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Preliminary Investigation of an Electrical Network Model for Ultrasonic Scattering

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PRELIMINARY INVESTIGATION OF AN ELECTRICAL NETWORK MODEL FOR ULTRASONIC SCATTERING

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ABSTRACT

This study relates the behavior of acoustic attenuation in a solid to the electrical transmission line model where the electrical shunt conductance, which is frequency dependent, represents the loss due to the scattering sites in the solid. Results indicate that the absolute value of attenuation at a given frequency depends on both the normalized mean square deviation of the density and bulk modulus of the scattering sites from the ambient medium and the spatial scattering correlation function. Besides establishing the absolute value of attenuation, the spatial correlation function determines the attenuation profile as a function of frequency.

INTRODUCTION

This report contains a study to simulate acoustic attenuation in a solid due to scattering by employing a one-dimensional electrical transmission line model. The electrical transmission model offers the advantage of studying attenuation as a function of frequency and scattering site density without the tedious process of preparing specimens.

By judiciously assuming that the scattering mechanism is isotropic and the scattering sites are randomly spatially distributed, it will be shown that the attenuation in a solid can be represented by a distributed frequency sensitive shunt conductance in the transmission line model. The density and bulk modulus of the solid will be represented by a distributed series inductance and shunt capacitance, respectively.

Since this study is attempting to relate a three-dimensional phenomena, scattering in a solid, to a one-dimensional model, electrical transmission line model, the concept of effective scattering cross section will be very important in the analysis. The collapsing of a multi-dimensional problem into a lower dimension simplifies the complexity of analysis providing that the desired salient features are retained. This is exactly what is done when measuring attenuation due to scattering in a physical specimen using a one-dimensional acoustic wave.

Assuming the above assumptions are valid, the concept of normalized mean square deviations of density and bulk modulus of the scatters from the ambient or host medium and the spatial correlation function of the scattering sites will be shown to be very important in describing the attenuation as a function of frequency. Results will predict Rayleigh and viscous type absorption.

The absolute level of attenuation will depend on the normalized mean square deviation of the density and bulk modulus as well as the spatial correlation function; while the profile of the attenuation as a function of frequency will depend strongly on the Fourier transform of the correlation function.

I. ACOUSTIC WAVES

Propagation of an acoustic wave is governed by two fundamental equations: the equation of motion and the equation of conservation of mass. The equation of motion is

$$\rho \frac{d\vec{v}}{dt} + \nabla p = 0 \tag{1-1}$$

where ρ is mass density, $\vec{v}$ is velocity and p is pressure. Because the total time derivative is used in Equation (1-1), it is the time rate of change in a coordinate system moving with a particular portion of the medium. This is called the Lagrangian description. The Eulerian description is the partial time derivative $\frac{\partial}{\partial t}$ at a "fixed point" in space as the medium moves past that point. The total and partial derivatives are related by the following operator expression

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v}\cdot\nabla. \tag{1-2}$$

The variables p, ρ, and $\vec{v}$ can be expressed in terms of their average values, p_o, ρ_o, and $\vec{v}_o$ and the small vibrating acoustic wave components p_1, ρ_1, and $\vec{v}_1$. Assuming the medium is stationary ($\vec{v}_o$=0) and the magnitudes of the small vibrating acoustic wave components, p_1 and ρ_1, are small compared with the average values p_o and ρ_o, Equation (1-1) becomes

$$\rho_o \frac{\partial\vec{v}_1}{\partial t} + \nabla p_1 = 0 \tag{1-3}$$

where $\vec{v}\cdot\nabla\vec{v} = (\vec{v}_o+\vec{v}_1)\cdot\nabla(\vec{v}_o+\vec{v}_1) \approx \vec{v}_o\cdot\nabla(\vec{v}_o) \approx 0$ because the medium is stationary.

The conservation of mass is given by

$$\nabla\cdot(\rho\vec{v}) + \frac{\partial\rho}{\partial t} = 0 \tag{1-4}$$

where $\nabla\cdot(\rho\vec{v}) = \vec{v}\cdot\nabla\rho + \rho\nabla\cdot\vec{v} \simeq \rho_o\,\nabla\cdot\vec{v}_1$ and $\partial\rho/\partial t = \partial\rho_1/\partial t$. Hence, Equation (1-4) can be rewritten

$$\rho_o\,\nabla\cdot\vec{v}_1 + \partial\rho_1/\partial t = 0. \tag{1-5}$$

In general, pressure p is a function of density ρ. For small p_1, it can be expressed by the following relationship

$$p = p_o + p_1 = p_o + (\partial p/\partial\rho)_{p_o}\rho_1. \tag{1-6}$$

Thus $p_1 = (\partial p/\partial\rho)_{p_o}\rho_1 = c^2\rho_1$ where $c^2 = (\partial p/\partial\rho)_{p_o}$. The value of c is the velocity of the acoustic wave in the medium. Note if the density is independent of pressure ($\partial\rho/\partial p=0$) the velocity of propagation is infinite. Equations (1-3) and (1-5) can be written in terms of two unknowns: p_1 and $\vec{v}_1$.

$$\rho_o\frac{\partial\vec{v}_1}{\partial t} + \nabla p_1 = 0 \tag{1-7-a}$$

$$\frac{1}{c^2\rho_o}\frac{\partial p_1}{\partial t} + \nabla\cdot\vec{v}_1 = 0 \tag{1-7-b}$$

Taking the divergence of $\vec{v}_1$ in Equation (1-7-a) and substituting in (1-7-b) it can be shown that

$$\frac{1}{c^2\rho_o}\frac{\partial^2 p_1}{\partial t^2} - \nabla\cdot\left(\frac{\nabla p_1}{\rho_o}\right) = 0. \tag{1-8}$$

Equation (1-8) is the basic acoustic wave equation for a stationary medium. If the medium is uniform (ρ_o = constant), Equation (1-8) can be simplified as

$$\frac{1}{c^2}\frac{\partial^2 P_1}{\partial t^2} - \nabla^2 P_1 = 0. \tag{1-9}$$

For a time harmonic case [$\exp(-j\omega t)$], Equation (1-9) becomes

$$(\nabla^2 + \frac{\omega^2}{c^2})P_1 = 0 \tag{1-10}$$

where P_1 represents the phasor pressure and is a function of space and ω the angular frequency ($\omega = 2\pi f$). The phasor velocity $\vec{V}_1$ can be determined from Equation (1-7-a)

$$\vec{V}_1 = \frac{\nabla P_1}{j\omega\rho_o}. \tag{1-11}$$

The power flux density vector $\vec{S}$ is given by

$$\vec{S} = \frac{1}{2} P_1 \vec{V}_1^{*} \tag{1-12}$$

where * represents the conjugate operator.

For a medium that is infinite in extent, a pressure wave traveling in the +x direction is given by the following expression

$$P_1 = P_{1o} \exp[\gamma x] \tag{1-13}$$

where P_{1o} represents the amplitude of the pressure wave and $\gamma = \alpha + jk$, the propagation constant associated with the pressure wave. The real part of γ (α = Real [γ]) measures the attenuation per unit length while imaginary part of γ (k=Im [γ]) measures the phase shift per unit length.

The amplitude of the velocity, $\vec{V}_{1o}$ can be expressed in terms of P_{1o} by substituting Equation (1-13) into Equation (1-11). The result is

$$\vec{V}_{1o} = \frac{\hat{x}\gamma P_{1o}}{j\omega\rho_o} \qquad (1\text{-}14)$$

where $\hat{x}$ represents a unit vector in the x-direction.

The characteristic impedance Z_o of the medium is defined as the ratio of $P_{1o}/\vec{V}_{1o}$ and in general will be a complex expression. The characteristic impedance is

$$Z_o = \frac{j\omega\rho_o}{\gamma} \qquad (1\text{-}15)$$

If the medium is lossless (α=0), the characteristic acoustic impedance will be a real expression ($Z_o = j\omega\rho_o/jk = \rho_o c$).

II. ACOUSTIC SCATTERING

Consider a particle, as shown in Figure 1-a, illuminated by an incident acoustic plane wave propagating in the x-direction

$$P_i(x) = P_{io} \exp(j\,k\,x) \tag{2-1}$$

where $k = \omega/c$. Assume in the discussion that follows that $P_{io} = 1$.

