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"DESIGN OF HELICOPTER ROTOR BLADES FOR OPTIMUM DYNAMIC CHARACTERISTICS"

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1. INTRODUCTION

This report is the fourth semi-annual report of a research project concerned with the optimal design of helicopter rotor blades. The goals of the project and the approach adopted have been described in previous reports, and these descriptions will not be repeated here. The present report will be confined primarily to describing several studies comparing the forced responses of an initial (i.e., non-optimized) blade to those of a final (optimized) blade.

At the 39th Annual Forum of the American Helicopter Society last spring, we presented a paper describing our work on optimal rotor blade design. In the ensuing discussion, several questions were raised about the manner in which the optimal design problem had been formulated. One question was whether or not the forced response of the blade can be adequately controlled, as we have assumed, by our approach of "frequency placement", that is, of restricting the natural frequencies of the blade to lie within narrow intervals located away from certain integer multiples of the rotor speed. A second question was whether or not aerodynamic damping substantially reduces the resonant peaks, in which case concern about avoiding resonances through proper selection of frequency windows would be unnecessary. Similar questions had been mentioned in the original proposal for this project, when it was stated that the sensitivity of the optimal design to the choice of frequency window would be studied, and if it was found that frequency placement did not in fact reduce vibrations, then some other objective such as minimizing root shear would be

explored. Because of the timely coincidence of the questions raised at the Forum and the intent expressed in our original proposal, we decided the time had arrived to investigate the appropriateness of "frequency placement".

The investigation of this question was carried out through two, somewhat overlapping, problems. First, the forced responses of an initial (i.e., non-optimized) design were compared to those of a final (optimized) design as the frequency of the forcing function was changed; cases with and without aerodynamic damping were considered. Next, the responses of initial and final designs were evaluated as one natural frequency was varied (the others were held fixed), and a forcing function containing harmonics of the rotor speed was applied. Again, cases with and without aerodynamic damping were considered. The general finding from these studies is that frequency placement is a viable means of reducing vibration, although it is by no means the only method and should be used in conjunction with others.

In the penultimate section of the report, several topics are briefly described in which studies have been initiated during the reporting period, but not yet completed. The final section of the report contains a sketch of plans for future work.

2. RESPONSE OF STARTING AND OPTIMAL DESIGNS FOR VARYING FORCING FREQUENCIES

In this section of the report, the response of both the initial and final (optimal) designs to an external forcing function is studied as the frequency of the forcing function is varied. Blades both with and without aerodynamic damping are

considered. To formulate these problems, consider the forced behavior of a rotor-blade.

The equation of motion for the finite-element representation of a rotor blade subjected to an external excitation may be written in matrix form as

$$[M]\{\ddot{X}(t)\} + [C]\{\dot{X}(t)\} + [K]\{X(t)\} = \{F(t)\},$$

where

$[M]$ = mass matrix,

$\{X(t)\}$ = column vector of nodal displacements,

$[C]$ = damping matrix,

$[K]$ = stiffness matrix, and

$\{F(t)\}$ = forcing function column vector.

The forcing function may in turn be expressed as

$$\{F(t)\} = \{V_0\} e^{i\omega t},$$

where

ω = forcing frequency, and

V_0 = forcing amplitude.

After some calculation, it can be shown that the amplitude of the response -- written as $\{X\}$, independent of t -- can be given as

$$\{X\} = [K + i\omega C - \omega^2 M]^{-1} \{V_0\}$$

In the present section, the flapping response is considered. The in-plane response is inferred from the results without

damping, since there is little aerodynamic damping in the inplane direction.

Fig. 1 shows a plot of the forcing amplitude V_0 [Ref.1] used in the study. Given the forcing amplitude, we can calculate the response of each node of the finite-element representation of the blade as the value of the forcing frequency, W , is varied. The tip (finite-element node farthest from the hub) response is of special interest. Before the results obtained from this study are presented, it is useful to examine the frequency placement results which are described in the Third Semi-Annual Report (pp.21-26). The results for the frequencies (in units of cycles/rev) are, for flapping mode only,

MODE	INITIAL DESIGN	FINAL DESIGN
1st	1.18	1.18
2nd	3.22	3.09
3rd	5.89	5.67

Blade dimensions are given in Table 8, p. 26, of the Third Semi-Annual Report.

The frequencies in the above table correspond to the symmetric modes of a teetering rotor. Thus, only even harmonics of the rotor speed have been considered as forcing frequencies. As a result, the optimized blade (Final Design) finds the third mode moved away from the critical 6.0/rev (from 5.89 to 5.67). Similarly, the movement of the second mode to 3.09/rev removes it from 2.0 and 4.0/rev. In the comparison study to follow, however, we will apply the entire spectrum of frequencies to this blade (not just even harmonics). Thus, the "Final Design" can no longer

be considered optimum. A comparison of the two blades, however, does indicate the strong effect of resonance because each case has a distinct resonance (6 and 3/rev)

We shall now consider the results of the present study. Fig. 2 shows the tip responses of both the initial design and final (i.e., optimized) design as functions of the forcing frequency. Aerodynamic damping has been neglected (Alternatively, the results can be interpreted as giving the inplane response.). It can be seen that near 1.18 cycles/rev, the responses of the two designs are very similar. However, the responses corresponding to the second and third modes differ significantly. For example, in the second mode the peak of 3.22 cycles/rev (initial design) moves to 3.09 cycles/rev (final design). Similarly, the peak of the third mode moves from 5.89 cycles/rev to 5.67 cycles/rev, which is especially important since it is highly desirable to keep the frequency away from the integer frequency of 6 cycles/rev. We conclude from these results that the frequency placement approach does have a significant effect on the forced tip response when damping is not considered.

Next, the effect of aerodynamic damping is considered, that is, the results to be presented correspond to flapping. Mathematical details of the damping formulation are available in the thesis by Ko [Ref.2]. The effect of aerodynamic damping on reducing the resonant peaks of the tip response of the initial blade is shown in Fig. 3. Fig. 4 shows the damped responses of both the initial and optimized blades so that the effect of frequency placement can be studied. It is interesting to observe

here that when damping is included, no apparent advantage is gained by optimizing the blade, at least in terms of reducing the tip response, except in the range of 3-4/rev, in which a thirty-five percent reduction occurs. However, we must also examine the effect of optimization when the response is measured by the average shear force existing in the blade.

Consequently, the shearing force in the blade is considered next. As a measure of the average shear in the rotor, we consider the sum of the squares of the shear force (abbreviated SSS),

$$SSS = Y_1^2 + Y_2^2 + \dots + Y_{10}^2$$

In this equation, Y_i represents the shear force at node i in the (ten-element) finite-element model. Note that the root shear is necessarily included as one of the terms on the right-hand side of the equation, so that a large value of root shear will cause SSS to also be large.

Fig. 5 shows the variation of SSS with respect to the forcing frequency for the initial design with and without aerodynamic damping. Fig. 6 shows the same quantities for the final (optimized) design. Fig. 7 compares the quantity SSS corresponding to initial and final designs when aerodynamics is considered. Inspection of these figures shows that, in contrast to behavior of the tip response, the shear response is significantly affected by optimizing the blade, even when aerodynamic damping is included. The 3/rev loads are increased by fifty percent due to the movement of w_2 from 3.22 to 3.09/rev. Similarly, the 6/rev loads are reduced by seventy percent due to the movement of w_3

from 5.89 to 5.67/rev. Thus, even with damping, frequency placement is a powerful driver of loads. It follows that frequency placement can be justifiably considered an important part of blade optimization.

