EIGENSPACE DESIGN TECHNIQUES FOR ACTIVE FLUTTER SUPPRESSION

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OBJECTIVE

The objective of this discussion is to examine the application of eigenspace design techniques to an active flutter suppression system for the DAST ARW-2 research drone. Eigenspace design techniques allow the control system designer to determine feedback gains which place controllable eigenvalues in specified configurations and which shape eigenvectors to achieve desired dynamic response. Eigenspace techniques have been applied to the control of lateral and longitudinal dynamic response of aircraft [1,2]. However, little has been published on the application of eigenspace techniques to aeroelastic control problems.

This discussion will focus primarily on methodology for design of full-state and limited-state (output) feedback controllers. We do not intend to address the significant difficulties associated with the realization of full- and limited-state controllers. Most of the states in aeroelastic control problems are not directly measurable, and some type of dynamic compensator is necessary to convert sensor outputs to control inputs. Compensator design can be accomplished by use of a Kalman filter modified if necessary by the Doyle-Stein procedure for full-state loop transfer function recovery [3], by some other type of observer, or by transfer function matching.

EIGENSPACE DESIGN TECHNIQUES

Eigenspace techniques allow the designer to place closed-loop eigenvalues (λ_i) and shape closed-loop eigenvectors (v_i). We will briefly review the theory. For a more detailed discussion see Refs. 1, 2, and 4. First we assume the system is controllable and observable and the matrices B and C are of full rank. (In the case of full-state feedback, C = I.) Later the controllability assumption will be relaxed. The above assumptions yield the results shown below. If we have full-state feedback and n controls, we can arbitrarily place all eigenvalues and shape all eigenvectors to any desired form. If we have full-state feedback and a single control, only pole placement is possible. Since any attainable eigenvector is in the subspace spanned by $(\lambda I-A)^{-1}B$, it is impossible to exactly achieve a desired eigenvector in most aircraft control problems. In practice this does not appear to be a serious problem.

Uncontrollable eigenvalues cannot be moved but an additional element in each eigenvector associated with these eigenvalues can be shaped.

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

Dim (x) = n

Dim (u) = m

Dim (y) = r

(1) max (m,r) closed-loop eigenvalues can be assigned

(2) max (m,r) closed-loop eigenvectors can be shaped

(3) min (m,r) elements of each eigenvector can be arbitrarily chosen

Attainable eigenvector in space spanned by

$$(I\lambda_i-A)^{-1}B$$

CALCULATION OF GAIN MATRIX-I

We will first describe the eigenspace design technique for full-state feedback for a system described in standard state-space form. The design procedure consists of determining a gain matrix K such that for all closed-loop eigenvalue and eigenvector pairs

$$(A+BK)v_i = \lambda_i v_i$$

where λ_i is the desired closed-loop eigenvector and v_i is the associated closed-loop eigenvector. This is equivalent to finding w_i such that

$$(I\lambda_i - A)\ v_i = Bw_i$$

Once all the w_i's have been found, the gain matrix can be calculated.

In order to arbitrarily place all the λ_i's and v_i's, the control vector will have to be of the same order as the state vector and B would have to be invertible. In general this is not the case, and the achievable eigenvalues must lie in the subspace spanned by

$$(\lambda_i I - A)^{-1}B$$

In general, the desired eigenvector v_i^d will not reside in this subspace.

$$\dot{x} = Ax + Bu$$

$$y = Cx$$

$$u = Ky$$

$$x = \sum_{i=1}^{n} a_i v_i e^{\lambda_i t}$$

$$(A + BK)v_i = \lambda_i v_i$$

$$(I\lambda_i - A)v_i = Bw_i$$

$$v_i = L_i w_i$$

$$L_i = (\lambda_i I - A)^{-1}B$$

$$K = W[CV]^{-1}$$

CALCULATION OF GAIN MATRIX-II

Since the desired eigenvalues are in general not achievable, the w_i's are selected to minimize the weighted sum of the squares of the difference between the elements of the desired and attainable eigenvectors given by the performance index J_i. The term P_i is a positive definite symmetric matrix whose elements can be chosen to weight the difference between certain elements of the desired and attainable eigenvalues more heavily than others. Setting the derivative of J_i with respect to w_i equal to zero gives w_i. The notation * denotes complex transpose. Once w_i is calculated, the achievable eigenvector v_i is obtained. If an eigensolution is not to be altered, setting $w_i = 0$ assures that the associated v_i and λ_i remain in their open-loop configurations. If we desire output feedback, the procedure is easily modified as shown.

