FEEDBACK LAWS FOR FUEL MINIMIZATION FOR TRANSPORT AIRCRAFT

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The Theoretical Mechanics Branch has as one of its long-range goals to work toward solving real-time trajectory optimization problems on board an aircraft. This is a generic problem that has application to all aspects of aviation from general aviation through commercial to military. Our overall interest is in the generic problem, but we must focus on specific problems to achieve concrete results. The problem is to develop control laws that will generate approximately optimal trajectories with respect to some criteria such as minimum time, minimum fuel, or some combination of the two. These laws must be simple enough to be implemented on a computer that can be flown on board an aircraft, which implies a major simplification from the two-point boundary value problem generated by a standard trajectory optimization problem. In addition, the control laws must allow for changes in end conditions during the flight, and changes in weather along a planned flight path. Therefore, a feedback control law that generates commands based on the current state rather than a precomputed open-loop control law is desired. This requirement, along with the need for order reduction, argues for the application of singular perturbation techniques.

- Solve Trajectory Optimization Problems on board the aircraft in real time
- Allow for changing conditions
  - Weather
  - Air traffic control changes
- Feedback control law
- Singular perturbation techniques

Singular perturbation techniques can sometimes be used to break a big problem down into more manageable parts. For example, a large-order numerical optimization problem can frequently be divided into a series of smaller subproblems that can be solved one by one in a serial fashion. The solutions to these subproblems are then combined to generate an approximation to the solution of the original large-order problem. This technique is very valuable and has been used successfully in a number of different areas. The validity of the technique depends on a separation of time scales for (or a decoupling of) the various states involved in the problem. For some problems, this separation of the states occurs naturally, and may even be made obvious by one or more small parameters of the problem. For flight problems, some of the time-scale separations are fairly easy to find and agree upon, but others are controversial at best. Another problem with the technique is that while stable feedback laws can be generated for the initial boundary layers which correspond to the ascent portion of a trajectory, the feedback laws are unstable in the descent portion.

- Subdivide large-scale numerical optimization problem into series of smaller subproblems
- Solve subproblems serially then put solutions together to approximate solution to original problem
- Difficulties with the technique:
  - Time-scale ordering / separation
  - Terminal boundary layers - descent

FUEL OPTIMAL TRANSPORT PROBLEM

The particular problem to be discussed here is a fuel optimization problem for a transport aircraft in the vertical plane. The cost function which is to be minimized is the integral over the flight time of fuel flow rate f. The state vector consists of range x; total energy per unit weight E; altitude h; and flight path angle γ. The controls for the problem are thrust T; and lift L. The remaining parameters are velocity V; the force of gravity g; drag D; and weight W (which is considered a constant for the problem). The fuel flow rate is modelled as a quadratic in thrust with coefficients that are, in general, functions of energy and altitude. The drag is modelled as a quadratic in lift with coefficients that are also functions of energy and altitude. The ε's on the left-hand sides of the state equations are "small" numbers that determine the ordering and separation of the states in the singular perturbation formulation of the problem.

Minimize $$J = \int_{t_0}^{t_f} f(h, E, T)\, dt$$

subject to:

