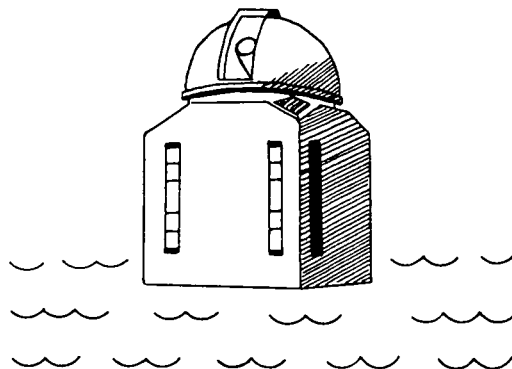


CALIFORNIA INSTITUTE OF TECHNOLOGY

BIG BEAR SOLAR OBSERVATORY



CONSEQUENCES OF A CHROMOSPHERIC TEMPERATURE
GRADIENT ON THE WIDTH OF $H\alpha$
IN LATE-TYPE GIANTS

D. M. Zarro

BBSO #0230

February 1984

Big Bear Solar Observatory

California Institute of Technology

ABSTRACT

An analytic expression for the integrated H α optical depth profile is derived for a one dimensional slab geometry model chromosphere, with electron temperature increasing as a power law with height. The formula predicts H α opacity and profile width to be sensitive functions of the thermal gradient. Application of the model to observation reveals that broad H α absorption widths in G and K giant stars are consistent with a mean H α chromospheric optical depth of 50, while narrower widths in M stars indicate slightly lower opacities. It is proposed that differences in H α width between late-type giants of similar spectral type may be due, in part, to differences in their chromospheric thermal gradient, and associated H α opacity.

Subject headings: line profiles-stars: atmospheres-stars: chromospheres

I. INTRODUCTION

Studies of the H α absorption width in late-type stars have been pursued by various workers. Kraft, Preston, and Wolff (1964) (hereafter KPW) demonstrated that H α full width at half absorption is correlated, in a spectral type dependent fashion, with stellar luminosity. On the assumption that the H α line is optically thick and Doppler broadened, KPW attributed the H α width-luminosity (W-L) relation to the systematic increase of non-thermal chromospheric velocities with decreasing stellar surface gravity. More recent investigations of the H α W-L broadening have relied upon similar interpretations. Fosbury (1973) and Reimers (1973) indicated that the H α width is comprised of thermal and non-thermal Doppler velocity components: the former providing a lower limit to the width in dwarfs, the latter responsible for the width enhancement in giant and supergiant stars.

A commonly evoked assumption in the above studies is that H α chromospheric Doppler width is a constant function of height. However, neglect of depth variations in this parameter is overly simplistic in view of the fact that, associated with a general chromospheric temperature rise, there is a corresponding increase in atomic hydrogen thermal velocities. For example, in the case of the solar chromosphere, mean hydrogen thermal velocities range from approximately 8 km s^{-1} (at the temperature minimum) to 13

km s^{-1} in the coronal transition zone ($T \sim 10^4 \text{ K}$).

The existence of thermal and, possibly, non-thermal velocity gradients (cf. Eriksson 1980) necessarily implies a depth dependence for the total hydrogen Doppler width. Moreover, depth variations in temperature will influence the hydrogen excitation state at different levels in the chromosphere and, in particular, the height distribution of the non-LTE level 2 population (Thomas and Athay 1961). The aim of the present investigation is to examine analytically the effects of such temperature dependencies on the H α optical depth and, by implication, on the H α line width.

Our approach is to represent the stellar chromosphere as a plane-parallel one dimensional region with electron temperature and microturbulence that increase as power law functions of height. The model is used to derive an analytic formula for the integrated H α optical depth (assuming a Doppler broadening coefficient) and an expression for the H α width (according to an Eddington-Barbier approximation). Inferred H α widths will be compared with observation in § III, where attention will be restricted to giant stars ($\log g \leq 3.5$), to avoid complications arising from the presence of photospheric damping wings. Details of the analysis are described in the following section.

