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A MODIFIED LIFTING LINE THEORY FOR WING-PROPELLER INTERFERENCE

By

R. K. Prabhu

and

S. N. Tiwari, Principal Investigator

Progress Report
For the period June 1 to September 30, 1983

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, Virginia

Under
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Submitted by the
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Norfolk, Virginia 23508

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FOREWORD

This constitutes a progress report on the work completed during the period June 1 to September 30, 1983 on the research project "Wing-Propeller Interference Studies." The work was supported by the NASA/Langley Research Center (Analytical Methods Branch of the Low-Speed Aerodynamics Division) through Cooperative Agreement NCC1-65. The cooperative agreement was monitored by Dr. Chen-Huei Liu of the Low-Speed Aerodynamics Division.

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A MODIFIED LIFTING LINE THEORY FOR WING-PROPELLER INTERFERENCE

By

R. K. Prabhu[1] and S. N. Tiwari[2]

SUMMARY

An inviscid incompressible model for the interaction of a wing with a single propeller slipstream is presented. The model allows the perturbation quantities to be potential even though the undisturbed flow is rotational. The governing equations for the spanwise lift distribution are derived and a simple method of solving these is indicated. Spanwise lift and induced drag distribution for two cases are computed.

INTRODUCTION

Sharp increases in the cost of aviation fuel and uncertainties regarding its supplies which came about as a result of the so-called oil-crisis in 1973, have prompted aircraft designers to look for highly fuel efficient

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modes of propulsion. This has led to the revival of interest in turbo-prop propulsion systems. It is well known that the old technology propellers are still the most efficient means of propulsion for speeds of up to M=0.6. Currently, work is in progress to develop propellers that could operate efficiently up to a flight Mach number of about 0.8. However, several major problems are associated with the use of propellers - one of which is the interference of the propeller slipstream with other parts of the airplane, in particular the wing. This interference brings about changes in the aerodynamic characteristics which have to be fully understood before making any design decisions. Therefore, the wing-propeller interference problem is being studied with renewed interest. The purpose of this paper is to present a method to determine the spanwise lift distribution on a large aspect ratio wing in the presence of a propeller slipstream.

The classical lifting line theory applied to the wing-propeller interaction problem (ref. 1) makes the following four assumptions in addition to those of the lifting line theory applied to large aspect ratio wings:

1. The propeller slipstream is confined within a stream tube of circular cross section.
2. The velocity in this stream tube is uniform (U_J).
3. The relation between the sectional lift and angle of attack is the same as that of an airfoil in uniform flow (with velocities U_J and U_∞ for wing sections inside and outside the slipstream respectively).
4. While computing the downwash, the stream tube representing the propeller slipstream is assumed to extend from upstream infinity to downstream infinity.

Whereas the assumption that the propeller slipstream to be a stream tube of circular cross section is reasonable, the assumption that the velocity inside this tube is uniform, is not realistic. The slipstream behind a propeller does neither have a uniform velocity distribution nor have a velocity discontinuity. The third assumption concerning the lift curve slope of the wing sections washed by the propeller stream, is also not realistic.

These rather drastic simplifications of the classical theory prompted several workers to make a detailed study of the problem. Rethorst and coworkers (ref. 2-5) made extensive studies and developed a lifting surface theory for this problem. They assumed, however, that the propeller slipstream was in the form of a circular jet in which the velocity was uniform. Kleinstein and Liu (ref. 6) scrutinized the assumptions of the classical theory and improved on some of the assumptions. For the sectional lift curve slope they used the data obtained by the solution of the Euler equations. For the computation of the downwash, however, they assumed that the slipstream was in the form of a uniform jet. Their results brought out the effects of the assumption (3) above. Ting et al. (ref. 7) made a new approach by applying the asymptotic method to the interference of a wing with multiple propellers. Their method required the lift data for wing sections in the nonuniform flow which had to be determined by solving the Euler equations. Lan (ref. 8) used a quasi-vortex-lattice method and a two vortex sheet representation of the slipstream to study the interference problem. Rizk (ref. 9) developed a model for the interaction between a thin wing and a nearly uniform jet. He developed a small disturbance model and allowed the perturbation to be potential although the undisturbed stream was rotational.

In the present analysis, the propeller slipstream is assumed to be in the form of a jet with a smooth velocity profile and without a distinct boundary. The relation between the sectional lift and the angle of attack is assumed to be obtained by a local two-dimensional analysis. For the purpose of computing the downwash due to the trailing vortices the slipstream is assumed to extend from far upstream to far downstream. In addition, the disturbance due to the wing, and the nonuniformity in the slipstream are assumed to be small. These assumptions are then used in the classical theory to derive a modified lifting line theory for wing propeller interference.

ANALYSIS

Consider a two-dimensional nonuniform flow past an airfoil. If the undisturbed stream having a non-uniform velocity field $U(y)$ is rotational, then the governing equations to be solved are the Euler equations. If the airfoil is thin and is at a small angle of attack, and the perturbation velocity components u and v may be assumed to be small, then the vorticity transport equation obtained by eliminating pressure from the Euler equations reduces to

$$U(u_y - v_x)_x - vU_{yy} = 0 \tag{1}$$

Further if it is assumed that the term U_{yy} which is the vorticity in the undisturbed stream, is small, then the second term in equation (1) may be neglected. Then, the governing equation reduces to

$$(u_y - v_x)_x = 0 \tag{2}$$

which is satisfied if the perturbation velocity field is irrotational. Therefore, under the assumption that the perturbations are small and the vorticity in the undisturbed stream is small, the perturbation velocity field can be described by a velocity potential.

This concept of embedding potential disturbances in a slightly rotational background flow was employed by Rizk (ref. 9) while considering the wing-propeller interaction problem. He, however, considered a more complex problem wherein the effects of the slipstream swirl and a compressibility were retained. He solved the resulting equations by a finite difference method.

The classical lifting line theory for the wing-propeller interaction given by Koning (ref.1) is an extension of Prandtls' lifting line theory for large aspect ratio wings. The equation governing the spanwise distribution of circulation $\Gamma(y)$ is

$$\Gamma(y) = 1/2 \; c(y) \; c_{\ell\alpha}(y) \; U\{\alpha(y) - w(y)/U\} \tag{3}$$

where $c(y)$ is the wing chord, $c_{\ell\alpha}(y)$ is the sectional lift curve slope and $\alpha(y)$ is the angle of attack; also $U = U_J$ for $|y| < R$, i.e., for stations inside the slipstream tube and $U = U_\infty$ for $|y| > R$, i.e., for stations outside, R being the slipstream tube radius. The downwash $w(y)$ is given by the relation

$$w(y) = \frac{1}{4\pi}\left\{\int_{-s}^{s}\frac{d\Gamma(\eta)}{y-\eta} - \varepsilon_2\left(\int_{-s}^{-R}-\int_{R}^{s}\right)\frac{d\Gamma(\eta)}{y-\eta} + \varepsilon_1\int_{-R}^{R}\frac{d\Gamma(\eta)}{y-R^2/\eta}\right\} \quad (4a)$$

$$|y| < R,$$

$$= \frac{1}{4\pi}\left\{\int_{-s}^{s}\frac{d\Gamma(\eta)}{y-\eta} - \varepsilon_2\left(\int_{-R}^{R}\frac{d\Gamma(\eta)}{y-\eta}\right) - \varepsilon_1\left(\int_{-s}^{-R}-\int_{R}^{s}\right)\frac{d\Gamma(\eta)}{y-R^2/\eta}\right\} \quad (4b)$$

