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DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS  
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OLD DOMINION UNIVERSITY  
NORFOLK, VIRGINIA

COMPONENT MODE SYNTHESIS AND  
LARGE DEFLECTION VIBRATIONS OF  
COMPLEX STRUCTURES

By

Chuh Mei, Principal Investigator

Final Report  
for the period November 1, 1982 to October 31, 1983

Prepared for the  
National Aeronautics and Space Administration  
Langley Research Center  
Hampton, Virginia 23665

Under  
Research Grant NAG1-301  
Joseph E. Walz, Technical Monitor  
Structural Dynamics Branch (SDD)



February 1984

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P. O. Box 6369  
Norfolk, Virginia



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## FOREWORD

The material presented in this final report was performed under Grant No. NAG-1-301 entitled "Component Mode Synthesis and Large Deflection Vibrations of Complex Structures." This report summarizes the research results on modal synthesis and nonlinear forced vibrations of beams. The study was performed at the NASA/Langley Research Center during the period from November 1, 1982 to October 31, 1983. The work was monitored under the supervision of Joseph E. Walz and Dr. Jerrold M. Housner, Structural Dynamics Branch, Structures and Dynamics Division, NASA/Langley Research Center.

COMPONENT MODE SYNTHESIS AND LARGE DEFLECTION VIBRATIONS  
OF COMPLEX STRUCTURES

By

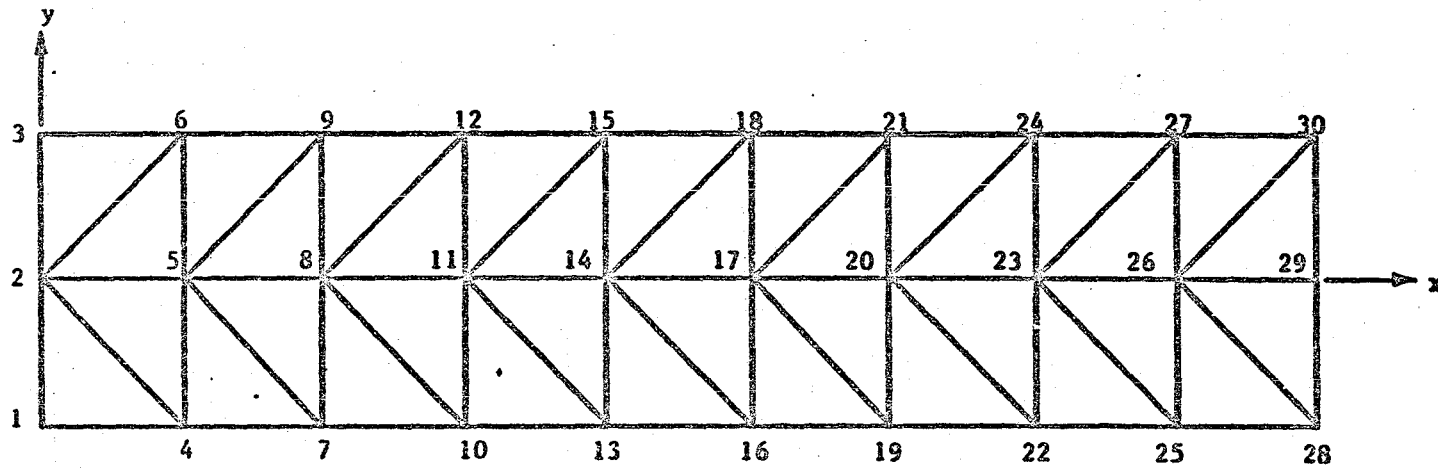
Chuh Mei\*

Part 1. Dynamic Analysis of Large Complex Structures  
Using Component Mode Methods in NASTRAN

