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(NASA-CR-173251) NUMERICAL SOLUTION OF THE
NAVIER-STOKES EQUATIONS FOR ARBITRARY
2-DIMENSIONAL MULTI-ELEMENT AIRFOILS Final
(Mississippi State Univ., Mississippi)

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Entitled
NUMERICAL SOLUTION OF THE NAVIER-STOKES EQUATIONS
FOR ARBITRARY TWO-DIMENSIONAL MULTI-ELEMENT AIRFOILS

by

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January 23, 1983
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Introduction

This research project covered a period of ten years, from 1 January 1974 through 31 December 1983. The overall purpose was to develop numerical solutions of the Navier-Stokes equations, with an algebraic turbulence model, for time-dependent two-dimensional flow about multi-element airfoils.

Fundamental to these solutions was the use of numerically-generated boundary-conforming curvilinear coordinate systems to allow bodies of arbitrary shape to be treated. This project, in fact, provided the initial support for the study of elliptic grid generation systems in two dimensions, and much of the development of numerical grid generation based on this technique has been done under this project. A general two-dimensional grid generation code for multiple-body configuration was written as a part of this project and made available through the COSMIC code library.

Numerical Grid Generation

The developments under this project in the area of numerical grid generation are summarized as follows:

1. Elliptic grid generation with coordinate line control. This initial work was covered in the Ph.D. dissertation of Thames (1975) and was reported in Thompson, Thames and Mastin, Journal of Computational Physics (1974), and in Thames, Thompson and Mastin, NASA SP-347 (1975). Results were also reported at the AIAA 2nd Computational Fluid Dynamics Conference in Hartford, Conn. (1975). The basic techniques of elliptic generation were developed and coded, including in particular the means of user-controlled attraction of coordinate lines and/or points to other coordinate...
lines and/or points to provide a priori concentration of coordinate lines in regions of expected high gradients, e.g., boundary layers.

(2) **Extension to multiple-body fields: the TOMCAT code.** This work culminated in the development of the TOMCAT code which generates two-dimensional boundary-conforming coordinate systems for fields containing any number of arbitrarily-shaped bodies, with a priori control of coordinate line spacing as mentioned above. This code was made available through the COSMIC code library, as reported in the *NASA Tech Briefs* (Spring 1978, p. 150). The code was described fully in a contractor report, CR-2729 (1977), and also was reported in Thompson, Thames and Mastin, *Journal of Computational Physics* (1977).

(3) **Automatic coordinate line concentration in boundary layers.** This allowed a prescribed number of lines to be placed within a specified boundary layer thickness on an airfoil, requiring only the specification of the Reynolds number. This technique was discussed in Thompson, et al., *NASA CP-2045* (1978); in Thompson, *Computational Fluid Dynamics* (1980); and in Thompson, *Numerical Grid Generation* (1982).

(4) **Evaluation of grid-induced truncation error.** Studies were made of the truncation error induced by rapid changes in coordinate line spacing, and several distribution functions were compared in this regard. Some of this work was involved in the MS thesis of D. Thompson (1980).

(5) **Grids with infinity transformed to the origin.** This type of grid is particularly appropriate to double-airfoil configurations. The work was discussed in the MS thesis of Long (1977) and was also reported in Thompson, et al., *NASA CP-2045* (1978).
(6) **Grid generation surveys.** This project provided support for extensive coverage of the literature on grid generation over the years, contributing to the surveys given by Thompson, Warsi and Mastin, *Journal of Computational Physics* (1982), and that presented by Thompson at the AIAA 21st Aerospace Sciences meeting in Reno (1983).

(7) **Exposition on grid generation.** In the course of this long project the principles of numerical grid generation from elliptic generation systems, and the use thereof in the numerical solution of partial differential equations, were formulated and refined to the point of consistent presentation for general use. This is reflected in the expositive presentations by Thompson in *Computational Fluid Dynamics* (1980), and in *Numerical Grid Generation* (1982).

**Incompressible Navier-Stokes Codes**

The codes developed for the incompressible Navier-Stokes equations are summarized below. All these, except the first, were based on the velocity-pressure formulation, using the Poisson equation for the pressure. The pressure boundary condition on the body surface was obtained from the normal momentum equation, although in later work a zero normal pressure gradient on the surface was found to be sufficient because of the very close spacing of the first coordinate line off the surface, typically set at about 1% of the expected boundary layer thickness.

In all these codes, provision was made for the use of artificial viscosity proportional to the divergence of the velocity, and hence analytically zero, to control the solution during the initial transients. The start was through continuous acceleration, instituted through the inclusion
of fictitious body force terms during the period of acceleration. The Baldwin–Lomax algebraic turbulence model was used in all cases.

(1) **Stream function-vorticity.** This code used point SOR iteration for the vorticity equation and the stream function Poisson equation. This work was discussed in the Ph.D. dissertation of Thames (1975) and was reported in Thames, Thompson and Mastin, NASA SP-347 (1975), and in Thames, et al. *Journal of Computational Physics* (1977). Results for application to boundary layers were discussed in the MS thesis of Walker (1974), also being reported in Thompson, et al., *Symposium on Unsteady Aerodynamics* (1975), and in Thames, et al., (1977) cited above. Applications to airfoils at higher Reynolds numbers were discussed in the MS thesis of Bearden (1977) and were reported in Thompson, et al., NASA CP-2045 (1978).

