Segmented Testing*

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ABSTRACT

The fraction of faults detected for a digital network is frequently high for the first few input combinations applied out of a set of test vectors. When the particular ordering of test patterns does not appreciably change the shape of the coverage curve, there appears to be an advantage to splitting the test into segments which are applied at different times. It is shown that the expected time to error detection and the probability of an undetected double error can be reduced. The amount of reduction is dependent on the shape of the fault coverage curve. It is conjectured that such a reduction can be obtained for VLSI networks.

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INTRODUCTION

The shape of a fault coverage curve; i.e. the cumulative fraction of detected faults as a function of the number of tests applied, is often a saturated curve. Recent work on fault-tolerant computing (1,2) and built in test (3) have reported such situations. This characteristic is often found in testing situations; fault table minimization, D algorithm, path sensitizing, random test, etc. (4).

The basic strategy proposed in this paper is to split up a test set into two or more segments; then apply these shorter tests at differed times so that there is a reduced time between periods of testing. If each test segment detects many or most faults then it is expected that faults should be detected sooner on the average. Consider the 15 gate combinational logic circuit in Figure 1. The function realized is a 3 out of 5 select followed by a majority vote. Three ones on the E inputs select three T lines out of the five using gates 1 to 5; gates 6 to 11 OR the selected lines in various pairs which are ANDed at gates 12 to 14. Gate 15 ORs these products to realize the majority of the three selected T inputs. This function was implemented in a Rockwell-Collins gate array as part of a VLSI project. Suppose the possible faults are single gate input or output stuck-at faults. There are 3 leads each for gates 1 to 14 and 4 leads for gate 15. The 46 total pins result in 92 single faults. Since there are no reconvergent fan-out paths of differing inversion parity, expanding and contracting faults can be considered separately. We start with tests for expanding faults. It is easy to show that the following 4 tests will detect all combinations of gate input or gate output stuck faults which increase the number of ones in the function.

