# Prediction of Vortex Shedding From Circular and Noncircular Bodies in Supersonic Flow 

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## Scientific and Technical Information Branch

## TABLE OF CONTENTS

Page
Section ..... No.
LIST OF SYMBOLS ..... v
SUMMARY ..... 1
INTRODUCTION ..... 1
GENERAL APPROACH ..... 3
METHODS OF ANALYSIS ..... 4
Conformal Mapping ..... 5
Analytic tranformation ..... 5
Numerical transformation ..... 6
Body Model ..... 7
Circular bodies ..... 7
Noncircular bodies ..... 8
Vortex Shedding Model ..... 9
Equations of motion ..... 10
Surface pressure distribution ..... 12
Separated wake ..... 14
Forces and moments ..... 16
Vortex smoothing ..... 18
Vortex core ..... 21
PROGRAM ..... 23
General Description ..... 23
Limitations ..... 25
Error Messages and STOPS ..... 26
Numbered STOP Locations ..... 28
Input Description ..... 31
Input Preparation ..... 53
Panel layouts ..... 53
Numerical mapping ..... 56
Integration interval ..... 57
Vortex core ..... 58
Section
Page
Sample Cases. ..... 58
Output ..... 60
RESULTS ..... 65
Circular Bodies ..... 65
Elliptic Bodies ..... 70
Arbitrary Bodies ..... 75
CONCLUSIONS ..... 76
RECOMMENDATIONS ..... 77
REFERENCES ..... 79
TABLES I AND II
FIGURES 1 THROUGH ..... 33

## LIST OF SYMBOLS

| a | semi-major or horizontal axis of elliptic cross section |
| :---: | :---: |
| ${ }^{A}{ }_{\ell}$ | ```coefficients of conformal transformation, Eq. (9)``` |
| b | semi-minor or vertical axis of elliptic cross section |
| $c_{n}$ | normal-force coefficient per unit length, Eq. (39) |
| $c_{y}$ | side-force coefficient per unit length, Eq. (43) |
| $\mathrm{C}_{\ell}$ | rolling-moment coefficient, Eq. (47) |
| $C_{m}$ | pitching-moment coefficient, Eq. (41) |
| $C_{n}$ | yawing-moment coefficient, Eq. (45) |
| $C_{p}$ | pressure coefficient, Eq. (29) |
| $c_{p_{I}}$ | incompressible pressure coefficient, Eq. (31) |
| $C_{y}$ | side-force coefficient, Eq. (44) |
| $\mathrm{C}_{\mathrm{N}}$ | normal-force coefficient, Eq. (40) |
| D | diameter of circular body, or equivalent diameter of noncircular body |
| G | complex velocity component, Eq. (18) |
| $\ell_{\text {ref }}$ | reference length |
| L | total number of Fourier coefficients used to describe transformation, Eq. (9), also model length |
| $M_{X}$ | rolling moment about the x -axis |
| ${ }^{M} \mathrm{Y}$ | pitching moment about the y -axis |
| $M_{z}$ | yawing moment about the z-axis |


| $M_{\infty}$ | free-stream Mach number |
| :---: | :---: |
| p | local static pressure |
| $p_{\infty}$ | free-stream static pressure |
| $q_{\infty}$ | free-stream dynamic pressure, $\frac{1}{2} \rho \mathrm{~V}_{\infty}^{2}$ |
| $r$ | radial distance from a vortex to a field point |
| $r^{\prime}$ | radial distance to a point on a noncircular body, Fig. 4 |
| $r_{c}$ | vortex core radius, Eqs. (50) and (51) |
| $r_{0}$ | radius of circle |
| $\operatorname{Re}_{\xi}$ | ```Reynolds number based on boundary layer run length and minimum pressure conditions, Um``` |
| S | reference area |
| $\mathrm{u}_{\mathrm{e}}$ | surface velocity in crossflow plane |
| $u_{r}$ | axial perturbation velocity |
| U | local velocity |
| $\mathrm{v}_{\theta}$ | vortex induced velocity |
| v,w | velocity components in real plane |
| $\mathrm{V}_{\infty}$ | free-stream velocity |
| W | complex potential, Eq. (16) |
| $x, y, z$ | body coordinate system with origin at the nose: $x$ positive aft along the model axis, y positive to starboard, and $z$ positive up |
| $\mathrm{x}_{\mathrm{m}}$ | axial location of center of moments |
| $\alpha$ | angle of attack |
| ${ }^{\text {c }}$ | angle between free-stream velocity vector and body axis |
| B | angle of sideslip; also polar angle in o-plane, Fig. 2 |

## Subscripts

| ()$\left.^{-}\right)$ | conjugate of complex quantity |
| :--- | :--- |
| ()$_{m}$ | vortex $m$ |
| ()$_{c p}$ | center of pressure |

## SUMMARY

An engineering prediction method and associated computer code NOZVTX to predict nose vortex shedding from circular and noncircular bodies in supersonic flow at angles of attack and roll are presented. The body is represented by either a supersonic panel method for noncircular cross sections or line sources and doublets for circular cross sections, and the lee side vortex wake is modeled by discrete vortices in crossflow planes. The three-dimensional steady flow problem is reduced to a two-dimensional, unsteady, separated flow problem for solution. Comparison of measured and predicted surface pressure distributions, flow field surveys, and aerodynamic characteristics are presented for bodies with circular and noncircular cross sectional shapes.

## INTRODUCTION

Current missile applications requiring high aerodynamic performance can involve noncircular body shapes in supersonic flow at high angles of attack and nonzero roll angles. The angle of attack range may be sufficiently high to cause formation of body separation vortices, and the body vortex shedding characteristics are directly influenced by the body cross-sectional shape and the flow conditions. It is desirable to model the body vortex wake by means of a rational method capable of considering a variety of body shapes over a wide range of flow conditions. It is important that the separation vortex wake induced effects on the nonlinear aerodynamic characteristics of the body be handled properly.

The phenomena of interest are the sheets of vorticity formed on the lee side of the body at high angles of attack. The vorticity is formed by boundary-layer fluid leaving the
body surface from separation points on both sides of the body (Fig. 1) and rolling up into a symmetrical vortex pair. A method to predict these flow phenomena in the vicinity of circular and noncircular bodies in subsonic flow is described in References 1 and 2. The extension of the method to supersonic flow is described in Reference 3.

The purpose of this report is to describe an engineering prediction method and associated computer code developed to calculate the nonlinear aerodynamic characteristics and flow fields of noncircular noses at high angles of attack in supersonic flow. The objectives of the method are to use a three-dimensional, attached flow, supersonic panel method, or a supersonic line source and doublet method to represent the body and a two-dimensional, incompressible, separated flow model to calculate the lee side vortex shedding from the body alone at angle of attack and angle of roll. The predicted pressure distribution on the body under the influence of the free stream and the separation vortex wake will be used to calculate the aerodynamic loads on the body. Conformal mapping techniques are used to transform noncircular cross sections to a circle for calculation purposes.

The following sections of this report include a discussion of the approach to the problem and a description of the analysis and flow models required to carry out the calculation. The prediction method is evaluated through comparison of measured and predicted results on a variety of body shapes, including circular and elliptical cross sections. A user's manual for the computer code is also included. The manual consists of descriptions of input and output, and sample cases.

## GENERAL APPROACH

Bodies at high angles of attack exhibit distributed vorticity fields on their lee side due to boundary-layer fluid leaving the body surface at separation lines. One approach for modeling these distributed vorticity fields has involved the use of clouds of discrete potential vortices. Underlying the basic approach is the analogy between two-dimensional unsteady flow past a body and the steady three-dimensional flow past an inclined body. In fact, the three-dimensional steady flow problem is reduced to the two-dimensional, unsteady, separated flow problem for solution. Linear theory for the attached flow model and slender body theory to represent the interactions of the vortices are combined to produce a nonlinear prediction method. The details of the application of this approach to the prediction of subsonic flow about circular and noncircular bodies are presented in References 1 and 2, and the extension to supersonic flow is described in Reference 3. Other investigators have also used this approach to successfully model the subsonic flow phenomena in the vicinity of circular cross section bodies (Refs. 4 and 5). The purpose of this report is to document the extension of the previous subsonic analysis of References 1 and 2 to supersonic flow.

The calculation procedure is carried out in the following manner. The body is represented by supersonic source panels (noncircular cross section) or supersonic line sources and doublets (circular cross section), and the strength of the individual singularities is determined to satisfy a flow tangency condition on the body in a nonseparated uniform flow at angle of attack and roll. Starting at a crossflow plane near the body nose, the pressure distribution on the body is computed using the full Bernoulli equation. The boundary layer in the crossflow plane is examined for separation
using modified versions of Stratford's separation criteria. The Stratford separation criteria are based on two-dimensional incompressible flow. At the predicted separation points, incompressible vortices with their strength determined by the vorticity transport in the boundary layer are shed into the flow field. The trajectories of these free vortices between this crossflow plane and the next plane downstream are calculated by integration of the equations of motion of each vortex, including the influence of the free stream, the body, and other vortices. The pressure and trajectory calculations are carried out by mapping the noncircular cross section shape to a circle using either analytical or numerical conformal transformations. The vortex-induced velocity contribution to the body tangency boundary condition includes image vortices in the circle plane. At the next downstream crossflow plane, new vortices are shed, adding to the vortex cloud representing the wake on the lee side of the body. This procedure is carried out in a stepwise fashion over the entire length of the body. The details of the individual methods combined into the prediction method are described in the following section.

## METHODS OF ANALYSIS

The development of an engineering method to predict the pressure distributions on arbitrary missile bodies in supersonic flow at high angles of incidence requires the joining of several individual prediction methods. In this section, the individual methods are described briefly, and the section concludes with a description of the complete calculation procedure.

## Conformal Mapping

The crossflow plane approach applied to arbitrary missile bodies results in a noncircular cross section shape in the presnece of a uniform crossflow velocity and free vortices in each plane normal to the body axis. The procedure used to handle the noncircular shapes is to determine a conformal transformation for mapping every point on or outside the arbitrary body to a corresponding point on or outside a circular body. The two-dimensional potential flow solution around a circular shape in the presence of a uniform flow and external vortices is well known and has been documented numerous places in the literature (Refs. 4 and 6). Thus, the procedure is to obtain the potential solution for the circular body and transform it to the noncircular body. Conformal transformations used are of two distinct types, analytical and numerical.

Analytic transformation.- For very simple shapes like an ellipse [Fig. 2(a)], the transformation to the circle can be carried out analytically as described in Reference 7. For example,

$$
\begin{equation*}
\sigma=v+\frac{a^{2}-b^{2}}{4 v} \tag{I}
\end{equation*}
$$

where

$$
\begin{equation*}
\sigma=y+i z \tag{2}
\end{equation*}
$$

in the real plane, and

$$
\begin{equation*}
\nu=\tau+\mathbf{i} \lambda \tag{3}
\end{equation*}
$$

in the circle plane. The derivative of the transformation is

$$
\begin{equation*}
\frac{d \sigma}{d \nu}=1-\left(\frac{a^{2}-b^{2}}{4 v^{2}}\right) \tag{4}
\end{equation*}
$$

which is required for the velocity transformation discussed later.

Numerical transformation.- For complex noncircular shapes, the transformation cannot be carried out analytically and a numerical transformation is required. The numerical transformation chosen is described in detail in Reference 8, and a brief summary of the conformal mapping procedure follows.

The sequence of events in the numerical mapping is shown in Figure $2(b)$. The arbitrary cross section shape of the body in the o-plane is required to have a vertical plane of symmetry. The transformation of interest will map the region on and outside the body in the o-plane to the region on and outside a circle in the $v$-plane. The first step is a rotation to the $\zeta$-plane so that the cross section is symmetric about the real axis. Thus,

$$
\begin{equation*}
\sigma=y+i z \tag{5}
\end{equation*}
$$

and

$$
\begin{align*}
\zeta & =i \sigma \\
& =\xi+i n \tag{6}
\end{align*}
$$

A mapping that transforms the outside of the body in the $\zeta$-plane to the outside of the unit circle is

$$
\begin{equation*}
\zeta=\int \frac{1}{\nu^{2}} \prod_{r=1}^{m}\left(\nu-\nu_{r}\right)^{\frac{\gamma_{r}}{\pi}} d \nu \tag{7}
\end{equation*}
$$

where $\gamma_{r}$ are the exterior angles of the $m$ segments of the body cross section. In Equation (7), $\pi$ denotes a product series. For a closed body

$$
\begin{equation*}
\sum_{r=1}^{m} \gamma_{r}=2 \pi \tag{8}
\end{equation*}
$$

The final transformation has the form

$$
\begin{equation*}
\sigma=-i\left[\nu+\sum_{\ell=1}^{L} \frac{r_{0}^{\ell+1} \mathrm{~A}_{\ell}}{v^{\ell}}\right] \tag{9}
\end{equation*}
$$

where the $A_{l}$ coefficients are obtained through an iterative scheme described in Reference 8 and $r_{0}$ is the radius of the equivalent circle in the $v$-plane. The derivative of the transformation is

$$
\begin{equation*}
\frac{d \sigma}{d \nu}=-i\left[1-\sum_{\ell=1}^{L} \frac{r_{0}^{\ell+1} \ell A_{\ell}}{v^{\ell+1}}\right] \tag{10}
\end{equation*}
$$

which is required for the velocity calculations described in a later section.

The above numerical mapping procedure has been applied to a wide range of general cross section shapes with good success.

Body Model
The prediction method described in this report is applicable to both circular and noncircular bodies. Separate body models are developed for circular and noncircular shapes because arbitrary noncircular shapes require panels and numerical conformal mappings and circular bodies can be represented by more computationally efficient means. Both flow models are described in this section.

Circular bodies.- A potential flow model of an axisymmetric body in supersonic flow is described in References 9 and 10. Such a body is represented by a distribution of line
sources and sinks and line doublets on the body axis to account for volume and angle of incidence effects, respectively. The strengths of the line singularities are determined from the flow tangency condition applied at points on the body surface. A downstream marching procedure from the nose tip to the body base is used for this calculation.

The axis singularity distribution for bodies of revolution provides an accurate and efficient means to calculate the induced velocity field associated with the body at angle of incidence. The calculation requires much less computation time than a panel method, and the resulting velocity distributions are quite smooth. One restriction on the body shape is the requirement that the nose contour lie inside the Mach cone. This constitutes the major limitation to the method.

Noncircular bodies.- The configurations considered in this study are bodies alone, without fins or control surfaces. The body geometry is described by the coordinates of the cross-sectional shape at a number of axial stations, and linear interpolation between specific points is used. The body is represented by a three-dimensional lattice of supersonic source panels distributed on the surface of the body. The panels account simultaneously for volume and angle of incidence effects. The attached flow solution for a panel inclined to the flow direction is based on supersonic linear theory as described in Reference 11 and the method used is a modified version of the program code presented in Reference 12. The pitch and yaw angles are related to the included angle of attack and angle of roll by the expressions

$$
\begin{align*}
& \sin \alpha=\sin \alpha_{c} \cos \phi  \tag{11}\\
& \sin \beta=\sin \alpha_{c} \sin \phi \tag{12}
\end{align*}
$$

The flow tangency condition is applied at the control point located at the panel centroid of each body panel. The result is a set of linear simultaneous equations in terms of the unknown panel source strengths. Solution by an iterative approach produces the singularity strength on each panel and thus provides a means to calculate the velocity field induced by the body. The nature of the solution is such that the velocity on the body surface is correct only at the control points; therefore, linear interpolation is used to determine body surface velocity components for use in pressure calculations at points lying between control points. The body panel induced velocities are calculated one time for the entire body to provide a table of velocity components for interpolation purposes. Using a slender-body theory approximation, these velocity components are combined with other velocity components for the pressure calculation described in a later section.

There is a basic limitation in the use of the above panel method. The angle of inclination between a body source panel and the body centerline must be less than the semiapex angle of the Mach cone associated with the free-stream Mach number. Thus, there is a limit to the body nose configurations which can be modeled by the body source panel method.

Vortex Shedding Model
The body vortex shedding model described in this section is nearly the same as the subsonic model presented in Reference 2 ; therefore, the following description will be brief, and only the differences between the two models will be discussed in detail.

Equations of motion.- The equations of motion of a shed nose vortex in the presence of other free vortices in the vicinity of a body in a uniform stream follow. In the circle (v) plane, the position of a vortex, $\Gamma_{m}$ is

$$
\begin{equation*}
\nu_{\mathrm{m}}^{\Gamma}=\tau_{\mathrm{m}}+i \lambda_{\mathrm{m}} \tag{13}
\end{equation*}
$$

and the image of $\Gamma_{m}$ is located at

$$
\begin{equation*}
\nu_{m_{-\Gamma}}=\frac{r_{0}^{2}}{\bar{v}_{m}} \tag{14}
\end{equation*}
$$

In the real plane, the position of the vortex $\Gamma_{m}$ is

$$
\begin{equation*}
\sigma_{m_{\Gamma}}=y_{m}+i z_{m} \tag{15}
\end{equation*}
$$

The complex potential in the real plane is

$$
\begin{equation*}
W(\sigma)=\Phi+i \Psi \tag{16}
\end{equation*}
$$

and the corresponding velocity at $\Gamma_{\mathrm{m}}$ is

$$
\begin{equation*}
v_{m}-i w_{m}=\frac{d w_{m}(\sigma)}{d \sigma}=\left.\frac{d}{d v}\left[w_{m}(\sigma)\right] \frac{d v}{d \sigma}\right|_{\substack{\sigma=\sigma_{m} \\ v=v_{m}^{m}}} \tag{17}
\end{equation*}
$$

The complex potential of $\Gamma_{m}$ is not included in Equation (17) to avoid the singularity at that point. The derivative of the transformation is obtained from Equation (4) or Equation (10).