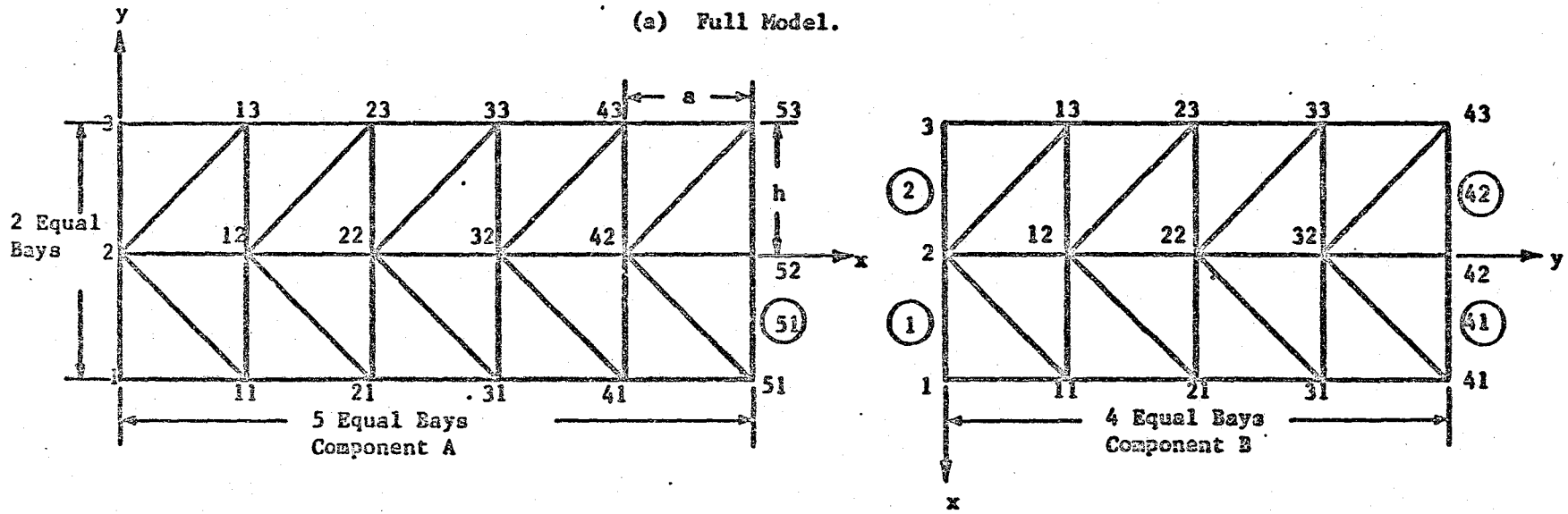
The complexity of aerospace structures has been increased enormously during the past decade. A new challenge has confronted the structural dynamists by the proposed space station to be in service by the year 1990. It will be an evolving structure (ref. 1), and it will not be possible for it to be ground tested because the final configuration may not be known when the first component is put into space. The component mode method, therefore, may be employed for the dynamic analyses for determining frequency, mode shape and transient response of such a large structure system in space.

The NASTRAN computer program, a structural analysis tool widely used in the aerospace industry, contains a modal synthesis capability. Other than the nine-bay truss structural problem presented in the NASTRAN demonstration manual, little is publicly known about its capabilities. Preliminary assessment of the accuracy of the NASTRAN modal synthesis analysis is accomplished by making a comparison of the NASTRAN modal synthesis with full structure NASTRAN and nine other modal synthesis results (ref. 2) using the nine-bay truss shown in Figure 1. Figures 2-4 show the relative accuracy obtained using the various modal synthesis procedures. The limited study indicates that the fixed-interface method in NASTRAN, fixed-interface

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(a) Full Model.



(b) Substructured Model.

Figure 1. Truss Model.