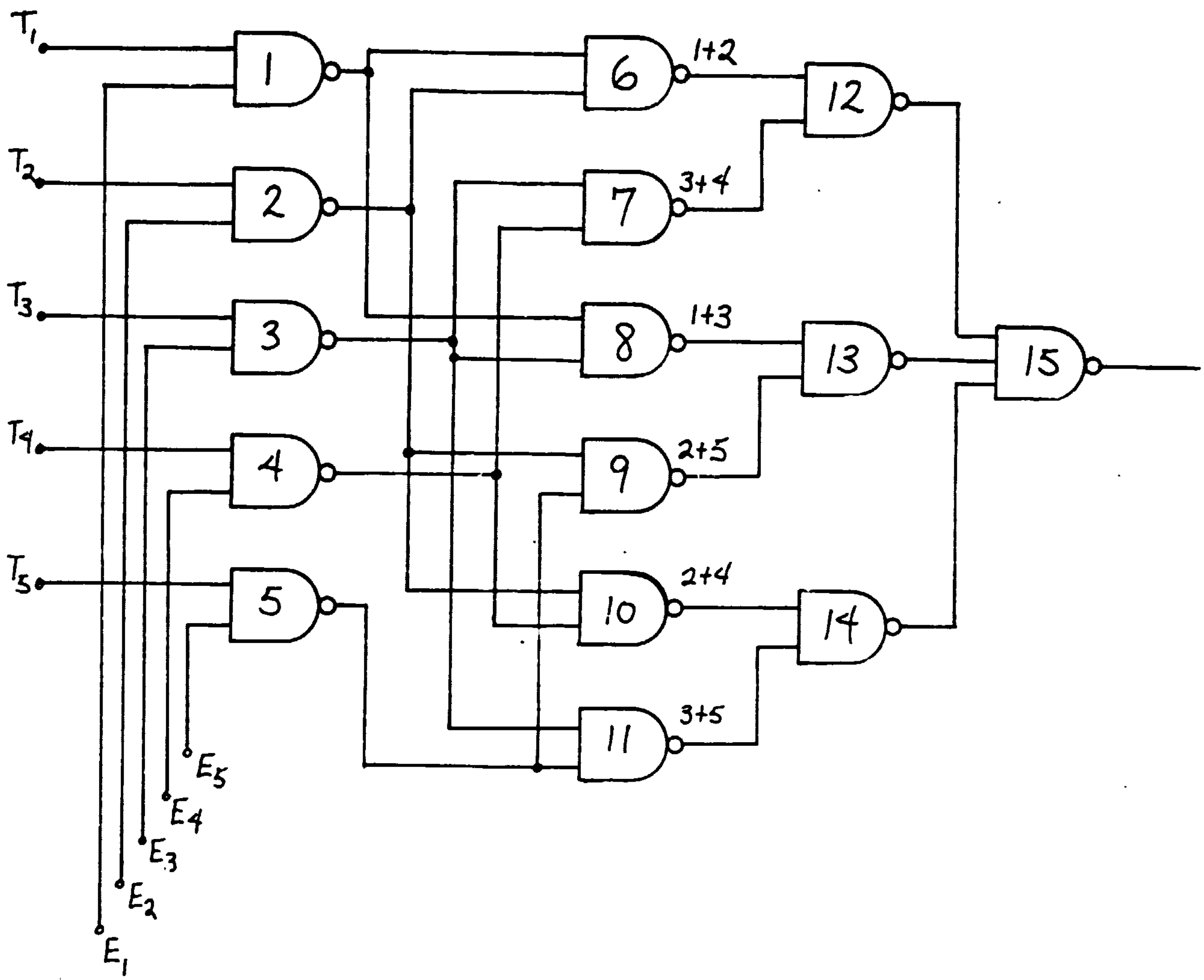

Figure 1 Gate example

T_{11} E = 11100

T = 01011

T_{12} E = 00111

T = 11100 (1)

T_{13} E = 01011

T = 11100

T_{14} E = 11100

T = 00111

Any single test from set (1) will detect 27 of the 46 expanding stuck faults while any two consecutive tests (including T_{11}, T_{14}) will detect 44 of the 46.

For contracting faults the following six tests will detect gate input or output stuck faults.

T_{01} E = T = 10010

T_{02} E = T = 00101

T_{03} E = T = 01001 (2)

T_{04} E = T = 01010

T_{05} E = T = 11000

T_{06} E = T = 00110

The first 3 tests detect 40 out of the 46 contracting faults.

Suppose we interlace sets (1) and (2) as follows:

$$T_{11}, T_{01}, T_{12}, T_{02}, T_{03}, T_{13}, T_{04}, T_{14}, T_{05}, T_{06} \tag{3}$$

It is easy to count the number of single stuck faults detected by (3) as individual patterns are applied. The resulting fault coverage curve is given as the solid line in Figure 2. Rotating (3) and considering other initial test patterns gives the other two curves in Figure 2.

Next we compare two different methods of applying set (3):

1. Complete Test: The entire set (3) is applied every I time units,

2. Segmented Test: The first 5 tests of (3) every odd multiple of I/2 time units and the last 5 tests of (3) every even multiple of I/2.

In the complete case any single fault in the first I time units is detected by the test at time I. We ignore faults which occur during the testing for the moment. This assumption does not significantly change the result. Suppose further that the fault process is stationary. The expected time to fault detection ETFD is thus half the interval or I/2.

For the segmented case consider single faults that occur in the first internal of I/2. Some faults are missed by the five tests at I/2 and are not detected until the end of the next I/2 segment by the remaining five tests. For this example it turns out that the same fraction of faults in the second subinterval are missed by the even set.

The faults detected by the odd set will have an average time to detection of half the subinterval or I/4. The 11 faults missed by the odd set will have their time to detection increased by I/2, the time between the odd and even subtests.