The scattered acoustic pressure wave is given by

$$P_s(\vec{r}) = f(\hat{i},\hat{o}) \frac{\exp(j\,k\,r)}{R} \text{ for } R > \frac{D^2}{\lambda} \tag{2-2}$$

where λ is the wavelength in the medium; D is the particle dimension; $f(\hat{i},\hat{o})$ represents the information about the amplitude, phase and polarization of the scattered wave in the far field ($R > D^2/\lambda$); and $\hat{i}$ and $\hat{o}$ are unit vectors associated with the incident and scattered wave respectively. [1].

The incident and the scattered power flux in a lossless medium are given by

$$\vec{S}_i = \hat{i}(|P_i|^2/2Z_o) \tag{2-3-a}$$

$$\vec{S}_s = \hat{o}(|P_s|^2/2Z_o) \tag{2-3-b}$$

where Z_o is assumed to be a real quantity. It will be shown later that for a low loss medium, Z_o is essentially a real quantity.

The differential scattering cross section is defined as follows,

$$\sigma_d(\hat{i},\hat{o}) = \lim_{r\to\infty}[R^2 S_s/S_i] = |f(\hat{i},\hat{o})|^2. \qquad (2\text{-}4)$$

The above equation represents an effective geometrical area that the incident wave "sees" and this area is equal to the magnitude squared of the scattering function $f(\hat{i},\hat{o})$. A more meaningful definition of a cross section is one that accounts for the scattered power at all angles (in all directions) surrounding the scattering site. This cross section is defined as the scattering cross section σ_s, and is given by

$$\sigma_s = \int_{4\pi} \sigma_d \, d\Omega = \int_{4\pi} |f(\hat{i},\hat{o})|^2 \, d\Omega \qquad (2\text{-}5)$$

where $d\Omega$ is the differential solid angle.

If both scattering and absorption are present, the sum of the scattering and the adsorption cross sections is

$$\sigma_t = \sigma_s + \sigma_a \qquad (2\text{-}6)$$

where σ_a represents the absorption cross section and is equal to the volume integral of the loss inside the scatterer:

$$\sigma_a = \int_V W \, dV/S_i \qquad (2\text{-}7)$$

where W represents the power density loss inside the scattering site.

III. THE FUNCTIONAL RELATIONSHIP OF σ_s WITH RESPECT TO WAVELENGTH AND THE VOLUME OF THE SCATTERING SITE

According to Equation (2-2) the amplitude of the far field pressure wave is inversely proportional to the distance R. This must be the case because the total power radiated from the scattering site must be independent of radius R (area of a sphere = $4\pi R^2$).

The scattered pressure wave P_s is a linear function of the incident pressure wave P_i and thus P_s at a distance R is proportional to P_i and the volume V of the scatterer:

$$|P_s| = |P_i| \frac{|f(\hat{i},\hat{o})|}{R} = |P_i| \frac{\text{constant } V}{R}. \tag{3-1}$$

The constant in Equation (3-1) should be of dimension L^{-2}, and since it is a function of wavelength, it should be proportional to $(\text{wavelength})^{-2}$. Hence,

$$\sigma_s \sim |f(\hat{i},\hat{o})|^2 \sim V^2/\lambda^4. \tag{3-2}$$

The above analysis assumes that the size of the scatterer is smaller than a wavelength and is referred to as Rayleigh scattering.

In order to determine if the geometry of the scattering site has an effect on the results as stated in Equation (3-2), let the scattering site be a cylinder whose diameter is small compared to λ and whose length approaches infinity. (Obviously there is no such site in the "physical world.") The amplitude of the far field pressure wave must be inversely proportional to $\sqrt{R}$ because the total power radiated per unit length of the cylinder must be independent of radius R (area of cylinder per unit length = $2\pi R$). Hence

$$|P_s| = |P_i| \frac{|f(\hat{i},\hat{o})|}{\sqrt{R}} = |P_i| \frac{(\text{constant})}{\sqrt{R}} (\pi a^2 \cdot 1) \qquad (3\text{-}3)$$

where a is the radius of the cylinder. The constant in Equation (3-3) should be of dimension $L^{-3/2}$, and since it is a function of wavelength, it should be proportional to $(\text{wavelength})^{-3/2}$. Hence,

$$\sigma_s \sim |f(\hat{i},\hat{o})|^2 \frac{(\text{cross sectional area of cylinder})^2}{\lambda^3} \qquad (3\text{-}4)$$

For very long wavelengths and first-order approximation, Morse and Ingard [2] have shown that the scattered power flux is

$$S_s = K\frac{f^3 a^4}{R} S_i (1-2\cos\Theta)^2 \qquad (3\text{-}5)$$

where K is a constant of proportionality and Θ is the angular displacement.

The results of the above analysis indicate that the scattering cross section is proportional to the frequency to the fourth power for a site whose dimensions are small compared to wavelength and the third power for a site that has one dimension (e.g. the length of the cylinder) approaching infinity.

IV. SCATTERING FROM AN INHOMOGENEITY

The energy lost by the incident wave may be absorbed or scattered by a scattering site. In Section II, it was shown (Equations (2-5) and (2-6)) that the amount of energy lost per second by an incident plane wave divided by the incident wave's intensity or power flux is called the cross section σ_t of the object. If there are, on the average, N scatterers per unit volume of the medium, the incident wave, for each unit volume of travel, looses $N\sigma_t$ of the wave's power flux. Hence the intensity of a plane wave propagating through a scatterer-filled medium is

$$|\vec{S}| = |\vec{S}_o| \exp(-N\sigma_t x) \tag{4-1}$$

where x is the distance of penetration. The attenuation factor is $\frac{1}{2} N \sigma_t$.

Morse and Ingard [3] have investigated the propagation of a pressure wave in an inhomogeneous medium. The essence of their investigation will be presented. Restricting the analysis to a single frequency $(\omega/2\pi)$ and supposing there is a region "R", which can be inscribed in a sphere of radius "a", the density and compressibility for the region inside and outside the sphere can be described by a set of quantities

$$\text{Density} = \begin{cases} \rho_e, & |\vec{r}| \leq R \\ \rho, & |\vec{r}| > R \end{cases} \tag{4-2-a}$$

$$\text{Compressibility} = \begin{cases} K_e = 1/(\rho_e c_e^2) & |\vec{r}| \leq R \\ K = 1/(\rho c^2) & |\vec{r}| > R \end{cases} \qquad (4\text{-}2\text{-}b)$$

where c_e and c and ρ_e and ρ are the wave velocities and mass densities inside and outside the sphere, respectively.

The time-independent equation for the pressure amplitude is

$$\nabla^2 P + k^2 P = -k^2 \gamma_K(\vec{r})\, P - \nabla \cdot [\gamma_\rho(\vec{r}) \nabla P] \qquad (4\text{-}3\text{-}a)$$

where

$$k = \omega/c \qquad (4\text{-}3\text{-}b)$$

$$\gamma_K = \begin{cases} \dfrac{K_e - K}{K}, & |\vec{r}| \leq R \\ 0 \;, & |\vec{r}| > R \end{cases}$$

$$\gamma_\rho = \begin{cases} \dfrac{\rho_e - \rho}{\rho_e}, & |\vec{r}| \leq R \\ 0 \;, & |\vec{r}| > R . \end{cases}$$

See Figure 1-b.

At large distances from the sphere ($r >> a$, the radius of the sphere enclosing region "R"), the pressure wave would be

$$P = P_{io}[\exp(jk_x x) + f(\hat{i},\hat{o})\exp(jkr)/r] \qquad (4\text{-}4)$$

where the first term in the brackets represents a plane wave traveling in the x-direction while the second term represents a scattered wave. The key term in Equation (4-4) is $f(\hat{i},\hat{o})$, the angle-distribution factor, and is given by the following expression,

$$f(\hat{i},\hat{o}) = \frac{k^2}{4\pi} \int_V (\gamma_K P - j(\gamma_\rho) \frac{\hat{o}}{k} \cdot \nabla' P) \exp(-jk\hat{o}\cdot\vec{r}')\, dv'. \quad (4-5)$$

If a small particle is assumed, the first term inside the integral in Equation (4-5) gives isotropic scattering and the second term gives scattering proportional to cosine of the angle between the unit vectors $\hat{i}$ and $\hat{o}$.