3. BLADE RESPONSE TO HARMONICS OF ROTOR SPEED

In the study just described, the response of the blade to changes in the forcing frequency was considered. Now we consider a different approach. In effect, we examine how the blade responds to a forcing function "during the optimization procedure" -- in the sense that during optimization, the optimization algorithm varies the natural frequency of the blade (to force it to satisfy the frequency constraints). In obtaining the results to be presented next, we simulated the optimization procedure by varying the natural frequency. Thus we can observe what happens to the forced response during frequency placement.

The formulation of the approach is as follows. Through appropriate transformations (described in the Appendix), the system mass matrix can be written as

$$[M] = [U]^{-T}[U]^{-1}$$

and the system stiffness matrix as

$$[K] = [M][U] \text{diag} [(w_i^2)] [U]^T [M]$$

in which w_i are the natural frequencies of the system, $[U]$ is a matrix whose columns are eigenvectors and the notation "diag" indicates a diagonal matrix (all off-diagonal terms vanish). From

examination of these expressions, it can be seen that the stiffness and mass matrices can be considered functions of the natural frequencies. Thus it becomes possible to fix all frequencies but one, and then study the response of the system as that one frequency is varied. In particular, the response to the following forcing function will be studied:

$$\{F(t)\} = \{V_1\}e^{iWt} + \{V_2\}e^{i2Wt} + \dots + \{V_n\}e^{inWt}$$

where

W = the rotor speed, and

$$\{V_n\} = (1/n)\{V_0\},$$

and $\{V_0\}$ was defined previously in Fig. 1. Since the arguments of the exponentials are integer multiples of W , resonance will occur at harmonics of the rotor speed. The particular forcing function given above is known from empirical observation to provide an approximate, but physically realistic representation of the radial and harmonic variations of the amplitude of the load on a real blade. As in Section 2, the blade response will be defined through the tip displacement and the sum of the squares of the shears, except that, here, the $n = 1$ term has been omitted from the expressions for calculating tip displacement and shears because this term represents a tip-path plane tilt that is controlled by the pilot for trimming purposes. It is not part of the true vibratory loads we are considering.

Results for the problem just formulated are shown in Fig. 8, where the sum of the squares of the shears is plotted as a function of w_2 , the second natural frequency, with the other

natural frequencies being fixed. This figure corresponds to the initial blade design (Blade dimensions are given in Table 8, p. 26, of the Third Semi-Annual Report.) Fig. 9 shows the same quantity for the case where the third natural frequency is varied. It is interesting to note that the response curve for the damped case in Fig. 9 lacks resonant peaks -- apparently the damped response is so completely dominated by the resonance of the second natural frequency, which is fixed near 3/rev, that the (damped) resonant peaks for the third frequency are negligible by comparison. It is worth mentioning, in passing, that the value of the damped response corresponding to $w_1 = 1.18$, $w_2 = 3.22$, and $w_3 = 5.89$ can be read from Fig. 8 as well as Fig. 9 and can be seen to be the same (approximately 6.0 lbs). This observation provides a re-assuring check that the figures represent actual behavior and not a programming error.

For the final (optimal) design, the analogous quantities are plotted in Figs. 10 and 11. Again, no resonant peaks are present in the damped response when the third natural frequency is varied. Comparison of magnitudes of ordinates in Figs. 8 and 10 (no damping) shows that the overall shear measure is reduced in the final design in the regions away from resonance. Also, the choice of scale on the vertical axis in Fig. 10 highlights the effect of frequency placement. Note that by inspecting Figs. 8-11, a designer may select the design frequency which minimizes the average shear as measured by the SSS.

One of the most interesting results of Fig. 10 is information about the width of valleys and peaks, since

this gives design information. First, let us examine the no-damping curve (inplane response). Here, the minimum points are nearly at the centers of the regions (2.55/rev) and (3.55/rev). The frequency windows to maintain no more than thirty percent increase in loads are 2.40 - 2.70/rev and 3.40 - 3.70/rev (plus or minus 0.15/rev) -- a fairly narrow window. For the damped curves (flapping response), minima are also near the one-half points, but the window for thirty-percent increases are much wider -- 2.20 - 2.90/rev and 3.20 to 3.80/rev (plus or minus 0.30/rev). Stated another way, inplane frequencies should be no closer than a 0.4/rev from integers, but flapping frequencies may be as close as 0.2 from an integer. It should be emphasized that these observations apply to this particular example and may not be generalized for other frequency constraints. In future work, we will apply similar reasoning to optimized articulated rotors for which the frequency spectrum is more meaningful.

Another conclusion to be drawn from the above results is that the undamped response curve has very flat-bottomed "valleys" when one of the fixed frequencies is near an integer value (cf. Figs. 10 and 11).

4. OTHER STUDIES CONDUCTED DURING THE REPORTING PERIOD

As an extension of work reported previously, data were obtained for six rotor-blades produced by several different helicopter companies, and considerable effort was expended in attempting to choose box-beam dimensions and other stiffness and mass parameters in our finite-element model in order to match the natural frequencies of these given blades. The motivation for

this work is, once a given blade can be properly represented by the finite-element model, to improve it through the application of optimization techniques. The task of matching frequencies necessarily proceeds by a certain amount of trial-and-error and is, as a result, time-consuming and tedious. The results of this frequency-matching effort will be described in the final report.

Another study conducted during the reporting period has been concerned with the application of an alternative optimization technique -- the optimality-criteria method. This method, a generalization of the traditional stress-ratio approach to improving a structural design, has received considerable attention among structural optimizers in the last five to ten years [Refs.3-5], and thus it appears appropriate to make at least a preliminary investigation of its applicability to rotor-blade design. At this writing, however, our efforts to implement the optimality-criteria method have not been successful; since there exist a number of different ways of implementing the method, one should not conclude that it cannot be made to work for rotor-blade design. Only the particular implementation we have chosen appears in doubt. At present, we have no plans for continuing work on this method, since it threatens to divert effort from more promising topics.

Yet another topic of study during the reporting period was the effect on natural frequency calculations when secondary structural items such as shear deformation, restraint of warping during twist, and filler stiffness are ignored. A simplified elliptic blade profile, approximating a true helicopter blade,

was studied to obtain rough estimates of modelling errors caused by neglect of the secondary items. It was concluded that the mathematical model we have been using should be quite accurate (perhaps only a one percent change at most due to the consideration of secondary items.). However, it is imperative that accurate filler properties, dimensions, and locations be known in order to represent the mass distribution with a reasonable degree of precision.

A final topic of study during the reporting period has been the implementation of a faster subroutine for eigenvalue calculation. Computer code for the subspace iteration algorithm has been obtained and integrated into our blade analysis program. Difficulties involving missing eigenvalues have surfaced, however, and further development is required.

5. FUTURE WORK

In addition to continuing work in the area described above (matching data for some actual rotor blades to finite-element models, and then optimizing the blades), we also plan to work in another area: that of investigating the validity of the dimensional constraints we have chosen. In previous reporting periods, we have described many examples in which constraints on the thicknesses of the walls of the box beam have been present. It can be concluded that such constraints can be handled in an optimization procedure with little difficulty. Another, less clearly defined, type of dimensional constraint is that of constructibility; for example, if an actual blade is to be constructed, abrupt thickness changes from one finite element to

the next present manufacturing difficulties. In the remaining year of the research project, one area of investigation will be to formulate, through consultation with helicopter manufacturers, realistic constraints for constructibility.

6. APPENDIX - Derivation of Mass and Stiffness Matrices
as Functions of Natural Frequencies

Define

$$[K^*] = [M]^{-1/2} [K] [M]^{-1/2},$$

and construct a square matrix $[U^*]$ by using the eigenvectors of $[K^*]$ as columns. If the eigenvectors are normalized to the identity matrix, that is, if

$$[U^*]^T [U^*] = [I],$$

it then follows that

$$[U^*]^T [K^*] [U^*] = \text{diag} [(w_i^2)],$$

where w_i^2 are the eigenvalues of $[K^*]$.