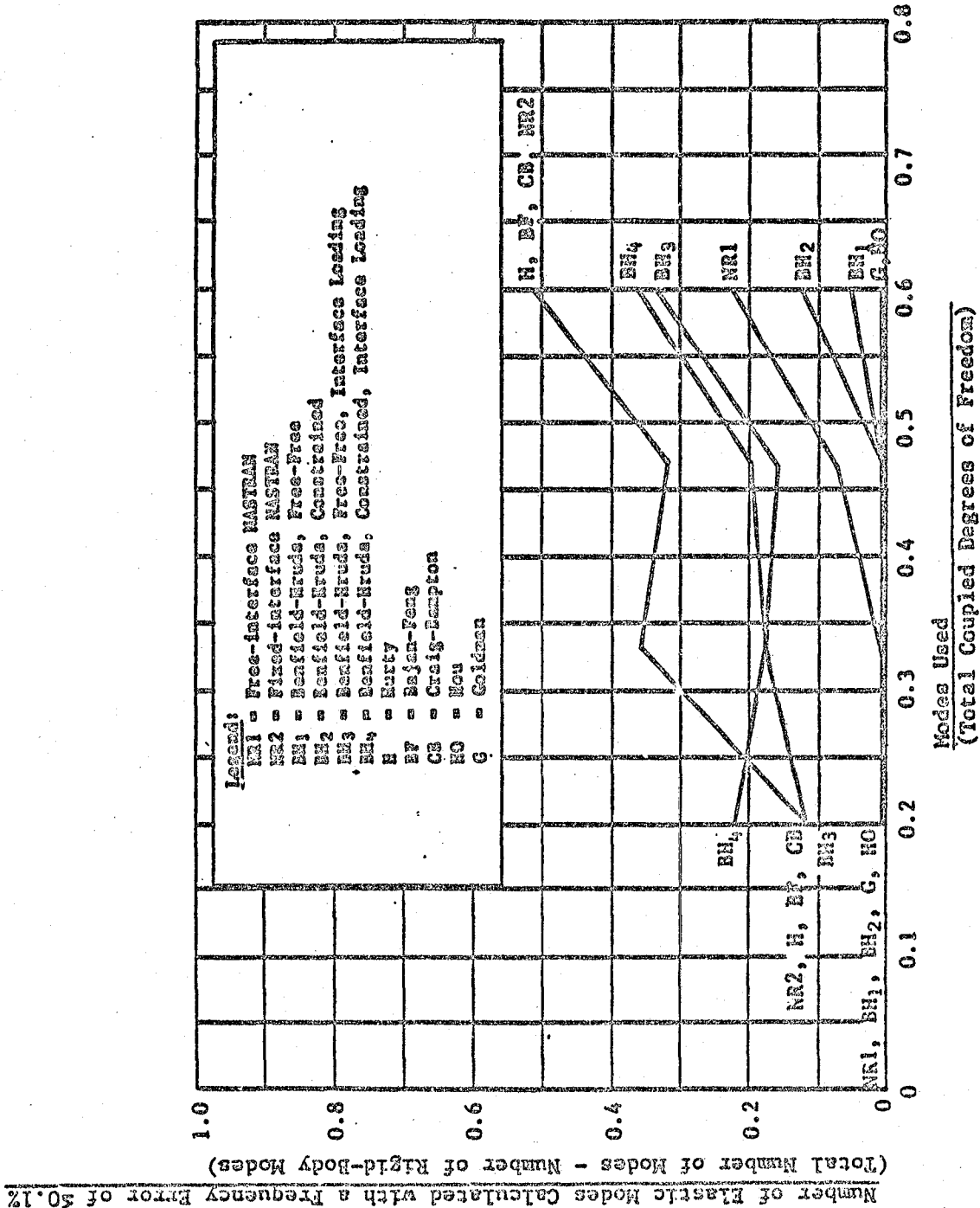


Figure 2. Comparison of Methods with Frequency Error of 0.1%.