In case of a complex eigenvalue, a real-gain matrix results from a simple transformation [1, 2, 4].

$$J_i = (v_i^d - v_i)^* P_i (v_i^d - v_i)$$

$$\frac{\partial J_i}{\partial w_i} = 0$$

$$w_i = (L_i^* P_i L_i)^{-1} L_i^* P_i v_i^d$$

If an eigensolution is not to be altered,

$$w_i = 0$$

UNCONTROLLABLE EIGENVALUES-I

In aeroelastic control problems, the states associated with the gust model are uncontrollable. Moore [4] showed that it is possible to use feedback to assign some components of eigenvectors associated with uncontrollable eigenvalues. An algorithm for performing such an assignment is given. For an uncontrollable eigenvalue λ_g,

$$[I\lambda_g - A]$$

is singular. We can partition the eigenvector v_g associated with this eigenvalue as shown below where v_g^{II} contains only the uncontrollable states.

λ_g uncontrollable

Partition v_g such that

$$[\lambda_g I - A] v_g = B w_g$$

Becomes

$$\left[\begin{array}{c|c} I\lambda_g - A^I & Q \\ \hline 0 & R \end{array}\right] \left[\begin{array}{c} v_g^{I} \\ \hline v_g^{II} \end{array}\right] = \left[\begin{array}{c} B^I \\ \hline 0 \end{array}\right] w_g$$

v_g^{II} contains only uncontrollable states

The equation

$$Rv_g^{II} = 0$$

is automatically satisfied if we select v_g^{II} to be equal to the open-loop portion of v_g which contains the uncontrollable states. Since λ_g is not an eigenvalue of A^I, $[\lambda_g I-A^I]$ is nonsingular, and by performing the indicated calculations, w_g can be determined in much the same way as for the controllable eigenvalues.

