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DETAILED DESIGN
OF A
VARIABLE VOLUME HYDROGEN MASER

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CHAPTER I

INTRODUCTION - REVIEW OF PERTINENT MASER EQUATIONS

When a hydrogen atom approaches the walls of the maser bulb, the attractive van der Waals forces decrease the amplitude of the wave function at the nucleus thus lowering the energy. Near the wall, the overlap of the electron clouds of the atom and the wall cause the atom to reverse its velocity and move away. The wave function overlap is increased and so is the energy [1]. The net phase shift thus created is given by $\phi = \frac{1}{\hbar} \int_{\Delta t} \Delta E(t)dt$ where Δt is the duration of the collision, ΔE the change in hyperfine energy. (I.1) The corresponding shift in the maser frequency is given by $\delta f = \frac{r\phi}{2\pi}$ where $\underline{r}$ is the collision rate. (I.2)

Hydrogen maser operation can be described by a quality parameter q given by [2]: $q = \frac{\sigma\bar{v}\hbar}{8\pi\mu^2} \frac{T_b}{T_t} \frac{V_c}{V_b} \frac{1}{\eta} \frac{1}{Q} \frac{I_{tot}}{I}$ where σ = transition cross-section = 2.6×10^{-15} cm^2 (I.3)

$\bar{v}$ = average speed of the H atoms (= 3.58×10^5 cm/sec at 308^{0}K; = 3.94×10^5 cm/sec at 373^{0}K)

$\hbar$ = Planck's constant $/2\pi = 10^{-27}$ erg sec

μ = magnetic dipole moment of the electron in the H atom = 9.2849×10^{-21} ergs/gauss

T_b = bulb time

T_t = total time

V_c = volume of the maser cavity

V_b = volume of the bulb

$\eta = \frac{\langle H_z \rangle^2_{bulb}}{\langle H^2 \rangle_{cavity}} \equiv$ filling factor

Q = quality factor of the cavity

I_{tot} = total flux of H atoms

I = flux of atoms in the right excited state.

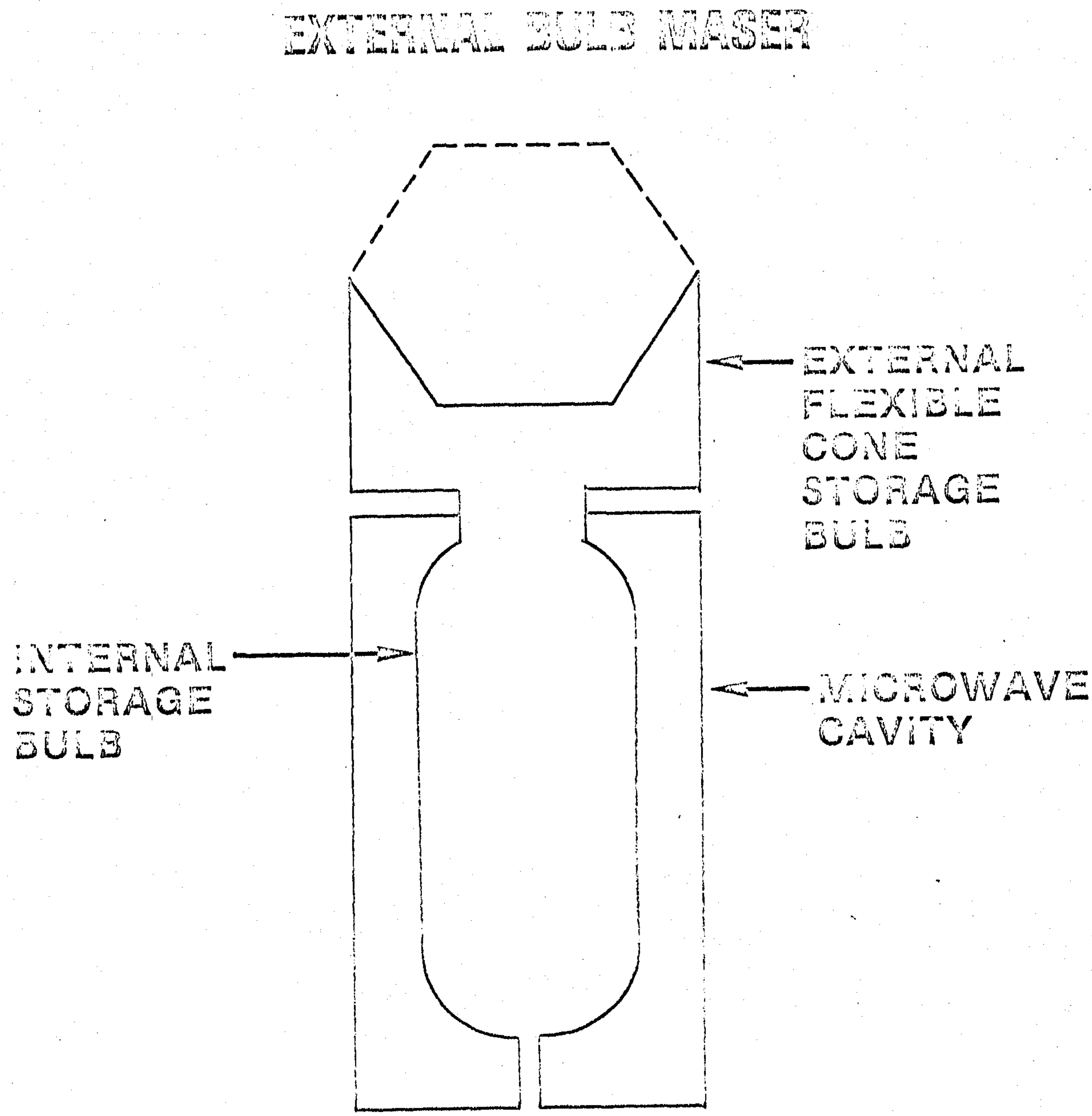

FIGURE 1 - SCHEMATIC OF VARIABLE VOLUME MASER

For maser operation, one must have $q < .172$.

In order to tune the maser accurately, the ratio of the high flux to low flux resonance line widths must be as large as possible. The theoretical maximum is given by

$$r = \frac{\Delta\nu_{max}}{\Delta\nu_{min}} = \frac{1+q+(1-6q+q^2)^{\frac{1}{2}}}{1+q-(1-6q+q^2)^{\frac{1}{2}}} \tag{I.4}$$

One also defines

$$\frac{1}{n'} = \frac{V_c}{V_b}\,\frac{1}{n} \tag{I.5}$$

Whenever a change of the bulb volume is involved, it is found useful to talk about a quantity B defined as

$$B = \frac{V_+}{V_-} \tag{I.6}$$

The (+) and (-) subscripts in the volume V refer to the <u>large</u> (open) and <u>small</u> (closed) configurations, respectively. In the variable volume hydrogen maser shown schematically in Figure 1, there is a frequency shift due to the change in the volume. It is our hope to design the system so that this shift will be less than one part in 10^{14}.

