NASA CR-175,369

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DYNAMICS AND CONTROL OF

FORWARD SWEPT WING AIRCRAFT

Final Report

Grant No. NAG-1-305

Project Period: November 1, 1982 - December 31, 1983

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February 10, 1984

(NASA-CR-175369) DYNAMICS AND CONTROL OF FORWARD SWEPT WING AIRCRAFT Final Report, 1 Nov. 1982 - 31 Dec. 1983 (Purdue Univ.) 43 p HC A03/MF A01 CSCL 01C

N84-17172

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1. Introduction

This constitutes the final report on the research being performed by the School of Aeronautics and Astronautics, Purdue University, for the NASA Langley Research Center under grant number NAG-1-305. The area of research is multi-input/multi-output control synthesis techniques motivated by applications such as forward-swept-wing aircraft, which exhibit significant rigid-body/aeroelastic modal coupling.

2. Comment on Personnel

A major portion of this research was performed by a doctoral graduate student (Mr. Mike Gilbert), while the student was in residence at the Langley Research Center. This was an important element of this program, providing the opportunity for this work to actually proceed "in situ" with the graduate researcher interacting daily with NASA's international experts on control of aeroelastic phenomena.

In addition to the research reported herein, the graduate student actively participated in experimental evaluations of several candidate control laws for stabilizing a dynamically scaled, statically unstable wind tunnel model of a forward-swept-wing aircraft tested in Langley's transonic dynamics test facility (TDT).

3. Summary

In the early phase of this research, the potential of cooperative game theory for multi-variable control synthesis was briefly explored, and a summary of key concepts are attached as Appendix A to this report. The key conclusion of this survey was that if a multivariable (multi-input/multi-output) control law is to be synthesized to be cooperative, this corresponds to a Pareto-optimal rather than a Nash solution to the mathematical game, and the Pareto solution may be found via Linear Quadratic Regulator control theory. On the other hand, if, for example, more than one interacting controller is not being synthesized by the same "designer," the optimal solution is the Nash equilibrium.

A second method of modern control law design was proposed that addresses the problems of selecting the cost functional which the control law is to optimize, and the lack of other useful design information. The approach recognizes that in any optimal control problem, there are many more design parameters to be selected than the gains of the optimal control law. These additonal design parameters may be part of the system dynamics or they may be part of the optimal control problem formulation. Either way, by calculating the sensitivity of the dynamical system time and frequency domain performance to these parameters, systematic ways of altering these parameters to improve the performance of the control law can be developed. For example, if the parameters are selected as the nominal values of the poles of a compensator, the compensator design can be adjusted to provide better performance. Similarly, if the parameters are entries of the optimal control problem cost functional, then the control problem itself can be altered to meet time domain design specifications. This technique titled the "Optimum Parameter Sensitivity" approach is presented in Appendix B of this report.

Appendix A

Aspects of Non-Zero Sum Differential Game Theory with Application to Multivariable Control Synthesis

M.G. Gilbert

April 11, 1983

Introduction:

Modern, optimal control theory and differential game theory (1-4) were developed concurrently but independently during the early 1960's. Both are concerned with obtaining optimal control strategies (open or closed loop) for multi-input systems describable by a set of differential equations. Optimal control problems are characterized by a single controller using a control law picked to minimize a single scaler objective function. Non-zero sum differential games on the other hand involve several players (controllers) each attempting to control the system to minimize it's own cost function in the presence of the control actions of the other players. Thus differential games are characterized by multiple controllers minimizing multiple cost functions, leading to a vector minimization problem in order to obtain optimal control strategies.

Modern optimal control theory has been studied extensively and has found application in developing control laws for state regulation, terminal guidance, and process control. Much less is known about differential games, whose primary applications have included pursuit-evasion studies and economics problems. It is generally recognized that optimal control can be viewed as a subset of differential game theory, as will be apparent in the linear, time invariant, game to be discussed in the next section.

Some Aspects of Differential Game Theory:

Generally, the average control law designer is much more familiar with optimal control theory than differential game theory; for that reason this section will highlight some important features of differential game theory. The discussion which follows pertains particularly to linear, time invariant, non-zero sum differential games which can be modeled mathematically as

$$\dot{x} = Ax + \sum_{i=1}^{m} B_i u_i \qquad 1)$$

It is assumed that each player or controller picks his control u_i to minimize a quadratic cost function of the form

$$J_i = \frac{1}{2} \int_0^{\infty} \{x^T Q_i x + \sum_{j=1}^{m} u_j^T R_{ij} u_j\} dt \qquad 2)$$

Note that the ith player may be penalized for the j^{th} player's use of control energy.

In general, it cannot be expected that a control strategy set $U \triangleq \{u_1, \ldots, u_m\}$ can be found which will absolutely minimize every player's cost function in the presence of the other player's control actions. If such a control set exists, it could be found by solving m uncoupled optimal control problems with each player independently controlling the system. Because such a control set cannot usually be found, definitions of the sense in which a control set U* is optimal with regard to another control set U are needed. There are three widely accepted definitions of when a particular control law solution set U* is optimal in differential game theory. These are the Nash Equilibrium solution, the min max solution, and the Pareto-optimal or non-inferior solution.

