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KINEMATICS, INFLUENCE FUNCTIONS AND FIELD QUANTITIES FOR DISTURBANCE PROPAGATION FROM MOVING DISTURBANCE SOURCES

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16. Abstract

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ABSTRACT /4

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Table of Contents Page /5

Definitions /6

Geometrical Quantities

	Surface area or occupation surface
	Radial distance of a disturbance source of a field point laterally to a reference axis in the x direction
h_o, h_ν	Radial distance of a field point from the trajectory of a disturbance source ($h_o = h_\nu$)
l_o	Length of a singularity element
m_o	Distance between the source-sink elements of a dipole
n_o	Thickness of a singularity element normal to l_o
P; P_o; P_ν	Location of the field point, the disturbance source and the emission point
r_o	Radial distance between the field point and the momentary position of the disturbance source at time t_o
r_ν	Radiation radius or emission radius in spherically symmetrical disturbance waves
r^*_ν	Effective emission radius when including the extension factor of source elements
v_o	A control volume or occupation volume
x, y, z	Cartesian coordinates
$\hat{\xi}_o$	Spatial parameter of compatibility in the kinematics of the disturbance propagation
θ_o, $\bar{\theta}_o$	Angle of inclination of the r_o line in relation to the trajectory of the singularities or the field point in the P_oPP_ν plane
θ_ν, $\bar{\theta}_\nu$	Angle of emission in relation to the trajectory of the singularities or the field point in the P_oPP_ν plane
θ^*_ν, θ_ν	Angle of emission in relation to the orientation of direction of the emission line or source-sink axis
χ_S	Angle of inclination of a source element to the x axis

u'; v'; w'	Components of the velocities of disturbance in the medium
V_S	Velocity of the disturbance source relative to the medium
V_E	Velocity of the field point relative to the medium
V	Total velocity of the medium in the disturbance field /8
β	Mach number parameter $\left[\sqrt{1 - Ma_S^2}\right]$
δ	Delta function for determination of the allocation between disturbance source, emission point and field point
ε	Angle of aberration between optical and acoustic position of the disturbance source $[\theta_o - \theta]$
Γ, γ	Basic function and unit disturbance quantity for the Green theorem in a disturbance field of moving disturbance sources, equation (4.7)
κ	Relationship of specific heat $[c_p/c_v]$
κ_o	Temporal compatibility parameter in the kinematics of the disturbance propagation
λ^*	Relationship between nonsteady and quasi-steady disturbance quantities
μ	Mach angle
ρ	Local density of the medium in the disturbance field
ρ_∞	Density in a resting or undisturbed medium
$\Delta\rho$	Density disturbance in a disturbance field $[\rho - \rho_\infty]$
σ_V, σ_L, σ_N	Correlation factors as a result of effective extension of source elements in emission
σ_R	Correlation factor as a result of effective shift in souce-sink elements in emission
σ_D, σ_D^*	Doppler factor in moving disturbance sources and a resting or moving field point
ϕ	Disturbance potential in the medium
ω_ν	Circular frequency in the nonsteady disturbance sources

χ_S	Angle of inclination of a source-sink axis to the x axis
ψ_o	Angle of inclination of the P_oPP_ν plane to the x-y plane /7
ψ_S	Angle of inclination of the radial h_S line to the disturbance source in relation to the x-y plane
ψ	Angle of inclination of the radial h_o line to the field point in relation to the x-h_X plane $[\psi_S - \psi_o]$

<u>Aerodynamic and Aeroacoustic Quantities</u>

a	Local velocity of Sound
a_∞	Velocity of sound in a medium at rest
F_i	Disturbance force per unit of volume $[\mathrm{kg/m^2 sec^2}]$
G_{ij}	Impulse transport parameter per unit of area $[\mathrm{kg/m \cdot sec^2}]$
k	Wave number $[\omega/a_\infty = 2\pi/\lambda]$
Ma_S	Mach number of the moving disturbance source $[V_S/a_\infty]$
$\overline{Ma}_E$	Mach number of the moving receiver field point $[V_E/a_\infty]$
$\overline{Ma}_E$	Mach number component of the moving receiver field point in the P-P_o-P_ν plane
p	Static pressure in the disturbance field
p_∞	Static pressure in the medium at rest
Δp	Pressure disturbance $[p - p_\infty]$
q_∞	nominal dynamic pressure (reference quantity) $\left[\frac{\kappa}{2} p_\infty Ma_S^2\right]$
Q	Source intensity per unit of volume $[\mathrm{kg/m^3 \cdot sec}]$
$\overline{s}$	Dimensionless density disturbance or pressure disturbance, equation (2.13)
$\overline{S}$	Singularity intensity of an element of volume, equation (4.9)
t	time
t_o, t_ν	momentary time of signal impact at the field point and retarded time of the emission of the disturbance signals

ω_o, ω_E — Circular frequency of the disturbance signals at a resting or moving field point

Ω, Ω^* — Disturbance function, equations (2.4) and (2.10) [1/sec] or [$1/m^2$]

Indices

o — For designation of the momentary time and orientation between disturbance source and field point

ν — For designation of the emission quantities for the impacting disturbance waves at the field point

i' j — Index of direction for x, y, a (i;j = 1;2;3)

μ — Emission positions (μ= 1 for Ma_S<1 and μ= 1;2 for Ma_S >1)

1. Introduction

/9*

I. Newton [1], J.B. Biot [2] and P.S. Laplace [3] were concerned with the theoretical calculation of sound propagation in homogeneous media at rest with the derivation in [2] and [3] completely in agreement with the physical process. The first formulation of the wave equation as well as the solution for the one-dimensional case was carried out by J.L. d'Alembert [4]. The wave equation was then applied extensively to propagation processes and vibration problems of various types. The solution of the radiation field with spherical symmetry in the case of sources fixed in space resulted from the contribution of S.D. Poisson [5]. The fundamental work carried out by A. Cauchy [6], H. v. Helmholtz [7] and G. Kirchhoff [8] on radiation fields with sources fixed in space of a general type are of considerable significance. Solutions for the two-dimensional or cylinder-symmetrical wave equations may be derived according to the method of T. Levi-Civita [9], H. Lamb [10] and J. Hadamard [11] from the case of spherical symmetry. In the standard texts of acoustics by Lord Rayleigh [12] and in hydrodynamics of H. Lamb [13], detailled and fundamental considerations are found on the disturbance propagation of disturbance sources of different types fixed in space.

The wave propagation of moving light sources was first studied by C. Doppler [14] and the space-time relationships in the disturbance propagation of moving singularities are termed Doppler kinematics after him. The strict mathematical treatment of the general Doppler principle was reported in the physics of W. Voigt [15]. The transformation indicated by Voigt contains two free constants with a suitable determination leading directly to the Lorentz transformation. The actual derivation according to H.A. Lorentz [16], or according to H. Poincare [17] was produced later within the framework of electromagnetic wave propagation of moving sources. After the introduction of the theory of relativity for moving systems, the derivation of the Lorentz

*Numbers in the margin indicate pagination in the foreign text.

transformation was presented in a more simple and understandable manner, see the discussions by A. Einstein [18], H. Minkowski [19] as well as M. Planck [20]. Further discussions on this subject /10 are found in the texts by A. Sommerfeld [21] as well as in the report on this topic by H. Küssner [22].

The disturbance propagation of moving singularities represents a fundamental subject in aerodynamics and in aeroacoustics. In contrast to the disturbance field in aeroacoustics with the disturbance quantities fundamentally dependent on time, there are non-stationary and stationary disturbance fields in aeordynamics. The latter represent a special case, since the disturbance fields or the field points accompany the moving reference system of singularities and the quantities of singularities themselves are not dependent on time. This manner of consideration is applied in stationary aerodymanics of moving bodies and is therefore identical when a reference system of solid bodies and relative flow is employed as a basis for observations. The first reports on this subject, considering the effect the Mach number in stationary subsonic areodynamics were written by O. Janzen [23], Lord Rayleigh [24], H. Glauert [25], L. Prandtl [26] and [27], Th. v. Karman [28], while the first fundamental reports on the area of stationary supersonic aerodynamics were presented by J. Ackeret [29], Th. v. Karman and N.B. Moore [30] and L. Prandtl [31]. A natural expansion of the subject then comprised the non-stationary aerodynamics of moving bodies. The first fundamental reports on this subject were written by H. Küssner [32], C. Possio [33] and I.E. Garrick [34] and [35].

Although the standard theory of acoustics was developed and expanded very early, the problems of aeroacoustics, especially the calculation of sound propagation from moving sources, was attacked much later. A simple observation and physical explanation of sound propagation of moving sources was reported by L. Prandtl [36]. Several important reports on this subject were written by H. Hönl [37], N. Rott [38]. H. Billing [39] and H.L. Oestreicher [40]. By converting the general Navier-Stokes equation into the standard wave equation for quadrupolar radiation sources fixed in space, the sound propagation of turbulent air streams was formulated by M.H. Lighthill [41]. The solution was subsequently applied to the case of moving radiation. Further contributions on sound fields of moving sources were provided by I.E. Garrick and C.A. Watkins [42], H.L. Oestreicher [43] and M.V. Lowson [44]. Some fundamental observations on energy density and energy flow in a sound field may be found in reports by N. Rott [38], M.J. Lighthill [41], D.I. Blokhintsev [45] and H.S. Ribner [46]. /11

When the standard methods for treating linearized field equations in aerodynamics or wave equations in aeroacoustics are considered, mainly integral methods are employed with the application of integral transformation or the Green theorem with a suitable basic function. Formal mathematical transformations analogous to the Prandtl-Glauert transformation are always included in the wave equation of moving sources or in the field equation with the Mach number terms in order to reconvert the equations to the known standard form. Therefore, no conclusions are obtained on the physical facts, resulting from the motion of the disturbance sources. Especially in aerodynamics, the effect of the Mach number is considered an effect of compressibility in the sense of a pushing together of the medium. In the aeroacoustics of moving disturbance sources, however, extensive analyses of the kinematics of disturbance fields have been carried out, as in the reports on the subject by N. Rott [38] and I.E. Garrick [34] and [35]. Several further conclusions in this respect are provided in the reports by H.L. Oestreicher [40] and M.J. Lighthill [41]. On the basis of the fact that the standard methods of solution were employed in these reports, there was no necessity for deriving the correlation functions of moving singularities in a general sense. Several fundamental observations on the origin of the correlation functions resulting from disturbance source motion were presented in two reports by A. Das [47] and [48].

In the present report, further relationships are defined from the basic equations on disturbance propagation and the resulting correlation functions from the movement of the disturbance sources are demonstrated physically. Through the introduction of the delta function in the disturbance elements, /12 the spatial and temporal relationships may be separated carefully in the derivation of the correlation functions. The presentation of the correlation functions has been generalized extensively by including surface occupations of singularitites in a random orientation in space in addition to the elementary volumes. The treatment and equations given apply equally to subsonic and supersonic velocities of the disturbance sources. The derived universal relationships and the resulting harmonic basic orders supply extensive information on this subject. Accordingly, several standard interpretations of aerodynamic processes in the disturbance field are provided in novel descriptions.

Some fundamentals of acoustics or aeroacoustics are provided in the texts by P.M. Morse and K.U. Ingard [49] as well as by M.E. Goldstein [50].

2. Basic Equations and Kinematics of the Disturbance Fields of Moving Singularities

In order to determine the disturbance dimensions of moving disturbance sources or those fixed in space, two conservation theorems are applied, specifically the continuity equation and the impulse equation, i.e. the general Bernoulli equation. By eliminating the quantity ρ through the isentropic relationship between pressure, density and velocity of sound, the complete fundamental equation of the disturbance field is obtained, appearing in the form of the wave equation.

2.1. Wave Equation in Spatially Fixed and in Moving Disturbance Sources in a Medium at Rest

The standard wave equation is applied to spatially fixed singularities in a medium at rest having infinite dimensions:

$$\nabla^2 \varphi - \frac{1}{a^2} \frac{\partial^2 \varphi}{\partial t^2} = \Omega(t)\, \delta(x_\nu - x_o) \quad , \tag{2.1}$$

where ϕ is the disturbance potential, Δ^2 is the Laplace operator, Ω a disturbance function and δ the Dirac-delta function with respect to space and time. /13

In the case of moving singularities with the velocity V_S relative to the medium, the following wave equation results in the accompanying coordinate system, designated as the wave equation of moving sources.

$$(2.2) \qquad \nabla^2\varphi - \frac{1}{a^2}\frac{D^2\varphi}{Dt^2} = \Omega(t)\ \delta[x_{\nu i} - x_{oi} - V_i(t_o - t_\nu)]$$

with i = 1, 2 and 3 for the three-axis directions and V_i for the components of velocity in these directions. Furthermore, D/Dt represents the substantial derivations. The position of the emission point P_ν (x_ν, y_ν, z_ν) of a disturbance wave with spherical symmetry and a radius r_ν is clearly defined with respect to the momentary position $P_o(s_o, y_o, z_o)$ with the delta function. It is demonstrated with this delta function that the path has a random orientation in space, i.e. to the three coordinate directions. Therefore, the operator assumes the following form for the substantial derivation:

$$(2.3) \qquad \frac{D^2}{Dt^2} = \left[\frac{\partial}{\partial t} + \vec{V}\cdot v\right]^2$$

with $\vec{V} = \vec{V}_S + \Sigma\vec{v}_i$, where $v_i = [\phi_x, \phi_y, \phi_z]$ representing the disturbance velocities in the medium. The disturbance function $\Omega(t)$ in equation (2.1) and equation (2.2) may be produced as a result of the source, dipolar or quadrupolar effect and therefore has the expression

$$(2.4) \qquad \Omega(t) = \frac{1}{\rho_\infty}Q(t) + \frac{1}{\omega_\nu\rho_\infty}[\nabla\cdot F_i](t) + \frac{1}{\omega_\nu\rho_\infty}[\nabla\cdot\nabla\cdot G_{ij}](t).$$

The local velocity of sound a in equations (2.1) and (2.2) has, with isentropic conditions, the relationship

$$(2.5) \qquad a^2 = a_\infty^2\left\{1 + \frac{\Delta\rho}{\rho_\infty}\right\}^{\kappa-1}$$

with $\Delta\rho = \rho - \rho_\infty$ as density disturbance. It follows from the prediction of a small disturbance that:

$$a^2 = a_\infty^2 \left[1 + \frac{\kappa-1}{2}\frac{\Delta\rho}{\rho_\infty} + \ldots\right] \approx a_\infty^2 \; . \quad (2.6)$$ /14

When the x axis of the coordinate system is positioned in the resulting path of the disturbance source relative to the medium and when the relationships according to equations (2.3) and (2.6) are introduced into equation (2.2), the linearized equation of the disturbance potential in a moving reference system is obtained after cancelling the members of higher order:

(2.7a)

$$\nabla^2\varphi - \frac{1}{a_\infty^2}\frac{D_o^2\varphi}{Dt^2} = \Omega\,\delta(t_o - t_\nu - \frac{r_\nu}{a_\infty})\,\delta\left[x_\nu - x_o - Ma_S r_\nu,\, y, z\right] ,$$

where $$\frac{D_o^2}{Dt^2} = \left[\frac{\partial}{\partial t} + v_S\frac{\partial}{\partial x}\right]^2$$

It also applies that

(2.7b)

$$(1 - Ma_S^2)\,\varphi_{xx} + \varphi_{yy} + \varphi_{zz} - \frac{2}{a_\infty} Ma_S\,\varphi_{xt} - \frac{1}{a_\infty^2}\varphi_{tt} = \Omega(t)\,\delta\left[x_\nu - x_o - Ma_S r_\nu, y, z\right]$$

with $$\Omega(t) = \Omega\,\delta(t_o - t_\nu - \frac{r_\nu}{a_\infty}) \; .$$

While equation (2.1) with the operator D/Dt in accordance with equation (2.3) supplies the complete potential equation of the disturbance field, the linearized equation (2.7) with the operator D_o/Dt is often employed in aerodynamics and aeroacoustics. When another coordinate representation such as cylinder-polar coordinates, spherical-polar coordinates, etc. is introduced for the moving reference system, maintaining the x direction, only the expression appropriate for the Laplace operator is altered in equation (2.7).