Next, let

$$[U] = [M]^{-1/2} [U^*],$$

from which it follows that

$$[U]^T = [U^*]^T [M]^{-1/2},$$

$$[U]^{-1} = [U^*]^T [M]^{1/2},$$

$$[U]^T [K] [U] = \text{diag} [(w_i^2)],$$

and

$$[U]^T [M] [U] = [I].$$

Finally, then, the stiffness and mass matrices can be written as functions of the eigenvalues, w_i^2 :

$$\begin{aligned}
[M] &= [U]^{-T}[U]^{-1} \\
&= [M]^{1/2}[M]^{1/2},
\end{aligned}$$

and

$$\begin{aligned}
[K] &= [U]^{-T}\text{diag}[(w_i^2)] [U]^{-1} \\
&= [M]^{1/2}[U^*]\text{diag}[(w_i^2)] [U^*]^T[M]^{1/2} \\
&= [M][U]\text{diag}[(w_i^2)] [U]^T[M].
\end{aligned}$$

Note that the eigenvectors, $[U]$, and eigenvalues, w_i^2 , appearing on the right-hand side were originally calculated from the stiffness and mass matrices, $[K]$ and $[M]$. If we consider only relatively small changes in the frequencies, w_i , then the eigenvectors should be relatively unchanged. Thus the last two equations for $[M]$ and $[K]$ with $[U]$ held fixed can be considered as expressing the mass and stiffness matrices as explicit functions of the natural frequencies.

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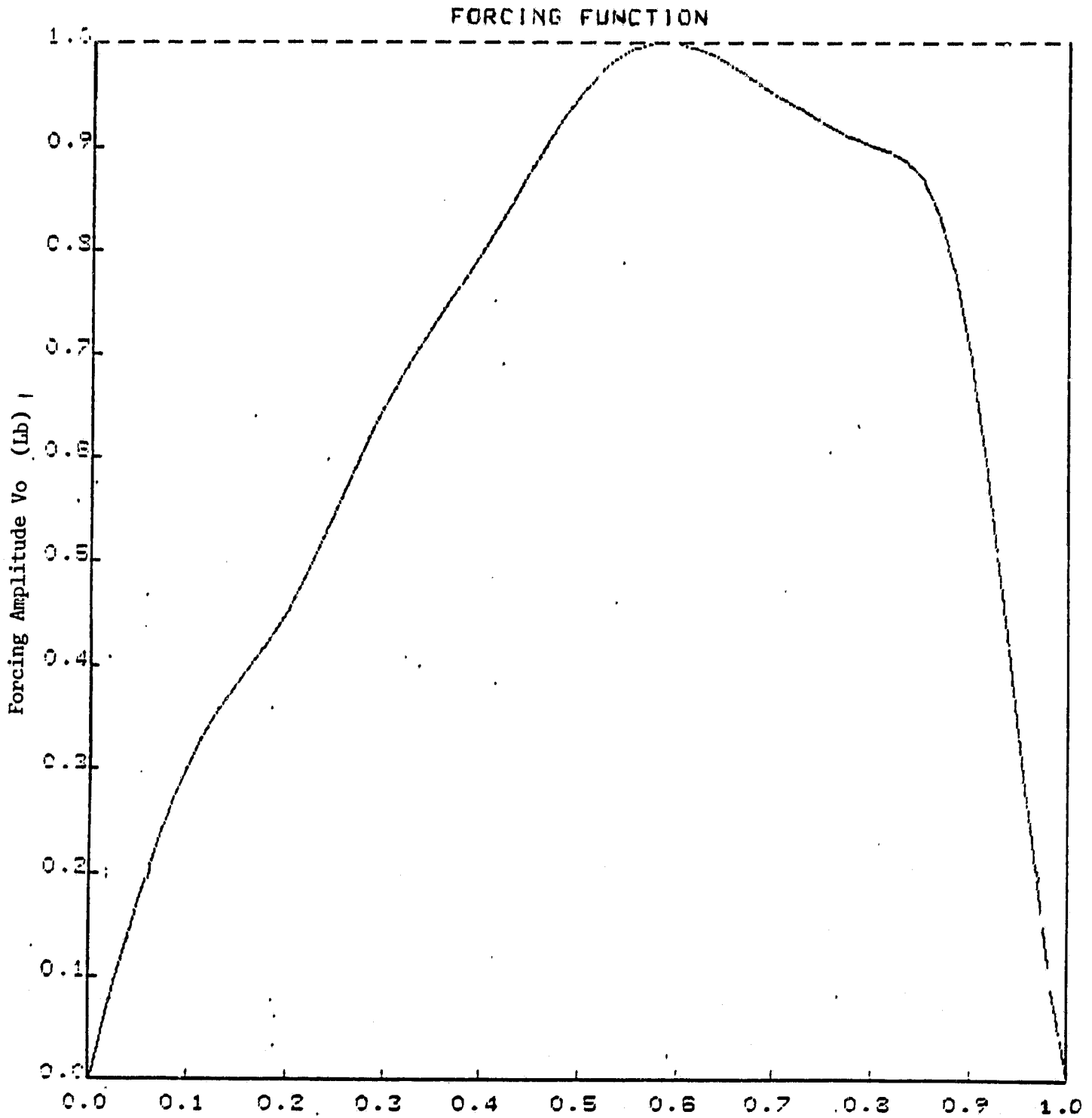


Fig 1 Blade Station (x/R)
Radial Variation of Forcing Function

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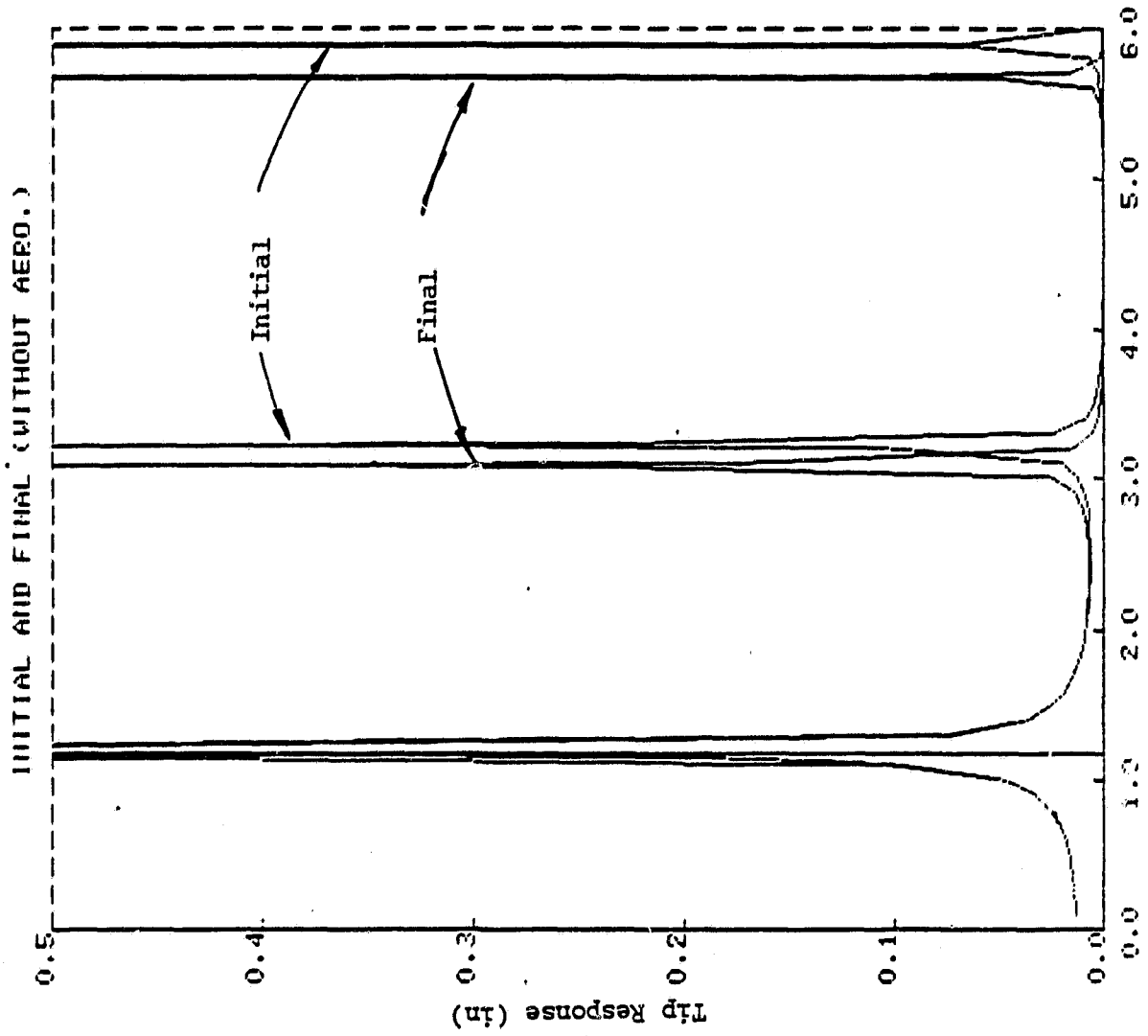