The total velocity at $\Gamma_{\mathrm{m}}$ in the crossflow plane is written as

$$
\begin{equation*}
\frac{v_{m}-i w_{m}}{V_{\infty}}=G_{\alpha}+G_{\beta}+G_{n}+G_{m}+G_{T}+G_{r} \tag{18}
\end{equation*}
$$

where each term in Equation (18) represents a specific velocity component in the $\sigma$-plane. The first term represents the uniform flow due to angle of attack.

$$
\begin{equation*}
G_{\alpha}=-\left.i \sin \alpha\left[1+\left(\frac{r_{0}}{v_{m}}\right)^{2}\right] \frac{d v}{d \sigma}\right|_{\sigma=\sigma_{m}} \tag{19}
\end{equation*}
$$

The second term represents the uniform flow due to angle of yaw.

$$
\begin{equation*}
G_{B}=-\left.\sin \beta\left[1-\left(\frac{r_{0}}{v_{m}}\right)^{2}\right] \frac{\partial \nu}{\partial \sigma}\right|_{\sigma=\sigma_{m}} \tag{20}
\end{equation*}
$$

The next term represents the influence of all vortices and their images, with the exception of $\Gamma_{m}$.

$$
\begin{align*}
G_{n}=i & \sum_{n=1}^{N} \frac{\Gamma_{n}}{2 \pi r_{o} v_{\infty}}\left[\frac{1}{\left(\nu_{m} / r_{0}\right)-\left(r_{0} / \bar{v}_{m}\right)}\right. \\
& \left.-\frac{1}{\left(v_{m} / r_{0}\right)-\left(v_{n} / r_{0}\right)}\right]\left.\left.\right|_{n \neq m} \frac{d v}{d \sigma}\right|_{\sigma=\sigma_{m}} \tag{21}
\end{align*}
$$

The next term is due to the image of $\Gamma_{m}$.

$$
\begin{equation*}
G_{m}=\left.i \frac{r_{m}}{2 \pi r_{0} V_{\infty}}\left[\frac{1}{\left(\nu_{m} / r_{0}\right)-\left(r_{0} / \bar{\nu}_{m}\right)}\right] \frac{d v}{d \sigma}\right|_{\sigma=\sigma_{m}} \tag{22}
\end{equation*}
$$

The fifth term in Equation (18) represents the potential of $\Gamma_{m}$ in the $\sigma-p l a n e$ and is written as

$$
\begin{equation*}
G_{T}=-\left.i \frac{\Gamma_{m}}{2 \pi V_{\infty}}\left(\frac{1}{2}\right) \frac{d}{d \nu}\left(\frac{d \nu}{d \sigma}\right)\right|_{\sigma=\sigma_{m}} \tag{23}
\end{equation*}
$$

The last term in Equation (18) represents the velocity components induced by the portion of the body panel singularities representing the body volume effects.

$$
\begin{equation*}
G_{r}=\frac{v_{r}-i w_{r}}{v_{\infty}} \tag{24}
\end{equation*}
$$

Since the body panel singularities are three dimensional, they contribute an induced axial velocity, $u_{r}$.

The differential equations of motion for $\Gamma_{m}$ are

$$
\begin{equation*}
\frac{d \bar{\sigma}_{m}}{d x}=\frac{v_{m}-i w_{m}}{v_{\infty} \cos \alpha_{c}+u_{r}} \tag{25}
\end{equation*}
$$

where

$$
\begin{equation*}
\bar{\sigma}_{\mathrm{m}}=y_{\mathrm{m}}-i z_{\mathrm{m}} \tag{26}
\end{equation*}
$$

Therefore, the two equations which must be integrated along the body length to determine the trajectory of $\Gamma_{m}$ are

$$
\begin{equation*}
\frac{d y_{m}}{d x}=\frac{v_{m}}{v_{\infty} \cos \alpha_{c}+u_{r}} \tag{27}
\end{equation*}
$$

and

$$
\begin{equation*}
\frac{d z_{m}}{d x}=\frac{w_{m}}{v_{\infty} \cos \alpha_{c}+u_{r}} \tag{28}
\end{equation*}
$$

There are a pair of equations like (27) and (28) for each vortex in the field. As new vortices are shed, the total number of equations to solve increases by two for each added vortex. These differential equations are solved numerically using a method which automatically adusts the step size to provide the specified accuracy.

Surface pressure distribution.- The surface pressure distribution on the body is required to calculate the forces on the body and the separation points. The surface pressure coefficient is determined from the Bernoulli equation in the form

$$
\begin{equation*}
c_{p}=\frac{2}{\gamma M_{\infty}^{2}}\left\{\left[1+\frac{\gamma-1}{2} M_{\infty}^{2}\left(C_{P_{I}}\right)\right]^{\frac{\gamma}{\gamma-1}}-1\right\} \tag{29}
\end{equation*}
$$

where

$$
\begin{equation*}
c_{p}=\frac{p-p_{\infty}}{\frac{1}{2} \rho v_{\infty}^{2}} \tag{30}
\end{equation*}
$$

and

$$
\begin{equation*}
c_{p_{I}}=I-\left(\frac{\mathrm{U}}{\mathrm{~V}_{\infty}}\right)^{2}-\frac{2 \cos \alpha_{c}}{V_{\infty}} \frac{\mathrm{d} \phi}{\mathrm{dx}} \tag{31}
\end{equation*}
$$

In Equation (31), $U$ is the total velocity (including $V_{\infty}$ ) at a point on the body. It consists of many of the components described in the previous section. The body induced velocity components from Equation (24) include the total effect of the panel solution at angle of attack and roll. As a consequence of using the full panel solution in this component, the twodimensional doublet part of Equations (19) and (20) are not required. The vortex wake contribution is included through Equations (21) and (22).

The last term in Equation (31) represents the axial velocity missing from the two-dimensional singularities in the flow model. In this case, the shed vortices are the only singularities contributing to this term. The complex potential of a number of vortices, $\Gamma_{n}$, and their images in the circle plane is

$$
\begin{align*}
W(v) & =\phi+i \Psi \\
& =\sum_{n+1}^{N} \frac{\Gamma_{n}}{2 \pi}\left[-i \ln \left(v-v_{n}\right)+i \ln \left(v-\frac{r_{0}^{2}}{\bar{v}_{n}}\right)\right] \tag{32}
\end{align*}
$$

The "unsteady" term in Equation (31), evaluated on the body surface, is

$$
\begin{equation*}
\frac{d \phi}{d x}=\left.\operatorname{Real} \frac{d W(v)}{d x}\right|_{r=r_{0}} \tag{33}
\end{equation*}
$$

Equation (33) becomes

$$
\begin{align*}
\frac{d \phi}{d x} & =\sum_{n=1}^{N} \frac{\Gamma_{n}}{2 \pi}\left\{-\left[\frac{\left(\tau-\tau_{n}\right) \frac{d \lambda_{n}}{d x}-\left(\lambda-\lambda_{n}\right) \frac{d \tau_{n}}{d x}}{\left(\tau-\tau_{n}\right)^{2}+\left(\lambda-\lambda_{n}\right)^{2}}\right]\right. \\
& -\left[\frac{\left(\tau r_{n}^{2}-\tau_{n} r_{0}^{2}\right)\left(2 \lambda \tau_{n} \frac{d \tau_{n}}{d x}+2 \lambda \lambda_{n} \frac{d \lambda_{n}}{d x}-r_{0}^{2} \frac{d \lambda_{n}}{d x}-2 r_{0} \lambda_{n} \frac{d r_{0}}{d x}\right.}{\left(\tau r_{n}^{2}-\tau_{n} r_{0}^{2}\right)^{2}+\left(\lambda r_{n}^{2}-\lambda_{n} r_{0}^{2}\right)^{2}}\right] \\
& \left.+\left[\frac{\left(\lambda r_{n}^{2}-\lambda_{n} r_{0}^{2}\right)\left(2 \tau \lambda_{n} \frac{d \lambda_{n}}{d x}+2 \tau \tau_{n} \frac{d \tau_{n}}{d x}-r_{0}^{2} \frac{d \tau_{n}}{d x}-2 r_{0} \tau_{n} \frac{d r_{0}}{d x}\right)}{\left(\tau r_{n}^{2}-\tau r_{0}^{2}\right)^{2}+\left(\lambda r_{n}^{2}-\lambda_{n} r_{0}^{2}\right)^{2}}\right]\right\} \tag{34}
\end{align*}
$$

where

$$
\begin{equation*}
r_{n}^{2}=\tau_{n}^{2}+\lambda_{n}^{2} \tag{35}
\end{equation*}
$$

Separated wake.- The separated wake on the lee side of the body is made up of a large number of discrete vortices, each vortex originating from one of two possible locations at each time step in the calculation. The major portion of the lee side vortex wake has its origin at the primary separation points on each side of the body. The remainder of the wake originates from the secondary separation points located in the reverse flow region on the lee side of the
body. Both of these points are illustrated in the sketch of a typical crossflow plane of an elliptic cross section body shown in Figure 3. The mechanics of the calculation of the individual vortices follows.

As described in References 1 and 2, the pressure distribution for the primary flow in the crossflow plane is referenced to the conditions at the minimum pressure point, and a virtual origin for the beginning of the boundary layer is assumed. The adverse pressure distribution downstream of the minimum pressure point is considered with either Stratford's laminar (Ref. l3) or turbulent (Ref. 14) separation criterion to determine whether or not separation has occurred. These criteria, based on two-dimensional, incompressible, flat plate data, are adjusted for threedimensional crossflow effects in Reference 2. The laminar separation criterion states that the laminar boundary layer separates when

$$
\begin{equation*}
c_{p}^{1 / 2}\left[\xi \frac{d c_{p}}{d \xi}\right] \simeq 0.087 \sin \alpha_{c} \tag{36}
\end{equation*}
$$

In a turbulent boundary layer, separation occurs when

$$
\begin{equation*}
c_{p}\left[\xi \frac{d c_{p}}{d \xi}\right]^{1 / 2}\left(\operatorname{Re}_{\xi} \times 10^{-6}\right)^{-0.1} \simeq 0.35 \sin \alpha_{c} \tag{37}
\end{equation*}
$$

If the criteria indicate a separation point, the vorticity flux across the boundary layer at separation is shed into a single point vortex whose strength is

$$
\begin{equation*}
\frac{\Gamma}{v_{\infty}}=\frac{u_{e}^{2}}{2 v_{\infty}^{2}} \frac{\Delta x}{\cos \alpha_{c}} \tag{38}
\end{equation*}
$$

assuming no slip at the wall.

The initial position of the shed vortex is determined such that the surface velocity at the separation point is exactly canceled by the shed vortex and its image. When this criterion results in a vortex initial position that is too near to the body surface, certain numerical problems can cause difficulty in calculating the trajectory of this vortex. If the intial position of the vortex is nearer than five percent of the body radius from the surface, the vortex is placed five percent of the equivalent body radius. from the surface. This position represents the approximate thickness of the boundary layer.

The calculation of secondary separation is carried out in the same manner as described above for primary separation. It is necessary that a reverse flow region exist on the lee side of the body and that a second minimum pressure point be found in this region. Surface flow visualization of secondary separation regions on bodies at high angles of attack in subsonic flow are shown in Reference 15. For purposes of this analysis, the reverse flow is assumed to be laminar from the lee side stagnation point to the secondary separation point, and Stratford's laminar criterion is used to locate the secondary separation point. Laminar separation in the reverse flow region is expected because of the low velocities on this portion of the body. The vortex released into the flow at the secondary separation point has the opposite sign of the primary vortex and is generally much weaker in strength.

Forces and moments.- The forces and moments on the body are computed by integration of the pressure distribution around the body. At a specified station on the body, the normal-force coefficient on a $\Delta x$ length of the body is

$$
\begin{equation*}
c_{n}=\frac{\left(\frac{\Delta n}{\Delta x}\right)}{q_{\infty} D}=\frac{1}{D} \int_{0}^{2 \pi} c_{p} r^{\prime} \cos \beta^{\prime} d \beta \tag{39}
\end{equation*}
$$

where $r^{\prime}$ is the distance from the axis of the body to the body surface and $\beta^{\prime}$ is the local slope of the body in the crossflow plane. This is illustrated in the sketch in Figure 4. For circular bodies, $r^{\prime}=r_{0}$ and $B^{\prime}=\beta$. The total normal force coefficient on the body is

$$
\begin{equation*}
c_{N}=\frac{N}{q_{\infty} S}=\frac{D}{S} \int_{0}^{L} c_{n} d x \tag{40}
\end{equation*}
$$

and the pitching-moment coefficient is

$$
\begin{equation*}
c_{m}=\frac{M_{y}}{q_{\infty} s l_{r e f}}=\frac{D}{s} \int_{0}^{L} c_{n}\left(\frac{x_{m}-x}{\ell_{r e f}}\right) d x \tag{41}
\end{equation*}
$$

The center of pressure of the normal force is

$$
\begin{equation*}
\frac{x_{c p_{n}}}{l_{\text {ref }}}=\frac{x_{m}}{l_{\text {ref }}}-\frac{c_{m}}{c_{N}} \tag{42}
\end{equation*}
$$

$$
\text { Similarly, the side-force coefficient on a } \Delta x \text { length }
$$ of the body is

$$
\begin{equation*}
c_{y}=\frac{\left(\frac{\Delta s}{\Delta x}\right)}{q_{\infty} D}=-\frac{1}{D} \int_{0}^{2 \pi} C_{p^{\prime}} r^{\prime} \sin B^{\prime} d B \tag{43}
\end{equation*}
$$

The total side-force coefficient on the body is

$$
\begin{equation*}
c_{y}=\frac{F_{y}}{q_{\infty} S}=\frac{D}{S} \int_{0}^{I} c_{y} d x \tag{44}
\end{equation*}
$$

and the yawing-moment coefficient is

$$
\begin{equation*}
c_{n}=\frac{M_{z}}{q_{\infty} S \ell_{r e f}}=-\frac{D}{S} \int_{0}^{L} c_{y}\left(\frac{x_{m}-x}{\ell_{\text {ref }}}\right) d x \tag{45}
\end{equation*}
$$

The center of pressure of the side force is

$$
\begin{equation*}
\frac{x_{c p_{y}}}{\ell_{r e f}}=\frac{c_{n}}{C_{y}}+\frac{x_{m}}{l_{r e f}} \tag{46}
\end{equation*}
$$

A noncircular body at an arbitrary roll angle may experience a rolling moment caused by the nonsymmetry of the loading around the body. The total rolling moment on the body is calculated by summing the moments of the individual components of normal force and side force around each cross section and integrating over the body length. Using Equations (39) and (43), the rolling moment coefficient is calculated as
$c_{\ell}=\frac{M_{x}}{q_{\infty} S \ell_{\text {ref }}}$

$$
\begin{equation*}
=\frac{1}{S \ell_{r e f}} \int_{0}^{L}\left[\int_{0}^{2 \pi}\left(y C_{p} r^{\prime} \cos \beta^{\prime}-z C_{p^{\prime}} r^{\prime} \sin \beta^{\prime}\right) d \beta\right] d x \tag{47}
\end{equation*}
$$

Vortex smoothing.- A problem common to discrete vortex models is the singularity in the vortex induced velocity distribution at the loction of a point vortex and the very large velocities induced near the vortex. This can lead to errors in the surface pressure distribution when there are vortices near the body surface and to errors in the vortex tracking when vortices pass in close proximity.

The vortex diffusion model described in Reference 2 has the effect of removing the singularity at the vortex, but very large induced velocities are still in evidence near the vortex, thus the noted roughness in the surface pressure distribution continues to be a problem. Several solutions to this problem have been investigated, and a discussion of these vortex flow models follows.

In Reference 16 , discrete vortices are replaced by a distribution of vortices on straight line segments connecting the discrete vortex positions. In this way, the algebraic singularity is reduced to a logarithmic singularity. The strength of the uniform distribution of vorticity on each line segment is determined from the strengths of the discrete vortices at the ends of the segment. Defining $G_{d}$ as the complex velocity induced at $v_{m}$ by the line segment, analogous to Equation (2l), the velocity becomes

$$
\begin{equation*}
G_{d_{j}}=\frac{i \Gamma_{n^{\prime}}}{2 \pi v_{\infty}}\left(\frac{\bar{v}_{n+1}-\bar{v}_{n}}{\left|v_{n+1}-v_{n}\right|}\right) \log \left(\frac{v_{n+1}-v_{m}}{v_{n}-v_{m}}\right) \tag{48}
\end{equation*}
$$

where $\nu_{m}$ is not on the line segment. The strength of the vortex segment is

$$
\begin{equation*}
\Gamma_{n},=\frac{1}{2}\left(\Gamma_{n}+\Gamma_{n+1}\right) \tag{49}
\end{equation*}
$$

except when $\Gamma_{n}$ or $\Gamma_{n+1}$ represents the last vortex in the chain. When this occurs, the remainder of the strength of the last vortex is included in the last line segment to avoid losing a portion of the shed vorticity.

This smoothing procedure was investigated for a number of calculations, and it accomplished the smoothing of the induced velocity field to a great extent. The surface pressure distributions had some residual roughness caused
by other effects, but the pressure spikes caused by locally high vortex-induced velocities were eliminated. The major disadvantage of the above smoothing technique is cost. The nature of the total calculation procedure is such that many velocity calculations are required; therefore, the additional effort to set up the smoothing procedure and calculate velocities at numerous body points causes a dramatic increase in computation time. For this reason, alternate smoothing techniques are considered.

The smoothing technique incorporated into the present version of NOZVTX is one in which a single point vortex is represented by a simple distribution of several point vortices. The smoothing calculation is carried out only for vortices which lie within $10 \%$ of the local body radius of the surface of the body, and it is used only for calculating pressures on the body surface. The vorticity is distributed according to the procedure shown in Figures 5 (a) and (b). In Figure $5(a)$, vortex 2 is the only vortex which lies within $10 \%$ of the local body radius of the surface. The "distributed vorticity" model consists of nine smaller vortices; one located at the actual position of vortex 2 , four located along the line joining vortices 1 and 2 , and four along the line joining vortices 2 and 3. Each vortex has a strength equal to one-ninth of the actual vortex strength. Figure $5(b)$ shows distributed vorticity models for vortices which are end points of the vortex cloud; that is, for the first vortex and the most recently shed vortex. In the case of the first shed vortex, $\Gamma_{1}$, the distributed vorticity model consists of five point vortices, each vortex having a strength equal to one-fifth that of the actual vortex. The model for the most recent shed vortex,
$\Gamma_{3}$, is shown in Figure $5(b)$. The vorticity is distributed along two separate lines. One line is defined by the actual vortex position and its shed position at the previous axial station, indicated by $3^{\prime}$ in Figure $5(b)$. The second line is defined as before, the end points being the locations of $\Gamma_{2}$ and $\Gamma_{3}$.

The smoothing technique described above was investigated for a number of geometry and flow condition combinations, and it was found to smooth pressure spikes caused by locally high vortex-induced velocities or values of $d \phi / d x$. As expected, use of the smoothing technique increases execution time, but the actual execution time is determined by the number of vortices which lie close to the body surface. This technique redistributes the vortices and tends to spread their effect over a larger area, but the model is still made up of discrete vortices, each of which has a singularity at its origin. A viscous core model for the individual vortices is required to remove the singularity in the induced velocity calculation. This model is described in the following section.

Vortex core.- The diffusion core model (Ref. 2) for the point-vortex induced velocities removes the singularity at the vortex origin and effectively reduces the velocities near the vortex. The tangential velocity induced by a single point vortex is written as

$$
\begin{equation*}
\frac{v_{\theta}}{\Gamma}=\frac{1}{2 \pi r}\left[1-\exp \left(\frac{r^{2} v_{\infty}}{4 x v}\right)\right] \tag{50}
\end{equation*}
$$

where $r$ is the distance from the vortex to the field point and $x$ is a measure of the age of the vortex. The induced velocity from this core model is illustrated in the following sketch.