CHAPTER II

IDENTIFICATION OF DESIGN PARAMETERS

The design parameters are:

(1) Shape of the Bulb: The bulb will be made up of two parts: one fixed and the other variable. The fixed part is going to be cylindrical. It will have an opening into a variable volume that will be a truncated cone.

(2) q or Quality Parameter: This will determine whether or note maser action will occur.

(3) Tuning Factors: R: flux tuning factor

B: variable volume tuning factor

(4) Material to be Used: FEP Teflon has been examined (see next section) and found quite suitable.

(5) Operational Temperature to Off-Set the Wall Shift: This temperature is believed to be approximately 100^{o}C and will have to be investigated experimentally once the maser is built.

Study of Teflon as Wall Material

We have used a 1 square foot piece of Teflon in order to study its vacuum behavior and suitability for use as wall material for the new maser design.

The sample was crumpled to simulate operational conditions and placed in a vacuum manifold. When the 20 L/sec Noble VacIon pump and sample were heated to 117^{o}C, the pressure reached 8×10^{-9} Torr after three days.

The pump and sample were then transferred to an environmental chamber where the temperature was maintained at 125^{o}C for one day. The temperature was then raised to 131^{o}C for another day. At approximately this temperature, FEP Teflon is known to change phase.

The temperature was then gradually decreased to 126^{0}C for one day. The annealing process took six days at which stage the pressure was 1.8 x 10^{-11} Torr at room temperature. When opened, the sample had not changed its physical shape or dimensions and was judged suitable for use in the new maser.

CHAPTER III

FREQUENCY ERROR

In the following sections, we derive expressions for the statistical error on the measured frequency and the following notation is used.

The subscripts H and L refer to high and low flux. The subscripts (-) and (+) refer to closed/small and open/large external volume respectively.

At volume V_- (corresponding to R_-) $F_{tuned} = F_{L-} - \dfrac{F_{H-} - F_{L-}}{R_- - 1}$

$$= F_{L-} \left\{\frac{R_-}{R_- - 1}\right\} - F_{H-} \qquad \text{(III.1)}$$

At volume V_+ (corresponding to R_+) $F_{tuned+} = F_{L+} \left\{\dfrac{R_+}{R_+ - 1}\right\} - \dfrac{F_{H+}}{R_+ - 1}$ (III.2)

We have $\Delta F = F - F_{tuned}$ (III.3)

$\Delta F_{H+} = R_+ \Delta F_{L+}$ (III.4)

and

$\Delta F_{H-} = R_- \Delta F_{L-}$ (III.5)

$\delta^2 F_{tuned+} = \Delta^2 F_{L+} \dfrac{R_+^2}{(R_+ - 1)^2} + \dfrac{\delta^2 F_{H+}}{(R_+ - 1)^2}$ (III.6)

But

$F_{tuned-} = F_o + \Delta F_{--}$ (III.7)

So

$^B F_{tuned+} = {}^B F_o + B \Delta F_{L+}$ (III.8)

and

$F_{tuned-} = F_o + B \Delta F_{L+}$ (III.9)

$$ {}^{B}F_{tuned+} - F_{tuned-} = F_o \, (B-1) \tag{III.10} $$

Case 1: The RMS noise is independent of the line-width so that $\delta^2 F_H = \delta^2 F_L$

$$ B \, \delta^2 F_{tuned+} + \delta^2 F_{tuned-} = \delta^2 F_o \, (B-1)^2 \tag{III.11} $$

We obtain after substituting

$$ B^2 \left\{ \delta^2 F_{L+} \frac{R_+^2}{(R_+-1)^2} + \frac{\delta^2 F_{H+}}{(R_+-1)^2} \right\} + \left\{ \delta^2 F_{L-} \frac{R_-^2}{(R_--1)^2} + \frac{\delta^2 F_{H-}}{(R_--1)^2} \right\} = $$

$$ \delta^2 F_o (B-1)^2 \tag{III.12} $$

which becomes, after one defines $\delta F = \delta F_H = \delta F_L$

$$ \frac{\delta F_o}{\delta F} = \frac{1}{(B-1)} \sqrt{ \frac{B^2 (R_+^2+1)}{(R_+-1)^2} + \frac{(R_-+1)}{(R_--1)^2} } \tag{III.13} $$

Case 2: The RMS noise is proportional to the line-width

$$ B^2 \left(\delta^2 F_{L+} \frac{R_+^2}{(R_+-1)^2} + \frac{\delta^2 F_{H+}}{(R_+-1)^2} \right) + \left(\delta^2 F_{L-} \frac{R_-^2}{(R_--1)^2} + \right. $$

$$ \left. \frac{\delta^2 F_{H-}}{(R_--1)^2} \right) = \delta^2 F_o \, (B-1)^2 \tag{III.14} $$

Assuming the line-width to be inversely proportional to the corresponding volume,

$$ \delta^2 F_{L-} = B^2 \delta^2 F_{L+} \; ; \; \delta^2 F_{L+} = \frac{\delta^2 F_{L-}}{B^2} \tag{III.15} $$

$$ \delta^2 F_{H+} = R_+^2 \delta^2 F_{L+} \tag{III.16} $$

So

$$ \delta^2 F_{H+} = \frac{R_+^2}{B^2} \; \delta^2 F_{L-} \tag{III.17} $$

and

$$\delta^2 F_{H-} = R_-^2 \delta^2 F_{L-} = R_-^2 B^2 \delta^2 F_{L+} \qquad \text{(III.18)}$$

Finally, defining $\delta F = \delta F_{L+}$ gives

$$\frac{\delta F_0}{\delta F} = \frac{B}{B-1} \sqrt{\frac{2R_+^2}{(R_+-1)^2} + \frac{2R_-^2}{(R_--1)^2}} \qquad \text{(III.19)}$$

CHAPTER IV

SYSTEMATIC FREQUENCY ERROR

The aim is the study of localized variations in the concentration of excited atoms and how this is affected by the shape of the container (the ratio of surface to volume in particular). This yields a frequency shift which can be used to develop criteria that can be used to design the variable volume part of the maser.

The principal assumption made is to consider that the walls act as a sink on the excited atoms. This effect is directly proportional to the wall area and is large compared to any other contribution.