Of the three solutions, the Nash Equilibrium and Pareto-optimal are the most useful for the development of control laws satisfying the game. This is because the min max solution assumes irrational behavior of the other m-1 players when solving for the ith player's control law. Specifically, the solution assumes the m-1 other players are ignoring their own costs and using their control to maximize the ith player's cost. The game then becomes zero sum (the ith player minimizing his cost and the m-1 players combined maximizing it). The solution is overly pessimistic and may fail to be finite for the ith player even in well posed games.

Alternatively, the Nash Equilibrium solution assumes rational behavior of the players. The ith player's Nash Equilibrium control minimizes his cost function when the other m-1 players play their own Nash controls. Mathematically, this is stated as

$$J_i(u_1^*, \ldots u_{i-1}^*, u_i, u_{i+1}^*, \ldots u_m) \geq J_i(u_1^*, \ldots u_m^*) \qquad 3)$$

where J_i is given by 2, $u_1^*, \ldots u_m^*$ are the components of the Nash control set $\theta^*(u_1^*, \ldots u_m^*)$, and u_i is any control strategy other than the Nash control for the i^{th} player. The Nash solution is a stable equilibrium solution, since a player cannot deviate from his Nash control without incurring increased cost if all the other players use their own Nash controls.

Necessary conditions for the Nash Equilibrium solution for the game given by equations 1) and 2) are

$$u_i^* = -R_{ii}^{-1} B_i^T S_i x \qquad 4)$$

where S_i are solutions of the m coupled Ricatti like equations

$$\dot{S}_i = -S_iA - A^TS_i - Q_i - \sum_{j=1}^{m} [S_jB_jR_{jj}^{-1}R_{ij}R_{jj}^{-1}B_j^TS_j$$
$$-S_iB_jR_{jj}^{-1}B_j^TS_j - S_jB_jR_{jj}^{-1}B_j^TS_i] \quad , \quad S_i(t_f) = 0 \qquad 5)$$

Unfortunately, these equations are difficult to solve and only a few sufficient conditions for the existence of solutions to the equations are known (Ref. 5). In the single player case (m=1), the equations 5) reduce to the single Ricatti equation of optimal control theory.

The Nash solution, stated mathematically by 3), is an uncooperative solution since each player is only concerned with minimizing his own cost, and cares nothing about the other player's costs. The question then becomes whether or not other solutions exist which simultaneously reduce the costs of the players from their Nash costs. Pareto-optimal or non-inferior solutions to the differential game may be solutions with that property. In fact, Pareto-optimal solutions have the property that

$$J_i(u_1, \ldots, u_m) = J_i(u_1^*, \ldots, u_m^*) \ \forall_i = 1, \ldots m \qquad 6A)$$

or there is at least one i=1, ... m such that

$$J_i(u_1, \ldots, u_m) > J_i(u_1^*, \ldots, u_m^*) \qquad 6B)$$

where u_1^*, ..., u_m^* are the components of the Pareto-optimal control set $\psi^*(u_1^*, \ldots, u_m^*)$, and $u_1, \ldots, u_m$ are controls other than the Pareto-optimal controls. There are many solutions satisfying 6A), 6B), some of which may have costs for each player lower than their corresponding Nash costs. Selection of a particular non-inferior solution in all cases involves trading off the costs of one player over another, thus the players are faced with a negotiation in order to obtain a solution, or else some level of cooperation among players must be enforced.

It has been shown (6, 7) that some* of the non-inferior solutions to the vector optimization problem can be obtained by solving an m-1 parameter family of optimal control problems, if the cost functions satisfy certain convexity requirements. Mathematically, this is stated as follows. The control set $\psi^*(u_1^*, \ldots, u_m^*)$ is Pareto-optimal (non-inferior) if

$$J(\psi^*) < J(\psi) \qquad \psi \neq \psi^* \tag{7A}$$

where $J(\psi) \triangleq \sum_{i=1}^{m} \alpha_i J_k(\psi)$, $\alpha_i \geq 0 \quad \forall\, i=1, \ldots, m$, and $\sum_{i=1}^{m} \alpha_i = 1$

or

$$J(\psi^*) \leq J(\psi) \qquad \psi \neq \psi^* \tag{7B}$$

where $J(\psi) \triangleq \sum_{i=1}^{m} \alpha_i J_i(\psi)$, $\alpha_i > 0 \quad \forall\, i = 1, \ldots, m$, and $\sum_{i=1}^{m} \alpha_i = 1$.

---

*For the linear, quadratic cost game, it is believed all could be found, however unrealistic it may be to do so.

By 7A, the sets ψ^* which give the unique minimum of any individual J_i is Pareto-optimal. Note that selection of a set of α_i, thereby defining a particular solution, is equivalent to solving the negotiating problem referred to previously.

For the linear, quadratic cost games of equations 1) and 2) (the J_i of 2) satisfy the necessary convexity requirements, see Reference 8), the Pareto-optimal solutions are

$$u_i^*(\alpha) = -\left[\sum_{j=1}^{m} \alpha_j R_{ji}\right]^{-1} B_i^T S(\alpha) x \qquad 8)$$

where $S(\alpha)$ is given by

$$\dot{S} = -SA - A^T S - \sum_{i=1}^{m} \alpha_i Q_i + S \sum_{i=1}^{m} B_i \left[\sum_{j=1}^{m} \alpha_j R_{ji}\right]^{-1} B_i^T S \qquad 9)$$

$$S(t_f, \alpha) = 0$$

with the α_j satisfying the conditions of 7A), i.e. $\alpha_i \geq 0$.