Unfortunately, Equation (4-5) depends on the total pressure wave (incident and scattered pressure waves) and therefore Equation (4-5) is not a complete description of the scattering amplitude in terms of known quantities.

Chernov [4] analyzes the problem by the method of small perturbations. Assuming small changes in density and the velocity of propagation, the scattered pressure wave can be expressed by the following equation,

$$P_s = \frac{-P_{io}}{4\pi} \int_V [2k^2 \frac{\Delta c}{c_o} + j\,k\, \frac{\partial(\Delta\rho/\rho_o)}{\partial x'}]\, \frac{1}{r} \exp(jk(r+x')dv \quad (4-6)$$

where r is the distance from the scattering element (x'y'z') to the observation point x,y,z, P_{io} represents the amplitude of the incident pressure wave, and $\Delta c/c_o$ and $\Delta\rho/\rho_o$ are the normalized change in the wave velocity and density. Assuming $\Delta\rho/\rho_o$ is independent of position within the scattering site, the second term in the bracket in Equation (4-6) is zero. Since the average velocity $c_o = \sqrt{B_{Ao}/\rho_o}$ where ρ_o and B_{Ao} are the average density and bulk modulus, respectively, the normalized velocity change is

$$\frac{\Delta c}{c_o} = \frac{1}{2}(\frac{\Delta\rho}{\rho_o} - \frac{\Delta B_A}{B_{Ao}}) \quad (4-7)$$

where $\Delta C = C - C_e$, $\Delta\rho = \rho - \rho_e$, $\Delta B_A = B_A - B_{A_e}$, and

$\Delta\rho/\rho_o$ and $\Delta B_A/B_{Ao}$ represent the normalized change in the density bulk modulus. See Appendix A. Treating $\Delta\rho$ and ΔB_A as random flucuations and introducing the normalized correlation functions $N_\rho(\vec{r})$ and $N_{B_A}(\vec{r})$ where $N_\rho(\vec{r}) = \overline{\Delta\rho_1 \Delta\rho_2}/\overline{\Delta\rho^2}$ and $N_{B_A}(\vec{r}) = \overline{\Delta B_{A1}\Delta B_{A2}}/\overline{\Delta B_A^2}$, and assuming zero correlation between density and bulk modulus, the mean-square scattering pressure $P_s P_s^*$ can be determined using Equation (4-6). The result is

$$|P_s|^2 = \frac{|P_{io}|^2}{32\pi R^2} \int_{V_1}\int_{V_2} k^4 [\frac{\overline{\Delta\rho^2}}{\rho_o^2} N_\rho(\vec{r}) + \frac{\overline{\Delta B_A^2}}{B_{Ao}^2} N_{B_A}(\vec{r})] \exp[j\vec{K}\cdot\vec{r}]\, dv_1\, dv_2 \tag{4-8}$$

Where $\frac{1}{r} \approx \frac{1}{R}$ (R is the distance from the origin of coordinates to the observation point. See Figure 2.

$\vec{r} = \vec{r}_1' - \vec{r}_2'$ the vector separation between two points inside volume V.

$\vec{K} = k(\vec{n} - \vec{n}_1)$

$|\vec{K}| = 2k \sin \Theta/2$ where Θ is the scattering angle.

$\overline{\Delta\rho^2}$ = mean-square density.

$\overline{\Delta B_A^2}$ = mean-square bulk modulus.

In order to obtain a better understanding of Equation (4-8), assume that $N_\rho(\vec{r}) = N_{B_A}(\vec{r})$. Using transformations similar to the types used by Chernov [4], the result is

$$|P_s|^2 = A|P_{io}|^2 \frac{k^4}{R^2} [\overline{\Delta\rho^2}/\rho_o^2 + \overline{\Delta B_A^2}/B_{Ao}^2] \int_0^\infty N(r) \sin(kr)\, r\,dr \tag{4-9}$$

where A is a constant and the upper limit on the integral has been set to equal infinity, since the dimensions of the medium are assumed to be large compared to the correlation distance. Note the mean-square pressure in Equation (4-9) is a function of the scattering angle Θ.

The scattering area σ_s, which is the ratio of the total power scattered to the incident power density, can be determined from Equation (4-9) assuming a statistically isotropic medium and integrating the scattering angle Θ from 0 to π radians. The result is

$$\sigma_s = Ak^2[\overline{\Delta\rho^2}/\rho_o^2 + \overline{\Delta B_A^2}/B_{Ao}^2] \int_0^\infty [1-\cos(2kr)\ N(r)]dr \qquad (4\text{-}10)$$

In order to gain insight about the behavior of σ_s as a function of k, let $N(r) = e^{-r/a}$ and $N(r) = e^{-(\frac{r}{a})^2}$ where "a" can be interpreted as the "space constant" of the medium. The results are

$$N(r) = e^{-|r/a|}:\ \sigma_s = A\,[\overline{\Delta\rho^2}/\rho_o^2 + \overline{\Delta B_A^2}/B_{Ao}^2]\ \frac{4a^3k^4}{1+4a^2k^2} \qquad (4\text{-}11\text{-}a)$$

$$N(r) = e^{-|r/a|^2}:\ \sigma_s = A[\overline{\Delta\rho^2}/\rho_o^2 + \overline{\Delta B_A^2}/B_{Ao}^2](\frac{\sqrt{\pi}}{2}a)k^2(1-e^{-(ka)^2}) \qquad (4\text{-}11\text{-}b)$$

See Appendix B.

For ka<<1 the scattering area is proportional to k^4 and for ka>>1 the scattering area is proportional to k^2 for both correlation functions. Figure 3 is a plot of the following two functions which are the frequency dependent portion of Equation (4-11).

$$f_1(ka) = \frac{(ka)^4}{1+(2ka)^2} \qquad (4\text{-}12\text{-}a)$$

$$f_2(ka) = (ka)^2(1-e^{-(ka)^2}). \qquad (4\text{-}12\text{-}b)$$

Comparing the slopes of $f_1(ka)$ and $f_2(ka)$ with the function $(ka)^n$ where $n = 2$, 3 and 4, it can be seen that there is a gradual change in n as ka varies; Rayleigh for $ka \ll 1$ ($n=4$) stochastic scattering for $ka \gg 1$ ($n=2$) and a transition region of $ka \sim 1$.

V. SIMULATING SCATTERING USING A ONE-DIMENSIONAL TRANSMISSION LINE

Propagation of longitudinal sound waves in a solid and the propagation of transverse electromagnetic waves on an electric transmission line obey the same second order partial differential equation assuming the amplitude of the sound wave is small. The two parameters that characterize the medium in which the wave exists will be attenuation per unit length (α) and phase shift per unit length (k). These two parameters can be combined together using complex notation to generate a term called the propagation constant of the medium which is expressed as

$$\gamma = \alpha + j\,k \tag{5-1}$$

where $k = 2\pi/\lambda = 2\pi f/c$.

A low-loss medium is one that absorbs very little energy from the wave as it propagates through the solid and can be mathematically defined when $\alpha << k$. For metals, $0.1 \leq \alpha \leq 10$ nepers/cm and $k \sim 625$ radians/cm at f=50MHz. Thus metals can be classified as essentially a low-loss material.

Before the model of a wave traveling in a low loss medium is discussed, a short discussion on low-loss transmission lines will be presented.