$$|y| > R$$

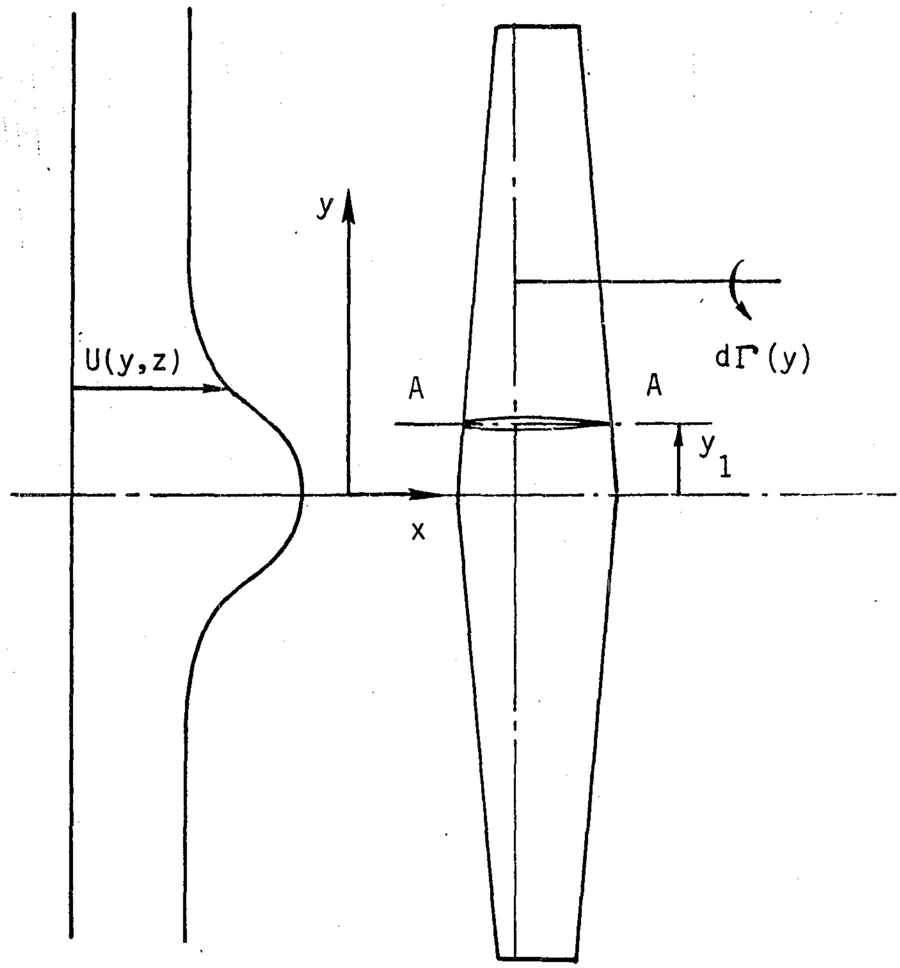
where $\varepsilon_1 = (\mu^2-1)/(\mu^2+1)$, $\varepsilon_2 = (\mu-1)^2/(\mu^2+1)$ and $\mu = U_J/U_\infty$. It may be recalled that in deriving these equations, the four assumptions mentioned in the previous section have been made. Further, when the propeller slipstream is absent $(U_J = U_\infty)$, the factors ε_1 and ε_2 become zero and the above equations reduce to those of the classical lifting line theory.

If the jet representing the propeller stream has a small excess velocity, i.e., $U_J - U_\infty = u \ll U_\infty$, then we may neglect terms of the order of $(u/U_\infty)^2$ in ε_1 and ε_2. In this case $\varepsilon_1 \approx u/U_\infty$ and $\varepsilon_2 \approx 0$; as a result, equations (4a) and (4b) get simplified to

$$w(y) = \frac{1}{4\pi}\left\{\int_{-s}^{s}\frac{d\Gamma(\eta)}{y-\eta} + \frac{u}{U_\infty}\int_{-R}^{R}\frac{d\Gamma(\eta)}{y-R^2/\eta}\right\}, \quad |y| < R \quad (5a)$$

$$= \frac{1}{4\pi}\left\{\int_{-s}^{s}\frac{d\Gamma(\eta)}{y-\eta} - \frac{u}{U_\infty}\left(\int_{-s}^{-R}\int_{R}^{s}\right)\frac{d\Gamma(\eta)}{y-R^2/\eta}\right\}, \quad |y| > R. \quad (5b)$$

Now consider a high aspect ratio wing with the propeller slipstream going past it symmetrically as shown in figure 1. Let the velocity profile be given by $U(y,z) = U(r) = U_\infty[1 + F(r^2)]$ where $r^2 = y^2 + z^2$. Outside the



$$U = U(y,z) = U_\infty\left(1 + F(y^2 + z^2)\right)$$

F IS AN ARBITRARY FUNCTION.

$F \rightarrow 0$, AS $y, z \longrightarrow \infty$

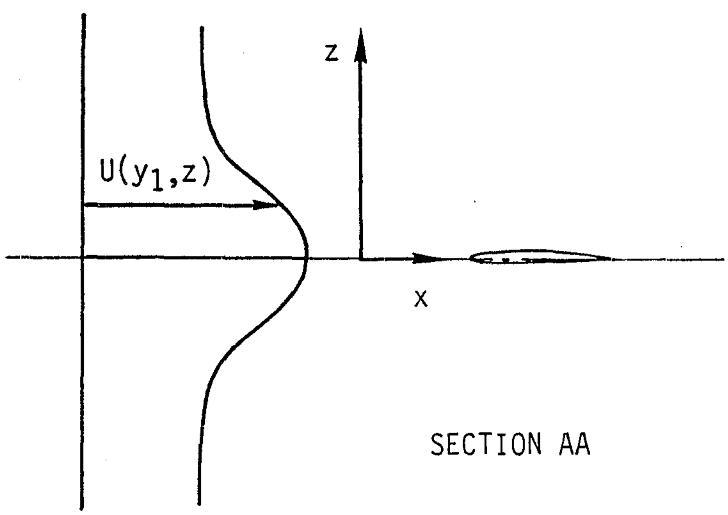


FIG.1 SCHEMATIC OF THE PROBLEM

slipstream i.e., for $r > R_{max}$, $F(r^2) = 0$ and $U(y,z) = U_\infty$.

Following the lifting line theory, the wing is replaced by a lifting line. The circulation at station y on the lifting line is given by

$$\Gamma(y) = 1/2\ c(y)\ c_{l_\alpha}(y)\ U(y,o)\ \{\alpha(y) - w(y)/U(y,o)\}. \qquad (6)$$

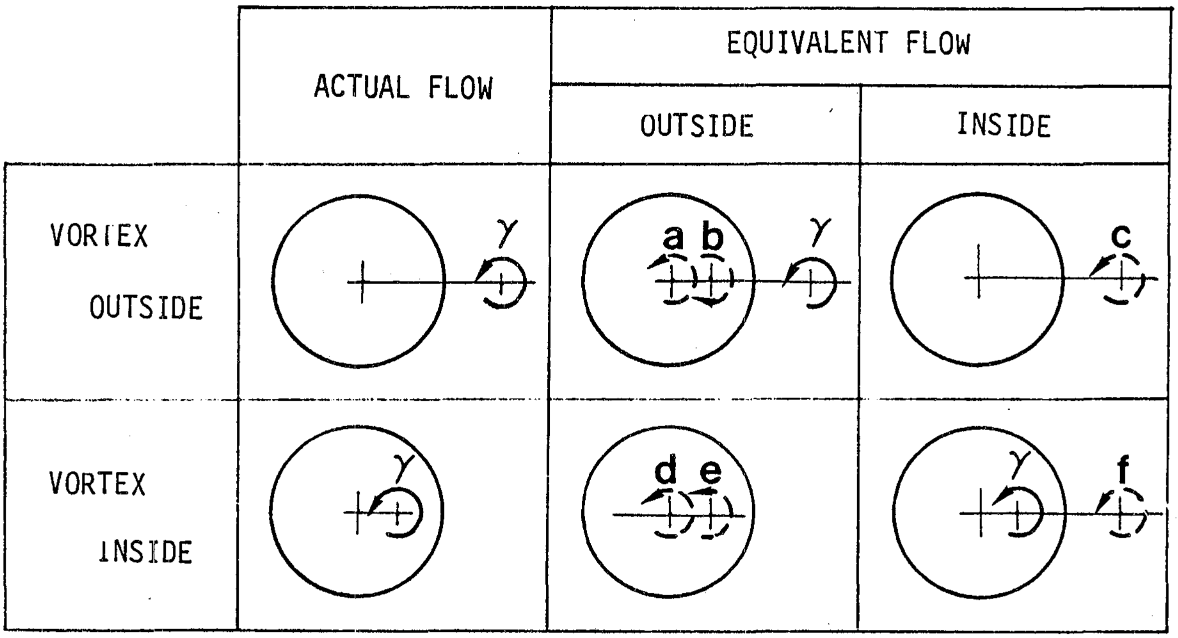
Lift curve slope is determined by considering the airfoil section in a uniform stream of velocity U_∞ for wing sections outside the propeller stream, and by considering the airfoil section in a nonuniform stream having a velocity profile at the corresponding spanwise station for wing sections within the propeller stream. In the present case, the wing section at the spanwise station $y = y_1$ would be in a stream of velocity $U(y = y_1;\ z)$. The lift curve slope for the wing sections in this nonuniform stream is to be obtained by a two-dimensional analysis. This can be done by solving the Euler equations which involves considerable computing effort. An approximate but simple method is the linearized potential flow method described in reference 10 (and summarized in Appendix A). However, for simplicity, only the second method has been used in the present analysis.