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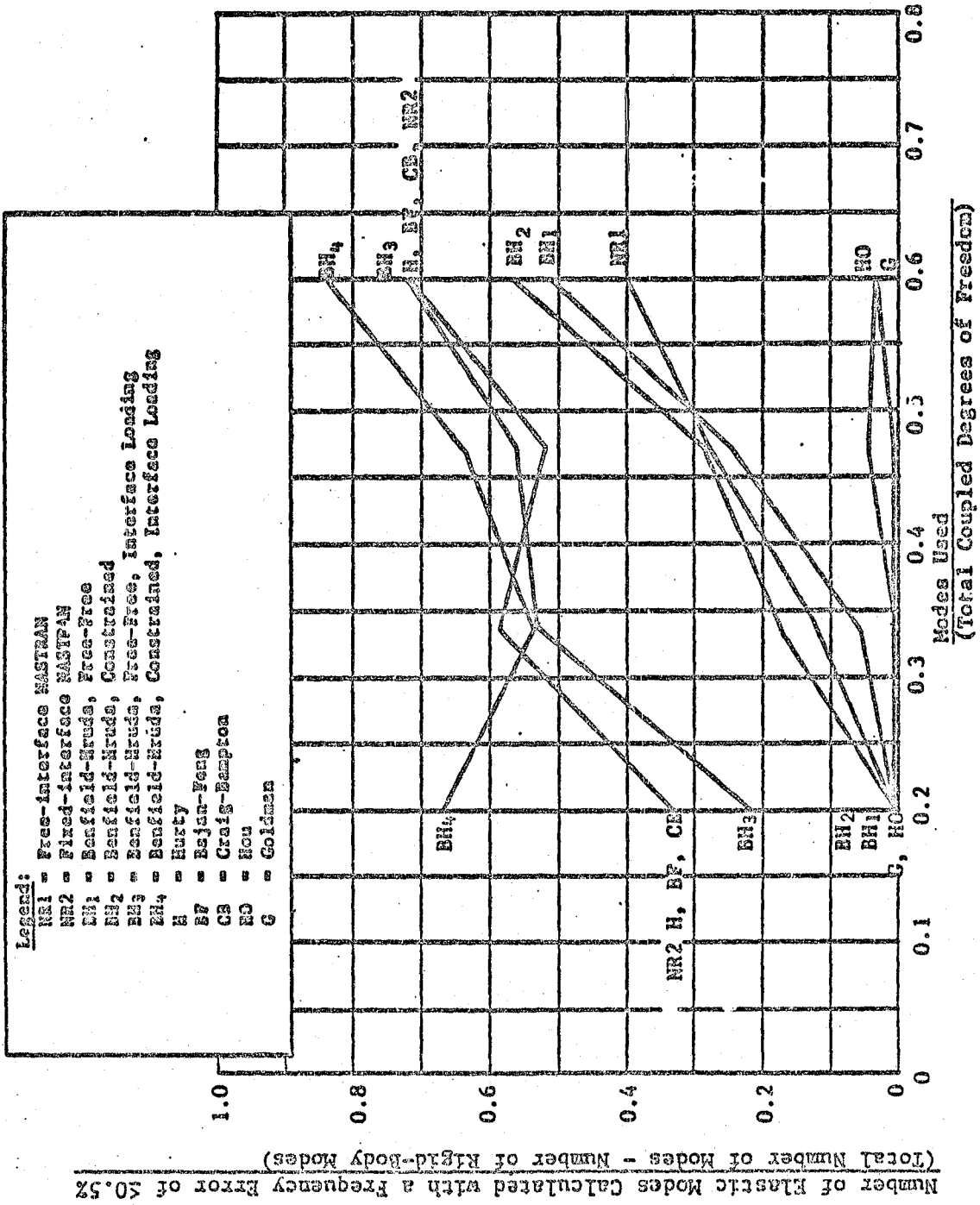


Figure 3. Comparison of Methods with Frequency Error of 0.5%.



