three force components X, Y, Z by pure interpolation formulas as a function of longitude and latitude, then more than 200 coefficients from observations would have to be calculated, that is, four times as many as when one uses the Gauss theory which only requires 46. This estimate is based on the following:

For example, the sine series (51) (Bessel series) page 73 of my paper 1897 are of the same kind as the formulas (10-21) for X, Y, Z given in Chapter 2 below, because the latter can also be given in the form of equations (51) in paper 1897*. Now, the general interpolation formula (52 and 53) (page 74 of the paper 1897) derived from (51), which represent the horizontal intensity T as a function of longitude λ and the inclination i contain about 50 coefficients and exponents which were calculated from the (observed) formulas (51). But since (52) and (53) already close with three λ, whereas the formulas for X, Y and Z go up to five λ inclusive, general empirical interpolation formulas which represent X, Y, Z as functions of longitude and latitude would contain about $3 \cdot 70 = 210$ constants to be determined from observations. In the paper 1897 I gave a fast and easy method for calculating the required number of coefficient gh (46) and I demonstrated high degree of agreement between the Gaussian theory and experience. This means that one can no longer raise the objection of incompleteness as long as we do not have much more and better observations than now. The predictions for the these new observations at the present time are not optimistic.

I would like to discuss the method mentioned in the atlas for the Earth's magnetism by Gauss and Weber paragraph 43 page 32. It suggests plotting differences between the calculation (using

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*For example, $i = +85^\circ$, $T = 0{,}46 + 0{,}07 \sin(147{,}7^\circ + \lambda) + 0{,}01 \sin(218{,}4^\circ + 2\lambda) + 0{,}02 \sin(45^\circ + 3\lambda)$

approximate coefficient gh and observations on maps or to present them in tables in order to make the foundation for an improved calculation of the gh. I remarked about this in the paper 1897 pages 1 and 2. I believe that this is unmanageable and this suggestion of Gauss and Weber made my predecessors take the wrong course when further developing the Gauss theory. Weber and Gauss intended* to improve the six elements $e_1\ e_2\ e_3\ e_4\ e_5\ e_6$ of a planetary trajectory which is calculated from three complete observations. In the event where more observations are made, the quantities are improved by the amount $\Delta e_1\ \Delta e_2\ \Delta e_3\ \Delta e_4\ \Delta e_5\ \Delta e_6$ using differential equations. One obtains the elements $e_1\ e_2\ e_3\ e_4\ e_5\ e_6$ by a tedious calculation using a large number of formulas without the method of the least squares because no excessive determination equations are present. The improvements $\Delta e_1\ \Delta e_2\ \Delta e_3\ \Delta e_4\ \Delta e_5\ \Delta e_6$ however, are found by using the method of least squares from six determination equations which are relatively easy to treat. This method not only represents a simplification but is a necessity for planetary calculations.

The problem of improving the coefficients gh of the Earth's magnetism of fourth order and the special calculation of the gh of the fifth and sixth, etc., orders is quite different. The coefficients gh themselves are obtained from linear equations of the form $(X)\ (Y)\ (Z)$ which are given in Chapter 2. The corrections $\Delta g\ \Delta h$ if one wishes to carry out such an operation, have to be calculated according to exactly the same formulas (X, Y, Z), by substituting $\Delta g\ \Delta h$

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*In his introduction to his atlas, Gauss states that Weber wrote the explanation for the maps, that is, the text, with the exception of paragraphs 21-25 which were written by Dr. Goldschmidt. Regarding his improvement methods, Gauss expresses himself more carefully in paragraph 33 of his general theory than does Weber. Gauss says there "there is no doubt that an improvement calculation according to these observations would result in a much better agreement and such a calculation would not be difficult. The length of this calculation would still be frightful, even if one would use similar measures for abbreviations as are used by astronomers in order to improve the elements of the planetary and cometary trajectories".

for gh and $\Delta k, \Delta l, \Delta m, \Delta K, \Delta L, \Delta M$ for klmKLM. By collecting differences between the calculations and the observations in maps or tables, which Weber, Neumeyer, etc. recommend, and for which A. Schmidt makes available his so-called normal elements, one has gained nothing. It is much better if the desired unknowns gh are calculated from the beginning if sufficient material is available. The comparison of the elements of the Earth's magnetism with those of geodesy which Schmidt carried out is not appropriate because the elements of the planets and geodesy are much more constant over time and their phenomena follow much more regular laws than those of the Earth's magnetism which in many aspects is similar to meteorology.