In order to represent the equation of the disturbance field of moving sources in reference systems with fixed medium, the Galilei transformation is employed. As presented in Fig. 1, it applies that:

$$\hat{x} = x - V_S t \; ; \quad \hat{y} = y \; ; \quad \hat{z} = z ; \quad \hat{t} = t \; . \tag{2.8}$$

Equation (2.7) therefore assumes the following form: /15

$$\nabla^2 \varphi - \frac{1}{a_\infty^2} \varphi_{tt} = \Omega \, \delta(t_o - t_\nu - \frac{r_\nu}{a_\infty}) \quad \delta\left[\hat{x}_\nu - \hat{x}_o - Ma_S \hat{r}_\nu , \hat{y}, \hat{z}\right] \; . \tag{2.9}$$

While the left-hand sides of equation (2.1) for spatially fixed disturbance sources and of equation (2.9) for moving disturbance sources have an identical form and are based on the propagation of the disturbance waves in spherical symmetry as a result of the compressibility of the medium, the right-hand side of the equation (2.9) includes the kinematic relationships, providing the effect of Mach number in the solution. Therefore, the effect of the Mach number in the linearized theory for moving disturbance sources is a purely kinematic effect.

When the relationship between the disturbance potential ϕ and the density or pressure disturbance $\bar{s}$ is linear, wave equations also result for the disturbance dimensions in the field from equation (2.7) or equation (2.9). These are

$$\nabla^2 \bar{s} - \frac{1}{a_\infty^2} \frac{D_o^2 \bar{s}}{Dt^2} = \Omega^* \; \delta(t_o - t_\nu - \frac{r_\nu}{a_\infty}) \; \delta\left[\hat{x}_\nu - \hat{x}_o - Ma_S \hat{r}_\nu , \hat{y}, \hat{z}\right]$$

(2.10)

or $$\nabla^2 \bar{s} - \frac{1}{a_\infty^2} \bar{s}_{tt} = \Omega^* \; \delta(t_o - t_\nu - \frac{r_\nu}{a_\infty}) \; \delta\left[\hat{x}_\nu - \hat{x}_o - Ma_S \hat{r}_\nu , \hat{y}, \hat{z}\right] .$$

In this case, $\Omega^* = \frac{\omega_\infty}{a_\infty^2} \Omega$ applies with a dimension of $[1/m^2]$.

2.2. The Density and Pressure Disturbance in a Disturbance Field of Moving Disturbance Sources

The density and pressure disturbances, produced at a field point at rest in the medium can be derived from the general

Bernoulli equation. The Bernoulli equation in a moving coordinate system is expressed as:

$$\frac{\partial\varphi}{\partial t} + \frac{V^2}{2} + \frac{a^2}{\kappa-1} = \frac{V_S^2}{2} + \frac{a_\infty^2}{\kappa-1} . \tag{2.11}$$

The following relationship exists for the pressure disturbance in the medium: /16

$$\frac{\Delta p}{p_\infty} = \frac{p}{p_\infty} - 1 = \left[\frac{a}{a_\infty}\right]^{\frac{2\kappa}{\kappa-1}} - 1 . \tag{2.12}$$

Utilizing the relationship according to equation (2.11) for a/a_∞ , a common expression for density and pressure disturbance results from equation (2.12).

In moving reference systems it applies that

$$\bar{s} = \frac{\Delta\rho}{\rho_\infty} = \frac{\Delta p}{\kappa p_\infty} = -\frac{1}{a_\infty^2}\left[\frac{\partial\varphi}{\partial t} + V_S\frac{\partial\varphi}{\partial x} + \frac{(\nabla\varphi)^2}{2}\right] \tag{2.13}$$

or in a resting reference system in the medium:

$$\bar{s} = \frac{\Delta\rho}{\rho_\infty} = \frac{\Delta p}{\kappa p_\infty} = -\frac{1}{a_\infty^2}\left[\frac{\partial\varphi}{\partial t} + \frac{(\nabla\varphi)^2}{2}\right] \tag{2.14}$$

with $(\nabla_\phi)^2 = u'^2 + v'^2 + w'^2$. This square term proves negligible in many cases.

2.3. Kinematics of Disturbance Propagation in Moving Disturbance Sources

When a disturbance source moves in a resting homogeneous medium of infinite dimensions with the velocity V_S, the disturbance propagation of spherical symmetry proceeds from the individual emision points. When a field point $P(t_o)$ is hit by a disturbance source at time t_o with the source located at the momentary position $P_o(t_o)$, the emission $P_\nu(t_\nu)$ of the wave on the path at a radial distance $r_\nu = a_\infty (t_o - t_\nu)$ from the field point and at a distance $x_\nu - x_o = Ma_S r_\nu = V_S(t_o - t_\nu)$ back from

from $P_o(t_o)$ (compare Fig. 2). The elapsed time t_ν of the emission is designated in comparison to the momentary time t_o as retarded time.

When the relative position of the disturbance source $P_o(t_o)$ and of a field point $P(t_o)$ are described at time t_o by r_o and θ_o, simple kinematic relationships for locating the emission position /17 P_ν (t_ν) result for the impacting disturbance wave at the field point through r_ν, θ_ν or x_ν and for determination of the emission time t_ν. The above-described kinematics of the disturbance field as presented in Fig. 2 is completely included in the following basic equations.

$$(2.15)\qquad (1 - Ma_S^2)\, r_\nu^2 - 2 r_o\, Ma_S \cos\vartheta_o\, r_\nu - r_o^2 = 0$$

$$(2.16)\qquad r_\nu \cos\vartheta_\nu - Ma_S\, r_\nu - r_o \cos\vartheta_o = 0 .$$

The following expressions result as solution for the radiation quantities:

$$(2.17)\qquad \frac{r_{\nu\mu}}{r_o} = \frac{1}{1-Ma_S^2}\left\{ Ma_S \cos\vartheta_o + (-1)^{\mu+1}\sqrt{1-Ma_S^2 \sin^2\vartheta_o} \right\}$$

$$(2.18)\qquad \cos\vartheta_{\nu\mu} = Ma_S + \frac{(1 - Ma_S^2)\cos\vartheta_o}{Ma_S \cos\vartheta_o + (-1)^{\mu+1}\sqrt{1 - Ma_S^2 \sin^2\vartheta_o}}$$

$$(2.19)\qquad x_{\nu\mu} = x_o + Ma_S\, r_{\nu\mu}$$

$$(2.20)\qquad t_{\nu\mu} = t_o - \frac{r_{\nu\mu}}{a_\infty} .$$

It applies: $\mu = 1$ for $Ma_S < 1$
$\mu = 1;2$ for $Ma_S > 1$.

It can be seen that all emission quantities determined in a dimensionless form according to equation (2.17) and (2.18) depend only on Ma_S and θ_o. Therefore, universal diagrams may be prepared for all field points in space, as presented in Fig. 3.

The kinematic relationships in disturbance propagation according to Fig. 2 produce several general relationships between the quantities r_o, θ_o and r_ν, θ_ν. While the field quantities in aerodynamics are generally described around the momentary source position P_o by means of r_o and θ_o, in aeroacoustics a representation is generally employed around the emission position P_ν by means of r_ν and θ_ν. In this case, the following relationships apply: /18

$$(2.21)\quad \begin{aligned} r_{\nu\mu}(\cos\vartheta_\nu - Ma_S) &= r_o \cos\vartheta_o \\ r_{\nu\mu}\sin\vartheta_\nu &= r_o \sin\vartheta_o \\ (-1)^{\mu+1} r_{\nu\mu}(1 - Ma_S \cos\vartheta_{\nu\mu}) &= r_o(1 - Ma_S^2 \sin^2\vartheta_o)^{1/2} \\ r_{\nu\mu}(1 + Ma_S^2 - 2\,Ma_S \cos\vartheta_{\nu\mu})^{1/2} &= r_o \\ \sin\varepsilon &= Ma_S \sin\vartheta_o . \end{aligned}$$

Furthermore, the following expression is obtained from equation (2.19) for $Ma_S > 1$ with $\mu = 1$ and 2:

$$(2.22)\quad \frac{x_{\nu 2} - x_{\nu 1}}{r_o} = \frac{V_S}{r_o}(t_{\nu 1} - t_{\nu 2}) = \frac{2\,Ma_S \sqrt{1 - Ma_S^2 \sin^2\vartheta_o}}{Ma_S^2 - 1} .$$

The relationship always applies for the propagation of disturbance waves in spherical symmetry:

$$(2.23)\quad (x - x_\nu)^2 + (y - y_\nu)^2 + (z - z_\nu)^2 - a_\infty^2(t - t_\nu)^2 = 0 .$$

Equation (2.23) remains invariant in the transition from a reference system at rest to a moving one, as well as in the coordinate transformation from physical space in the Lorentz space.

When a source element is shifted from the disturbance center $P_o(x_o, y_o, z_o)$ along or across the path line, a corresponding shift of the emission point occurs according to the relationships in equations (2.17) to (2.21) under the condition that the disturbance signals reach the field point at time t_o. The following relationships in the differential form have proven very useful.

In the shift of a disturbance source along the path line, the following relationships result: /19

$$\frac{\partial x_\nu}{\partial x_o} = \frac{1}{(1 - Ma_S \cos \vartheta_\nu)}$$

$$\frac{\partial h_\nu}{\partial x_o} = 0 \tag{2.24}$$

$$\frac{\partial r_\nu}{\partial x_o} = \frac{\cos \vartheta_\nu}{(1 - Ma_S \cos \vartheta_\nu)}$$

$$r_\nu \frac{\partial \vartheta_\nu}{\partial x_o} = - \frac{\sin \vartheta_\nu}{(1 - Ma_S \cos \vartheta_\nu)} .$$

When a shift dh_S of the disturbance source is undertaken in a random direction Ψ_S in the space across the path line with the field point located at $P(x, h_o, \Psi_o)$, the relationship applies:

$$\frac{dh_o}{dh_S} \approx \cos \psi \tag{2.25}$$

with

$$h_o = r_o \sin \vartheta_o = r_\nu \sin \vartheta_\nu = h_\nu . \tag{2.26}$$

In the shift of a disturbance source across the path line, the following expressions therefore apply:

$$\frac{\partial x_\nu}{\partial h_S} = - \frac{Ma_S \sin \vartheta_\nu \cos \psi}{(1 - Ma_S \cos \vartheta_\nu)}$$

$$\frac{\partial h_\nu}{\partial h_S} = \cos \psi \tag{2.27}$$

$$\frac{\partial r_\nu}{\partial h_S} = - \frac{\sin \vartheta_\nu \cos \psi}{(1 - Ma_S \cos \vartheta_\nu)}$$

$$(2.27) \qquad r_\nu \frac{\partial \vartheta_\nu}{\partial h_S} = \frac{(\cos \vartheta_\nu - Ma_S) \cos \psi}{(1 - Ma_S \cos \vartheta_\nu)}$$

The relationships according to equations (2.24) and (2.27) are used later for determining the correlation functions.

3. Derivation of the Correlation Functions in Disturbance Fields of Moving Singularities

In order to determine the disturbance quantities at a field point at time t_o, the radiation radius r_ν of the impacting disturbance wave is required, of course, the strength $\Omega(t_\nu)$ of the disturbance source at the retarded time t_ν of the emission and all the correlation functions, resulting from the motion of the disturbance source. As discussed in detail in [47], the correlation functions are derived from three physical observations:

a) the effective extension of source elements in emission, with the disturbance signals simultaneously reaching the field point at time t_o. The effective extension results in a longitudinal or transverse direction, therefore producing a volume expansion.

b) the effective shift of source-sink elements in emission with the disturbance signals simultaneously reaching the field point at time t_o.

c) the standard Doppler effect as a result of the alteration in wave passage duration dt_o through the field point in comparison to the duration dt_ν of the emission of the disturbance signals.

All these correlation functions may be derived directly from the basic equations, (2.17) to (2.27).

The following designations are introduced for the correlation functions:

The Spatial Correlation Functions

$\sigma_L = \frac{dl_\nu}{dl_o}$ correlation factor as a result of effective extension of the emission elements in a longitudinal direction coinciding with the Mach number component $Ma_e = Ma_S \cos \chi_S$. /21

$\sigma_N = \frac{dn_\nu}{dn_o}$ correlation factor as a result of effective extension of the emission elements in a transverse direction coinciding with the normal components of the Mach number, $Ma_n = Ma_S \sin \chi_S$.

$\sigma_V = \frac{dv_\nu}{dv_o}$ correlation factor as a result of effective extension of the volume elements of the disturbance sources in emission.

$\sigma_R = \frac{dr_\nu}{dr_{\nu o}}$ correlation factor as a result of effective shifting in the source-sink elements during emission. ($dr_{\nu o} = dr_\nu$ at $Ma_S = 0$).

The Temporal Correlation Functions

$\sigma_D = \frac{dt}{dt_o}$ Doppler factor in medium-fixed field point

$\sigma_D^* = \left.\frac{dt_\nu}{dt_o}\right|_{Ma_E}$ Doppler factor in moving field point.

These correlation factors are now derived for source elements with random orientation in space and to the path line.

3.1. The Spatial Correlation Functions as a Result of Effective Extension and Shifting of Source Elements in Emission

The Correlation Function σ_L

When the disturbance effect of a moving elementary source path dl_o is observed in space with random slanted position χ_S

to the path line, a correlated emission path dl_ν results according to the kinematic relationships of section 2. This extension in length occurs in the direction $Ma_e = Ma_S \cos \chi_S$. The process described is illustrated in Fig. 4. The effective emission path is obtained by combining the effective partial elements ϕl_ν, with disturbance signals reaching the field point P simultaneously at time t_o. It then follows /22

$$\frac{dl_\nu}{dl_o} = \frac{\Sigma\delta l}{dl_o} = \frac{dx_\nu}{dl_o}\cos\chi_S + \frac{dh_{S\nu}}{dl_o}\sin\chi_S . \tag{3.1}$$

Through utilization of the partial derivations according to the relationships in section 2, the following exact equations for the correlation functions σ_L result. It then applies that

$$\sigma_L = \frac{dl_\nu}{dl_o} = 1 + \frac{Ma_e \cos\vartheta_{\nu e}}{1 - Ma_S \cos\vartheta_\nu} \tag{3.2}$$

$$\sigma_L = \frac{dl_\nu}{dl_o} = \frac{1}{1 - Ma_e^* \cos\vartheta_\nu^*}, \tag{3.3}$$

where $Ma_e = Ma_S \cos \chi_S$ and $Ma_e^* = Ma_c/\cos(\chi_S - \chi_\nu)$ representing the Mach number components along dl_ν and dl_ν^*.

The Correlation Function σ_N

When a moving source element exhibits a random orientation in space or to the path line, an altered transverse extension dn_ν of the emission element is obtained in comparison to the actual width dn_o from the same observation as in the previous section. This fact is illustrated in Fig. 4. It follows from the kinematic relationships in this figure:

$$\sigma_N = \frac{dn_\nu}{dn_o} = \frac{1}{1 - Ma_n \cos\vartheta_{\nu n}}, \tag{3.4}$$

with $Ma_n - Ma_S \sin \chi_S$, representing the Mach number component in the direction normal to dl_o and forming $\theta_{\nu n}$ the emission angle between Na_n and r_ν .

The Correlation Function σ_V /23

When a moving element of volume of disturbance sources is observed with random orientation in space and to the path line, the effective emission volume dv_ν results with disturbance signals reaching the field point P simultaneously at time t_o. On the basis of the fact that the element is not extended normally to the direction of motion Ma_S, the alteration in volume consists of the longitudinal and transverse extension as already described. It therefore follows that

$$\sigma_V = \frac{dv_\nu}{dv_o} = \frac{dl_\nu}{dl_o} \cdot \frac{dn_\nu}{dn_o} = \sigma_L \cdot \sigma_N ; \tag{3.5}$$

and may apply ~~that~~

$$\sigma_V = \left\{ 1 + \frac{Ma_e \cos \vartheta_{\nu e}}{1 - Ma_S \cos \vartheta_\nu} \right\} \frac{1}{1 - Ma_n \cos \vartheta_{\nu n}} . \tag{3.6}$$

From the relation

$$Ma_S \cos \vartheta_\nu = Ma_e \cos \vartheta_{\nu e} + Ma_n \cos \vartheta_{\nu n} \tag{3.7}$$

it follows for the effective volume extension of a moving source element with random orientation in space:

$$\sigma_V = \frac{1}{1 - Ma_S \cos \vartheta_\nu} . \tag{3.8}$$

The type of origin of the correlation function σ_V is illustrated in Fig. 5 for various orientations of an element of volume.