After exhaustive computer modeling, it is found that in order to obtain an upper bound on the frequency error, that part of the external volume farthest away from the fixed volume contributes the most if it is considered to include the sharp tapered part up to the knee bend in the volume. Referring to Figure 2, this would include the region marked (II) up to C_1. The other region considered is marked (I) on the same figure. The fixed volume V_0 is connected to the variable volume via C_0, a communication part which consists of a hole.

This is shown schematically in the following section.

VARIABLE VOLUME EXTERNAL STORAGE BULB

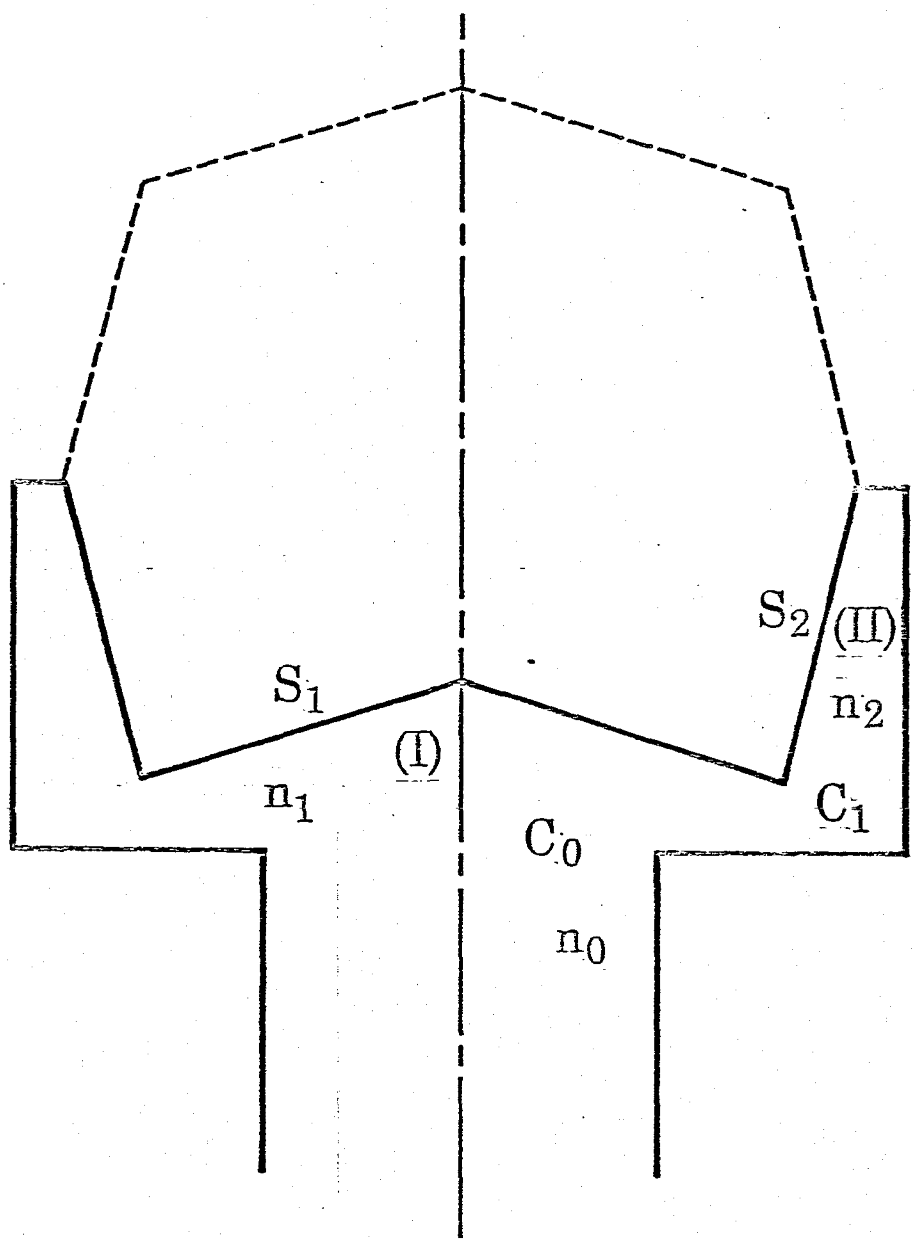

FIGURE 2 - VARIABLE VOLUME EXTERNAL STORAGE BULB

IV. 1 Two Region Model

TWO REGION MODEL

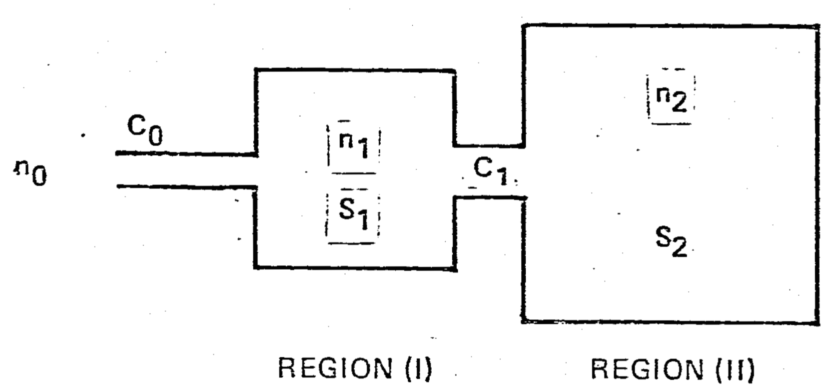

FIGURE 3 - TWO REGION MODEL

Let the n's refer to the density of excited atoms in the region under consideration. The S's refer to an equivalent pumping speed due to interaction with the walls and the C's are communication terms.

At equilibrium, $\dot{n}_{in} = \dot{n}_{relaxation} + \dot{n}_{escape}$

$$n_2 C_1 + n_o C_o = n_1 \left(C_o + C_1 + S_1 \right) \tag{IV.1}$$

and

$$n_1 C_1 = n_2 \left(C_1 + S_2 \right) \tag{IV.2}$$

Then

$$n_1 = n_2 \left(\frac{C_1 + S_2}{C_1} \right) \tag{IV.3}$$

and

$$n_2 C_1 + n_o C_o = n_2 \left(\frac{C_1 + S_2}{C_1} \right) \left(C_o + C_1 + S_1 \right) \tag{IV.4}$$

Thus

$$\frac{n_2}{n_o} = \frac{1}{1 + \frac{S_1}{C_o} + \frac{S_2}{C_1} + \frac{S_2}{C_o} + \frac{S_2 S_1}{C_o C_1}} \tag{IV.5}$$

Assuming that the communication terms (or C's) are large compared to the S's

$$\frac{n_2}{n_o} = \frac{1}{1 + \frac{S_1}{C_o} + S_2 \left(\frac{1}{C_1} + \frac{1}{C_o} \right)} \tag{IV.6}$$

or

$$\frac{n_2}{n_o} - 1 - \frac{S_1}{C_o} - S_2\left(\frac{1}{C_o} + \frac{1}{C_1}\right) \qquad \text{(IV.7)}$$