$$Rv_g^{II} = 0$$

$$v_g^I = [\lambda_g I-A^I]^{-1}B^I w_g - [\lambda_g I-A^I]^{-1} Pv_g^{II}$$

or

$$v_g^I = L_g w_g + \Delta v_g$$

minimizing

$$J_g = (v_g^{Id} - v_g^I)^* P_g (v_g^{Id} - v_g^I)$$

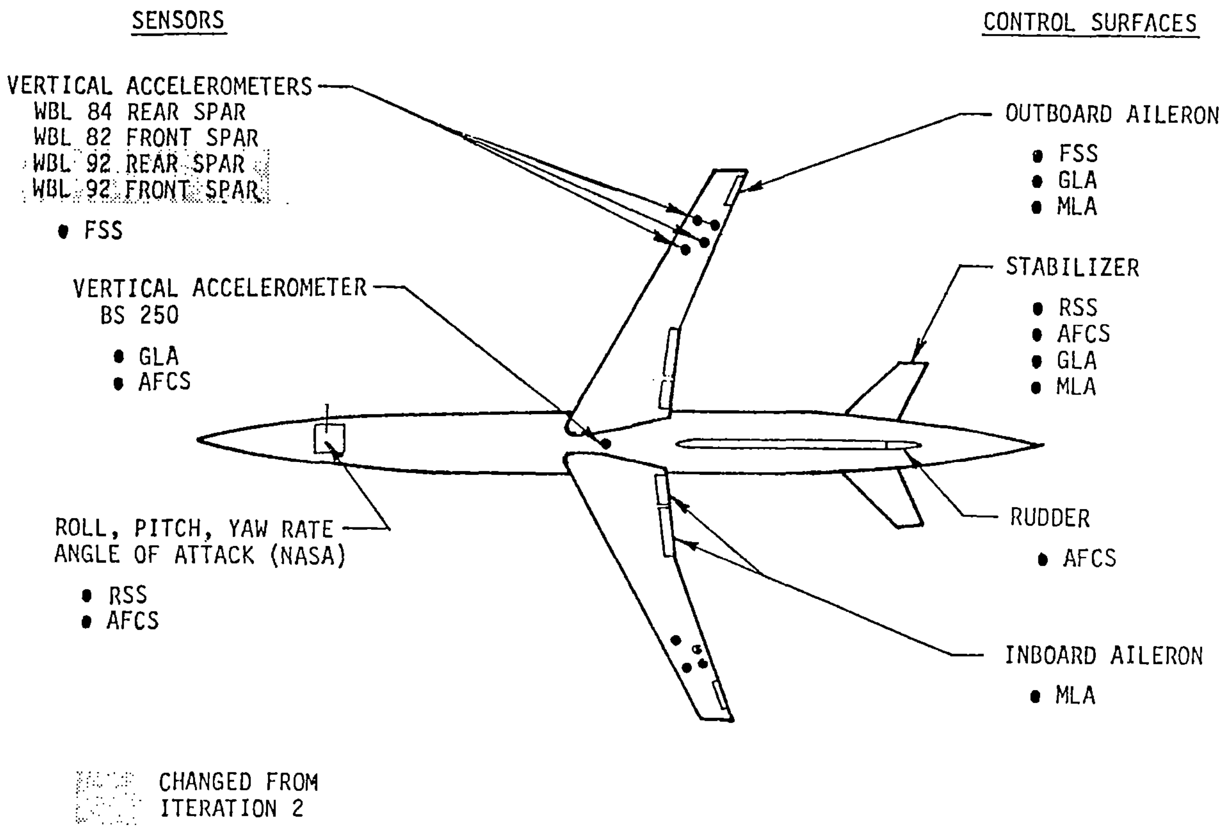
yields

$$w_g = (L_g^* P_g L_g)^{-1} L_g^* P_g (v_g^{Id} - \Delta v_g)$$

DAST ARW-2 FLIGHT TEST VEHICLE

The DAST ARW-2 flight test vehicle shown below is a Firebee II Drone which has been modified by replacing the conventional wing with a high aspect ratio supercritical wing designed to flutter within the flight envelope. Two control surfaces, an inboard and outboard aileron, are available on the wing. Current plans are to use the outboard aileron for flutter control and gust load alleviation and to use the inboard aileron for maneuver load control. Since two control inputs are needed if eigenvector shaping is to be accomplished, it was decided to use both the inboard and outboard control surfaces for flutter control. As will be shown, the inboard aileron is not effective for flutter suppression. However, it is possible to demonstrate eigenspace techniques using this control surface. Research currently in progress uses the elevator for control of rigid-body modes and the outboard aileron for control of flutter. Planned research will incorporate a leading-edge and a trailing-edge surface.

DAST ARW-2 SENSOR AND CONTROL SURFACE LOCATIONS



PERFORMANCE SPECIFICATIONS

The design flight condition for the flutter control system is M=0.86 (275 m/s or 908 ft/s) and an altitude of 15000 ft (475 m). At this flight condition, the uncontrolled wing flutters, and the flutter control system is required to stabilize the wing without exceeding specified limits on rms control surface activity. The control surfaces saturate if these limits are exceeded. Gain and phase margins must be adequate. The wing flutters at M=0.75 at this altitude, and the flutter control system must be activated at M=0.7. It must be verified that activation of the control system does not destabilize the wing at this flight condition. Also the flutter controller must not result in excessive increases in bending, shear, or torsional loads compared with the uncontrolled wing.

Design Condition

M = 0.86 h = 15000 ft.

Maximum RMS Control Surface Activity for 12 ft/s Gust

	Deflection	Deflection Rate
Inboard	10°	130°/s
Outboard	15°	740°/s

Mininum Stability Margins

Gain - 6 dB

Phase - 45°

No large increases in bending, torsion, and shear at M = 0.7

AEROELASTIC MODEL

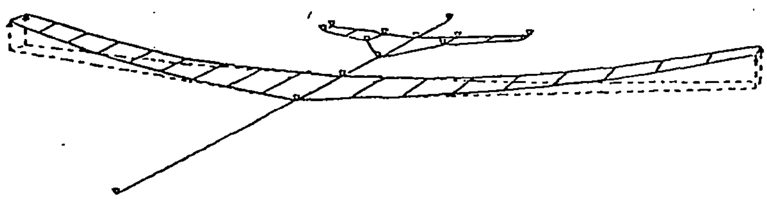
In the aeroelastic model of the wing given below, y_f is the vector of displacements of the various flexural modes, y_c is the vector of control surface deflections, y_g is the gust velocity, M_s is the structural mass matrix, C_s is the structural damping matrix, K_s is the structural stiffness matrix, q is dynamic pressure, and Q_c is the matrix of aerodynamic influence coefficients. Q_c is calculated as a function of reduced frequency by a doublet-lattice procedure and is approximated by Q_A, a matrix of rational polynomials in the Laplace operator s. The matrices A_i are selected to give the best least-squares fit to Q_c over a range of reduced frequencies.

$$([M_s]s^2 + [C_s]s + [K_s])\ [y_f] + q[Q_c(s)] \begin{bmatrix} y_f \\ y_c \\ y_g \end{bmatrix} = 0$$

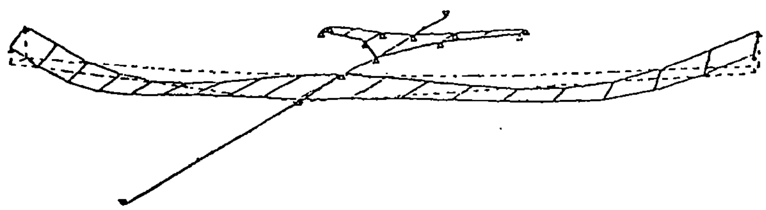
$$[Q_c(s)] \cong [A_0] + [A_1]\frac{cs}{2v} + [A_2]\left[\frac{cs}{2v}\right]^2 + \sum_{m=1}^{L} \frac{[A_{m+2}]s}{s+\frac{2v}{c}K_m}$$