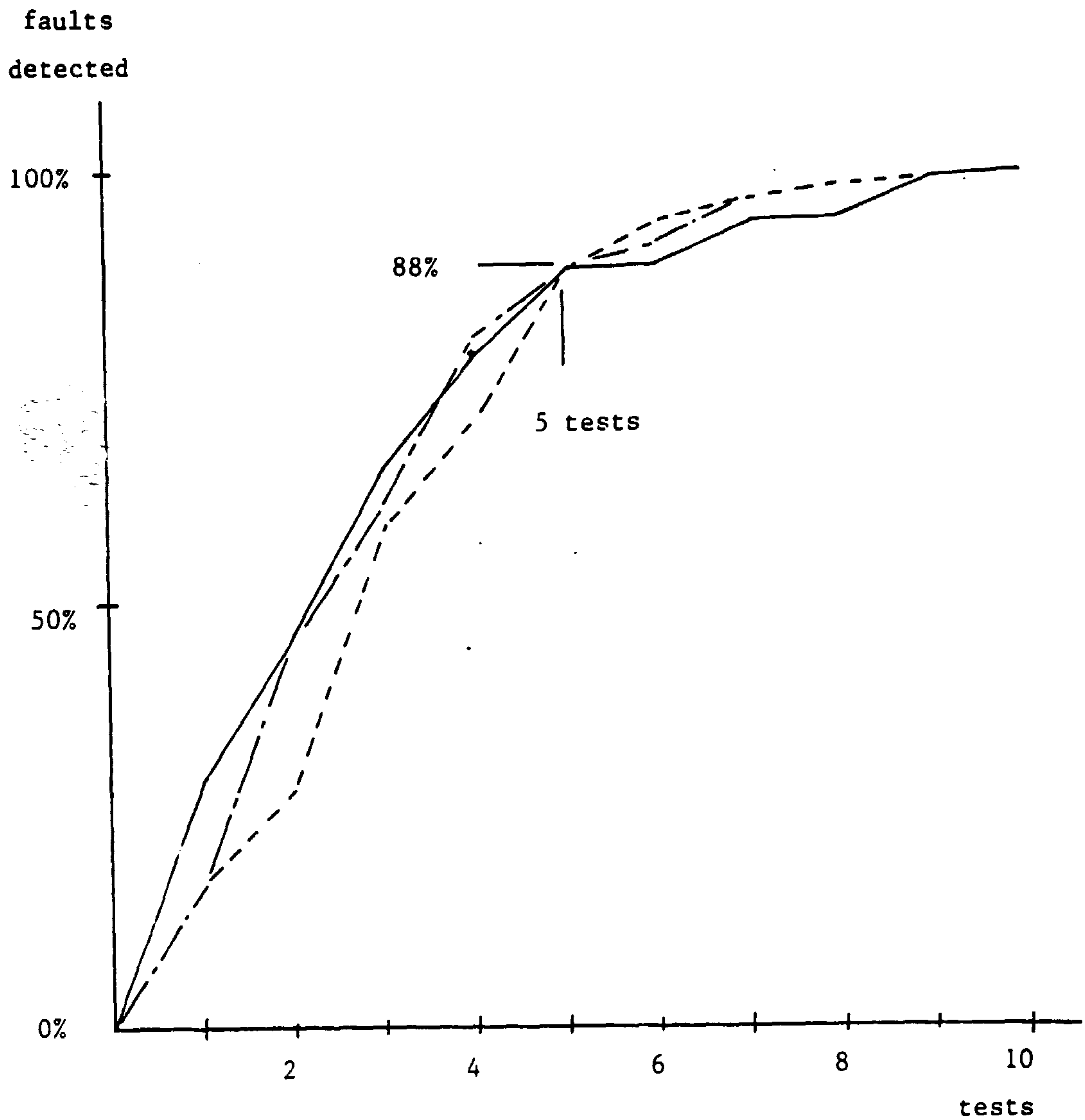

Figure 2 Coverage curves for gate example

Thus we can express the expected time to fault detection for this segmented test as

$$\mathrm{ETFD} = \frac{I}{4}\,\frac{81}{92} + \left(\frac{I}{4} + \frac{I}{2}\right)\frac{11}{92}$$
$$= \frac{I}{2}\,\frac{57}{92} \qquad (4)$$

The second factor of (4) is a factor by which we have reduced the ETFD by partitioning the complete test into two segments. This reduction is bought at a cost of testing twice as often. The overhead in shifting to testing mode has been doubled.

Another measure of testing goodness applicable to fault tolerant system is the probability of an undetected double error. The idea is that a single fault tolerant system can either adapt or flag the rest of the world when a single error is detected yet still compute correctly. Undetected double errors on the other hand might lead to overall system failure (1, 2). We assume a Poisson fault process (5) to estimate the probability of an undetected double error.

For a Poisson process with rate λ, the probability of exactly k occurrances in time interval t is

$$p(k,t) = e^{-\lambda t}\,\frac{(\lambda t)^k}{k!} \qquad (5)$$

To simplify the example we assume that λt is very small so that $e^{-\lambda t}$ can be treated as the value 1. This assumption does not appreciably change the character of the results. Expression (5) thus is reduced to

$$p(k,t) = \frac{(\lambda t)^k}{k!} . \tag{6}$$

Also we note that p(k,t) is much larger than p(k+1,t), i.e. single errors are much more likely than double errors which in turn are much more likely than triple errors, etc.