But

$$\frac{n_1}{n_o} = \frac{n_1}{n_2}\frac{n_2}{n_o} \qquad \text{(IV.8)}$$

and

$$\frac{n_1}{n_2} = \left(\frac{C_1 + S_2}{C_1}\right) \qquad \text{(IV.9)}$$

After substitution and elimination of terms in $\left(\frac{S}{C}\right)^2$ which are considered small compared to $\frac{S}{C}$, the result is:

$$\frac{n_1}{n_o} - 1 - \frac{S_1 + S_2}{C_o} \qquad \text{(IV.10)}$$

If r(A) is the collision rate with the wall and n(A) is the density of excited atoms near an element of wall area dA, the wall shift can be written using (I.2)

$$\delta F = \frac{1}{N}\int \phi(A)\ r(A)\ dA \qquad \text{(IV.11)}$$

where N is the total number of atoms in volume V and $\phi(A)$ is the average phase shift in area dA.

Then

$$\delta F = \frac{\bar{\nu}}{8\pi V_{-}} \int \phi(A)\ \frac{n(A)}{\bar{n}}\ d(A) \qquad \text{(IV.12)}$$

The largest contribution to $\Delta \bar{V}w$ comes from the smallest volume configuration which we call V_-.

$$E_i = \frac{n_i}{\bar{n}} - 1 \qquad \text{(IV.13)}$$

one has

$$\delta F = \frac{\bar{\nu}}{8\pi V_-} \int \phi_i(A)\, E_i(A)\, dA \qquad \text{(IV.14)}$$

i = regions (0), (I), (II)

IV.2 First Approximation

Suppose

$$\bar{n} = n_o \quad \text{so } E_o = o \qquad \text{(IV.15)}$$

$$\text{and} \int \Delta\phi \, dA = o \qquad \text{(IV.16)}$$

$$\int (\phi - \bar{\phi})\, dA = o \qquad \text{(IV.17)}$$

When the phase shift ϕ_i is constant over the region (i)

$$\phi_o + A_1 \phi_1 + A_2 \phi_2 = \bar{\phi} A_{total} \qquad \text{(IV.18)}$$

When the bulb temperature is such that to zeroth approximation the wall shift vanishes,

$$\bar{\phi} = o \qquad \text{(IV.19)}$$

$$\text{and } A_1 \phi_1 = -A_2 \phi_2 - A_o \phi_o \qquad \text{(IV.20)}$$

$$\delta F = \frac{\bar{\nu}}{8\pi V_-} \left\{ E_1 \left(-A_2 \phi_2 - A_o \phi_o \right) + E_2 A_2 \phi_2 \right\} \qquad \text{(IV.21)}$$

After substitution for the E's in term of the S's and C's

$$\delta F = \frac{-\bar{\nu}}{8\pi V_-} \left(A_2 \phi_2 \frac{S_2}{C_1} - A_o \phi_o \frac{\left(S_1 + S_2 \right)}{C_o} \right) \qquad \text{(IV.22)}$$

Another way of writing this is

$$\delta F = \frac{-\bar{\nu}}{8\pi V_-} \left\{ \phi_1 A_1 \left(\frac{S_1 + S_2}{C_o} \right) + \phi_2 A_2 \left[\frac{S_1}{C_o} + S_2 \left(\frac{1}{C_o} + \frac{1}{C_1} \right) \right] \right\} \tag{IV.23}$$

IV.3 Second Approximation (Using the Proper $\bar{n}$)

Let us compute $\bar{n}$ accurately in terms of n_o, n_1 and n_2. This gives

$$E_i = \frac{n_i}{\bar{n}} - 1 = \frac{n_i - n_o + n_o - \bar{n}}{\bar{n}} \tag{IV.24}$$

$$E_i \sim \frac{n_i - n_o}{n_o} + \frac{n_o - \bar{n}}{n_o} \tag{IV.25}$$

$$E_i \sim \frac{n_i - n_o}{n_o} + \left(1 - \frac{\bar{n}}{n_o} \right) \tag{IV.26}$$

Also by definition of the average

$$\bar{n} = \frac{1}{V_t} \left(n_o V_o + n_1 V_1 + n_2 V_2 \right) \tag{IV.27}$$

$$\bar{n} = n_o + n_1 \frac{V_1}{V_o} + n_2 \frac{V_2}{V_o} \tag{IV.28}$$

$$\text{Calling } \alpha_1 = \frac{V_1}{V_o} \text{ and } \alpha_2 = \frac{V_2}{V_o} \tag{IV.29}$$

$$\frac{\bar{n}}{n_o} = \frac{n_o}{n_o} + \frac{n_1}{n_o} \alpha_1 + \frac{n_2}{n_o} \alpha_2 \tag{IV.30}$$

SO

$$E_i = \left(\frac{n_i - n_o}{n_o} \right) - \left(\frac{n_1}{n_o} \alpha_1 + \frac{n_2}{n_o} \alpha_2 \right) \tag{IV.31}$$

But

$$\int \Delta\phi \; dA = o; \quad \int (\phi - \bar{\phi}) \; dA = o \tag{IV.32}$$

$$A_o \phi_o + A_1 \phi_1 + A_2 \phi_2 = \bar{\phi} A_{total} \tag{IV.33}$$

At the null, $\bar{\phi} = o$ (IV.34)

$$\delta F = \frac{\bar{\nu}}{8\pi V_-} \left\{ E_o \phi_o A_o + E_1 \phi_1 A_1 + E_2 \phi_2 A_2 \right\} \tag{IV.34}$$

After substitution for the E's the expression is

$$\delta F = \frac{-\bar{\nu}}{8\pi V_-} \left\{ \phi_o A_o J + \phi_1 A_1 \left[\left(\frac{S_1 + S_2}{C_o} \right) + J \right] + \phi_2 A_2 \left[\frac{S_1}{C_o} + S_2 \left(\frac{1}{C_o} + \frac{1}{C_1} \right) + J \right] \right\}$$

where

$$J = \frac{n_1}{n_o} \alpha_1 + \frac{n_2}{n_o} \alpha_2 \tag{IV.36}$$

$$J = \left[1 - \left(\frac{S_1 + S_2}{C_o} \right) \right] \frac{V_1}{V_o} + \left[1 - \frac{S_1}{C_o} - S_2 \left(\frac{1}{C_o} + \frac{1}{C_1} \right) \right] \frac{V_2}{V_o} \tag{IV.37}$$

The wall shift error can be written

$$\delta F = \frac{-\bar{\nu}}{8\pi V_-} \left\{ \phi_1 A_1 \left(\frac{S_1 + S_2}{C_o} \right) + \phi_2 A_2 \left[\frac{S_1}{C_o} + S_2 \left(\frac{1}{C_o} + \frac{1}{C_1} \right) \right] \right\} \tag{IV.38}$$

Since $\bar{\phi} = o$ and hence

$$\left(\phi_o A_o + \phi_1 A_1 + \phi_2 A_2 \right) J = o \tag{IV.39}$$

With reference to Figure 4 for the meaning of some of the nomenclature, let us describe some details of the computations. The volumes labelled V_1, V_2 and V_3 are shown in the same Figure and the corresponding areas will have the same subscript (i.e., A_2 refers to the area of volume V_2 etc).