$$\dot{x} = V \cos \gamma$$

$$\varepsilon_1 \dot{E} = (T - D) V / W$$

$$\varepsilon_2 \dot{h} = V \sin \gamma$$

$$\varepsilon_3 \dot{\gamma} = g(L - W \cos \gamma) / W V$$

where

$$f = \alpha_0(E,h) + \alpha_1(E,h) T + \alpha_2(h) T^2$$

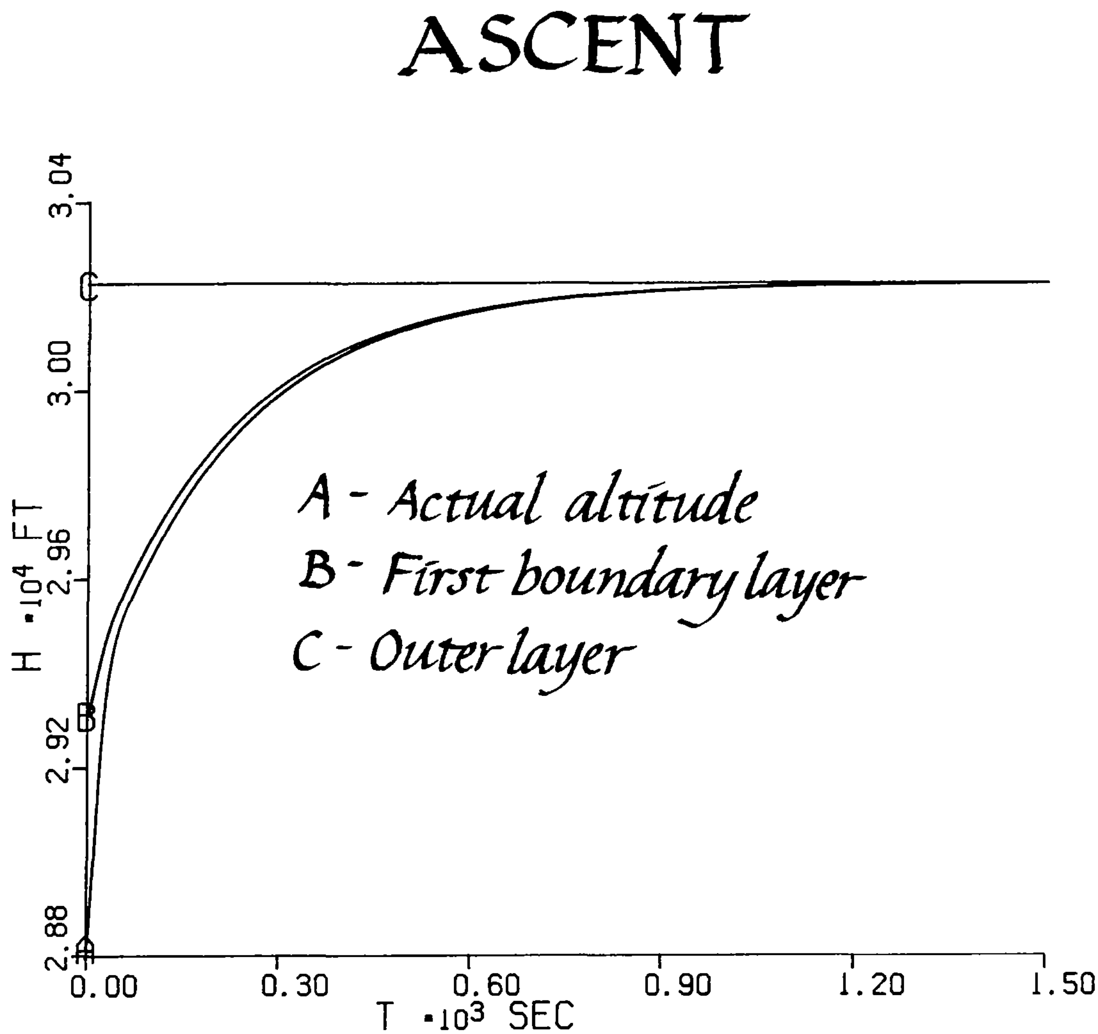
$$D = \beta_0(E,h) + \beta_1(E,h) L + \beta_2(E,h) L^2$$