II. CALCULATIONS

a) Model Chromosphere

The stellar chromosphere is modelled as a semi-infinite slab of plasma (predominantly hydrogen), with the lower boundary at the photospheric temperature inversion (designated T_*). Integrated mass column density (m : gm cm^{-2}) is adopted as height variable. This parameter is related to gas density (ρ) and geometrical height (z) according to

$$dm = -\rho dz , \quad (1)$$

where $\rho = \mu n_H m_H$, and $\mu = 1.4$ for a 10% helium composition. On the above height scale, the upper boundary of the slab corresponds to $m = 0$ and the lower boundary at $m = m_*$.

We parameterize the chromospheric temperature rise (and associated thermal broadening velocity) by a power law function of m

$$T = T_* (m/m_*)^{-1/\alpha} , \quad (2)$$

where the exponent α is a free parameter to be determined from observation (§ III). Note that for $\alpha > 0$, T increases for m decreasing, becoming infinite for $m=0$. The situation of constant chromospheric temperature corresponds to

infinitely large α . In addition, we incorporate a chromospheric microturbulent velocity (V_{turb}) to account for random non-thermal mass motions occurring on length scales smaller than a photon mean free path (Edmunds 1978). For simplicity, we follow Thomas (1973) in equating this largely ad hoc quantity to the local sound speed: $V_{\text{turb}} = c_s = (kT/\mu m_H)^{1/2}$. This component is combined with the mean atomic hydrogen thermal velocity [$V_{\text{th}} = (2kT/m_H)^{1/2}$] to give the following dependence of hydrogen doppler width (V_d : km s^{-1}) upon m

$$V_d = (V_{\text{th}}^2 + c_s^2)^{1/2} = V_{d*} (m/m_*)^{-1/2\alpha}, \quad (3)$$

where V_{d*} denotes the value at T_* .

b) The Hydrogen Level Two Population

In the region where Lyman continuum optical depth is ≥ 100 , Ly α opacity is sufficiently large to drive hydrogen levels 1 and 2 into radiative detailed balance. Under these circumstances, it can be shown that the departure coefficients for these levels are approximately equal ($b_1 \sim b_2$) and the corresponding non-LTE population ratio (n_2/n_1) follows closely the LTE Boltzmann relation (Thomas and Athay 1961). Furthermore, if hydrogen is less than 50% ionized its ground state population can be equated approximately to the total hydrogen density (n_H). According to these assumptions, we infer

$$n_2 = (n_2/n_1) n_1 = (n_2^*/n_1^*) (b_2/b_1) n_1 ,$$

$$\text{i.e., } n_2 \sim (g_2/g_1) \exp(-X_{12}) n_H , \quad (4)$$

where $g_2/g_1 = 4$ and $X_{12} = h\nu_{12}/kT$ is the Ly α excitation potential in units of thermal energy. (Asterisks denote LTE values).

Differentiation of the logarithm of equation (4) with respect to z yields

$$\partial \log n_2 / \partial z = \partial \log n_H / \partial z + (X_{12}/T) \partial T / \partial z . \quad (5)$$

For a steep chromospheric temperature rise ($\partial T / \partial z \gg 0$), this relation implies that an outward decrease in n_2 - arising from a hydrostatic density gradient in n_H - can be partially compensated by increased T . This behavior is borne out formally in the following calculation.

Assuming hydrostatic equilibrium, we can formulate momentum conservation as

$$gm = 1.1n_H kT + 0.5n_H \mu m_H V_{\text{turb}}^2 \quad (g: \text{surface gravity}), \quad (6)$$

where the left hand term corresponds to the weight of chromospheric material at height m , and the right hand side represents the sum of partial pressures in a perfect hydrogen (and 10% helium) gas containing turbulence. We

employ equation (6) to eliminate n_H from (5), and utilize relations (1), (2) and (3) to express the z derivative of n_2 in terms of m , ρ , and α . Thus,

$$\partial \log n_2 / \partial z = (\rho / m \alpha) (X_{12} - 1 - \alpha) . \quad (7)$$

In order to ensure that n_2 be monotonically increasing with height, we require that $\alpha < X_{12} - 1$, which (for a representative $T = 6000$ K) implies $\alpha < 19$. Therefore, a sufficiently steep temperature rise (small α) can effectively counteract a density gradient in n_2 and enhance the n_2 chromospheric column density above its isothermal value.