Fig 2 Forcing Frequency (No./Rev)
Tip Response Versus Forcing Frequency for Both Initial and Final Designs Without Damping

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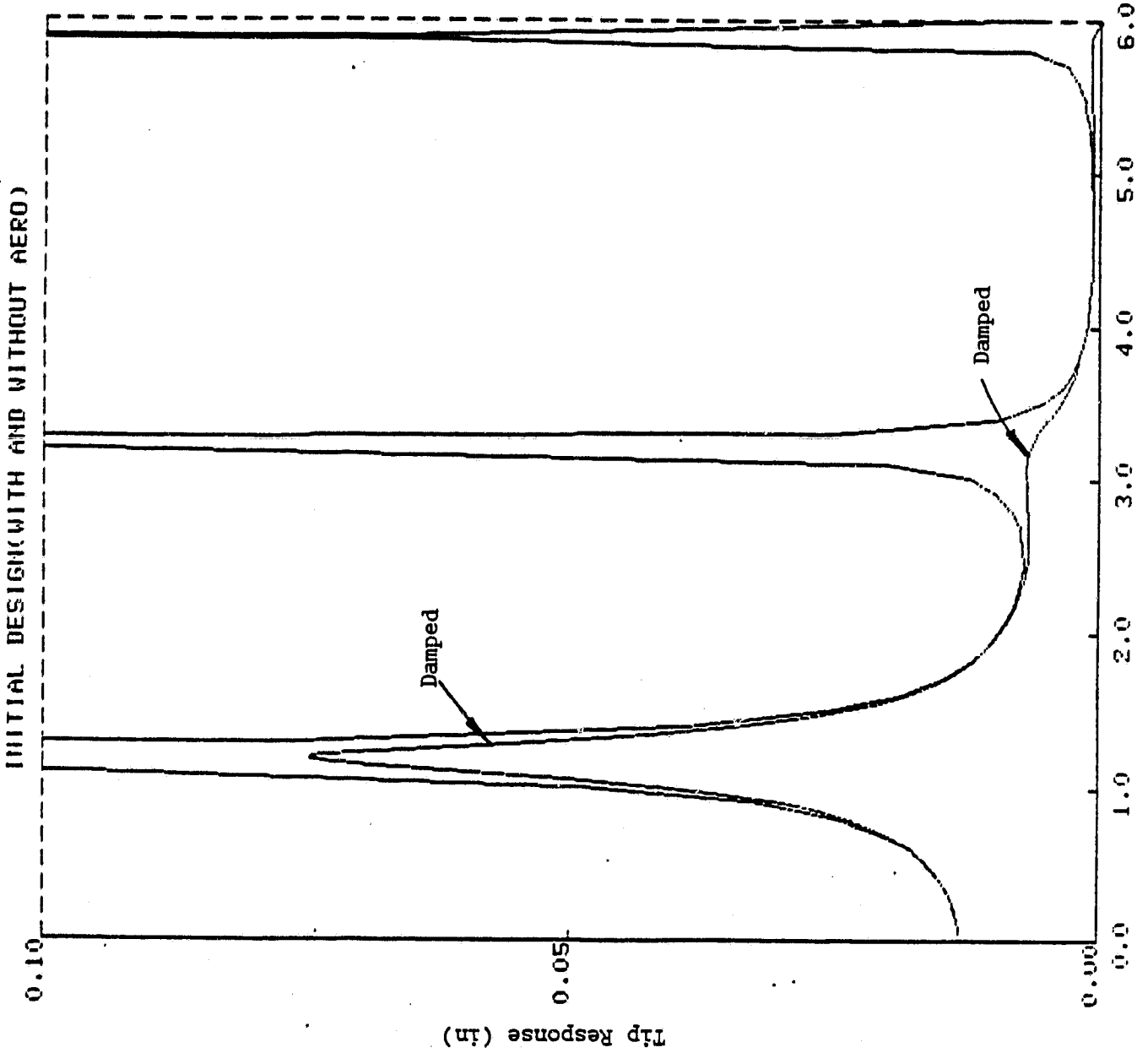


Fig 3 Forcing Frequency (No./Rev)
Tip Response Versus Forcing Frequency for Initial Design Both With and Without Damping