SEPARATION OF STATES

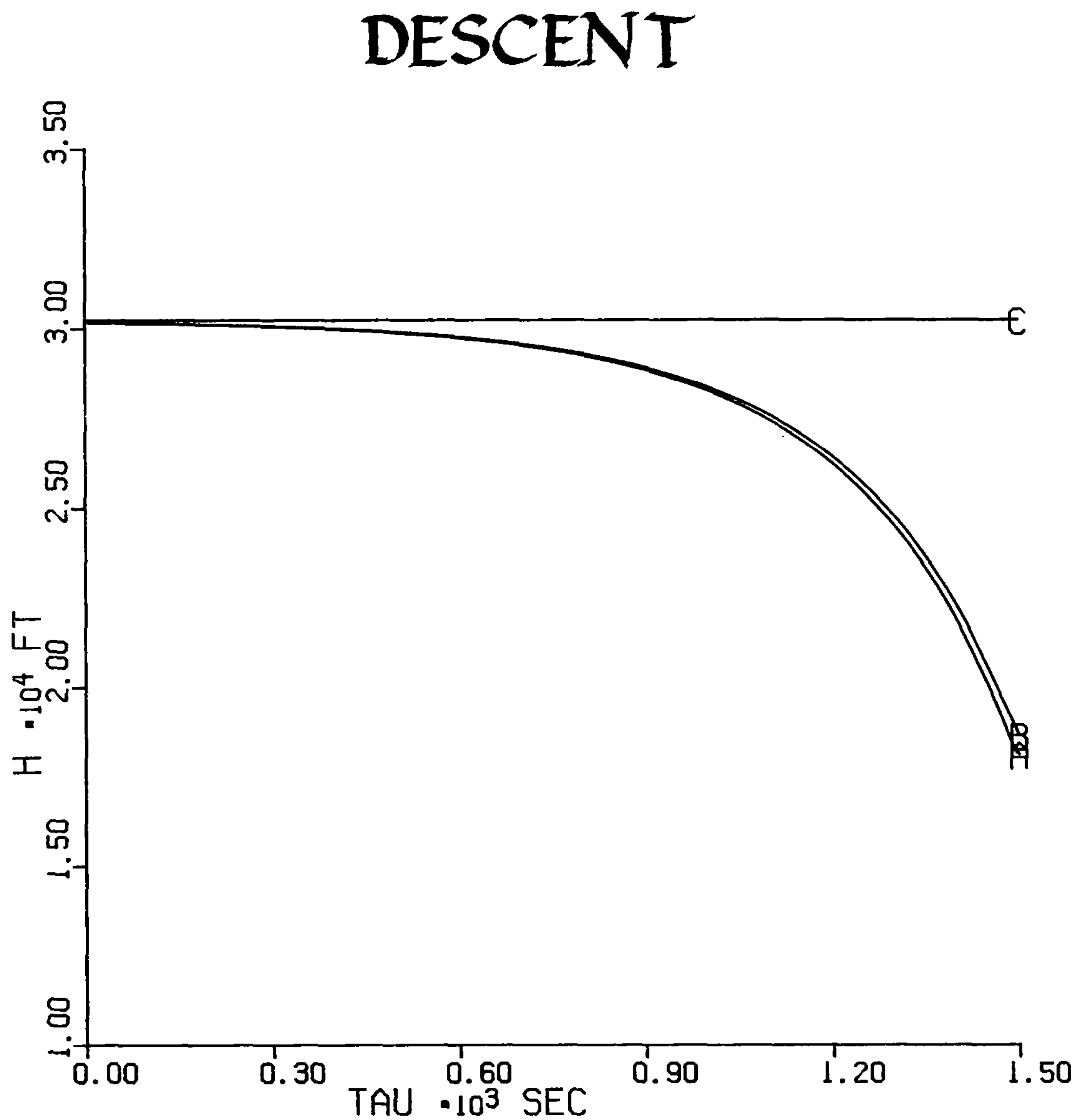
One way to approach the question of the separation of the various states in the problem is to assume that they can be separated into mutually exclusive time scales, each consisting of one state. This is, of course, ad hoc and does not bother with the realities of the aircraft dynamics, but has the virtue that it makes the equations easy to solve. The resulting feedback control law is easy to implement, at least for the initial boundary layers. The modelling of the aircraft dynamics under this assumption is unsatisfactory because altitude and flight path angle are highly coupled. An alternate approach is to recognize this coupling and separate the states into three groups: range as the outer layer, energy as the first boundary layer, and altitude and flight path angle as the second boundary layer. The equations for altitude and flight path angle are linearized about the solution from the first boundary layer subproblem so that a feedback solution can still be obtained.