c) Calculation of the H α Optical Depth Profile

We represent the H α line absorption coefficient by a gaussian profile. This approximation is valid provided that H α photon transfer is confined primarily within three to four Doppler widths of line center (Fosbury 1973). From equation (3), we deduce $4V_d = 46 \text{ km s}^{-1}$ at a mean chromospheric temperature of 6000 K. Examination of the KPW data reveals that H α half widths in late-type giants range typically from 30 to 35 km s^{-1} and, hence, fall within this nominal $4V_d$ limit separating the Doppler core from the outer line damping wing. Accordingly, we express the integrated H α optical depth at velocity shift ΔV from

line center by (Mihalas 1978)

$$\tau_{\Delta V} = (\pi^{1/2} e^2 / mc) f_{12} \lambda_{12} \int_{z_*}^{\infty} (n_2 / V_d) \exp[-(\Delta V / V_d)^2] dz, \quad (8)$$

where stimulated emission has been neglected. In the above, f_{12} , denotes the H α transition oscillator strength and λ_{12} , the corresponding wavelength. The integral over height ranges from the upper boundary of the slab chromosphere ($z \rightarrow \infty$) to the slab base at $z = z_*$.

Substitution of expressions (1), (2), (3), and (4) into (8) yields

$$\tau_{\Delta V} = (K / V_{d*}) \int_0^{m_*} (m / m_*)^{1/2 a} \exp[-a(m / m_*)^{1/a}] dm, \quad (9)$$

$$\text{where } a = (h\nu_{12} / kT_*) + (\Delta V / V_{d*})^2 \quad (9a)$$

$$\text{and } K = (\pi^{1/2} e^2 / mc) (f_{12} \lambda_{12} / \mu m_H) (g_2 / g_1)$$

$$= 1.07 \times 10^{13} \quad (\text{Allen 1973}).$$

Note that, following elimination of n_H via (1), the variable of integration is transformed from z to m . The above integral is reduced to more tractable form by a second change of variable to $u = a(m / m_*)^{1/a}$. Thus,

$$\tau_{\Delta V} = (K a m_* / V_{d*}) a^{-a-1/2} \int_0^a u^{a-1/2} \exp(-u) du \quad (10)$$

or, equivalently,

$$\tau_{\Delta V} = (K \alpha m_*/V_{d*}) a^{-\alpha-1/2} \gamma(\alpha+1/2, a) , \quad (11)$$

where $\gamma(x, a)$ denotes the incomplete gamma function.

For $\Delta V > 0$ and $T_* < 4500$ K (appropriate to giant stars later than about spectral type G0), equation (9a) implies $a > 26$. Series expansion of $\gamma(x, a)$ for large a gives (Abramowitz and Stegun 1964)

$$\gamma(\alpha+1/2, a) \sim \Gamma(\alpha+1/2) - a^{\alpha-1/2} \exp(-a) [1 + \text{terms order}(\alpha/a)] , \quad (11a)$$

where $\Gamma(x)$ is the complete gamma function. For $a > 26$ and $\alpha < 19$ (cf II b), we find that the expansion terms in (11a) can be neglected relative to $\Gamma(\alpha+1/2)$ with an error of less than 3%. Hence, we obtain the following approximation for the integrated H α opacity profile

$$\tau_{\Delta V} = (K \alpha m_*/V_{d*}) a^{-\alpha-1/2} \Gamma(\alpha+1/2) . \quad (12)$$

This formula indicates that H α chromospheric opacity is directly proportional to the mass column density at the temperature minimum or, equivalently, the chromospheric "thickness."

It is important to recognize that in the upper and considerably hotter chromospheric layers ($T > 7000$ K), where hydrogen is greater than 50% ionized, the Lyman continuum optical depth approaches unity. In this region, it can be shown that n_2 no longer follows the pseudo-Boltzmann relation (4) and is, instead, a decreasing function of height which depends sensitively upon electron density (Heidemann and Thomas 1980). Consequently, integration of (9) over this domain would tend to overestimate $\tau_{\Delta V}$. Allowance for this effect is, however, beyond the scope of this investigation.