Before proceeding to determine the downwash $w(y)$ in the present case, it is useful to recall the basis for the results in the classical setting, where the velocity within the slipstream tube (assumed to be of circular section of radius R) is constant.

Consider a vortex (representing the wing trailing vortex) of strength γ located at a distance η from the center 0 of the circle representing the slipstream tube. Let $|\eta| < R$. It can be shown by applying the interface conditions of continuity of pressure and streamline direction across the surface of the slipstream tube, (see ref. 4) that the flow within the circle is described by a vortex of strength γ at η together with its refracted image of strength $\varepsilon_1 \gamma$ at the inverse point R^2/η, whereas the flow outside the circle is described by a vortex of strength $(1 - \varepsilon_2)\gamma$ at η along with an additional vortex of strength $(\mu\ \varepsilon_1\ \gamma)$ at the center of the circle. Similarly, if the vortex is located outside the slipstream boundary i.e., $|\eta| > R$, it can be shown that the flow within the circle is described by a vortex of strength $(1 - \varepsilon_2)\gamma$ located at η, and the flow outside the circle is described by the vortex along with its refracted image of strength $(-\varepsilon_1\gamma)$ located at the inverse point R^2/η and another vortex of strength $(\varepsilon_1\gamma)$ at the center of the circle*. These results are illustrated in figure 2.

Now consider a propeller stream with a smooth axisymmetric velocity profile. For the purpose of analysis let this stream be divided into a large number of concentric annular cylinders of width ΔR. Consider a vortex of strength γ situated at $Q(OQ = \eta)$. Let the axial velocities in the adjacent annular jets with the interface at the radial stations R be U and $U + u$, respectively. It is easy to see that the change u in the jet velocity results in an image system as described in the previous paragraph. For $u \ll U$, as noted earlier, $\varepsilon_2 = 0$. Hence if $|\eta| < R$, the flow in $|y| < R$ is described by the vortex at Q with its refracted image of strength $\varepsilon_1\gamma$ at the inverse point T $(OT = R^2/\eta)$ and the flow in $|y| > R$ is described by the only vortex at Q; whereas if $|\eta| > R$, the flow in $|y| < R$ is described by the vortex at Q, and the flow in $|y| > R$ is

*If the spanwise loading is assumed to be symmetric, then the vortex at the center can be neglected for the purpose of determining the wing loading. However, if the effect of swirl on the propeller stream is considered, the spanwise load distribution will not be symmetric; consequently, the effect of this vortex needs to be taken into account.



$$a = \epsilon_1 \gamma \qquad b = \epsilon_1 \gamma \qquad c = (1-\epsilon_2)\gamma$$

$$d = \mu \epsilon_1 \gamma \qquad e = (1-\epsilon_2)\gamma \qquad f = \epsilon_1 \gamma$$

$$\epsilon_1 = (\mu^2-1)/(\mu^2+1) \qquad \epsilon_2 = (\mu-1)^2/(\mu^2+1)$$

$$\mu = \frac{\text{Jet Velocity}}{\text{Outer velocity}}$$

FIG.2 IMAGE VORTICES FOR A UNIFORM JET

described by the vortex at Q together with its refracted image of strength $(-\varepsilon\gamma)$ at the inverse point T. See figure 3.

As a result, the downwash at the spanwise station $P(PO = y)$ due to the vortex of strength γ located at Q (with $OQ = \eta$) and its image (whenever applicable) resulting from the surface of velocity discontinuity at the radius R is given by

$$\Delta w(y,\eta) = \frac{\gamma}{4\pi}\left\{\frac{1}{y-\eta} + \frac{\varepsilon_1}{y-R^2/\eta}\right\}, \quad |y| < R, \tag{7a}$$

$$= \frac{\gamma}{4\pi}\left\{\frac{1}{y-\eta}\right\}, \quad |y| > R, \tag{7b}$$

for the region $|\eta| < R$, and

$$\Delta w(y,\eta) = \frac{\gamma}{4\pi}\left\{\frac{1}{y-\eta}\right\}, \quad |y| < R, \tag{8a}$$

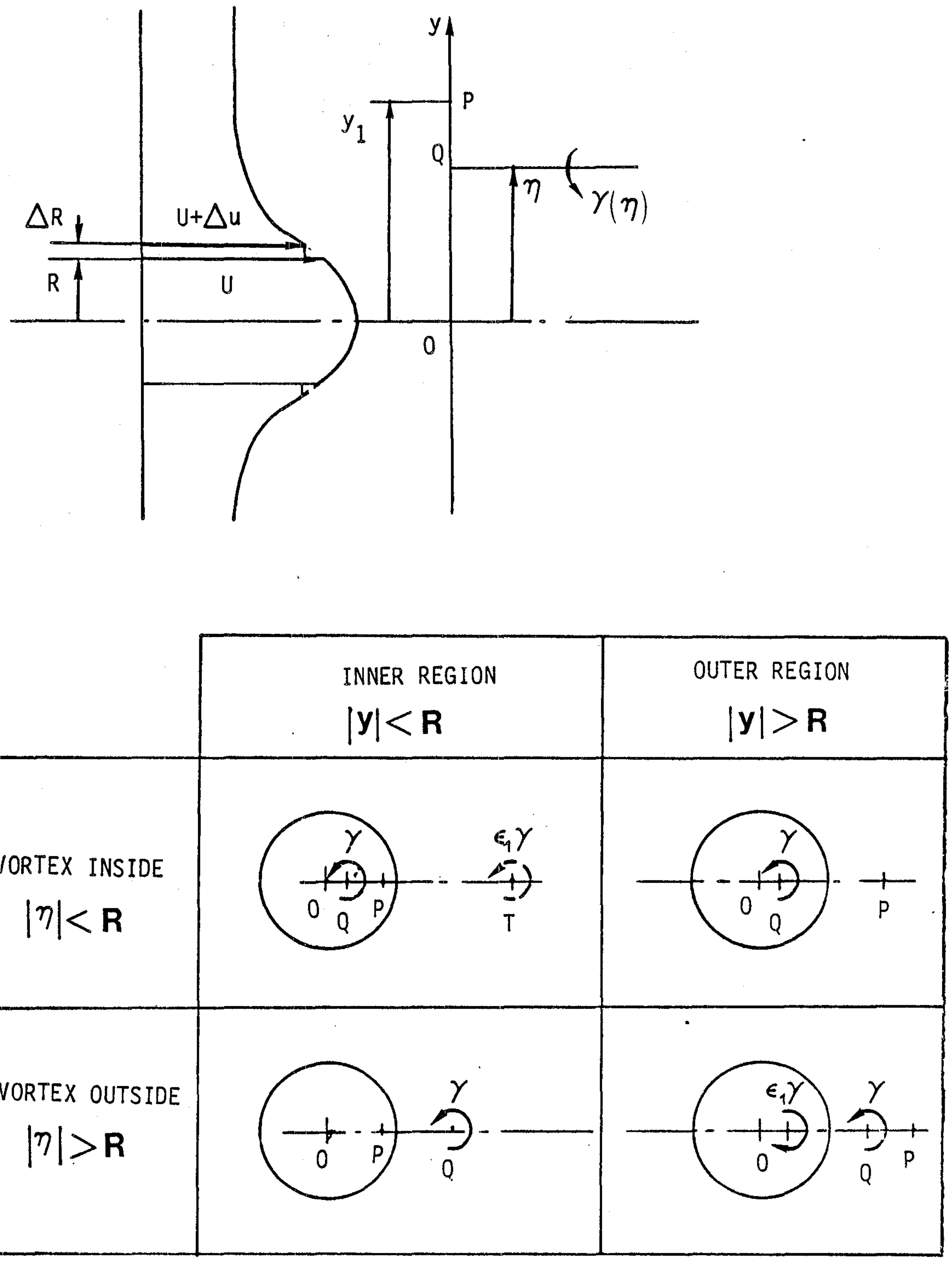
$$= \frac{\gamma}{4\pi}\left\{\frac{1}{y+\eta} - \frac{\varepsilon_1}{y-R^2/\eta}\right\}, \quad |y| > R \tag{8b}$$

for the region $|\eta| > R$. In the limit as ΔR tends to zero, we have

$$u = U(R) - U(R+\Delta R) \approx -\frac{dU}{dR}\cdot dR = -U'dR \tag{9}$$

so that $\varepsilon_1 = -(U'/U)\,dR$.