There is a geometrical interpretation, first given by (3), which is useful in understanding the Nash and Pareto-optimal solutions. Consider a two player game, with contours of constant cost plotted in the u_1, u_2 control space, as shown in Figure 1.

The short-dash lines are the minimizing control for one player when the other plays every other control in his admissable control range. The intersection of the two dash lines is the Nash Equilibrium solution, which occurs at the intersection of two cost contours. The cross hatched area represents admissable controls u_1 and u_2, which if played would

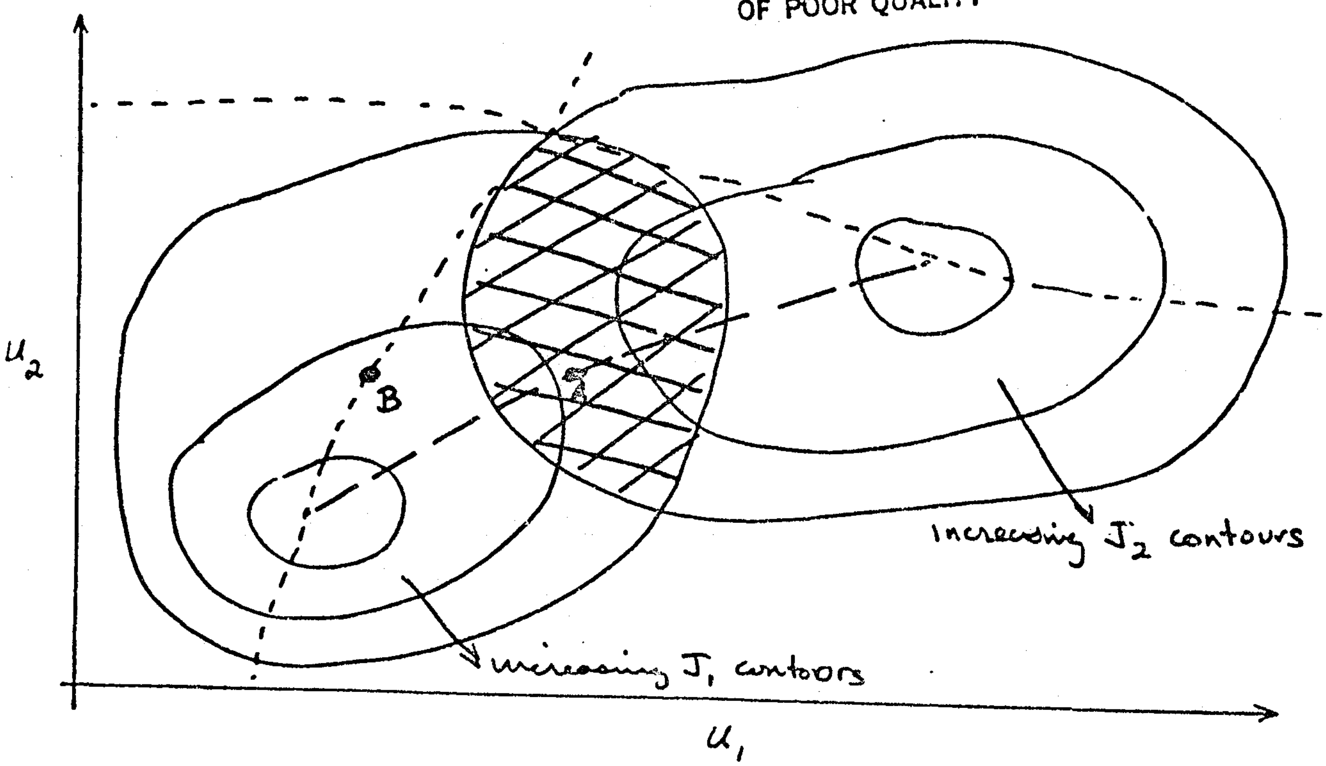

Figure 1

result in reduced cost (from the Nash cost) for both players. The long-dash line, which follows the tangent points of the cost contours, are the Pareto-optimal solutions. These are the minimal cost solutions for each player. They require cooperation, since both players must trust the other not to play a minimizing control in response to the first's use of control in the shaded area, and therefore these solutions are vulnerable to cheating. For example, say player two uses a control on the tangency line in the cross-hatch region in anticipation of player one using his control to arrive at solution A, but player one cheats, playing his minimizing control resulting in solution B. This is the so called "prisoners dilemma" (1), which most differential games, particularly linear games with quadratic costs, have. Note too what would happen if the Nash solution were also non-inferior; the non-inferior solution would be stable and invulnerable to cheating. This desirable situation does not occur in general.

Application to Control Law Development:

The last section discussed the types of solutions which occur in differential games in general, and linear, time invariant, quadratic cost games in particular. There are many more important properties of these solutions which were not discussed. Rather than devote much more time to these important properties, the potential of differential game theory to improve control law development for multi-input linear systems will be discussed.

Optimal control law synthesis for multi-input systems has in the past been primarily performed using modern optimal control theory, that is by minimizing a single scaler-valued quadratic cost function. This is because modern control theory takes advantage of powerful matrix methods for algaebraic manipulation, because most control system designs have in mind a single overall objective, and because of the difficulty of obtaining solutions to vector-valued optimization problems. For example, primary aircraft flight control systems have as their main objective improvement of the aircraft handling qualities. In those cases where several systems were desired, say an automatic flight control system and an active structural control system, the nature of the system plant (aircraft) was such that the systems might be designed separately. These are cases where a control set uniquely minimized both cost functions.