This results in

$$\begin{aligned}
&\sigma_V = \sigma_L = \frac{1}{1 - Ma_S \cos \vartheta_\nu} ; \quad \sigma_N = 1 && \text{for } \chi_S = 0 \\
&\sigma_V = \sigma_L \cdot \sigma_N = \frac{1}{1 - Ma_S \cos \vartheta_\nu} && \text{for } 0 < \chi_S < \tfrac{\pi}{2} \\
&\sigma_V = \sigma_N = \frac{1}{1 - Ma_S \cos \vartheta_\nu} ; \quad \sigma_L = 1 && \text{for } \chi_S = \tfrac{\pi}{2} .
\end{aligned} \tag{3.9}$$

It is noteworthy that the volume expansion and therefore σ_V in the present three cases has the identical value. /24

The Correlation Function σ_R

When a shift is undertaken from the amount dm_o of the source element at P_o in the direction $\hat{\chi}_S$ to the path line, a shift results at the emission point P_ν for the source-sink components with signals simultaneously reaching the field point at time t_o. This situation is illustrated in Fig. 6.

In a similar procedure to that in the previous section, the difference in emission radii of source-sink elements, following a path at a distance dm_o, can easily be determined at a field point P. It applies that:

$$\frac{dr_\nu}{dm_o} = \frac{\partial r_\nu}{\partial x_o} \cdot \frac{\partial x_o}{\partial m_o} + \frac{\partial r_\nu}{\partial h_o} \cdot \frac{\partial h_o}{\partial m_o} \,. \tag{3.10}$$

By employing the partial derivations according to the relationships in section 2, the exact expression is obtained:

$$\frac{dr_\nu}{dm_o} = \frac{\cos \hat{\vartheta}_\nu}{1 - Ma_S \cos \vartheta_\nu} \,, \tag{3.11}$$

where θ_ν represents the angle of inclination between r_ν and dm_o at the emission point P_ν and exemplifies the following relationship.

$$\cos \hat{\vartheta}_\nu = \cos \vartheta_\nu \cos \hat{\chi}_S - \sin \vartheta_\nu \cos(\psi_o - \psi_S) \sin \hat{\chi}_S \,. \tag{3.12}$$

When the actual source-sink arrangement at P_o is applied as spatially-fixed system at the emission point P_ν , the difference results there in the radiation radii of these elements to the field point P at

$$\left.\frac{dr_\nu}{dm_o}\right|_{Ma_S=0} (P_\nu) = \cos \hat{\vartheta}_\nu \,. \tag{3.13}$$

The correlation function σ_R follows from equations (3.11) and (3.13) and supplies the generally valid expression: /25

$$\sigma_R = \frac{dr_\nu/dm_o}{|dr_\nu/dm_o|_{Ma_S=0}} = \frac{dr_\nu}{dr_{\nu o}} = \frac{1}{1 - Ma_S \cos \vartheta_\nu} \tag{3.14}$$

The expression σ_R according to equation (3.14) represents a universal formula and is applied for elements of volume as well as for surface occupation of singularities.

3.2. The Temporal Correlation Functions Resulting from the Alteration in Wave Passage Duration at the Field Point in Comparison to the Emission Duration of Disturbance Signals

In the disturbance propagation of moving disturbance sources, the wave passage duration dt_o generally differs from the emission duration dt_ν of the disturbance signals by a field point. Therefore, a temporal extension effect dt_ν/dt_o, the so-called Doppler effect, arises, defined in the case of medium-fixed field points by σ_D and in the moving field point by σ_D^*. While the correlation factor σ_D also determined the field dimensions in the medium, σ_D^* is employed for frequency determination of the received signals at the moving field points. This situation is illustrated is Fig. 7. The derivation of equation (2.20) for t_o results in the universal expression:

$$\begin{aligned} \frac{dt_\nu}{dt_o} = 1 &- \frac{\partial r_o/\partial t_o}{a_\infty(1-Ma_S^2)} \left\{ Ma_S \cos \vartheta_o + (-1)^{\mu+1} \sqrt{1-Ma_S^2 \sin^2\vartheta_o} \right\} + \\ &+ \frac{r_o}{a_\infty} \frac{\partial\vartheta_o/\partial t_o}{(1-Ma_S^2)} \left\{ Ma_S \sin \vartheta_o + (-1)^{\mu+1} \frac{Ma_S^2 \sin^2\vartheta_o}{2\sqrt{1-Ma_S^2 \sin^2\vartheta_o}} \right\}. \end{aligned} \tag{3.15}$$

The temporal alteration of r_o and θ_o is given by: /26

$$\begin{aligned} \frac{1}{a_\infty} \frac{\partial r_o}{\partial t_o} &= - (Ma_S \cos \vartheta_o + \overline{Ma}_E \cos \bar{\vartheta}_o) \\ \frac{r_o}{a_\infty} \frac{\partial \vartheta_o}{\partial t_o} &= (Ma_S \sin \vartheta_o + \overline{Ma}_E \sin \bar{\vartheta}_o) . \end{aligned} \tag{3.16}$$

In this case $\overline{Ma}_E$ is the component of the field point Mach number relative to the medium in the P_oPP_ν plane and $\overline{\theta}_o$ is the angle between the vector $\overline{Ma}_E$ and r_o in the same plane. In the case of a medium-fixed field point, $\overline{Ma}_E = 0$.

The Correlation Function σ_D

From equations (3.15) and (3.16) the result for the Doppler factor at a medium-fixed field point:

$$\sigma_D = \left.\frac{dt_\nu}{dt_o}\right|_{\overline{Ma}_E=0} = \frac{1}{1-Ma_S^2}\left\{1 + (-1)^{\mu+1}\frac{Ma_S \cos\vartheta_o}{\sqrt{1-Ma_S^2 \sin^2\vartheta_o}}\right\}. \quad (3.17)$$

The relationships according to equation (2.21) result in:

$$\sigma_D = (-1)^{\mu+1}\frac{1}{(1 - Ma_S \cos\vartheta_{\nu\mu})} \quad (3.18)$$

with $\mu = 1$ for $Ma_S < 1$
$\mu = 1;2$ for $Ma_S > 1$.

The Correlation Function σ_D^*

In a moving disturbance source and moving field point with a finite value of $\overline{Ma}_E$, the universal Doppler factor σ_D^*, results, decisive for the frequency determination of disturbance signals at the field point. According to equations (3.15) and (3.16) it applies that: /27

$$\sigma_D = \left.\frac{dt_\nu}{dt_o}\right|_{\overline{Ma}_E} = \frac{1}{1 - Ma_S^2} + \frac{Ma_S\,\overline{Ma}_E \cos(\vartheta_o + \overline{\vartheta}_o)}{1 - Ma_S^2} + \quad (3.19)$$

$$+(-1)^{\mu+1}\frac{(Ma_S\cos\vartheta_o + \overline{Ma}_E\cos\overline{\vartheta}_o)}{(1-Ma_S^2)\sqrt{1-Ma_S^2\sin^2\vartheta_o}} - (-1)^{\mu+1}\frac{Ma_S^2\,\overline{Ma}_E\sin\vartheta_o\,\sin(\vartheta_o+\overline{\vartheta}_o)}{(1-Ma_S^2)\sqrt{1-Ma_S^2\sin^2\vartheta_o}},$$

where $\overline{Ma}_E$ and $\overline{\theta}_o$ are explained in Fig. 2. It is often practical to rewrite the Doppler factor σ_D^* in the expression of θ_ν and $\overline{\theta}_\nu$. For this purpose, the relationships according to equation (2.21) are employed as well as the following relationships:

$$(3.20)\qquad \begin{aligned} r_o \cos \overline{\vartheta}_o &= \left[r_\nu \cos \overline{\vartheta}_\nu - Ma_S r_\nu \cos(\vartheta_\nu + \overline{\vartheta}_\nu) \right]_\mu \\ \vartheta_o + \overline{\vartheta}_o &= \left[\vartheta_\nu + \overline{\vartheta}_\nu \right]_\mu . \end{aligned}$$

The conversion of the expression of σ_D^* according to equation (3.19) to the reference dimensions at the emission point supplies:

$$(3.21)\qquad \sigma_D^* = \Bigg[\frac{1}{1 - Ma_S^2} + \frac{Ma_S \, \overline{Ma}_E \cos(\vartheta_\nu + \overline{\vartheta}_\nu)}{1 - Ma_S^2} + \frac{Ma_S (\cos \vartheta_\nu - Ma_S) + \overline{Ma}_E \left\{ \cos \vartheta_\nu - Ma_S \cos(\vartheta_\nu + \overline{\vartheta}_\nu) \right\}}{(1 - Ma_S^2)(1 - Ma_S \cos \vartheta_\nu)} - \frac{Ma_S^2 \, \overline{Ma}_E \sin \vartheta_\nu \sin(\vartheta_\nu + \overline{\vartheta}_\nu)}{(1 - Ma_S^2)(1 - Ma_S \cos \vartheta_\nu)} \Bigg]_\mu$$

with $\mu = 1$ for $Ma_S < 1$

and $\mu = 1;2$ for $Ma_S > 1$.

When $\overline{Ma}_E = 0$ is placed in this equation, then σ_D^* converts /28
into the expression of σ_D according to equation (3.19). The general expression of the Doppler factor σ_D according to equations (3.19) and (3.21) includes all simple cases of the moving field points or receivers, occurring in literature. A representation of the results according to equation (3.18) for subsonic and supersonic Mach numbers of the disturbance source is carried out in Fig. 8. It is noteworthy that the Doppler factor σ_D for $\theta_\nu = 0$ assumes the maximum value at $Ma_S = 0$ and always becomes smaller with decreasing or increasing Mach numbers of the disturbance source, as the figure on the right demonstrates. While the Doppler factor constantly remains at 1 in the case of

$\theta_\nu = \pi/2$, a continuous decrease in the σ_D values is recorded for $\theta_\nu = \pi$ with increasing Mach number.

3.3. Universal Representation of the Correlation Functions and Several Special Characteristics in Disturbance Propagation

The spatial and temporal correlation functions, arising as the result of the disturbance source motion, may be compiled in a universal representation for random field points in space. When the correlation functions σ_V, σ_R and σ_D for random medium-fixed field points in space are plotted as a function of $Ma_S \cos\theta_\nu$, a universal curve results, consisting of two hyperbolic branches valid for a Mach numbers and all field points. This is explained in Fig. 9. The appropriate range of validity of the curve is extended by the value $\sigma = 1$ for $Ma_S = 0$ with increasing Mach number, as can be seen from this illustration. In the case of $Ma_S > 1$, the range extends beyond the line $Ma_S \cos\theta_\nu = 1$, representing the point of contact of the wave having spherical symmetry with the Mach cone. When a wave having spherical symmetry is observed during propagation around an emission point P_ν, the correlation factors σ in this wave remain at the same value for each θ_ν = const.

When the correlation factor σ_V is combined with the radiation radius r_ν, a further concept is created in the form of the actual radiation radius according to the definition $r^*_\nu = r_\nu/\sigma_V$. It then follows according to equation (3.8): /29

$$r^*_\nu = \frac{r_\nu}{\sigma_V} = r_\nu \, (1 - Ma_S \cos \vartheta_\nu) \, . \tag{3.22}$$

The alternate expression then obtained according to the relationship in equation (2.21)

$$r^*_\nu = r_o \, (1 - Ma_S^2 \sin^2 \vartheta_o)^{1/2}$$

(3.23)

$$\text{or.} \quad \frac{r^*_\nu}{r_o} = (1 - Ma_S^2 \sin^2 \vartheta_o)^{1/2} \, .$$

A universal curve, i.e. an ellipse, is then obtained for all Mach numbers and for field points in the entire space, as is illustrated in Fig. 10. The range of validity of this curve increases with Mach number, achieving the limit value $r_\nu^* = 0$ for $Ma_S \geqq 1$ at $Ma_S \sin \theta_o = 1$. An important characteristic can be seen in Fig. 10, as the quantities r_ν/r_o, θ_ν and r_ν^*/r_o always remain constant on a radial line of the momentary source position P_o, i.e. for θ_o = constant. It therefore follows that the impacting disturbance waves always have the same correlation functions at field points on this line.

The radiation process in an evenly moving source path of infinite length and random inclination to the path line is illustrated in Fig. 11. In this case, a field point P is observed on the same plane as the source path when the disturbance signals impact there at time t_o. The emission points of this signal can easily be determined according to equation (2.17) or equation (2.19). Since the position of the source path at time t_o is known, the position and shape of the emission line L may be completely defined from the guideline relationships. It applies that

$$\frac{r_\nu}{x_\nu - x_o} = \frac{r_\nu}{Ma_S r_\nu} = \frac{1}{Ma_S} . \tag{3.24}$$

This equation supplies a hyperbola for $Ma_S < 1$, a parabola for $Ma_S = 1$ and an ellipse for $Ma_S > 1$, where the field point is correlated as focus of these curves. For random field points in space, corresponding L_ν lines are obtained, the envelops of these base curves. The base curves are altered with respect to position and extension with the slanted position of the source path to the path line by assuming a symmetrical course around the slanted x_1 axis. The methods for solving the disturbance fields in standard area dynamics are essentially supported on this characteristic, by basing the solutions on the Mach number component $Ma_n = Ma_S \sin \chi$ of the source path. As is clearly demonstrated in Fig. 11, this procedure is an infraction of physics at $Ma_S \leqq 1$, since a portion of the disturbance sources situated /30 outside of the actual Mach cone is considered effective. The situation can easily be seen in Fig. 11. Therefore, it is clear that the division of the motion Mach number Ma_S into the normal and tangential components Ma_n and Ma_e, as is undertaken in standard aerodynamics, is a completely artificial method to achieve a solution. The actual sweepback effect of a slanted source path can be represented as follows.

In moving sources paths of infinite length, the actual extension of the emission elements in the longitudinal direction dl_o has no additional effect, since the entire length is already occupied by the sources. In contrast, the actual extension of the emission elements in the transverse direction is completely effective and is included entirely by the correlation function σ_N. This situation is in complete agreement with the physical processes in the disturbance field as shown in Fig. 4 and supplies the true basis for the sweepback effect.

A further noteworthy process in disturbance propagation of moving source paths in the case of supersonic Mach numbers is demonstrated in Fig. 12. In this case, it is shown that the disturbance waves impacting at the field point P in each case at times t_{o1} and t_{o2} originate at the emission lines $L_{\nu 1}$ and $L_{\nu 2}$. It is remarkable that the entire effect of the disturbance signals remains constant at the field point for all times t_o, since the radiation process with r_ν, σ and L_ν always supplies the same amount for the surface integrals of the $1/r^*$ curves. It can be demonstrated that the conditions remain in all field points in space behind the source path, specifically under the influence of the above-described dynamic processes and continuous expan- /31
sion of the emission lines.

When a source path of finite length moves in a longitudinal direction with the source elements situated along the path line $\chi_S = 0$, as is the case in axis-symmetrical disturbance fields, several remarkable phenomena occur in the propagation process along the path line. When a field point on the path line of the source path with $\chi_S = 0$ is observed, the effect of Mach number disappears there with the effective emission radius r_ν^*. The reason for this lies in the fact that in the appropriate $\theta_o = 0$ or π of the extension factor σ_V of a source element and the dimensionless emission radius r_ν/r_o of the disturbance wave at the field point assume identical values as is demonstrated in Fig. 13. Although both quantities σ_V and r_ν are subject to the entire effect of Mach number, the total effect is cancelled at r_ν^* with the result that r_ν^* is identical to r_o. It applies according to equations (2.21) and (3.13):

$$(3.25) \qquad \frac{r_\nu}{r_o} \equiv \sigma_V = \frac{1}{1 \pm Ma_S} \qquad \text{for } \vartheta_o = 0, \quad \pi .$$

Therefore it follows that:

$$(3.26) \qquad r_\nu^* = \frac{r_\nu}{\sigma_V} = r_o \qquad \text{for } \vartheta_o = 0 \text{ and } \pi .$$

This situation supplies the physical basis for the theory of thin bodies, postulated in literature by other means.