CHAPTER 2

A. Formulas for the calculations 5

I will now give the formulas which I have used and the observations which are the foundation of my calculations. The formulas were almost all given in the paper 1897 which is why I will only recapitulate here. The symbols are those used by Gauss. The potential V divided by the Earth's radius R is $\frac{V}{R} = P^0 + P' + P'' + P''' + P''''$. The northern component X, the western component Y and the vertical Z are expanded in the following cosine and sine series[*]

$$X = k_0 + k_1 \cos\lambda + K_1 \sin\lambda + k_2 \cos 2\lambda + K_2 \sin 2\lambda + k_3 \cos 3\lambda + K_3 \sin 3\lambda + \ldots$$
$$Y = l_0 + l_1 \cos\lambda + L_1 \sin\lambda + l_2 \cos 2\lambda + L_2 \sin 2\lambda + l_3 \cos 3\lambda + L_3 \sin 3\lambda + \ldots$$
$$Z = m_0 + m_1 \cos\lambda + M_1 \sin\lambda + m_2 \cos 2\lambda + M_2 \sin 2\lambda + m_3 \cos 3\lambda + M_3 \sin 3\lambda + \ldots$$

where the quantities $k\,l\,m\,K\,L\,M$ are functions of geographic latitude ϕ and are determined bv the following formulas (X), (Y), (Z). $\cos u = e,\ \sin u = f$ is used for abbreviation, and we use the letters $\mathfrak{A}\,\mathfrak{B}\,\mathfrak{C}\,\mathfrak{D}\,\mathfrak{E}\,\mathfrak{F}$ for the following functions which only depend on

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[*] λ is the eastern longitude from Greenwich and u is the angular separation from the astronomic North Pole so that u + ϕ = 90°.

even powers of the quantity ε which are simple symbols, that is:

$\varepsilon^2 - 0{,}2 = \mathfrak{A}$
$\varepsilon^2 - 0{,}4286 = \mathfrak{B}$
$\varepsilon^2 - 0{,}3333 = \mathfrak{C}$
$\varepsilon^2 - 0{,}1429 = \mathfrak{D}$
$\varepsilon^2 - 0{,}1111 = \mathfrak{E}$
$\varepsilon^2 - 0{,}2727 = \mathfrak{F}$
$\varepsilon^2 - 0{,}0909 = \mathfrak{H}$
$\varepsilon^2 - 0{,}5 = \mathfrak{A}_0$
$\varepsilon^2 - 0{,}7333 = \mathfrak{B}_0$
$\varepsilon^2 - 0{,}5714 = \mathfrak{C}_0$
$\varepsilon^2 - 0{,}6 = \zeta$
$\varepsilon^2 - 0{,}25 = \mathfrak{P}_0$
$\varepsilon^2 - 0{,}4667 = \mathfrak{a}_0$

$\varepsilon^2 - 0{,}3939 = \mathfrak{A}_0'$
$\varepsilon^2 - 0{,}1667 = \zeta'$

$\varepsilon^4 -$ [illegible] $\cdot \varepsilon^2 + 0{,}0076 = \mathfrak{A}_1$
$\varepsilon^4 -$ [illegible] $\cdot \varepsilon^2 + 0{,}1515 = \mathfrak{B}_1$
$\varepsilon^4 -$ [illegible] $\cdot \varepsilon^2 - 0{,}0303 = \mathfrak{C}_1$

$\varepsilon^4 -$ [illegible] $\cdot \varepsilon^2 + 0{,}1077 = \mathfrak{C}_0$
$\varepsilon^4 -$ [illegible] $\cdot \varepsilon^2 + 0{,}2362 = \mathfrak{D}_0$
$\varepsilon^6 -$ [illegible] $\cdot \varepsilon^4 +$ [illegible] $\cdot \varepsilon^2 -$ [illegible] $= \mathfrak{E}$
$\varepsilon^4 -$ [illegible] $\varepsilon^2 + 0{,}0667 = \mathfrak{H}$
$\varepsilon^4 -$ [illegible] $\varepsilon^2 + 0{,}1919 = \mathfrak{I}$
$\varepsilon^6 -$ [illegible] $\varepsilon^4 +$ [illegible] $\cdot \varepsilon^2 - 0{,}0246 = \mathfrak{H}_0'$

$\varepsilon^4 -$ [illegible] $\varepsilon^2 + 0{,}0857 = \mathfrak{F}'$
$\varepsilon^4 -$ [illegible] $\varepsilon^2 +$ [illegible] $= \mathfrak{B}_1$

$\varepsilon^4 -$ [illegible] $\varepsilon^2 +$ [illegible] $= \mathfrak{C}_0'$

Values of $m_0^{(4)} - g_4^{0} \cdot \varepsilon = \mathfrak{s}$ Gauss, units of the third decimal..