Consider P_2, the probability of an undetected double error in an interval I between complete test sets. For L successive frames of I this probability is approximately

$$P_2 = L \frac{(\lambda I)^2}{2} . \tag{7}$$

Expression (7) assumes that $L\lambda I$ is small compared to 1.

Next consider the complete test divided in two and applied every I/2 time units. Double errors within the shorter interval I/2 are undetected. In addition some single errors in the first interval are undetected after the partial test at I/2. If such an undetected fault is followed by another fault in the next interval before time I then an undetected double error has occured. In a similar fashion there are some single errors between time I/2 and I which when followed by another single error in the next half interval result in an undetected double error. Note that these cases are in essence the same as those in the discussion leading to expression (4). Adding the probability of these three situations for L frames of I we find a total probability

$$P_2 = L \left(\frac{(\lambda I)^2}{2}\right) \left[1 + \frac{11}{92} + \frac{L-1}{L} \frac{11}{92}\right] . \tag{8}$$

Replacing (L-1) in the last term of (8) by L simplifies the expression with the following upper bound.

$$P_2 = L \frac{(\lambda I)^2}{2} \left(\frac{1}{2}\right) \left(1 - \frac{11}{92} + \frac{11}{92}\right) \qquad (9)$$

The particular arrangement of (9) is intentional. The form is extended to the general case in the appendix A. The left most portion, $L(\lambda I)^2/2$, is the probability of the complete test (7). Next the 1/2 term corresponds to one over the number of segments. Finally, the right-most term is 1 plus twice the fraction of undetected faults. Comparing (9) and (7) we see that the segmented test probability is reduced by a factor of (57/92).

GENERAL CASE

In appendix A expressions are developed for the mean time to fault detection and for the probability of an undetected double error when segmented testing is used. Each of the M test segments is assumed to be applied at uniform time intervals. The extension to nonuniform application is straightforward and could be advantageous in some cases. The forms of these expressions allow a segmenting gain to be defined, assuming the same average testing effort for segmented and for nonsegmented test. This gain g(M) is the ratio of the probability for the segmented to the nonsegmented case. For both mean time to detection and probability of an undetected double error, the same gain expression results,

$$g(M) = M/\left[1 + 2 \sum_{i=1}^{M-1} \overline{\alpha}_i\right] \tag{10}$$

The parameter M is the number of test segments and α sub i bar denotes the fraction of faults missed by i consecutive test segments averaged over all M starting positions.

If the coverage curve is a straight line, then it is easy to show that the gain is always 1. In this case there is no advantage to segmenting. When the curve is concave (e.g., where there is a lot of initialization of state variables in a sequential network) then the gain is less than one and segmentation makes things worse. Fortunately the convex shape seems much more prevelent and improvement is often possible.

Examining the gain expression (10) we see that the numerator and denominator both increase with M. Whether an optimum value of M exists depends on the coverage curve. Practically speaking one would expect test overhead to eventually become significant as M is increased. In addition, M is limited to the number of individual tests in the complete test T. With these factors in mind it is instructive to consider the theoretical case where the coverage function is an exponential. Let

$$\alpha_i = \frac{1}{2^{\frac{Ki}{M}}} \tag{11}$$

When the number of faults times some α_t is less than one we assume that the test is complete. Using (11) in (10) and letting i become large will merely lower bound the possible gain. Substituting (11) into (10) we obtain a lower bound on g(M),

$$\frac{M(2^{K/M} - 1)}{(2^{K/M} + 1)} \, . \tag{12}$$

Supposing that M is a continuous variable and maximizing (12) with respect to M we find that the maximum occurs for M very large and that

$$\lim_{M \to \infty} g(M) = \frac{\ln 2}{2} K$$

or approximately

$$\bar{g} = 0.34657 \text{ K} \tag{13}$$

we let $\bar{g}$ denote the unrealizable gain maximum.