When three-dimensional singularity occupations of any type are located in smooth motion in space, all singularities with r_ν^* = constant may be combined for determining the disturbance quantities at a field point at time t_o. The momentary position of these singularities in relation to the field point are described by r_o and θ_o, necessarily fulfilling the relationships according to equation (3.23). It follows that:

$$(3.27) \qquad r_o = \frac{r_\nu^*}{(1 - Ma_S^2 \sin^2 \vartheta_o)^{1/2}} .$$

In the case of r_ν^* = const. and Ma_S = const., r_o describes surfaces or shells for the following form according to this equation: /32

a rotational ellipsoid for $Ma_S < 1$

a vertical plane for $Ma_S = 1$

a rotational hyperboloid for $Ma_S > 1$,

where the x axis forms a symmetrical axis through the field point. These functional characteristics supply the basis, designated in literature as surface rule. Several examples of this are calculated for three Mach numbers from equation (3.27) and illustrated in Fig. 14.

In aerodynamics and aeroacoustics, problems in surrounding flow or radiation are often simulated with the surface occupation of singularities. When the correlation functions are determined for this purpose as a result of the motion of the surface elements, the singularity elements must be considered as elements of volume of thin shells, because physical line occupation or surface occupation of the thickness of zero cannot be realized. The extension effects in emission are supplied by the correlation function σ_V according to equation (3.9). According to this it applies ~~that~~:

$$\sigma_V = \sigma_L \cdot \sigma_N = \frac{1}{1 - Ma_S \cos \vartheta_\nu} . \tag{3.28}$$

The analysis of this expression with the aid of equation (3.9) supplies some important new knowledge.

While a pure surface extension of the disturbance elements at $\chi_S = 0$ is recorded with $\sigma_V = \sigma_L$ and $\sigma_N = 1$, an alteration in flow amount or source density results at $\chi_S = \pi/2$ through the emission surface $dl_\nu = dl_o$, because $\sigma_L = 1$ remains. This situation is caused by the fact that no surface extension occurs in the element but only a transverse extension through $\sigma_V = \sigma_N$. In the case of $0 < \chi_S < \pi/2$, both effects occur together with emission. This situation is demonstrated in Fig. 5. According to the above-described physical process, the following relationships result for moving surface elements in a random orientation to the path line: /33

$$\begin{aligned} S_\nu &= \sigma_N S_o \\ dF_\nu &= \sigma_L dF_o . \end{aligned} \tag{3.29}$$

According to equation (3.28) it also applies that:

$$\sigma_V S_o dF_o = S_\nu dF_\nu . \tag{3.30}$$

Further fundamental analyses of the disturbance fields may be undertaken with the derived kinematic relationships and correlation functions, and many standard interpretations may be formulated in accordance with the facts in a new representation.

4. Solution of the Wave Equation and Derivation of the Field Quantities in Moving Singularities

The propagation processes of spatially fixed and moving disturbance sources in a medium at rest of infinite extension may be considered in the same manner as medium-fixed or moving reference system. The medium always serves as a carrier for the wave system with the disturbance centers and field points, and therefore the wave image, remaining identical in both systems. The linearized wave equation in the moving reference system has the general expression:

$$(4.1) \qquad \nabla^2 \varphi - \frac{1}{a_\infty^2} \frac{D_0^2 \varphi}{Dt^2} = \Omega \, \delta(\hat{\kappa}_0) \, \delta(\hat{\varepsilon}_0) \, .$$

In the medium-fixed reference system, the wave equation assumes the following form:

$$(4.2) \qquad \nabla^2 \varphi - \frac{1}{a_\infty^2} \frac{\partial^2 \varphi}{\partial t^2} = \Omega \, \delta(\hat{\kappa}_0) \, \delta(\hat{\varepsilon}_0) \, .$$

The delta functions in these equations include the temporal and spatial kinematic relationships in a universal form, since they apply both to spatially fixed and to moving disturbance sources. The designations introduced in this represent the following relationships: /34

$$(4.3) \qquad \begin{aligned} \hat{\varepsilon}_0 &= x_\nu - x_0 - Ma_S \, r_\nu \\ \hat{\kappa}_0 &= t_0 - t_\nu - \frac{r_\nu}{a_\infty} \, . \end{aligned}$$

In spatially fixed disturbance sources, $Ma_S = 0$, while the motion Mach number of the singularity is Ω in moving disturbance sources Ma_S. The expression for ϱ_0 is identical in both cases. As already described in section 2.1, the equations (4.2) and (4.3) can be converted to one another through the application of the

Galilei transformation. The delta functions in both equations remain invariant in this process.

In order to solve the wave equation of moving disturbance sources, generally integral methods are applied, utilizing integral transformations or through the application of the Green formula with a suitable base function. In both these methods, first the field equation is converted into the standard form. For this purpose a mathematical transformation analogous to the Prandtl-Glauert transformation or a Lorentz transformation is employed. In these transformed spaces, the equivalent radiation radii assume the same dimensions, identical to r_ν^* in equation (3.23). In standard aerodynamics, this situation is attributed to the effect of compressibility.

In the following section, the integral method in the actual physical space is employed for solving the wave equation of moving disturbance sources, specifically in such a manner that the correlation functions are included appropriately in the Green function, i.e. in the base function.

4.1. Solution for the Wave Equation According to the Expanded Green Theorem

In order to treat the wave equation in the case of moving singularities, the two expression according to equations (4.1) and (4.3) can be equally applied. For the subsequent suitable steps,the equation is employed here in a medium-fixed reference system. It states: /35

$$\nabla^2\varphi - \frac{1}{a_\infty^2}\varphi_{tt} = \Omega\,\delta(\hat{\kappa}_0)\,\delta(\hat{\varepsilon}_0)\,. \tag{4.4}$$

In comparison to the wave equation of spatially fixed singularities, two important characteristics must be underlined here, included in the solution. The motion of the disturbance source produces a shift in disturbance centers and causes a spatial extension effect of the emission elements with the singularities Ω. These two effects, determining the disturbance potential at the field point, are always completely included in $\hat{\varepsilon}$ and $(Ma_S, r_\nu, \theta_\nu)$.

The disturbance potential $\phi(t)$ dependent on time at the field point may be constructed by means of a Fourier series from a number of harmonic functions with the form

$$\varphi(t) = \varphi_o e^{-i\omega t}. \tag{4.5}$$

In order to solve the wave equation by means of the Green theorem, a basic function is now introduced such that it satisfies the wave equation for harmonic disturbance functions and includes the spatial correlation created at the emission center. The base function is ~~ex~~pressed as

$$\Gamma(P) = \frac{1}{4\pi} \frac{\sigma_V}{r_\nu} e^{ikr_\nu} \tag{4.6}$$

with $k = \omega / a_\infty$, r_ν as radiation radius of the disturbance wave at the field point. Furthermore, σ_V according to equation (3.8) represents the extension function of the correlated emission element at the disturbance center. While the correlation function $\sigma_V(Ma_S, \theta_\nu)$ is not dependent on r_ν and Ψ, this assumes a fixed value corresponding to equation (3.8) for an infinitesimal surface element of the disturbance wave at field point $P(r_\nu, \theta_\nu)$, because the actual emission element for the disturbance wave at the field point is completely defined. Therefore, the basic function fulfills the Helmholtz equation in the following form:

$$(\nabla^2 + k^2)\Gamma = \gamma\, \delta(\hat{\kappa}_o)\, \delta(\hat{\varepsilon}_o) \tag{4.7}$$

with γ as a unified disturbance function. When the same procedure is applied as according to Helmholtz [7] and Kirchhoff [8] and the delta function $\phi(\hat{\varepsilon}_o)$ is introduced into the disturbance quantities in an integral form in the application of the Green theorem, the solution of the disturbance potention results in the following form: /36

$$\varphi(P) = \sum_{\mu} \frac{(-1)^{\mu+1}}{4\pi} \left[\int_{V_o} \Omega(t) \frac{\sigma_{V\mu}}{r_{\nu\mu}} e^{ikr_{\nu\mu}} \, dv_o + \int_{F_o} \left\{ \frac{\sigma_{V\mu}}{r_{\nu\mu}} e^{ikr_{\nu\mu}} \frac{\partial\varphi}{\partial n_o} + \varphi \frac{\partial}{\partial n_o} \left(\frac{\sigma_{V\mu}}{r_{\nu\mu}} e^{ikr_{\nu\mu}} \right) \right\} dF_o \right], \tag{4.8}$$

where μ signifies the number of positions on the path line, fulfilling the condition $\hat{\varepsilon}_o = 0$ and n_o representing the surface normal value for F_o. It follows from section 2 that $\mu = 1$ for $Ma_S < 1$ and $\cancel{\mu} = 1$ and 2 for $Ma_S > 1$. In spatially fixed disturbance sources with $Ma_S = 0$, $\sigma_V = 1$ and, as a result, equation (4.8) is converted to the solution according to Helmholtz [7] and Kirchhoff [8]. The wave equation of moving disturbance sources may also be solved in an elegant manner by the introduction of the Lorentz transformation. This subject will be treated in more detail elsewhere.

The first integral in equation (4.8) includes the amount of the element of volume of singularities with the strength Ω (t), while the second integral represents the amount resulting from the source and dipolar occupation on the surface F_o.

4.2. Determination of the Disturbance Potential in Moving Singularities of Various Types

The solution of the wave equation of moving disturbance sources with their general form according to equation (4.8) includes the disturbance function $\Omega(t)$ in the volume integral, where these may represent a source, dipolar or quadrupolar singularity according to equation (2.4). The dipolar and quadrupolar singularities may in turn assume various arrangements in space or in relation to the path line.

When the three disturbance functions in equation (2.4) are /37 considered harmonic functions, where Q_0, F_o and G_o form the maximum amplitude of the disturbance dimensions, the following integral quantities are introduced in the volume integral:

$$\bar{S}_o = \frac{1}{\rho_\infty} \int_{V_o} Q_o \, dv_o = M_o \quad \left[\frac{m^3}{sec}\right] \quad \text{(source)}$$

(4.9)

$$\bar{S}_1 = \frac{1}{\omega_\nu \rho_\infty} \int_{V_o} F_o \, dv_o \equiv D_o \quad \left[\frac{m^4}{sec}\right] \quad \text{(dipole)}$$

$$\bar{S}_2 = \frac{1}{\omega_\nu \rho_\infty} \int_{V_o} G_{oij} \, dv_o \equiv T_o \quad \left[\frac{m^5}{sec}\right] \quad \text{(quadrupole)}$$

A general solution of the volume integral in equation (4.8) is obtained ~~by~~ the application of partial integration resulting in a unified expression of the following form:

(4.10)

$$\varphi(P) = \sum_\mu (-1)^{\mu+1} \frac{\bar{S}_{l+m}}{4\pi} \frac{\partial^{l+m}}{\partial x_o^l \, \partial h_o^m} \left[\left\{ \frac{e^{-i\omega_\nu t_{\nu\mu}}}{r_{\nu\mu} \, (1 - Ma_S \cos \vartheta_{\nu\mu})} \right\}\right]$$

with $\sigma_{V\mu} = 1/(1 - Ma_S \cos \vartheta_{\nu\mu})$,

where the following combinations result for the disturbance functions:

Sources: $l = m = 0$

Dipole: $l = 1, m = 0$; $l = 0, m = 1$

Quadrupole: $l = 2, m = 0$; $l = 1, m = 1$; $l = 0, m = 2$.

The surface integrals in equation (4.8) may be calculated analytically or numerically according to the singularity occupation with the surface being divided in the latter case into individual elements. The determination of the induced effect of these surface elements at the field point may be carried out such that the integral expressions are first described in the form of contour integrals along the edges of the surface elements. /38
The desired field dimensions result from this.

4.3. Determination of the Density and Pressure Disturbances in the Disturbance Field

The field quantities with respect to density and pressure disturbances can be calculated with a unified formula according to equation (2.14). It applies in a medium-fixed reference system that:

$$\bar{s}(P) = \frac{\Delta\rho}{\rho_\infty} = \frac{\Delta p}{\kappa p_\infty} = -\frac{1}{a_\infty^2}\left[\frac{\partial\varphi}{\partial t_o} + \frac{(\nabla\varphi)^2}{2}\right], \tag{4.11}$$

where the functional relationship of $\phi = \phi(Ma_S, r_\nu, \theta_\nu, t_\nu)$ is included in the derivation of ϕ for t_o. The determination of $\bar{s}$ can be carried out in a few steps, if the appropriate delta function $\delta(\hat{\kappa}_o)$ is introduced in integral form in the derivation according to equation (4.11). It then applies that:

$$\bar{s}(P) = \frac{\Delta\rho}{\rho_\infty} = \frac{\Delta p}{\kappa p_\infty} = -\frac{1}{a_\infty^2}\left[\frac{\partial}{\partial t_o}\int_{\infty}^{\infty}\left\{\varphi(t_o - \hat{\kappa}_o)\,\delta(\hat{\kappa}_o)\right\}_{\hat{\kappa}_o \to 0} d\hat{\kappa}_o + \frac{(\nabla\varphi)^2}{2}\right]. \tag{4.12}$$

After carrying out the derivation for t_o and after partial integration of the delta function, it follows in the moving reference system that:

$$\bar{s}(P) = -\frac{1}{a_\infty^2}\left[\frac{\partial\varphi}{\partial t_o} + V_S\frac{\partial\varphi}{\partial x_o} + \frac{(\nabla\varphi)^2}{2}\right]_{\hat{\kappa}_o=0} \tag{4.13}$$

or

$$\bar{s}(P) = -\frac{1}{a_\infty^2}\left\{\left(\frac{\partial\varphi}{\partial t_\nu}\,\frac{\partial t_\nu}{\partial t_o}\right)_{\hat{\kappa}_o=0} + V_S\left[\frac{\partial\varphi}{\partial r_\nu}\,\frac{\partial r_\nu}{\partial x_o} + \frac{\partial\varphi}{\partial\vartheta_\nu}\,\frac{\partial\vartheta_\nu}{\partial x_o}\right]_{\hat{\varepsilon}_o=0} + \frac{(\nabla\varphi)^2}{2}\right\}. \tag{4.14}$$

The derivations achieved when fulfilling the kinematic relationships $\hat{\kappa}_o = 0$ and $\hat{\varepsilon}_o = 0$ were already derived in sections 2 and 3. The field quantities are clearly described there and the resulting terms reproduced as near field and distant field portions. Exact relationships between instationary, quasi-stationary and stationary field quantities may be prepared from these figures.

In a moving source volume, the following density and pressure /39
disturbances occur in the disturbance field:

Near Field Term

$$(4.15) \quad \bar{s}_N = \frac{1}{4\pi} \sum_{\mu} (-1)^{\mu+1} \frac{\bar{S}(t_{\nu\mu})}{a_\infty r_o^2} Ma_S \cos\vartheta_o \left\{ \frac{1}{(1-Ma_S^2 \sin^2\vartheta_o)^{3/2}} \right\} .$$

Distant Field Term

$$\bar{s}_F = \frac{1}{4\pi} \sum_{\mu} (-1)^{\mu+1} \frac{i\omega_\nu \bar{S}(t_{\nu\mu})}{a_\infty^2 r_\nu} \left\{ \frac{1}{(1 - Ma_S \cos\vartheta_\nu)^2} \right\} .$$

Equations (4.15) and (4.16) produce the instationary disturbance quantities in the field. The disturbance quantities $\bar{s}_N$ and $\bar{s}_F$ on the unit spherical surface around x_o and x_ν are illustrated in Fig. 17. When $Ma_S << 1$, these quations convert into the quasi-stationary forms by replacing the expressions in parentheses by 1. The relationship of the instationary to the quasi-stationary disturbance quantities produces

$$(4.16) \quad \lambda_N^* = \frac{1}{(1 - Ma_S^2 \sin^2\vartheta_o)^{3/2}} \qquad \text{(near field)}$$

and

$$(4.17) \quad \lambda_F^* = \frac{1}{(1 - Ma_S \cos\vartheta_\nu)^2} \qquad \text{(distant field)}$$

These factors are reproduced for various Mach numbers and all field points in space in Fig. 18. They supply some interesting conclusions with respect to the effect of Mach number and field point orientations with the field points located on beams with constants θ_o or θ_ν .