u =	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°	150°	160°	170°
Epoch																	
1600	-105	-62	-8	+32	+50	+44	+21	-7	-30	-41	-37	-20	+4	+26	+38	+38	+33
1650	-99	-57	-8	+32	+43	+44	+21	-8	-31	-42	-39	-22	+5	+26	+40	+43	+41
1700	-100	-56	-5	+35	+52	+44	+19	-9	-34	-45	-41	-24	+4	+27	+43	+50	+52
1780	-111	-61	-2	+40	+57	+46	+18	-10	-33	-44	-44	-30	-7	+22	+48	+69	+81
1842	-137	-90	-30	+22	+54	+55	+37	+5	-23	-39	-35	-19	+4	+20	+26	+22	+15
1885	-132	-80	-13	+41	+69	+65	+34	-10	-49	-68	-60	-27	+15	+49	+59	+48	+30

Values of $m_0 - g_4^{0} \cdot 2\varepsilon = \mathfrak{I}$, Gauss. units of the third decimal.

u =	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°	150°	160°	170°
Epoch																	
1600	-511	-256	-43	+147	+225	+191	+87	-33	-127	-169	-145	-65	+39	+119	+141	+108	+55
1650	-482	-279	-40	+144	+222	+190	+86	-35	-132	-175	-152	-70	+36	+120	+148	+125	+82
1700	-497	-282	-32	+153	+234	+192	+82	-41	-137	-179	-156	-76	+27	+115	+158	+159	+129
1780	-565	-308	-17	+187	+251	+190	+70	-45	-125	-165	-158	-108	-23	+79	+171	+232	+263
1842	-605	-390	-120	+113	+241	+244	+147	+6	-118	-172	-147	-58	+43	+107	+110	+58	-1
1885	-615	-383	-76	+177	+305	+284	+153	-35	-206	-289	-245	-88	+105	+235	+232	+111	-35

Values of $k_0 - g_4^{0} \cdot f = \mathfrak{I}_1$ Gauss.units of the third decimal.

u =	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°	150°	160°	170°
Epoch																	
1600	-187	-295	-285	-178	-33	+92	+158	+159	+106	+21	-64	-130	-139	-98	-33	+17	+55
1650	-177	-280	-275	-171	-29	+92	+160	+163	+108	+21	-70	-132	-145	-107	-45	+6	+17
1700	-186	-292	-280	-172	-23	+102	+167	+164	+108	+22	-69	-132	-149	-122	-68	-20	+2
1780	-220	-338	-322	-175	-7	+118	+170	+176	+102	+32	-92	-168	-173	-163	-140	-96	-46
1842	-198	-326	-341	-252	-98	+50	+161	+186	+133	+36	-63	-122	-120	-68	-3	+38	+36
1885	-226	-362	-366	-280	-67	+101	+231	+256	+182	+34	-129	-237	-235	-132	+40	+102	+92

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As to be expected according to the equations above for [illegible] the quantities s_1 and s_2 are much larger than s on the average and all three cannot be ignored.

B. Observations forming the foundation of the calculations

The observations which I used for the Epoch 1885 are given in my paper 1897. The observations which I used as a basis for my calculations about the theory for the Epoch 1842 are taken from the known collection of Sabine. In all of the previous and following tables and formulas, we have the following definitions: λ eastern longitude from Greenwich; ϕ the geographic latitude, u the angular separation from the astronomic North Pole so that $\phi + u = 90°$; $P = \frac{V}{R}$ the potential divided by the Earth's radius R $F = \frac{1}{4\pi}\left(\frac{V}{R} - 2Z\right)$, the ideal distribution of the magnetism on the Earth's surface; X the northern, Y the western, Z the vertical component, δ the declination, i the inclination, T the horizontal intensity and finally J the total intensity.