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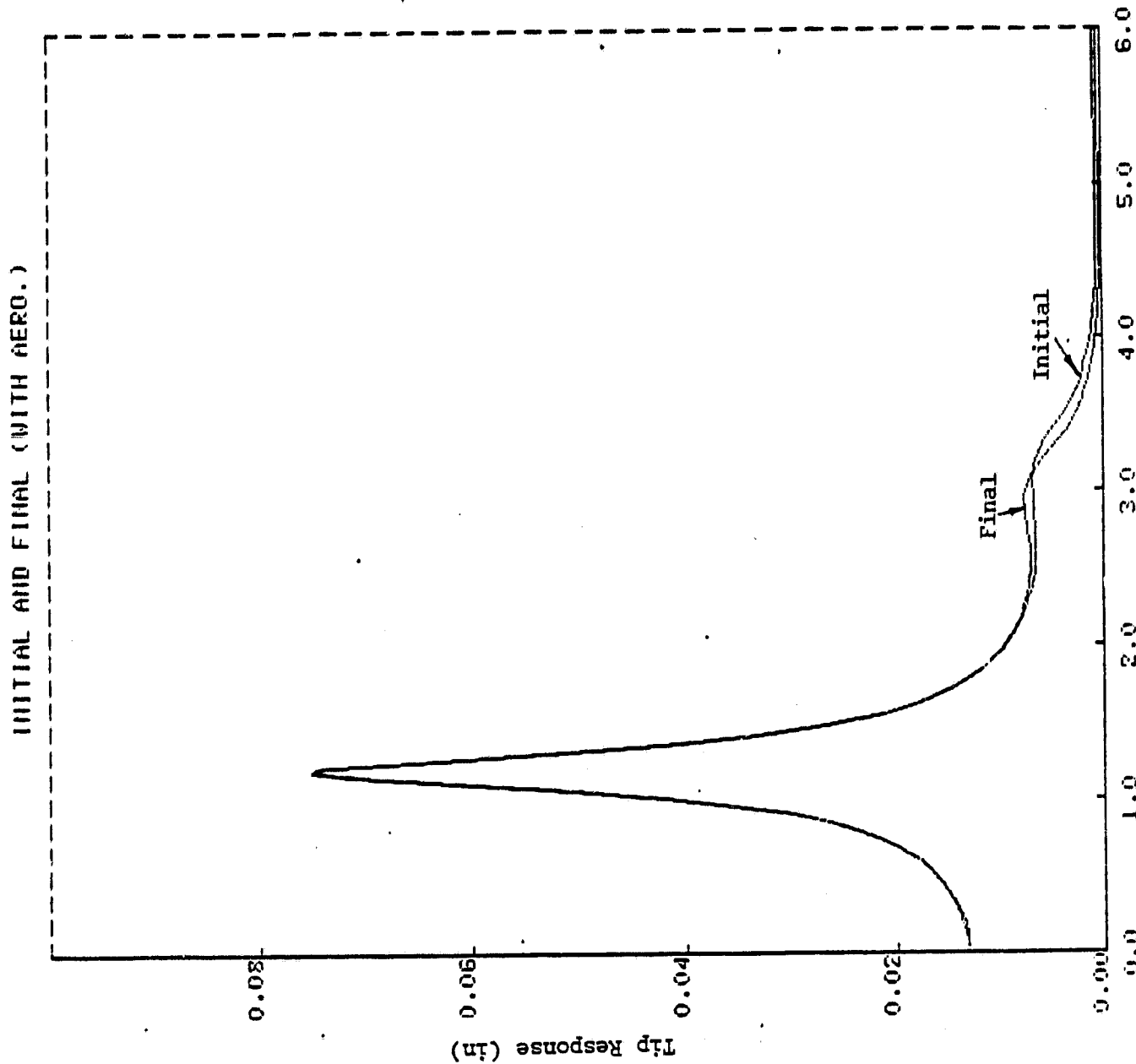


Fig 4 Forcing Frequency (No./Rev)
Tip Response Versus Forcing Frequency for Both Initial and Final Designs With Damping

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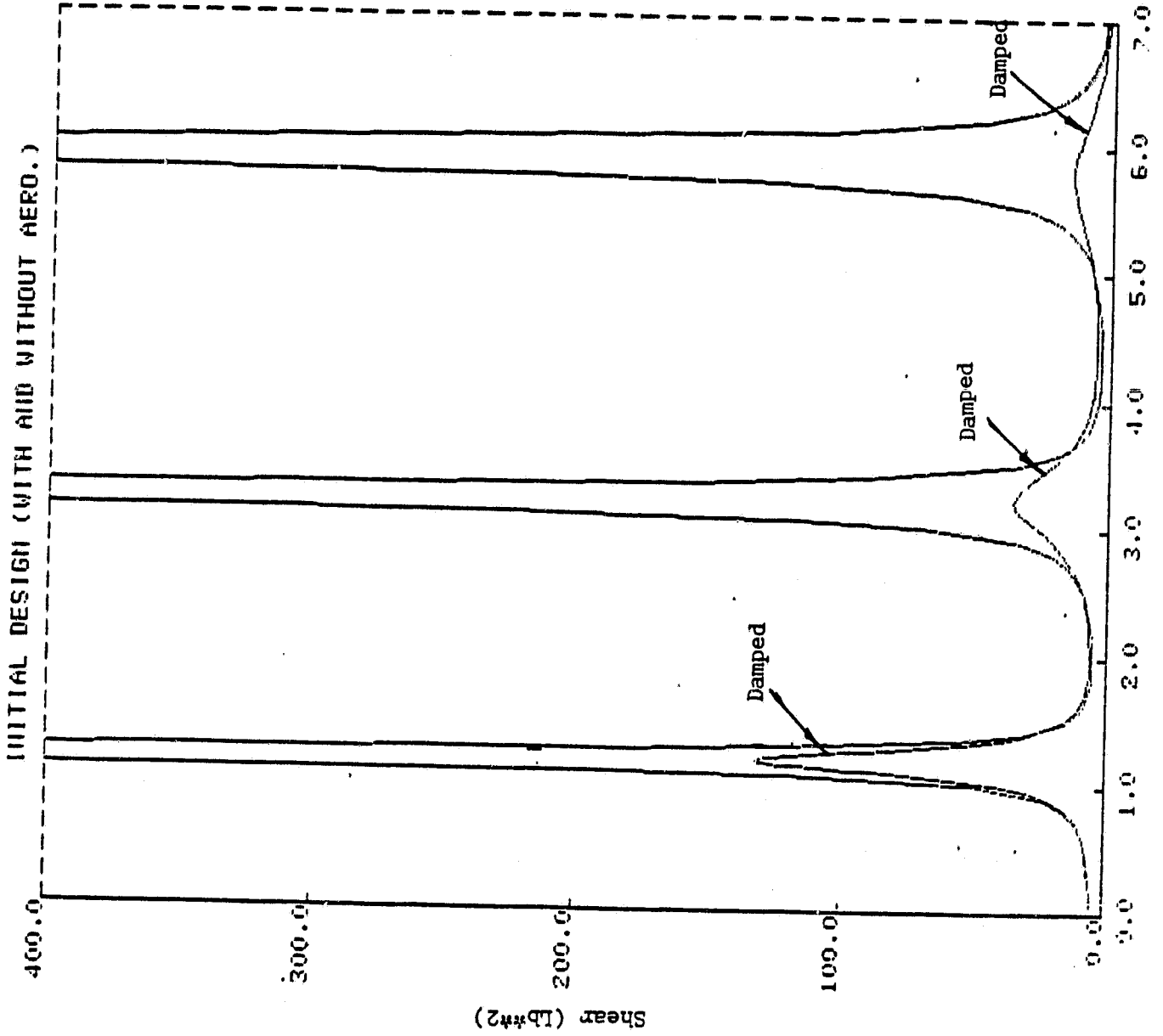


Fig 5 Forcing Frequency (No./Rev)
Sum of Squares of Shears Versus Forcing Frequency for Initial Design With and Without Damping