With the increasing complexity of aircraft design has come instances where several control systems could not be designed separately, the Grumman forward-swept-wing (FSW) demonstrator being an example. Because of the coupled rigid-body and structural dynamics of the FSW configuration, the separate flight and structural control systems of that aircraft have interacted unfavorably with each other. This is a case where an integrated approach, satisfying two different, broad, overall objectives and making use of differential game theory solutions might lead to better control laws for both systems.

Of course the question remains as to what is meant by better? All differential game theory has promised is that the individual cost functions, which may or may not have physical significance, will be minimized taking into account the control action of the other. The important physical properties of these solutions, like closed-loop eigenvalue locations, robustness properties, frequency response, etc., remain unknown. On the plus side however, there are now more solutions (control laws) to evaluate which at least have known properties among several scaler parameters than there are with an optimal control solution.

There exists at least one example where a Nash equilibrium solution (while not explicitly labeled as such) was proposed for control system development (9). The methodology attempted to account for the control action of the aircraft pilot in the design of an aircraft longitudinal stability augmentation system. The pilot was assumed to be an optimal regulator, leading to a two player, linear, quadratic cost differential game. Properties of the solution like eigenvalue location were not investigated as the scaler cost of the pilot was directly related to the Cooper-Harper handling qualities rating scale, the primary objective of the methodology being the best (lowest) Cooper-Harper rating possible. This example does serve however as an incentive to investigate the potential of differential game theory for control law design, since it was advantageous to use the theory in this case.

Questions:

The previous section has vaguely called for an "investigation" of differential game theory in the context of multi-input control system development. Some work is known to have been done in this area (10). The author is puzzled though that apparently a lot more has not been done. Either there exists unknown work which dispels any advantage of game theory in these situations, or else they have never been considered due to their complexity, the lack of a need to integrate several systems, or whatever. If the latter is the case, then enough questions arise about the differential games solutions and their applications to advise many investigations. Some questions are listed here.

1) Only a few sufficiency conditions for the existence of Nash solutions are known, given in terms of the norms of defined matrices when either a) the system A matrix has a prescribed degree of stability, or b) the solution to certain auxillary

control problems exist (5). Are there conditions on the weighting matrices of the cost functions (like observability and controllability concepts from optimal control) which determine directly if a Nash solution exists, rather than through the solution of an auxillary problem?

2) Are there weighting matrices in the cost functions such that the Nash solution is also non-inferior? Such a solution would have many desirable properties. In addition, the mathematical question of when a set of coupled Ricatti like equations could be solved by a single Ricatti equation would be answered.

3) Are there enough relationships between the individual cost functions and the physical properties of the closed-loop system, i.e. eigenvalue locations, frequency response, etc., to justify considering Pareto-optimal solutions over the Nash solutions? If so, can the problem be further reduced to a simple modern control problem with pre-defined structure? Or is the Pareto-optimal solution just another way of picking weighting matrices in modern control problem cost functions, similiar to methods used in Bryson and Ho and Kwakernaak and Sivan?

4) What kind of solution is a linear, quadratic cost, multi-input modern control problem in the context of differential games? Are the individual controls in the problem in a Nash Equilibrium, are they cooperating in a Pareto-optimal sense, or are they in a "prisoners dilemma" situation?

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Appendix B

OPTIMAL LINEAR CONTROL LAW DESIGN USING OPTIMUM PARAMETER SENSITIVITY ANALYSIS

by

Michael G. Gilbert

A Proposal for Doctoral Dissertation Research

I - INTRODUCTION

The use of feedback is a well known and effective means of altering the dynamics of a system in order to improve stability, reduce sensitivity to model errors, and meet performance specifications. The design methods used to develop these closed-loop control laws can be broadly classified into two categories, classical and modern. Classical methods provide systematic design information that is used by the designer to develop single loop controllers, possibly with their own dynamics, to meet performance specifications. They are less than easy to use if the system is multi-input/multi-output in nature. Modern methods, on the other hand, determine high order control laws easily, and they optimize a quadratic functional of the system states. The disadvantage of modern methods is the difficulty of writing the cost functional to reflect the performance specifications of the system, and the lack of systematic redesign information if the original design is unsatisfactory.

A method of modern control law design has been developed which addresses the problems of writing the cost functional which the control law is to optimize, and the lack of useful design information. The method recognizes that in any optimal control problem, there are many more design parameters to be selected than the gains of the optimal control law. These additional design parameters may be part of the system dynamics or they may be part of the optimal control problem formulation. Either way, by calculating the sensitivity of the dynamical system time domain performance to these parameters, systematic ways of altering these parameters to improve the performance of the control law can be developed. For example, if the parameters are selected as the nominal values of the poles of a compensator, the compensator design can be adjusted to provide better performance. Similarly, if the parameters are entries of the optimal control problem cost functional, then the control problem itself can be altered to meet time domain design specifications.