In the temporal, constant source intensity, stationary fields result with accompanying reference system by setting $\bar{S} = \bar{S}_o$ = const. in equation (4.15). It can be seen that the distant field portion of the field quantities disappears in this case, because the quantity $\omega_\nu = 0$ is the equation for $\bar{s}_F$. When a moving disturbance source is observed in an incompressible medium, it is practical to replace the quantity $\bar{s}_N$ by the coefficient of pressure c_p. It then applies that

/40

$$(4.18) \qquad c_p = \frac{2}{Ma_S^2}\,\bar{s}_N = \frac{2}{V_S}\,\frac{\bar{S}_o \cos\vartheta_o}{4\pi\, r_o^2} = \frac{2u'}{V_S}\,.$$

In the case of instationary disturbance fields, the determination of the signal frequency at the resting or moving field point is carried out such that the emission and reception of a defined number of waves is taken into consideration. It is necessary that

$$(4.19) \qquad \frac{\omega_o dt_o}{2\pi} = \frac{\omega_E dt_o^*}{2\pi} = \frac{\omega_\nu dt_\nu}{2\pi} = \text{const.}$$

Subsequently

$$(4.20) \qquad \omega_o = \sigma_D \omega_\nu \quad \text{and} \quad \omega_E = \sigma_D^* \omega_\nu\,.$$

The temporal extension functions σ_D and σ_D^* are already known from equations (3.18) and (3.19).

5. Summary

First all the correlation functions resulting from the motion of singularities are derived in a generally valid form, using the kinematics of disturbance propagation from moving disturbance sources by permitting random orientation in space, i.e. to the path line, for the source elements. It follows that the resulting correlation functions have spatial and temporal extension effects in the emission of the disturbance signals.

The correlation effects may be obtained in a universal representation for a random number of field points in space and for all Mach numbers in individual diagrams. All physical processes in the disturbance field are factually included by the the present manner of observation and several standard interpretations are presented in a new manner. Thereafter, the actual situations on the effect of Mach number, sweepback effect and effect of thinness of moving disturbance volumes and disturbance surfaces are formulated on a new basis. The solution of the wave equation is then described on a physical basis and the field quantities produced from this in a general form. The present unified treatment applies as well to aerodynamics and aeroacoustics as it does for subsonic and supersonic Mach numbers.

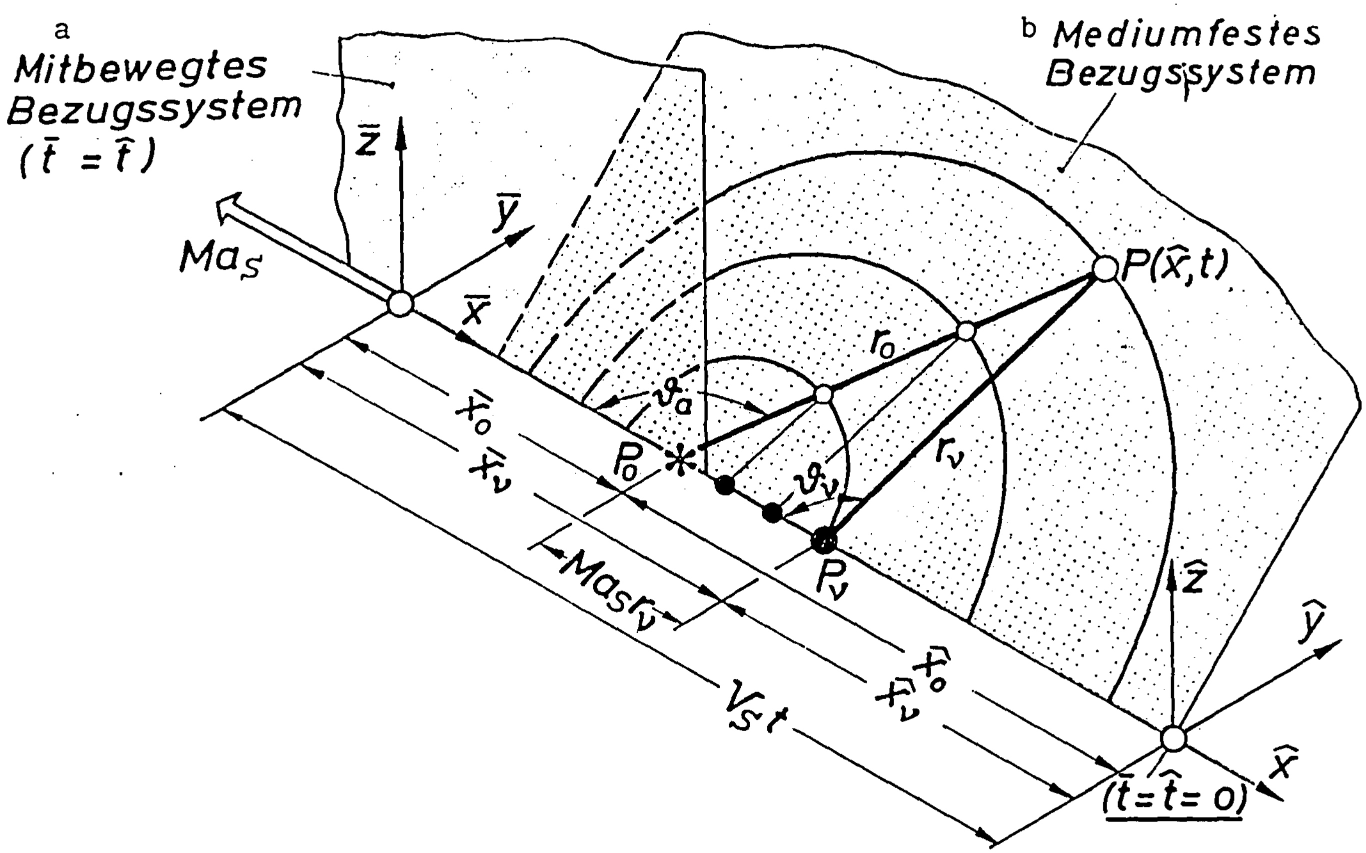

Fig. 1: Wave Propagation of a Disturbance Source with Smooth Motion, Represented in a Medium-Fixed Reference System with the Same Motion, Related to One Another Through the Galilei Transformation.

Key: a. Reference System with same motion.
b. Medium-Fixed Reference System

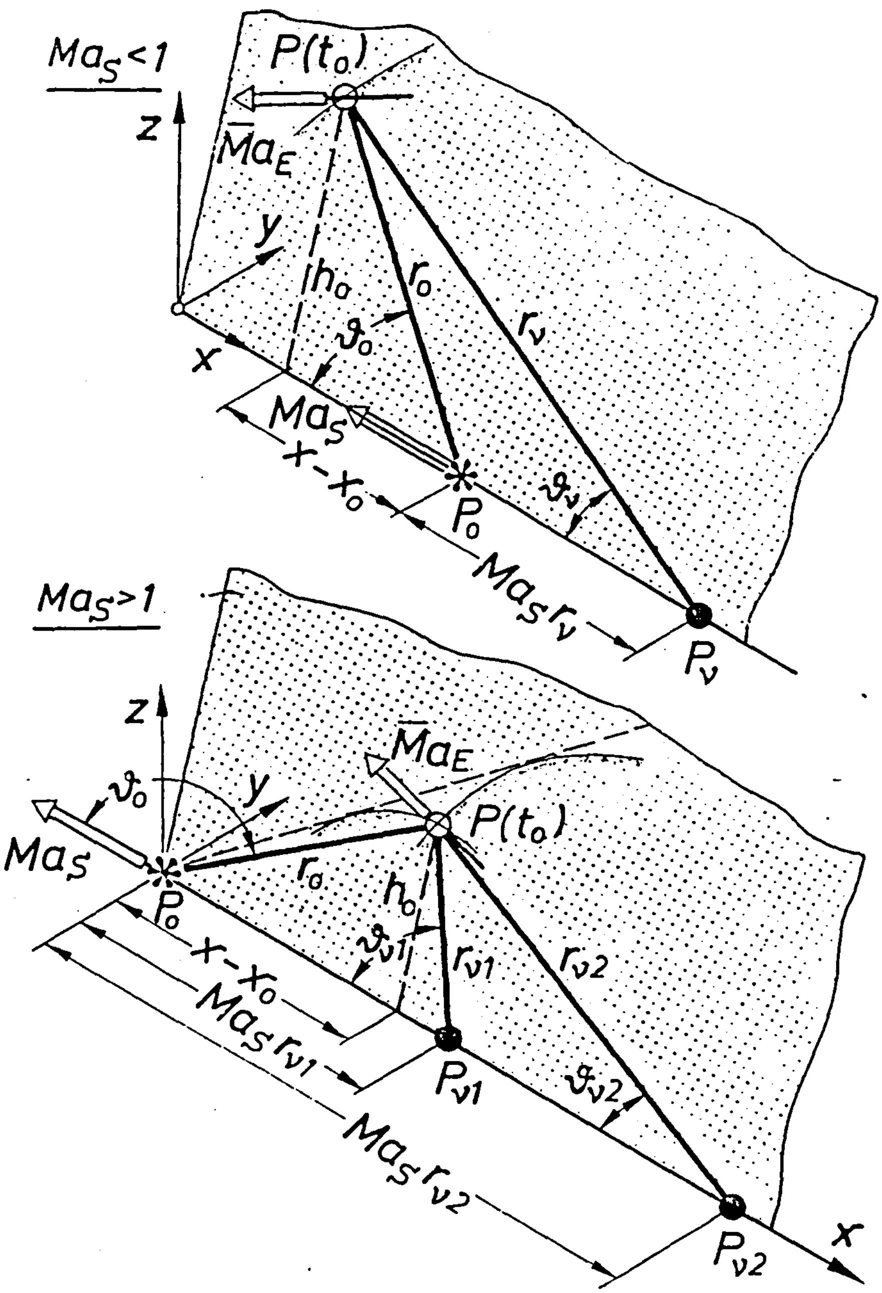

Fig. 2: Kinematic Relationships in the Disturbance Propagation of Moving Disturbance Sources in a Homogeneous Medium at Rest with Infinite Dimensions.

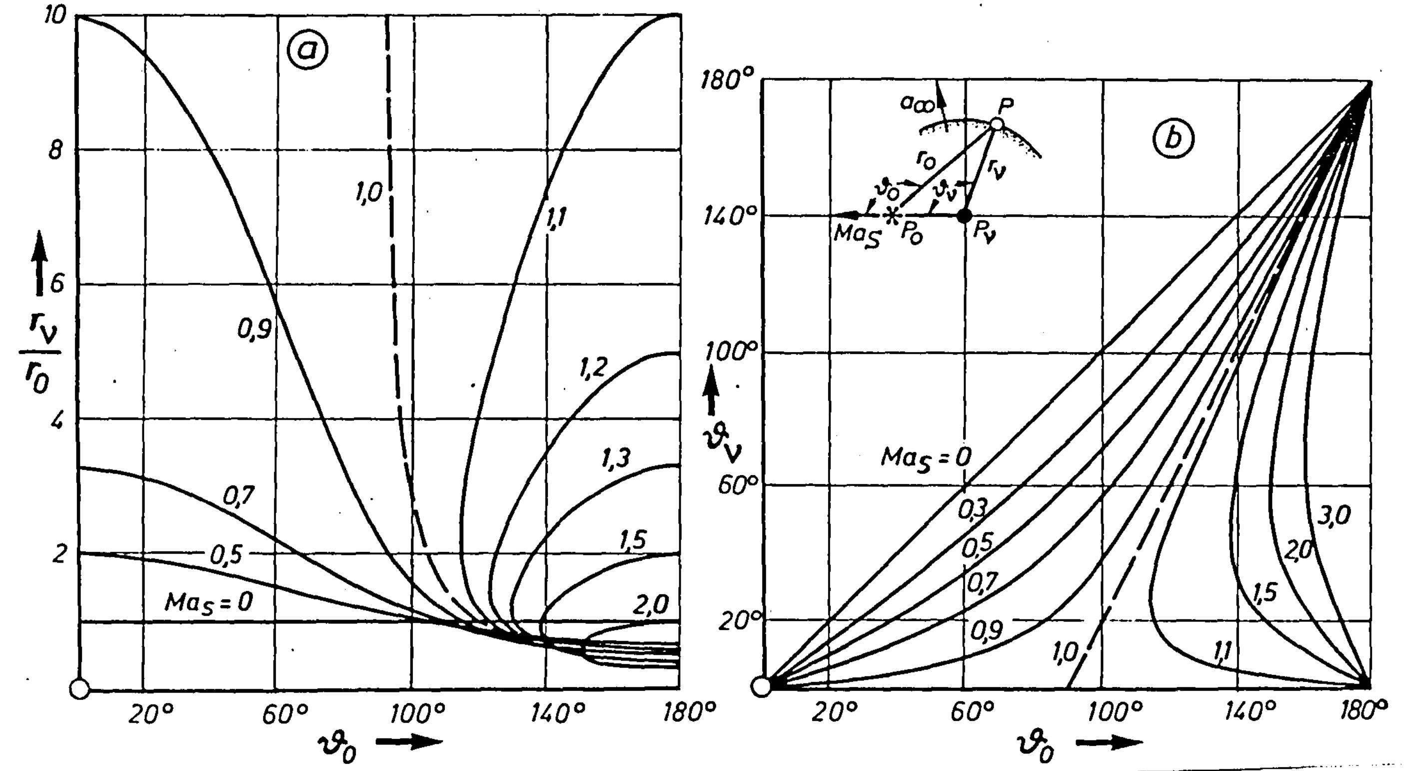

Fig. 3: Universal Relationships for Determination of Radiation Quantities r_ν and θ_ν of Disturbance Signals, Hitting a Moving Disturbance Source at Varying Mach Numbers Ma_S at the Field Points $P(r_o, \theta_o)$ at Time t_o.

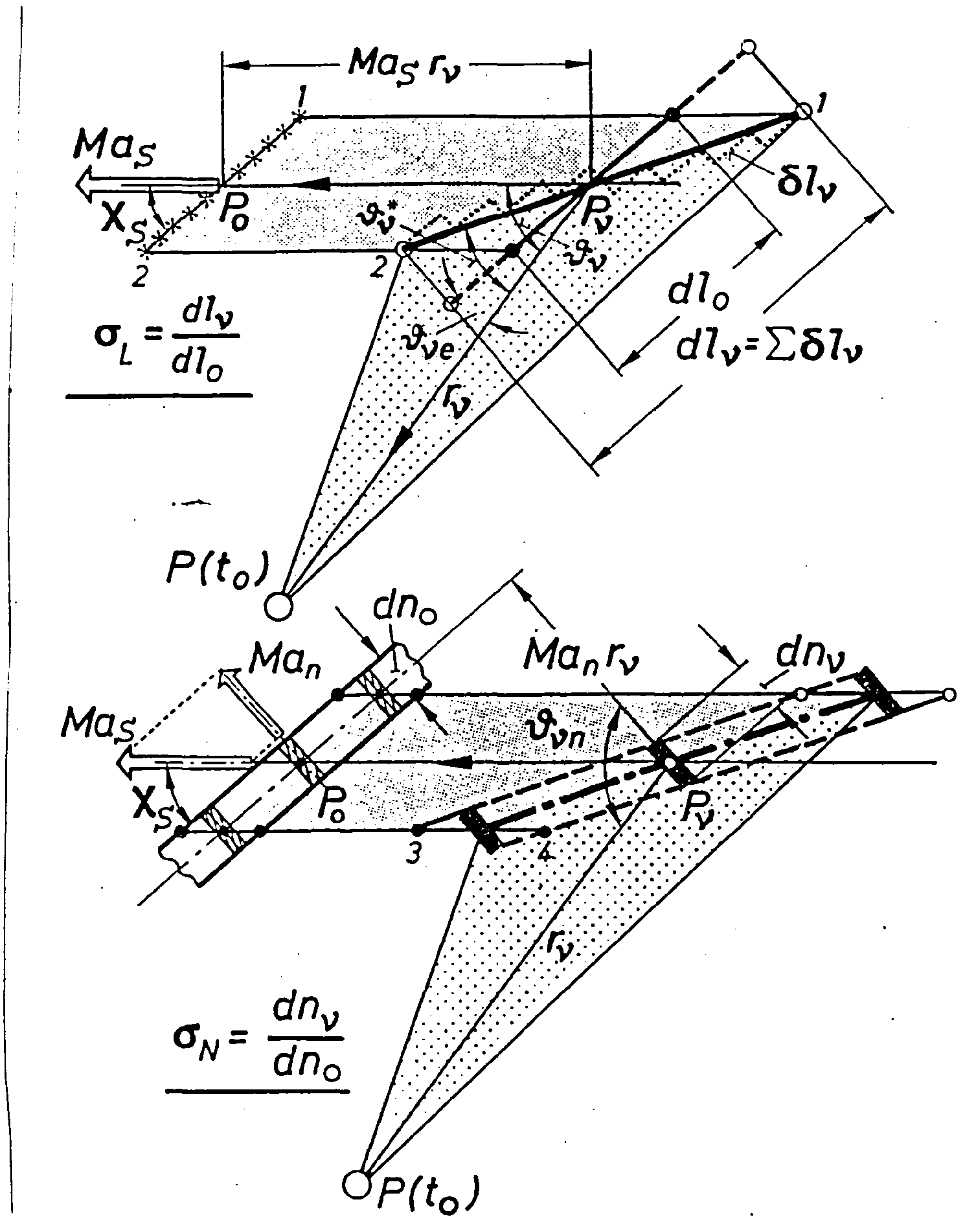

Fig. 4: Production of the Spatial Correlation Functions σ_L and σ_N as a Result of the Actual Longitudinal or Transverse Extension of the Emission Elements with all Disturbance Signals Reaching Field Point P Simultaneously at Time t_o.
($\sigma_L = dl_\nu/dl_o$; $\sigma_N = dn_\nu/dn_o$)