Figure 4 illustrates an electrical T-network that is terminated in its characteristic impedance, where the acoustic impedance ρc will be characterized by analogy with the electrical impedance Z_o ($Z_o=\rho c$). It can be shown [5] that when the circuit is terminated in its characteristic impedance, the input impedance is equal to the characteristic impedance which in turn is a function of the per unit impedance Z, per unit admittance Y and the physical length Δx. The characteristic impedance is

$$Z_o = \left[\frac{Z}{Y}\left(1 + \frac{ZY}{4}\Delta x^2\right)\right]^{\frac{1}{2}}. \tag{5-2}$$

The propagation constant can be expressed in terms of Z and Y as

$$e^{\gamma\Delta x} = e^{\alpha\Delta x}\, e^{jk\Delta x} = 1 + \sqrt{ZY}\,\Delta x + \frac{1}{2}(ZY)\Delta x^2 + \ldots. \tag{5-3}$$

From Equations (5-2) and (5-3), it is obvious that the characteristic impedance and the propagation constant depend on the per unit series impedance, per unit shunt admittance and the length of the transmission line section in a rather complicated manner. If the low-loss constraint is imposed by deleating the series per unit resistance, then the per unit acoustic impedance and admittance are:

$$Z = j\omega\rho_o \tag{5-4-a}$$

$$Y = G + j\omega/B_{Ao} \tag{5-4-b}$$

where ρ_o, B_{Ao}, ω and G represent the density of the material, the bulk modulus, angular frequency ($2\pi f$) and the per unit conductance, respectively. If $G \ll \omega/B_{Ao}$ and $\gamma\Delta x \ll 1$, then

$$Z_o \approx \sqrt{\rho_o B_{Ao}} \tag{5-5-a}$$

and

$$\exp(\gamma\Delta x) = \exp(\alpha\Delta x)\exp(jk\Delta x) \approx 1 + \frac{G}{2}\sqrt{\rho_o B_{Ao}}\,\Delta x + j\omega\sqrt{\frac{\rho_o}{B_{Ao}}}\,\Delta x. \tag{5-5-b}$$

The left side of Equation (5-5-b) can be approximated by $1 + \gamma\Delta x = 1 + \alpha\Delta x + jk\Delta x$ assuming $\gamma\Delta x \ll 1$. The attenuation and phase shift per unit length is

$$\alpha = \frac{1}{2}\sqrt{\rho_o B_{Ao}}\; G \tag{5-6-a}$$

and

$$k = \omega\sqrt{\frac{\rho_o}{B_{Ao}}}. \tag{5-6-b}$$

The results shown in Equation (5-6-a) indicate that α is linearly related to G and k is equal to ω/c where $c = \sqrt{B_{Ao}/\rho_o}$.

From Section IV, it was shown that

$$\alpha = \frac{1}{2}N\,\sigma_s\,. \tag{5-7}$$

Equating Equations (5-6-a) and (5-7) and solving for G, the result is

$$G = \frac{N\sigma_s}{\sqrt{\rho_o B_{Ao}}}. \tag{5-8}$$

The conductance G can be expressed in terms of Equation (4-11).

$$N(r) = e^{-|r/a|}:\; G = \frac{NA}{\sqrt{\rho_o B_{Ao}}}\left[\frac{\overline{\Delta\rho^2}}{\rho_o^2} + \frac{\overline{\Delta B_A^2}}{B_{Ao}^2}\right]\frac{4a^3k^4}{1+4a^2k^2} \tag{5-9-a}$$

$$N(r) = e^{-|r/a|^2}:\; G = \frac{NA}{\sqrt{\rho B_{Ao}}}\left[\frac{\overline{\Delta\rho^2}}{\rho_o^2} + \frac{\overline{\Delta B_A^2}}{B_{Ao}}\right]\left(\frac{\sqrt{\pi}a}{2}\right)k^2\left(1-e^{-(ka)^2}\right). \tag{5-9-b}$$

The conductance in Equation (5-9) reflect the scattering loss only and not absorption loss. As the mean-square values of $\overline{\Delta\rho^2}$ and $\overline{\Delta B_A^2}$ approach zero (perfect medium) the conductance and attenuation become zero (no losses due to scattering). Equation (5-9) also indicates that the conductance will be a function of frequency and the exact relationship will depend on the correlation function. From Figure 3, it appears for the two correlation functions assumed, the behavior of the conductance as a function of frequency ($ka = 2\pi fa/c$) would be approximately the same.

VI. CALCULATION OF ATTENUATION VERSUS FREQUENCY USING THE ONE-DIMENSIONAL TRANSMISSION LINE MODEL

A computer program was written based on the electrical circuit model shown in Figure 4. Table I lists the parameters used in the program and their corresponding numerical values. These values represent a medium having the approximate characteristics of steel.

TABLE I

	Symbol	
Density	ρ	$7.84 \cdot 10^3$ kg/m^3
Young's modulus	B_A	$19.6 \cdot 10^{10}$ newton/m^2
Length of Transmission line	Δx	10^{-6} meter
Velocity of Propagation	v	$5 \cdot 10^3$ meters/sec.
Average separation between scatterers	a_1 a_2	$5 \cdot 10^{-6}$ meter $15 \cdot 10^{-6}$ meter

Two average correlation spacings (a_1, a_2) were chosen in order to investigate the effect that superimposing two media had upon each other. The numerical choice was based primarily on having the attenuation exhibit the Rayleigh and stochastic scattering in the 20 to 100MHz frequency range. The strength of each correlation spacing was controlled by weighting factors W_1 and W_2 which are associated with a_1 and a_2, respectively.

The weighting factors W_1 and W_2 can be introduced in the electrical transmission line model by replacing the conductance G in Figure 4 by two parallel conductances such that $G = W_1G_1 + W_2G_2$ where G_1 and G_2 account for each set of scatterers associated with a_1 and a_2 respectively. By controlling the values of W_1 and W_2 (with the constraint that $W_1 + W_2 = 1$ for all cases), a blend of each set of scatterers can be obtained.

In order to circumvent the statistical determination of the structure of the medium (for example the variance of density and Young's modulus $\overline{\Delta\rho^2}$ and $\overline{\Delta B_A^2}$, respectively), it was assumed that a medium with a_1 = 5 microns has an attenuation of 1 neper/cm at a frequency of 20 MHz. This assumption is based on the results of Vary [6]. The variances $\overline{\Delta\rho^2}$ and $\overline{\Delta B_A^2}$ were assumed to be independent of the correlation spacing. This was based on the fact that the variances act as a scaling factor for attenuation, while the correlation function contributes to the attenuation profile when the behavior of attenuation is studied as a function of frequency.

Figure 5 represents attenuation versus frequency for the two average separations in Table I. The medium with the higher scattering density (a_1 = 5 microns) exhibits the higher attenuation at any frequency. The general attenuation profile for both correlation separation is consistent with theory. At low frequency the exponent of the frequency is slightly less than 4 (Rayleigh) and greater than 2 (stochastic scattering) at high frequencies. Tripling the average correlation distance reflects a decrease in the scatterer density and a change in the attenuation profile. This is consistent with the results shown in Figure 5.

The effect on attenuation of combining two media via weighting factors W_1 and W_2 are shown in Figure 6. Three different frequencies were chosen to study the effect different weighting factors have on attenuation. All three curves exhibit a negative slope which is consistent with changing from a medium of high scattering density to a medium with a low scattering density. Defining the attenuation-weighting factor sensitivity as a change in attenuation per change in weighting factor, one immediately sees, according to Figure 6, that the largest negative sensitivity occurs at f = 100 MHz and when the weighting factor W_1 is zero and W_2 is unity.

VII. DISCUSSION

An electrical transmission line model of a one-dimensional acoustic wave propagating in a solid has been developed to study the behavior of attenuation as a function of frequency. The equivalence of energy loss due to scatterers in the solid can be modeled as a conductance that is dependent on frequency. This conductive element causes the electrical wave to experience attenuation as it propagates along the transmission line which is similar to the phenomenon that occurs when the acoustic wave propagates in a solid.

Theoretical results developed in this report indicate that if the distribution of the scattering sites is random, attenuation as a function of frequency depends on the functional behavior of the spatial correlation function and the normalized mean square deviation of the density and bulk modulus of the scatters from the ambient solid. The absolute level of the attenuation is primarily determined by the sum of normalized mean square deviation of both density and bulk modulus of the scatterers and the spatial correlation function, while the profile of the attenuation versus frequency is determined principally by the spatial correlation function. Assuming correlation functions of the form $e^{-|r/a|}$ and $e^{-|r/a|^2}$, the results indicate that attenuation depends on the fourth power of frequency if $ka \ll 1$ where k is the wave number and "a" is the average separation of the scattering sites; while for $ka \gg 1$ the attenuation depends on the second power of frequency. Between these two extremes, attenuation is a rather complex function of frequency.