FLEXURAL MODES

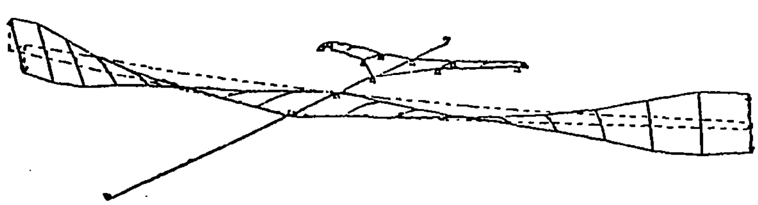
Initially, seven structural modes were used in the math model of the wing; however, by comparing eigenvalues calculated using lower order models with eigenvalues resulting from a model which included seven structural modes, it was found that flutter could accurately be modeled by including only three modes. These modes are shown below. The first mode will be labeled first mode bending, the second mode will be labeled second mode bending although it contains some torsion, and the third mode will be labeled first mode torsion. Rigid-body modes were not included.



First Bending



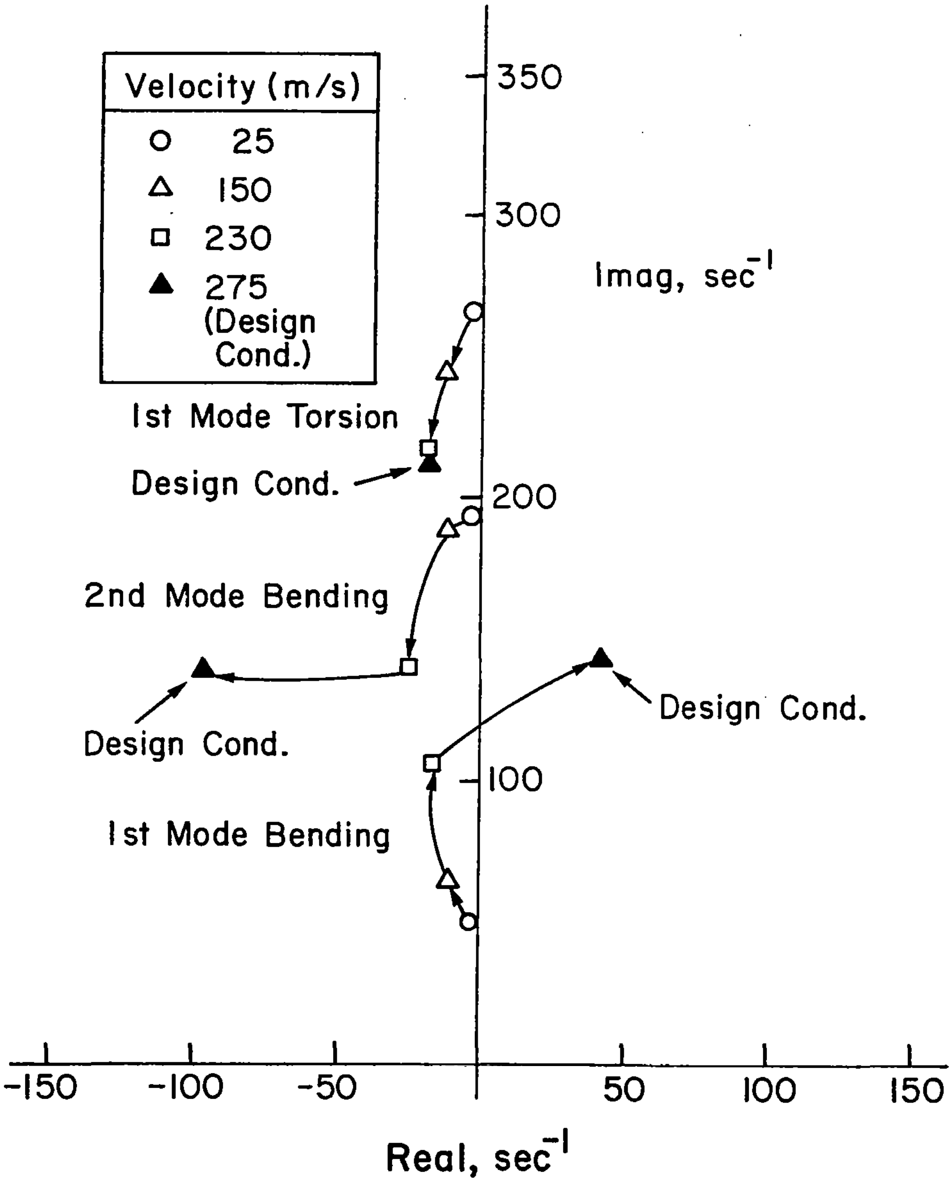
Second Bending



First Torsional

OPEN-LOOP ROOT LOCUS

The locus of the roots associated with the flexure modes is shown below. The lowest frequency mode is associated with first mode bending, the next highest with second mode bending and the highest with first mode torsion. It can be seen that first mode bending is the unstable mode while second mode damping increases with velocity. The frequencies of these modes approach one another with increasing velocity. The first torsion mode is not affected by velocity as much as the other two modes, but it is necessary to include this mode in order for the system to flutter. The wing flutters at a velocity of 787 ft/s (240 m/s). This is about Mach 0.75.