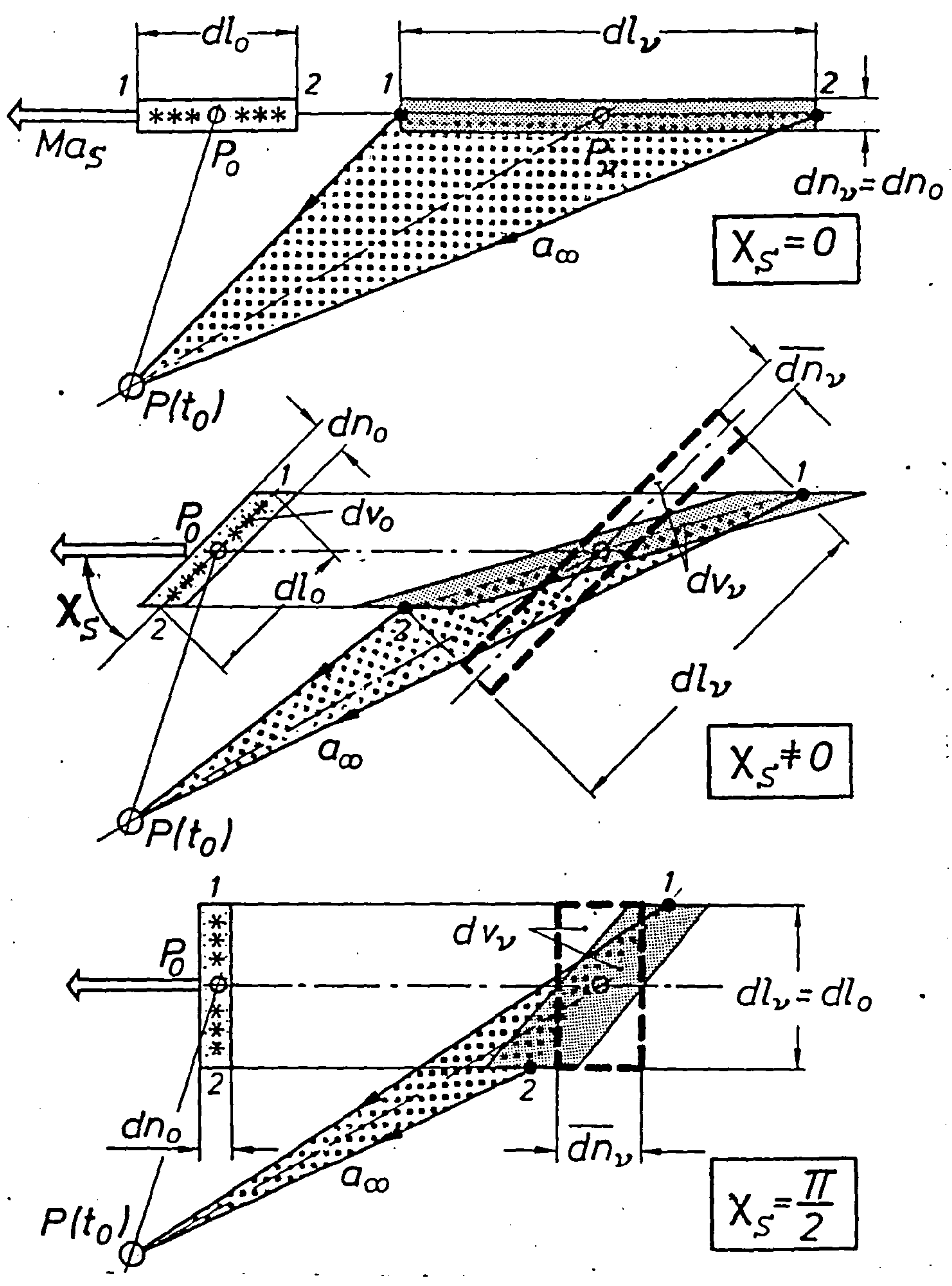

Fig. 5: Production of the Spatial Correlation Functions σ_V as a Result of the Actual Volume Expansion of the Emission Elements in the Case of Various Orientations of the Disturbance source in Space or to the Path Line ($\sigma_V = \sigma_L \; \sigma_N$).

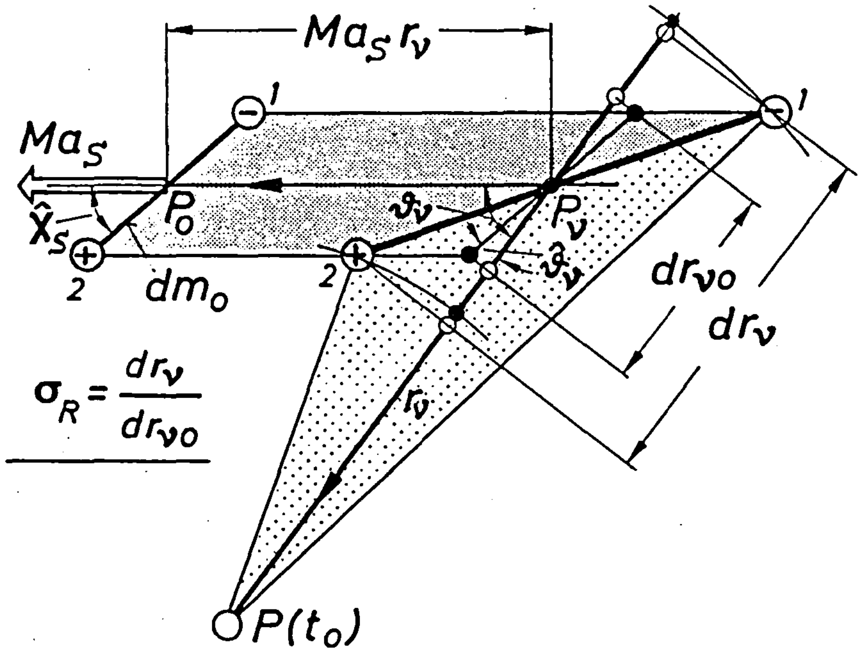

Fig. 6: Production of the Spatial Correlation Function σ_R as a Result of the Actual Shift in Emission Points of Moving Source-Sink Elements of a Dipole, with the Disturbance Signals Reaching Field Point P Simultaneously at Time t_o

($\sigma_R = dr_\nu/dr_{\nu o}$).

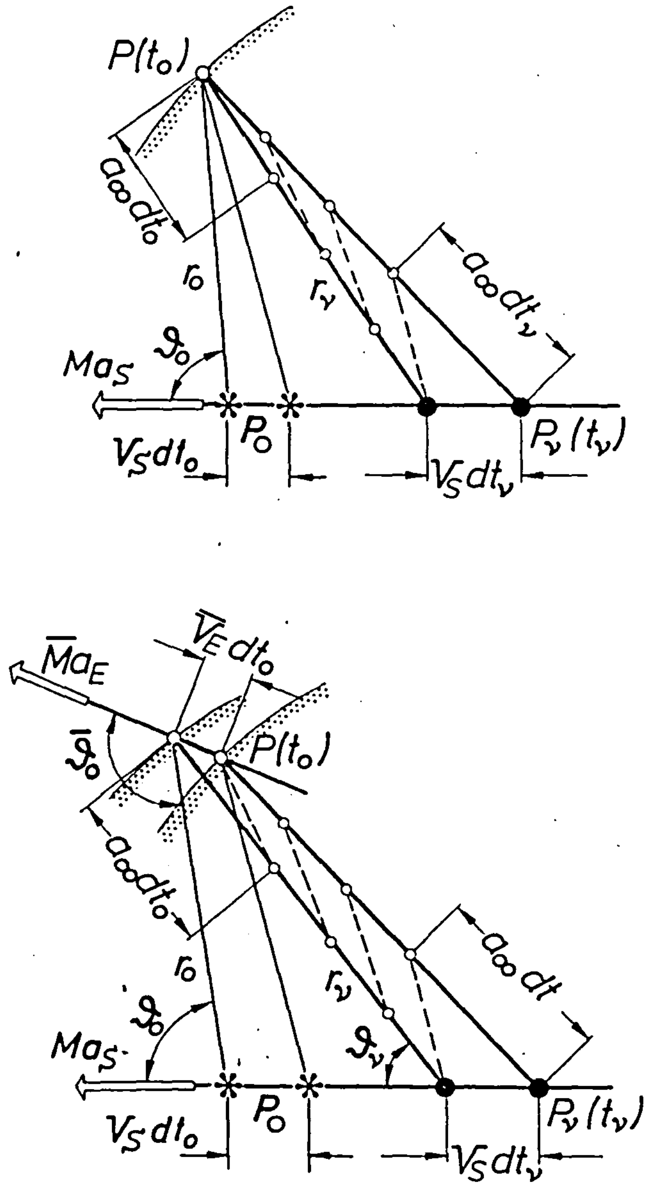

Fig. 7: Production of the Doppler Factor σ_D and σ_D^* as a Result of the Change in Wave Passage Duration dt_o through a resting or moving Field Point in Comparison to Duration dt_ν of the Emission of Disturbance Signals.
$(\sigma_D,\ \sigma_D^* = dt_\nu/dt_o)$.

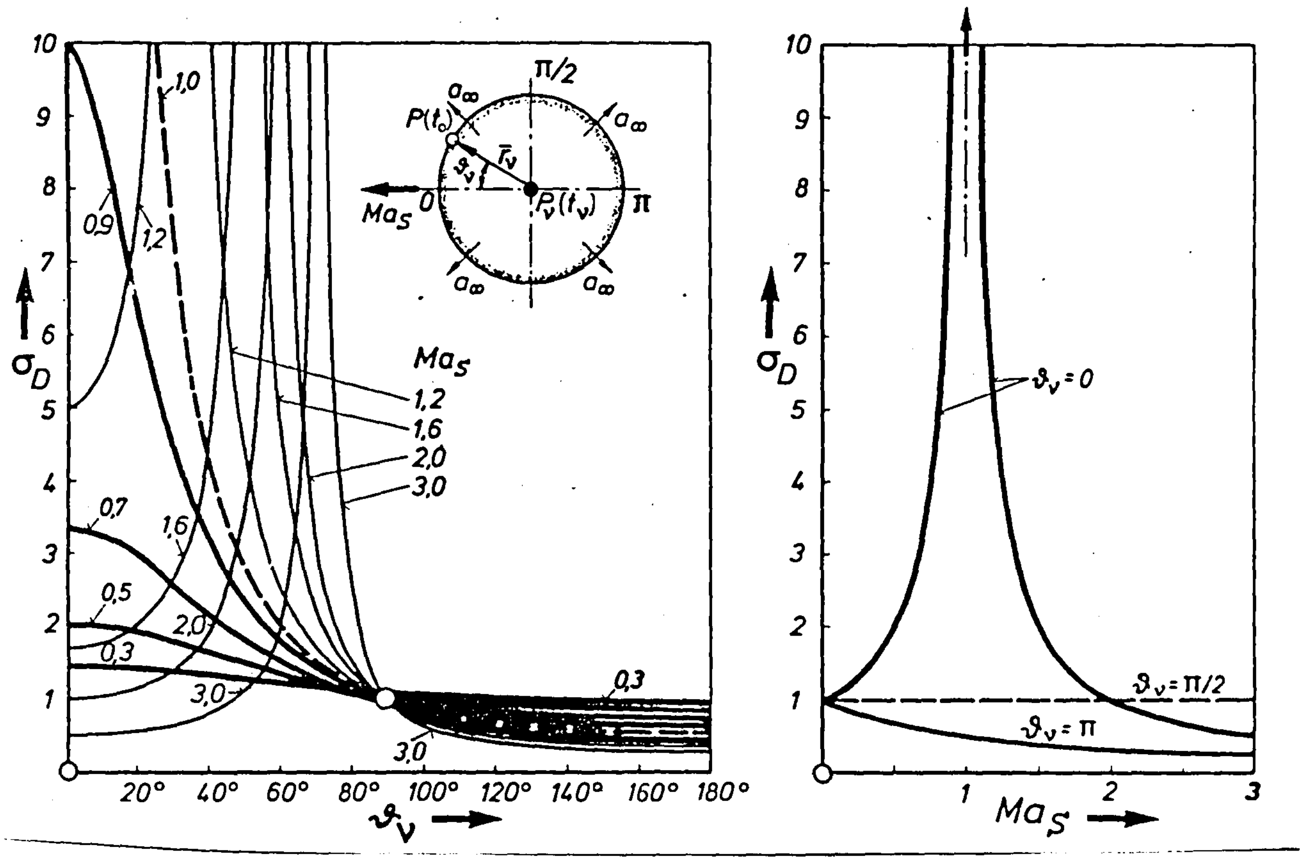

Fig. 8: The Doppler Factor σ_D in the Propagation of Spherically Symmetrical Waves from Moving Disturbance Sources in a Homogeneous Medium at Rest.

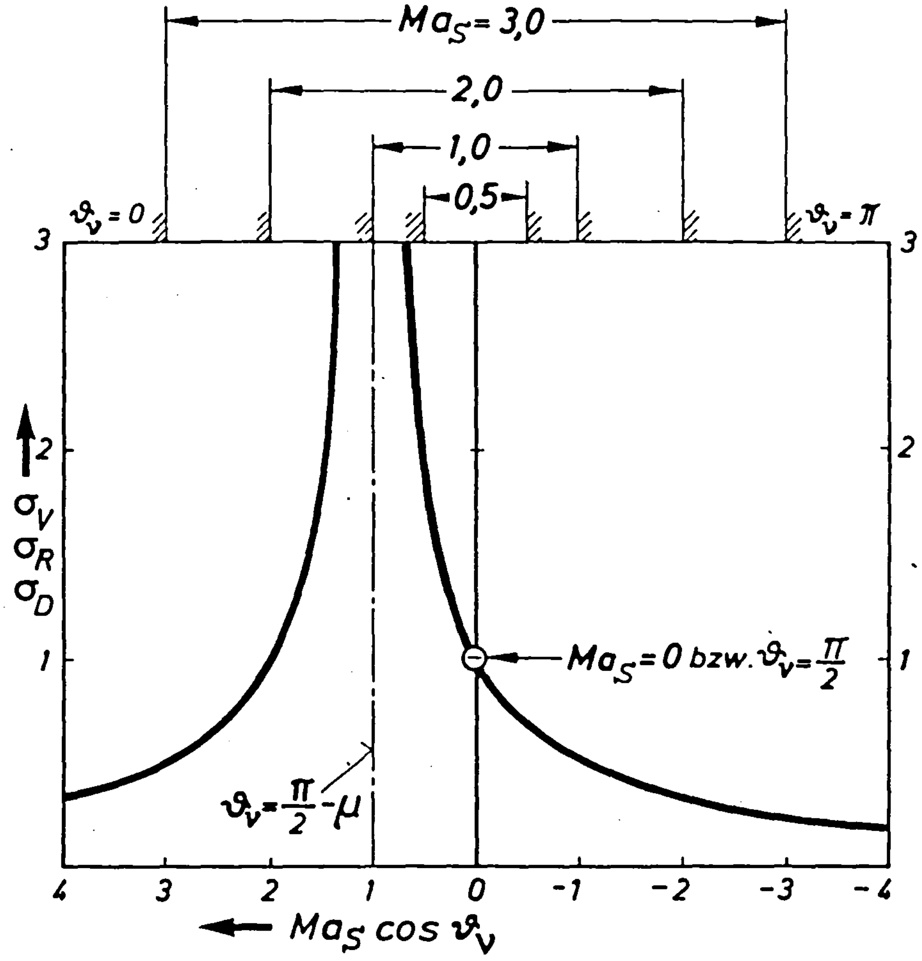

Fig. 9: A Universal Representation of the Correlation Functions σ_V, σ_R and σ_D in the Propagation of Spherically Symmetrical Disturbance Waves from Moving Singularities.