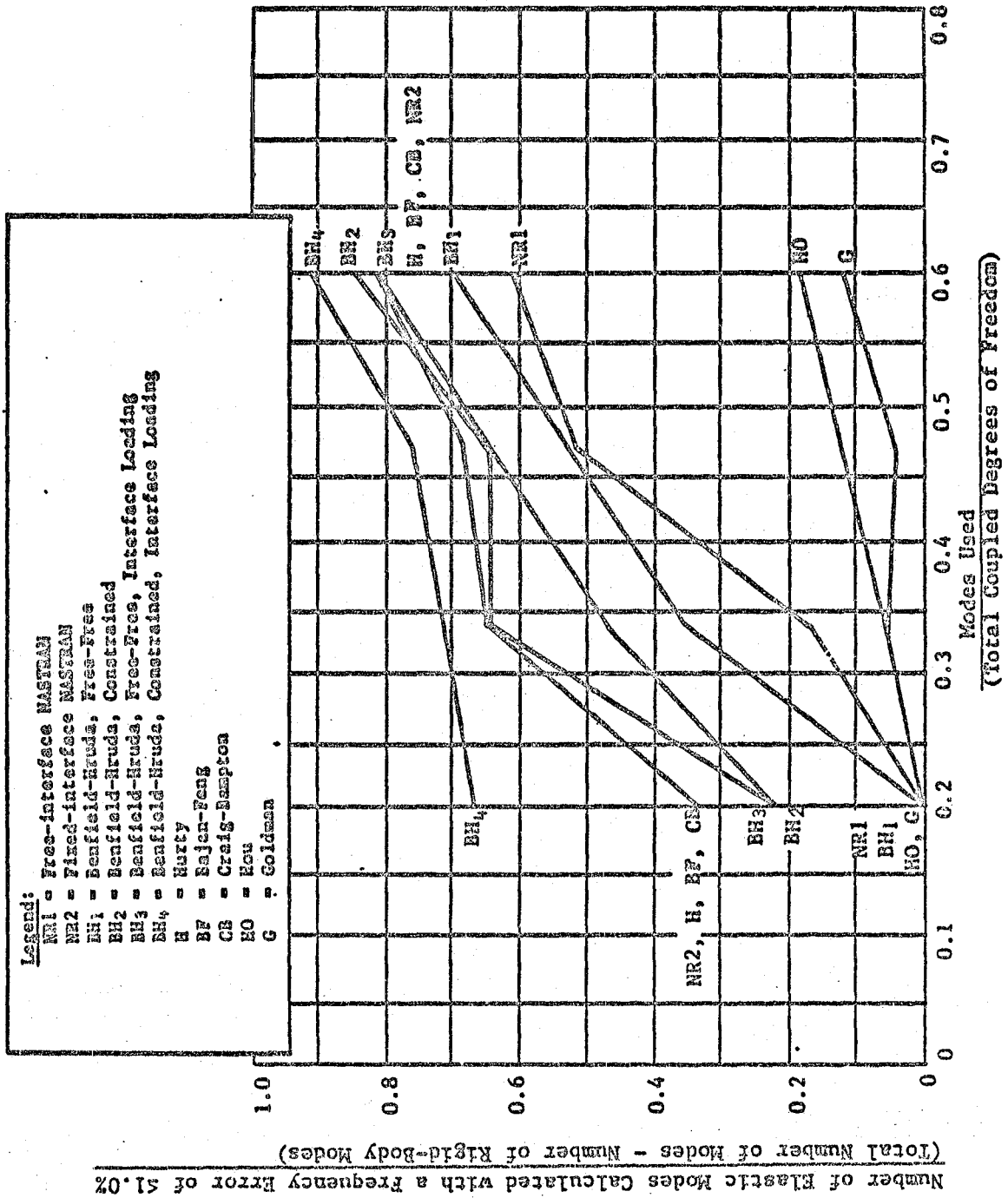


Figure 4 Comparison of Methods with Frequency Error of 51.0%

method of Hurty (ref. 3) and the equivalent Craig-Bampton method (ref. 4) produce the most accurate results for a given number of degrees-of-freedom. Slightly better accuracy was achieved by the procedures introduced by Benfield and Hruda (ref. 5). But these latter methods (ref. 5) suffer the disadvantage that the modes of one substructure are not independent of the modes of other substructures. More detailed results are documented in the progress report entitled, "NASTRAN Modal Synthesis Capability (ref. 6)."

A NASTRAN component mode transient response analysis was also performed on the free-free truss structure. A concentrated force was applied at grid point 42 of component B for 0.12 seconds and then removed. Tables 1 and 2 give the displacements and stresses and are compared with the full structure NASTRAN results. It demonstrates that excellent transient response can be obtained using component modal synthesis. Detail results, DMAP alters, solution sequences and computer CPU time can be found in reference 6.