I.a - Background

The concept of sensitivity of optimum solutions to problem parameters has recently been developed by Sobieski, Barthelemy, and Riley [1], and Barthelemy and Sobieski [2,3] in the field of parameter optimization theory. This concept differs from the traditional idea of sensitivity analyses in that the results are used in the design process rather than the performance assessment stage. What this means is that the sensitivity data is used to redefine the original optimization problem so as to improve performance by altering preselected parameters of the problem (parameters not selected by the optimization). Prior to this development, sensitivity analyses were used to determine performance changes to arbitrary, uncontrollable perturbations in system parameters. The optimal sensitivity derivatives calculated by Sobieski, Barthelemy, and Riley were obtained by differentiating the necessary conditions of optimality for non-linear programming problems with inequality constraints. One of the most interesting applications of the optimal sensitivity results has been in the development of multi-level optimization schemes for large structure parameter optimization problems (Sobieski, James, and Dovi [4]).

Sensitivity analyses are commonly used in both classical and modern control theories to assess the effects uncontroled variations in the system parameters will have on system performance, primarily in stability margins. Methods have been developed by various researchers (e.g. Yedavalli and Skelton [5]) for designing control systems which are insensitive to parameter variations. Only recently, time response sensitivity was introduced as a means of assessing system performance changes to parameter variations by Schaechter [6]. Neither of these applications of sensitivity uses the information as part of the design process however. There also exist several methods for defining the modern control theory problem so as to meet certain system objectives, besides the commonly used intuition of the designer. These include selecting weighting matrices on the basis of asymptotic properties of linear regulators (Harvey and Stein [7] and Stein [8]).

Other areas of control system design worth mentioning are output feedback theory, reduced order compensator design, and frequency shaped linear regulator design. Output feedback theory was developed by Levine and Athans [9] and solves the problem of feeding back fewer outputs of the system than there are states. Reduced order compensator design is receiving attention because it recognizes the actual control law structure that is usually implemented on working hardware. One method of designing reduced order compensators has been parameter optimization methods (Mukhopadhyay, Newsom, and Abel [10]). Frequency shaping techniques of linear regulator design were developed by Gupta [11], by making the weighting matrices of the optimal control problem functions of frequency. These areas of control theory are noted because they have the potential to make use of optimal sensitivity analyses as will be presented in this paper.

1.b Proposed Methodology

The proposed method of control law design makes use of the idea of optimal sensitivity to problem parameters to develop and use systematic design data to further improve controlled system time domain performance. This is to be done by computing the sensitivity or derivative of the time domain response, either absolute or mean square, to parameters of the problem not selected as part of the optimization process. These sensitivities are then to be used to redefine the parameters so the problem can be resolved, with an expected improvement in performance. The sensitivities are computed in a two step process. First, the sensitivity of the necessary condition for the control to be optimal is obtained, and then the sensitivity of the time response is computed as a function of the sensitivity of the necessary condition. The result is a connection between the time response of the system and the parameters which specify the optimal control problem being solved. With this connection, it is possible to specify changes in the designer selected parameters using a first order Taylor series, and the optimal control problem resolved, leading to a two stage optimal control design process.

II - METHODOLOGY

This section presents a discussion of the use of parameter sensitivity analysis in the development of linear regulator control laws. In this section, the time domain state, output, and control responses, both absolute and RMS, are of interest. The derivation of the sensitivity of these time responses to arbitrary parameters of the dynamical system or to parameters of the control law formulation will developed in the first subsection, followed by a review of linear regulator theory. The following subsection will then combine the parameter sensitivity results and linear regulator theory to give a systematic method for either defining the linear regular problem or altering the system dynamics to achieve desired time domain responses. This section will then be concluded by a discussion of practical aspects of this methodology, as it applies in the case of time domain design criteria.

II.a - Parameter Sensitivity of Time Domain Responses

The sensitivity of time domain state, output, and control responses to arbitrary parameters that appear in the model or control law formulation, for a linear, time invariant dynamical system is derived in this subsection. The derivation begins by considering the state space model of the system in the form

$$\dot{x} = Ax + Bu + Dv$$
$$y = Cx \qquad \text{(II.1)}$$

where x is an n-dimensional state vector, u is an m-dimensional control vector, y is an l-dimensional output vector, and v is a k-dimensional disturbance vector which may be either deterministic or stochastic. The matrices A, B, C, and D are constant coefficient matrices of the appropriate dimensions. Assuming that a linear, full-state feedback control law exists and is specified as $u = -Gx$, then the closed-loop dynamics of the system are given by

$$\dot{x} = \bar{A}x + Dv \qquad \text{(II.2)}$$

where $\bar{A}$ is defined as

$$\bar{A} = A - BG \qquad \text{(II.3)}$$

The closed-loop system given by eqn. (II.2) is a function of the original model dynamics matrix A, input matrix B, and the gain matrix G. Each of these matrices may themselves be functions of parameters which may vary due to a natural process of the system, may be imprecisely known, or may be specified by the designer. For example, the entries of the A matrix may be subject to modeling errors, and the entries of the gain matrix G, if they are obtained using optimal control theory, are functions of designer selected weightings. It is possible to determine the effects these parameters have on the state, output, and control response of the closed-loop system (II.2) in the following manner.