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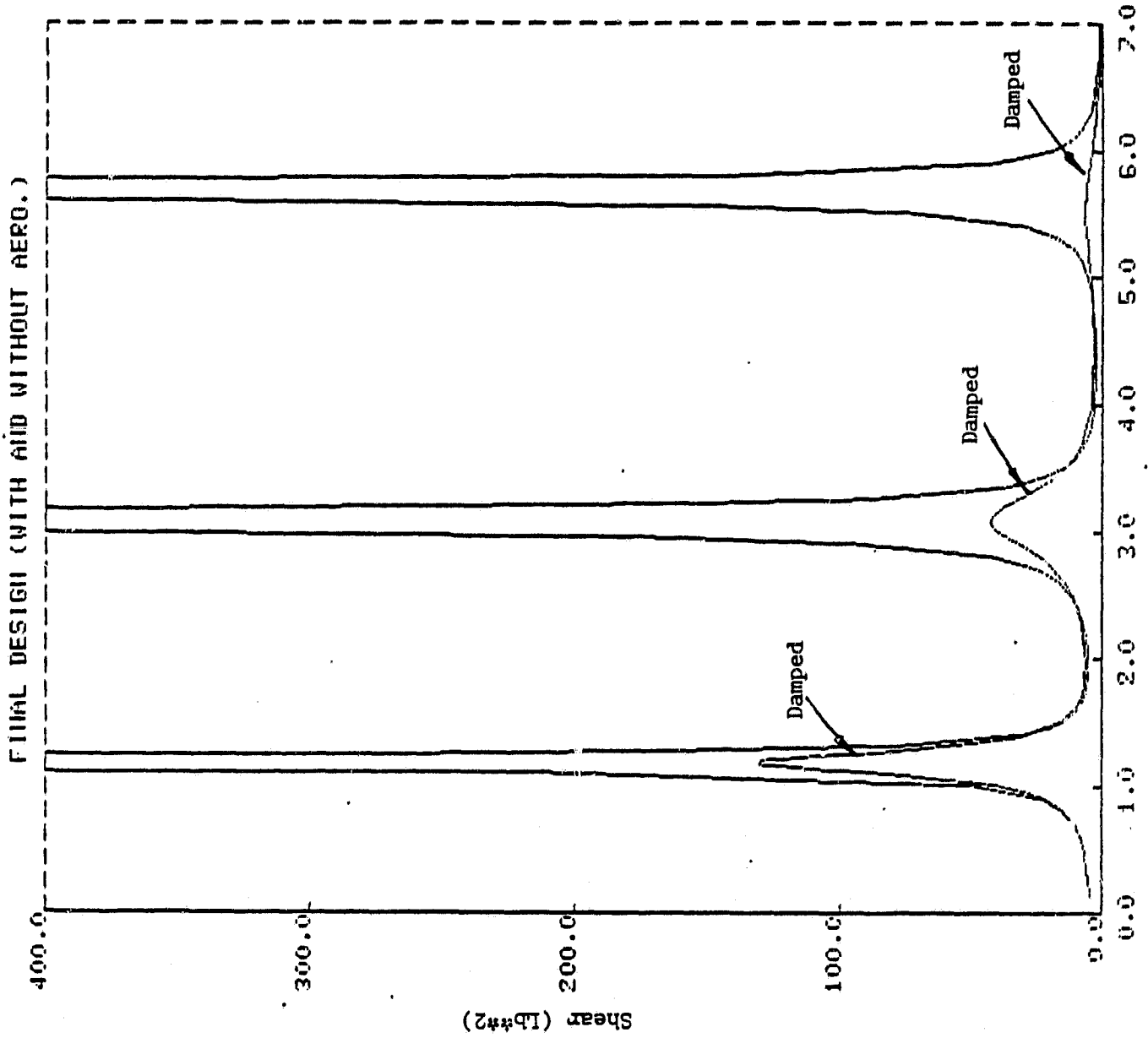
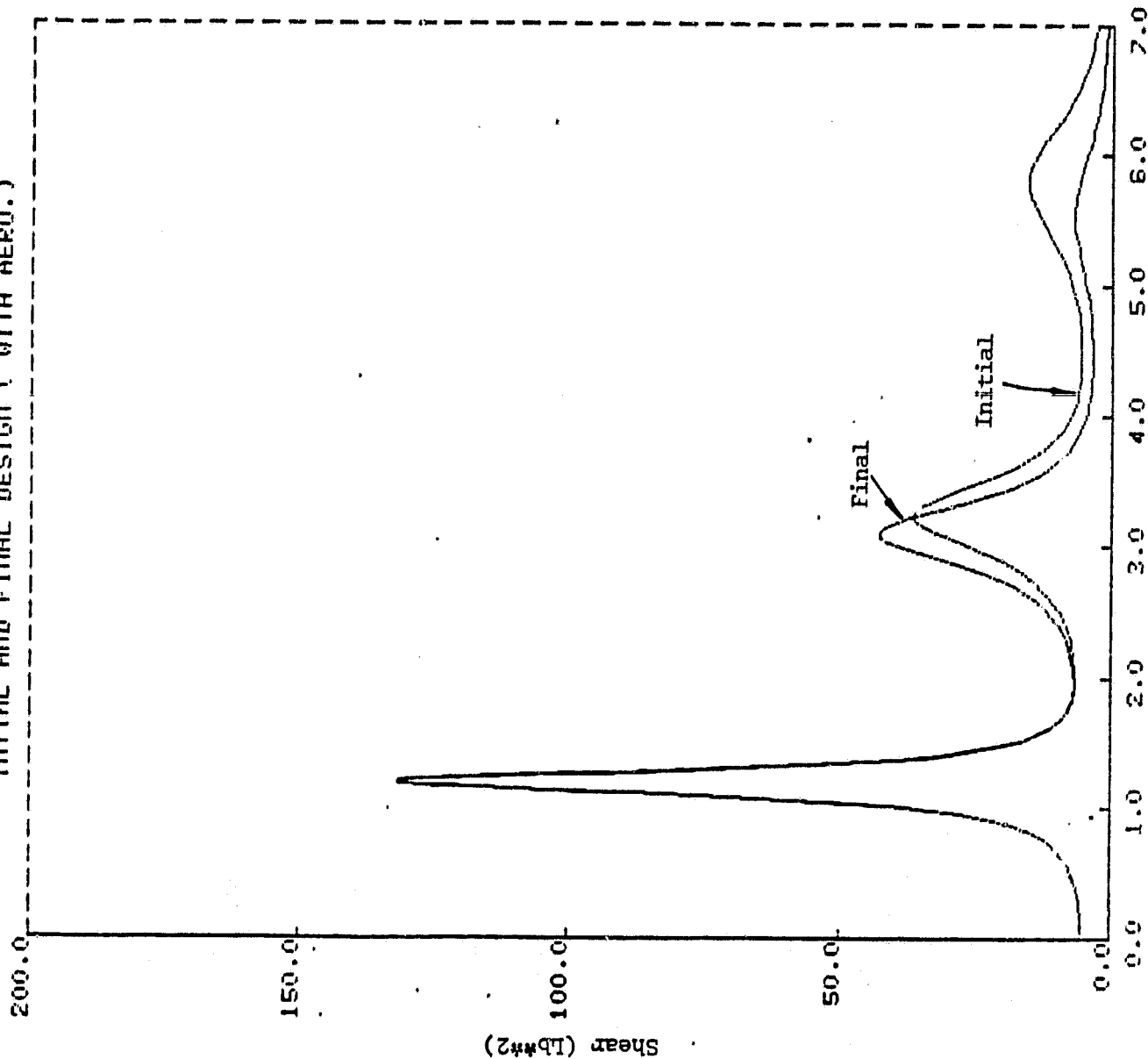


Fig 6 Forcing Frequency (No./Rev)
Sum of Squares of Shears Versus Forcing Frequency for Final Design With and Without Damping

INITIAL AND FINAL DESIGN (WITH AERO.)