Vortex Induced Velocity

The vortex core model represented by Equation (50) has received considerable attention in the context of body vortex induced effects, and it has a number of shortcomings. Since the exponential term is a function of $r$, the flow medium ( $v$ ), and the age of the vortex ( $x$ ), the core is constantly changing size as the vortex moves through the field. Under certain conditions, the core radius, denoted as $r_{c}$ in the sketch, can become very small and the induced
velocity becomes unrealistically large. In an attempt to further modify the core model to keep the induced velocities within physically realistic limits, the following modification was made. The location of the maximum induced velocity, $r_{c}$ in the sketch, is fixed at a specific radius to be selected by the user. Given a core radius, the vortex induced velocity is

$$
\begin{equation*}
\frac{v_{\theta}}{\Gamma}=\frac{1}{2 \pi r}\left[1-\exp \left[-1.256 \frac{r^{2}}{r_{c}^{2}}[]\right.\right. \tag{51}
\end{equation*}
$$

The core model described by Equation (51) is included in the version of the code described in this report, and in a later section, guidelines for the selection of an appropriate core radius are presented. Results indicate that this simple core model provides adaquate smoothing for the necessary velocity calculations.

Other investigators of discrete vortex models must use a core model of some type. Although there are many different core models, all serve the same purpose as those described above, and nearly all are directed at eliminating the singularity and reducing the maximum induced velocity. The code described in a later section is easily modified to incorporate another core model if the user desires to do so.

## PROGRAM

## General Description

The computer code, NOZVTX, presented in this section has application as an engineering prediction method for circular and noncircular bodies at high angles of attack in supersonic flow. The code is in effect a combination of
two separate codes. The first is the subsonic body vortex shedding method described in References 1 and 2, and the second is the supersonic body model for bodies alone described in References 10,11, and 12. The combined code provides a means to predict the nonlinear aerodynamic characteristics and flow fields for bodies with circular and noncircular cross sections. The effect of fins is not included.

The code consists of a main program, NOZVTX, and 50 subroutines. The overall flow map of the program is shown in Figure 6 where the general relationship between the subroutines and external references can be seen. An alphabetical listing of the subroutines is shown in Table I. It was an objective of this effort to keep the two codes as distinct as possible to maintain their individual identity so that modifications to either code could be carried out without major changes in the other code. Note that the main subroutine of the body vortex shedding calculation, ELNOSE, includes the body panel code via the calls to subroutines GEOM, VELCMP, and SOLVE. The line singularity model for axisymmetric bodies is included through a call to subroutine BDYGEN. The communication of information between segments of the program is handled by name common blocks. A cross reference table for the calling relationship between the program subroutines and the external references is shown in Figure 7. A similar table for the common blocks is shown in Figure 8.

The program is written in standard FORTRAN language. Operation requires four (4) auxiliary files in addition to the usual input and output units. Core storage for execution is approximately 230,000 octal on a CDC CYBER 760 computer. Execution times on the same computer can vary from 10 to 400 or more seconds. Execution time depends on many factors such as the number of panels required for the
body, the integration interval (DX), the number of shed vortices, the use of symmetry, and even the flow conditions.

## Limitations

Program NOZVTX is applicable to bodies alone at high angles of attack in supersonic flow, with some limitations. Certain of the limitations to follow are basically rules-ofthumb for the use of the program that have evolved through experience with the program and verification of the program by comparison with data. The range of applicability of the program may be extended by additional verification with appropriate data.

There is a Mach number/geometry limitation in the body model that requires the body surface to lie behind the Mach cone. This has greatest effect at high Mach numbers or on blunt noses at low Mach numbers, and a message is printed if the geometry condition is violated. It has been the experience of the authors that, when this condition arises, the nose can generally be made slightly more pointed so that it lies within the Mach cone. Often this modification is required only on the first few percent of body length, and the effect on the total nose forces and moments and shed vortex field is minimal.

An associated Mach number limitation involves the quality of the predicted surface pressure distribution at high Mach numbers. As will be shown in comparisons of experiment and theory, the predicted pressure coefficients on the windward side are much lower than those measured at body points which are near to the shock wave. This is a common problem for linear body models like the axis singularities and the panels considered in NOZVTX, and it is possible that the windward pressure can be corrected by use of nonlinear pressure prediction techniques.

Another less rigid limitation involves the Mach number/ angle of attack range. It is suggested that the crossflow Mach number ( $M_{\infty} \sin \alpha_{c}$ ) not exceed unity. A stricter limitation is that the crossflow Mach number should not exceed the critical value; that is, the local Mach number in the crossflow plane should not exceed unity at any point in the plane. The root of this limit is the vortex shedding calculation which uses an incompressible vortex model and an incompressible separation criterion. Also, when the crossflow Mach number exceeds the critical value, a complex shock near the shoulder may contribute to separation. It is important that the user be aware of these limits, particularly when the program is being used in that range. However, the author has used the program for flow conditions in which the crossflow Mach number slightly exceeded unity, and the results were quite resonable. Some of these results are presented in a later section.

## Error Messages and STOPS

The program code has numerous internal error messages. These are generally explicit and they will be described below only if an explanation will provide some insight into the problem and its correction. There are several execution terminations at numbered STOP locations within the program. These are described in the table below along with some suggestions to correct the problem.

The most common message is "CONVERGENCE NOT OBTAINED IN INTEGRATION." This message, along with the axial station, $X I$, and the integration interval, $H$, is printed when the trajectory calculation does not converge within the specified tolerance, E 5 . One message will terminate execution at STOP 40 described in the table to follow.

The program contains several error messages pertaining to the number of panels used to describe the body. The message "ERROR - NUMBER OF ROWS OF PANELS IN BODY SECTION 1 EXCEEDS $30^{\prime \prime}$ is self explanatory and results in STOP 20. A similar message "ERROR - NUMBER OF ROWS OF SINGULARITY PANELS ON BODY EXCEEDS $30^{\prime \prime}$ results in STOP 50. The message "ERROR - NUMBER OF BODY PANELS EXCEEDS 600" results in STOP 60. The message "ERROR - BODY HAS MORE THAT 20 CIRCUMFERENTIAL PANELS" refers to the number of panels describing the body cross section circumference. This error results in STOP 200. All of the errors associated with the body panels are caused by incorrect values being input in items 32 and 41 of the input list.

If a pivot element of the matrix of panel influence coefficients is zero, the message "ERROR - THE MATRIX IS SINGULAR" is printed and the program terminates execution at STOP 300. This is a very uncommon error and is not seen for normal body panel arrangements.

As mentioned in a previous section, the slope of any panel on a noncircular body must be less than the Mach angle. If a panel has a greater slope, the message "ERROR - BODY PANEL SLOPE EXCEEDS MACH ANGLE" is printed and execution terminates at STOP 210. A modification of the body geometry to decrease the panel slopes is required to correct this situation.

The surface of an axisymmetric body must lie inside the Mach cone from the nose tip. If certain body points lie outside the Mach cone, subroutine BDYGEN will make an effort to correct the definition of the body and revise the radius distribution. If the Mach number is too high to correct the body within its specified length, the message "BODY RADIUS TOO LARGE IN RELATION TO BODY LENGTH" is printed and execution terminates at a STOP. If, in the process of
correcting the body points outside the Mach cone, the subroutine cannot locate an appropriate point, the message "MACH CONE - BODY MERIDIAN INTERSECTION NOT FOUND AFTER 100 TRIALS" is printed and execution terminates at a STOP. These difficulties are corrected by modifying the body shape or lowering the Mach number of the run.

The numerical conformal mapping procedure can generate several error messages from subroutine CONFOR. The message "***ERROR IN SUM OF EXTERIOR ANGLES***" is printed if the routine has difficulty calculating appropriate body angles within a specified tolerance. Another message "***COEFFICIENT AN $(I)=\ldots$ GREATER THAN +- . 001 AFTER __ ITERATIONS OUT OF __" is printed on rare occasions when the mapping procedure is having difficulty converging on a solution. Both of the above messages are informational, and they do not result in a program STOP. However, the user is advised to check the transformed shape carefully with the actual shape before using the results. If there is a difficulty in the numerical mapping calculation, refer to the Input Preparation section for assistance.

Numbered STOP Locations

Termination
Message

## Description

STOP Normal program stop or the axisymmetric body lies inside the Mach cone from the nose tip. More than 30 rows of panels on body. Revise input values of NFORX and KFORX in input items 32 and 41, respectively.

STOP $40 \quad$ Vortex trajectory integration has failed to converge at an axial station. This problem is usually caused by (I) two or more vortices in close proximity such that they are rotating
about each other so rapidly that the integration subroutine cannot converge on a solution, or (2) a vortex too near the body surface. These situations are often dependent on the body shape and the specific flow conditions, but the following steps should be tried to resolve the problem. Increase the integration tolerance E5, or increase the size of the combination parameter RGAM so that it is greater than the distance between the troublesome vortices. It often helps to increase the integration interval DX and restart the calculation from a previous station.

The vortex trajectory calculation has resulted in a vortex inside the body. This is a rare, but serious, problem; and it signifies a major problem with the solution. It usually means that the body paneling scheme is in error. If no obvious problem exists, change the paneling layout to increase the number of circumferential and axial panels.

Total number of shed vortices exceed 200. Combine some vortices and restart the calculation and run to completion. An alternate solution is to increase the integration interval DX so that fewer vortices are shed.

The number of positive or negative primary separation vortices exceeds 70. Use the same remedy as for STOP 44.

The number of positive or negative secondary or reverse flow separation vortices exceeds 30 . Combine the secondary vortices to reduce their total number and restart the calculation;
increase DX to reduce numbers of vortices shed, or set $N S E P R=0$ and eliminate the secondary vortices from the calculation.

STOP 50
More than 30 rows of panels on body. See STOP 20.

STOP 60

STOP 200

STOP 202

STOP 210

STOP 300
More than 600 panels on body. Revise requested number of panels in input items 32 and 41.

More than 20 panels on the body circumference. Revise values of NRADX and KRADX in input items 32 and 41 , respectively.

There is an error in the vortex tables in subroutine VTABLE. Check input values of NBLSEP, NSEPR, NVP, NVM, NVA, NVR, and RGAM in items 1 and 6.

A body panel has a slope greater than the Mach angle.

There is a singularity in the panel influence coefficient matrix. Check panel layout geometry for errors.

An additional comment is required here about a situation which can occur which results in a fatal execution error and termination at a TIME IIMIT with no other message. This is not a usual occurrence, but it has been observed in certain special cases. The typical case is near the end of a very long body from which a large number of vortices have been shed. The difficulty is that two differential equations of motion must be integrated for each vortex and the derivatives involve all vortices in the field. Solution by a KuttaMerson scheme can involve many derivative calculations. The minimum time required for each axial step in the calculation is proportional to the square of the total number of
vortices. As the number of vortices increases, convergence becomes more difficult and the time for each step can increase dramatically. If this situation should occur, the best approach is to combine adjacent vortices and restart the calculation with a larger convergence criterion E5. It is sometimes useful to increase the axial interval DX.

## Input Description

The purpose of this section is to describe the input to program NOZVTX. The input formats are shown in Figure 9, and the definitions of the individual variables are provided in the following paragraphs. The input to the body panel and axis singularity portion of the program is maintained nearly identical to the input requirements for program WDYBDY of Reference l0. The purpose for this was to minimize changes in WDYBDY; however, it leads to some redundancy in the preparation. The small amount of redundancy is acceptable in light of the simplification in the use of the current program. If there is a question about the input in the panel portion of the program, the user is referred to Appendix $K$ of Reference 10 for additional information.

The remainder of this section includes a description of each of the input variables and indices required for the use of program NOZVTX. The order follows that shown in Figure 9 where the input format for each item and the location of each variable on each card is presented. Data input in I-format are right justified in the fields, data input in F-format may be placed anywhere in the field and must include a decimal point, and data input in E-format are right justified in the fields and must include the decimal point and the complete exponent designation.

Note that all length parameters in the input list are dimensional variables; therefore, special care must be taken that all lengths and areas are input in a consistent set of units.

Item 1 is a single card containing sixteen integers, each right justified in a five-column field. The variables are defined as follows.

Item

| Variable | Description |
| :---: | :---: |
| NCIR | ```Cross section shape index. =0, circular cross section =l, elliptic cross section =2, arbitrary cross section``` |
| NCF | Numerical transformation index. $=0$, mapping coefficients are calculated based on body shape input =1, mapping coefficients are input by user in items 18 through 22 |
| ISYM | ```Symmetry index. =0, right-left flow symmetry =l, no symmetry (required if \phi }\not=\mp@subsup{0}{}{\circ}\mathrm{ )``` |
| NBLSEP | ```Body vortex separation index. =0, no separation (required if 的 = 00 in item 5) =1, laminar separation =2, turbulent separation``` |
| NSEPR | ```Reverse flow separation index. =0, no separation =l, laminar separation in reverse flow region``` |

NSMOTH Vortex induced velocity smoothing index. $=0$, no smoothing
$=1$, smoothing of vortex-induced velocities used in pressure calculation
$=2$, smoothing of vortex-induced velocities used in pressure calculation and smoothing of $\partial \phi / \partial x$ calculation

NDFUS Vortex core model index.
$=0$, potential vortex
$=1$, diffusion core model; preferred (see RCORE in item 5)

NDPHI Unsteady pressure term index.
$=0$, omit $\partial \phi / \partial x$ from $C_{p}$ calculation
$=1$, include $\partial \phi / \partial x$ term; preferred
INP

NXFV

NFV

NVP
Nose force index.
$=0$, slender body theory force on portion
of nose ahead of starting point (XI)
$=1$, zero force on nose ahead of XI; preferred value for restart option

Number of x-stations at which flow field is calculated or special output generated. See item 23.
$0 \leq N X F V \leq 8$
Number of field points for flow field calculation. See item 24.

$$
0 \leq N F V \leq 200
$$

Number of $+\Gamma$ vortices on $+y$ side of body to be input for restart calculation. See item 26.

$$
0 \leq N V P \leq 70
$$

| NVR | ```Number of -\Gamma reverse flow vortices on +y side of body to be input for restart calculation. See item 27. 0 < NVR < 30``` |
| :---: | :---: |
| NVM | Number of $-\Gamma$ vortices on $-y$ side of body to be input for restart calculation. See item 28. $N V M=0$ if $I S Y M=0$. $0 \leq N V M \leq 70$ |
| NVA | ```Number of +\Gamma reverse flow vortices on - y side of body to be input for restart calculation. See item 29. NVA = 0 if ISYM = 0. 0\leqNVA \leq 30``` |
| NASYM | ```Asymmetric vortex shedding index. See item 7. =0, no forced asymmetry; preferred =l, forced asymmetry``` |

Item 2 is a single card defining seven integer output option indices. Each index is right justified in a five column field. The indices are defined as follows.
NHEAD $\geq 1$
NPRNTP Pressure distribution print index.
$=0$, no pressure output except at special
x-stations specified by item 23
$=1$, pressure distribution output at each
x-station

| NPRNTS | Vortex separation print index. <br> $=0$, no output <br> =1, output at each x-station; preferred <br> $=2$, detailed separation calculation <br> output; for debugging purposes only |
| :---: | :---: |
| NPRNTV | ```Vortex cloud summary output index. =0, no vortex cloud output =l, vortex cloud output; preferred``` |
| NPLOTV | ```Vortex cloud printer-plot option. =0, no plot =l, plot full cross section on a constant scale =2, plot upper half cross section on a constant scale =3, plot full cross section on a variable scale; preferred``` |
| NPLOTA | ```Plot frequency index. =0, no plots =1, plot vortex cloud at x-stations specified by item 23 =2, plot vortex cloud at each x-station``` |
| NPRTVL | Velocity calculation auxiliary output for debugging purposes only (Sub. VLOCTY). <br> $=0$, no output; preferred <br> =1, print velocity components at field points; see items 23 and 24 <br> $=2$, print velocity components at body control points during pressure calculations. This option can produce massive quantities of output (not recommended). |

Item 3 is a series of cards containing hollerith information indentifying the run. The cards are reproduced in the output just as they are input.

Item
Variable
Description
3 TITLE NHEAD cards of identification. Information to be printed at top of output sheets.

Item 4 is a single card containing reference information used in forming aerodynamic coefficients. The variables are defined as follows.

Item
Variable
Description
4
REFS
Reference area. REFS > 0
REFL Reference length. REFL > 0
XM Moment center.
SI Body length (L).
SD Body maximum diameter, or equivalent noncircular body diameter (D). Used in definition of RE .

Item 5 is a single card containing the flow condition parameters. Each variable is input in a ten column field. The flow parameters are:

Item
Variable
Description
5 ALPHAC Angle of incidence, degrees.

$$
\begin{aligned}
& 0^{\circ} \leq \alpha_{c}<90^{\circ} \\
& \text { If } \alpha_{c}=0^{\circ}, \text { NBISEP }=0 \text { in item } 1 .
\end{aligned}
$$

PHI
RE Angle of roll, degrees ( $\phi$ ). Reynolds number ( $V_{\infty} D / v$ ).

RCORE Ratio of the local vortex core radius to the local body diameter. Default value is .025, and maximum allowable value permitted in the code is . $05 * \mathrm{SD} / \mathrm{I}_{0}$.
$\mathrm{XMACH} \quad$ Mach number, $\mathrm{M}_{\infty}$ :

Item 6 is a single card containing the specification of the axial extent of the run and certain parameters associated with the vortex wake. Typical values are shown with the variables.

| Item | Variable | Description |
| :---: | :---: | :---: |
| 6 | XI | Initial x-station. XI > 0 |
|  | XF | Final x-station. $\mathrm{XF} \geq \mathrm{XI}$ |
|  | DX | ```Increment in x for vortex shedding calcula- tion. Typical value, DX \approx D/2 when FDX = 0.0. For variable DX, set DX = 0.0 and FDX > 0.0.``` |
|  | EMKF | ```Minimum distance of shed vortex starting position from body surface. =1.0, vortices positioned such that separation point is a stagnation point in the crossflow plane >1.0, minimum radii away from body surface for shed vortices. Typical value, EMKF = 1.05``` |
|  | RGAM | Vortex combination factor. <br> $=0.0$, vortices not combined; preferred <br> $>0.0$, radial distance within which vortices are combined. Typical value, $\mathrm{RGAM}=0.05 \mathrm{D}$ |

VRF Vortex reduction factor to account for observed decrease in vortex strength. $=0.6$, for subsonic flow $=1.0$, for closed bodies, or supersonic flow: preferred

FDX Factor for variable x-increments. When FDX > 0.0, DX = FDX * (local body radius). $D X$ should be input as zero when this option is used.

Item 7 contains only one variable that is of general use in program NOZVTX, and that is the integration error tolerance, E5. The variable XTABL is for diagnostic purposes only. The next three variables concern the use of forced asymmetry for bodies at very high angles of attack. The variables are defined as follows.