(2) **Point SOR.** This code was based on point SOR iteration with under-relaxation, using a variable acceleration parameter field calculated from the local velocity. This code was discussed in Thompson, et al., NASA CP-2045 (1978); in Thompson, *Computational Fluid Dynamics* (1980); and in the MS thesis of D. Thompson (1980).

(3) **Approximate factorization.** This code used approximate factorization on the two momentum equations and line SOR iteration on the pressure Poisson equation, thus requiring only one pass of the momentum equation at each time step, although the pressure equation still was iterated. This work was discussed in the Ph.D. thesis of Bernard (1981), and results were presented at the AIAA/ASME 3rd Joint Thermophysics, Fluids, Plasma and Heat Transfer Conference in St. Louis (1982) and also in Mueller,

(4) **Vectorized checkerboard SOR.** This code was based on checkerboard SOR iteration in all equations, and was fully vectorized on the CYBER 203. The vectorization was explicit using one vector FORTRAN with bit vectors to control the manipulation of the arrays. This code was discussed in the Ph.D. dissertation of Patel (1983), and a paper has been submitted for presentation at the AIAA 17th Fluid Dynamics, Plasma Dynamics, and Lasers Conference in Snowmass, Colorado (1984). This was the most promising of the incompressible codes, showing considerable speed advantage because of the vectorization.

**Compressible Navier-Stokes Codes**

The compressible Navier-Stokes codes are summarized as follows. All used backward-time and central-space differences and were second-order in time and space. Again the Baldwin-Lomax turbulence model was used.

(1) **Point SOR.** This code used point SOR iteration of all equations at each time step, with a variable acceleration parameter field calculated from the local velocity. Since these parameters proved to be very small for the continuity equation, this code was unreasonably slow. The code was discussed in the Ph.D. dissertation of Turner (1979).

(2) **Density Gradients as dependent variables.** A small investigation of the use of density gradients as dependent variables was made, as reported in the MS thesis of Kwon (1977), but little promise of advantage was found.
(3) Approximate factorization. This code was based on approximate factorization in the delta form for all equations and was discussed in the Ph.D. dissertation of Cooper (1980). Results were presented at the AIAA 14th Fluid and Plasma Dynamics Conference in Palo Alto (1981).

Incompressible Potential Code

In addition to the Navier-Stokes codes, an incompressible potential flow code for multiple-body fields was developed in the early stages of the project. This solution was also discussed in the Ph.D. dissertation of Thames (1975). Some results were reported in Thames, et.al., Journal of Computational Physics (1977), and more extensive results were collected later in the MSU report of Thompson and Thames (1982).

Conclusion

The major accomplishments of this long project were the fundamental development of elliptic grid generation and the establishment thereof as a basic component in the numerical solution of partial differential equations, particularly in computational fluid dynamics. A significant additional accomplishment was the development of a vectorized code for the incompressible Navier-Stokes equations. Finally, perhaps the real significance of this project was the basic support it provided over the years for the development, correlation and dissemination of ideas on numerical grid generation and the graduates it produced.
PUBLICATIONS

Journal Articles


Conference Presentations


Reports


Thesis


Dissertations


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<tr>
<th>Name</th>
<th>Degree</th>
<th>Date</th>
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<tr>
<td>Ray L. Walker</td>
<td>M.S.</td>
<td>1974</td>
<td>CIA</td>
</tr>
<tr>
<td>Frank C. lhames</td>
<td>Ph.D.</td>
<td>1975</td>
<td>NASA Langley Research Center</td>
</tr>
<tr>
<td>John H. Bearden</td>
<td>M.S.</td>
<td>1977</td>
<td>Cabot Corp. (entering Ph.D. program at Georgia Tech)</td>
</tr>
<tr>
<td>W. Serrill Long</td>
<td>M.S.</td>
<td>1977</td>
<td>United Airlines</td>
</tr>
<tr>
<td>J. H. Kwon</td>
<td>M.S.</td>
<td>1977</td>
<td>unknown (last known as Ph.D. student at Cornell)</td>
</tr>
<tr>
<td>Louie Turrer</td>
<td>Ph.D.</td>
<td>1979</td>
<td>Boeing</td>
</tr>
<tr>
<td>David S. Thompson</td>
<td>M.S.</td>
<td>1980</td>
<td>Ph.D. student at Iowa State</td>
</tr>
<tr>
<td>G. Kyle Cooper</td>
<td>Ph.D.</td>
<td>1980</td>
<td>Sverdrup Corp.</td>
</tr>
<tr>
<td>Robert S. Bernard</td>
<td>Ph.D.</td>
<td>1981</td>
<td>U.S. Army Engineer Waterways Experiment Station</td>
</tr>
<tr>
<td>Nisheeth Patel</td>
<td>Ph.D.</td>
<td>1983</td>
<td>U.S. Army Ballistic Research Laboratory</td>
</tr>
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Of these ten graduates, all but two were U.S. citizens.