- "Straight" Singular Perturbations – separate layers for altitude and flight path angle
  - $\varepsilon_3 \ll \varepsilon_2$
  - leads to implementable feedback law
  - unsatisfactory modelling of A/C dynamics
- Linearize altitude/flight-path angle subproblem about solution from first boundary layer
  - $\varepsilon_3 = \varepsilon_2$
  - Implementable feedback law
  - good model of dynamics

This figure shows a plot of the solution for altitude from the separate boundary layers discussed on the previous slide for a trajectory with initial altitude at point A. The curve labeled C is the altitude solution from the outer layer and represents the cruise altitude for a transport trajectory. It is a horizontal line because the outer layer solution is a constant altitude cruise. The curve labeled B represents the first boundary layer solution for altitude. It converges to the cruise equilibrium nicely but does not meet the initial condition. The actual altitude for the trajectory comes from the second boundary layer solution and is shown as the curve starting at A. It meets the initial condition and approaches the first boundary layer solution as they both converge to the cruise altitude.



The difficulty with singular perturbation techniques for descent trajectories can be shown in this figure which shows the altitude resulting from a singularly perturbed solution for a descent. The horizontal line again represents the cruise altitude and is an equilibrium for the boundary layer equations. The other two curves are the altitude solutions from the first and second layer subproblems computed in the backward direction from the endpoint. However, for descent (and for any terminal boundary layer), the trajectory is moving away from the stable equilibrium as time increases. For this reason, the feedback law that was stable for ascent is unstable for descent. Therefore, any inaccuracies at the beginning of or along the trajectory will result in very large errors at the endpoint. The only reliable way to use the same control law for descent as ascent is to precompute the descent in the negative direction from the desired endpoint. This precludes taking into account any changes in the end conditions after the descent is initiated.



ANOTHER APPROACH FOR DESCENT

In order to accommodate the terminal part of a trajectory while maintaining commonality with the analysis used so far, it was decided to change the problem statement by modifying the cost function to include an altitude term multiplied by an adjustable coefficient. The cost function becomes the integral of a convex combination of the fuel flow rate, as before, and the additional altitude term. When the weighting parameter k is zero, this is the original cost function for the problem. Values of k between 0 and 1 change the equilibrium altitude for the problem to a value lower than that for the original cruise. Thus, the aircraft can be made to descend by changing the parameter k to a nonzero value. The desirable property of always approaching a stable equilibrium is maintained with this technique, and it becomes just an extension of the technique used for ascent. The cost function is no longer that for a fuel optimal problem when k is not zero, but the fuel flow rate is still a part of the cost function. This cost function represents a trade-off between fuel optimization and simplicity of the overall control law.

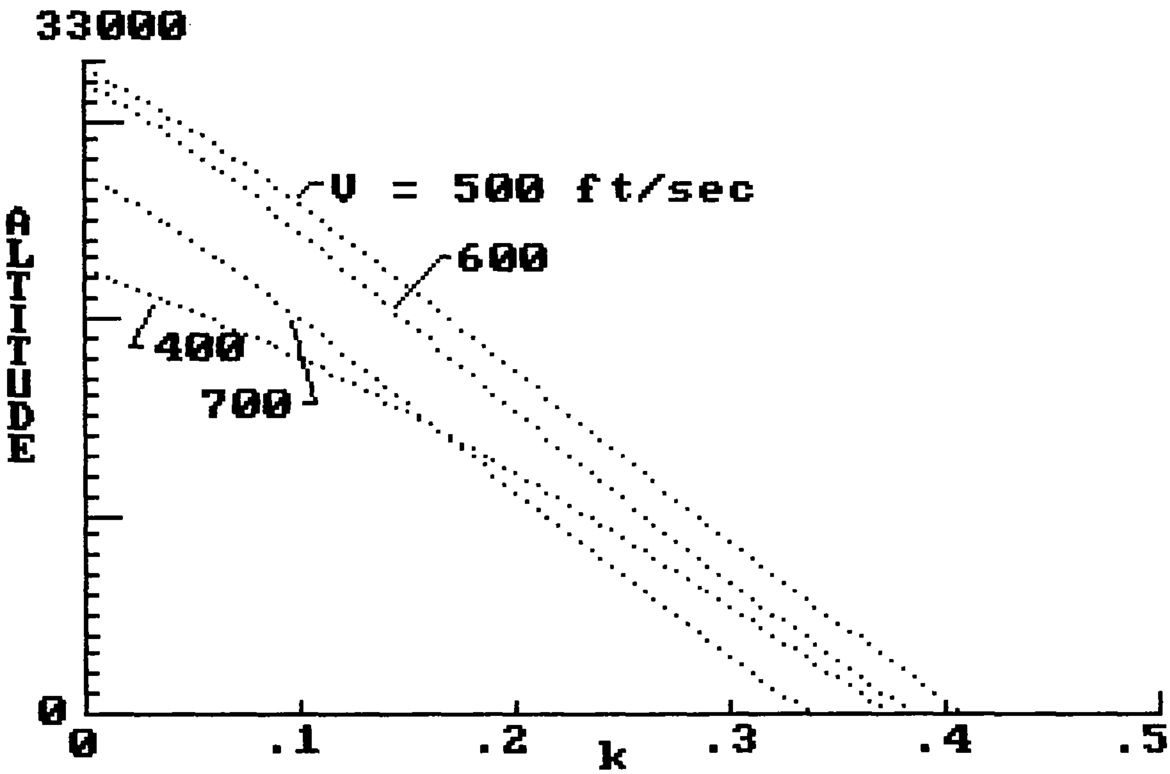
- Change problem to get stable feedback law for descent trajectory
- Change cost function – add altitude term