Letting R to vary from zero to R_{max} (which can be theoretically



OP = y_1 ;   OQ = η ;   OT = R^2/η

FIG. 3 IMAGE VORTICES FOR A SMOOTH JET

infinity) we obtain the following expression for the downwash at y due to a trailing vortex of strength γ located at η.

$$\Delta w(y,\eta) = \frac{\gamma}{4\pi}\left\{ \frac{1}{y-\eta} + \left(\int_0^{|\eta|} - \int_{|y|}^{R_{max}}\right) \frac{U'}{U} \frac{dR}{y-R^2/\eta} \right\}, \quad |y| > |\eta| \tag{10a}$$

$$= \frac{\gamma}{4\pi}\left\{ \frac{1}{y-\eta} + \left(\int_0^{|y|} - \int_{|\eta|}^{R_{max}}\right) \frac{U'}{U} \frac{dR}{y-R^2/\eta} \right\}, \quad |y| < |\eta| \tag{10b}$$

If $\Gamma(y)$ is the unknown circulation distribution along the lifting line, then

$$\gamma = -d\Gamma(\eta).$$

We can now write down the expression for the downwash $w(y)$ at the spanwise station y due to the trailing vortices resulting from the distribution $\Gamma(y)$ as influenced by the axisymmetric jet placed symmetrically with respect to the wing as follows:

$$w(y) = \int \Delta w(y,\eta)$$

$$= -\frac{1}{4\pi}\left[\int_{-s}^{s} \frac{d\Gamma}{y-\eta} + \left\{ \int_{-s}^{-y} - \int_{y}^{s} \right\}\left\{ \int_0^{|y|} - \int_{|\eta|}^{R_{max}} \right) \frac{U'}{U} \frac{dR}{y-R^2/\eta} \right\} d\Gamma(\eta)$$

$$+ \int_{-y}^{y} \left\{ \left(\int_0^{|\eta|} - \int_{|y|}^{R_{max}} \right) \frac{U'}{U} \frac{dR}{y-R^2/\eta} \right\} d\Gamma(\eta). \tag{11}$$

This expression along with the relation

$$\Gamma(y) = 1/2\ c(y)\ c_{\ell\alpha}(y)\ U(y,0)\ \{\alpha(y) - w(y)/U(y,0)\} \tag{12}$$

forms the integro-differential equation for the unknown $\Gamma(y)$. For a given wing $c(y)$ and $\alpha(y)$ are known; in addition the velocity distribution in the propeller slipstream is assumed known. The sectional lift curve slope can be determined by the method of reference 10 or 11. With these information equations (11) and (12) can be solved for the unknown $\Gamma(y)$.

METHOD OF SOLUTION

A simple method of solving the equations (11) and (12) is to assume $\Gamma(y)$ to be a piecewise constant so that there is a certain finite number (say N) of trailing vortices. First, it is found convenient to introduce the following transformation

$$y = s \cos\theta,\ \eta = s \cos\phi,\ \text{and}\ r = R/s \tag{13}$$

Next, trailing vortices are placed at the following N spanwise locations

$$\phi_k = (2K-1)\pi/2N,\ K=1,2,\ldots,\ N, \tag{14}$$

the strength of the trailing vortices being $\gamma(K)\ 4\pi s U_\infty$.

The control points are chosen at the following locations

$$\theta_I = I\pi/N, \qquad I = 1,2,\ldots, N. \tag{15}$$

The circulation at the spanwise station θ_I is given by

$$\Gamma(I) = 4\pi s\, U_\infty \sum_{K=1}^{I} \gamma(K). \tag{16}$$

With this, equation (12) gets transformed into

$$4\pi s\, U_\infty \sum_{K=1}^{I} \gamma(K) = 1/2 \;\; c(I)\, c_{\ell\alpha}(I)\, U(I)\, \{\alpha(I) - w(I)/U(I)\} \tag{17}$$

where $c(I)$, $c_{\ell\alpha}(I)$, $U(I)$, $\alpha(I)$ and $w(I)$ are the corresponding values at the spanwise station $s\cos\theta_I$. The downwash at the control point θ_I due to the N trailing vortices (together with their images) on one side of the wing centerline is given by

$$w_1(I) = U_\infty \sum_{K=1}^{N} \gamma(K) \left\{ \frac{1}{\cos\theta_I - \cos\phi_K} + \left(\int_0^{|\cos\phi_K|} - \int_{|\cos\theta_I|}^{\infty} \right) \frac{U'}{U} \frac{dr}{\cos\theta_I - \frac{r^2}{\cos^2\phi_K}} \right.$$

$$\text{for } |\cos\theta_I| > |\cos\phi_K| \tag{18a}$$

and
$$= U_\infty \sum_{K=1}^{N} \gamma(K) \left\{ \frac{1}{\cos\theta_I - \cos\theta_K} + \left(\int_0^{|\cos\theta_I|} - \int_{|\cos\phi_K|}^{\infty} \right) \frac{U'}{U} \frac{dr}{\cos\theta_I - \frac{r^2}{\cos\phi_K}} \right\}$$

$$\text{for } |\cos\theta_I| < |\cos\phi_K| \tag{18b}$$

which can be written as

$$w_1(I) = U_\infty \sum_{K=1}^{N} G(U,I,K) \tag{19}$$

$$\text{where } G(U,I,K) = \frac{1}{\cos\theta_I - \cos\phi_K} + \left(\int_0^{|\cos\phi_K|} - \int_{|\cos\theta_I|}^{\infty} \right) \frac{U'}{U} \frac{dr}{\cos\theta_I - \dfrac{r^2}{\cos\phi_K}}$$

$$\text{for } |\cos\theta_I| > |\cos\phi_K| \tag{20a}$$

$$\text{and} \qquad = \frac{1}{\cos\theta_I - \cos\phi_K} + \left(\int_0^{|\cos\theta_I|} - \int_{|\cos\phi_K|}^{\infty} \right) \frac{U'}{U} \frac{dr}{\cos\theta_I - \dfrac{r^2}{\cos\phi_K}} \tag{20b}$$

$$\text{for } |\cos\theta_I| < |\cos\phi_K|$$

There is a similar contribution from the trailing vortices from the other half of the wing, so that the total downwash at the station is

$$w(I) = U_\infty \sum_{K=1}^{N} \gamma(K) \left[G(U,I,K) - G(U,I,-K) \right] \tag{21}$$

$$I = 1,2,\ldots, N.$$

On using this expression for $w(I)$ in equation (17) the following set of simultaneous equations is obtained.

$$\sum_{K=1}^{I} \gamma(K) \left[1 + \mu(I) \{ G(U,I,K) - G(U,I,-K)\} \right]$$

$$+ \sum_{K=I+1}^{N} \gamma(K)\, \mu(I) \{ G(U,I,K) - G(U,I,-K)\} = \mu(I) \frac{U}{U_\infty} \alpha(I) \tag{22}$$

$$I = 1,2,\ldots,N.$$

where
$$\mu(I) = \frac{c(I)\; c_{1\alpha}(I)}{8 \pi s} \tag{23}$$

By solving this set of simultaneous equations using any standard procedure, the unknowns $\gamma(K)$, $K=1,2,\ldots,N$ can be determined. The circulation $\Gamma(I)$, lift, downwash and other quantities at the control points can then be computed.

RESULTS

As the first example, a rectangular wing of aspect ratio 6.0 is chosen. The velocity distribution in the propeller stream is assumed as

$$U(y,z) = U_\infty[1 + a \exp \{ -(y^2 + z^2)/d^2\}] \tag{24}$$

with $a = 0.5$ and $d = 0.3$. The resulting spanwise lift distribution is shown in figure 4a along with the lift distribution for the wing in uniform flow. The effect of the jet is two-fold; first it changes the section lift curve slope and second it modifies the downwash close to the centerline distribution. The c_ℓ based on the free stream velocity shows the expected distribution. Figure 4b shows the spanwise distribution of the induced drag coefficient.

