NONLINEAR SYSTEMS APPROACH TO CONTROL SYSTEM DESIGN

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First Annual NASA Aircraft Controls Workshop
NASA Langley Research Center
Hampton, Virginia
October 25-27, 1983

Consider some of the control system design methods for plants with nonlinear dynamics. If the nonlinearity is weak relative to the size of the operating region, then the linear methods apply directly. Fixed-gain design may be feasible even for significant nonlinearities. It may be possible to find a single gain which provides adequate control of the linear models at several perturbation points. If the nonlinearity is restricted to a sector, that fact may be used to obtain a fixed-gain controller. Otherwise, a gain may have to be associated with each perturbation point p_i. A gain schedule $K(p(v))$ is obtained by connecting the perturbation points by a function, say $p(v)$, of the scheduling parameter v (i.e., speed). When the scheduling parameter must be multidimensional, this approach is difficult; the objective of our research is to develop an easier procedure.

$$\dot{x} = f(x, u, z)$$

1. SMALL OPERATING REGION — LINEAR METHODS

   $$p = (x_o, u_o),\ \delta\dot{x} = \left(\frac{\partial f}{\partial x}\right)_p \delta x + \left(\frac{\partial f}{\partial u}\right)_p \delta u,\ \delta u = K(p)\,\delta x$$

2. FIXED-GAIN DESIGN
   a. ONE GAIN FOR ALL $\{p_i\}$
   b. ONE GAIN FOR SECTOR NONLINEARITIES

3. GAIN SCHEDULING

   $p_i = p(v_i)$, $K(p(v))$, v = SCHEDULING PARAMETER

4. NONLINEAR TRANSFORMATION OF STATE AND CONTROL

SIMPLIFICATION THROUGH COORDINATE CHANGE

We attempt to simplify the design problem by simplifying the representation of the plant. Here is an example. The state equation is nonsingular, but a fixed-gain design is impossible. The nonsingular matrix E represents rotation in the plane of x through the angle ψ. The change of control coordinates from u to v results in a globally constant, linear system. Even an approximate cancellation of E could be quite helpful, since the resulting system may be nearly constant and the fixed-gain design may be applicable.

$x \epsilon R^2$, $u \epsilon R^2$, $EE^T = I$

$$\dot{x} = E\,(\psi(t))\,u$$

$\psi = 0 \longrightarrow \dot{x} = u,\ \dot{x} = K_0 x$, STABLE

$\psi = \pi \longrightarrow \dot{x} = -u,\ \dot{x} = -K_0 x$, UNSTABLE

NEW CONTROL COORDINATES:

$v = E(\psi(t))\,u \longrightarrow \dot{x} = v$, GLOBALLY

APPROXIMATE TRANSFORMATION:

$v = E\,(\psi)\,u \longrightarrow \dot{x} = E\,(\delta\psi(t))\,v$ FIXED GAIN MAY WORK

BRUNOVSKY CANONICAL FORM

The first step in the design approach is to try to transform the given system into something more simple, i.e., ideally, a set of decoupled strings of integrators. This set is called the Brunovsky canonical form for controllable, constant, linear systems. The number of strings is given by the number of control axes m, and the number of integrators (dots in the figure) is given by a Kronecker index K_i. In general, $\Sigma K_i = n$. Two examples are shown for the case of $n = 12$ and $m = 4$. According to linear theory, the set of Kronecker indexes is invariant under nonsingular transformations and feedback.

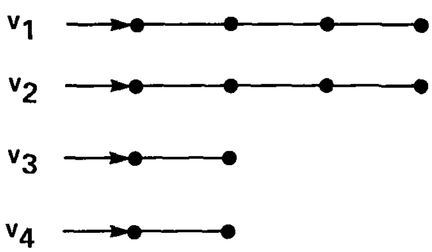
GIVEN $\dot{x} = Ax + Bu$, $x \epsilon R^n$, $u \epsilon R^m$, (A, B) CONTROLLABLE

CAN BE TRANSFORMED WITH

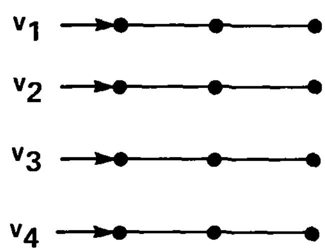
$y = Tx$, $v = Rx + Wu$, (T, W) NONSINGULAR