$$J = \int_{t_0}^{t_1} [(1-k)\, f(E,h,T) + kh]\, dt$$

- Different values of k give different values for equilibrium altitude

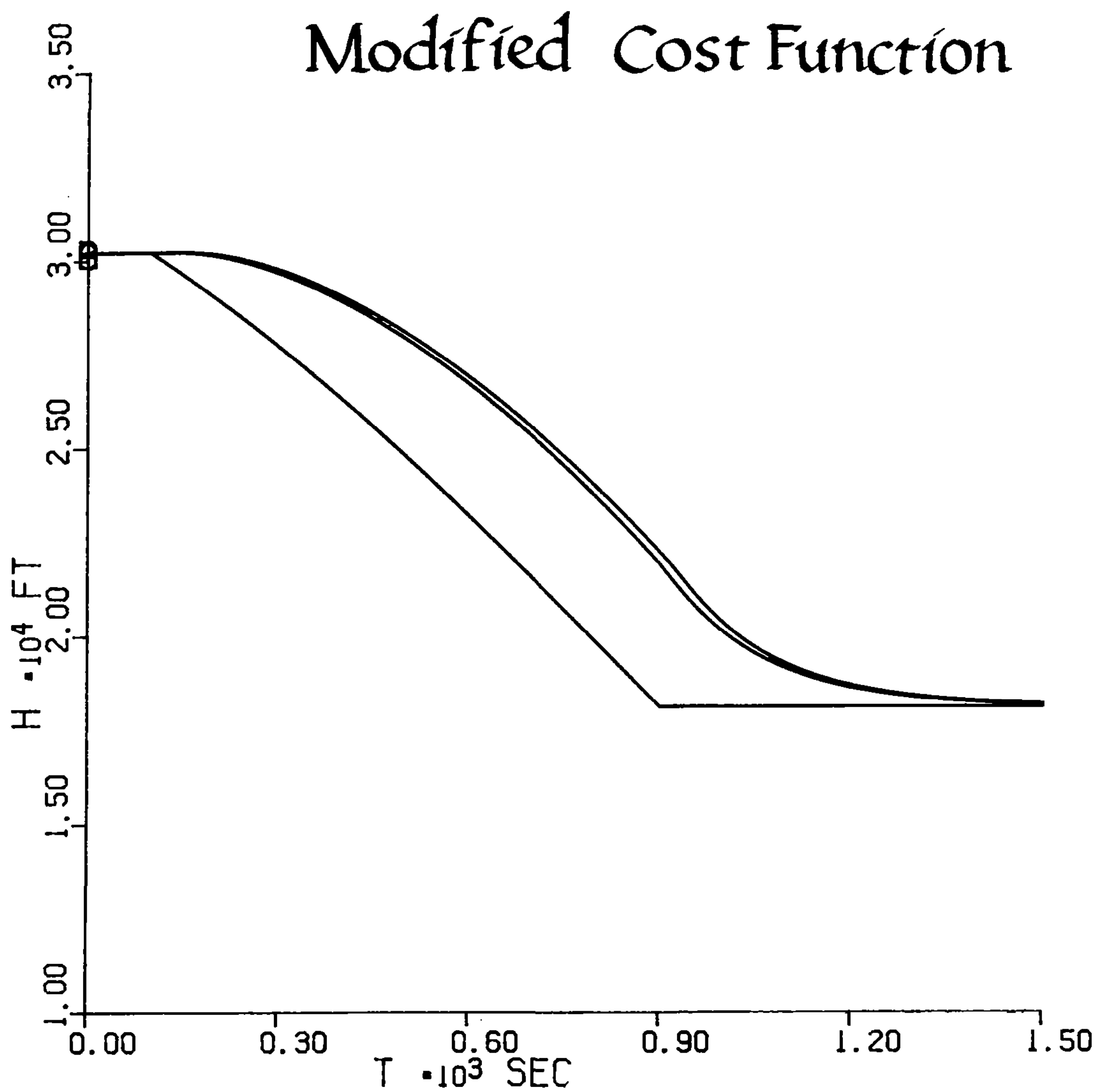
EQUILIBRIUM ALTITUDES

This figure demonstrates that the equilibrium altitude may be changed in an almost linear fashion by changing the constant k multiplying altitude in the cost function. The equilibrium altitude for this problem is a function of cruise velocity. The outer layer subproblem (with k = 0) consists of determining the cruise altitude that corresponds to a given cruise velocity. The choice of cruise velocity must be made from other considerations. This figure shows equilibrium altitudes for four different cruise velocities plotted against the constant k. It can be seen that as k varies butween 0 and .4, the equilibrium altitude changes from the fuel optimal altitude to the ground. The variation for each value of velocity is nearly linear, but the change with velocity is obviously nonlinear. These curves were generated by solving the outer layer subproblem for four different values of cruise velocity for different values of the parameter k.



DESCENT TRAJECTORY

This figure shows a descent trajectory generated by the technique described previously. It is a plot of altitude vs time with three different curves plotted. The curve with the straight line segments represents the altitude from the outer layer subproblem. It starts at the fuel optimal cruise level with k = 0. At 100 seconds into the flight, k is varied linearly from 0 to .15, and the equilibrium altitude follows it down almost in a straight line. The other two curves are the altitude from the first and second boundary layers. The secondary boundary layer altitude represents the actual altitude achieved by the simulated aircraft using the feedback control law under discussion. By changing the way the parameter k is varied, descents with different characteristics can be achieved. One common characteristic is that the bottom of the descent is always an exponentially stable approach to the new equilibrium altitude.