SCHEMATIC OF ONE HALF OF THE VARIABLE VOLUME

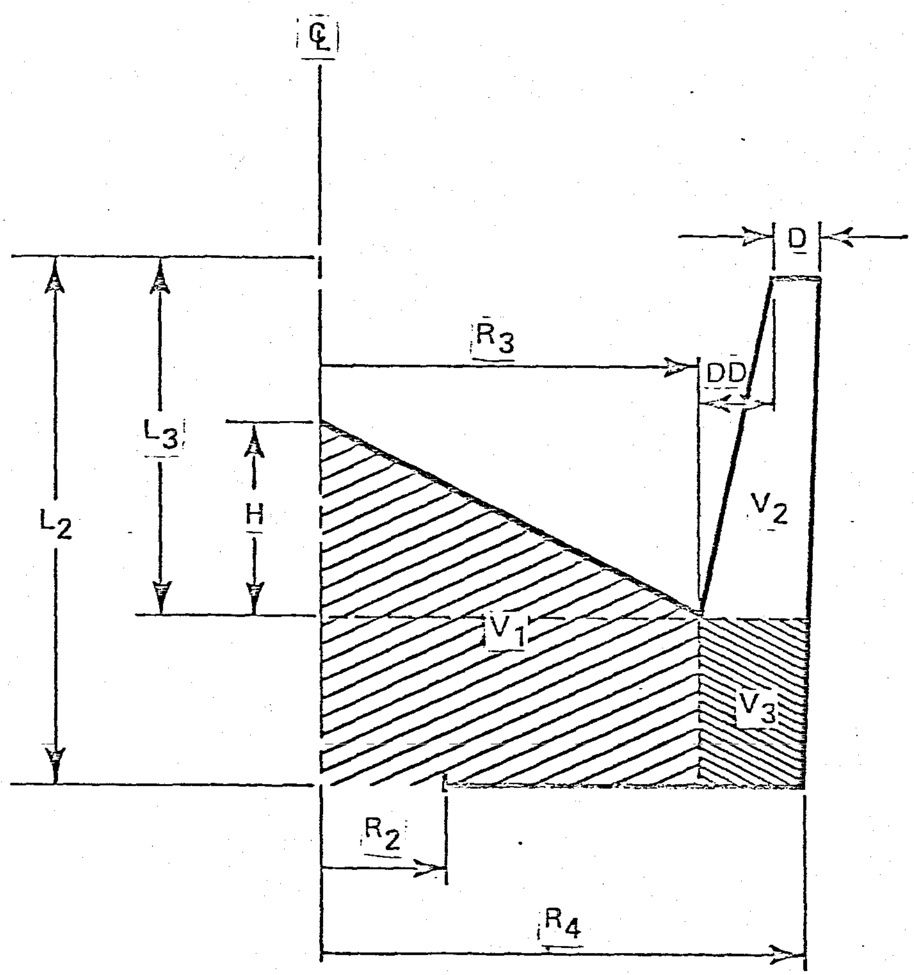

FIGURE 4 - SCHEMATIC OF ONE HALF OF THE VARIABLE VOLUME

So one obtains

$$V_1 = \pi R_3^2 \left(L_2 - L_3 \right) + \frac{1}{3} \pi R_3^2 H \tag{IV.40}$$

$$V_2 = \frac{2}{3} \pi R_4^2 L_3 - \frac{1}{3} \pi L_3 \left(R_3^2 + D^2 - 2R_4 D + R_3 R_4 - R_3 D \right) \tag{IV.41}$$

$$V_3 = \pi \left(L_2 - L_3 \right) \left(R_4^2 - R_3^2 \right) \tag{IV.42}$$

$$A_1 = \pi R_3 \sqrt{H^2 + R_3^2} + \pi \left(R_3^2 - R_2^2 \right) \tag{IV.43}$$

$$A_2 = \pi \left(R_3 + R_4 - D \right) \sqrt{\left(R_4 - D - R_3 \right)^2 + L_3^2} + 2\pi R_4 L_3 - \pi D^2 + 2\pi R_4 D \tag{IV.44}$$

$$A_3 = 2\pi R_4 \left(L_2 - L_3\right) + \pi \left(R_4^{\ 2} - R_3^{\ 2}\right) \qquad \text{(IV.45)}$$

For the equivalent pumping speed we use the expression

$$S_i = \frac{\bar{v}}{4} \; k \; A_i \qquad \text{(IV.46)}$$

together with formula (IV.38) obtained in the second approximation.

CHAPTER V

FILLING FACTOR

$$\eta = \frac{V_b}{V_c}\,\frac{\langle H_z\rangle^2_{bulb}}{\langle H^2\rangle_{cavity}} \tag{V.1}$$

where
$$\langle H_z\rangle_{bulb} = \frac{1}{V_b}\int_{bulb} H_z\,dV \tag{V.2}$$

and
$$\langle H^2\rangle_{cavity} = \frac{1}{V_c}\int H^2 dV \tag{V.3}$$

$$= \frac{1}{V_c}\int_{cavity}\left(H_z^{\,2} + H_r^{\,2}\right) dV$$

For operation in the TE_{011} mode [3]:

$$H_z = J_o\left(S_{01}\frac{r}{a}\right)\sin\left(\frac{\pi z}{d}\right) \tag{V.4}$$

$$H_r = \left(\frac{\pi a}{S_{01} d}\right) J_o'\left(S_{01}\frac{r}{a}\right)\cos\left(\frac{\pi z}{d}\right)$$

$$= -\left(\frac{\pi a}{S_{01} d}\right) J_1\left(S_{01}\frac{r}{a}\right)\cos\left(\frac{\pi z}{d}\right) \tag{V.5}$$

where a = cavity radius

d = cavity length

S_{01} = 3.8317 (first zero of J_1)