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Table I. Transient Response and Percent Error in Displacement

| Grid<br>Pts.<br><br>Times | Full Truss | B Substr. | $\frac{F-B}{F}$ | Full Truss | B. Substr. | $\frac{F-B}{F}$ | Full Truss | B Substr. | $\frac{F-B}{F}$ |
|---------------------------|------------|-----------|-----------------|------------|------------|-----------------|------------|-----------|-----------------|
|                           | 28         | 41        |                 | 29         | 42         |                 | 30         | 43        |                 |
| 0.0                       | 0.0        | 0.0       | 0.0             | 0.0        | 0.0        | 0.0             | 0.0        | 0.0       | 0.0             |
| 0.0015                    | 0.4786243  | 0.4785015 | 0.02566         | 0.499332   | 0.4993054  | 0.00533         | 0.4786243  | 0.4785015 | 0.02566         |
| 0.0030                    | 2.070906   | 2.070847  | 0.00285         | 2.089915   | 2.089938   | -0.00110        | 2.070906   | 2.070847  | 0.00285         |
| 0.0045                    | 4.794056   | 4.793882  | 0.00363         | 4.813719   | 4.813558   | 0.00334         | 4.794056   | 4.793882  | 0.00363         |
| 0.0060                    | 8.662573   | 8.662563  | 0.00012         | 8.682071   | 8.683060   | -0.00218        | 8.662573   | 8.662563  | 0.00012         |
| 0.0075                    | 13.65921   | 13.65901  | 0.00146         | 13.67806   | 13.67781   | 0.00183         | 13.65921   | 13.65901  | 0.00146         |
| 0.0090                    | 19.79589   | 19.79587  | 0.00010         | 19.81609   | 19.81619   | -0.00050        | 19.79589   | 19.79587  | 0.00010         |
| 0.0105                    | 27.07146   | 27.07133  | 0.00048         | 27.09119   | 27.09122   | -0.00011        | 27.07146   | 27.07133  | 0.00048         |
| 0.0120                    | 35.47588   | 35.47573  | 0.00042         | 35.49501   | 35.49476   | 0.00070         | 35.47588   | 35.47573  | 0.00042         |

Table II. The Axial Force in Elements of B Substructure

| Times | Element No. |          |          |
|-------|-------------|----------|----------|
|       | 111         | 112      | 113      |
| 0.0   | 0.0         | 0.0      | 0.0      |
| 0.003 | 235.1038    | 282.6908 | 235.1038 |
| 0.006 | 252.9327    | 304.9388 | 252.9327 |
| 0.009 | 179.1082    | 254.5373 | 179.1082 |
| 0.012 | 137.4126    | 223.7419 | 137.4126 |

| Times | Element No. |          |          |
|-------|-------------|----------|----------|
|       | 141         | 142      | 143      |
| 0.0   | 0.0         | 0.0      | 0.0      |
| 0.003 | 185.0618    | 951.0315 | 185.0618 |
| 0.006 | 199.0282    | 1021.000 | 199.0282 |
| 0.009 | 177.9509    | 1021.209 | 177.9509 |
| 0.012 | 159.4311    | 959.688  | 159.4311 |

## PART 2. Large Deflection Vibrations of Beams using Finite Element Methods

Since space structures will be mostly large, lightweight and flexible, large deflection analysis methods are urgently needed to study the vibratory responses of complex structures.