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Fig 7 Forcing Frequency (No./Rev)

Sum of Squares of Shears Versus Forcing Frequency for Both Initial and Final Designs With Damping

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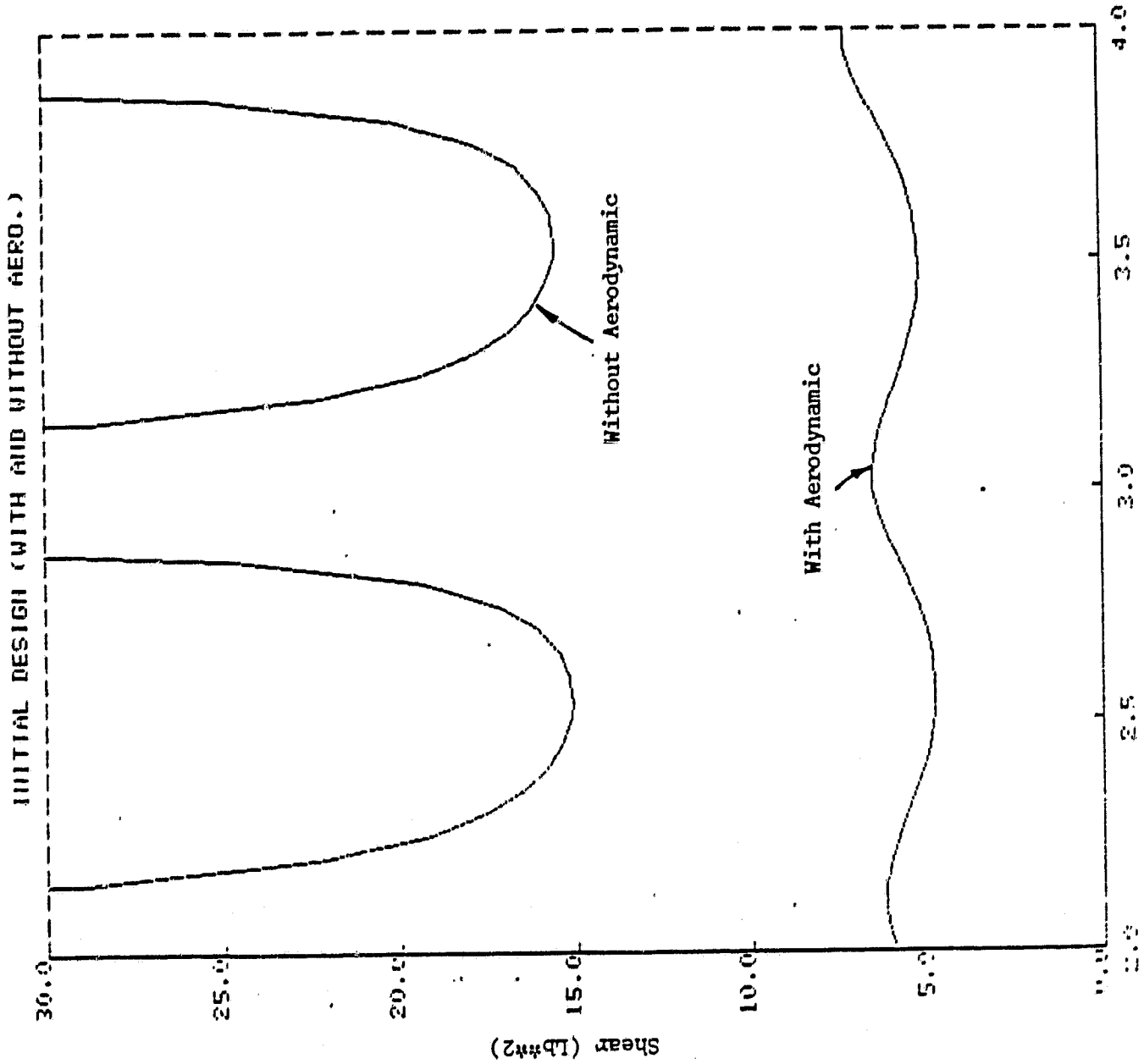


Fig 8 W W2 (No./Rev) W1 and W3 are fixed.
Sum of Squares of Shears Versus Second Natural Frequency for Initial Design Both With and Without Damping

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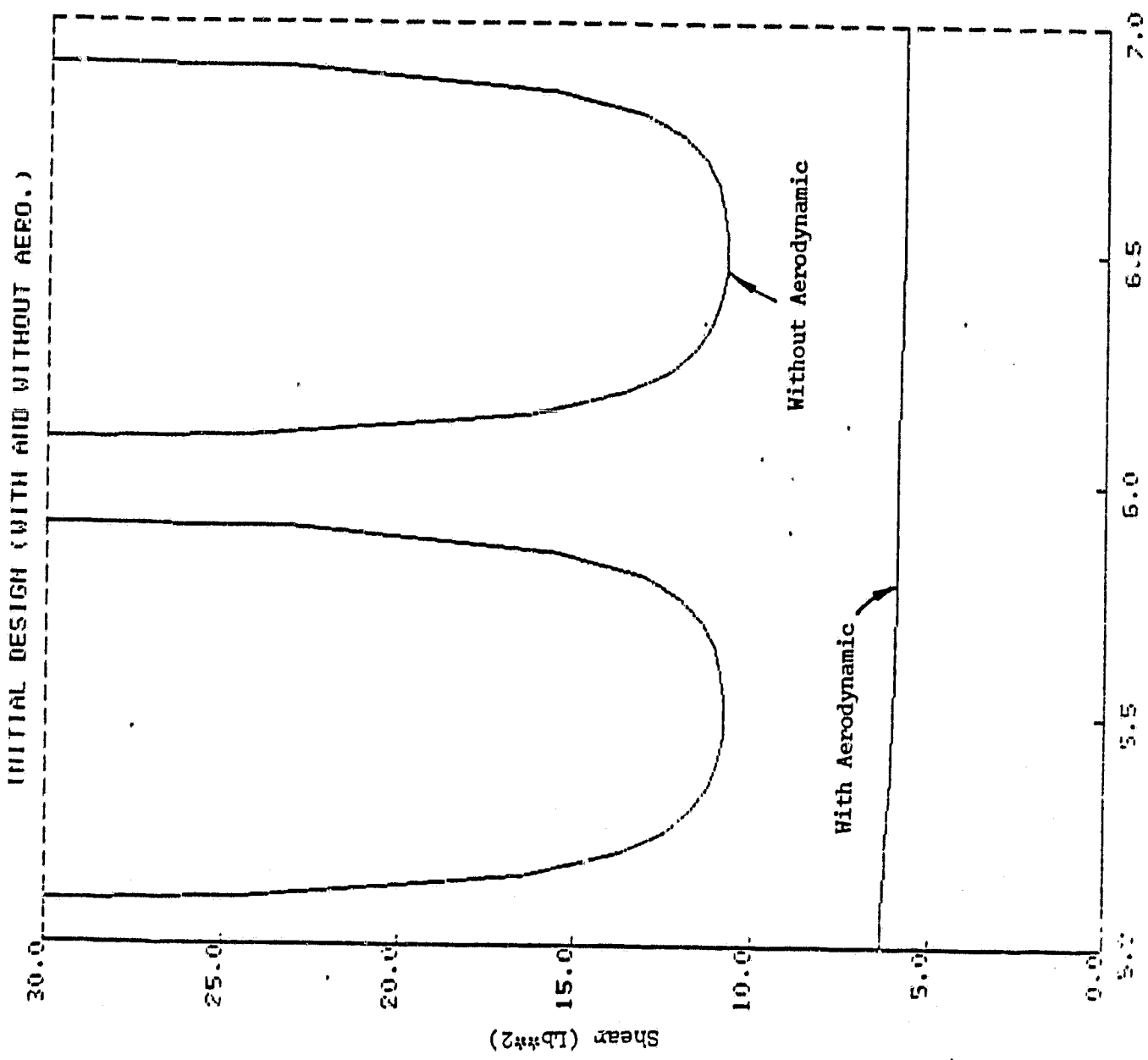


Fig 9 W3 (No./Rev) W1 and W2 are fixed.
Sum of Squares of Shears Versus Third Natural Frequency for Initial Design Both With and Without Damping