/56

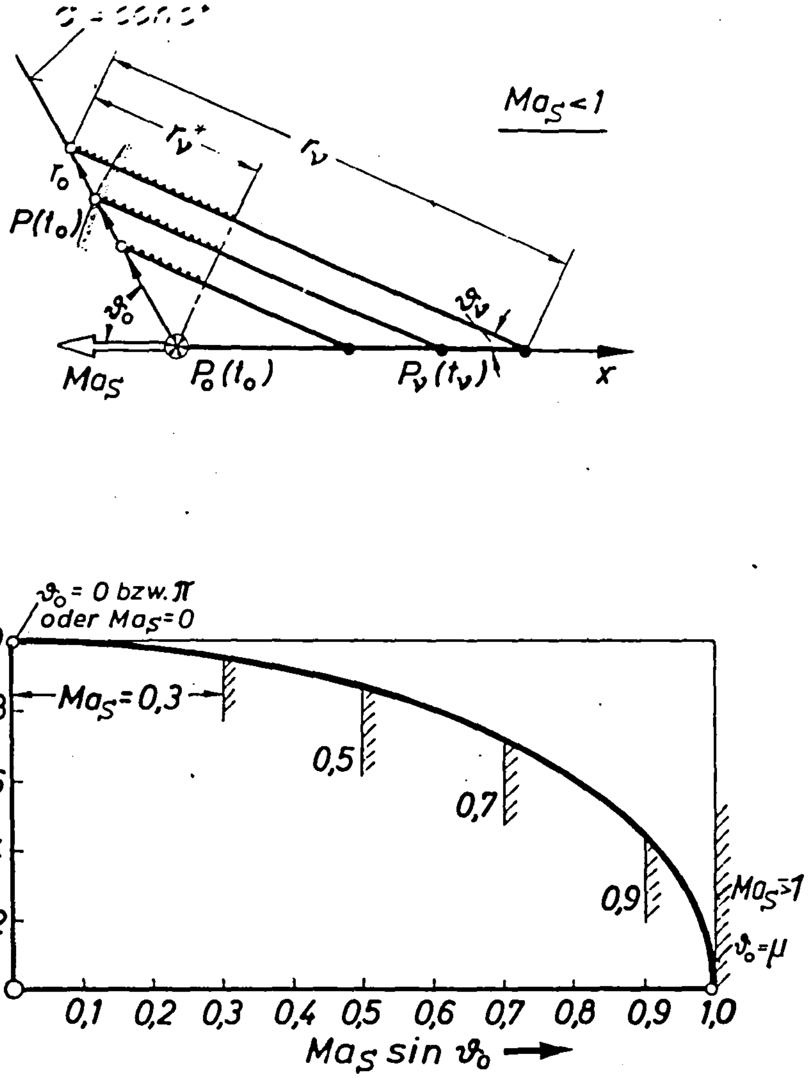

Fig. 10: Universal Representation of the Actual Emission Radii r_ν^*/r_o for Random Field Points in Space with Subsonic and Supersonic Mach Numbers of the Disturbance Sources.

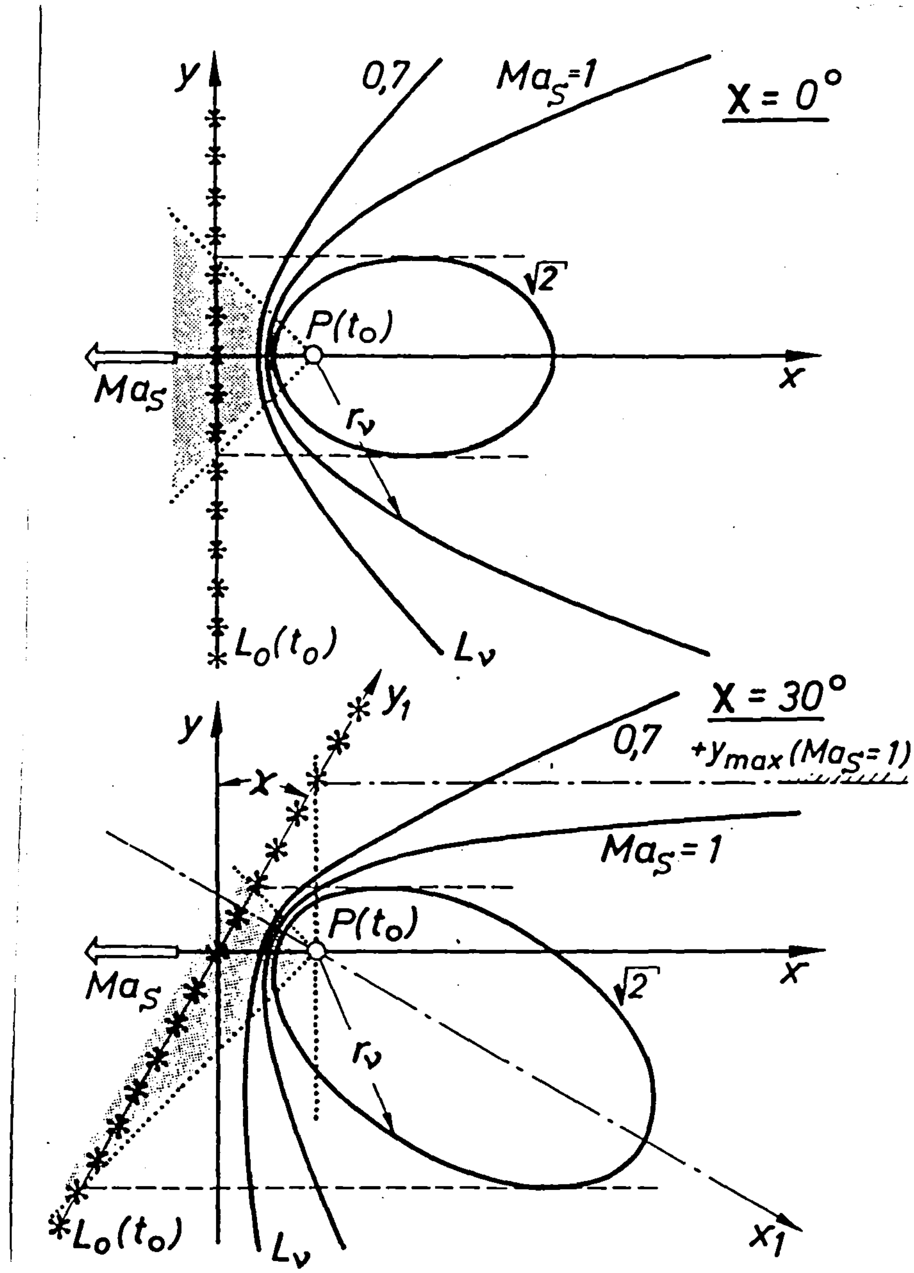

Fig. 11: Radiation Processes of Moving Source Lines of Infinite Length with Various Mach Numbers and Various Orientations to the Path Line with the Disturbance Signals from the L_ν lines Reaching the Field Point P(x,0,0) Simultaneously at time t_o.

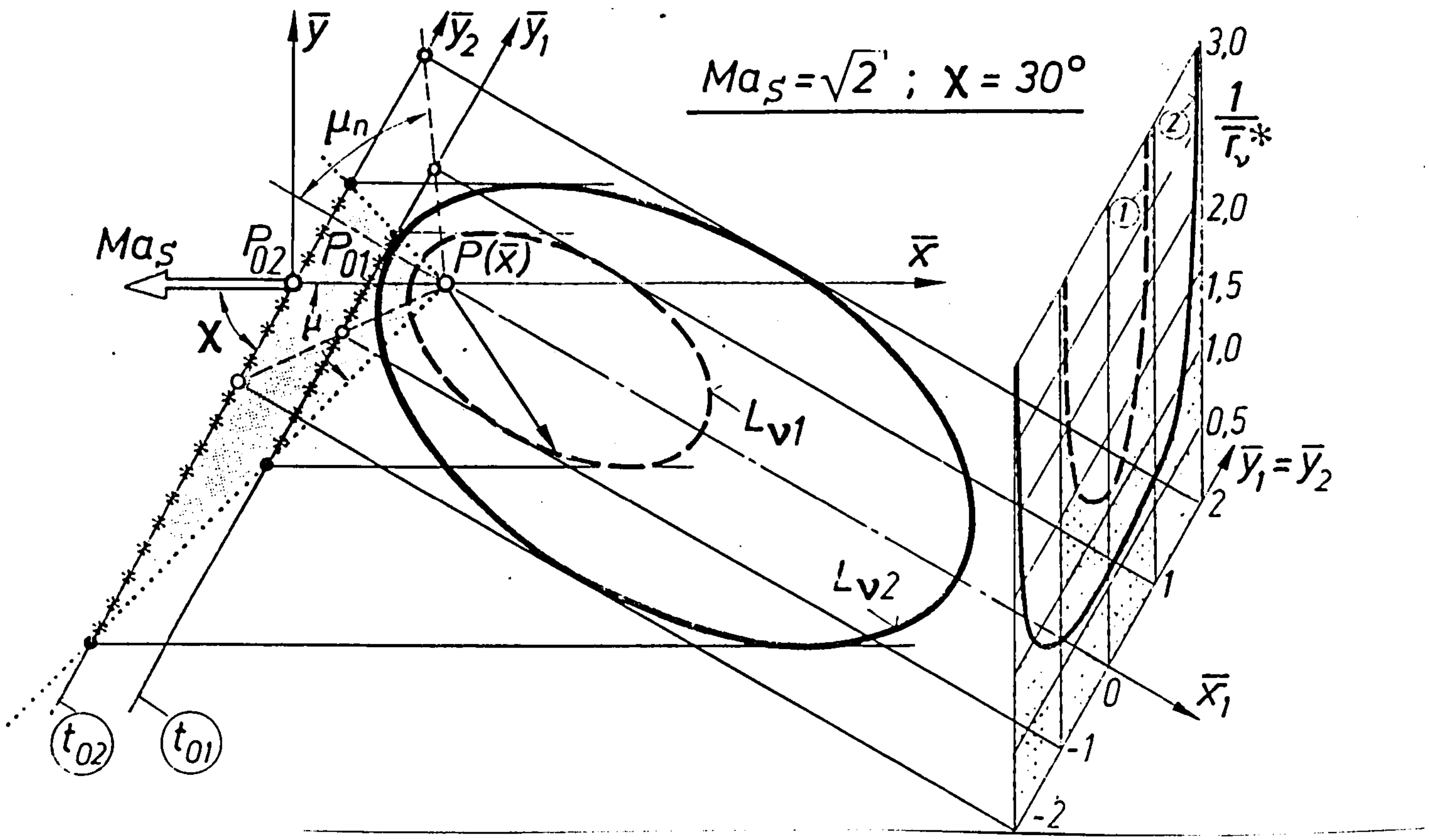

Fig. 12: Radiation Processes from a Moving Source Line in the Case of Supersonic Mach Numbers and Correlated Emission Lines $L_{\nu 1}$ and $L_{\nu 2}$ with Disturbance Signals reaching Field Point P at Times t_{o1} and t_{o2} and always producing Constant Field Quantities for $\bar{x} \geq x_o$.

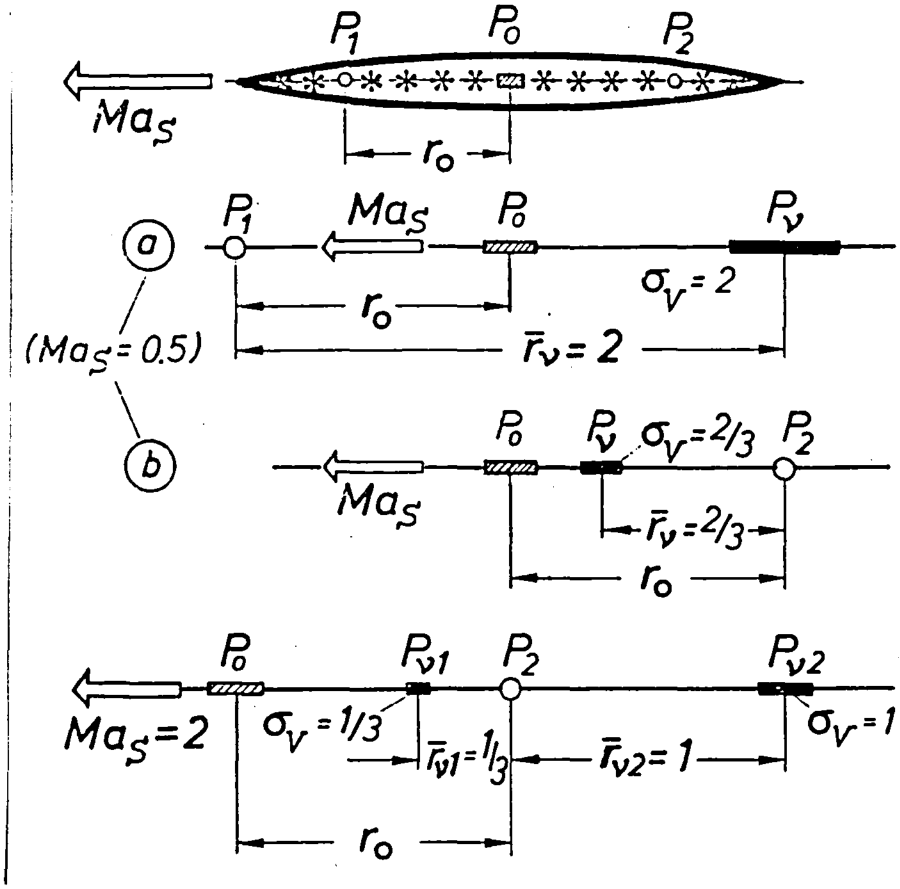

Fig. 13: Radiation Processes of Moving Source Elements in Field Points on the Orbit and the Physical Phenomena, Leading to the Exact Cancellation of the Effect of Mach Number in the Field Quantities ($\bar{r}_\nu = r_\nu / r_o = \sigma_V$).

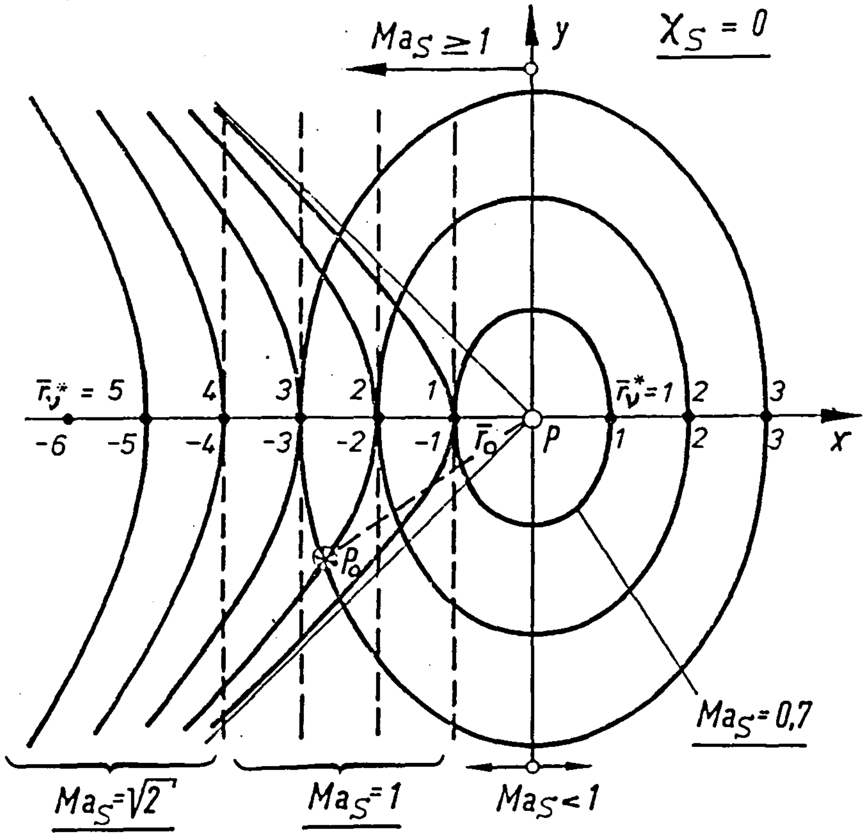

Fig. 14: Surfaces in Space with Constant Values of r_ν^* at Subsonic, Sonic and Supersonic Velocities of the Disturbance Sources with the Axis x of Symmetry Through the Field Point P.