TABLE 1. Epoch 1842. Observed declination δ

u	λ = φ	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330° λ
10°	+80	+32 30	+4 0	-46 40	-30 30	-30 30	-13 0	-45 30	-47 0	-46 0	-45 0	-42 30	+50 0
20	+70	+29 0	+4 0	-44 0	-17 20	-7 0	-9 0	-40 15	-42 0	-44 15	-33 10	-18 0	+17 15
30	+60	+26 0	+5 15	-40 0	-40 55	+1 20	-1 0	-46 25	-43 30	-34 [illegible]	[illegible]	[illegible] 0	+13 30
40	+50	+23 10	+7 10	-4 35	-6 30	+3 40	+0 30	-20 0	-23 30	-18 45	-7 30	[illegible]	[illegible]
50	+40	+20 25	+8 50	-1 50	-2 0	+3 20	+0 50	-11 30	-17 50	-17 40	-7 30	+8 35	+14 0
60	+30	+19 11	+9 6	+0 8	-1 57	+1 3	-2 17	-11 40	-12 56	-11 51	-7 33	+6 30	+19 4
70	+20	+17 20	+10 15	+2 8	-1 54	+0 6	-3 44	-10 58	-8 17	-2 14	-7 43	+1 30	+15 13
80	+10	+20 0	+10 36	+3 4	-1 35	-6 44	-4 30	-9 0	-6 41	-5 15	-7 30	-1 45	[illegible]
90	0	+20 40	+13 30	+4 42	-1 0	-1 3	-5 36	-7 0	-7 0	-5 0	-7 30	-3 40	[illegible] 33
100	-10	+22 50	+10 14	+6 30	+0 51	-0 11	-5 51	-8 40	-5 52	-5 54	-10 7	-5 50	+9 30
110	-20	+24 30	+22 45	+10 8	+9 5	+1 33	-7 37	-10 0	-7 46	-7 37	-12 1	-7 57	+8 5
120	-30	+23 30	+26 45	+17 45	+10 40	+3 28	-0 36	-11 30	-8 41	-9 5	-13 35	-10 30	+6 46
130	-40	+21 42	+31 6	[illegible]	[illegible]	+5 55	-10 31	-19 30	-10 30	-11 11	-16 0	-13 33	+3 30
140	-50	+18 36	+33 30	+30 54	+31 0	+20 42	-12 25	-17 36	-13 6	-13 40	-22 36	-17 36	[illegible]
150	-60	+15 30	+33 30	+42 0	+46 10	+49 30	-15 40	-21 40	-17 40	-20 36	-27 30	-21 6	+33 40
160	-70	+13 0	+31 54	+47 30	+44 15	+47 30	-22 30	-38 0	-33 6	-32 0	-34 30	-43 24	[illegible] 0
170	-80	+12 0	+29 0	+47 30	[illegible]	[illegible]	[illegible]	-43 30	-35 0	-43 30	-43 0	-32 0	+25 30
	λ =	0°	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°

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TABLE 2. Epoch 1842. Observed inclination i_0.

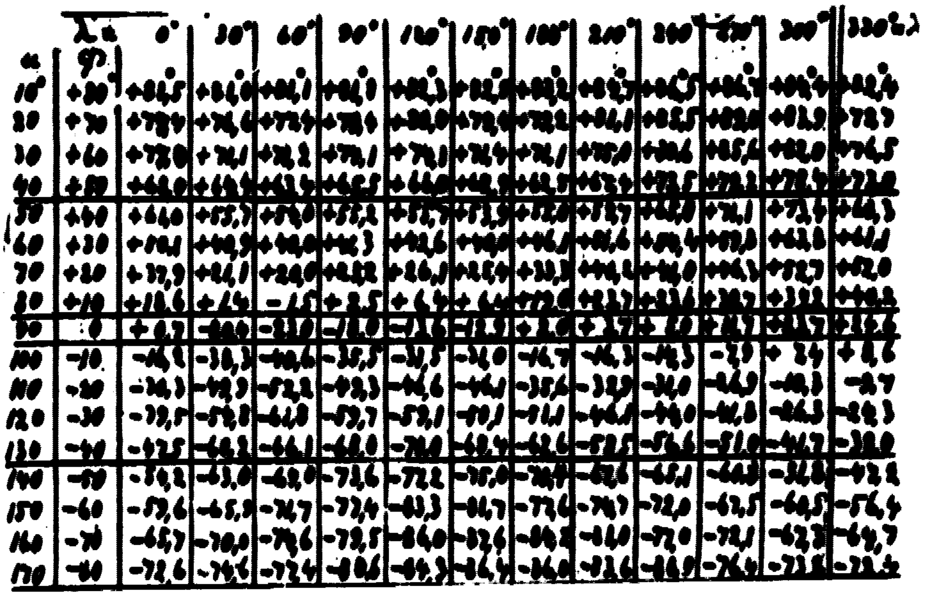
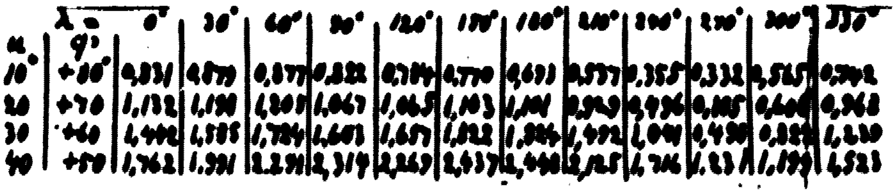


TABLE 3. Epoch 1842. Observed horizontal intensity T_0



Dr. Fritsche. Elements of the Earth's magnetism 1600-1885.
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