If the cavity squatness is described by

$$g = \frac{d}{a} \tag{V.6}$$

Then

$$w = c\left[\left(\frac{S_{01}}{a}\right)^2 + \left(\frac{\pi}{d}\right)^2\right]^{\frac{1}{2}} \tag{V.7}$$

$$d = \frac{pc}{w} \quad \text{and } a = \frac{pc}{gw} \tag{V.8}$$

$$\text{where } p = \sqrt{S_{01}{}^2 g^2 + \pi^2} \tag{V.9}$$

$$S_{01}{}^2 = 14.6819; \; \pi^2 = 9.8696 \tag{V.10}$$

$$\frac{c}{w} = 3.359142360 \text{ cm} \tag{V.11}$$

It is found that the optimum value for $\frac{\rho}{a} = 0.450$

when one writes

$$q_{p_m} = \frac{\sigma\bar{v}\,h}{8\pi\mu^2}\left(\frac{\gamma_t}{\gamma_b}\right)\frac{1}{\eta_0'\,\frac{V_{p_m}}{V_0}}\left(\frac{I_{tot}}{I}\right)\frac{1}{Q} \tag{V.12}$$

$$q_{p_m} = \left(\frac{\gamma_t}{\gamma_b}\right)\left(\frac{I_{tot}}{I}\right)\frac{1}{Q}\;\frac{V_o}{\eta_0'}\left(\frac{\sigma\bar{v}\,h}{8\pi\mu^2 V_{p_m}}\right) \tag{V.13}$$

Then one can define the new factor U

$$q_{p_m}' = \frac{1}{U}\, q_{p_m} \tag{V.14}$$

Using the formulae put forth in section IV one can obtain the systematic frequency shift (section 10 in the program listed in Appendix A). Using the two new values of q obtained when the variable volume is in the open and closed configurations and substituting into equation (III.13) gives the statistical error for different values of U (section 12 of the program listing - Appendix A).

Such that

$$\frac{1}{U} = \left(\frac{\gamma_t}{\frac{\gamma_b}{2}}\right)\left(\frac{\frac{I_{tot}}{I}}{2}\right) \quad \frac{\frac{35000}{Q}\left(V_0 \times .5\right)}{\eta_0 \times 5600} \tag{V.15}$$

We have plotted the statistical and systematic frequency errors as a function of L_3, the external volume length for g = 2, 3 and 4 for U = 1.5 and 2. These plots are shown in Figures 5, 6 and 7. The values of g = 3 or 4 for U = 1.5 or 2 and $L_3 \sim 10$ cm give error values that are acceptable for the present work.