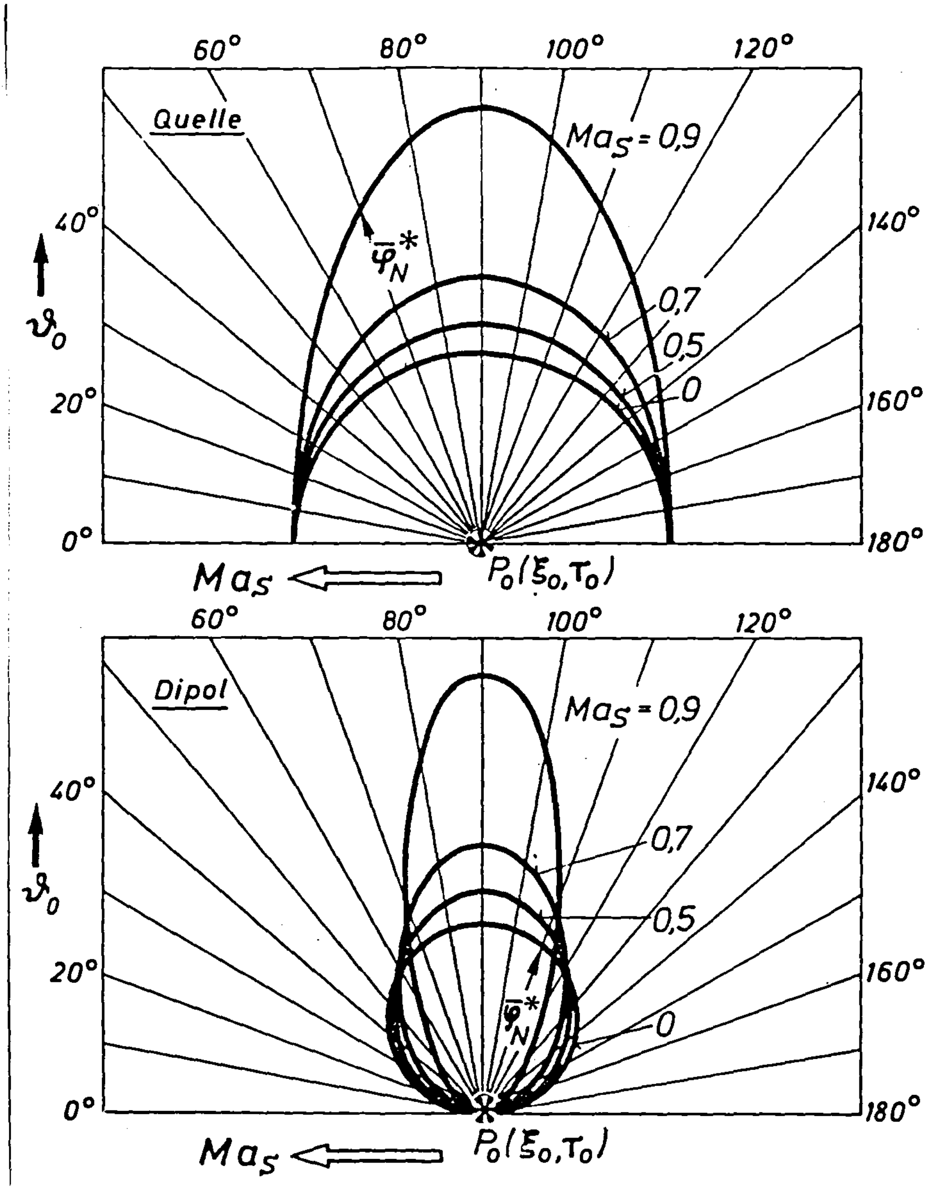

Fig. 15: Near Field Protions of the Disturbance Potential on a Unit Sphere with $r_o = 1$ around ξ_o with Various Mach Numbers of a Moving Source and a Moving Dipole.
($\overline{\Phi}_N^* = \overline{\Phi}_N/\overline{S}$ or $\overline{\Phi}_N/\overline{D}\ \cos\psi$).

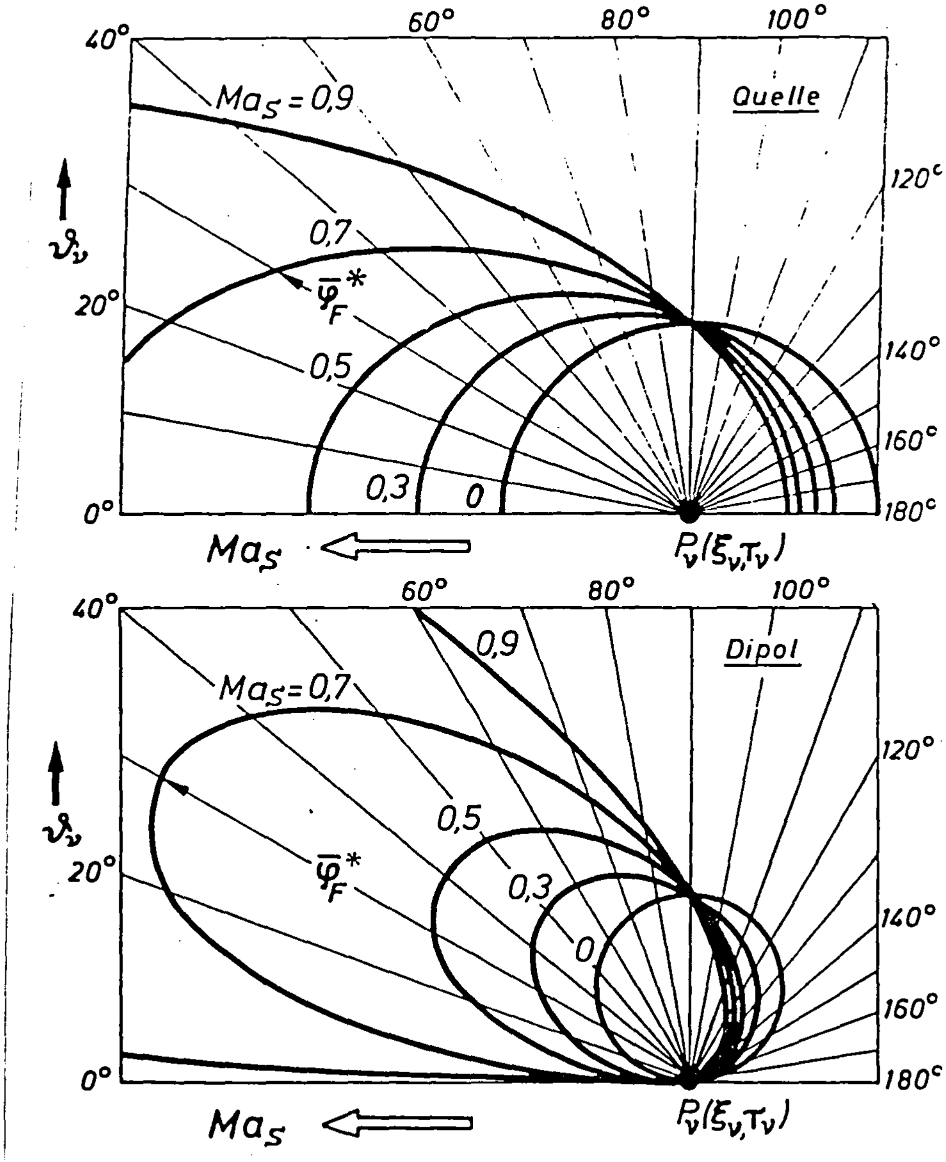

Fig. 16: Distant Field Portion of the Disturbance Potential in the Disturbance Waves with $\overline{r}_\nu = 1$ around ξ_ν at Various Mach Numbers of a Moving Source and a Moving Dipole.
($\overline{\phi}_F^* = \overline{\phi}_F/\overline{S}$ or $\overline{\phi}_F/\omega_\nu \overline{D}\cos\psi$).

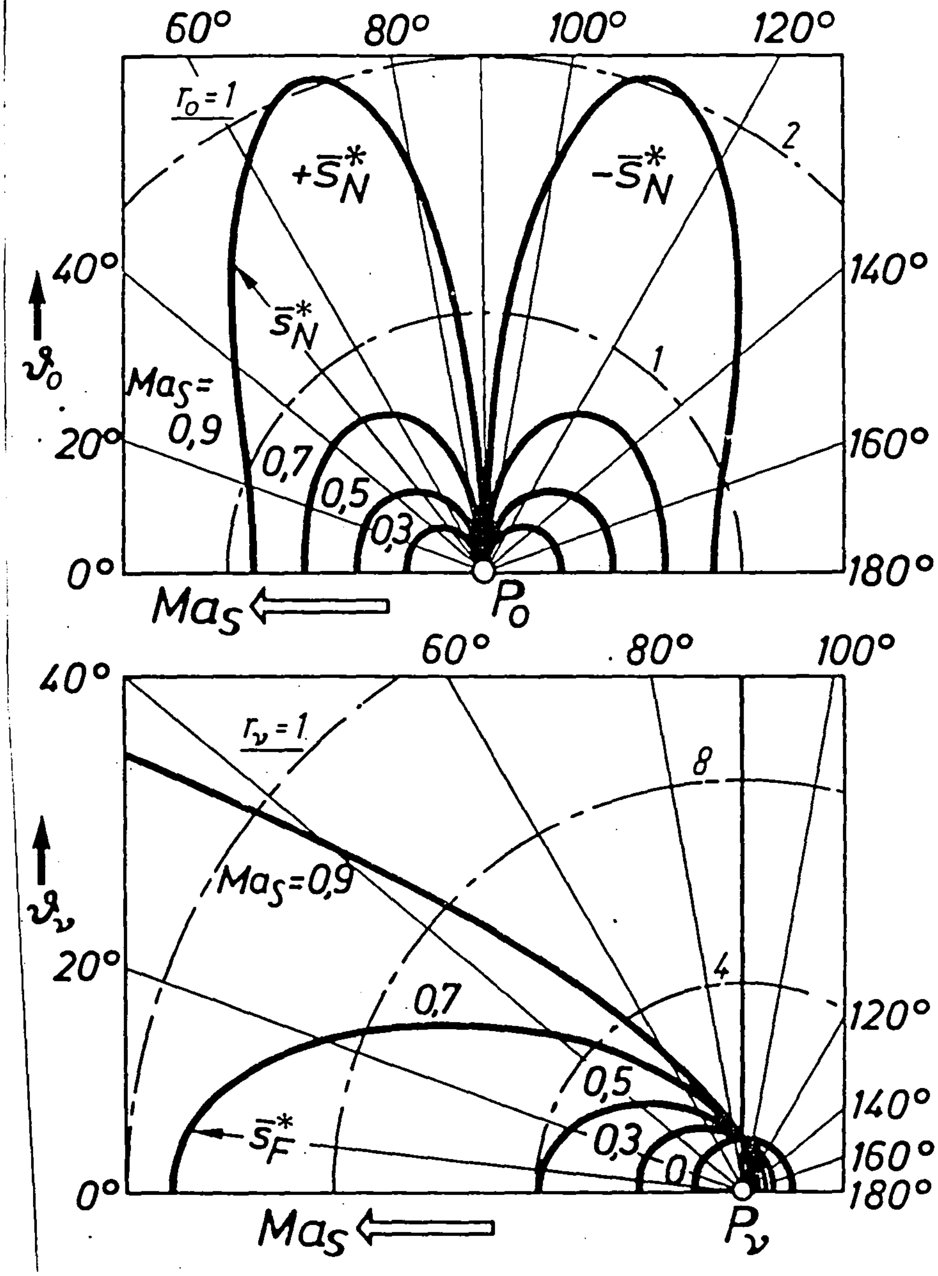

Fig. 17: Near Field and Distant Field Portions of the Disturbance Quantities on a Unit Sphere with $\bar{r}_o = 1$ around ξ_o or with $\bar{r}_\nu = 1$ around ξ with Various Mach Numbers of a Moving Source. ($\bar{s}_N^* = \bar{s}_N/\omega_\nu \bar{S}$ or $\bar{s}_F^* = \bar{s}_F/\omega_\nu^2 \bar{D}\cos\psi$).

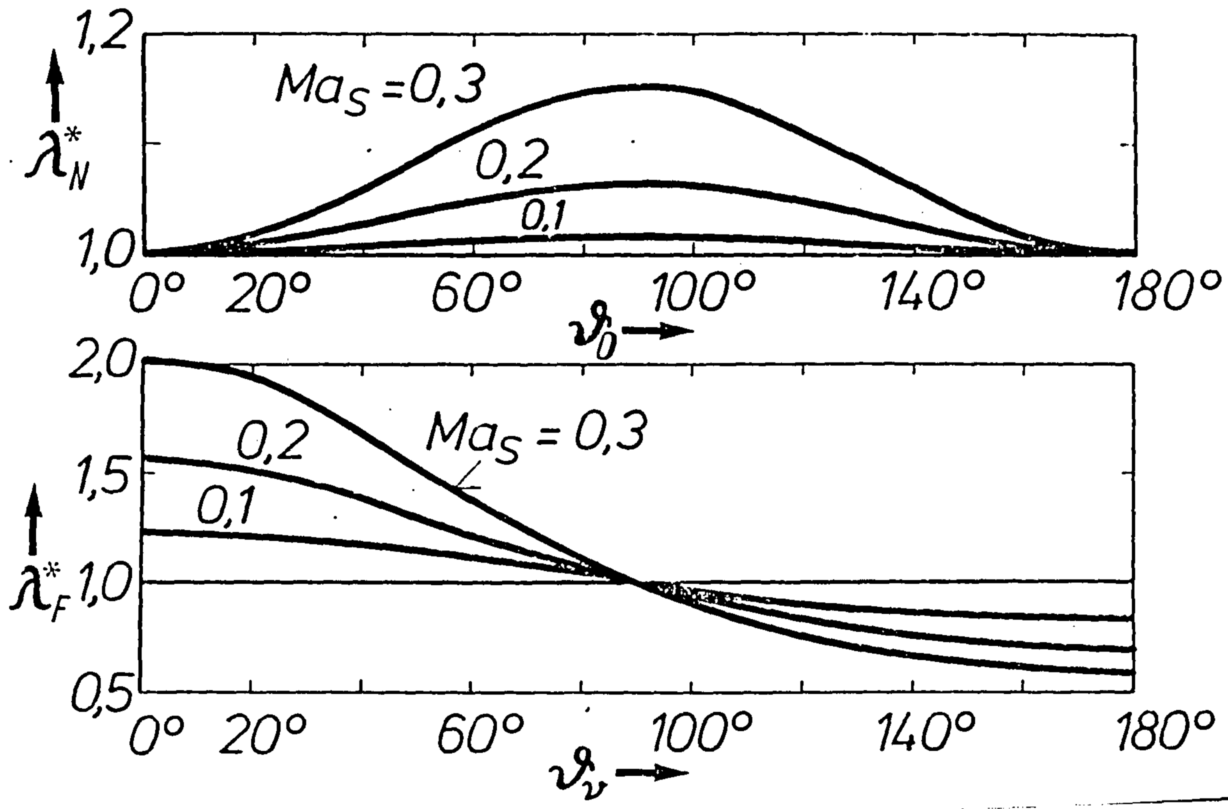

Fig. 18: The Ratio of the Nonsteady to the Quasi-Steady Field Quantities in the Near Field and Distant Field Portions of a Moving Source ($\lambda^* = \bar{s}_{nonst.}/\bar{s}_{q.st.}$).

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