Item 7 E5

XTABI

XASYMI

XASYMF

Description
Error tolerance for vortex trajectory calculation. (Relative error in vortex position.) Typical range, $\mathrm{E} 5=0.01$ to 0.05 .
$x$-location after which a table of corresponding points between the real body and the transformed circle is not printed. For diagnostic purposes on noncircular shapes. Typical value, XTABL $=0.0$.

Initial $x$-location at which forced asymmetry of separation points is used. Typical value, XASYMI $=0.0$.

Final x-location at which forced asymmetry of separation points is used. Typical value, XASYMF $=0.0$.

DBETA
Amount of forced asymmetry for separation points on body, degrees. Typical value, DBETA $=0.0$.

Items 8, 9, 10 , and 11 provide a table of geometry characteristics that must be input for all bodies, circular or noncircular in cross section.

| Item | Variable | Description |
| :---: | :---: | :---: |
| 8 | NXR | Number of entries in body geometry table (1 $\leq$ NXR $\leq 50$ ). |
| 9 | XR | x-stations for geometry table (NXR values, 8 per card). |
| 10 | R | $r_{0}$, circular body radius at x-stations, or radius of tranformed circle (NXR values, 8 per card). For an ellipse, $r_{0}=(a+b) / 2$ |
| 11 | DR | dro/dx, body slope of transformed shape at $x$-stations (NXR values, 8 per card). |
| Items 12 and 13 are included only if NCIR $=1$; that is, |  |  |
| for elliptical bodies. |  |  |
| Item | Variable | Description |
| 12 | AE | a, horizontal half-axis of elliptic cross section (NXR values, 8 per card). |
| 13 | BE | b, vertical half-axis of elliptic cross section (NXR values, 8 per card). |

Items 14 through 22 are included only if NCIR=2; that is, for a body with arbitrary cross section that must be handled with the numerical transformation and conformal mapping proceđures.

## Item

14

## Variable

MNFC Number of coefficients describing transformation of arbitrary body to a circle ( $1 \leq$ MNFC $\leq 25$ ).
MXFC Number of x-stations at which transformation coefficients are defined ( $1 \leq$ MXFC $\leq 10$ ). For a similar shape body at all cross sections, MXFC $=1$.

15 XFC

## Description

x-stations at which transformation coef- ficients are defined. For a similar cross section body, MXFC=1, XFC(1) $\leq X I$ (MXFC values, 8 per card).

Items 16 and 17 are included when NCF $=0$. This block of cards is repeated MXFC times, once for each x-station input in Item 15.

Item
16 NR

17

Variable

XRC (I,J) Coordinates of right-hand side of body. YRC (I,J) The convention for ordering the coordinates from bottom to top in a counter-clockwise fashion, as shown in Figure 10 , is observed. Right/left body symmetry is required in the numerical mapping. (NR cards with one set of coordinates per card.)

Items 18 through 22 are input in lieu of Items 16 and 17 when NCF > 0. The values of these variables must be obtained from a previous run of the numerical mapping portion of the code. The only purpose for providing the option to input the mapping parameters is to eliminate the need to recompute the numerical mapping in subsequent runs and save computation time. This set of items is repeated MXFC times, once for each x-station input in Item 15.

| Item | Variable | Description |
| :---: | :---: | :---: |
| 18 | $\begin{aligned} & \operatorname{ZZC}(J) \\ & \operatorname{RZC}(J) \end{aligned}$ | Axis shift and radius of mapped cross section (one card with two values). |
| 19 | NR | Number of sets of body coordinates at a particular axial station. |
| 20 | XRC, YRC | Coordinates of right side of body defining cross section starting at $\beta=0^{\circ}$ and ending at $\beta=180^{\circ}$ (NR cards with one set of coordinates per card). See Figure 10. |
| 21 | THC | Angle of defined body points in circle plane (NR values, 8 per card). |
| 22 | AFC | Mapping coefficients (MNFC values, 6 per card). |

Items 23 and 24 are included only if NXFV $>0$. Item 23 specifies the axial stations at which additional output or plots are requested. Item 24 is included only if NFV $>0$. Each card of this item contains the nondimensionalized $y, z$-coordinates of a field point at which the velocity field is calculated at each axial station specified in Item 23.

Item Variable Description
23 XFV
x-station at which field point velocities are calculated or at which additional output is printed (NXFV values).

Omit Item 24 if NFV $=0$.
$24 \mathrm{YFV}, \mathrm{ZFV} \quad \mathrm{y} / \mathrm{r}_{\mathrm{o}}, \mathrm{z} / \mathrm{r}_{0}$-coordinates of field points at which velocity field is calculated, expressed as a fraction of the local body radius (NFV cards with one set of coordinates per card). It is important that the field points lie outside the body surface.

Items 25, 26, 27, 28, and 29 are included for a restart calculation only, and the condition NVP + NVR + NVM + NVA $\neq 0$ must be satisfied. It is not necessary to input all four types of vorticity in a restart calculation.

Item 25 is a single card containing the nose force and moment coefficients at the restart point; that is, $\mathrm{X}=\mathrm{XI}$. These values may be set equal to zero, but the resulting forces and moments from the current calculation will be in error.

| Item | Variable | Description |
| :---: | :---: | :---: |
| 25 | CN | Normal-force coefficient. |
|  | CY | Side-force coefficient. |
|  | CM | Pitching-moment coefficient. |
|  | CR | Yawing-moment coefficient. |
|  | CSL | Rolling-moment coefficient. |

Item 26 is a block of NVP cards to specify the positive separation vorticity on the right side of the body. Omit this item if NVP $=0$. The variable XSHEDP associated with each vortex is used only to identify individual vortices and permit the user to follow individual vortices during the calculation.

| Item | Variable | Description |
| :---: | :---: | :---: |
| 26 | GAMP | $\Gamma / V_{\infty}$, positive separation vorticity on right side of body. |
|  | YP, ZP | coordinates of discrete vortices on right side of body at starting point (XI). |
|  | XSHEDP | x -location at which individual vortex was shed (may be 0.0). |

(Item 26 consists of NVP cards.)

Item 27 is a block of NVR cards to specify the secondary or additional vorticity on the right side of the body or at any other position in the field. This array is a convenient way to put an arbitrary cloud of additional vorticity which is to be maintained separately from the other body vorticity in the field. Omit this item if NVR $=0$.

Item
Variable

## Description

27 GAMR
$\Gamma / V_{\infty}$, reverse flow or additional vorticity on right side of body.
$\mathrm{XR}, \mathrm{ZR} \quad$ coordinates of discrete vortices on right side of body at starting point (XI).
x-location at which individual vortex was shed (may be 0.0).
(Item 27 consists of NVR cards.)

Item 28 specifies the negative separation vorticity on the left side of the body, analogous to Item 26. Omit this item if $N V M=0$ or if $I S Y M=0$. In the case of a symmetric flow field (ISYM=0), this block of vorticity is automatically defined as the mirror image of the positive vorticity input in Item 26.

| Item | Variable | Description |
| :---: | :---: | :---: |
| 28 | GAMM | $\Gamma / V_{\infty}$, negative separation vorticity on left side of body. |
|  | YM, ZM | Coordinates of discrete vortices on left side of body at starting point (XI). |
|  | XSHEDM | x-location at which individual vortex was shed (may be 0.0). |
| (Item | 8 consist | of NVM cards.) |

Item 29 specifies the secondary or additional vorticity on the left side of the body or at any other position in the field. This block of vorticity is analogous to Item 27, and it is omitted if NVA $=0$ or if ISYM $=0$. As with the previous item, this block of vorticity is automatically defined as the mirror image of the negative vorticity input in Item 27 if symmetry is required by ISYM $=0$.

| Item | Variable | Description |
| :---: | :---: | :---: |
| 29 | GAMA | $\Gamma / V_{\infty}$, reverse flow or additional vorticity on left side of body. |
|  | YA, ZA | Coordinates of discrete vortices on left side of body at starting point (XI). |
|  | XSHEDA | $x$-location at which individual vortex was shed (may be 0.0). |
| (Item | 29 consist | of NVA cards.) |

The above 29 items make up the body vortex shedding portion of the input deck.

Items 30 through 43 are associated with the paneling of bodies with noncircular cross sections and are taken from the input specifications for program WDYBDY (Ref. 10). Item 44 is used to describe bodies with circular cross sections. As described above, it was desired to keep the panel code input as unchanged as possible from the original WDYBDY code; however, the resulting input deck requires the specification of several variables over which the user has no control. The input deck considered herein contains only those variables which are optional to the user. Other nonoptional variables have been hard coded with their appropriate value, and these should not be changed by the user.

Omit Items 30 through 43 if NCIR $=0$.

| Item Variable | Description |
| :--- | :--- |
| 30 | TITLEl |
|  | One card of identification information to <br> be printed ahead of the paneling output. |

Item 31 is a single card containing two indices, each index is right justified in a three column field. The purpose of IXZSYM is to define the manner in which the body paneling is carried out for different flow conditions. A symmetric body in a symmetric flow $\left(\phi=0^{\circ}\right)$ can benefit from the IXZSYM $=0$ option, which panels only one half of the body and then utilizes the symmetry characteristics to reduce computation time. The same symmetric body in an asymmetric flow condition $\left(\phi \neq 0^{\circ}\right)$ must be paneled in total, IXZSYM $=1$, to pick up the nonsymmetry in the loading around the body. The body symmetry is used to reduce the input required as described later.

IXZSYM Panel symmetry option.
$=0$, panel symmetric half of body for symmetric flow ( $\phi=0^{\circ}$ )
$=1$, panel complete symmetric body using +y side geometry for a symmetric body in asymmetric flow ( $\phi \neq 0^{\circ}$ )
$=-1$, panel complete configuration using geometry of both sides

ITBSYM $\quad=0$, configuration has top/bottom symmetry
=l, no top/bottom symmetry (e.g., see Fig. 10)

Item 32 contains indices to specify the body geometry.
Item Variable

## Description

## 32 <br> J2

Body geometry specification option.

$$
=0 \text {, no body (not recommended) }
$$

=l, geometry for arbitrary shaped body
$=-1$, circular body defined by crosssectional area at XFUS stations (not recommended)
$=-2$, circular body defined by radius at XFUS stations
$=-3$, elliptic body defined by both semiaxes at XFUS stations

J 6
$=0$, cambered body - not available
$=1$, body symmetrical with respect to XY-plane (uncambered body)
$=-1$, circular or elliptic body with J2 $\neq 0$ (preferred value)

| NRADX | Number of points used to represent the body segment about the circumference. If the configuration is symmetric <br> (IXZSYM $=0,1$ ), NRADX is input for the half section. If the entire configuration is input (IXZSYM $=-1$ ), NRADX is input for the full section. The half section or full section is divided into NRADX-l equal angles, and the $y$ - and $z$-coordinates of the panel corner points are defined by the intersections of these meridian angles with the body surface ( $3 \leq$ NRADX $\leq 20$ ). <br> It is the experience of the authors that the best panel representation of the body is obtained when NRADX is an even number; that is, there should be an odd number of panels on the half-body. It is very important for the pressure calculation that there be a panel control point on the $B=90^{\circ}$ meridian. |
| :---: | :---: |
| NFORX | Number of axial body stations used to define geometry ( $2<\mathrm{NFORX}<30$ ). |

The next block of input specifies the geometry of the body for purposes of laying out the panels. This input is somewhat redundant with the input in Items 8 through 13; however, it is important that the paneling geometry be separate from the previous geometry but consistent with it. In many cases, the paneling geometry may be specified in a more coarse grid than that required by the separation portion of the program.

Item 33 is a block of cards containing the axial stations at which body geometry is input for paneling purposes. The values are input ten to a card in seven column fields with a decimal point required.

## Item Variable Description

33 XFUS x-stations at which body paneling geometry is input (NFORX values).

Item 34 is inlcuded only if $\mathrm{J} 2=-1$ (Item 32). The circular body cross-sectional areas are specified at the XFUS stations defined in Item 33.

Item
Variable
Description
34
FUSARD Circular body cross-sectional areas at XFUS stations (NFORX values).

Item 35 is included only if $\mathrm{J} 2=-2$ (Item 32). The circular body radii are specified at the XFUS stations defined in Item 33.

Item
Variable
Description
35 FUSRAD R, circular body radii at XFUS stations (NFORX values).

Items 36 and 37 are included only if $J 2=-3$ (Item 32). The elliptic body horizontal and vertical semi-axes are specified at the XFUS stations defined in Item 33.

| Item | Variable | Description <br> 36 |
| :--- | :--- | :--- |
|  | FUSBY | Elliptic body horizontal semi-axes at <br> XFUS stations (NFORX values). |
| 37 | FUSAZ | Elliptic body vertical semi-axes at |
|  |  | XFUS stations (NFORX values). |

Item 38 is included only if J2 $=1$ (Item 32). The y,z-coordinates of points on an arbitrary cross section body are specified in Item 38, and there is one set of Item 38 for each of the axial stations defined by XFUS in Item 33. The convention for ordering the coordinates from bottom to top is observed (e.g., Fig. l0). If the full cross section is to be specified (IXZSYM $=-1$, Item 31) the ordering continues counter-clockwise from the top of the body back to and including the bottom point to close the cross section.

## Item

Variable

## Description

38
YJ, ZJ
y,z-coordinates of arbitrary body at XFUS stations (NRADX cards).

Item 38 is repeated NFORXtimes, once for each XFUS station.

The following portion of the input deck is described as Auxiliary Input Cards in Appendix K of Reference 10. The purpose of this input is to provide a simple means to adjust the actual panel layout keeping the geometry specified above unchanged.

Item 39

Variable
TITLE2 One card of identification information for paneling geometry.

Item 40 contains an index to specify the output for the body panel portion of the program. Because of the quantity or output possible, it is recommended that $|I P R I N T| \leq 1$ be the highest level of output specified. Note that the values of the first two indices on this card are fixed.


If IPRINT < 0 , the panel geometry is included with the output specified by |IPRINT|. This option provides a useful means to check the paneling arrangement.

Item 41 is a single card containing two indices in three column fields. These indices provide a conventional means to change the panel layout without changing the geometry input.

Item
41 KRADX

## Description

Number of meridian lines used to define panel edges on the body segment (see note for NRADX in Item 32).

| KRADX $\quad=$ | meridian lines are defined by NRADX |
| ---: | :--- |
|  | in Item 32 |$\quad$| $>0$, | meridian lines are calculated at |
| ---: | :--- |
|  | KRADX equally spaced points |
|  | $(3 \leq$ KRADX $\leq 20)$ |
| $<0$, | meridian lines are calculated at |
|  | specified values of PHIK in Item 42 |
|  | $(3 \leq\|K R A D X\| \leq 20)$ |

For symmetric configurations (IXZSYM $=0,1$ ), KRADX is the number of meridians on the half section. For full configurations (IXZSYM $=-1$ ), KRADX is the number of merdians on the full section including meridians at $0^{\circ}$ and $360^{\circ}$.

KFORX Number of axial stations used to define leading and trailing edges of panels on the body.
$=0$, the panel edges are defined by NFORX and XFUS in Items 32 and 33 , respectively
$>0$, the panel edges are defined by XFUSK in Item $43(2 \leq K F O R X \leq 30)$

Item 42 is included if $K R A D X \leq-3$ in Item 41. This item provides a means to easily modify the panels by changing the body merdian angles which specify the panel corner points. The purpose of this option is to allow the user to pack additional panels into a region of the body which is changing rapidly.
Item Variable

## Description

42 PHIK
Body merdian angles (degrees) with PHIK $=0^{\circ}$ at the bottom of the body and PHIK $=180^{\circ}$ at the top (|KRADX| values).

Item 43 is included only if KFORX is nonzero. The variable XFUSK defines the axial positions of the panel edges. If this item is not included, the panels are specified by XFUS in Item 33. The purpose of this item is to simplify the changing of a panel layout without changing the entire geometry input deck.

| Item Variable | $\frac{\text { Description }}{}$Axial stations of leading and trailing |
| :--- | :--- |
|  | edges of body panels (KFORX values). |

Item 44 is included only if NCIR $=0$; that is, for bodies with circular cross sections.

Item Variable
44
NXBODY

NCODE Integer control index for specifying forebody shape over length of XNOSE.

$$
\begin{aligned}
\text { NCODE } & =0, \text { Parabolic } \\
& =1, \text { Sears-Haack } \\
& =2, \text { Tangent ogive } \\
& =3, \text { Ellipsoidal } \\
& =4, \text { Conical }
\end{aligned}
$$

XNOSE Length of nose part of body, measured from nose tip.

XLBODY Length of body.

This concludes the input deck for a single run of program NOZVTX. The program is not set up to stack input for multiple cases because of the sometimes long and generally unpredictable run times.

## Input Preparation

In this section, some additional information is provided to assist the user in the preparation of input for certain selected problems. The previous section on the input description must be used to understand the individual variables which go into NOZVTX, and this section will permit the user to select the appropriate input to get optimum results from the code.

Panel layouts.- The choice of panel arrangement to represent the noncircular body shape is perhaps the most important part of the input selection as the quality of the panel layout can determine the quality of the predicted results. Simply using the largest possible number of panels is one solution; however, this will not guarantee an optimum panel layout, nor will it guarantee the best results. In the following paragraphs, some guidelines are presented for various situations to optimize both results and computation time whenever possible.

The elliptic cross section will be considered first because the code has been checked out for a number of different elliptic bodies. In each case to follow, right/ left symmetry of the panels (IXZSYM $=0$ or 1 in Item 31) is assumed; therefore, only one side of the body will be shown to illustrate the panel layout.

A 2:l elliptic cross section is shown in Figure 11 where several different panel arrangements have been selected. In Figure $11(a)$, nine program-determined panels (KRADX $=10$
in Item 41), each specified by a $20^{\circ}$ central angle, are shown. The only problem area is near the shoulder of the ellipse where the outer three panels miss the actual surface by a considerable amount. Increasing the number of panels to IO (KRADX $=11$ ) produces the arrangement shown in Figure ll(b). Even though an additional panel has been included, the region near the shoulder is not modeled better than before. In fact, this layout is missing an important physical effect because no panel has the same orientation as the shoulder ( $d z / d y=\infty$ ), and the entire effect of the shoulder is lost.

An improved panel layout is shown in Figure ll(c) where nine panels are specified by their meridian angles (KRADX $=-10$ ). In this layout, the panels are concentrated in the region of highest curvature near the shoulder and spread out over the lessor curved region. Note that an odd number of panels is specified so that the outer panel nearest the shoulder has the proper slope. Top and bottom symmetry of the panels is shown, and this is a recommended arrangement for cross sections with top/bottom symmetry. For this particular cross section, additional panels can be specified for better resolution at the cost of increased execution time. It is important that the user check the panel layout used by the code to ensure reasonable modeling of the actual surface.