The time response of the closed-loop system (II.2) due to the disturbance v is well known and is given in terms of the state transition matrix and matrix superposition integral [12] as

$$x(t) = \Phi(t,t_0) + \int_{t_0}^{t} \Phi(t,\tau)Dv(\tau)d\tau \qquad \text{(II.4)}$$

where $\Phi(t_2,t_1)$ is the state transition matrix of the closed-loop dynamics matrix given in eqn. (II.3). In order to calculate the sensitivity of the state to arbitrary parameters, it is of course possible to diferentiate eqn. (II.4) with respect to those parameters, but because the A matrix of eqn. (II.3) is in general not symmetric, the diferentiation will prove difficult. The alternative is to differentiate eqn. (II.2) first, and then write the superposition integral for that expression. Proceeding

$$\frac{\partial \dot{x}}{\partial p} = \frac{\partial \bar{A}}{\partial p}x + \bar{A}\frac{\partial x}{\partial p} \qquad \text{(II.5)}$$

where p is the arbitrary parameter, and it has been assumed that the disturbance v and the way it enters the system (through the matrix D) are independant of the parameter. The time response of eqn. (II.5) is similiar to the system time response in that the transition matrix of (II.5) is identical to that of (II.4), with eqn. (II.5) driven by the state of the closed-loop system;

$$\frac{\partial x}{\partial p} = \Phi(t,t_0)\frac{\partial x}{\partial p}(t_0) + \int_{t_0}^{t} \Phi(t,\tau)\frac{\partial \bar{A}}{\partial p}x(\tau)\,d\tau \tag{II.6}$$

In general, the initial state of the system will also be independent of the parameter p, so that eqn. (II.6) will reduce to the calculation of the integral term only.

The above results give the sensitivity of the state to problem parameters; if the sensitivity of the system outputs or controls are of interest, the results of eqn. (II.6) can be used to calculate these quantities as well, since both are linear functions of the state. For the system controls, the sensitivity to problems parameters is obtained using $u = -Gx$ as

$$\frac{\partial u}{\partial p} = -\frac{\partial G}{\partial p}x - G\frac{\partial x}{\partial p} \tag{II.7}$$

and the sensitivity of the system outputs to the parameters is

$$\frac{\partial y}{\partial p} = \frac{\partial C}{\partial p}x + C\frac{\partial x}{\partial p} \tag{II.8}$$

The sensitivity results that have thus far been obtained are for cases where the sensitivity of the absolute time response is of interest. If the covariance of the system state is of interest, a differant set of results can be obtained using the closed-loop system covariance equations. The covariance response of the system given by eqn. (II.2) to the disturbance v is [13]

$$\dot{X} = \bar{A}X + X\bar{A}^T + DVD^T; X(t_0) = X_0 \tag{II.9}$$

where X is defined as the system covariance matrix, $\bar{A}$ is the closed-loop dynamics matrix, and V is the intensity matrix of v if v is a zero-mean, Gaussian, white noise, random vector, or the square disturbance if v is deterministic (if v is deterministic, then X is the square response). The sensitivity of the system covariance to the parameter p is just

$$\frac{\partial \dot{X}}{\partial p} = \bar{A}\frac{\partial X}{\partial p} + \frac{\partial X}{\partial p}\bar{A}^T + \frac{\partial \bar{A}}{\partial p}X + X\frac{\partial \bar{A}^T}{\partial p} \tag{II.10}$$

where it has again been assumed that the disturbance and the way it enters the system are independent of the parameters. Notice that eqn. (II.10) has the same form as eqn. (II.9), and contains the system covariance explicitly.

If the closed-loop control law yields an asymptotically stable system (all the eigenvalues of (II.2) strictly in the left-half-plane), then the covariance equation (II.9), for the case of a stationary random input v, will achieve a steady-state solution [13] given by

$$0 = \bar{A}X + X\bar{A}^T + DVD^T \tag{II.11}$$

and the sensitivity equation (II.10) will also have a steady-state solution [6] given by

$$0 = \bar{A}\frac{\partial X}{\partial p} + \frac{\partial X}{\partial p}\bar{A}^T + \frac{\partial \bar{A}}{\partial p}X + X\frac{\partial \bar{A}^T}{\partial p} \qquad \text{(II.12)}$$

The sensitivity of the control and output covariances are obtained from the state covariance sensitivity in the following manner. The control covariance is, in terms of the state covariance, [13]

$$U = GXG^T \qquad \text{(II.13)}$$

Differentiating eqn. (II.13) with respect to the parameter p gives

$$\frac{\partial U}{\partial p} = \frac{\partial G}{\partial p}XG^T + G\frac{\partial X}{\partial p}G^T + GX\frac{\partial G^T}{\partial p} \qquad \text{(II.14)}$$

The sensitivity of the output covariance [13] is obtained similarly as

$$Y = CXC^T \qquad \text{(II.15)}$$

and

$$\frac{\partial Y}{\partial p} = \frac{\partial C}{\partial p}XC^T + C\frac{\partial X}{\partial p}C^T + CX\frac{\partial C^T}{\partial p} \qquad \text{(II.16)}$$

II.b - Optimal Linear Regulator Theory

In this subsection, a brief review of linear regulator theory is presented for the infinite time, constant coefficient, full state feedback case. This material will be used in the next subsection along with sensitivity results to develop a methodology of control law design to meet time domain response criteria.

Consider again the open-loop dynamical system given by eqn. (II.1). It is desired to find a state feedback control law, that is, a control policy which is a function of the states of the system, such that the states of the system stay close to zero in the presence of the disturbance v. In fact, the objective of the design can be generalized to achieve desired output performance in the presence of the disturbance, by using the output equation in (II.1). The optimal solution for this problem, when the objective is stated as a quadratic function of the states and controls, is the standard linear, quadratic regulator. The LQ regulator problem is formulated as follows.