INTO m DECOUPLED STRINGS OF INTEGRATORS, LENGTH = KRONECKER INDEX

EXAMPLE: $n = 12$, $m = 4$



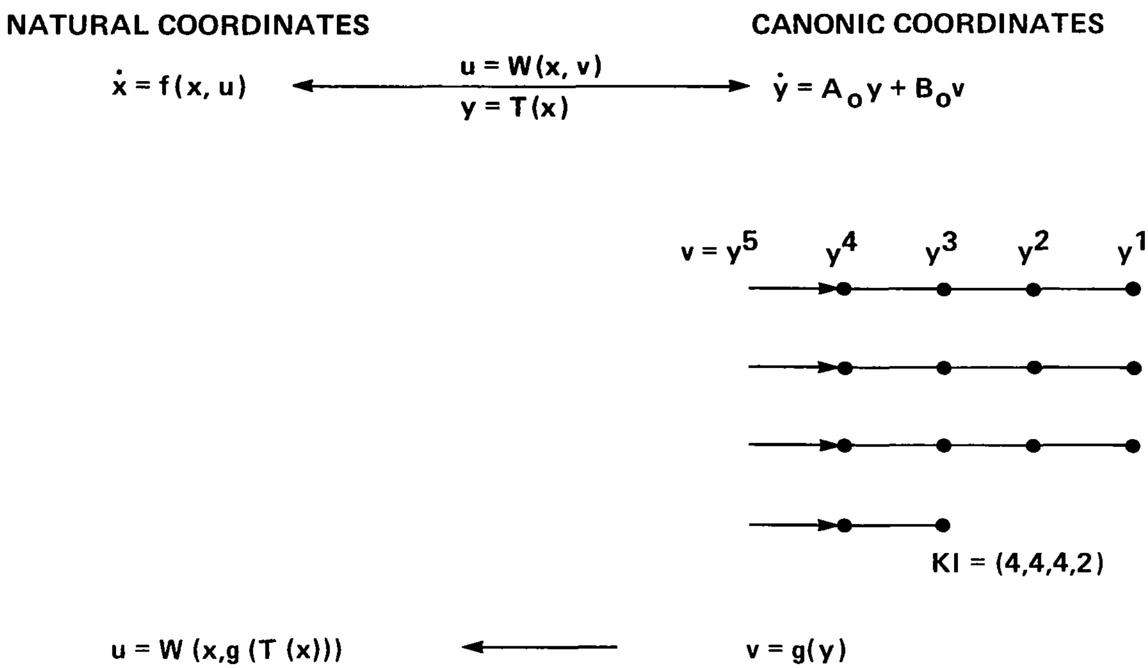
KI = (4,4,2,2)



KI = (3,3,3,3)

OUTLINE OF DESIGN PROCEDURE

In principle, the design procedure is to transform the natural representation of the plant into the corresponding Brunovsky form, then design a control law $v = g(y)$ for the canonical system, and finally pull the law back into the natural coordinates in terms of which law must be implemented.



DEVELOPMENT OF THE DESIGN APPROACH

Before this design approach becomes practical, several issues must be resolved.

(1) When can a given system be transformed into a linear model, how does one construct the required transformation, and how feasible is it to implement the resulting algorithm on flight computers?

(2) What is the structure of the complete control system that includes the linearization step?

(3) How robust is the resulting design?

(4) How can the constraints on control and state be enforced?

These issues have been explored both theoretically and experimentally. The results are summarized in the following figures.

1. TRANSFORMATION
    a. EXISTENCE
    b. COMPUTATION
    c. IMPLEMENTATION
2. STRUCTURE OF COMPLETE CONTROL SYSTEM
3. COMPLEXITY AND ACCURACY OF MODEL – ROBUSTNESS
    a. EXACT STATE SPACE
    b. TRUNCATED STATE SPACE
4. ENFORCEMENT OF DESIGN CONSTRAINTS
5. EXPERIMENTS
    a. FORTRAN
    b. REAL-TIME (FLIGHT COMPUTER, HYDRAULICS) SIMULATION
    c. FLIGHT

The key theoretical result from the existence of linearizing transformations is the result of work by Krener (ref. 1), Brockett (ref. 2), Jakubcyzk and Respondek (ref. 3), and Hunt and et al. (ref. 4). The necessary and sufficient conditions are best expressed in terms of lie brackets. A lie bracket (f, g) constructs a new vector field from the old ones f and g. A set is involutive if the brackets do not create new directions.