It was shown that the attenuation, which is directly proportional to the net cross sectional area of the scattering site, is proportional to both the square of the wave number k and the difference between the Fourier transforms of the scattering site correlation function. The Fourier transforms of the correlation function that form this difference are evaluated at $k = 0$ and $k \neq 0$, respectively. Results indicate that for the Fourier transform evaluated at $ka \gg 1$, the attenuation is due to viscous type absorption ($\alpha \propto f^2$). For the case when $ka \ll 1$, the absorption is Rayleigh ($\alpha \propto f^4$). In essence, the type of absorption mechanism depends strongly on frequency components that are contained in the correlation function.

The results of this investigation were tested by assuming a medium with a base level of 1 neper/cm at a frequency of 20MHz and a = 5 microns. The spatial correlation function, $e^{-|r/a|}$, was assumed in all cases. Based on the above attenuation constraint, attenuation curves were generated using the electrical transmission line model for a = 5 and 15 microns over a frequency range from 20 to 100 MHz. The attenuation profiles developed follow the typical measured attenuation of a physical medium. For a = 15 micron medium, the behavior of attenuation as a function of frequency was less than for the 5 micron medium. This is consistent with a physical medium since "a" represents the average separation between scattering sites. An increase in "a" represents a decrease in scattering site density.

Two media, a = 5 and 15 microns, were superimposed with the mixture controlled by weighting factors W_1 and W_2. The weighting factors W_1 and W_2 are associated with media a = 5 and 15 microns, respectively. By varying W_1 and W_2 with the sum of W_1 and W_2 equaled to unity, the blend of the media can be varied. Attenuation versus weighting factors was investigated for frequencies of 30, 50 and 100 MHz. Results indicate that the greatest attenuation sensitivity occurs at f = 100 MHz, $W_1 \to 0$ and $W_2 \to 1$. This is consistent with the physical medium, since the greatest change in attenuation occurs when the scattering site density is slightly greater than zero and the frequency is at its upper bound (f = 100MHz in this investigation).

This study indicates that future efforts should focus on comparing the attenuation as determined by using the pulse-echo-technique with the above approach. Assuming the medium has single type scattering sites and the scattering is isotropic, the normalized mean square deviation of the density and bulk modulus scattering sites can be determined by identifying the density and bulk modulus a priori and the corresponding spatial ensemble of these sites. The spatial ensemble can be determined by studying functional behavior of density and bulk modulus along radial lines eminating from a point on the surface of the specimen. This assumes that the surface configuration of the scattering sites will be representative of the distribution of these sites inside the medium. Details will have to be investigated on exactly how this information is to be obtained.

Knowing the functional behavior of the spatial ensemble as described in the above paragraph, the spatial correlation function can be determined using techniques found in statistical analysis. In this investigation, two correlation functions that are commonly found in physical systems, $e^{-|r/a|}$ and $e^{-|r/a|^2}$, were chosen. It seems plausible that correlation functions similar in behavior to the above functions should occur, with modifications, in order to account for specific spatial distribution of scattering sites.

VIII. CONCLUSION

It has been demonstrated that using the electrical transmission line model for studying the scattering mechanism by ultrasonic waves in a solid is feasible if a frequency sensitive shunt conductance is introduced into the electrical model. The presence of the shunt conductance causes the electrical signal to be attenuated as it propagates, which is similar to the behavior of a ultrasonic wave propagating in a solid.

Mathematical results indicate that if a solid medium is viewed as realization of an ergodic spatial random process, the attenuation is related to the Fourier transform of the correlation function.

APPENDIX A - MODEL OF HOST MEDIUM

Consider the model of a medium shown in Figure A-1-a. The host medium can be characterized by a mass density ρ and a Young's modulus B_A, while the scattering material by ρ_e and B_{Ae}. The sectional view shown in the Figure depicts the scattering sites by islands surrounded by the host medium. Only one hand of scattering material is assumed which implies that all islands are identical.

Assuming the scattering sites are randomly distributed spatially, the statistical characteristics of the distribution should be independnet of the orientation of the slice through the medium. Also, it will be assumed that the realization shown in Figure A-1-a is one realization from an infinite number of possibilities of a random process and that this process is stationary and ergodic to at least the second moment. This implies that ensemble and spatial averages are the same and the ensemble and spatial correlation functions are equivalent. In other words, all statistical information up to and including the second moment is contained in any specimen composed of the same host medium and the same randomly distributed scattering sites.

An ensemble of realizations as a function of space can be generated by moving along a line through the composite medium such as is shown in Figure A-1-b. Here the mass density is plotted as a function of space. Young's modulus could be studied in a similar manner. Because the random process was assumed to be ergodic to the second moment, just one of the realizations would be sufficient to determine the average, variance and correlation function for the random process provided that the dimensions of the medium is large compared to the spatial fluctuations in mass density or Young's modulus.

The following mathematical development will center on the behavior of the mass density in the specimen. Parallel results can be obtained for Young's modulus.

There is an average density, ρ_o, associated with the random process and it is shown in Figure A-2. The variance $\overline{\Delta\rho^2}$ measures the amount of spread ρ has about ρ_o.

In order to completely describe the process to the second moment, the correlation function must be introduced. It measures how well the density at one point in the specimen compares to another point. Since the process is assumed to be ergodic, it is the separation distance that is important and not the absolute distance as measured with respect to some origin. The correlation function can be expressed in the following manner

$$\overline{\Delta\rho_1\Delta\rho_2} = \overline{\Delta\rho^2}\, N_\rho(\vec{r}) \qquad \text{(A-1)}$$

where $\overline{\Delta\rho^2}$ and $N_\rho(\vec{r})$ represents the variance and the normalized correlation function, respectively. These concepts were used in Section IV.

APPENDIX B - DEVELOPMENT OF EQUATION (4-11-a)

This appendix presents the development of Equation (4-11-a) starting with Equation (4-9). For purposes of simplification, let $B = [\overline{\Delta\rho^2}/\rho_o^2 + \overline{B_A^2}/B_{Ao}^2]$. Thus Equation (4-9) can be expressed in the following manner:

$$\sigma_{s_\Theta} = \frac{A_1 B}{\pi R^2} k^4 \int_0^\infty \frac{N(r) \sin (2Kr \sin (\frac{\Theta}{2}))}{2k \sin (\Theta/2)} r dr \qquad \text{(B-1)}$$

where Θ is the scattering angle, σ_{s_Θ} represents the cross sectional area for a given scattering angle and A_1 equals A/2 in Equation (4-9). The net cross sectional area is determined by integrating Equation (A-1) over a sphere of radius R. Hence

$$\sigma_s = \int_0^\pi \sigma_{s_\Theta} \, 2\pi R^2 \sin \Theta d\Theta =$$

$$\frac{A_1 B}{2\pi R^2} k^3 \int_0^\infty N(r) \int_0^\pi \frac{\sin(2kr \sin (\frac{\Theta}{2}))}{\sin(\frac{\Theta}{2})} 2\pi R^2 \sin\Theta d\Theta \, r dr. \qquad \text{(B-2)}$$

Using the relationship $\sin\Theta = 2\sin(\frac{\Theta}{2}) \cos (\frac{\Theta}{2})$, Equation (B-2) can be rewritten as:

$$\sigma_s = 2A_1 B k^3 \int_0^\infty N(r) \int_0^\pi \sin(2kr \sin (\frac{\Theta}{2}))\cos(\frac{\Theta}{2}) d\Theta \, r dr. \qquad \text{(B-3)}$$

Letting $u = 2kr \sin(\frac{\Theta}{2})$, Equation (B-3) can be expressed as:

$$\sigma_s = 2A_1 B\, k^3 \int_0^\infty N(r) \left(\int_0^{2kr} \sin(u)\, \frac{du}{kr} \right) r\,dr =$$

$$2A_1 B\, k^2 \int_0^\infty N(r)\,[1-\cos(2kr)]\,dr. \qquad \text{(B-4)}$$

Assuming $N(r) = e^{-|r/a|}$, the net cross sectional area is

$$\sigma_s = 2A_1 B\, k^2 \left[\int_0^\infty e^{-r/a}\, dr - \int_0^\infty e^{-r/a} \cos(2rk)\, dr \right] \qquad \text{(B-5)}$$

Performing the above integration and simplifying the result, the net cross sectional area is

$$\sigma_s = 2A_1 B\, k^2 a \left[1 - \frac{1}{1+(2ka)^2} \right] = \frac{A\, B\, 4a^3 k^4}{1+(2ka)^2} \qquad \text{(B-6)}$$

where A in Equation (4-11-a) is $2A_1$ and B is $[\overline{\Delta\rho^2}/\rho_o^2 + \overline{\Delta B_A^2}/B_{Ao}^2]$.