M larger than the number of tests has no meaning. For M = K/2 in (12),

$$g(K/2) = 0.3 \text{ K} \tag{14}$$

and

$$g(K/2) = 0.8656\ \bar{g}.$$

For the gain to be half of $\bar{g}$ we find numerically that

$$M = \frac{L}{5.525}\ .$$

Example 2

Suppose that a network requires $L = 48$ tests and that the coverage curve satisfies (11) with $K = 12$. the limiting value for the gain $\bar{g}$ is 4.159. When $M = 6$ there are 6 subsets of 8 tests each and the α coeficients would be

$$\alpha_i = \frac{1}{(4)}\ i$$

Any 8 test segment detects 3/4 of the faults.

The gain from (12) or (14) computes to

$$g(6) = 3.6$$

Evaluating (10) directly assumeing α_i is zero for $i > 5$ gives

$$g(6) = 3.601$$

Table 1 lists the gain function for example 2 as a function of M.

M Number of Segments	g(M) Improvement Ratio or Gain
2	1.998
3	2.647
4	3.111
6	3.601
12	4.000
24	4.118
48	4.149

Table 1. Gain for Example 2

Case Study

The segmented testing idea was applied to a particular subsystem of SSI and MSI logic called the bus guardian unit (BGU). A brief description of this subsystem and the testing assumptions are given in Appendix B. The unit has a complexity of 1296 equivalent gates and 655 package pins. Assuming single pin stuck at 1 or 0 faults, a lower bound on the fraction of missed faults was made for M=6. From these estimates (Tables B1) we can compute a lower bound on the segmenting gain (expression 10) for 2,3 and 6 segments (Table 2). it is felt that these lower bounds are within 10% of the actual valves for the test set considered.

M Number of Segments	g(M) Improvement Ratio or Gain
2	1.49
3	1.79
6	2.22

Table 2. Segmenting Gain for the Case Study

DISCUSSION

The segmenting of a test set presented may also be applied when a processor is tested by executing self-test code. Segmenting should be beneficial whenever the overhead associated with switching to test mode is small and little initialiation is associated with test subsequences.

It is possible to specifically design test sets to enhance the gain obtained from segmenting. Examples have been constructed where a slightly longer test which is constructed to be segmented yields a lower mean time to detection than the shortest test set. In both cases the average number of tests per unit time was held constant. Finally it may be possible to design networks to maximize the gains from a segmented testing environment.

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Appendix A

Expressions for General Coverage

In this appendix we develop expressions for the probability of a double error occuring before the first error is detected, P_2, and the mean time between single fault occurrence and detection MTFD. The fault process is assumed to be Poisson with the fault rate λ very small.

Initially suppose that a complete fault detection set T of length L is divided into M segments and is applied to a unit under test uniformly spaced in time and repeated periodically. The total time for one complete test is denoted by I. The fraction of time actually spent applying test patterns is assumed to be small. If not, the results are not changed significantly but the analysis is considerably more complex. Let T_j denote the jth test segment. The total test T is then

$$T = T_0 T_1 \cdots T_{M-1} \; . \tag{A1}$$

Any rotation of T, e.g.,

$$T_j \; T_{j+1} \cdot\cdot \; T_{M-1} \; T_0 T_1 \cdot\cdot \; T_{j-1} \tag{A2}$$

is assumed to also be a complete test and to have the same coverage curve as T. This is for simplicity of notation and will be relaxed later. Let α_j denote the fraction of faults which are undetected by the first j test segments. Clearly α_j is nondecreasing in j. Figure A1 indicates α_j on a typical coverage curve.