As the second example, a tapered wing of aspect ratio 6.67 and taper ratio 0.5 is chosen. The velocity distribution in the propeller stream is assumed as

$$U(y,z) = U_\infty \left[1 + a_1 \exp\left\{-(y^2+z^2)/d_1^2\right\} - a_2 \exp\left\{-(y^2+z^2)/d_2^2\right\}\right]. \qquad (25)$$

With $0 < a_2 < 1 + a$ and $d_1 > d_2$, this profile has the maximum velocity not on the axis, and is a better approximation to the actual axial velocity distribution in the slipstream of a propeller. In the example chosen $a_1 = 0.6$, $a_2 = 0.75$, $d_1/s = 0.3$ and $d_2/s = 0.05$. The spanwise lift and induced drag distribution computed for this example are shown in figure 5a and 5b, respectively.

CONCLUDING REMARKS

By improving some of the assumptions of the classical lifting line theory for the wing-propeller interference problem, a new theory is developed to determine the spanwise lift distribution of large aspect ratio wings as influenced by a single propeller. The essential difference between the present and the classical theory is that whereas the classical theory

RECTANGULAR WING, AR=6.0

$U(y,z)/U_\infty = 1 + a\exp(-(y^2+z^2)/d^2)$

a=0.5, d=0.3

Angle of attack =0.1 radians

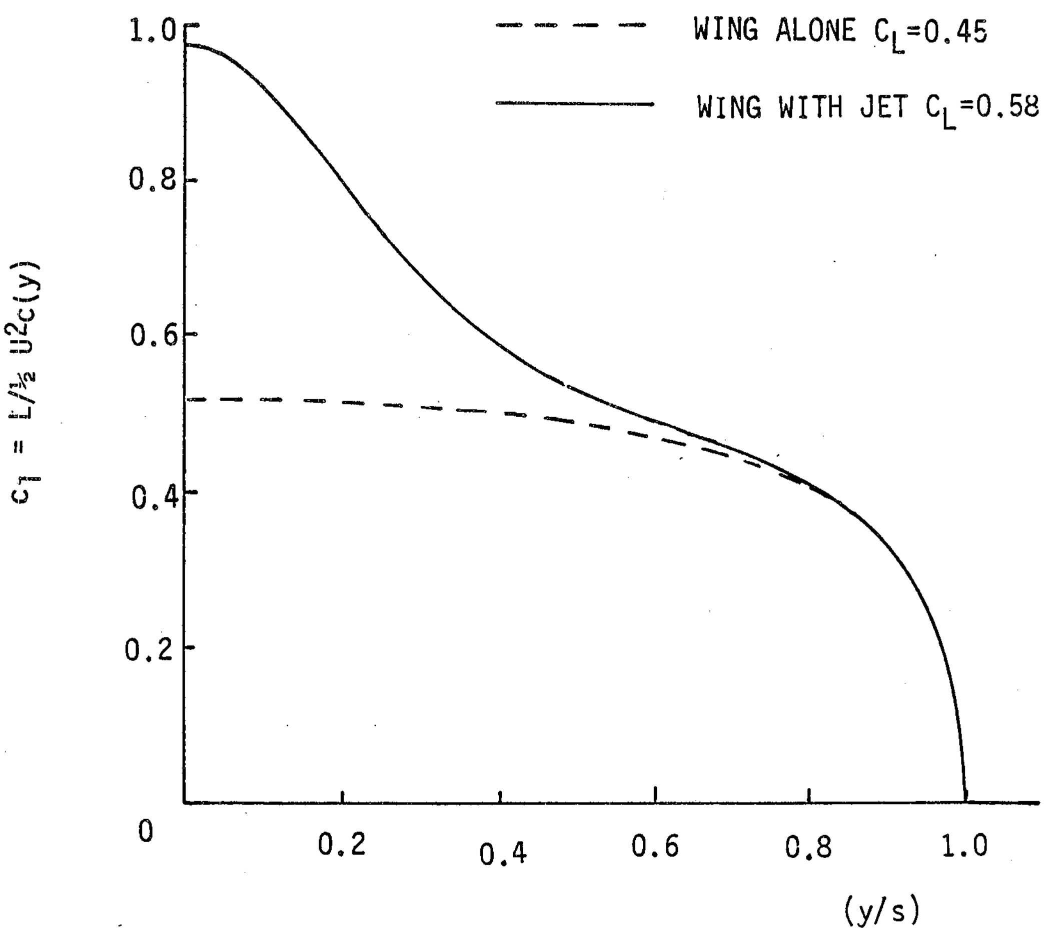


FIG. 4a SPANWISE LIFT DISTRIBUTION

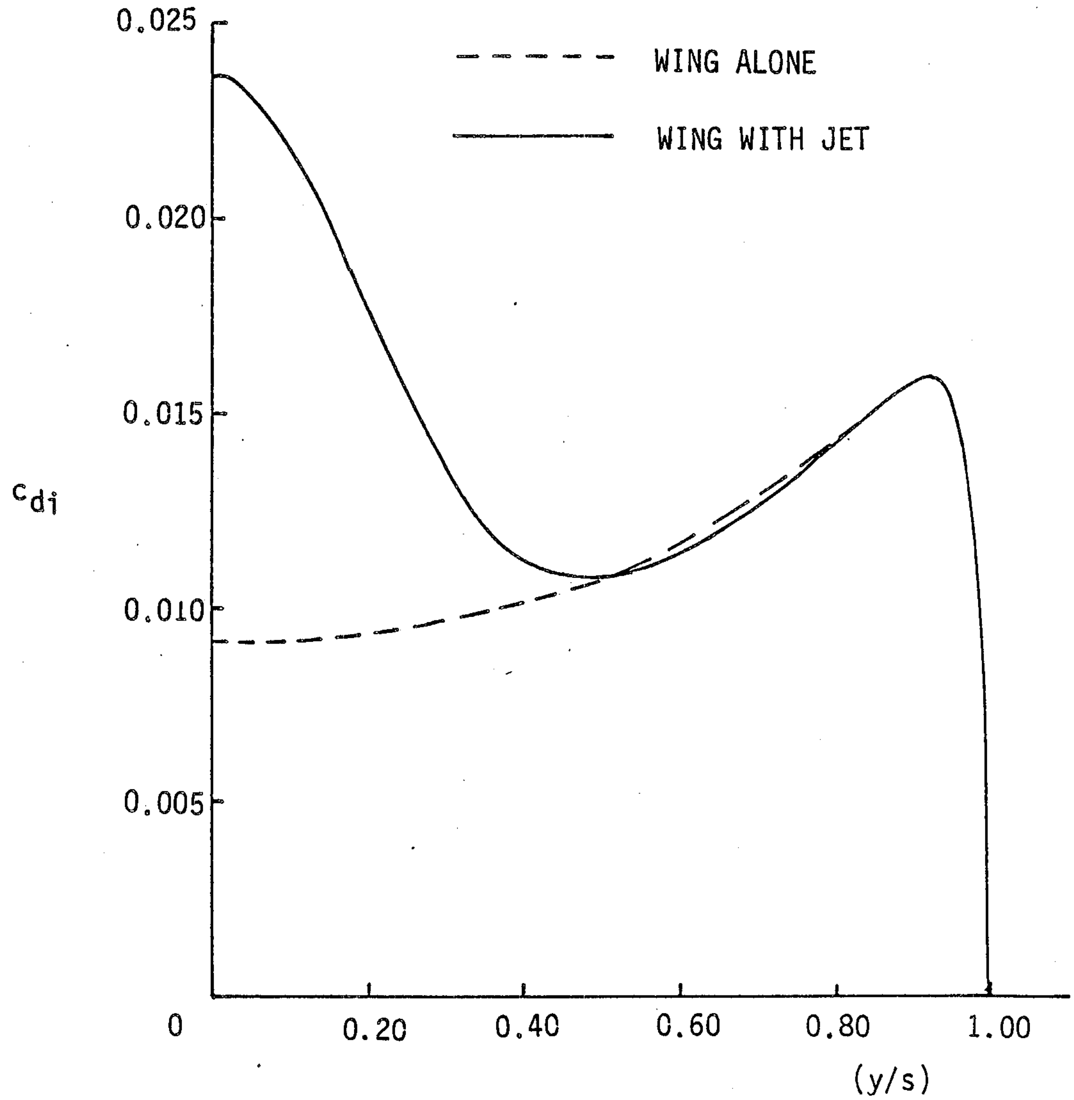


FIG. 4b SPANWISE INDUCED DRAG DISTRIBUTION ON THE RECTANGULAR WING

TAPERED WING $\quad AR = 6.67 \quad TR = 0.5$

Axisymmetric jet

$$U(y,z)/U_\infty = 1 + a_1 \exp(-(y^2 + z^2)/d_1^2) + a_2 \exp(-(y2 + z^2)/d_2^2)$$