According to Equation (B-5) the first term in the brackets is independent of frequency and a function of the average correlation distance "a", while the second term is a function of frequency ($k=2\pi f/c$) and "a". It is the second term in Equation (B-5) that determines the attenuation dependency on frequency through parameter k.

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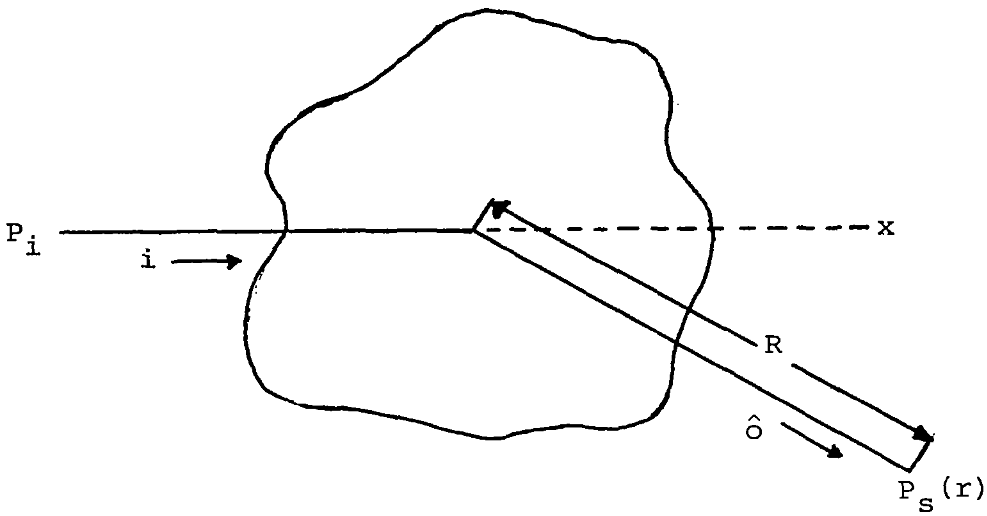


FIGURE a. A plane wave incident upon a scatterer.

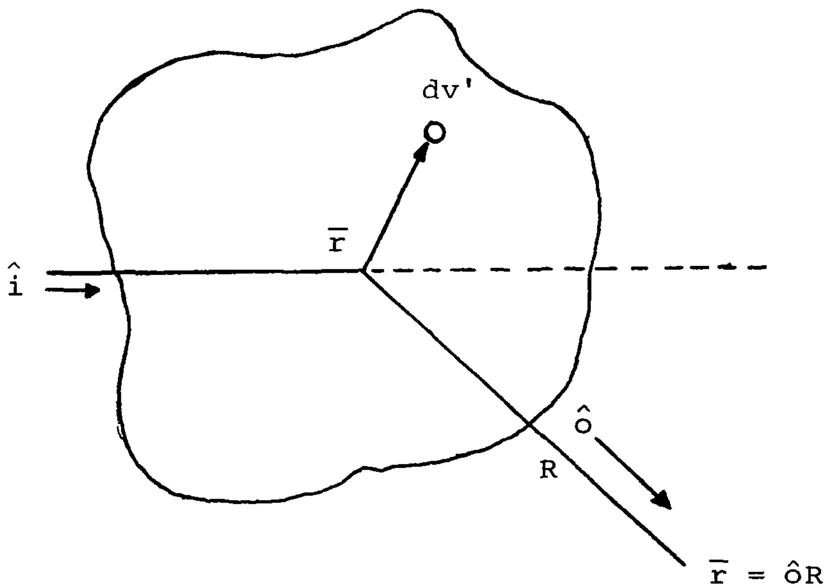


FIGURE b. Geometry for defining the scattering amplitude function $f(\hat{i},\hat{o})$.

FIGURE 1. A scattering site.

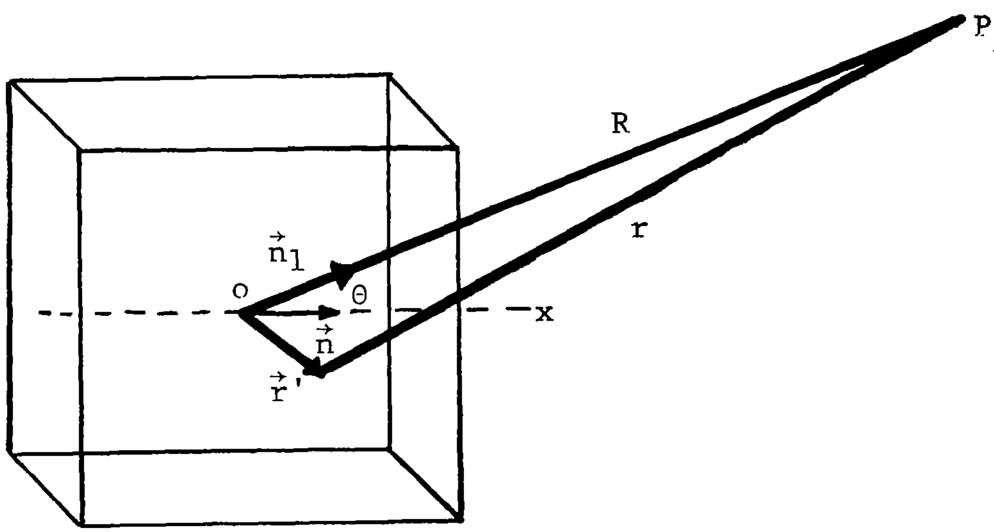


FIGURE 2. The scattering configuration.