WIND GUST AND ACTUATOR MODELS AND STATE SPACE FORMULATION

A second-order model forced by white noise was used to simulate the vertical gust. Both inboard and outboard ailerons are driven by high bandwidth actuators. In the range of frequencies covered by the three-mode structural model, a fourth-order transfer function was shown to give a very close approximation of the actual inboard actuator/aileron transfer function. A third-order transfer function was used for the outboard aileron. The details of these models are given in Ref. 5. The state space model of the combined system is given below. The vector X_F includes the displacements and velocities associated with flexure modes and the aerodynamic lag states. The vector X_c includes the states associated with the inboard and outboard actuator models. The vector X_g includes the states associated with the gust model. The vector U is the control input to the actuators, and w is the scalar white noise input to the gust model. The total system model is 18th order. Note that the open-loop responses of the actuators are decoupled from one another and are not influenced by the motion of the wing (small inertial cross-coupling terms have been neglected) and that the gust states are uncontrollable.

$$y_g/w = G_g(s)$$

$$y_{c_i}/u_i = G_i(s)$$

$$y_{c_o}/u_o = G_o(s)$$

$$\begin{bmatrix} \dot{X}_F \\ \dot{X}_c \\ \dot{X}_g \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ 0 & A_{22} & 0 \\ 0 & 0 & A_{33} \end{bmatrix} \begin{bmatrix} X_F \\ X_c \\ X_g \end{bmatrix} + \begin{bmatrix} 0 \\ B \\ 0 \end{bmatrix} U_c + \begin{bmatrix} 0 \\ 0 \\ G \end{bmatrix} w$$

EIGENSPACE DESIGN APPROACHES

The initial eigenspace controller was designed by rotating the unstable eigenvalues about the imaginary axis and leaving all other eigenvalues and eigenvectors in their open-loop positions. The results are shown in the table on the following page. Although this initial design stabilized the wing at both the flutter test condition (M=0.86) and at the condition at which the flutter controller would be initially activated (M=0.7), the rms inboard deflection rate is near its maximum (saturation) value at the flutter condition. It was felt that the performance might be enhanced by redesign of the control system to reduce the inboard deflection rate. Also the initial design approach did not use the capability of eigenspace techniques to shape eigenvectors as well as assign eigenvalues, and it was desired to exercise this capability. Since the aircraft exhibits satisfactory response at velocities less than the flutter speed, it was decided to force the closed-loop response of the wing at the design condition to approach the open-loop response of the wing at a velocity 656 ft/s (20% less than the flutter speed). The closed-loop eigenvalues associated with the flexure modes and aerodynamic lag states were moved to their open-loop positions at 656 ft/s. The open-loop actuator eigenvalues were the same for both flight conditions and were not moved, and the gust eigenvalues were uncontrollable and could not be moved. The desired eigenvectors were selected to be the open-loop eigenvectors of the wing at 656 ft/s. The weighting matrices in the performance index were initially set at one. This procedure reduced all surface activity only slightly. It was decided to use eigenvector shaping to shift control surface activity from the inboard to the outboard actuator. The components of the open-loop aeroelastic eigenvectors in the actuator directions are zero for all flight conditions. Since the desired eigenvectors were chosen to be the open eigenvectors, penalizing the difference between the achievable and desired aeroelastic eigenvectors in the direction of the inboard actuator would reduce inboard activity. This was accomplished by increasing the weights on these components to 2.5×10^3 while all other weights remained at one. The results are shown in the table on the following page.