A scaler cost functional of the outputs and the controls is formed as

$$J = \int_0^\infty \left(y Q' y + u Ru\right) dt \qquad \text{(II.17)}$$

where Q' and R are designer selected weighting matrices which reflect the importance of the responses to the designer. The Q' matrix must be positve semidefinite, and the R matrix must be positive definite [14]. The cost functional J given by (II.17) can be rewritten in terms of the system states by defining $Q = C\,Q'C$,

$$J = \int_{t_0}^{\infty} (x' Qx + u' Ru)\,dt \qquad \text{(II.18)}$$

The necessary conditions for the control u to minimize the cost functional of eqn. (II.18) yield the equations [14]

$$u = -R^{-1}B^{T}P(t)x(t) \qquad \text{(II.19a)}$$

$$\dot{P} = -PA - A^{T}P + PBR^{-1}B^{T}P - Q\ ;\ P(t_0) = 0 \qquad \text{(II.19b)}$$

If the matrix pair $(A, \sqrt{Q})$ is completely observable [15], then the matrix differential Riccati equation given by (II.19b) will have a steady-state solution for P which is given by

$$0 = -PA - A^{T}P + PBR^{-1}B^{T}P - Q \qquad \text{(II.20)}$$

In the steady-state case, the optimal control policy becomes time invariant, the gain matrix defined as $G = R^{-1}B^{T}P$ is constant, and the closed-loop system dynamics is given by eqn. (II.2).

II.c - Regulator Design Using Parameter Sensitivity

With the results of the two previous subsections, a methodology of control law design to meet time domain response criteria, using an optimal control law, will now be presented. This method addresses the main drawback of LQ regulator theory, and that is the need for the LQ designer to select certain design parameters without the benefit of a physical relationship between those parameters and the actual time responses of the system. For example, by selecting elements of the Q' and R matrices as parameters, the parameter sensitivity results of subsection II.a can be used to define the optimal LQ problem to meet the time domain criteria. The fact that a physical relationship between the designer selected weights and the time domain responses can be established is seen by extending the parameter sensitivity results.

To begin, recall that both the absolute time response sensitivity, eqn. (II.6), and the covariance sensitivity, eqn. (II.10), are functions of the partial derivative of the closed-loop dynamics matrix $\bar{A}$ with respect to the parameter p of interest. For the case of the optimal LQ control law the partial derivative can be calculated, using eqn. (II.3). Thus

$$\frac{\partial \bar{A}}{\partial p} = \frac{\partial A}{\partial p} - \frac{\partial B}{\partial p} G - B \frac{\partial G}{\partial p} \tag{II.21}$$

Now, the partial derivative of the gain matrix G with respect to the parameter p is calculated as

$$\frac{\partial G}{\partial p} = \frac{\partial R^{-1}}{\partial p} B^T P + R^{-1} \frac{\partial B^T}{\partial p} P + R^{-1} B^T \frac{\partial P}{\partial p} \tag{II.22}$$

An expression for the partial derivative of the steady-state matrix Riccati equation (eqn. II.20) can be obtained by adding and subtracting 2PBR B P to eqn. (II.20) to get

$$0 = (A - BR^{-1}B^T P)^T P + P(A - BR^{-1}B^T P) + Q + PBR^{-1}B^T P \tag{II.23}$$

Differentiating eqn. (II.23) with respect to p, canceling terms, and replacing $R^{-1}B^{T}P$ by G gives

$$0 = \bar{A}\frac{\partial P}{\partial p} + \frac{\partial P}{\partial p}\bar{A} + \left\{\frac{\partial Q}{\partial p} + \frac{\partial A}{\partial p}P + P\frac{\partial A}{\partial p} - P\left(\frac{\partial B}{\partial p}R^{-1}B^{T} + B\frac{\partial R^{-1}}{\partial p}B^{T} + BR^{-1}\frac{\partial B^{T}}{\partial p}\right)P\right\} \tag{II.24}$$

With eqn.'s (II.21), (II.22), and (II.24), it is now possible to calculate the sensitivity of the absolute time response to parameters of the optimal LQ regulator problem using eqn. (II.6), and the sensitivity of the state covariance to the parameters using eqn. (II.10). Control and output absolute time response sensitivities are calculated from eqn.s (II.7) and (II.8) once the absolute state sensitivity is obtained. Control and output covariance sensitivities are obtained from eqn.s (II.14) and (II.16) once the state covariance sensitivity is calculated from eqn. (II.10). These sensitivities are the physical links between the designer selected parameters of the LQ design problem and the actual time responses of the closed-loop system.

To illustrate the use of optimal parameter sensitivity analysis in the design of optimal linear regulators, the following situation is considered. Assume that a dynamical system modeled by a linear state space representation is to be controlled by feeding back all the system states, including states which are used in the model to represent the dynamics of the control actuators. A quadratic cost function of the form given by eqn. (II.18) is written, which weights the states and controls. The optimal linear regulator problem is solved (eqn.s II.19), and the system state covariance response to a random input calculated from eqn. II.11. If the RMS response of one or more of the states is unacceptable to the designer, sensitivity analysis can be used to change the design, in the following manner. A set of one or more parameters is selected, say for example the time constants of the actuators and elements of the state weighting matrix Q. The sensitivity of the unacceptable state response to these parameters is calculated using eqn.s (II.24) and (II.12), and the unacceptable response expanded in first order Taylor series about the nominal value of one of the parameters. This expansion is then used to make a change in the nominal value of the parameter such that the state response is improved. The optimal linear regulator problem is then resolved with the new value of the parameter. The result is an optimal control law which more nearly meets the state response criteria of the designer.