$a_1 = 0.6; \quad d_1 = 0.3; \quad a_2 = -0.75; \quad d_2 = 0.05$

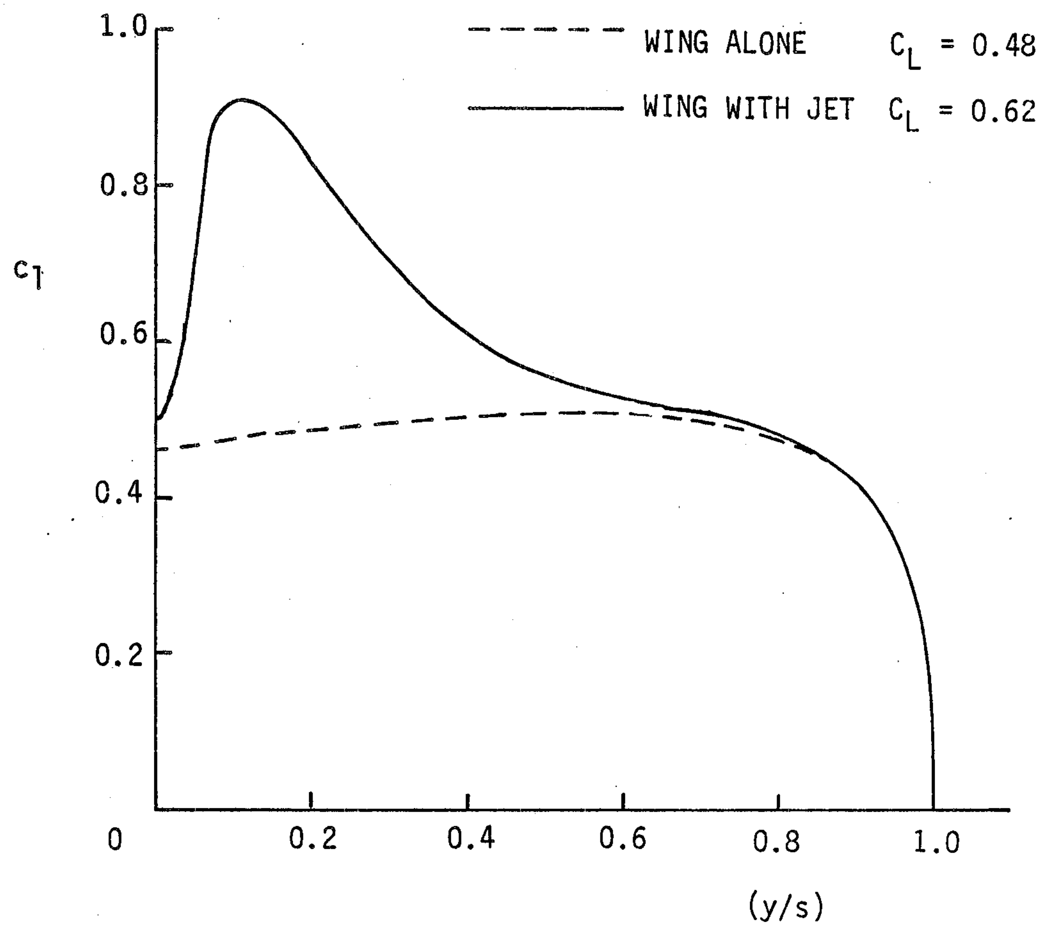


FIG. 5a SPANWISE LIFT DISTRIBUTION ON A TAPERED WING

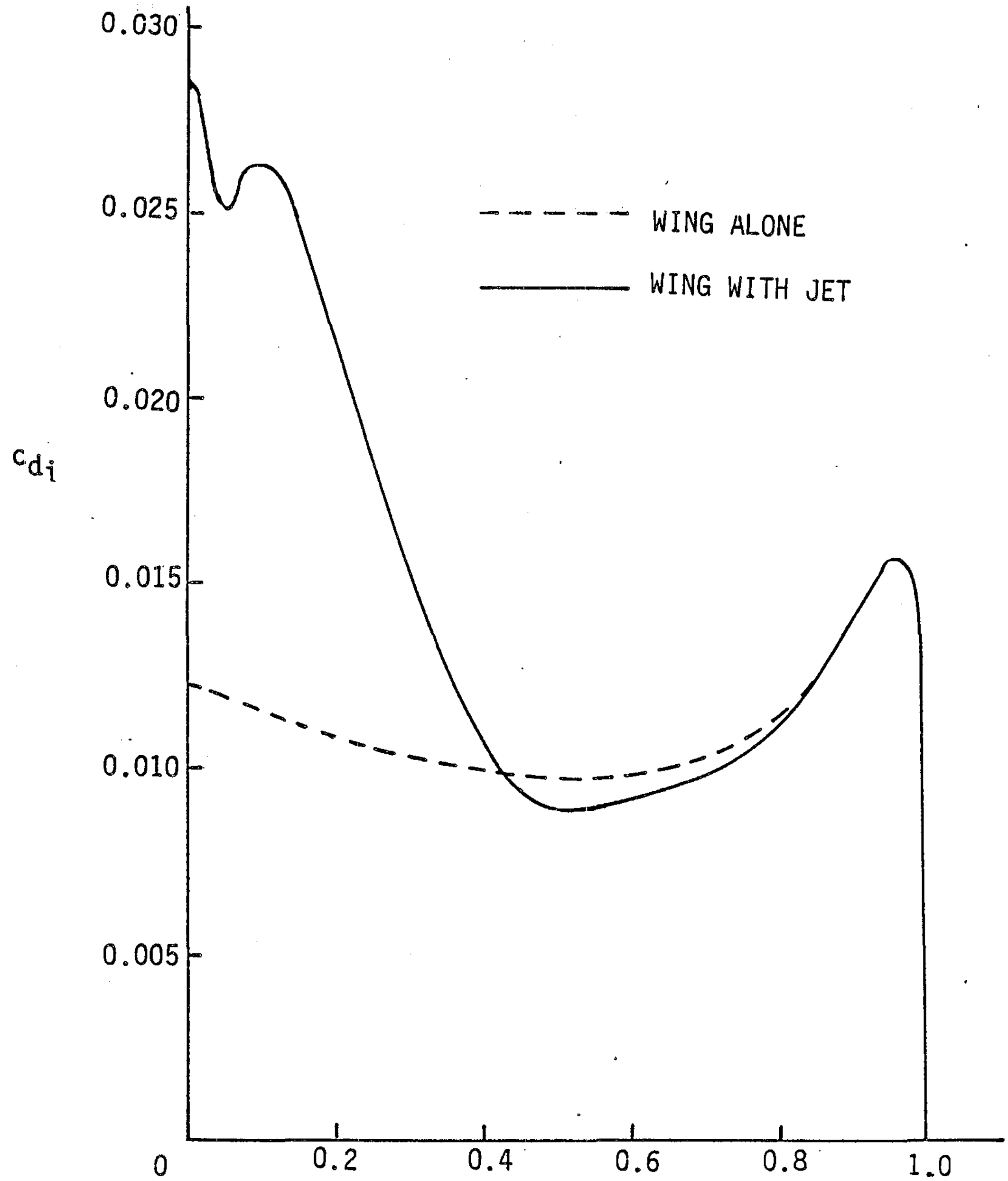


FIG. 5b SPANWISE INDUCED DRAG DISTRIBUTION ON THE TAPERED WING

idealizes the propeller slipstream as a circular jet with constant velocity, the present theory replaces this assumption by a more realistic one. The present study covers a single propeller placed symmetrically ahead of the wing. The underlying method can be applied, without much difficulty, to the case of an unsymmetrically placed propeller and non-overlapping multipropellers. Results of these studies will be reported soon.

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APPENDIX

A THIN AIRFOIL IN A STREAM OF SMOOTH VELOCITY PROFILE

Consider an infinite series of jets of the same width h as an approximation to a given smooth velocity profile. In the limit as h tends to zero, the velocity profile tends to the given smooth velocity profile. Let the velocity the nth jet being denoted by U_n. Let a thin airfoil be placed on the axis of the primary jet in which the velocity is U_0. If the airfoil is represented by a single vortex Γ, then the strength ${}^nK_s$ of the image vortices is given by the relation

$$\beta_{n+1}^2 \; {}^{n+1}K_{n+s+1} = {}^nK_{n+s+1} - \alpha_{n+1} \; {}^nK_{n-s}, \quad \begin{array}{l} -\infty < n < \infty \\ -\infty < s < \infty \end{array}$$

$$\alpha_n = (U_n^2 - U_{n+1}^2)/(U_n{}^2 + U_{n+1}^2) \; ; \; \beta_n = \quad 1 - \alpha_n^2,$$

with ${}^nK_n = 0, \qquad n \neq 0,$

and ${}^0K_0 = \Gamma.$

It may be noted that the first index (n) in ${}^nK_s$ corresponds to the stream under consideration and the second index (s) corresponds to the stream in which the image vortex is located.