Different panel arrangements on a 3:l elliptic cross section body are shown in Figure 12. The sharper curvature of the shoulder of this shape creates a bigger modeling problem than the 2:l ellipse shown previously. In Figure $12(a)$, a l3-panel arrangement (KRADX $=14$ ) with equal spacing is shown, and it is obvious that a large portion of the shoulder region is not modeled properly for the prediction of flow field characteristics near to the body.

For this particular shape, more panels are required for better resolution. Additional panels are added in the region $69^{\circ}<\beta<111^{\circ}$ to produce the 19-panel layout (KRADX $=-20$ ) shown in Figure $12(\mathrm{~b})$.

An example of reasonable panel arrangements on a lobed cross section body without top/bottom symmetry is shown in Figure 13. The shape shown in this figure is that described in References 17 and 18. The panel edges do not lie precisely on the actual body shape, a problem which is discussed in the numerical mapping section to follow. The 9-panel layout in Figure $13(\mathrm{a})$ is a resonable representation for preliminary calculations; however, certain detail is missing in the regions of greatest curvature. Additional panels are included in these regions, and the resulting layout is shown in Figure $13(\mathrm{~b})$. Custom panel layouts are shown for both lobed-body examples to take advantage of the fewer panels required on the flat portion of body.

It is difficult to specify rigid guidelines for panel layout on arbitrary cross sections. The examples shown above were determined by this author after several trials, and proper panel selection becomes easier as the user gains experience with the code. The user should remember that linear interpolation of panel characteristics is used between panel control points in the current version of the code. Based on the experience of the authors, it is always useful to compare the paneling layout with the actual body shape, and generally, good engineering judgement will allow the user to select a reasonable panel layout.

Nothing has been said about axial panel layout (NFORX or KFORX) in the above discussion. Axial spacing can be critical to the predicted results, and as mentioned above, good engineering judgement will usually produce an adequate
layout. Linear interpolation is also used in the axial direction to determine panel characteristics between panel control points. This procedure can be used to great advantage for a specific class of body, the cone. Since the predicted linear results on a cone are the same at each axial cross section, fewer panels are required in the axial direction. The major requirements are the following.

1. There should be one row of panels sufficiently close to the nose tip or starting point (XI) for the calculation that a control point can fall between the starting point and the nose tip.
2. There should be a row of panels with control points located just aft of the last axial station of the calculation (XF).
3. If a conical nose is attached to the cylindrical afterbody, there should be enough axial stations specified near the junction of the nose and afterbody that adequate body characteristics are available for interpolation purposes.

An example of a panel arrangement for a cone forebody is presented in the sample cases.

Numerical mapping.- The specification of the appropriate numerical mapping parameters (Items 14 through 22) depends on the shape of the cross section. In the interest of optimum computation time, the fewest possible terms in the transformation series (MNFC in Item 14) should be used which will produce an adequate mapping. Each different shape should be checked by the user to determine the proper number of coefficients. For example, a simple cross section which is similar to a circle may require as few as 10 coefficients. The shape shown in Figure 13 required 20 coefficients for an excellent mapping. If a large number of calculations are
to be made for a noncircular shape, it is worthwhile for the user to try different numbers of coefficients in an effort to find an optimum number. Not only should the shapes be compared carefully, the unseparated flow pressure coefficients should be compared to evaluate the quality of the mapping.

The numerical mapping now included in NOZVTX has been used for a variety of cross section shapes, and very few problems have been observed. One general problem area is bodies with concave sides. The lobed body in Figure 13 has a moderate concave section on each side, and the mapping procedure had difficulty converging on a set of coefficients. When the body shape was modified slightly, as shown in Figure l3(a), to give the concave region a flat or slightly convex shape, the mapping converged easily. This small change in the shape probably has very little effect on the predicted pressure distribution. This is an example of the type of problems to which the user should be alert.

Integration interval.- Choice of the appropriate integration interval (DX in Item 6) depends to a large extent on the type of results of interest. The code permits the interval to be a constant (FDX $=0$ in Item 6) or a function of the local radius (FDX > 0). If the user is interested in detailed nose loads and separation on the nose, where the radius is changing rapidly, the latter option should be used with $0.5 \leq F D X \leq 1.0$. This option can lead to a wide range of vortex strengths which, on occasion, has been determined to be the cause of certain vortex tracking difficulties. The user should be alert to this possibility. The constant interval option ( $D X=$ constant) is the best choice when the results of a major interest are the vortex clouds near the end of the body.

Vortex core.- The core size specification is referenced to the local body diameter (RCORE in Item 5). The default value is 0.025 , and this value is adequate for most cases. The larger the core radius specified, the more the vortex effects are reduced. To eliminate the possibility of negating the vortex effect entirely, a maximum vortex core of .05 times the maximum body diameter is built into the code. Except in unusual situations, the default value is recommended, and this is the value used in the comparisons with data shown in a later section.

## Sample Cases

In this section, 11 sample input decks are described to illustrate the input preparation and use of the program for a wide range of configurations and flow conditions. Insofar as possible, each major option in the program is illustrated in the set of sample cases. The major features of each sample case are identified in Table II to assist the user in finding an appropriate example to suit a specific need.

Sample Cases 1 through ll shown in Table II are designed to exercise the most common options of program NOZVTX, and Sample Case 1 is included specifically to provide a short computer run for testing purposes. The input decks for the sample cases are shown in Figures $14(a)$ through ( $k$ ). The sample cases are based on typical bodies used in the data comparisons to follow in a later section, and the reference in which the configuration is defined is noted on each sample case.

Sample Case 1 is a test case based on the $3: 1$ elliptic missile body in Reference 19; however, only a portion of the nose is considered to keep the execution time and output at a minimum. The purpose of this run is to provide the
user with a short run to check out the code. The output from this case is described in the next section. Sample Case 2 in Figure $14(\mathrm{~b})$ is a restart of the previous run, and it illustrates the continuation of a calculation. Sample Case 3 in Figure 14 (c) is the input deck for a complete run of this 3:l elliptic missile. The first three sample cases do not necessarily use the optimum panel arrangement in an effort to minimize execution time. For production runs, the authors recommend a panel layout similar to that shown in Figure $12(\mathrm{~b})$.

Sample Case 4, Figure l4(d), illustrates the missile body in a rolled condition $\left(\phi=45^{\circ}\right)$. The lack of flow symmetry and consequently the lack of right/left panel symmetry dictate the use of a smaller number of panels because the entire cross section must be paneled. This run is typical of any noncircular body in a rolled flow condition.

Sample Cases 5 and 6, Figures $14(e)$ and (f), illustrate input decks for circular cross section bodies (Refs. 20 and 21). Since axis singularities are used in place of panels, the input is shorter and easier to prepare. The primary objective of Sample Case 6, an axisymmetric body at $\alpha_{c}=0^{\circ}$, is the prediction of the surface pressure distribution with no separation effects included.

Sample Cases 7 and 8, Figures $14(g)$ and (h), are input decks for a 2:l elliptic cross section forebody from Reference 18. The first case is for a standard panel layout determined by the code. The panels for the second case are tailored to the cross section shape by specification of the central angles of the panel edges.

Sample Case 9 is a 3:l elliptic cross section conical forebody (Ref. 17). The input specifies a variable axial
integration interval, and the axial panel distribution is determined by the conical shape. Because of the character of conical flow, a very smail number of axial panels is required as discussed in a previous section on input preparation.

Sample Case l0, Figure $14(j)$, is a lobed cross section forebody from Reference 18. This is an arbitrary noncircular body and numerical mapping is required for solution as indicated in the input. Since the cross section is changing at each $x$-station, the cross section shape must be input at a number of stations thus accounting for the large quantity of input. The number of axial statiors must be sufficient to provide acceptable accuracy in the body shape using linear interpolation between input stations.

Sample Case ll, Figure l4(k), is set up for the 2:1 elliptic cross section missile body in Reference 22. The panel layout for this body is the same as that shown in Ficure ll(c).

## Output

A typical set of output from program NOZVTX is described in this section. In general, the output quantities are labeled and each page is headed with appropriate descriptive information. The actual output obtained is determined by the output options selected by the user. For purposes of this description, representative pages from Sample Case 1, described in a previous section, are presented in Figure 15.

The total output from Sample Case 1 is contained on 30 pages, including pages which are continuations of another page. The first page shown in Figure $15(\mathrm{a})$ is headed by the title cards input as Item 3. The remainder of this
page, the following two pages shown in Figures 15 (b) and (c), and the top of the fourth page shown in Figure 15 (d) contain all the input data with appropriate labels. Body source panel geometry is shown on pages 5 through 13, Figures $15(\mathrm{e})$ through ( m ). Page 5, shown in Figure $15(e)$ and the following two pages shown in Figures $15(f)$ and ( $g$ ) indicate the panel corner coordinates. Pages 8,9 , and 10 , shown in Figures $15(h),(i)$, and (j), contain a table of the body panel centroid coordinates. This is followedby a table of body panel areas and inclination angles. The inclination angles DELTA and THETA orient the panels in the body coordinate system. Detailed descriptions of these angles are given in Reference 10. This table continues onto page 12 and the top of page 13, shown in Figures $15(\ell)$ and ( $m$ ). The next item on page $13, G B(N)$, is the list of source strengths for the body at angle of attack. This is followed by a table of velocity components at the control point of each panel. Each component is labeled, and the table continues onto pages 14 and 15 shown in Figures $15(n)$ and (0). The last item on page 15 , under the heading MODIFIED SOURCE STRENGTH, is the source strength on each panel of the body with no crossflow influence. This panel strength represents the volume solution of the body for purposes of tracking vortices. This list of source strengths continues onto page 16 shown in Figure $15(\mathrm{p})$.

The next two pages, shown in Figures $15(q)$ and ( $r$ ), contain the predicted pressure distribution around the body at the first axial station. A summary of the geometric characteristics and the Mach number is printed at the top of page 17, followed by the pressure distribution. The columns are labeled, but a brief explanation of the printed items follow. The first three columns, $Y, Z$, and BETA, locate the specific points on the body cross section. U/VO is the local axial velocity referenced to the free-stream
velocity. V/VO and $W / V O$ are the local lateral and vertical velocities, respectively. These velocities include all perturbation effects including vortex induced velocity components. The column VT/VO is the total velocity in the crossflow plane given as the vector sum of the $V / V O$ and $W / V o$ velocities. A positive VT/VO represents flow in a counterclockwise direction when viewed from the rear. $C P(M)$ is the compressible pressure coefficient as calculated from Equation (29). DPHI/DT is the unsteady part of the incompressible pressure relation as represented by the last term in Equation (31). CPZ is the steady part of Equation (31), and $C P(I)$ is given by Equation (3I) in its entirety. VCP/VO is the local two-dimensional crossflow plane velocity whose positive sense is the same as that for VT/VO.

The local force and moment coefficients are printed at the middle of page 18. $C N(X)$ and $C Y(X)$ are local force coefficients calculated by Equations (39) and (43), respectively. The remainder of the forces and moments represent total quantities on the nose between the origin and the local axial station. The normal force and pitching moment coefficients, CN and CM , are calculated with Equations (40) and (41), and the axial center of pressure of the normal force, XCPN, is given by Equation (42). XCPN is not normalized by the reference length on page 7. The corresponding side force and yawing-moment coefficients and center of pressure of the side force are given by Equations (44), (45), and (46), respectively. The rollingmoment coefficient, CSL, is positive in a counterclockwise sense when viewed from the rear of the body. The small value shown in this output is due to numerical round-off error during the calculation. The same is true for all lateral coefficients which appear as very smałl, nonzero,
values when a symmetric body and symmetric flow conditions dictate that they are identically zero.

Page 19 of the output from Sample Case l, Figure 15(s), summarizes the important points in the pressure distribution and the separation calculation. The last two lines of output contain specific information on the vortices shed at this station. The vortex numbers, NV, assigned here are the cumulative numbers of vortices in the flow field. The vortex strength, GAM/V, is obtained from Equation (38) where the boundary layer edge velocity $u_{e} / V_{\infty}$ is shown as $V T / V$ in this line of output. $M(K)$ is the actual distance of the vortex from the body surface. The position of the vortex in the body plane is given by $Y, Z$, and BETA. The position of the vortex in the circle plane is given by the coordinates YC, ZC. The radial position of the vortex in the transformed circle plane is RG/R. The latter quantity is useful in providing a quick estimate of the relative positions of new. vortices being shed into the wake.

Page 20, shown in Figure $15(t)$, summarizes the vortex field after the trajectories have been calculated over one axial interval. The x-station at the top of this page is exactly $D X$ downstream of the previous $x$-station. Information to locate each individual vortex in both the body plane and the circle plane is provided. One additional piece of information for each vortex is XSHED, the x-station at which the individual vortex originated. This is printed to allow the user to keep track of any.individual vortex throughout the trajectory calcułation. Each vortex maintains its individual identity and the right and left side vortices are grouped together on this page. Centroids of the individual groups of vorticity also show on page 20. The only exception to the above discussion of individual vortices occurs when vortices are allowed to combine through use of the parameter

RGAM in Item 6 of the input list. When vortices are combined, the particular vortices are identified in a special line of output.

This completes one cycle of output. The next page of output is the pressure distribution at the new axial station, and the format is the same as page 17 shown in Figure l5 (q). At each axial station, the output repeats through the cycle shown in Figures l5(q), (r), (s), and (t). At specified axial stations, certain additional output is available. A simple plot of the body cross section and the vortex field at $x=4.2$ is shown in Figure $15(u)$. This particular output is on page 23 of the actual output from Sample case I. Each type of vortex is shown with a different symbol in this figure. Postive body vorticity shed from the right side of the body is shown as an $X$, and negative vorticity from the left side is represented by an 0 . If it is requested, the calculated velocity field appears as shown in Figure l5(u). This output appears on page 24 of the actual output. The coordinates and velocities shown on this page are in the body plane. The normalizing diameter, $D$, in the last two columns is the equivalent diameter of the noncircular body.

The last page of output is shown in Figure l5(w). The top portion of the output is the summary page for the separation calculation at the last x-station, $X=X F$. This part of the output is similar to that shown in Figure 15 (s). The lower part of the page is printed only at the completion of the calculation, and it contains a summary of the forces and moments and the vortex strengths and centroids at the end of the body.

## RESULTS

For purposes of evaluating the accuracy and range of applicability of the engineering prediction method and associated program code NOZVTX described above, comparisons of measured and predicted aerodynamic characteristics of various configurations have been made. The objective of the method is to predict the characteristics for noncircular cross section bodies, but selected data for bodies of revolution will also be considered to illustrate certain features of the prediction method. Typical results from the prediction method are presented in the following sections.

## Circular Bodies

The prediction method was applied to an ogive-cylinder model (Ref. 2l) in supersonic flow to evaluate the ability of the prediction method to calculate pressure distributions. The configuration has a circular cross section and a three caliber ogive nose followed by 3.67-caliber cylindrical afterbody. Circumferential pressure distributions at a large number of axial stations are available for a range of angles of attack and supersonic Mach numbers. The free-stream Reynolds number, based on diameter, is $0.5 \times 10^{6}$ for the results to follow.

The model is represented by 51 line sources on the body body axis for the calculation at $\alpha_{c}=0^{\circ}$. The measured and predicted pressure distributions at three Mach numbers are compared in Figure 16 where the multiple data symbols at each axial station represent the range of scatter in the data. The purpose of this comparison is to verify the body model and the pressure calculation technique under conditions in which there are neither crossflow nor separation effects.

In Figure $16(a)$, the agreement is very good at $M_{\infty}=1.6$ except in the region very near the nose. As seen in Figures $16(b)$ and (c), the agreement between measured and predicted pressures on the nose gets progressively poorer as the Mach number increases. It is possible that nonlinear effects caused by the close proximity of the shock wave are responsible for this disagreement. For purposes of this investigation, a small error in the pressure very near the nose will have very little effect on the vortex shedding from the remainder of the body. As a further check on the body model, a panel distribution of 13 rings of 14 equally spaced circumferential panels was used for the same calculation. The pressure results are virtually identical to those obtained from the line singularity model.

The prediction method was next applied to the same model at $\alpha_{c}=20^{\circ}$ angle of attack and $M_{\infty}=1.6$. At this angle of incidence, the separation vortex wake on the lee side of the body is well developed. Measured and predicted pressure distributions at two axial stations on the nose and one station on the cylinder are compared in Figure 17. The agreement between experiment and theory at $x / D \approx 0.8$ is very good except on the windward side of the body upstream of the minimum pressure point. Similar results are illustrated in the lower portion of Figure $17(a)$ for a station near the end of the nose ( $x / D=2.8$ ) . The predicted minimum pressure point is upstream of the measured value, but the general trend of the predicted pressure distribution is correct. The slightly irregular behavior of the predicted curve near $\beta=130^{\circ}$ is caused by local interference of the lee side separation vortices.

Measured and predicted pressures on the cylindrical portion of the body $(x / D=5.1)$ are shown in Figure $17(b)$. A large vortex wake, also shown in the figure, has developed
on the lee side, and it has a dominant effect on the pressure distribution. The solid curve, representing the predicted pressure distribution in the presence of the vortex wake, is in good agreement with the measured results near the minimum pressure point and on the lee side. The predicted pressures are lower than those measured on the windward side of the body. There is also some disagreement between experiment and theory near $\beta=130^{\circ}$ where it appears that some of the vortex induced effects are missing from the predictions. Contrast the solid curve with the dashed curve representing a predicted pressure distribution with no separation effects included.

The prediction method was next applied to an ogivecylinder model for which detailed lee side flow field measurements are available (Ref. 20). This model is an ogive-cylinder with a two-caliber nose and a thirteen-caliber cylindrical afterbody. The tests were conducted at Mach numbers of 2 and 3 at a free-stream Reynolds number, based on diameter, of approximately $2 \times 10^{6}$. The data consist of lee side velocity components and separation vortex strengths and positions. Neither pressure nor force and moment data are available from these tests.

The model is represented by 51 line source and doublet singularities on the body axis. The predicted vortex cloud pattern at $x / D=10$ is shown in Figure 18 for $M_{\infty}=2$ and $\alpha_{c}=15^{\circ}$. The vortex shedding is symmetric on both sides of the body. Also shown in this figure are the separation points located at approximately 86 degrees measured from the windward stagnation point. The measured and predicted vortex strength and centroid of vorticity are shown to be in reasonable agreement in this figure. The predicted vortex wake is approximately 9 percent stronger than the measured value, and the centroid of the predicted vortex cloud is
slightly lower and more outboard than the measured results. Again, as a further check on the body model, the body was represented by 15 rings of 14 equally-spaced circumferential panels. The predicted results were nearly identical in each case; however, the execution time for the line singularity model was only 20 percent of that required for the panel solution.