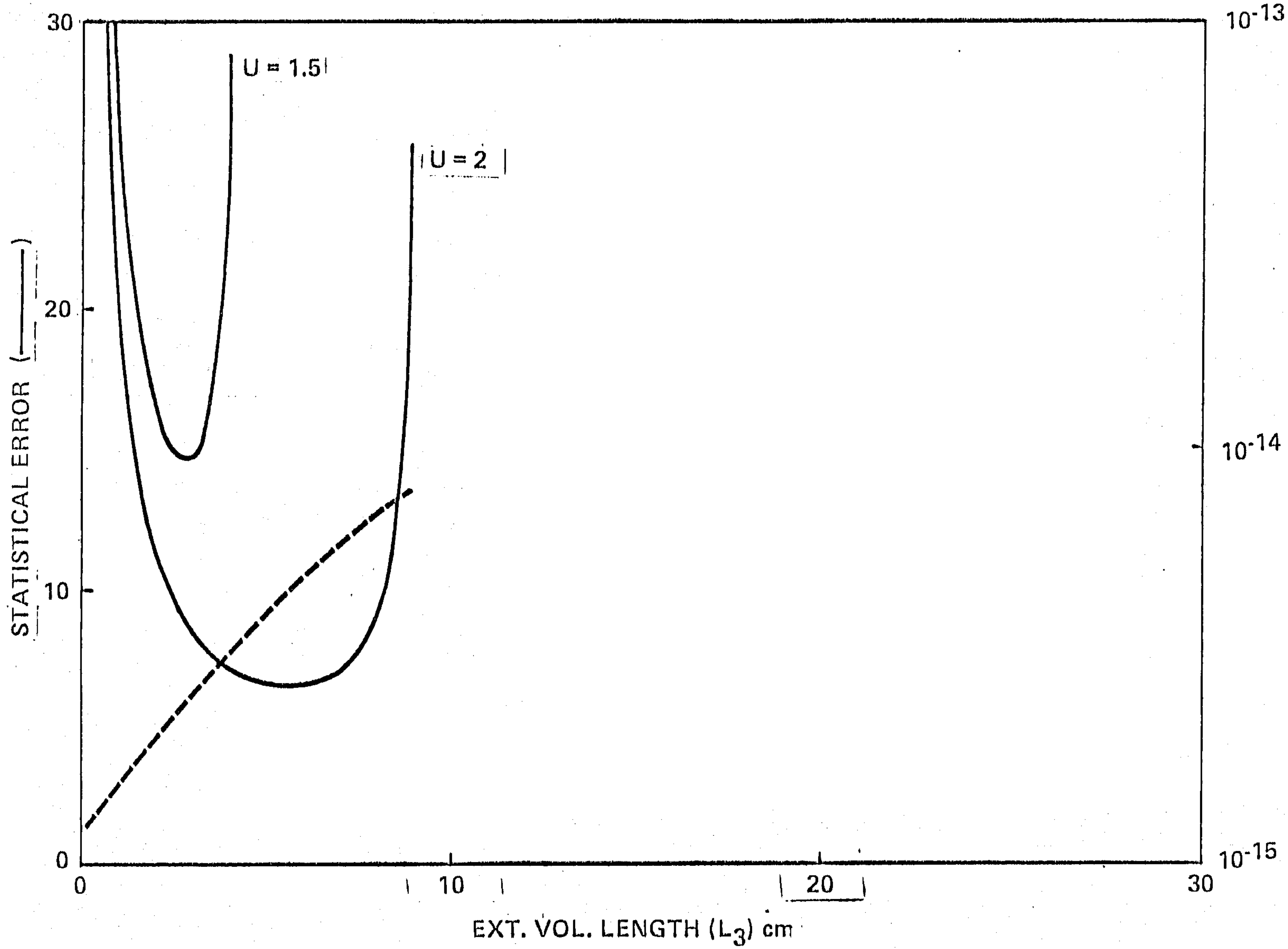

FIGURE 5 - STATISTICAL AND SYSTEMATIC ERROR FOR G = 2

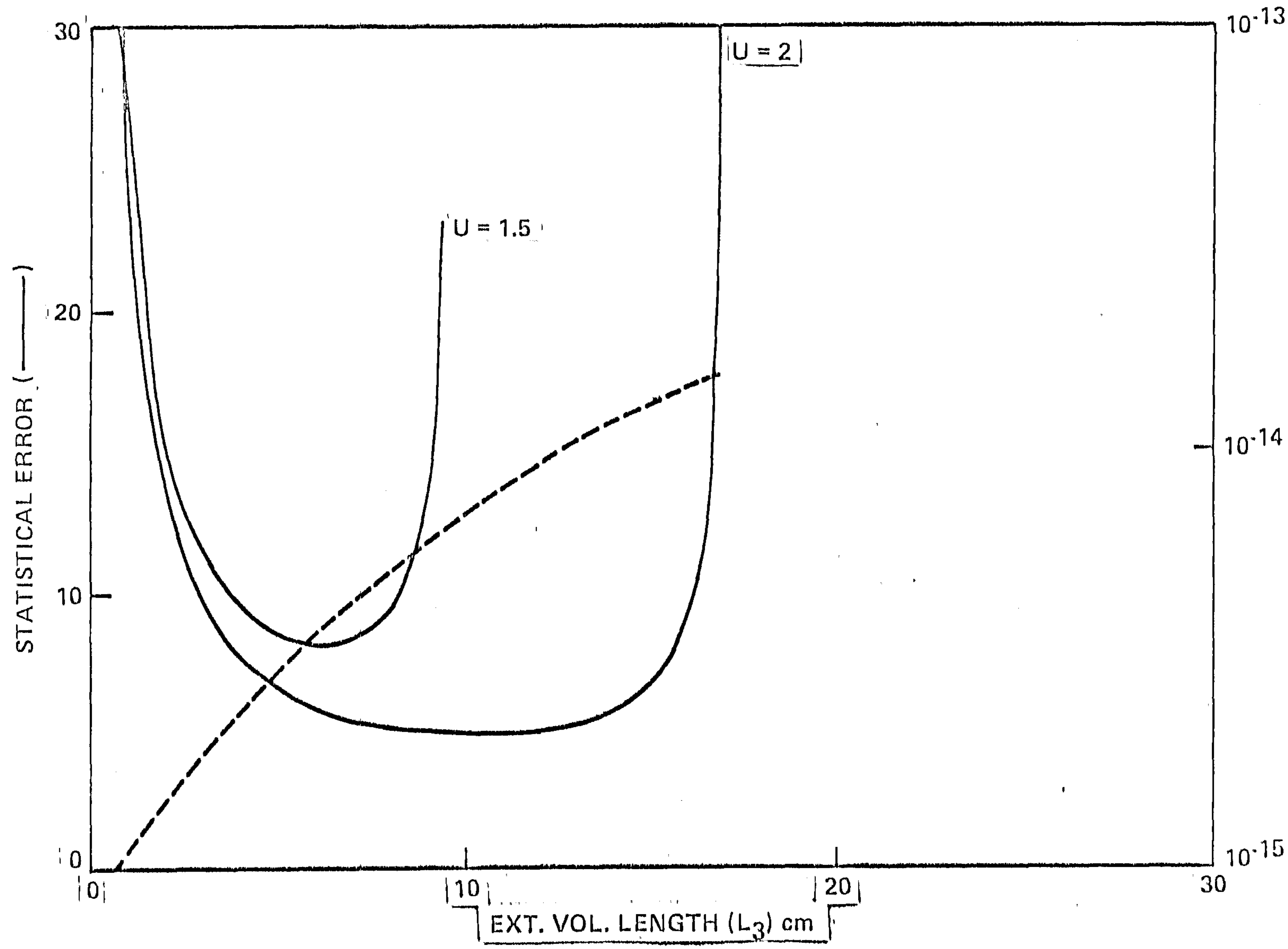

FIGURE 6 - STATISTICAL AND SYSTEMATIC ERROR FOR G = 3

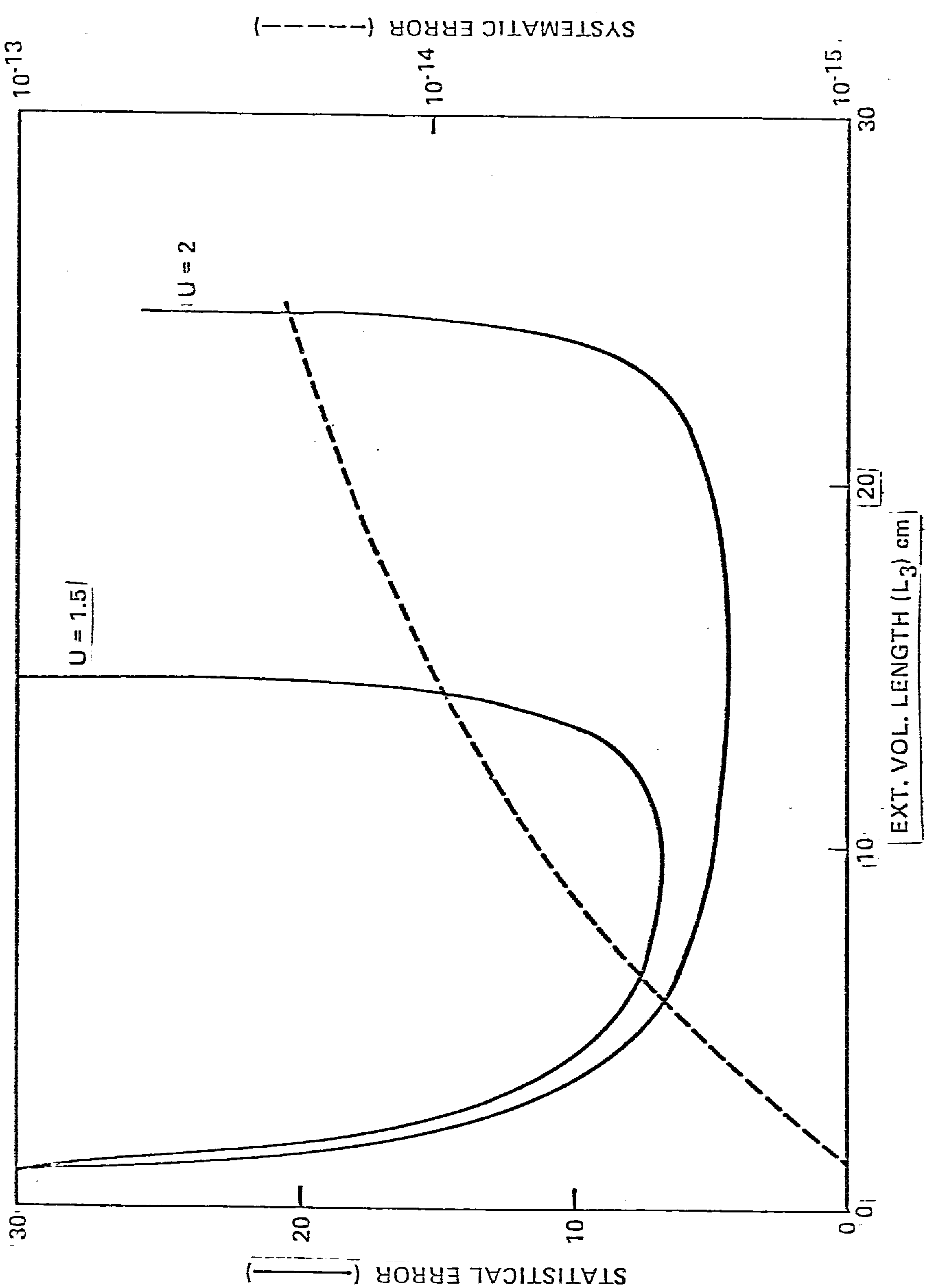

FIGURE 7 - STATISTICAL AND SYSTEMATIC ERROR FOR G = 4

CHAPTER VI

ACTUAL DESIGN - CONCLUSIONS

Referring to Figure 2 for the meaning of the various quantities, the dimensions for a trial configuration are listed below.

A. Fixed Volume

The cavity is to be made of aluminum and the bulb will be cylindrical with a radius equal to .45 that of the cavity. The maximum length of the cavity can be about 30" (limited to the length of Permalloy sheets commercially available) and is equal to the length of the bulb.

g is the ratio of length to radius of the cavity. For

$$g = 4 = \frac{d}{a_{cavity}} = \frac{d \times .45}{a_{bulb}} \qquad \text{(VI.1)}$$

So $a_{bulb} = \frac{d \times .45}{4} \simeq \frac{30 \times 2.54 \times .45}{4} = 8.6$ cm

is the radius of the bulb.

B. Variable Volume

For a maser stable to one part in 10^{14}, the dimensions for the variable volume can be:

$V_0 = 6850$ cc (for $g = 4$) $\eta' = .442$

$R_2 = 7$ cm; $H = 3$ cm; $R_3 = 11$ cm; $R_4 = 13$ cm

$D = .5$ cm; $DD = .5$ cm

$L_2 = 12$ cm; $L_3 = 10$ cm

Bulb Material: FEP Teflon.

APPENDIX A.-VARIABLE VOLUME HYDROGEN MASER FOCAL PROGRAM

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 2.10 C PROGRAM HMD18           DATE:12-JAN-77
 2.20 C FOR A VARIABLE VOL. CONTAINING A CUT-OFF IN THE
 2.30 C NARROW PART*
 2.40 C
 2.50 C THIS PROGRAM ALSO HAS CHANGING ETA PRIME
 2.60 C PROGRAM MODIFICATION DATE: 17-JAN-77

 8.90 C [THE FOLLOWING SECTION SETS UP ALL THE DEFAULT VALUES]

 9.01 S W1=-3.9E-06
 9.03 S U=1
 9.05 S I=1
 9.10 S W2=W1
 9.15 S R2=7
 9.25 S VB=2.5E+05
 9.30 S W3=W1
 9.40 S V0=3425