III. RESULTS AND DISCUSSION

a) Dependence of H α Width upon Temperature Gradient

Implications of the optical depth formula (12) for the general behavior of H α widths are now discussed. A relatively straightforward technique of linking wavelengths of characteristic features in a line profile with atmospheric optical depths is provided by the Eddington-Barbier (E-B) approximation (Mihalas 1978). Physically, the E-B relation implies that at wavelength shift $\Delta\lambda$ from line center, the emergent radiation is characteristic of the source function at an atmospheric layer corresponding approximately to unit integrated optical depth (Athay 1972). Thus, for example, solution of (12) for the velocity displacement ΔV (with $\tau_{\Delta V} = 1$) yields

$$H_0 = 2V_{d*} (h\nu_{12}/kT_*)^{1/2} [\tau_0^{2/(2\alpha+1)} - 1]^{1/2}, \quad (13)$$

where H_0 refers to the line full width ($2\Delta V$: km s^{-1}) and τ_0 denotes the H α optical depth evaluated at line center ($\Delta V = 0$ in equation 12). This equation admits a power law dependence of H_0 upon τ_0 (or m_*), modified by a temperature sensitivity arising from the Boltzmann excitation term. Note that the formula becomes invalid when $\tau_0 < 1$. In the latter optically thin cases, the H α width loses its sensitivity to chromospheric opacity and is, instead, dominated by Doppler broadening velocities (cf. Goldberg 1957).

The above relationship can be converted into an approximate dependence upon surface gravity and effective temperature (T_{eff}) by invoking the theoretical scaling law of Ayres (1979)

$$m_* \sim 3.6 \times 10^{-13} g^{-1/2} T_{\text{eff}}^{7/2} \quad (14a)$$

and the empirical relation (Linsky and Ayres 1973)

$$T_* \sim 0.77 T_{\text{eff}}, \quad (14b)$$

where the constants of proportionality are derived from solar values of g , T_{eff} , m_* , and T_* (Ayres et al. 1976). Utilizing these expressions in (12) and (13), we compute the variation of H_0 with respect to α for a sequence of

giant stars of representative spectral types G0, G5, K0, K5, and M0. Gravities and effective temperatures (see Table 1) employed for the calculation are obtained from Allen (1973). Figure 1 illustrates the resulting curves for α in the range $6 < \alpha < 18$.

Examination of this diagram reveals the following important features:

1. For a fixed exponent α , H_0 decreases with decreasing T_{eff} . This behavior is qualitatively consistent with observations reported by KPW which indicate generally narrower H α widths in cool giants (e.g., M type) relative to hotter counterparts (e.g., G type) of similar luminosity. Fosbury (1973) attributed this temperature sensitivity to a reduction in chromospheric H α opacity with advancing spectral type. Within the context of the present model, the latter effect is a consequence of the Boltzmann temperature sensitivity of the n_2 population in the low chromosphere (cf. equation 4). Cooler temperature minima in late-type stars ($T_* \sim 0.77 T_{\text{eff}}$) imply lower n_2 and, hence, decreased integrated H α opacity.

2. The H α profile width is a sensitive function of the thermal gradient. For each spectral class, H_0 increases with decreasing α . This variation can be understood in terms of the enhancement of n_2 column density (and corresponding increase in τ_0) with steepening temperature rise above T_* (cf. eqs [7] and [12]). Thus,

within the framework of our model, the H α absorption width can be considered a diagnostic of the chromospheric thermal gradient.

b) Comparison With Observation

Further insight into the dependence of H_0 upon α is gained by comparing predictions of formula (13) with H α data of KPW. The H α width inferred from the E-B relation applied to (12) refers to that part of the line profile (probably the line wing) formed in the vicinity of T_* . By contrast, the measurements of KPW are defined in terms of the full width at half absorption, which likely is representative of hotter chromospheric layers ($T \sim 6000$ K; Fosbury 1973). However, if we restrict our attention to late-type giant stars which (unlike G and early K dwarfs) do not exhibit broad photospheric damping wings, the difference between the above widths is probably small (KPW; Lo Presto 1971).

We bin H α width measurements of KPW according to the following criterion. Giants falling within ± 2 sub-divisions of the reference spectral types in Table 1 are considered to be of approximately equal T_{eff} . Thus for example, G3, G4, G5, G6, and G7 are treated as G5. Similarly G8, G9, K0, K1, and K2 are grouped as K0 and so on. Each width point is then plotted on the appropriate spectral curve in Figure 1, at its corresponding value of

H_0 . This procedure allows a graphical solution for values of α and τ_0 that produce consistency between our model and observation. For the plotted sample of 24 stars, we infer the range $9 < \alpha < 12$; with cooler giants tending to require smaller α relative to hotter giants of the same H_0 .