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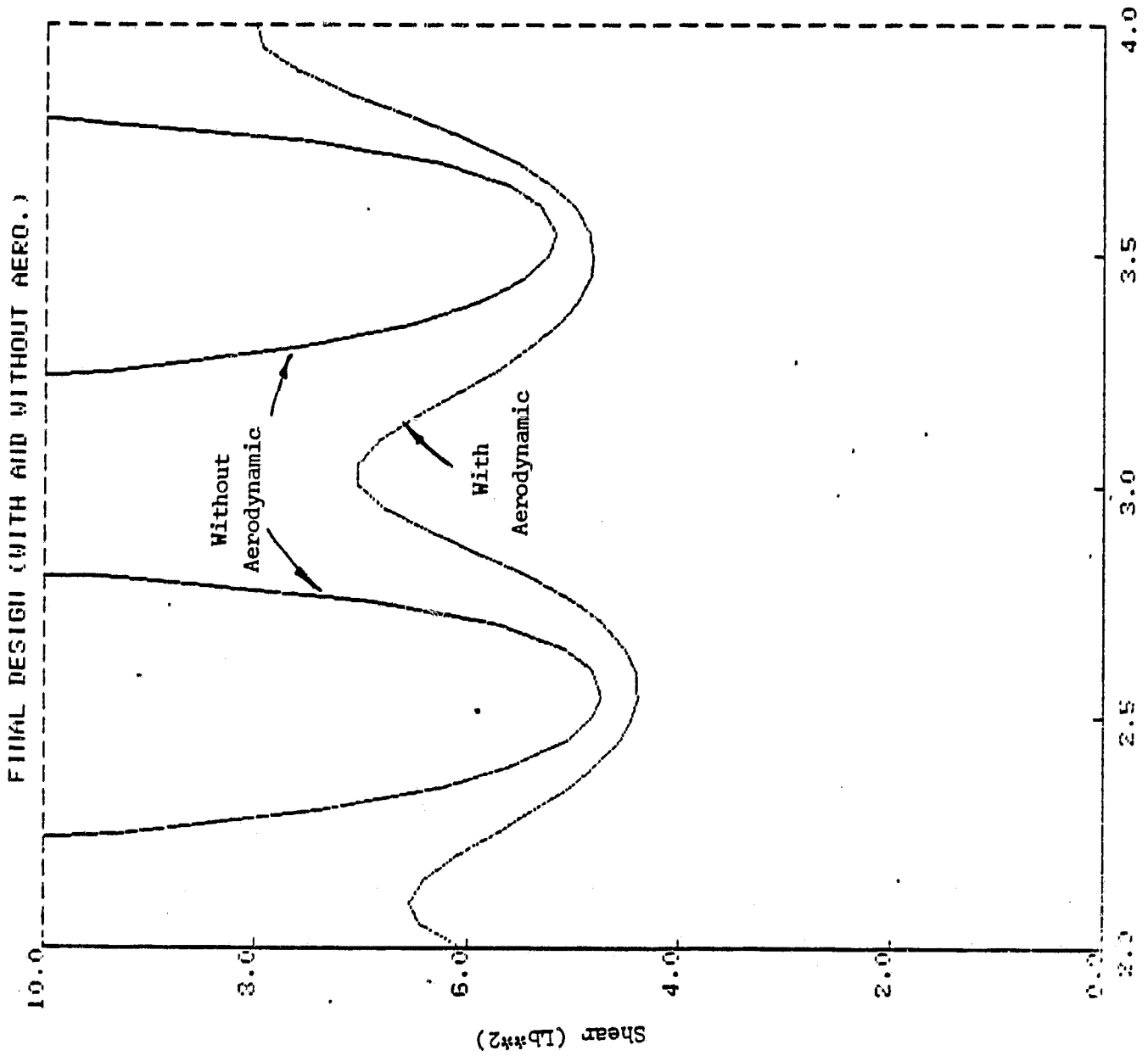


Fig 10 . W2 (No./Rev) W1 and W3 are fixed.
Sum of Squares of Shears Versus Second Natural Frequency for Final Design Both With and Without Damping

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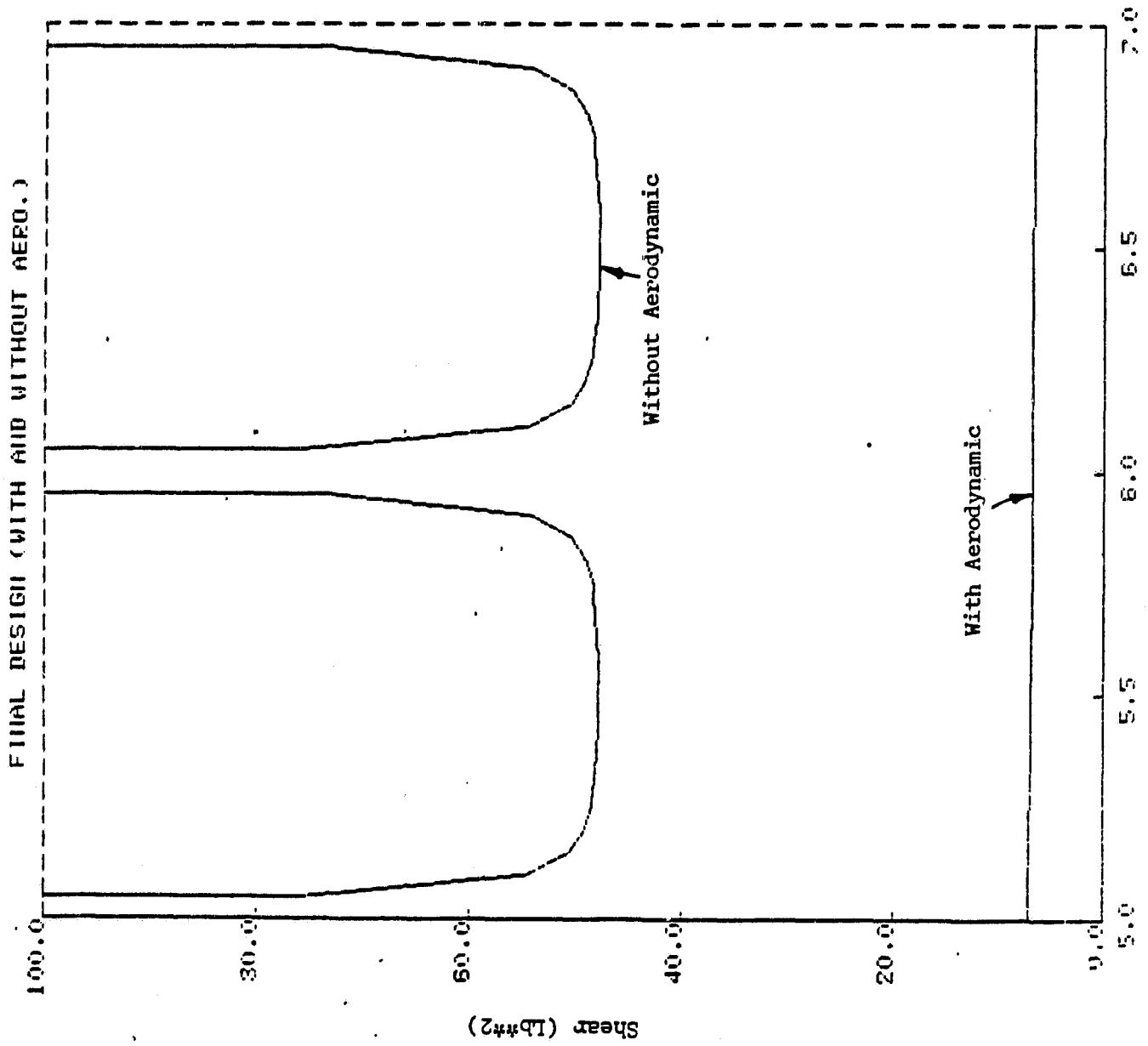


Fig 11 W3 (No./Rev) W1 and W2 are fixed.
Sum of Squares of Shears Third Natural Frequency for Final Design Bith With and Without Damping