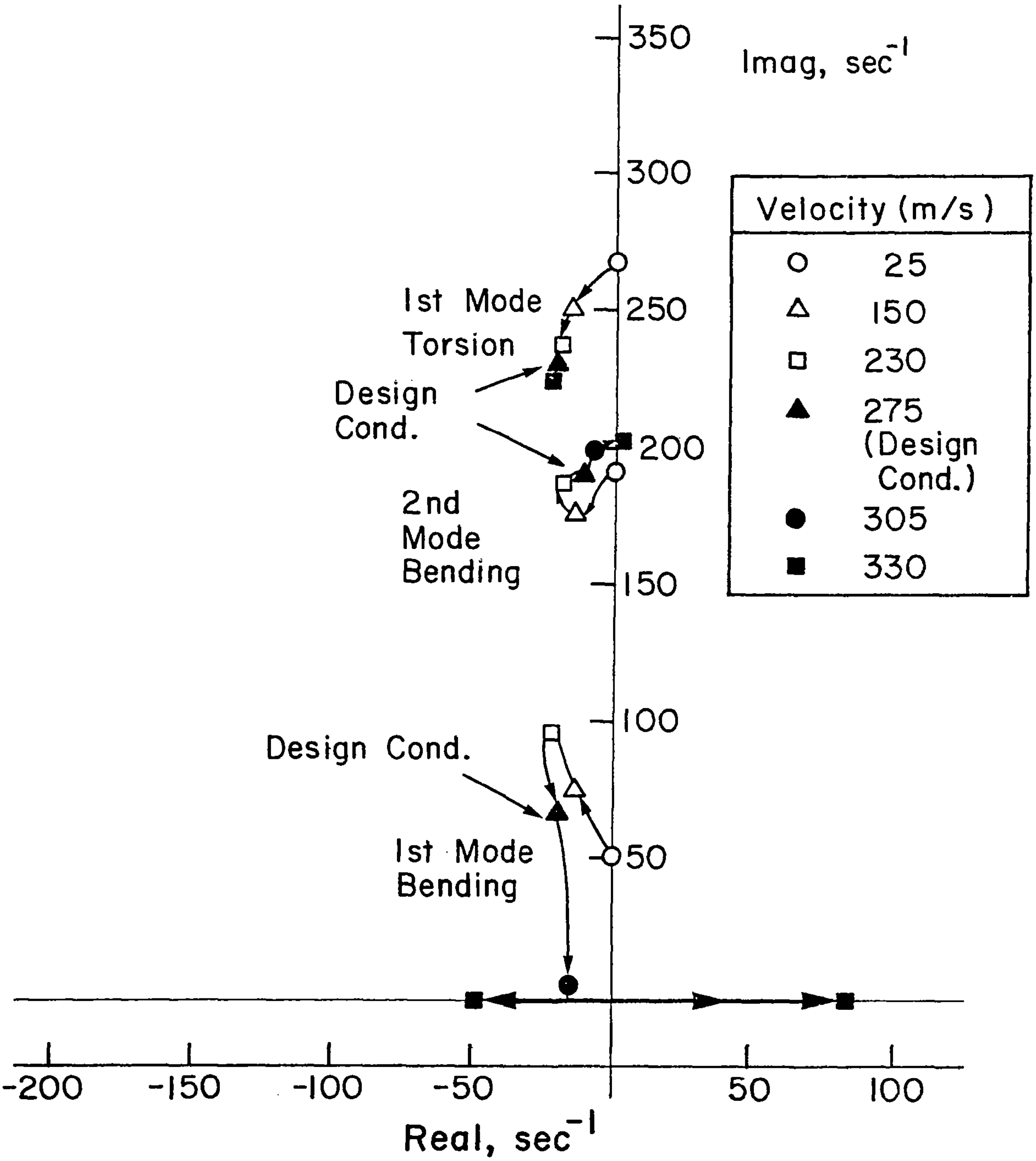
COMPARISON OF RMS CONTROL SURFACE ACTIVITY FOR THE EIGENSPACE CONTROLLERS

The performance of the initial and final eigenspace controllers is summarized in the table below. The inboard actuator rate for the initial eigenspace design is almost saturated while, with eigenvector shaping, the control effort is shifted from the inboard to the outboard control surface. Failure of both the inboard and outboard aileron was simulated. When the inboard actuator failed, performance was not affected very much; however, failure of the outboard actuator resulted in an unstable response. This indicates that the inboard actuator is not a good choice for use in flutter suppression.

Controller Design	Inbd Defl	Inbd Rate	Outbd Defl	Outbd Rate
Unstable Roots Rotated About Imag Axis (Initial Design)	0.5	108.0	3.3	509.0
Eigenvector Shaping to Reduce Inbd Rate (Final Design)	0.9	86.0	4.7	612.0
Initial Design Inbd Failed	-	-	3.5	519.0
Final Design Inbd Failed	-	-	4.4	572.0
Max Allowable	10	130	15	740