Measured and predicted body vortex strengths on the same ogive cylinder at Mach numbers 2 and 3 and angles of attack 10, 15 , and 20 degrees are compared in Figure 19. At $M_{\infty}=2$, the predicted results are slightly higher than those measured. At $M_{\infty}=3$, the predicted results are lower than those measured on the forward portion of the body and in good agreement on the rear portion. The slopes of the predicted curves are in good agreement with the experimental results, an indication that the predicted shedding rate and strengths of the shed vortices are correct. The difference in magnitude between the measured and predicted vortex strength in Figure 19 is attributable in part to the location of the onset of separation. If separation is predicted to start further aft on the body than the actual start of separation, the predicted vortex strength will be less than measured. The maximum error in vortex strength exhibited in Figure 19 represents an error in the predicted axial position of onset of separation of less than one body diameter except at the highest angles of attack.

Measured and predicted downwash velocity fields in the plane of symmetry of the model at $M_{\infty}=2$ and $\alpha_{c}=15^{\circ}$ are shown in Figure 20 for three axial stations. The results are generally in good agreement, and the predicted velocity distribution exhibits the correct trend in every case. The vortex field used to calculate the induced velocities at $x / D=10$ is the distribution shown in Figure 18.

Additional comparisons of measured and predicted downwash velocities on the lee side of the body at $x / D=10$ are shown in Figure 21. In Figure $21(a)$, the velocities are calculated along a line at $z / D=.75$ which passes through the predicted centroid of vorticity. The agreement is only fair in the vicinity of the centroid ( $\mathrm{y} / \mathrm{D} \approx 0.4$ ) , an indication that the present discrete vortex cloud model is not completely reliable inside the cloud. In Figure $21(b)$, the measured and predicted velocities are compared along a line at $z / D=1.05$ which passes above both the measured and predicted centroid of vorticity. These results indicate the correct trend in the predicted velcoity distribution; however, since the strength of the wake vorticity is nearly correct, it appears that a cloud model that is higher off the body is required to bring the predicted velocities into better agreement with measurements. The need for a higher cloud is verified in Figure 18 where it is shown that the actual centroid of vorticity is approximately .15D above the predicted centroid. As shown in Figure l9, the strength of the vortex cloud is in good agreement with experiment.

Similar comparisons of measured and predicted downwash velocity fields on the lee side of the ogive cylinder at $M_{\infty}=3, \alpha_{c}=15^{\circ}$ are shown in Figures 22 and 23 . The results on the plane of symmetry on the model lee side are shown in Figure 22 where it is obvious that the comparisons are not as good as those shown in Figure 20 for $M_{\infty}=2 \because$ The general trends are correct, but the magnitude of the velocity components is in error. This is likely due to a combination of circumstances. For example, in Figure l9, the predicted vortex strength corresponding to $M_{m}=3, \alpha_{c}=15^{\circ}$ is less than that measured at $x / D=7$ and in good agreement with measurements at $x / D=13$. As seen in Figure 22 , the vortex induced velocity is less than that measured at $x / D=7$ and nearly correct at $x / D=13$. The fact that the predicted
velocity profiles are not in agreement may be due, in part, to the problem discussed previously, the error in the predicted centroid of vorticity. The velocity profiles indicate that the predicted centroid of the separation vorticity may be too low, a possible problem at higher Mach numbers.

Analogous to the $M_{\infty}=2$ results in Figure 21 , comparisons of measured and predicted velocity profiles on the lee side of the body at $x / D=10$ are shown in Figure 23 for $M_{\infty}=3$. The results look very good in Figure $23(a)$ at $z / D=.75$ where the region of comparison is in the vortex cloud and near the vertical centroid of vorticity. In Figure $23(b)$, the agreement between the velocity profiles is poor with the predicted velocities being much less than those measured.

## Elliptic Bodies

The prediction method has been applied to a number of different elliptic cross section bodies. A variety of results are presented for several configurations under a range of flow conditions, the major objective being to examine a large number of configurations to identify the strengths and weaknesses of the code.

The 3:1 elliptic cross section missile of primary interest in these comparisons is the sharp-nose body described in References 19, 23, and 24. The tests were conducted at $M_{\infty}=2.5$ and a free-stream Reynolds number of $2 \times 10^{6}$ per foot. The sharp-nose body considered in the comparisons to follow was modeled with 18 circumferential panels around the cross section and 29 axial rings of panels. Sample Cases 3 and 4 describedin Table II and Figures $14(c)$ and (d), respectively, produce the results to follow.

Flow visualization via vapor-screen photographs (Ref. 23) and pressure data (Ref. 24) are available for comparison with the theoretical results. For better visualization of the vortex development along the body, the photographs were digitized. These results are presented in the upper left corner of Figure 24 for the missile at $M_{\infty}=2.5, \alpha_{c}=20^{\circ}$, and $\phi=0^{\circ}$ and $45^{\circ}$. Cross sections through the body at three axial stations are shown. The extent of the vortices from the vapor screen photographs are shown by the solid curve, and the predicted discrete vortex wake is shown by the individual vortices connected by a dashed line. The unrolled flow condition ( $\phi=0^{\circ}$ ) shown in Figure $24(a)$ illustrates very good agreement between the predicted vortex cloud and the vapor screen results. The strength of the predicted vortex cloud is shown at each axial station, and because of symmetry, the left and right vortices are of equal magnitude and opposite sign.

The rolled flow condition ( $\phi=45^{\circ}$ ) is shown in Figure $24(b)$. To be consistent with the output from the code, the body is shown unrolled with the flow angle rolled 45 degrees. The general character of the predicted vortices is in fair agreement with the experimental results; however, there are certain areas of disagreement. The theoretical vortex on the right side has rolled up much tighter than the vapor screen indicates, and the left theoretical vortex has not moved out away from the body as far as the actual vortex, nor has it rolled up as tightly as the actual vortex.

Comparison of measured and predicted pressure distributions at $x / L=0.60$ on the body are shown in Figure 25 for the unrolled and rolled condition. The unrolled results shown in Figure $25(a)$ illustrate some of the same problem areas discussed for circular bodies. The predicted
stagnation pressure on the windward side is in good agreement with experiment, but the predicted pressure around the shoulder region of the cross section is much lower than the measurements. The pressure on the lee side is also underpredicted, possibly due to vortex interference effects. As shown in Figure $24(a)$, there is a strong concentrated vortex field developed on the lee side at this axial station ( $\mathrm{x} / \mathrm{L}=0.60$ ). The irregularities in the predicted pressures near the shoulder region are caused by local vortex interference effects. As noted in the discussion for the circular bodies, the discrepancy in the pressure comparisons on the windward side of the body near the shoulders may be caused by nonlinear effects induced by the close proximity of the shock wave.

The measured and predicted pressures on the $3: 1$ elliptic body in a rolled flow condition are compared in Figure 25 (b). The qualitative agreement is very good, but a problem on the windward side of the body is in evidence. The vortex induced effects cause the irregularities in the predicted pressures, and it appears that the induced effects are too large on the positive lee side $\left(\theta \approx 320^{\circ}\right)$. In Figure $24(\mathrm{~b})$ it is obvious that the close proximity of the predicted wake will have a large effect on the pressure distributions, and it is likely that the predicted wake is causing the lack of agreement in this region.

NOZVTX was also applied to a conical forebody with a 3:1 elliptic cross section shape (Ref. 17). The major axis, in planform, is defined by a 20 -degree half angle, and the total length of the forebody is 14 inches. The test was conducted at a free-stream Reynolds number of $2 \times 10^{6}$ per foot. The panel model representation consists of 17 circumferential panels on the half body ( 34 panels on the total cross section) and four axial rings of panels. The limited number of axial
panels is permissible because of the similar characteristics on a conical configuration in supersonic flow described in the previous section on input preparation. The detailed panel layout is described in Sample case 9.

Comparisons of measured and predicted pressure distributions on the conical forebody are shown in Figure 26 for two axial stations. Except for small differences on the lee side of the body, the measured pressure distributions are nearly identical at the two axial stations. In Figure 26(a), the theoretical pressure distribution is not in good agreement with experiment on the windward side of the body, particularly near the shoulders. On the lee side, two prediced curves are shown; the dashed curve represents no separation vortex effects, the solid curve has vortex effects included. There is considerable difference in these results, all of the difference attributable to the vortex induced effects. The vortex effects improve the predicted pressures in the region around $\beta=120^{\circ}$ on the body, but they detract from the agreement near the lee side stagnation point, $150^{\circ}<\beta<180^{\circ}$. This may be an indication that the actual lee side vorticity is rolled up tightly around $B=120^{\circ}$, whereas the predicted vortex cloud is distributed over the entire lee side of the body as illustrated in Figure 27.

The prediction method has been applied to the elliptic cross section body described in Reference 22 and illustrated in Figure 28. The major axis of the cross section is specified by an ogive distribution on the nose, and the cross section is a $2: l$ ellipse over the entire body length. The equivalent base diameter of this body is 1.125 inches. Circumferential pressure measurements at numerous axial stations are available at $M_{\infty}=1.5$ and a range of angles of incidence and angles of roll. The tests were conducted at a free-stream Reynolds number, based on the equivalent diameter, of approximately $0.3 \times 10^{6}$.

This 2:1 elliptic cross section body is modeled with 18 circumferential panels around the complete cross section and 15 axial rings of panels. No smoothing is used in any of the calculations to follow. The general layout of the geometry is illustrated in Sample Case ll in Figure l4(k).

Measured and predicted pressure distributions on the elliptic cross section body at $M_{\infty}=1.5, \alpha_{c}=15^{\circ}$, and several roll angles are compared in Figure 29 at two axial stations on the nose. The agreement between experiment and theory for $\phi=0^{\circ}$ is better away from the tip of the nose as shown in Figure 29(a). This is consistent with the circular body results described previously. The sparse nature of the measurements make it difficult to fully evaluate the prediction method other than to observe that the basic trends of the pressure distribution are predicted. At $x / D$ ~ 2.1 , the separation vorticity has grown in strength and scope, and the major region of vortex wake influence covers the region between $\beta=90^{\circ}$ and $130^{\circ}$.

The predicted pressure distributions on the same body at roll angles of $22.5^{\circ}$ and $45^{\circ}$ are shown in Figures 29 (b) and 29(c), respectively. The predicted results are generally in good agreement with experiment, and the correct trends of the data are modeled by the theory in each case. The local irregularities in the predicted curves are due to the influence of the separation vortices.

NOZVTX was next applied to a $2: 1$ elliptic cross section cone (forebody 4 in Ref. l7) in an unrolled flow condition. The pressure results for $M_{\infty}=1.7$ and $\alpha_{c}=20.35^{\circ}$ are shown in Figure 30 where it is obvious that agreement is poor over much of the surface. The lack of agreement on the windward side and near the body shoulder is typical of previous results on elliptical cross section bodies. On the lee side, there is a large effect of the predicted vortex field which
is spread out over much of the lee side. The effects of the vortices on the measured pressures appear to be more limited in scope. These results are the same as those shown on the 3:1 elliptic cone forebody in Figure $2^{\circ} 6(a)$.

The final 2:l elliptic cross section body considered in these comparisons is forebody 2 from Reference 18, and the predicted results are from Sample Case 8 in Figure l4(f). The comparison of measured and predicted pressure distributions in Figure 31 are in good agreement over the entire body surface, even on the windward side. The pressure distribution on the body is such that separation does not occur over most of the length of the forebody. Separation begins at approximately $x / L=0.8$, but in contrast to the elliptic cross section bodies with their major axis horizontal, the vortices are very weak and located near the lee side stagnation point.

## Arbitrary Bodies

The arbitrary noncircular body option in NOZVTX has been checked out for a number of different cross section shapes to verify the numerical transformation. Some of the shapes considered are a square cross section with rounded corners, a diamond cross section with rounded corners, and a lobed or pear shape cross section. The latter cross section is the only shape for which pressure data are available, therefore, results on this configuration are described as follows.

The configuration selected for these comparisons is the lobed cross section body, forebody 3 from Reference 18, shown in Figure 13. The panel arrangement shown in Figure $13(b)$ is used for the predicted results included herein, and the specific calculations were made using Sample Case 10. Comparison of measured and predicted pressure distributions
at two axial stations are shown in Figure 32. At $x / L^{\prime \prime}=.14$ in Figure $32(a)$, the agreement is very good on the windward side of the body, only fair in the region of minimum pressure, and poor near the lee side stagnation point. This particular location near the nose of the body has minimal vortex interference effects. At the mid-point of the forebody length, $x / L=0.5$, shown in Figure $32(b)$, the agreement is only fair over the entire circumference even though the trend of the data is predicted well. One unusual result in Figure 32 (b) is the overpredicted pressure on the windward side of the body, a situtation which has not occurred on any other configuration. The roughness in the predicted pressure distribution on the lee side of the body is caused by local vortex interference effects. The vortices are distributed along the body surface, and they have not started to roll up. A sketch of actual predicted vortex positions is shown in Figure 33.

## CONCLUSIONS

An engineering method and the program code NOZVTX to predict the vortex shedding from circular and noncircular bodies in supersonic flow at angles of attack and roll are described. The coupling of a supersonic panel method, an axisymmetric source and doublet model and a two-dimensional crossflow vortex shedding method has proved to be a successful means to predict the general flow characteristics about slender, circular and noncircular bodies at large angles of incidence and moderate Mach numbers. Comparisons of measured and predicted pressure distributions and associated flow fields on a variety of bodies indicate that the principal features of the flow phenomena are modeled. The predicted vorticity distributions in the wake of a body of
revolution in supersonic flow are in reasonable agreement with experiment. This. also results in reasonable agreement between measured and predicted velocity components in the flow field of the body. The ability to model the correct flow field near the body leads to the capability to calculate wake induced interference effects on fins and other control surfaces as well as surface pressure distributions. The vortex shedding analysis described herein can be incorporated into an overall computation method for missiles and aircraft.

## RECOMMENDATIONS

In the course of development of the program code NOZVTX, several specific areas for needed improvements to the method were discovered. These improvements were beyond the scope of the present study, but they are noted here as specific recommendations for future work.

The first, and possibly the most important, recommendation involves a thorough testing of program NOZVTX. Limited numbers of comparisons with experimental results on a variety of bodies are included herein, but many other comparisons are needed. The program should be tested over a wide range of angles of attack and roll and Mach numbers to better define the operational limits of the method. The program should also be applied to a wide range of noncircular body shapes for which data are available.

A critical weakness in the prediction method appears to be the difficulty in calculating accurate pressure distributions on the windward side of certain bodies. For example, from the comparisons with experiment described in a previous section, the method has problems with conical
forebodies at all Mach numbers and most shapes at high Mach numbers. These difficulties may be caused by nonlinear effects induced by the shock wave when it is relatively near to the windward body surface. Further investigation into this problem is recommended.

Comparisons of measured and predicted velocity components on the lee side of an axisymmetric body at high angle of attack indicate the method has a potential capability to handle this problem. However, some additional work on the details of the vortex cloud are required. This may involve a different procedure to blend the discrete vortices when velocities in the midst of the cloud are to be calculated. With regard to the cloud itself, qualitative results on a 3:1 missile body are in excellent agreement with experiment; but based on one set of experimental results, predicted vortex clouds on an axisymmetric body are typically lower than those measured. Additional investigation into the mechanics of the vortex tracking procedure may explain this disagreement.

The final set of recommendations involve improvements to various parts of the program. The Stratford separation criteria have proved to work very well even though they are based on incompressible flow; however, similar criteria or corrections to the existing criteria should be developed for compressible flow. In this same area, it has been observed in surface flow visualization studies that the separation line on a body exhibits characteristics of laminar, turbulent, and transitional flow. At the present time, the crossflow plane separation may be either laminar or turbulent but not transitional or a combination of laminar and turbulent. It may be possible to use surface flow visualization information to correlate the three separation regions on the body and build this feature into the prediction method.

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## TABLE I - NOZVTX SUBROUTINE LIST

| Subroutines | Subroutines |
| :--- | :--- |
| NOZVTX* |  |
| AMAP | ITRATE |
| ASUM | NEWRAD |
| BDYGEN | PANEL |
| BMAP | PARTIN |
| BODPAN | PLOTA2 |
| BODVEL | PLOTA5 |
| BODYR | PLOTA6 |
| CMAP | PLOTA7 |
| COMBIN | PLOTA8 |
| CONFIG | PLOTV |
| CONFOR | SEPRAT |
| DFEQKM | SHAPE |
| DIAGIN | SMOOTH |
| DOUBLT | SOLVE |
| DPHIDT | SORPAN |
| DTABLE | SOURCE |
| DZDNU | SRFVEL |
| ELNOSE | SUM |
| F | VCENTR |
| FLDVEL | VELCAL |
| FORCEP | VELCMP |
| FPVEL | VLOCTY |
| GEOM | VOLUME |
| INPT | VTABLE |
| INVERT |  |
| Main PrOgram. |  |
|  |  |

TABLE II - SAMPLE CASES FOR PROGRAM NOZVTX

| Sample Case | Ref. | Fig. | Body Cross Section | $M_{\infty}$ | ${ }^{\alpha}{ }_{c}$ | 中 | Gencral Comments | $\begin{aligned} & \text { Execution } \\ & \text { Time } \\ & (\text { CDC-CYBER } \\ & 760) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 19 | 14(a) | $\begin{gathered} 3: 1 \\ \text { Ellipse } \end{gathered}$ | 2.5 | $20^{\circ}$ | $0^{\circ}$ | An elliptic missile body case with flow symmetry, panel symmetry; surface pressure calculation, vortex field plots at each axial station, and velocity field calculation at selected axial stations (TEST CASE). | 8.6 sec . |
| 2 | 19 | 14 (b) | $\begin{gathered} 3: 1 \\ \text { Ellipse } \end{gathered}$ | 2.5 | $20^{\circ}$ | $0^{\circ}$ | Restart of Sample Case 1 | N/A |
| 3 | 19 | 14(c) | $\begin{gathered} 3: 1 \\ \text { Ellipse } \end{gathered}$ | 2.5 | $20^{\circ}$ | $0^{\circ}$ | Full run for missile body in Sample Case 1 | -- |
| 4 | 19 | 14(d) | $\begin{gathered} 3: 1 \\ \text { Ellipse } \end{gathered}$ | 2.5 | $20^{\circ}$ | $45^{\circ}$ | Same elliptic missile body from Sample Case 1; no flow symmetry, surface pressure calculation and vortex field plots at each axial station. | N/A |
| 5 | 20 | 14(e) | Circle | 2.0 | $15^{\circ}$ | $0^{\circ}$ | An ogive cylinder body case with line sources and doublets, flow symmetry; surface pressure calculation and vortex field plots at each axial station, flow field calculation at selected axial stations. | 15.5 sec. |

panutyuod - II atqea

| Sample Case | Ref. | Fig. | Body Cross Section | $M_{\infty}$ | ${ }^{n}$ | $\phi$ | General Comments | $\begin{gathered} \text { Execution } \\ \text { Time } \\ \text { (CDC-CYBER } \\ 760) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 6 | 21 | 14(f) | Circle | 1.6 | $0^{\circ}$ | $0^{\circ}$ | An ogive cylinder body case with line sources and doublets and variable DX; surface pressure calculation and no separation. | 6.0 sec . |
| 7 | 18 | 14(g) | $\begin{gathered} 2: 1 \\ \text { Ellipse } \end{gathered}$ | 1.7 | $20.36^{\circ}$ | $0^{\circ}$ | An elliptic cross section forebody with panel symmetry: surface pressure calculation and vortex field plots at each axial station. | 28.1 sec. |
| 8 | 18 | 14 (h) | $\begin{gathered} \text { 2:l } \\ \text { Ellipse } \end{gathered}$ | 1.7 | $20.36^{\circ}$ | $0^{\circ}$ | Same as Sample Case 6 with body panels tailored to the shape by specification of the central anqles of the panel edges. | 28.1 sec. |
| 9 | 17 | 14 (i) | $\begin{gathered} 3: l \\ \text { Ellipse } \end{gathered}$ | 1.7 | $20.35^{\circ}$ | $0^{\circ}$ | An elliptic cross section conical forebody with panel symmetry, flow symmetry, and variable integration interval; sus:face pressure calculation with vortex smonthing. | 61.8 sec. |

Table II - Concluded

| Sample Case | Ref. | Fig. | $\begin{gathered} \text { Body } \\ \text { Cross } \\ \text { Section } \end{gathered}$ | $M_{\infty}$ | ${ }^{\alpha} \mathrm{c}$ | 中 | General Comments | $\begin{gathered} \text { Execution } \\ \text { Time } \\ (\mathrm{CDC}-\mathrm{CYBER} \\ 760) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 18 | 14(j) | Lobed | 1.7 | $20.36^{\circ}$ | $0^{\circ}$ | A lobed cross section forebody case with left/right panel symmetry, flow symmetry, variable DX, and numerical mapping; surface pressure calculation with vortex smoothing. | 47.0 sec . |
| 11 | 22 | 14(k) | $\begin{gathered} 2: 1 \\ \text { Ellipse } \end{gathered}$ | 1.5 | $15^{\circ}$ | $0^{\circ}$ | An elliptic cross section missile body with panel symmetry, flow symmetry; surface pressure calculation, periodic vortex field plots. | N/A |



REAL PLANE


CIRCLE PLANE
(a) Analytical transformation procedure

(b) Numerical transformation procedure

Figure 2.- Conformal mappins nomenclature.