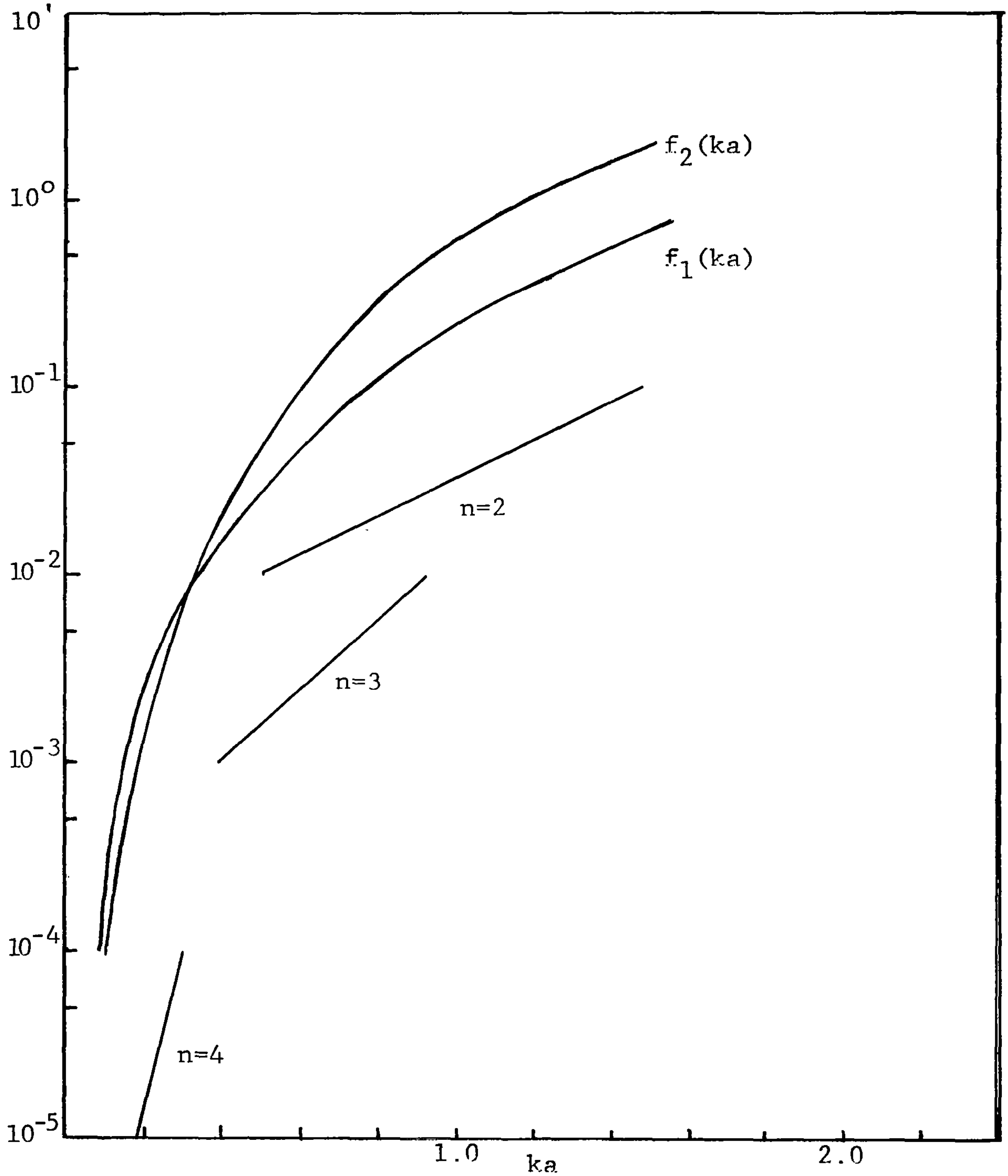
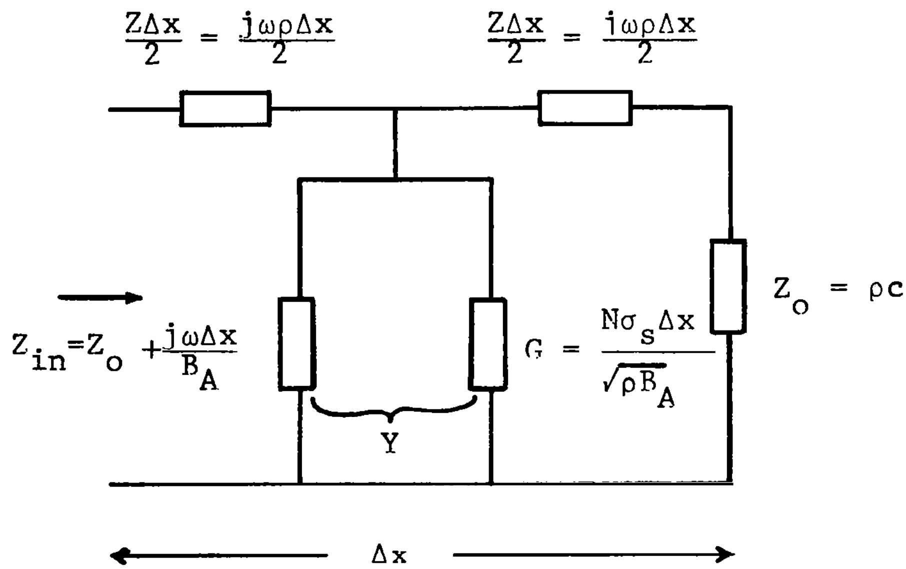


FIGURE 3. Plot of f_1(ka) and f_2(ka) versus ka.



Z = per unit impedance

Y = per unit admittance

G = per unit conductance

$\rho\Delta x$ is equivalent to per unit inductance

$\Delta x/B_A$ is equivalent to per unit capacitance

FIGURE 4. A one-dimensional electrical model of a solid material.

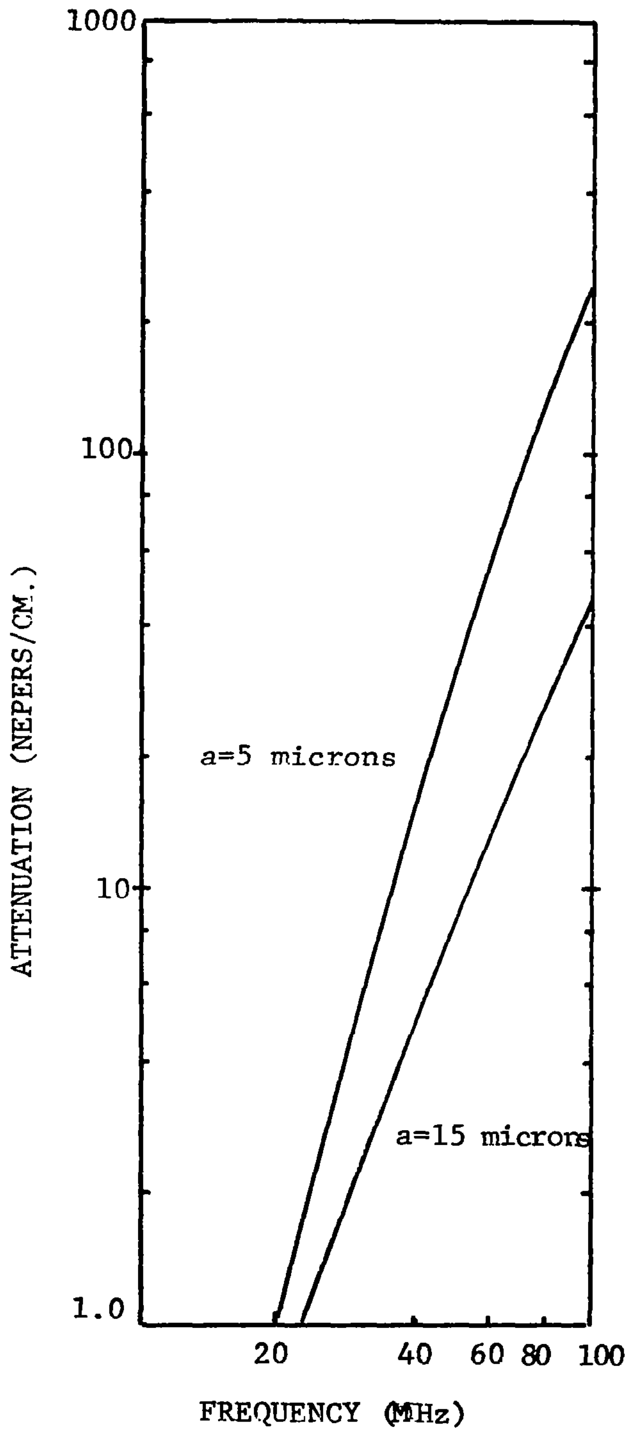
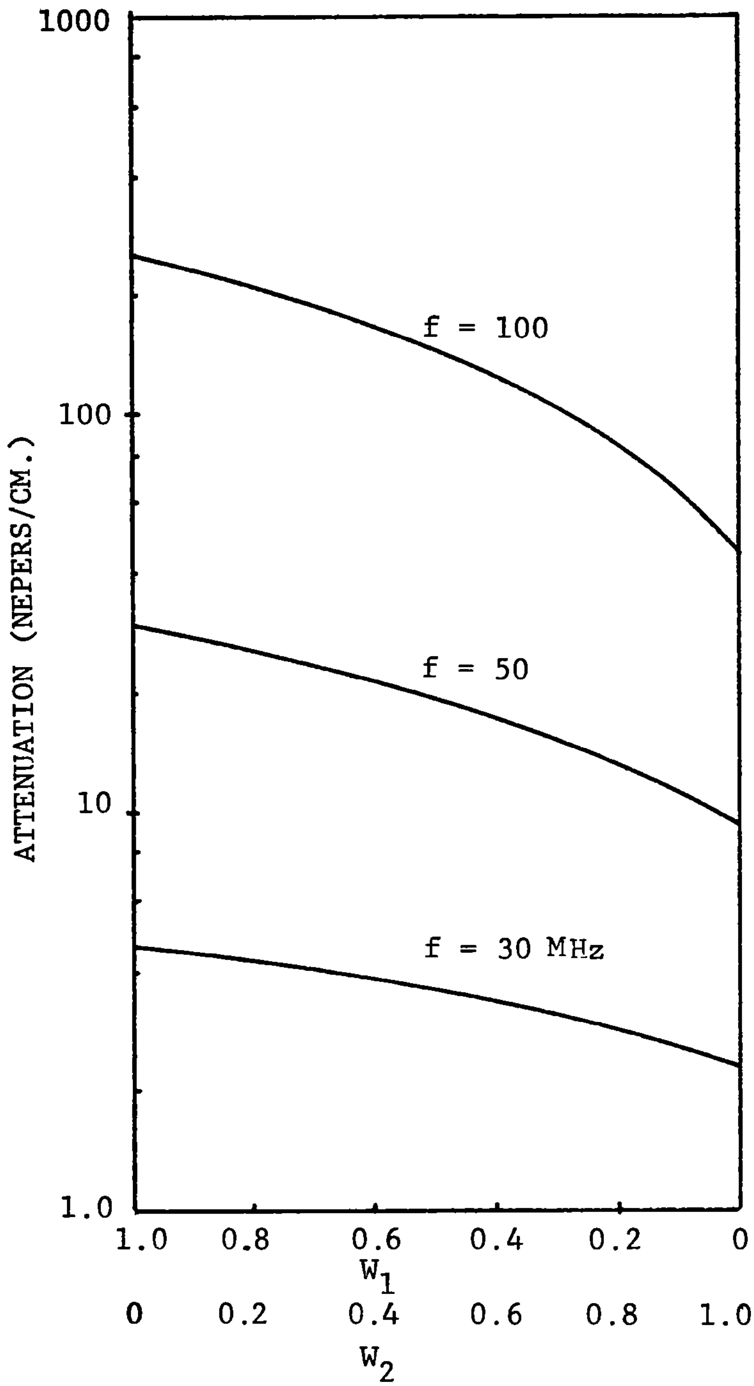
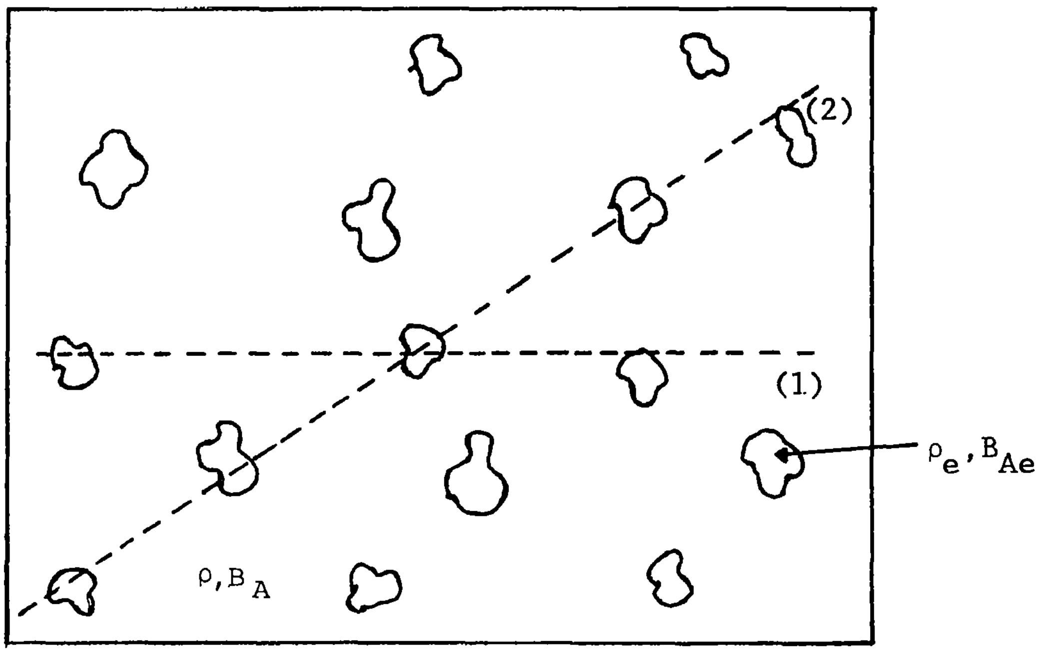