It is instructive to compare the above constraints with results of radiative transfer calculations. Table 2 lists the quantities T_* , m_* , and m_0 derived from studies of Ca II (and Mg II) resonance line formation in β Ceti (G9 III), β Gem (K0 III), α Boo (K2 III), and α Tau (K5 III). The parameter m_0 designates the mass column density at $T = 8000$ K. (This point corresponds approximately to the location of Lyman continuum optical depth unity [Athay 1981]). For each star, we use equation (2) to derive an equivalent α exponent for a power law temperature rise between (m_*, T_*) and $(m_0, 8000 \text{ K})$. The resulting values are listed in Table 2. Given the approximations implicit in equation (13), and the uncertainties associated with the boundary temperatures and densities in Table 2, we find reasonable overlap between the range of computed exponents and the range deduced from Figure 1.

However, it should be emphasized that temperature-density profiles in late-type stars do not follow simple constant-exponent power laws, but characteristically display a temperature gradient that is variable with height (refer to Linsky 1980 and references

cited in Table 2). For the case of the solar chromosphere, Athay (1981) has demonstrated that height variations in plasma radiative properties (e.g., changes related to a switch from continuum to spectral lines as the dominant radiative loss mechanism) can lead to perturbations in the chromospheric temperature gradient.

In view of the above caveat, the α exponent inferred from equation (13) should only be interpreted as a parametric (rather than definitive) description of the chromospheric temperature gradient required to produce an observed H α width. As an application of this formalism, we use empirical H α widths to delimit the τ_0 parameter. With the aid of equation (12), we connect the spectral curves in Figure 1 with lines of constant $\tau_0 = 20$ and 100, respectively. Data points for G5, K0, and K5 fall predominantly within these extremes, while cooler M giants tend to cluster about $\tau_0 \sim 20$. Hence, we deduce that (for the case of a sonic microturbulence) the observed H α widths of G and K giants are consistent with an average τ_0 of 50 - 60. The latter estimates conform with a mean τ_0 of 50 derived independently by Fosbury (1973) from a study of the H α W-L relation in G and K stars.

IV. CONCLUSIONS

The H α absorption profile is not normally considered to be a useful diagnostic of chromospheric thermodynamic conditions in late-type giants (Fosbury 1974). Cool low gravity stars are characterized by relatively small chromospheric electron densities ($N_e \leq 10^9 \text{ cm}^{-3}$) that guarantee radiative control of the H α line. In other words, the H α source function is essentially uncoupled from chromospheric thermal structure and is, instead, dominated by fixed photospheric Balmer and Paschen radiation fields (Gebbie and Steinitz 1974).

However, our theoretical calculations indicate that - due to the temperature dependence of the hydrogen Doppler width and non-LTE level 2 population - H α optical depth scales can possibly couple to thermal structure. In particular, this coupling is found to induce a sensitivity of H α width to the chromospheric temperature gradient. On the basis of this result, we conclude that future observational and numerical investigations of H α , in conjunction with the more traditional diagnostics Ca II H and K, may prove viable in constraining chromosphere temperature models of late-type stars.

In addition, we have demonstrated that the H α absorption width (despite its Doppler broadened nature) can be a potentially sensitive function of chromospheric optical depth in a region of steep temperature rise (cf.

power law in equation 13). By contrast, the slab formula of Goldberg (1957) (derived for the case of a constant chromospheric Doppler parameter) predicts a much weaker logarithmic association between H α width and opacity. The latter distinction suggests that opacity effects may be an important consideration in the understanding of the H α W-L relation; particularly in giants exhibiting strong Ca II K emission and, hence, possibly enhanced chromospheric temperature gradients (Kelch et al. 1978). Furthermore, we conjecture that differences in observed H α width between giants of similar T_{eff} (and equal chromospheric non-thermal broadening), may likely reflect differences in their chromospheric temperature gradient and associated H α opacity.

Finally, it is important to realize that chromospheric velocity fields in the forms of microturbulence (Fosbury 1973; Eriksson 1980), macroturbulence (Smith and Dominy 1979) and stellar winds (Mallik 1982) are known to profoundly influence the widths of spectral lines such as H α in late-type giants (and supergiants). Clearly, further detailed calculations are required to determine the extent to which these dynamical processes combine with opacity effects in governing the behavior of the H α profile.