II.d - Extensions and Practical Considerations

In the previous subsections, the idea of parameter sensitivity was discussed, LQ regulator theory reviewed, and the means of using parameter sensitivity as a redesign tool in optimal control theory presented. This subsection will discuss potential extensions of the use of optimal sensitivity analysis, consider the practical aspects of computing the sensitivities, and give the reasoning behind the assumptions and restrictions that were made during the course of the development.

The use of optimal sensitivity analysis in other kinds of optimal control design processes is readily apparent, particulary in the three areas mentioned in subsection I.a. The first of these was the output feedback problem, where a number of system outputs, less than the number of states, are fedback. Optimal sensitivity results can be derived for this problem, since the necessary conditions for the optimal solution are known [9]. Thus optimal sensitivity analysis can be used in the same manner for this problem as for the full state feedback problem. Perhaps more appealing is using the sensitivity results for the output feedback to design reduced order control laws. This could be done by augmenting the system state equations with a set of dynamics which represents a dynamic compensator. Treating the coefficients of the compensator, which define it's dynamics, as parameters, and using output feedback theory to find gains for the states of the compensator, sensitivity analysis could be used to iterate on a solution for the compensator. Another possible extension is to the frequency shaping problem of Gupta [11], where the sensitivity analysis could perhaps be used to define the elements of the frequency dependant weighting matrices.

As far as the computation of sensitivities is concerned, it should be apparant that for each differant scaler parameter of interest, a set of vector/matrix equations must be solved. At first, this seems like a large computational burden, since these kinds of equations can be difficult and expensive to solve. Fortunately however, the coefficients of the time response equations and their associated sensitivity equations are exactly the same, and further they are independent of the parameters. This means that once the coefficient matrices have been factored to solve the covariance equation for example, they need not be factored again. Rather, each new equation can be solved merely by forward and back substitution of a new right-hand-side. In addition, a method similiar to that used in [14] to calculate gradients of cost functions may be adapted to this analysis technique, further simplifying the computational burden. Similarly for the absolute time response equations, the state transition matrix needs to be computed only once for each control law.

Most of the assumptions and restrictions made during this development are related to solving the LQ regulator problem. The restrictions that the Q' matrix be positive semi-definite and the R matrix be positive definite insure that a solution exists that will stabilze the system if it is initially unstable, if the matrix pair (A,B) is at least stabilizable, and the matrix pair (A,C) is at least detectible. The requirement that the pair $(A,\sqrt{Q})$ be completely observable is desirable so that a steady-state, symmetric solution to the matrix Riccati equation exists. In light of the desire for a symmetric solution to the Riccati equation, the selection of parameters from the Q and R matrices should be made in such a way so as to make sure Q and R are always symmetric matrices.

III PROPOSED RESEARCH

In section II, the idea of sensitivity of optimum solutions to problem parameters was adapted to optimal control problems. A derivation of optimal sensitivity analysis for these type problems was presented to illustrate that the methodology could be used to aid the design of linear feedback control laws. This section will propose a systematic investigation of optimal senstitivity analysis as design tool in optimal control design. This investigation has as an objective the development of a research program satisfying the requirements of a doctoral dissertation.

III.a - Proposed Research Program

Section II presented the initial development of a design methodology based on optimal sensitivity analysis. Clearly this development was incomplete in the sense that only the infinite time, constant coefficient linear regulator problem was addressed. As the first step in a logical development of a methodology, a broader class of problems must be considered. To this end, the general mathematical formulation of optimal sensitivity analysis in optimal control theory would be sought. This would include extension to finite time cases, including final time state weightings, and to the time varying system dynamics cases. Certainly the method must also be extended to consider cases where the feedback variables are contaminated with noise. Thus the optimal sensitivity of linear-quadratic-Gaussian (LQG) regulators would be a result of this study.

Also in section II, a design methodology, using optimal sensitivity analysis, was suggested. This method too needs to be formalized and exercised on appropriate, representative problems. One suggested problem would involve the integrated control of flight dynamics and structural dynamics of a highly coupled, flexible aircraft. With this example, it is expected that the method will prove advantageous since the designer will be afforded more information with which to select design parameters in an systematic manner.

An interesting adaptation of optimal sensitivity analysis would be in the area of reduced-order dynamic compensators. One previous method for solving these type problems das been direct parameter optimization of the control laws. Optimal sensitivity analysis may be able to used for these kinds of designs without resorting to direct optimization. To do this, the sensitivity analysis would have to be developed for the output feedback problem. Then, by augmenting the system dynamics by a canonical form of a compensator, and selecting as the output feedback variables the states of the compensator, the sensitivity of the system response to the elements of the compensator could be calculated. These sensitivities could then be used to redefine the compensator.

As a final area of proposed research, the sensitivity of frequency domain reponses to parameters of the optimal control law formulation should be investigated. This study would help establish a relationship between these variables and the frequency response of the system. In the course of this study, it may be necessary to consider the frequency shaping quadratic forms of Gupta [11].

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