A complete solution of the above equation is complex; but for small variation of velocity from jet to jet, it is possible to write

$$\alpha_n = (U_n^2 - U_{n+1}^2)/(U_n^2 + U_{n+1}^2) \approx - u_n/U_n,$$

where $u_n = (U_{n+1} - U_n) << U_n$.

With $\alpha_n << 1$, a first order solution for ${}^nK_s$ can be obtained. The resulting image system for the primary stream is

$${}^0K_0 = \Gamma, \qquad \text{at } y = 0,$$

$${}^0K_{2s} = 0,$$

$${}^0K_{2s-1} = \alpha_s \Gamma \qquad \text{at} \quad y = (2s-1)h, \quad s \geq 1,$$

$$= - \alpha_s \Gamma \quad \text{at} \quad y = (2s-1)h, \quad s \leq 0.$$

If the airfoil is represented by a vortex distribution $\gamma(x)$, $0 \leq x \leq c$ instead of a single vortex Γ, a similar image vortex system results. The downwash at the airfoil with this image system is then given by

$$w(x) = \frac{1}{2\pi} \int_0^c \left\{ \frac{1}{x-\xi} + \sum_{s=1,3,\ldots} \frac{\alpha_s(x-\xi)}{(x-\xi)^2 + (2s-1)h^2} - \sum_{s=1,-3,\ldots} \frac{\alpha_s(x-\xi)}{(x-\xi)^2 + (2s-1)h^2} \right\} \gamma(\xi)d\xi$$

For small h (=dz), $u_n = -(dU/dz)\ dz$, and the expression for α_s becomes

$$\alpha_s = -\frac{1}{U} \cdot \frac{dU}{dz} \cdot dz$$

where $\frac{dU}{dz}$ and U are measured at z. Consequently the equation for the downwash at the airfoil becomes

$$w(x) = \frac{1}{2\pi} \int_0^c \left\{ \frac{1}{x-\xi} - \int_0^\infty \frac{U'}{U} \frac{(x-\xi)\, dz}{(x-\xi)^2 + 4z^2} + \int_{-\infty}^0 \frac{U'}{U} \frac{(x-\xi)\, dz}{(x-\xi)^2 + 4z^2} \right\} \gamma(\xi) d\xi$$

For any given smooth velocity profile $U(z)$, with $U(z) \neq 0$, $-\infty < z < \infty$, the integrals in the parenthesis can be evaluated using any standard technique and by satisfying the flow tangency condition on the mean camber line, the unknown vortex distribution $\gamma(x)$ can be determined. The lift coefficient of the section can then be determined.

If the given velocity profile $U(z)$ is symmetric and the airfoil is placed on the line of symmetric, then the expression for $w(x)$ simplifies to

$$w(x) = \frac{1}{2\pi} \int_0^c \left\{ \frac{1}{x-\xi} - 2(x-\xi) \int_0^\infty \frac{U'}{U} \frac{dz}{(x-\xi)^2 + 4z^2} \right\} \gamma(\xi) d\xi$$

FOREWORD

This constitutes a progress report on the work completed during the period June 1 to September 30, 1983 on the research project "Wing-Propeller Interference Studies." The work was supported by the NASA/Langley Research Center (Analytical Methods Branch of the Low-Speed Aerodynamics Division) through Cooperative Agreement NCC1-65. The cooperative agreement was monitored by Dr. Chen-Huei Liu of the Low-Speed Aerodynamics Division.

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A MODIFIED LIFTING LINE THEORY FOR WING-PROPELLER INTERFERENCE

By

R. K. Prabhu[1] and S. N. Tiwari[2]

SUMMARY

An inviscid incompressible model for the interaction of a wing with a single propeller slipstream is presented. The model allows the perturbation quantities to be potential even though the undisturbed flow is rotational. The governing equations for the spanwise lift distribution are derived and a simple method of solving these is indicated. Spanwise lift and induced drag distribution for two cases are computed.

INTRODUCTION

Sharp increases in the cost of aviation fuel and uncertainties regarding its supplies which came about as a result of the so-called oil-crisis in 1973, have prompted aircraft designers to look for highly fuel efficient

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modes of propulsion. This has led to the revival of interest in turbo-prop propulsion systems. It is well known that the old technology propellers are still the most efficient means of propulsion for speeds of up to M=0.6. Currently, work is in progress to develop propellers that could operate efficiently up to a flight Mach number of about 0.8. However, several major problems are associated with the use of propellers - one of which is the interference of the propeller slipstream with other parts of the airplane, in particular the wing. This interference brings about changes in the aerodynamic characteristics which have to be fully understood before making any design decisions. Therefore, the wing-propeller interference problem is being studied with renewed interest. The purpose of this paper is to present a method to determine the spanwise lift distribution on a large aspect ratio wing in the presence of a propeller slipstream.

The classical lifting line theory applied to the wing-propeller interaction problem (ref. 1) makes the following four assumptions in addition to those of the lifting line theory applied to large aspect ratio wings:

1. The propeller slipstream is confined within a stream tube of circular cross section.
2. The velocity in this stream tube is uniform (U_J).
3. The relation between the sectional lift and angle of attack is the same as that of an airfoil in uniform flow (with velocities U_J and U_∞ for wing sections inside and outside the slipstream respectively).
4. While computing the downwash, the stream tube representing the propeller slipstream is assumed to extend from upstream infinity to downstream infinity.

Whereas the assumption that the propeller slipstream to be a stream tube of circular cross section is reasonable, the assumption that the velocity inside this tube is uniform, is not realistic. The slipstream behind a propeller does neither have a uniform velocity distribution nor have a velocity discontinuity. The third assumption concerning the lift curve slope of the wing sections washed by the propeller stream, is also not realistic.

These rather drastic simplifications of the classical theory prompted several workers to make a detailed study of the problem. Rethorst and coworkers (ref. 2-5) made extensive studies and developed a lifting surface theory for this problem. They assumed, however, that the propeller slipstream was in the form of a circular jet in which the velocity was uniform. Kleinstein and Liu (ref. 6) scrutinized the assumptions of the classical theory and improved on some of the assumptions. For the sectional lift curve slope they used the data obtained by the solution of the Euler equations. For the computation of the downwash, however, they assumed that the slipstream was in the form of a uniform jet. Their results brought out the effects of the assumption (3) above. Ting et al. (ref. 7) made a new approach by applying the asymptotic method to the interference of a wing with multiple propellers. Their method required the lift data for wing sections in the nonuniform flow which had to be determined by solving the Euler equations. Lan (ref. 8) used a quasi-vortex-lattice method and a two vortex sheet representation of the slipstream to study the interference problem. Rizk (ref. 9) developed a model for the interaction between a thin wing and a nearly uniform jet. He developed a small disturbance model and allowed the perturbation to be potential although the undisturbed stream was rotational.

In the present analysis, the propeller slipstream is assumed to be in the form of a jet with a smooth velocity profile and without a distinct boundary. The relation between the sectional lift and the angle of attack is assumed to be obtained by a local two-dimensional analysis. For the purpose of computing the downwash due to the trailing vortices the slipstream is assumed to extend from far upstream to far downstream. In addition, the disturbance due to the wing, and the nonuniformity in the slipstream are assumed to be small. These assumptions are then used in the classical theory to derive a modified lifting line theory for wing propeller interference.

ANALYSIS

Consider a two-dimensional nonuniform flow past an airfoil. If the undisturbed stream having a non-uniform velocity field $U(y)$ is rotational, then the governing equations to be solved are the Euler equations. If the airfoil is thin and is at a small angle of attack, and the perturbation velocity components u and v may be assumed to be small, then the vorticity transport equation obtained by eliminating pressure from the Euler equations reduces to

$$U(u_y - v_x)_x - vU_{yy} = 0 \qquad (1)$$

Further if it is assumed that the term U_{yy} which is the vorticity in the undisturbed stream, is small, then the second term in equation (1) may be neglected. Then, the governing equation reduces to

$$(u_y - v_x)_x = 0 \qquad (2)$$

which is satisfied if the perturbation velocity field is irrotational. Therefore, under the assumption that the perturbations are small and the vorticity in the undisturbed stream is small, the perturbation velocity field can be described by a velocity potential.