A finite element method has been developed for nonlinear vibrations of beam structures subjected to harmonic excitation. Longitudinal deformation and inertia are both included in the formulation. A harmonic force matrix [h] was developed for a beam element for nonlinear oscillations under uniform harmonic excitation. Formulation of the harmonic force matrix is based on the mathematical basis (ref. 7) that the simple harmonic force  $P_0 \cos \omega t$  is simply the first order approximate solution of the simple elliptic forcing function  $BA \operatorname{cn}(p\tau, k)$ . Also the well known perturbation solution

$$\left(\frac{\omega}{\omega_L}\right)^2 = 1 + \frac{3}{4} \beta A^2 - \frac{P_0}{A}$$

of a Duffing system  $q_{,\tau\tau} + q + \beta q^3 = P_0 \cos \omega \tau$  is the first order approximate solution of the simple elliptic response  $q = A \operatorname{cn}(p\tau, k)$ . Derivation of the element harmonic force and nonlinear stiffness matrices are given in detail in progress report entitled, "Finite Element Analysis of Nonlinear Free and Forced Vibrations of Beams (ref. 8)."

Table 3 shows the finite element free vibration results with and without considering effects of longitudinal deformation and inertia (ELDI). It clearly demonstrates the remarkable agreement between the present finite element with ELDI and Rayleigh-Ritz solutions.

Table 4 shows the frequency ratios for a simply supported and a clamped beam ( $L/R = 100$ ) subjected to an uniform harmonic force of  $P_0 = 2.0$  ( $F_0 = 1322 \text{ lb/in.}$ ) and  $1.0$  ( $3277 \text{ lb/in.}$ ), respectively. It demonstrates the

TABLE III

Free Vibration Frequency Ratios  $\omega/\omega_1$   
for a Simply Supported Beam with Immovable Axial Ends

| A   | Without ELDI <sup>a</sup>                    |                    |                 | With ELDI (L/s = 100)                     |                    |                        |
|-----|--|--------------------|-----------------|---|--------------------|------------------------|
|     | Elliptic<br>Function<br>Solution<br>(ref. 9) | Finite Element     |                 | Rayleigh<br>Ritz<br>Solution<br>(ref. 10) | Finite Element     |                        |
|     |  | First<br>Iteration | Final<br>Result |   | First<br>Iteration | Final<br>Result        |
| 1.0 | 1.0392                                       | 1.0395             | 1.0388          | 1.0607                                    | 1.0613             | 1.0613(3) <sup>b</sup> |
| 2.0 | 1.3178                                       | 1.3203             | 1.3119          | 1.2246                                    | 1.2270             | 1.2269(4)              |
| 3.0 | 1.6257                                       | 1.6295             | 1.6022          | 1.4573                                    | 1.4620             | 1.4617(4)              |
| 4.0 | 1.9760                                       | 1.9761             | 1.9216          | 1.7309                                    | 1.7383             | 1.7375(6)              |
| 5.0 | 2.3501                                       | 2.3396             | 2.2544          | 2.0289                                    | 2.0393             | 2.0378(7)              |

a. Effects of longitudinal deformation and inertia.

b. Number in brackets denotes the number of iterations to get a converged solution.

TABLE IV

Forced Vibration Frequency Ratios  $\omega / \omega_L$   
for a Simply Supported and a Clamped Beam with Immovable Axial Ends