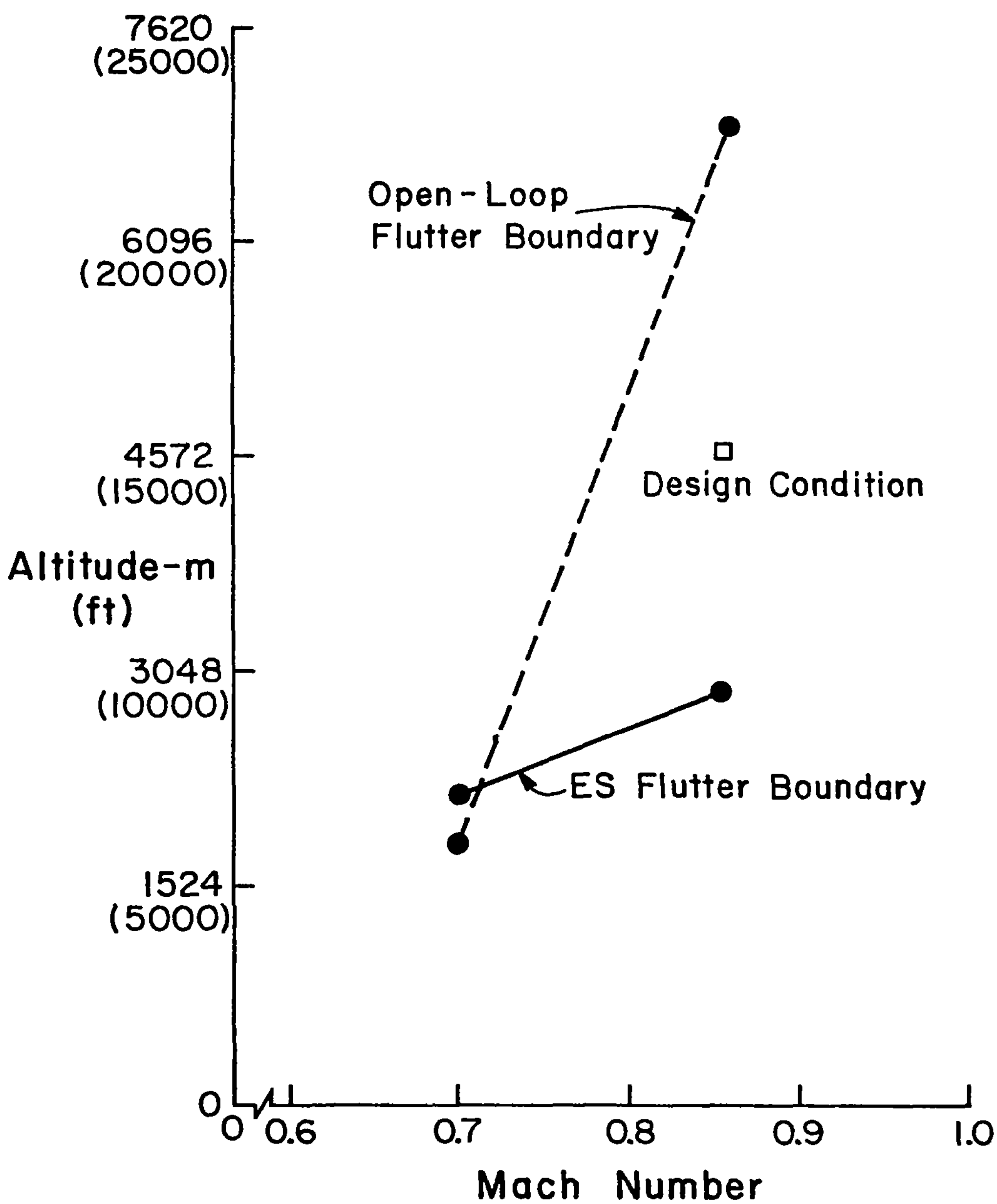
CLOSED-LOOP ROOT LOCUS

The root locus of the aeroelastic modes for the final eigenspace design is shown below. The wing goes unstable at about 1017 ft/s (M=0.96). The open-loop flutter speed is 787 ft/s (M=0.75); therefore, the control system results in an increase in flutter speed of about 29%. As in the open-loop case, the first bending mode goes unstable, but in the closed-loop case, the eigenvalues associated with this mode move to the real axis where one real root goes unstable. The roots associated with the second bending mode are almost unstable at this velocity.