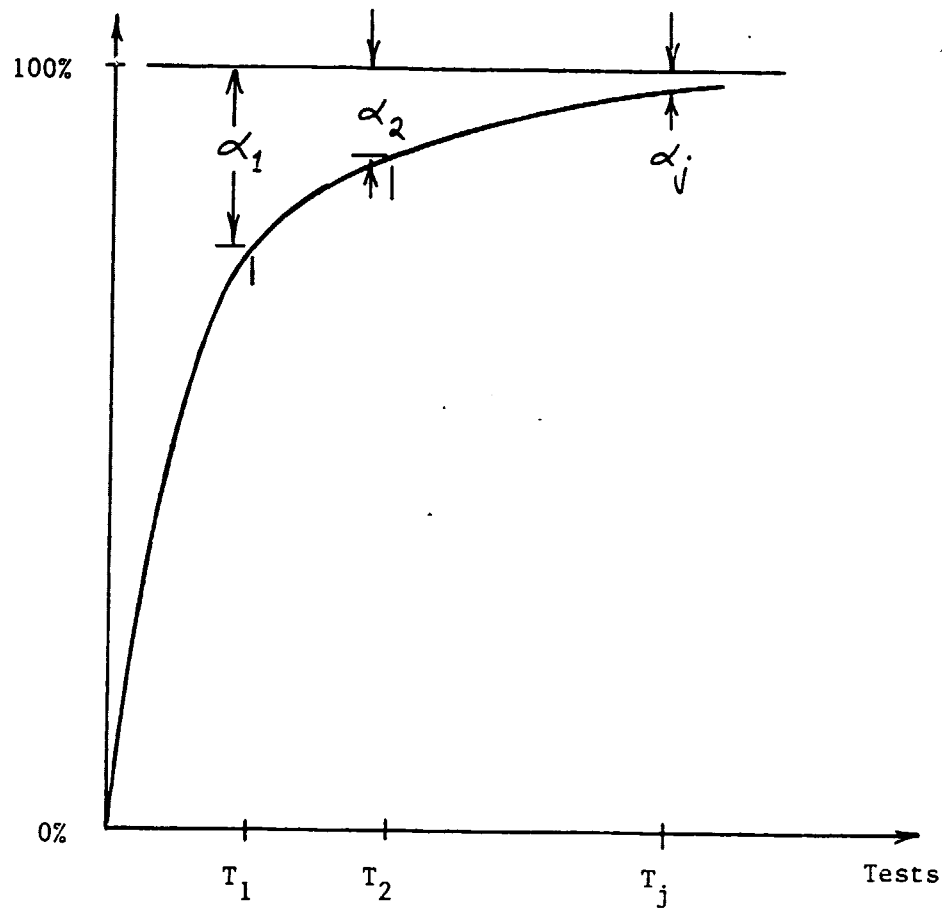

Figure A1 Coeficient α_i for a coverage function

For a Poisson process the probability of exactly k occurrences in a time t is given by

$$e^{-\lambda t}\,\frac{(\lambda t)^k}{k!} \tag{A3}$$

where λ is the rate parameter. Consider the system for L complete testings. In this total time there are LM intervals between test segments. Consider the instances where a single error in the first interval is undetected before some other error occurs. A double error before any testing has probability

$$e^{-\frac{\lambda I}{M}}\left(\frac{\lambda I}{M}\right)^2 \frac{1}{2}\,. \tag{A4}$$

We suppose that λI is very small compared to 1 so that the exponential term can be replaced by 1 in the various expressions. Thus a double error in the first interval has probability given by $(\lambda I)^2/2M^2$.

A single fault in the first interval which is missed by test segment T_o has probability $\alpha_1 \frac{\lambda I}{M}$ assuming equally likely faults. A single fault in the second interval has probability $\frac{\lambda I}{M}$. This situation has probability $\alpha_1 \frac{(\lambda I)^2}{M^2}$. In a similar fashion an error in the first block which is undetected in the first j intervals followed by an error in interval j+1 has probability $\alpha_j \frac{(\lambda I)^2}{M^2}$.

In the LM intervals the first case occurs LM times, the second occurs LM-1 times, and the last occurs LM-j times. Adding all such terms results in the probability of two errors in an interval or a single error which is undetected before a second error occurs, P_2

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$$P_2 = \frac{(\lambda I)^2}{M^2} [\frac{LM}{2} + \alpha_1(LM-1) + \alpha_2(LM-2) + \cdot\cdot + \alpha_j(LM-j) \cdot\cdot] =$$

$$\frac{(\lambda I)^2}{M^2} [\frac{LM}{2} + \sum_{j=1}^{M-1} \alpha_j(LM-j)] =$$

$$P_2 = \frac{L(\lambda I)^2}{M} [\frac{1}{2} + \sum_{j=1}^{M-1} \alpha_j(1 - \frac{j}{LM})] \ . \qquad \text{(A5)}$$

For L large we can replace $(1 - \frac{j}{LM})$ by 1 and approximate this expression by

$$P_2 = \frac{L(\lambda I)^2}{M} [\frac{1}{2} + \sum_{j=1}^{M-1} \alpha_j]$$

$$P_2 = \frac{L(\lambda I)^2}{2} \frac{1}{M} (1 + 2 \sum_{j=1}^{M-1} \alpha_j) \ . \qquad \text{(A6)}$$

Expressions (A5) and (A6) omit triple and higher order faults since we have assumed that λI is very small. The first factor $L(\lambda I)^2/2$ is the probability of two errors within an interval I for L intervals. This is the case where the total test is applied once in I time units. The factor $1/M$ is the maximum reduction possible for M segments when the coverage curve is very steep, i.e., the sum of the α_j is small. The last term is the dependency on the shape of the coverage curve.