| A   | Without ELDI <sup>a</sup>                  |                          |   | With ELDI        |                                   |
|---|--|--------------------------|---|------------------|-----------------------------------|
|   | Simple<br>Elliptic<br>Response<br>(ref. 9) | Perturbation<br>Solution | Finite Element<br>First      Final<br>Iteration    Result |                  | Finite Element<br>Final<br>Result |
| Simply Supported Beam Subjected to $P_0 = 2.0$ ( $F_0 = 1322$ lb/in.) |  |                          |   |                  |                                   |
| - 1.0   | 1.7852                                     | 1.7854                   | 1.7852  | 1.7856           | 1.7682(3) <sup>b</sup>            |
| ± 2.0   | 0.8472<br>1.6557                           | 0.8660<br>1.6583         | 0.8621<br>1.6563  | 0.8460<br>1.6512 | 0.7108(4)<br>1.5829(4)            |
| ± 3.0   | 1.4003<br>1.8217                           | 1.4216<br>1.8314         | 1.4102<br>1.8226  | 1.3760<br>1.8002 | 1.2123(4)<br>1.6743(4)            |
| ± 4.0   | 1.8413<br>2.1013                           | 1.8708<br>2.1213         | 1.8453<br>2.0988  | 1.7846<br>2.0495 | 1.5871(6)<br>1.8759(6)            |
| ± 5.0   | 2.2606<br>2.4361                           | 2.2995<br>2.4673         | 2.2325<br>2.4236  | 2.1619<br>2.3432 | 1.9371(7)<br>2.1337(7)            |
| Clamped Beam Subjected to $P_0 = 1.0$ ( $F_0 = 3277$ lb/in.)          |  |                          |   |                  |                                   |
| ± 1.0   | 0.2118<br>1.4307                           | 0.2165<br>1.4307         | 0.2096<br>1.4297  | 0.2091<br>1.4297 | 0.1772(3)<br>1.4251(3)            |
| ± 2.0   | 0.8279<br>1.2987                           | 0.8292<br>1.2990         | 0.8215<br>1.2942  | 0.8203<br>1.2936 | 0.7905(4)<br>1.2743(4)            |
| ± 3.0   | 1.0401<br>1.3232                           | 1.0433<br>1.3248         | 1.0279<br>1.3127  | 1.0239<br>1.3099 | 0.9726(5)<br>1.2694(5)            |
| ± 4.0   | 1.2183<br>1.4101                           | 1.2247<br>1.4142         | 1.1979<br>1.3910  | 1.1888<br>1.3836 | 1.1151(6)<br>1.3197(6)            |
| ± 5.0   | 1.3938<br>1.5322                           | 1.4042<br>1.5401         | 1.3619<br>1.5016  | 1.3457<br>1.4874 | 1.2513(8)<br>1.4014(8)            |

a. Effects of longitudinal deformation and inertia.

b. Number in brackets denotes the number of iterations to get a converged solution.

closeness between the earlier finite element results without ELDI, the simple elliptic response and the perturbation solution. The present finite element results indicate clearly that the ELDI are to reduce the nonlinearity.

Beams with various boundary conditions, including movable axial ends, are given in reference 8. Results obtained will be presented at the Second International Conference on Recent Advances in Structural Dynamics, to be held April 9-13, 1983, at the Institute of Sound Vibration Research, Southampton, England.



## REFERENCES

1. Hedgepeth, J.M.: Survey of Future Requirements for Large Space Structures. NASA CR-2621, 1976.
2. Benfield, W.A., Bodley, C.S., and Morosow, G.: Modal Synthesis Method. NASA Symposium on Substructuring Testing and Synthesis. Marshall Space Flight Center, Alabama, August 1972.
3. Hurty, W.C.: Dynamic Analysis of Structural Systems Using Component Modes. AIAA J. 3:678-685, April 1965.
4. Craig, R.R., Jr. and Bampton, M.C.C.: Coupling of Substructures for Dynamic Analysis. AIAA J. 6:1313-1319, July 1968.
5. Benfield, W.A. and Hruda, R.F.: Vibration Analysis of Structures by Component Mode Substitution. AIAA J. 9:1255-1261, July 1971.
6. Mei, C.: NASTRAN Modal Synthesis Capability. Progress report for the period November 1, 1982-June 30, 1983, Research Grant NAG1-301, June 1983.
7. Hsu, C.S.: On the Application of Elliptic Functions in Nonlinear Forced Oscillations. Quarterly Applied Mathematics. 393-407, 1960.
8. Mei, C.: Finite Element Analysis of Nonlinear Free and Forced Vibrations of Beams. Progress report for the period November 1, 1982 to June 30, 1983, Research Grant NAG1-301, June 1983.
9. Woinowsky-Krieger, S.: The Effect of an Axial Force on the Vibration of Hinged Bars. J. Appl. Mech. 17:35-36, March 1950.
10. Raju, I.S., Rao, G.V., and Raju, K.K.: Effect of Longitudinal or Inplane Deformation and Inertia on the Large Amplitude Flexural Vibrations of Slender Beams and Thin Plates. Journal Sound and Vibration. 415-422. 1976.

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