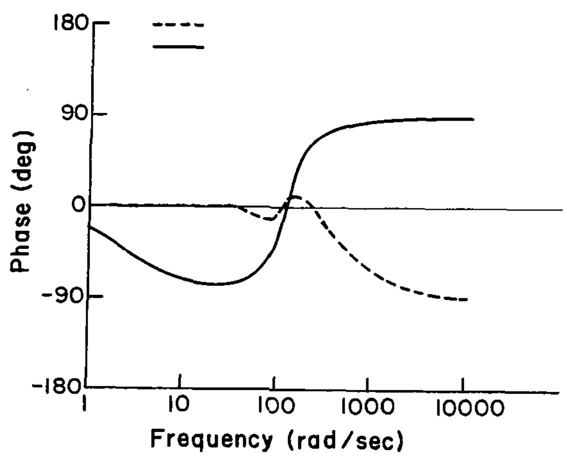
FLUTTER BOUNDARY

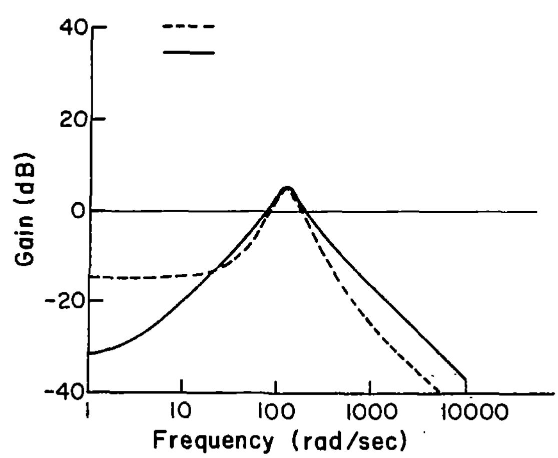
The results of varying altitude while maintaining Mach number constant are used to define the flutter boundary for the open-loop wing and the wing controlled with the final eigenspace design. At M=0.86, the uncontrolled wing is unstable until an altitude of 6700 m is reached. At the same Mach number, the controlled wing is stable for altitudes above 2900 m. At M=0.7, the uncontrolled wing is stable for altitudes above 1800 m, whereas the controlled wing is stable for altitudes above 2100 m.



STABILITY MARGINS

Since the inboard aileron was ineffective in stabilizing the wing, and the outboard aileron is critical, stability margins with only the outboard loop closed are shown below. The initial eigenspace design results from rotating the unstable eigenvalues about the imaginary axis and, with the inboard aileron inactive, this is identical to a design resulting from linear quadratic regulator theory. This is guaranteed to have excellent stability margins (Ref. 6). The gain margin is 6 dB, and phase margins are greater than 60°. The frequency response characteristics of the final eigenspace design are considerably different from those of the initial design. Gain margins are 6 dB or better, but phase margins are less than 20°. The solid lines represent the initial eigenspace design, and the dotted lines represent the final eigenspace design.





OUTPUT FEEDBACK

Since the inboard actuator was ineffective for flutter control, it was eliminated from the design. This resulted in a system with a single control input and allowed only eigenvalue assignment. An accelerometer was used to measure the motion of the wing, and a feedback compensator that approximated the frequency response characteristics of the full-state loop transfer function was designed. Also the effects of restricting the number of states in the feedback controller were investigated. The design approach was to treat this as a problem in output feedback as described earlier. The C matrix was selected to eliminate various states from the output, and a feedback controller was designed using eigenvalue placement techniques. Since the gust states are uncontrollable if P states are fed back, only P-2 eigenvalues could be placed. The following sets of states were eliminated from being fed back: (a) gust (2 states), (b) aerodynamic lags (3 states), (c) actuator (3 states), (d) first bending (2 states), (e) second bending (2 states), (f) first torsion (2 states), and (g) first torsion and aerodynamic lags (5 states). Except in the case where the first bending mode was eliminated, unstable eigenvalues were rotated about the imaginary axis, and eigenvalues associated with the retained states were maintained in their open-loop positions. The positions of the other eigenvalues were not assigned. Since the eigenvalues of the first bending mode were unstable, these eigenvalues were rotated, and the eigenvalues of the first torsional mode were not assigned. This mode then went unstable. The results are summarized below. The rms response was very sensitive to the gust states, and attempts to remove these states or alter their eigenvectors resulted in large rms responses. Loads at the wing root were reduced using the full-state design. Note a slightly different gust model was used so the rms results below are not directly comparable with those presented earlier.

States Eliminated	Otbd Defl	Otbd Rate	Gain Margin Db	Phase Margin (Degrees)
Gust	32	456	6.0	±60
Actuators	2.0	265	4.1	±36
Aero. Lags	6.6	394	5.7	-50,60
Mode 1	Unstable	Unstable	Unstable	Unstable
Mode 2	2.0	260	2.8	-27,18
Mode 3	2.0	271	5.8	-63,+54
Mode 3 + Aero. Lags	6.6	430.0	6.1	-54,+72 +81,+45
Full State	2.0	259	6	±60
Compensator	2.72	250.8	6.0	±60

RMS Responses M = .86

CONCLUSIONS

Eigenspace techniques can provide a powerful tool for the design of feedback control systems for aircraft. The design techniques described in this paper are easily implemented and are computationally inexpensive. The basic problems facing the designer are those of determining where to place eigenvalues and of selecting appropriate eigenvectors. This usually requires some insight into the system to be controlled; however, in this respect all control system design techniques are the same. Stability margins must be carefully examined since there are no guaranteed margins as is the case for linear quadratic regulators. On the other hand, modal decoupling is easily achieved, and certain roots can be maintained in their open-loop configurations if desired. Output or limited-state controllers can easily be designed.

	Shear ($\text{lbs} \times 10^2$)	Torsion ($\text{in} - \text{lbs} \times 10^2$)	Bending ($\text{in} - \text{lbs} \times 10^4$)
Open Loop	4.131	6.928	2.365
Closed Loop	3.775	1.029	2.321

Wing Root Loads $M = 0.7$

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