Next suppose that a rotation of the test set (A2) gives a different coverage curve than the initial order (A1). Up to M different fractional probabilities could result. But in our summation leading up to (A5) we can replace α_1 by the average of the M possibility different fractional coefficients, one for each phase of (A2). Thus we define an average fractional coefficient $\bar{\alpha}_j$,

$$\bar{\alpha}_j = \frac{1}{M} \sum_{i=0}^{M-1} \text{fraction of faults undetected by } T_i \ T_{i+1} \cdot\cdot \ T_{i+j-1}$$

where the subscripts are added modulo M. Using the same procedure as led to (A6) we find the probability P_2,

$$P_2 = \frac{L(\lambda I)^2}{2} \frac{1}{M} (1 + 2 \sum_{j=1}^{M-1} \bar{\alpha}_j) \ . \tag{A7}$$

The computation of the mean time between single fault occurrence and detection, MTFD, is quite similar to that of P_2. Suppose initially that any rotation of T has the same coverage curve. A fault can occur uniformly within the first interval I/M. For those faults that are detected by T_0 the mean time is just half the interval or I/2M. The fraction of such faults is $1 - \alpha_1$. The fraction $\alpha_1 - \alpha_2$ of faults in the first interval is detected by T_1. For this second class the mean time is I/M longer or 3I/2M. Forming the expected value we find

$$\begin{aligned} \text{MTFD} &= \frac{I}{2M} [(1 - \alpha_1) + 3(\alpha_1 - \alpha_2) + 5(\alpha_2 - \alpha_3) + \cdots] \\ &= \frac{I}{2} \frac{1}{M} [1 + 2 \sum_{j=1}^{M-1} \alpha_j] \ . \end{aligned} \tag{A8}$$

Again the interpretation of (A8) is similar to that of (A6). The factor I/2 is the value expected for no segmenting, the second factor 1/M is the maximum conceivable reduction for M segments, and the last factor is the coverage curve coefficient.

As was the case for P_2 (A8) is easily extended to different coverage curves for rotations and

$$\text{MTFD} = \frac{I}{2} \frac{1}{M} [1 + 2 \sum_{j=1}^{M-1} \bar{\alpha}_j] \ . \tag{A9}$$

Appendix B - Case Study

Introduction

The bus guardian unit (BGU) of the fault-tolerant multi-processor (FTMP) is specialized module designed to enable or decouple a bus drive to a system bus line. Two BGU's are used for each processor/memory unit. A BGU has an equivalent complexity of about 1200 logic gates. In operation, 3 out of 5 bus lines are selected on the basis of an internally stored select code SEL. Each selected bus line is input to a synchronizing sub-unit called a deskewer. The three synchronized bit streams are voted to yield a serial message. if the message is recognized as being addressed to the particular BGU, then one of 5 storage registers are updated. Four of these registers generate the 20 BGU output enable lines while the fifth register contains the select code, SEL.

Because of the fault-tolerant nature of the communication scheme and the limited output visibility, the BGU is an interesting module to test. To simplify the discussion we will ignore the specialized BGU operations associated with power-on, master-resest, and power-fail and consider normal operation.

At the highest level there are 3 types of BGU behavior;

Case 1. Correct response to a valid message,

Case 2. Failure of the BGU to recognize a valid message,

Case 3. Change by the BGU when not commanded.

These last two correspond to a miss or a false alarm respectively. These cases require separate tests to detect. The following three facets of the BGU add to the testing problem.

The FTMP communication system is designed to tolerate single failures, hence the three seperate serial inputs which are voted. But with three correct bus inputs, many BGU interval faults are also tolerated when they occur prior to voting. BGU voter discrepencies (2 of 3 or 1 of 3) are not visible as outputs. This situation requires test inputs which are also 2 of 3 or 1 of 3 to propagate faults to a visible output.