COMMENTS

The general area of singular perturbation techniques has been shown to offer a good framework for on-board optimal trajectory control. The large numerical problem of computing optimal trajectories requires some simplification and order reduction in order to hope for an on-board solution. The haphazard use of order reduction techniques without considering the implications of separating states which may actually change on the same time scale can lead to control laws based on an improper model of flight dynamics. In particular, altitude and flight path angle must be considered on the same time scale for the fuel optimal transport problem, since they are highly coupled. By linearizing the altitude and flight path angle equations about the solution to the first boundary layer subproblem, they can be considered on the same time scale, and a feedback control law can be developed that accurately reflects the dynamics of the aircraft.

A major problem with singular perturbation techniques has been the inability to derive stable feedback laws for terminal layers without precommuting the terminal boundary layer trajectory. It has been shown that by adding an altitude term to the cost function with a variable multiplier, feedback laws can be generated that always fly toward a stable equilibrium. These laws, though ad hoc in nature, can be used to approximate optimal trajectories. A nice feature of this technique is that the control law used for ascent is continued throughout the trajectory with the only change for descent being a nonzero multiplier on the altitude term.

Singular perturbation techniques offer a good framework for on-board optimal trajectory control

Altitude and flight-path angle must be considered on same time scale

Altitude term in cost function leads to feedback law that is good for whole trajectory

Need optimal trajectories for comparisons