VECTOR FIELDS	$f, g: R^n \rightarrow R^n$
LIE BRACKETS	$[f, g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g$
INVOLUTIVE SET	$\{h_1, \ldots, h_r\}$ IF $[h_i, h_j] \in \text{SPAN}\{h_1, \ldots, h_r\}$

CONDITIONS FOR LINEARIZABILITY

1. It is necessary to find control coordinates which appear linearly in the state equation. For aircraft this means angular acceleration instead of ailerons, elevator, and rudder.

2. The resulting system must have linear-like controllability.

3. The fields $\{f, g_1, g_2 \ldots, g_m\}$ must satisfy an involutivity condition. For example, let $n = 6$, $m = 2$, and the Kronecker index set $KI = (3, 3)$. Then, the six vector fields must span the state space and the first four must be involutive

1. $\dot{x} = f(x, u) \longrightarrow \dot{x} = f(x) + \Sigma\, g_i(x)\, u^*$

2. **LINEAR – CONTROLLABLE**

3. **INVOLUTIVE**

EXAMPLE: $\dot{x} = f(x) + g_1(x)u_1^* + g_2(x)u_2^*$

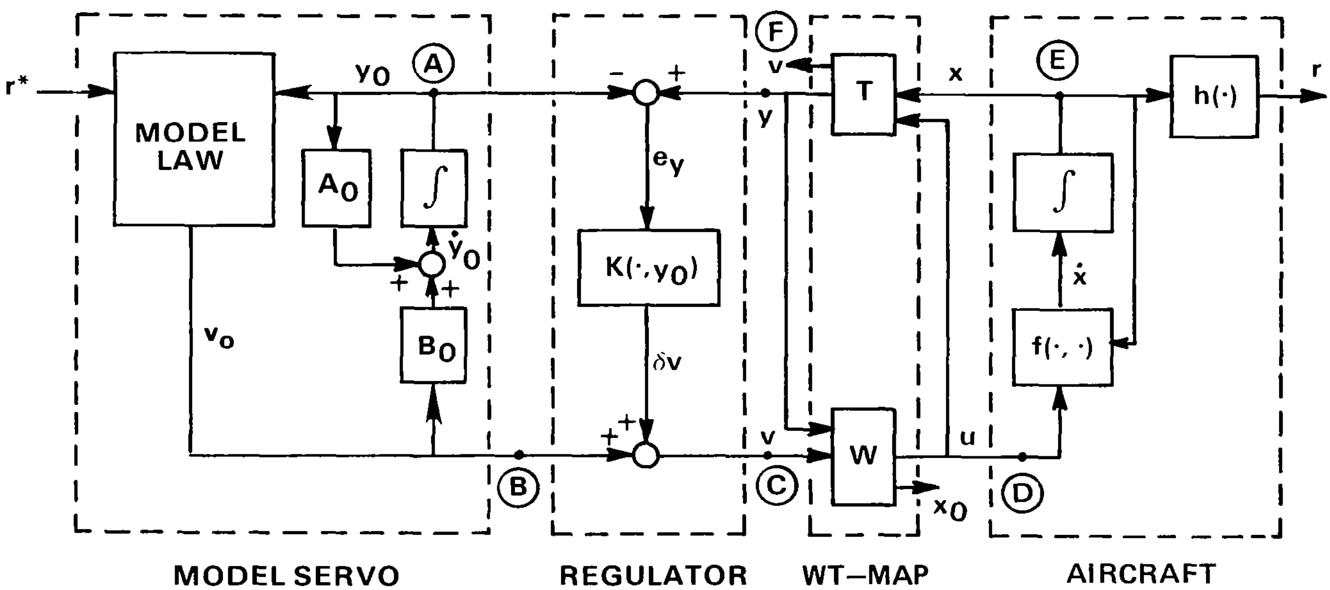
$KI = (3, 3)$

$$\boxed{\begin{array}{cc} \boxed{\begin{array}{cc} g_1 & [f, g_1] \\ g_2 & [f, g_2] \end{array}} & \begin{array}{c} [f, [f, g_1]] \\ [f, [f, g_2]] \end{array} \end{array}} \text{ — 6 DIMENSIONAL SPAN}$$

INVOLUTIVE

STRUCTURE OF THE COMPLETE CONTROL SYSTEM

The structure has the form of an exact model follower. The linearizing transformation is done in the WT-map. Through this map the plant is seen as a set of decoupled strings of integrators. The same set is employed as the dynamics (A_o, B_o) of the model servo, where the desired motion is defined by means of the input r^* and the, in general, nonlinear control law. The control of modeling inaccuracies and other disturbances is accomplished by the regulator which operates on the error e_y and outputs corrective control δv. It may be noted that the regulator works into the simple canonical dynamics and, in effect, the gain scheduling is done automatically by the WT-map.