Figure 3. - Sketch of crossflow plane separation points.


Figure 4. - Body crossflow plane nomenclature.


Figure 5.- Vortex smoothing procedure.


Figure 6.- Flow map of program NOZVTX.


Figure 6.- Continued.


Figure 6.- Continued.

(d) Page 4

Figure 6.- Continued.


PROGRAM

END
(e) Page 5

Figure 6.- Concluded.


(b) Page 2
Figure 7.- Continued.



(e) Paqe 5
Figure 7.- Continued.

(f) Page 6

Figure 7.- Concluded.




$$
\text { (b) Page } 2
$$

Figure 8.- Continued.


(d) Page 4
Figure 8.- Continued.

(e) Page 5
Figure 8.- Continued.


(a) Page 1
Figure 9.- Input forms for Progran NOZVTX.

(b) Page 2
Figure 9.- Continued.

(18)* *):
(c) Page 3
Figure 9.- Continued.



[^0]\[

$$
\begin{gathered}
\text { (d) Page } 4 \\
\text { Figure 9.- Continued. }
\end{gathered}
$$
\]

ITEM

| (27)* | GMMR | YR | 20 | ZM | 30 | XSIIEDR |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |




(38) $\times$ A

(g) Page 7
Figure 9.- Concluded.


Figure 10.- Convention for ordering coordinates for a noncircular cross section at $\mathrm{X}=\mathrm{XFC}(\mathrm{J})$.

(a) 9 panels, equal spacing

--- Body surface
10 panels (KRADX = 11)
(b) 10 panels, equal spacing

--- Body surface

- 9 panels
(KRADX $=-10$ )
(c) 9 panels, custom spacing

Figure ll.- Panel layout on a $2: 1$ elliptic cross section.


Figure 12.- Panels layout on a $3: 1$ elliptic cross section.

(a) 9 panels, custom spacing
(b) 17 panels, custom spacing

Figure 13.- Panel layout on a lobed cross section.

(a) Sample Case 1

Figure 14.- Sample input decks for program NOZVTX.

(b) Sample Case 2

Figure 14.- Continued.

(c) Sample Case 3

Figure l4.- Continued.

(d) Sample Case 4

Figure l4.- Continued.

| $\begin{aligned} & 0 \\ & 4 \end{aligned}$ | 0 | 0 | 0 | 1 | 10 | 109 | 0 | 0 | 00 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 1 | 3 | 0 |  |  |  |  |  |  |
| NIELSEN ENGINEERING A RESEARCH. INC. PRGGRAM NOZVTX |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| SIARPULAR OGIVE |  |  | Crlindep | megrees. |  |  |  |  |  |  | REF: $\triangle F A T L$ TR-78-127 BY OBERKAMPF OHI = 0 CIEGREES |  |  |  |  |  |
| MACL NO. $=2.0$. |  |  | ALHMAC $=15$ |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7.06858 |  | 3.0 | 6.0 | 39. | 3.0 |  |  |  |  |  |  |  |  |  |  |  |
| 15.0 |  | 0.0 | 1750000. | 0.0 | 2.0 |  |  |  |  |  |  |  |  |  |  |  |
| 3.0 |  | 39. | 1.5 | 1.05 | 0.0 | 1.0 |  | 0.0 |  |  |  |  |  |  |  |  |
| . 01 |  | 0.0 | 0.0 | 0.0 | 0.0 |  |  |  |  |  |  |  |  |  |  |  |
| 23 |  |  | . |  |  |  |  |  |  |  |  |  |  |  |  |  |
| $n .0$ |  | - 25 | - 50 | .75 | 1.00 | 1.25 |  | 1.50 | 1.75 |  |  |  |  |  |  |  |
| 3.00 |  | 2.25 | 2.50 | 2.75 | 3.00 | 3.25 |  | 3.50 | 3.75 |  |  |  |  |  |  |  |
| 4.00 |  | 4.25 | 4.50 | 4.75 | 5.00 | 5.25 |  | 5.50 | 5.75 |  |  |  |  |  |  |  |
| 4.00 |  | 36.0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| D. 0 |  | . 12981 | . 25272 | . 36895 | -47870 | -58216 |  | . 0794 H | . 77082 |  |  |  |  |  |  |  |
| -R 5630 |  | . 93606 | 1.01020 | 1.07883 | 1.14203 | 1.19940 |  | 1.25250 | 1.29990 |  |  |  |  |  |  |  |
| 1.34216 |  | 1.37933 | 1.41144 | 1.4385 e | 1.46772 | 1.47792 |  | 1.49019 | 1.49755 |  |  |  |  |  |  |  |
| 1.570 |  | 1.500 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 53333 |  | . 50528 | .47815 | . 45195 | . 42630 | . 40145 |  | . 37722 | . 35355 |  |  |  |  |  |  |  |
| . 33041 |  | . 30773 | . 29548 | . 263 hl | .24209 | - 221199 |  | . 17950 | . 17928 |  |  |  |  |  |  |  |
| -15883 |  | . 13857 | .11847 | . 09851 | .07867 | . 05893 |  | . 03925 | .01761 |  |  |  |  |  |  |  |
| ก.0 |  | 0.0 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 21.0 |  | 30. | 30. |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0.0 |  | 1.2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0.0 |  | 1.50 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0.0 |  | 1.80 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0.0 |  | 2.10 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9.0 |  | 2.40 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| n. 0 |  | 2.70 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 0.0 |  | 3.00 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 30 |  | 1.2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 3 |  | 1.50 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 3 |  | 1.8n |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 3 |  | 2.10 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 3 |  | 2.40 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 3 |  | 2.70 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 3 |  | 2.00 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 1.05 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 1.10 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 1.2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 1.30 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 1.40 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| -45 |  | 1.50 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 1.60 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 1.70 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 1.90 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 1.90 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 2.00 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 2.10 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 2.20 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 2.30 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 2.40 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| .45 |  | 2.70 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 45 |  | 3.00 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 60 |  | 1.05 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 50 |  | 1.10 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 60 |  | 1.2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 60 |  | 1.30 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| .60 |  | 1.40 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| . 6 |  | 1.50 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| - 6 |  | 1.60 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

(e) Sample Case 5

Figure 14.- Continued.

(e) Sample Case 5, Continued.

Figure l4.- Conṭinued.

| 1.20 | 0.0 |
| :---: | :---: |
| 1.20 | .10 |
| 1.20 | . 2 |
| 1.20 | . 3 |
| 1.20 | -4 |
| 1.20 | . 5 |
| 1.20 | . 6 |
| 1.20 | . 7 |
| 1.20 | . 8 |
| 1.20 | - 0 |
| 1.20 | 1.0 |
| 1.20 | 1.1 |
| 1.20 | 1.2 |
| 1.20 | 1.3 |
| 1.20 | 1.4 |
| 1.2 | 1.50 |
| 1.2 | 1.6 |
| 1.2 | 1.7 |
| 1.2 | 1.9 |
| 1.2 | 1.9 |
| 1.2 | $2 . n$ |
| 1.2 | 2.10 |
| 1.2 | 2.2 |
| 1.2 | 2.3 |
| 1.2 | 2.40 |
| 1.2 | 2.70 |
| 1.2 | 3.00 |
| 1.35 | 0.0 |
| 1.35 | . 1 |
| 1.35 | . 2 |
| 1.35 | . 3 |
| 1.35 | . 4 |
| 1.35 | . 5 |
| 1.35 | . 6 |
| 1.35 | . 7 |
| 1. 35 | . 8 |
| '. 35 | . 9 |
| 1.35 | 1.0 |
| 1.35 | 1.1 |
| 1.35 | 1.2 |
| 1.35 | 1.3 |
| 1.35 | 1.4 |
| 1.35 | 1.50 |
| 1.35 | 1.6 |
| 1.35 | 1.7 |
| 1.35 | 1.80 |
| 1.35 | 1.9 |
| 1.35 | 2.0 |
| 1.35 | 2.10 |
| 1.35 | 2.2 |
| !. 35 | 2.3 |
| 1.35 | 2.40 |
| 1.35 | 2.70 |
| 1.35 | 3.00 |
| 1.50 | 0.0 |
| 1.50 | . 3 |
| 1.50 | . 6 |
| 1.50 | . 9 |
| 1.50 | 1.2 |
| 1.50 | 1.50 |
| 1.50 | 1.80 |
| 1.50 | 2.10 |
| 1.50 | 2.40 |
| 1.50 | 2.70 |
| 1.50 | 3.00 |
| 61 | $=6.0$ |

70. 

(e) Sample Case 5, Concluded.

Figure 14.- Continued.

(f) Sample Case 6

Figure 14.- Continued.

(g) Sample Case 7

Figure 14.- Continued.

(h) Sample Case 8

Figure 14.- Continued.
(i) Sample Case 9

Figure l4.- Continued.

(j) Sample Case 10

Figure l4.- Continued.

| $n .0$ | 0.0 |
| :---: | :---: |
| 0.0 | 0.0 |
| $n .0$ | 0.0 |
| $n .0$ | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| $n .0$ | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 9.0 | 0.0 |
| 9.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| 0.0 | 0.0 |
| . 00000 | -. 17874 |
| -01558 | -. 17809 |
| . 03105 | -. 17612 |
| . 04629 | -. 17275 |
| - 08110 | -. 16784 |
| . 0.9347 | -.15312 |
| -11049 | -.131ba |
| -12508 | -.10496 |
| .13147 | -. 07590 |
| -13097 | -.04757 |
| - 12607 | -.02223 |
| .11470 | . 019 |
| . 10000 | .058 |
| .08365 | . 09970 |
| .r7212 | . 12492 |
| .0547a | . 15044 |
| . 04333 | . 16171 |
| . 03011 | . 17077 |
| . 91545 | . 1760 A |
| . 00000 | .1787 |
| . 00000 | -. 35097 |
| . 03059 | -. 34970 |
| . 06098 | -. 74574 |
| . 09089 | -. 33922 |
| - 11997 | -. 32962 |
| . 17359 | -. 30067 |
| . 21698 | -. 25857 |
| . 26561 | -. 20009 |
| . 25816 | -. 14905 |
| . 25719 | -. 09361 |
| . 24756 | -.014365 |
| . 2239 | . 0373 |
| . 1964 | . 1139 |
| . 16427 | . 19577 |
| . 14162 | . 24530 |
| . 10752 | . 29541 |
| . 08509 | - 21755 |
| . 05913 | . 33534 |
| . 03035 | . 34693 |
| $0.0000 n$ | . 35097 |
| . 00000 | -. 51671 |
| . 04504 | -.51484 |
| . 08978 | -. 50915 |
| . 13381 | -.4994n |
| - 17662 | -. 48527 |
| . 75557 | -. 44265 |
| . 31942 | -. 38067 |
| - 78160 | -.30367 |

## (j) Sample Case 10, Continued.

Figure 14.- Continued.

| $\begin{array}{r} .38006 \\ .37 B 63 \end{array}$ | -.21943 -.13781 |
| :---: | :---: |
| . 36447 | -. 06427 |
| . 22956 | . 05493 |
| - 28909 | . 1674 |
| . 24184 | -2AR21 |
| . 20850 | . 36114 |
| . 15830 | . 43492 |
| . 12527 | .4675n |
| . 08705 | . 49360 |
| . 04460 | . 51076 |
| 0.00000 | . 51671 |
| . 00000 | -.67594 |
| .05892 | -. 67350 |
| .11744 | -. 666306 |
| . 17505 | -. 65331 |
| .23106 | -. 63482 |
| . 33432 | . 57977 |
| .41786 | -. 49798 |
| . 47304 | -. 39692 |
| .49710 | -. 28705 |
| . 49532 | -.1pn2a |
| .4767R | -. 08407 |
| 3113 | . 17185 |
| 7618 | . 21935 |
| . 31637 | . 37703 |
| . 27275 | . 47243 |
| . 20709 | . 56895 |
| . 16387 | - 51158 |
| . :1389 | . 54583 |
| . 0.5846 | .66817 |
| 0.00000 | . 67594 |
| . 00000 | -. 82868 |
| . 07224 | -. 82569 |
| . 14398 | -. F1656 |
| . 21461 | -. 80093 |
| . 23327 | -. 77827 |
| .40987 | -. 70991 |
| . 51229 | -.61051 |
| . 57992 | -.48561 |
| . 60953 | -. 35101 |
| . 60724 | -. 22102 |
| . 58452 | -. 10307 |
| 2856 | . 08809 |
| 6364 | .26891 |
| . 38785 | .46222 |
| . 33439 | . 57919 |
| . 25387 | . 69751 |
| .20090 | . 74977 |
| .13961 | .79177 |
| . 37167 | . 91915 |
| 0.00000 | .828*8 |
| .00000 | -. 97492 |
| . 03499 | -. 97139 |
| . 15939 | -. 06066 |
| . 2524 A | -.94227 |
| . 33325 | -.91561 |
| .4822n | -.83519 |
| . 60268 | -. 71825 |
| . 68226 | -. 57249 |
| .71710 | -.414n2 |
| . 71440 | -. 26002 |
| . 68767 | -. 12125 |
| - 22182 | . 10364 |
| . 54545 | . 31636 |
| . 4567n | . 54770 |

(j) Sample Case 10, Continued.

Figure 14.- Continued.

(j) Sample Case 10, Continued.

Figure 14.- Continued.

(j) Sample Case 10, Continued.

Figure 14.- Continued.

| 1．12299 | －1．33833 |
| :---: | :---: |
| 1.2712 a | －1．06673 |
| 1.33620 | －． 77145 |
| 1.33117 | －．48451． |
| 1.28136 | －． 22594 |
| 1.15865 | ． 19311 |
| 1.01636 | ． 58949 |
| ． 85023 | 1.01327 |
| ． 73304 | 1．26966 |
| ． 55653 | 1.52904 |
| ． 44040 | 1.64361 |
| ． 30605 | 1.73567 |
| .15710 | 1.79571 |
| 0.00000 | 1.81560 |
| ． 00000 | －1．91084 |
| ． 16657 | －1．97393 |
| － 332 n | －1．88289 |
| ． 49486 | －1．84685 |
| ． 6531 ¢ | －1．79459 |
| ． 94511 | －1．6369A |
| 1.18125 | －1．40775 |
| 1.33723 | －1．12207 |
| 1．40551 | －． 81147 |
| 1.40023 | －． 50964 |
| 1.34783 | －． 23766 |
| 1.21876 | ． 20313 |
| 1.06009 | ． 52007 |
| ． 89434 | 1.0 .5583 |
| ． 77107 | 1.33553 |
| ． 58540 | 1.51837 |
| ． 46325 | 1.72888 |
| ． 32192 | 1.82572 |
| ． 16525 | 1.88887 |
| 0.00000 | 1.91094 |
| .00000 | －1．99459 |
| .17422 | －1．99135 |
| ． 34725 | －1．96935 |
| ． 51758 | －1．93165 |
| ．68317 | －1．87700 |
| ．98851 | －1．71215 |
| 1.23540 | －1．47241 |
| 1.39854 | －1．1736 |
| 1.47005 | －．84E74 |
| 1.46453 | －． 53304 |
| 1.40972 | －． 24857 |
| 1.27473 | ． 21245 |
| 11818 | ．64855 |
| ． 93541 | 1.1147 R |
| ． 80647 | 1.39685 |
| －6122A | 1.68222 |
| ． 48452 | 1.80826 |
| ． 33671 | 1.90955 |
| ． 17286 | 1.97560 |
| 0.00000 | 1.99859 |
| .00000 | －2．07983 |
| .18130 | －2．07231 |
| ． 36137 | －2．04940 |
| ． 53862 | －2．01017 |
| ． 71094 | －1．95330 |
| 1.02869 | －1．78175 |
| 1.28572 | －1．53226 |
| 1.45540 | －1．22130 |
| 1.52981 | －．88324 |
| 1.52405 | －．55471 |
| 1.46703 | －．25868 |
| 37世55 | ．アフ109 |

（j）Sample Case 10，Continued．
Figure l4．－Continued．

| 1.16364 | .67491 |
| :---: | :---: |
| .97343 | 1.16009 |
| .A3926 | 1.45364 |
| .63717 | 1.75061 |
| . 50622 | 1.88177 |
| . 35039 | 1.98718 |
| . 17987 | 2.05591 |
| 0.00000 | 2.07983 |
| . 00000 | -2.15457 |
| .18782 | -2.14678 |
| - 37435 | -2.12305 |
| . 55798 | -2.08241 |
| . 73649 | -2.02350 |
| 1.n6569 | -1.84578 |
| 1.33192 | -1.58732 |
| 1.50780 | -1.26519 |
| 1.58479 | -.91498 |
| 1.57883 | -. 57465 |
| 1.51975 | -. 26797 |
| 1.37422 | . 22904 |
| 1.20545 | . 69916 |
| 1.00841 | 1.20178 |
| . 86942 | 1.50589 |
| . 66007 | 1.8135? |
| . 52234 | 1.94940 |
| . 36299 | 2.15859 |
| . 19633 | 2.12979 |
| 0.10000 | 2.1545\% |
| . 00000 | -2.222¢2 |
| . 19377 | -2.21478 |
| - 38621 | -2.19030 |
| . 57565 | -2.14837 |
| . 75982 | -2.08759 |
| 1.09941 | -1.90424 |
| 1.37411 | -1.6376n |
| 1.55556 | -1.30527 |
| 1.63499 | -. 94396 |
| 1.62884 | -.59285 |
| 1.56789 | -. 27646 |
| 1.41775 | . 23629 |
| 1.24364 | . 72131 |
| 1.04036 | 1.23985 |
| - 89696 | 1.55357 |
| . 58097 | 1.5709 n |
| . 53888 | 2.01114 |
| . 37448 | 2.12379 |
| . 19223 | 2.19725 |
| 0.00000 | 2.22292 |
| . 00000 | -2.28456 |
| . 19915 | -2.2763n |
| . 39694 | -2.2511* |
| . 59165 | -2.20805 |
| . 78093 | -2.14558. |
| 1.12995 | -1.95714 |
| 1.41228 | -1.683n9 |
| 1.59877 | -1.34153 |
| 1.68040 | -.97018 |
| 1.67409 | -.50932 |
| 1.61144 | -.28414 |
| 1.45713 | . 24285 |
| 1.27818 | . 74135 |
| 1.06925 | 1.27429 |
| . 92187 | 1.59673 |
| . 69989 | 1.92293 |
| . 55385 | 2.06701 |
| -78488 | 2.18270 |

(j) Sample Case 10, Continued.