This concept of embedding potential disturbances in a slightly rotational background flow was employed by Rizk (ref. 9) while considering the wing-propeller interaction problem. He, however, considered a more complex problem wherein the effects of the slipstream swirl and a compressibility were retained. He solved the resulting equations by a finite difference method.

The classical lifting line theory for the wing-propeller interaction given by Koning (ref.1) is an extension of Prandtls' lifting line theory for large aspect ratio wings. The equation governing the spanwise distribution of circulation $\Gamma(y)$ is

$$\Gamma(y) = 1/2\ c(y)\ c_{\ell\alpha}(y)\ U\{\alpha(y) - w(y)/U\} \tag{3}$$

where $c(y)$ is the wing chord, $c_{\ell\alpha}(y)$ is the sectional lift curve slope and $\alpha(y)$ is the angle of attack; also $U = U_J$ for $|y| < R$, i.e., for stations inside the slipstream tube and $U = U_\infty$ for $|y| > R$, i.e., for stations outside, R being the slipstream tube radius. The downwash $w(y)$ is given by the relation

$$w(y) = \frac{1}{4\pi}\left\{\int_{-s}^{s}\frac{d\Gamma(\eta)}{y-\eta} - \varepsilon_2\left(\int_{-s}^{-R} - \int_{R}^{s}\right)\frac{d\Gamma(\eta)}{y-\eta} + \varepsilon_1\int_{-R}^{R}\frac{d\Gamma(\eta)}{y-R^2/\eta}\right\} \qquad (4a)$$

$$|y| < R,$$

$$= \frac{1}{4\pi}\left\{\int_{-s}^{s}\frac{d\Gamma(\eta)}{y-\eta} - \varepsilon_2\left(\int_{-R}^{R}\frac{d\Gamma(\eta)}{y-\eta}\right) - \varepsilon_1\left(\int_{-s}^{-R} - \int_{R}^{s}\right)\frac{d\Gamma(\eta)}{y-R^2/\eta}\right\} \qquad (4b)$$

$$|y| > R$$

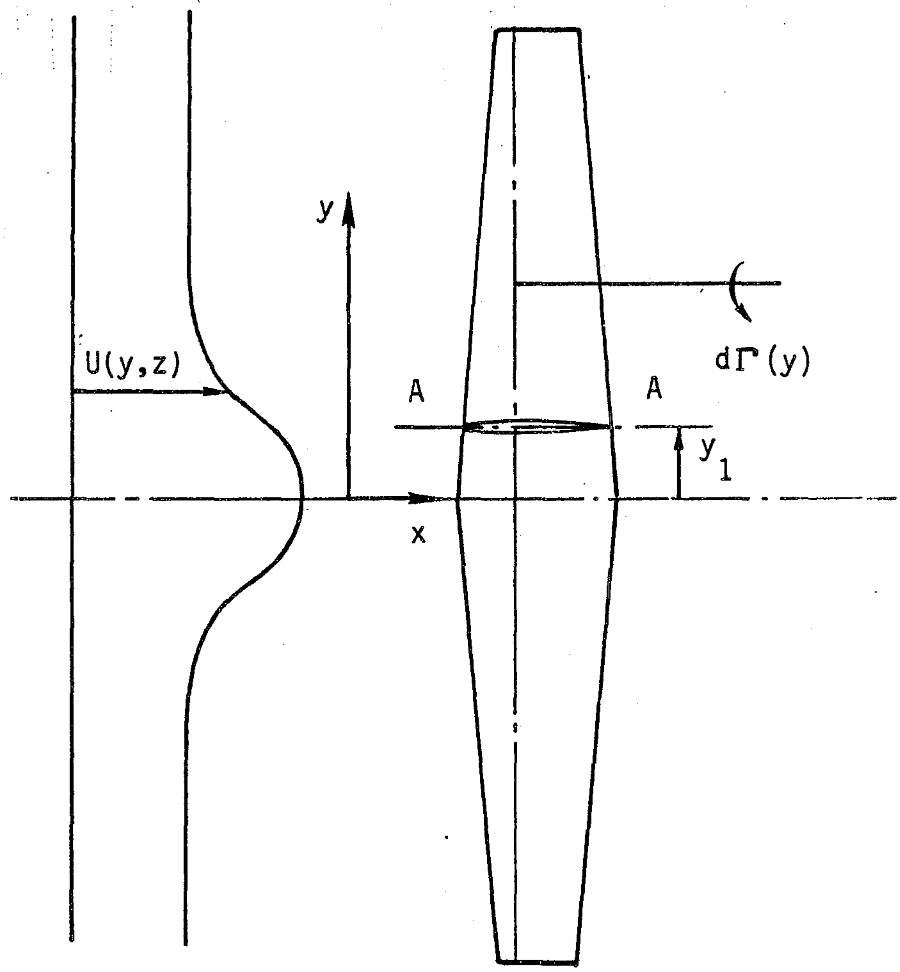
where $\varepsilon_1 = (\mu^2-1)/(\mu^2+1)$, $\varepsilon_2 = (\mu-1)^2/(\mu^2+1)$ and $\mu = U_J/U_\infty$. It may be recalled that in deriving these equations, the four assumptions mentioned in the previous section have been made. Further, when the propeller slipstream is absent $(U_J = U_\infty)$, the factors ε_1 and ε_2 become zero and the above equations reduce to those of the classical lifting line theory.

If the jet representing the propeller stream has a small excess velocity, i.e., $U_J - U_\infty = u \ll U_\infty$, then we may neglect terms of the order of $(u/U_\infty)^2$ in ε_1 and ε_2. In this case $\varepsilon_1 \approx u/U_\infty$ and $\varepsilon_2 \approx 0$; as a result, equations (4a) and (4b) get simplified to

$$w(y) = \frac{1}{4\pi}\left\{\int_{-s}^{s}\frac{d\Gamma(\eta)}{y-\eta} + \frac{u}{U_\infty}\int_{-R}^{R}\frac{d\Gamma(\eta)}{y-R^2/\eta}\right\}, \quad |y| < R \qquad (5a)$$

$$= \frac{1}{4\pi}\left\{\int_{-s}^{s}\frac{d\Gamma(\eta)}{y-\eta} - \frac{u}{U_\infty}\left(\int_{-s}^{-R} - \int_{R}^{s}\right)\frac{d\Gamma(\eta)}{y-R^2/\eta}\right\}, \quad |y| > R. \qquad (5b)$$

Now consider a high aspect ratio wing with the propeller slipstream going past it symmetrically as shown in figure 1. Let the velocity profile be given by $U(y,z) = U(r) = U_\infty[1 + F(r^2)]$ where $r^2 = y^2 + z^2$. Outside the



$U = U(y,z) = U_\infty(1 + F(y^2 + z^2))$

F IS AN ARBITRARY FUNCTION.

$F \to 0$, AS $y, z \to \infty$

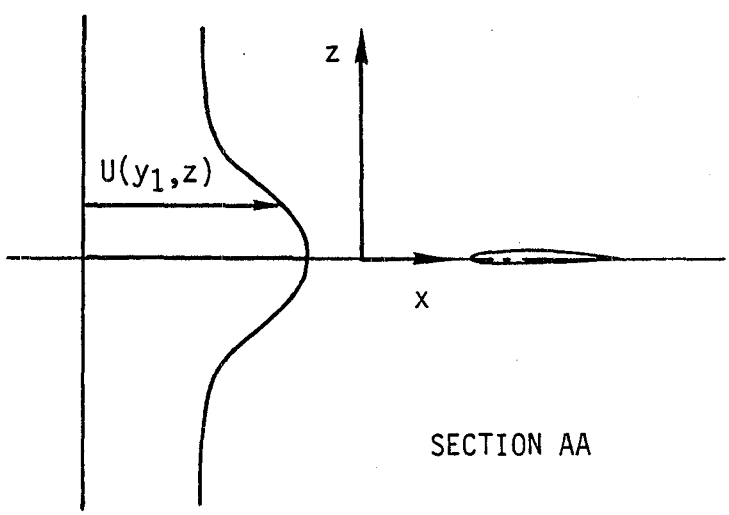


FIG.1 SCHEMATIC OF THE PROBLEM

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