My appreciation to Professor A. W. Rodgers for many useful discussions concerning this investigation, and to Dr. D. Gary for critically reading the manuscript. This work was supported by NASA under grant NGL 05 002 034 and by the NSF under grant ATM-8211002.

TABLE 1
GRAVITIES AND EFFECTIVE TEMPERATURES EMPLOYED
FOR CALCULATION OF H α WIDTHS

Spectral Type	T _{eff} (K)	Log (g) (cm s ⁻²)
G0	5600	3.3
G5	5000	3.0
K0	4500	2.6
K5	3800	1.9
M0	3200	1.4

TABLE 2

SELECTED MODEL PARAMETERS FOR FOUR LATE-TYPE GIANTS

Star	Spectral Type	T_* (K)	m_* (gm cm^{-2})	m_0	α	ref ^a
β Gem	K0 III	3805	0.30	1.6(-5)	13.2	1
β Cet	G9 III	3575	0.185	2.1(-5)	11.3	2
α Boo	K2 III	3200	1.78	3.2(-5)	11.9	3
α Tau	K5 III	2700	0.30	7.9(-5)	7.6	1

^aReferences.- (1) Kelch et al. 1978. (2) Eriksson, Linsky and Simon 1983. (3) Ayres and Linsky 1975.

REFERENCES

- Allen, C. W. 1973, Astrophysical Quantities (3rd. ed.; University of London: Athlone Press).
- Ambramowitz, M., and Stegun, I. 1968, Handbook of Mathematical Functions (N.B.S. Applied Mathematics Series # 55).
- Athay, R. G. 1972, Radiation Transport in Spectral Lines (Dordrecht: Reidel).
- Athay, R. G. 1981, Ap. J., 250, 709.
- Ayres, T. R. 1979, Ap. J., 228, 509.
- Ayres, T. R., and Linsky, J. L. 1975, Ap. J., 200, 660.
- Ayres, T. R., Linsky, J. L., Rodgers, A. W., and Kurucz, R. L. 1976, Ap. J., 210, 199.
- Edmunds, M. G. 1978, Astr. Ap., 64, 103.
- Eriksson, K. 1980, Uppsala Astronomical Report, No. 16.
- Eriksson, K., Linsky, J. L., and Simon, T. 1983, Ap. J., 272, 665.

Fosbury, R. A. E. 1973, Astr. Ap., 27, 1.

----- 1974, M.N.R.A.S., 169, 147.

Gebbie, K. B., and Steinitz, R. 1974, Ap. J., 188, 399.

Goldberg, L. 1957, Ap. J., 126, 318.

Heidmann, N., and Thomas, R. N. 1980, Astr. Ap., 87, 36.

Kelch, W. L., Linsky, J. L., Basri, G. S., Chiu, H. Y.,
Chang, S. H., Maran, S. P., and Furenlid, I. 1978, Ap. J., 220, 962.

Kraft, R. P., Preston, G. W., and Wolff, S. C. 1964, Ap. J., 140, 235.

Linsky, J. L. 1980, Ann. Rev. Astr. Ap., 18, 439.

Linsky, J. L., and Ayres, T. R. 1973, Ap. J., 180, 473.

Lo Presto, J. C. 1971, Pub.A.S.P., 83, 674.

Mallik, S. V. 1982, J. Astrophys. Astr., 3, 39.

Mihalas, D. 1978, Stellar Atmospheres (2nd. ed.; San
Francisco: Freeman).

Reimers, D. 1973, Astr. Ap., 24, 79.

Smith, M. A., and Dominy, J. F. 1979, Ap. J., 231, 477.

Thomas, R. N. 1973, Astr. Ap., 29, 297.

Thomas, R. N., and Athay, R. G. 1961, Physics of the Solar Chromosphere (New York: Interscience).

Figure Caption

Fig. 1. $\log (H_0)$ versus α for a series of representative giant stars of spectral type G0, G5, K0, K5, and M0, respectively. Solid curves are generated according to equation (13). Superimposed filled points denote $H\alpha$ width measurements (from KPW) for selected G, K and M giants. The procedure for grouping the latter data is discussed in the text. Note that H_0 for G0 III is not given by KPW. Upper and lower dashed lines correspond to constant $H\alpha$ line center optical depths of 100 and 20, respectively.