HELICOPTER EXPERIMENT

The objective of this experiment is to investigate the effectiveness and realism of the design approach and to uncover potential problem areas. The current work is with the UH1H helicopter equipped with the VSTOLAND avionics system including the Sperry 1819B flight computer. The model used in the design is a rigid-body nonlinear force (f^F), and moment (f^M) generation process. Inertial coordinates (r, v) of position and velocity vectors, body attitude matrix C, and angular velocity ω form the state. The moment controls u^M are the roll and pitch cyclic and the pedals. The collective is the power control u^P.

STATE:

$$x = \begin{pmatrix} r \\ v \\ C \\ \omega \end{pmatrix} \in X \subset R^3 \times R^3 \times SO(3) \times R^3$$

CONTROL:

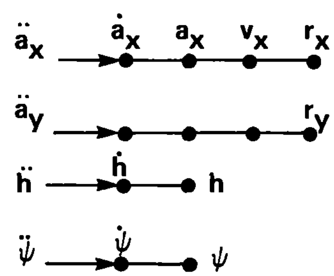
$$u = \begin{pmatrix} u^M \\ u^P \end{pmatrix} \in U \subset R^3 \times R$$

STATE EQUATION:

$$\begin{aligned} \dot{r} &= v \\ \dot{v} &= f^F(x, u) \\ \dot{C} &= S(\omega)\, C \\ \dot{\omega} &= f^M(x, u) \end{aligned}$$

The canonical model chosen is shown in the figure. There is a pair of strings, each four integrators long. This pair represents the two horizontal channels (r_x, r_y). In addition, there is a two-integrator string for altitude h. The fourth string is for the heading ψ. The canonical controls are the second derivatives of horizontal acceleration, vertical acceleration, and yaw acceleration. The transformation is computed by means of two Newton-Raphson trim routines. First, the controls are computed and yield the given accelerations $(\dot{w}_{bc}, \ddot{h})$ at the given state. Then the attitude C is computed for the given horizontal acceleration. The Jacobian matrices needed by Newton-Raphson are computed numerically.

CANONICAL MODEL



KI = (4,4,2,2)

TRANSFORMATION

a. **ON-LINE NEWTON-RAPHSON MOMENT TRIM**

$$\left.\begin{array}{l} f^M(r,v,C,w_b,u) = \dot{w}_{bc} \\ f^F_3(r,v,C,w_b,u) = \ddot{h}_c \end{array}\right\} \longrightarrow u_c = h^M(v, C, w_b, \ddot{h}_c, \dot{w}_{bc})$$

b. **ON-LINE N-R FORCE TRIM**

$$f^F_h(r,v,C,o,u_o) = a_h \longrightarrow (\phi,\theta) = h^F(r,v,\psi,a_h,o,o)$$

SUMMARY — UH1 EXPERIMENT

The experiment has progressed to the point that the following observations can be made.

(1) The implementation of the control scheme is practical. The complete code takes less than 22 msec on the Sperry 1819B flight computer

(2) Both FORTRAN and real-time simulations (real flight computer and hydraulics) are consistent with theory. The tracking accuracy along the trajectory including hover, climb, descent and high-speed flight indicates that the UH1 model routinely used for manned simulations is linearizable to a degree where a fixed-gain controller is possible. There is robustness with respect to weight, center of mass, moment of inertia, and force and moment models, but there is sensitivity to errors in wind estimates. The actuator dynamics (15 rad/sec) may be commuted with the TW-map for regulator bandwidth below 2 rad/sec

(3) Further research is needed to develop rigorous methods for including high-frequency dynamics and explicit enforcement of control and state constraints

1. IMPLEMENTATION IS PRACTICAL
   - SAMPLING TIME = 50 msec
   - NEW JACOBIANS FOR N - R TRIM EVERY 5TH SAMPLE
   - COMPLETE CODE = 22 msec ON SPERRY 1819B

2. FORTRAN AND MANNED SIMULATIONS ARE CONSISTENT WITH THEORY
   - TRACKING ACCURACY IMPLIES LINEARIZABILITY
   - ROBUSTNESS
   - ACTUATOR DYNAMICS

3. PROBLEM AREAS
   - METHOD FOR INCLUDING SERVO DYNAMICS
   - METHOD FOR EXPLICIT ENFORCEMENT OF CONSTRAINTS

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