FIGURE 5. Attenuation versus frequency.

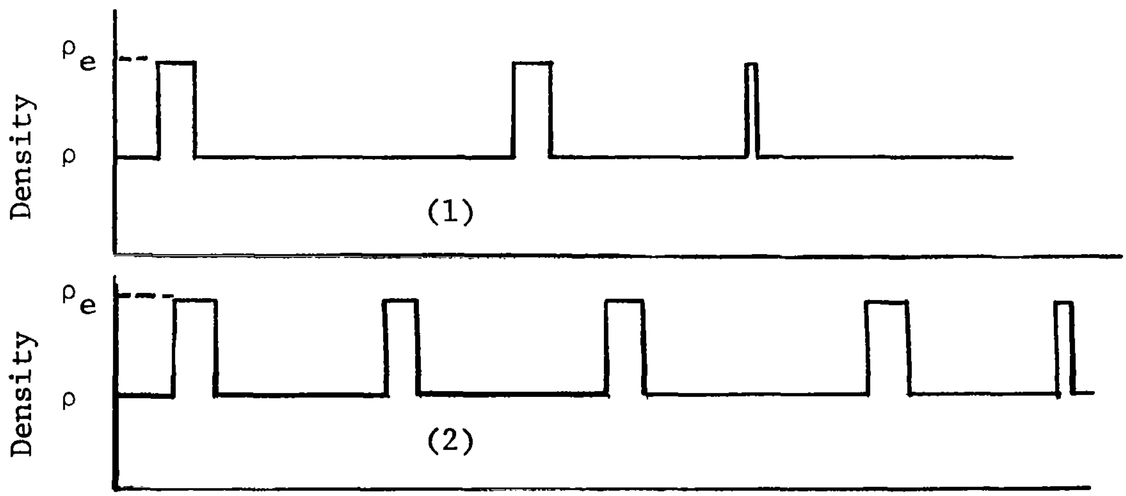


W_1 measures the effect of a medium with a_1=5 microns
W_2 measures the effect of a medium with a_2=15 microı

FIGURE 6. Attenuation versus weighting factors W_1 and



(a) Specimen with scattering sites.



(b) Two realizations of mass density.

FIGURE A-1. Model of a solid medium.

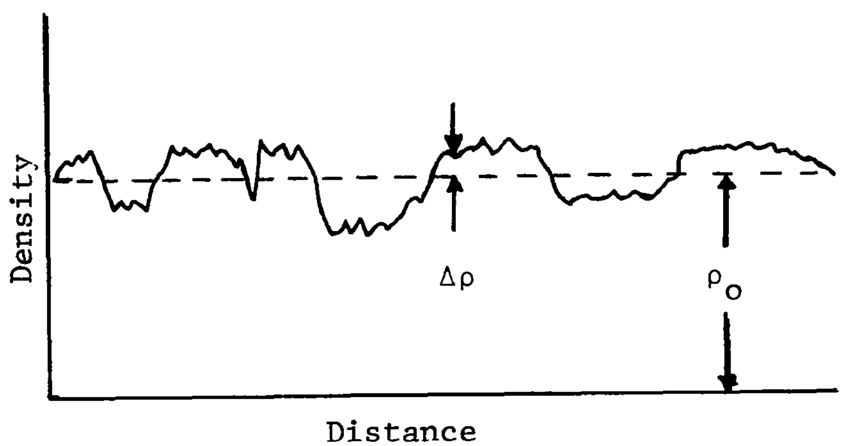


FIGURE A-2. Typical realization with the average and fluctuation in mass density versus distance.

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15. Supplementary Notes

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16. Abstract

This study relates the behavior of acoustic attenuation in a solid to the electrical transmission line model where the electrical shunt conductance, which is frequency dependent, represents the loss due to the scattering sites in the solid. Results indicate that the absolute value of attenuation at a given frequency depends on both the normalized mean square deviation of the density and bulk modulus of the scattering sites from the ambient medium and the spatial scattering correlation function. Besides establishing the absolute value of attenuation, the spatial correlation function determines the attenuation profile as a function of frequency.

17. Key Words (Suggested by Author(s)) Ultrasonics; Ultrasonic scattering; Ultrasonic attenuation; Scattering model; Scattering correlation function; Frequency dependent scattering	18. Distribution Statement Unclassified - unlimited STAR Category 38

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