Closely related are input selection faults. The 3 of 5 select logic and SEL code assignment interact. Many single bit changes in a SEL code result in 2 of the three desired bus inputs still selected. Thus many select logic faults and SEL register faults are not visible with 3 correct inputs. Since the SEL register contents can only be infered from other register outputs, a series of tests are needed.

Finally the BGU address decoder utilizes 20 message positions. A single stuck position at the correct value can occur in 20 ways, hence 20 tests are needed to detect these Case 3 failures. In addition there are other faults associated with the BGU timing logic which result in Case 3 behavior which can not be overlapped with addressing false alarm faults.

To illustrate the segmenting idea we make the following testing assumptions. The assumptions 2 and 3 correspond, approximately, to the manufacturing test environment.

1. The fault class is single pin stuck-at 1 or 0 faults.
2. The 5 system bus lines are available as inputs. Bus inputs can be freely chosen.
3. The 20 enable ouputs are observable.
4. Fault dectection is the objective.

The BGU is constructed from 50 digital integerated circuits (Table B1), 3 delay units, two op amp comparitors and some discrete components such as pullup resistors, decoupling capacitors, etc. The unit is assembled on a single circuit board. The digital circuits include 26 SSI packages, 24 MSI packages, and 3 delay units. The SSI accounts for 327 logic pins with 193 equivalent gates; the MSI for 328 pins with 1103 equivalent gates. As compared to LSI or VLSI the gate to pin ratio is quite small.

Description		Per Ckt Pins	Per Ckt Gates	No. Ckts	Total Pins	Total Gates
54LS00	Quad NAND	12	4	2	24	8
54LS02	Quad NOR	12	4	1	12	4
54LS04	Hex Inverter	12	6	1	12	6
5404	Hex Inverter	12	6	1	12	6
54LS09	Quad AND-OC	12	4	1	12	4
54LS10	Triple NAND	12	3	1	12	3
54LS30	8 Input NAND	9	1	1	9	1
54LS32	Quad OR	12	4	2	24	8
54LS74	Dual D Flip-Flop	12	12	7	84	84
54LS112	Dual J-K Flip-Flop	14	16	1	14	16
54LS240	Octal Buffer	18	10	1	18	10
4001B	Quad CMOS NOR	12	4	2	24	8
70C96	HexCMOS Buffer	14	7	5	70	35
				Subtotal	327	193
54LS138	3-8 Decoder	14	16	1	14	16
54LS164	Octal Shift Regs	12	52	4	48	208
54LS191	4 Bit Up/Down Ctr	14	58	5	70	290
54LS251	8-1 Mux	14	17	3	42	51
54LS253	Dual 4-1 Mux	14	16	2	28	32
54LS259	Octal Latch	14	58	3	42	174
7136	6 Bit Comparator	14	22	1	14	22
54C174	Hex Register CMOS	14	62	5	70	310
				Subtotal	328	1103
				Total	655	1296

Pins = Logic Pins Gates = Equivalent Gates

Table B1- Bus Guardian Digital Integrated Circuits

Examining the design in detail there are five key areas which require particular test inputs:

A. The five output registers and associated tri-state buffers.

B. The address decoder.

C. The select code register and 3 of 5 select logic.

D. The two voters.

E. The timing decade logic.

Area C has ten distinct paths through the select logic which require two messages for each path, one to change the SEL code and a second message to verify the updated SEL code by changing an observable output. Another five messages are required to complete the testing of area A.

Area B requires 20 test messages for Case 3 (false alarm) failures as noted earlier. The BUSY voter in Area D requires 6 additional false alarm messages. Finally Area E can be tested for Case 3 faults with 7 more messages. The total set of 58 messages is sufficient to test for single pin s-a faults. It can be shown that at least 54 messages are necessary.

The sufficient message set can be divided into 6 segments with each of the previously mentioned subsets as evenly as possible yielding 9 or 10 test messages per segment. An upper bound on the α_i values for this segmented test set can be determined by counting the number of faults that are always detected. Exact values could be determined by simulation. From upper bounds on the α_i a lower bound on the segmenting gain g(6) expression 10 can be found. The following table lists these values.

i	α_i
1	0.29
2	0.23
3	0.17
4	0.11
5	0.05

Table B1. Miss Fractions α_i for the Case Study