Figure 14.- Continued.

| -19757 | 2.25829 |
| :---: | :---: |
| 0.00000 | 2.28456 |
| . 00000 | -2.33981 |
| . 20397 | -2.33135 |
| . 40654 | -2.30559 |
| . 60595 | -2.26145 |
| . 79981 | -2.19746 |
| 1.15728 | -2.00447 |
| 1.44643 | -1.72370 |
| 1.63743 | -1.37397 |
| 1.72104 | -. 99364 |
| 1.71457 | -.624n5 |
| 1.65041 | -. 29101 |
| .49236 | . 24873 |
| 1.30909 | . 75927 |
| 1.09511 | 1.30510 |
| .04415 | 1.63534 |
| . 71681 | 1.96943 |
| . 56725 | 2.11099 |
| . 39419 | 2.23557 |
| . 20235 | 2.31290 |
| 0.00000 | 2.33981 |
| - 00000 | -2.38855 |
| - 20822 | -2.37992 |
| .41501 | -2.35361 |
| -A185a | -2.3085 |
| . 81647 | -2.24324 |
| 1.18139 | -2.14623 |
| 1.47857 | -1.7597n |
| 1.67154 | -1.4i259 |
| 1.75680 | -1.01434 |
| 1.75020 | -.63705 |
| 1.68470 | -. 29707 |
| . 52345 | . 25301 |
| . 33636 | . 77509 |
| 1.11793 | 1.33229 |
| . 96383 | 1.66941 |
| . 73175 | 2.01046 |
| - 97906 | 2.16110 |
| .4024n | 2.28215 |
| . 20657 | 2.36108 |
| 0.00000 | 2.38855 |
| .00000 | -2.43080 |
| . 21190 | -2.42201 |
| .42235 | -2.39524 |
| . 62952 | -2.34439 |
| . 83091 | -2.28292 |
| 1.20228 | -2.08242 |
| 1.50268 | -1.79083 |
| 1.70111 | -1.4274n |
| 1.78797 | -1.03228 |
| 1.78125 | -. 64832 |
| 1.71459 | -. 30233 |
| . 55040 | . 25840 |
| . 36000 | . 78880 |
| 1.13770 | 1.35586 |
| . 99088 | 1.69896 |
| . 74469 | 2.04602 |
| . 58931 | 2.19932 |
| .4n952 | 2.32251 |
| . 21022 | 2.40284 |
| 0.00000 | 2.43080 |
| . 00000 | -2.46655 |
| -21501 | -2.45763 |
| . 42854 | -2.43045 |
| - 63877 | -7.78394 |

(j) Sample Case 10, Continued.

Figure 14.- Continued.

(j) Sample Case 10, Continued.

Figure l4.- Continued.
(j) Sample Case 10, Concluded. Figure l4.- Continued.

(k) Sample Case 11

Figure 14.- Concluded.
phutinam nu／vtx
SJAVHS AH 6LODL－WL VSVN ：九十斤

DEFINITIONS
$C N(X)$
$C Y(X)$
$C N$
$C Y$
$C M$
$C R$
$C R$
$C L L$

$$
\begin{aligned}
& \text { REFERE MCE QUANTITIES } \\
& \text { REF. ARFA ....... } \\
& \text { RFF. LFNGTH..... } \\
& \text { XIMOMFNT CENIEHI. } \\
& \text { PODY LENGTH ILI... } \\
& \text { BAGE IIAMETER IDI. }
\end{aligned}
$$

$$
\begin{aligned}
& \text { FLOW CONDITIONS } \\
& \text { MACH NUMRE. }
\end{aligned}
$$

$$
\begin{aligned}
& \text { MACH NUMPF.H....... } \\
& \text { ALPHA } C \text { (DEG.) ... } \\
& \text { PHI (NEG.) ........ }
\end{aligned}
$$

$$
\begin{aligned}
& \text { REYNOLDS NO. (D). } \\
& \text { VORIEX FACTOR } \\
& \text { QCNOF }
\end{aligned}
$$

ALPHA (NEG.) ......

$$
\begin{aligned}
& \text { RE (2-DIM.). } \\
& \text { FIRE) CROSSFLOH... }
\end{aligned}
$$

$$
\begin{aligned}
& \text { BETA (NFG.) *..... } \\
& \text { U/V } \\
& \text { RET2-DIM. }
\end{aligned}
$$

$$
\begin{aligned}
& 12.57 \\
& 4.00 \\
& 15.80 \\
& 2 H .00 \\
& 4.00 \\
& \\
& 2.50 \\
& 20.00 \\
& 0.00 \\
& 6.065 E+05 \\
& 1.00 \\
& .03 \\
& 20.00 \\
& 0.00 \\
& .940 \\
& 2.219 E+05 \\
& 5.699 E+04
\end{aligned}
$$

$$
\begin{gathered}
x 1 \\
2.800
\end{gathered}
$$

$\div$ $\frac{5}{2}$ 。

$$
\frac{\pi}{2} \circ
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Po

$$
\begin{gathered}
\text { NCIR } \\
1 \\
\\
\text { NPRNIP } \\
1
\end{gathered}
$$



$$
\begin{aligned}
& \text { UP } \\
& \text { LEFT } \\
& \text { IEHILUCKHISF }
\end{aligned}
$$

$$
\text { (a) Page } 1
$$

$$
\begin{aligned}
& \text { EMKF } \\
& 1.050
\end{aligned}
$$

$$
\begin{gathered}
\text { RGAM } \\
0.000 \\
\\
\text { NVP } \\
0
\end{gathered}
$$

Figure 15．－Output for sample case 1.

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\begin{gathered}
\text { UJN } \\
\because \text { SNOIIdO }
\end{gathered}
$$

Snolitanos רilini
若
INPUT GEOMF IHY CHADACTEMISTICS

ADDITIONAL OUTPIIT PRINTED AT THE FOLLDWING AXIAL STATIONS 4.20
 INPUT (GE
NXH $=$

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| $\times$ | $p$ | 09/nx | $\wedge$ | H |
| :---: | :---: | :---: | :---: | :---: |
| 0.0000 | 0.0000 | - 2 und | 0.0000 | 0.0000 |
| .4?00 | .11an | - нпо $^{\text {a }}$ | . 1750 | - |
| . 7000 | .19an | . 2800 | . 2940 | .09\%0 |
| 2.1000 | -5870 | . 2340 | . 8810 | . 2440 |
| 3.5000 | . 8500 | -1750 | 1.2750 | -4250 |
| 4.9000 | 1.0790 | -1940 | 1.6170 | - 5390 |
| 5.6000 | 1.142n | .1450 | 1.1740 | - 4911 |
| 6.3000 | 1.2810 | .1350 | 1.4280 | . 6410 |
| 7.7000 | 1.46550 | -1240 | 2.1980 | . 1330 |
| 8.4000 | 1.5500 | . 1190 | 2.3260 | .1750 |
| 9.1000 | 1.6310 | . 1100 | 2.4470 | .H160 |
| 10.5000 | 1.7810 | -1020 | 2.6720 | . H 419 |
| 11.9800 | 1.91 ma | -0930 | 2.8760 | -95911 |
| 12.6000 | 1.4770 | - OASO | 2.9650 | .9880 |
| 13.3000 | 2.0340 | -0760 | 3.0510 | 1.9170 |
| 14.7000 | 2.1380 | .0690 | 3.2030 | 1.0680 |
| 15.4000 | 2.1790 | . 0550 | 3.2690 | 1.0900 |
| 16.8000 | 2.2520 | . 0470 | 3.3780 | 1.1260 |
| 17.5000 | $2.279 n$ | . 0.340 | 3.4180 | 1.1400 |
| 18.2000 | 2.2990 | .0700 | 3.44An | 1.1490 |
| 19.0400 | 2.3090 | . 0040 | 3.4640 | 1.1550 |
| 19.1000 | ?. 3040 | -. 0170 | 3.4560 | 1.1520 |
| 20.3000 | 2.2n80 | -. 0300 | 3.4320 | 1.1440 |
| 21.0000 | 2.2650 | -. 0410 | 3.3970 | 1.1320 |
| 22.4000 | 2.2020 | -.0470 | 3.3030 | 1.1010 |
| 23.1000 | 2.1650 | -.0550 | 3.2480 | 1.0830 |
| 24.5000 | 2.0460 | -. 0570 | 3.1280 | 1.0430 |
| 25.9000 | 2.0060 | -. 0550 | 3.0090 | 1.0030 |
| 26.6000 | $1.970 n$ | -.0390 | 2.9550 | . 9850 |
| 28.0000 | 1.973 | -.0340 | 2.8850 | . 4 ¢, 20 |

Flow fielo veloritifs abe calculated at fielo points
.250
.250
.570
.550
.750
.750
1.000
1.050
1.250
1.250

* GEOM **
* CONFIG *:
3:1 ELLIPTICAL CROS:-SFCTIUN hUOPY
aliotitional paneling inpuit

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2.1 \text { OOO }
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3.5000
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$$
5.6000
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$$
6.3000
$$

$$
\begin{aligned}
& \\
& \stackrel{\circ}{\circ} \\
& \vdots \\
& \vdots
\end{aligned}
$$

$$
\text { (d) Page } 4
$$

Figure 15.- Continued.





空

 1 AND 3 INDICATE BODY






















[^2][^3]



|  |
| :---: |
|  |  |
|  |  |


(h) Page 8 Figure 15.- Continued.







(j) Page 10
Figure 15.- Continued.

[^4]









[^5]07452
.144

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.04719
$\stackrel{\sim}{2}$
$\stackrel{8}{8}$

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## 0 0 0 0 0



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\text { Stratford separation cultadion (LAMILAK) }+(S)=\text { ofigth }
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\begin{aligned}
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\end{aligned}
$$
\]

SUMMARY OF PRESSIIDE UISTRIRUITIC
or siot:
-r SIDE:

$$
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& \text { NV } \\
\text { OY SIOE: } & 1 \\
-Y \text { SIDE: } & 2
\end{array}
$$

$$
\begin{aligned}
& \text { ION PI. } \\
& \text { FSSIMRF } \\
& \text { ION } \\
& \text { ION PI. } \\
& \text { ISSURF } \\
& \text { ION } \\
& \text { AND STRE } \\
& \text { GAM/V } \\
& .4503 \\
& -.45 n 3
\end{aligned}
$$

Figure 15.- Continued.

RG
1.32024
1.32024
$2 C$
.94354
.94354
$r C$
.92345
-.92345


₹


| $J$ | $\checkmark$ | 2 | l/vo | v/vo | w/vo | vitvo |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | .2410 | . 7230 | 9.1765E-01 | -3.4614E-02 | 1.2126E-01 | 1.2610E-01 |
| 2 | .2610 | .9840 | 9.1947E-01 | -3.03?4E-02 | 1.4284E-01 | 1.4606E-01 |
| 3 | .4AP\% | . 7 130 | 9.158SE-01 | -5.7109E-02 | 1.0H2SE-01 | 1.2239E.01 |
| 4 | .4R2n | . 98400 | 9.16R4F-01 | -6.8524E-02 | 1.4.33E-01 | 1.5891E-01 |
| 5 | .7230 | . 4420 | 9.1441E-01 | -3.5174E-02 | H. $14.34 \mathrm{E}-02$ | 8.8797t-02 |
| 6 | . 72.30 | .964n | $9.1474 E-01$ | -1.2153E-01 | 1.3185E-01 | 1.8377E-01 |
| 7 | . 9840 | -4820 | $9.1022 E-01$ | 5.2814E-02 | 5.2321E-02 | $7.4343 E-02$ |
| 8 | . 9647 | - 7640 | 9.1417F-01 | -2.3093E-01 | 1.4919E-01 | 2.1493E-01 |
| 9 | 1.2050 | -4A20 | $9.0519 E-01$ | 1.5568t-01 | 3.4164E-01 | $3.7544 \mathrm{E}-01$ |
| 10 | 1.2050 | . 9640 | R.9R54E-01 | -3.0422E-01 | $3.634 \mathrm{E}-01$ | 4.7720E-01 |

$$
\begin{array}{r}
2 \\
.63474 \\
.63474
\end{array}
$$

$$
\begin{array}{lr}
Y 0 / 4 & 20 / 4 \\
.7509 & .3174 \\
-.7509 & .3174
\end{array}
$$

$$
\begin{gathered}
=\quad b . \\
\mathrm{cH} \\
.401 \\
-.2<4 \\
-.210 \\
\mathrm{cp} \\
-.158 \\
-.229 \\
-.210 \\
\\
V T / V \\
.1369 \\
.7359
\end{gathered}
$$

force and moment coefficients in unrolled body couruinates
$\quad \mathrm{cm} \mathrm{\times 0}$
-.00000

$$
\begin{aligned}
& 8 \\
& 0 \\
& i
\end{aligned}
$$

(w) Page 30
Figure 15.- Concluded.

$$
\begin{gathered}
\text { OCP } 10 \mathrm{Dx} \\
0.000 \\
.343 \\
\text { OCPI } 10 \mathrm{DX} \\
0.000 \\
.343 \\
2 C \\
.1429 \\
.1429
\end{gathered}
$$



Figure 16. - Measured and predicted pressure distribution on an ogive-cylinder at $\alpha=0^{\circ}$.


Figure 16. - Continued.


Figure 16. - Concluaded.


Figure 17.- Measured and predicted circumferential pressure distribution on an ogive-cylinder body at $\mathrm{M}_{\infty}=1.6, \alpha_{\mathrm{C}}=20^{\circ}$.


Figure 17.- Concluded.


Figure 18.- Predicted vortex wake on the lee side of an ogive cylinder.


Figure 19.- Measured and predicted body vortex strength on an ogive cylinder.


Figure 20.- Measured and predicted downwash distribution on the lee side of an ogive cylinder at $M_{\infty}=2, \alpha_{C}=15^{\circ}$.


Figure 21.- Measured and predicted downwash distribution on the lee side of an ogive cylinder at $M_{\infty}=2, \alpha_{c}=15^{\circ}, x / D=10$.


Figure 22.- Measured and predicted downwash distribution on the lee side of an ogive cylinder at $M_{\infty}=3, \alpha_{c}=15^{\circ}$.


Figure 23.- Neasured and predicted downwash distribution on the lee side of an ogive cylinder at

$$
M_{\infty}=3, \alpha_{c}=15^{\circ}, x / D=10 .
$$



Figure 23.- Concluded.

(a) $\phi=0^{\circ} \quad x / L \cong 1.0$
Figure 24.- Measured and predicted vortex patterns on a 3:1 elliptic cross section body at $M_{\infty}=2.5$ and $\alpha_{c}=20^{\circ}$.
O

Figure 24.-Concluded.

(a) $\phi=0^{\circ}$

(b) $\phi=45^{\circ}$

Figure 25.- Measured and predicted circumferential distribution on a 3:l elliptic cross section missle at $M_{\infty}=2.5, \alpha_{c}=20^{\circ}, x / L=0.60$.






$$
\begin{aligned}
M_{\infty} & =1.5 \\
\alpha_{c} & =15^{\circ} \\
\phi & =0^{\circ}
\end{aligned}
$$



(a) $\phi=0^{\circ}$

Figure 29.- Measured and predicted circumferential pressure distribution on a 2:l elliptic cross section body at $M_{\infty}=1.5, \alpha_{c}=15^{\circ}$.


$$
\begin{aligned}
M_{\infty} & =1.5 \\
\alpha_{c} & =15^{\circ} \\
\phi & =22.5^{\circ}
\end{aligned}
$$



(b) $\phi=22.5^{\circ}$

Figure 29.- Continued.

(c) $\phi=45^{\circ}$

Figure 29.- Concluded.

Figure 30 - Measured and predicted circumferential pressure distribution on a 2:1 elliptic cone forebody at $M_{\infty}=1.7, \alpha_{C}=20.35^{\circ}, \mathrm{x} / \mathrm{L}=0.5$.

Figure 31.- Measured and predicted circumferential pressure distribution on a 2:1 elliptic forebody at $M_{\infty}=1.7, \alpha_{c}=20.36^{\circ}, \mathrm{x} / L=0.5$.




Figure 33.- Predicted vortex wake on a lobed body at $M_{\infty}=1.7, \alpha_{C}=20.36^{\circ}, x / L=0.50$.


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[^0]:    $(24)+\ddagger$| YFV | ${ }^{10}$ |
    | :--- | :--- |
    |  | ZFV |
    |  |  |
    |  |  |
    |  |  |

    

[^1]:    

[^2]:    
    

[^3]:    

[^4]:    
    
    

[^5]:    

[^6]:    force and moment coffficients - pagssurf integration