 9.60 S A0=1094
 9.65 S H=3
 9.70 S K=1/50000
 9.75 S D=.5
 9.80 S L2=10
 9.90 S L3=8
 9.92 S R4=13
 9.95 S R3=R4-1-D
 9.99 S PI=3.1415926

10.10 C [THIS PART CALCULATES THE SYSTEMATIC ERROR]
10.11 C
10.20 S L3=I;S L2=2+I
10.23 S V1=PI*(R3^2*(L2-L3)+.3333*R3^2*H)
10.25 S V2=PI*(R4^2*L3-.3333*L3*(R3^2+(R4-D)^2+R3*(R4-D)))
10.26 S V3=PI*(L2-L3)*(R4^2-R3^2)
10.28 S VM=V0+V1+V2+V3
10.30 S C1=2*PI*R3*(L2-L3)
10.35 S C2=PI*(R4^2-R3^2)
10.40 S C0=PI*(R2^2)
10.50 S A1=(PI*R3*FSQT(R3^2+H^2)+PI*(R3^2-R2^2))
10.60 S A2=FSQT((R4-D-R3)^2+L3^2)
10.61 S A2=(R3+R4-D)*A2+2*R4*L3
10.62 S A2=A2+R4^2-(R4-D)^2
10.63 S A2=PI*A2
10.65 S A3=PI*(2*R4*(L2-L3)+(R4^2-R3^2))
10.70 S ZZ=(A1+A2+A3)/C0
10.75 S ZX=W1*(A1+A3)*ZZ+W2*A2*(ZZ+A2/C2)
10.80 S ZY=-VB*K*ZX/(8*PI*VM)
10.85 S ZR=W1*A1*ZZ+W2*(A2+A3)*(ZZ+(A2+A3)/C1)
10.90 S ZS=-VB*K*ZR/(8*PI*VM)
10.92 I (ZS-ZY)10.95,10.95,10.97
10.95 S SE=ZY;G 10.98
10.97 S SE=ZS
10.98 S SE=SE/1.42E09
10.99 D 11
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11.05 C [CALCULATION OF B]
11.06 C
11.10 S VN=R4^2*L2+L3/3*(R3^2+(R4-D)^2+R3*(R4-D))
11.12 S VN=PI*(VN+R3^2*H/3)
11.30 S VP=V0+VN
11.50 S B=VP/VM
11.70 S QA=VP/V0
11.90 S EX=0
11.95 D 12

12.10 C [COMPUTE THE STATISTICAL ERROR ON THE FREQUENCY USING
12.20 C THE NEW VALUE OF B WITH THE ERROR FORMULA FROM HMD01]
12.30 C
12.40 S EP=.5
12.50 S C=.213669E+03/EP
12.52 S C=C/U
12.60 S Q0=C*8/35000;
12.62 S Q=Q0*VM/V0
12.70 S SQ=1-6*Q+Q^2;I (-SQ)12.72,12.72;Q
12.72 S SQ=FABS(SQ)
12.73 S J=FSQT(SQ)
12.80 S RM=(1+Q+J)/(1+Q-J)
12.81 S Q=Q0*VP/V0
12.82 S SQ=1-6*Q+Q^2;I (-SQ)12.83,12.83;Q
12.83 S J=FSQT(SQ)
12.84 S RP=(1+Q+J)/(1+Q-J)
12.85 S G=(B^2*(RP^2+1))/((RP-1)^2)
12.90 S H=(RM^2+1)/((RM-1)^2)
12.95 S ER=FSQT(G+H)/(B-1)
12.99 S Y=ER
```


[INSERT AT THIS LOCATION A SUITABLE PLOTTING PROGRAM]

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