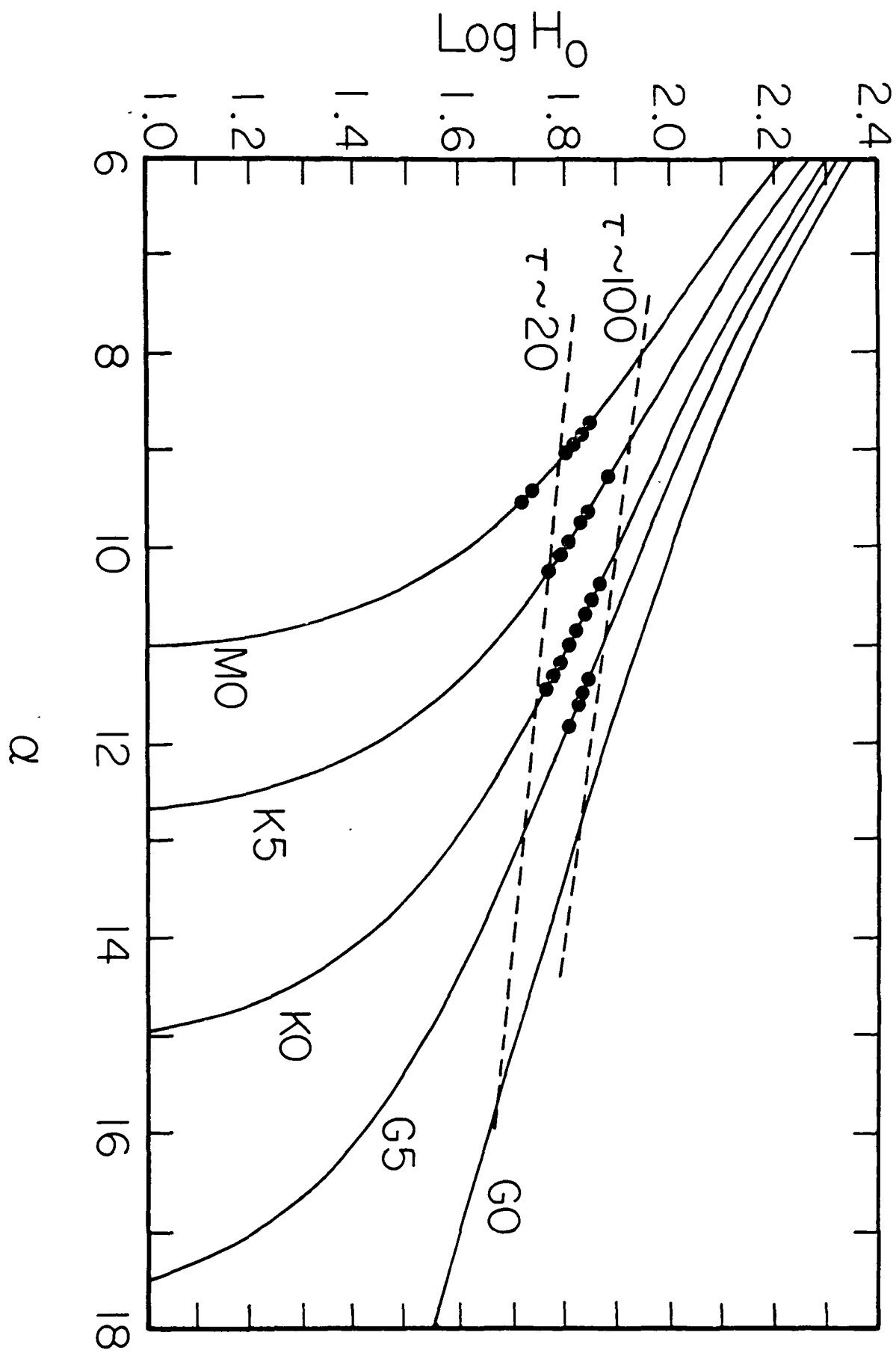


Figure 1

SOLAR ASTRONOMY 264-33
CALTECH
PASADENA, CA 91125