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THE ORBIT OF LAGEOS AND SOLAR ECLIPSES

INTRODUCTION

We wish to point out the importance of the effect of solar eclipses on the orbit of the Lageos satellite.

Solar radiation pressure perturbs Lageos' orbit. The orbit determination computer programs currently in use include this effect when they integrate the orbit. They also take into account the interruption of sunlight when Lageos moves into the earth's shadow.

These programs do not, in most cases, at present take into account the diminution of radiation pressure when Lageos moves into the moon's shadow, i.e., suffers a solar eclipse by entering the moon's umbra or penumbra. This diminution will affect Lageos' orbit by weakening for a time the radiation pressure acting on the satellite, thus perturbing the orbit differently from what it would if full sunlight were shining. The importance of this effect must be assessed for Lageos' orbit. An accurate orbit is necessary for Lageos to accomplish its mission of monitoring tectonic plate motion, polar motion, and earth rotation. For more information on Lageos, see Smith and Dunn (1980) and Rubincam (1982).

In particular, we examine how the eclipses that occurred between launch on 4 May 1976 and the end of 1983 affected the semimajor axis. We show that some eclipses have perturbed the orbit at the one centimeter (0.01 m) level. This is significant, since a 1 cm change in the semimajor axis translates into an along-track error of 9 m over a period of 15 days.

SEMIMAJOR AXIS CHANGE

We now derive an approximate equation for the change in the semimajor axis due to an eclipse. We first assume that the acceleration $\ddot{\vec{r}}$ due to solar radiation pressure has magnitude $\pi R_L{}^2 B F_S C_R / c M_L$ and is directed away from the center of the sun. B is fraction of the area of the sun not obscured by the moon, so that B = 1 for full sunlight and 0 when the moon completely covers the sun.

The other quantities are given in Table 1. Next, we assume that Lageos' orbit is circular, so that the change in semimajor axis a with time t is given by da/dt = 2S/n, where n is the mean motion and S is the tangential acceleration (e.g., Blanco and McCuskey, 1961, p. 178). S may be found from $\ddot{\vec{r}}$ by using the transformation given in Rubincam (1982, p. 370). Substituting the resulting equation for S in the equation for da/dt and integrating gives

$$\Delta a = \frac{2\pi R_L{}^2 F_S C_R a^3}{cGM_E M_L} \left\{ [\cos\Omega \cos U_S + \sin\Omega \cos I_S \sin U_S] \int_{U_1}^{U_2} (B-1) \sin U \, dU \right.$$
$$+ [\sin\Omega \cos I \cos U_S - \cos\Omega \cos I \cos I_S \sin U_S - \sin I \sin I_S \sin U_S]$$
$$\left. \int_{U_1}^{U_2} (B-1) \cos U dU \right\} \quad (1)$$

where we have used dU = ndt. The other quantities appearing here are explained in Table 1.

This equation gives the difference between the change in a due to full sunlight and that due to an eclipse over the eclipsed arc of the orbit; hence the factor B−1. This factor is given by

$$B - 1 = (1/\pi)\left[(\theta/\theta_S)\sqrt{1-(\theta^2/4\theta_S{}^2)} - 2 \operatorname{Arcsin}\sqrt{1-(\theta^2/4\theta_S{}^2)}\right] \quad (2)$$

where θ is the angular separation of the sun and moon, while θ_S is the angular radius of the sun. This equation comes from considering two disks of equal size overlapping each other, so that the angular radius of the moon θ_M is assumed to be equal to θ_S. All quantities appearing outside the integrals in (1) are taken to be constant during the course of each eclipse. Also, the orbit of the earth about the sun is assumed to be circular.

LAGEOS ECLIPSES

How many times was the sun eclipsed by the moon as seen by Lageos, for the period between launch and the end of 1983? To answer this question, we looked at Lageos, solar, and lunar positions every 30 seconds from one day before to one day after each new moon. The Lageos positions came from two long-arc orbit solutions, one from 1976 to 1981, and the other from 1982 to 1983.

Both solutions assumed that the along-track acceleration due to charge drag (Rubincam, 1982) was -4.23×10^{-12} m s^{-2}. The solar and lunar positions came from a Jet Propulsion Laboratory ephemeris tape. Times of new moon were taken from the Nautical Almanacs for the appropriate years. The angle θ between the sun and the moon was computed from the Cartesian positions via dot products. Whenever θ was less than $\theta_M + \theta_S$, and Lageos was not in the earth's shadow, the moon at least partially obscured the sun as seen by Lageos. (In this calculation θ_M was not assumed to be equal to θ_S as in (2); rather θ_M and θ_S had their actual values.)

To find the change in semimajor axis, we computed the integrals appearing in (1) numerically for each eclipse by computing $B-1$ and U at each 30 second time step. The values for Ω in (1) came from the Lageos GEODYN positions, while U_S came from the Nautical Almanacs.

RESULTS

The results are summarized in Table 2. There were 30 solar eclipses seen by Lageos between launch on 4 May 1976 and the end of the year on 31 December 1983, an average of 4 per year. On eight occasions there were two eclipses during the same new moon (numbers 3 and 4, 6 and 7, etc.). There were only three occasions (numbers 3, 15, and 19) on which Lageos spent time in the earth's shadow while an eclipse was occurring.

All of the eclipses were penumbral; Lageos never entered the moon's umbra. Interestingly, number 27 was annular. All eclipses occurred within 4 hours of new moon. An eclipse lasted an average of 18 minutes. The shortest, number 15, was 2½ minutes long; Lageos spent most of this eclipse in the earth's shadow (it would have been 21½ minutes long had the earth been transparent). The longest eclipse was number 11 at 57 minutes; it also gave the biggest change in the semimajor axis.

Most of the eclipses had little effect on the semimajor axis of Lageos' orbit, as can be seen from Table 2. Only seven eclipses changed a by more than 2 mm from what it would have been due to full sunlight. However, the eclipses on 28 March 1979 (number 11) and 15 December 1982 (number 27) changed a by more than a centimeter. The one on 28 March 1979 was the biggest, giving $\Delta a = +17.6$ mm. It effectively cancelled about 16 days' worth of charge drag on the satellite. (Charge drag decreases a at the rate of about 1.1 mm day^{-1}; see Rubincam, 1982 and Afonso *et al.*, 1984.)

DISCUSSION

Figure 1 shows the currently unmodeled variations in along-track acceleration from launch to about the middle of 1983 (Christodoulis and Smith, 1983). The average acceleration of -3.3×10^{-12} m s^{-2} is due to charged particle drag (Rubincam, 1982; Afonso *et al.*, 1984).

These unmodeled variations limit the accuracy of our Lageos solutions, because we assumed a constant along-track acceleration of -4.23×10^{-12} m s^{-2}. To estimate their effect, suppose we made a constant error of 3×10^{-12} m s^{-2} in the along-track acceleration. Over 7½ years the error in position would be roughly $(3 \times 10^{-12}$ m s$^{-2}) \times (7½$ yrs$)^2/2 = 84$ km. This would give an error in the moon's position of about 1 minute of arc. Since the moon's diameter is about 30 minutes of arc, it is clear that the errors involved will not seriously affect out eclipse calculations.

Obviously the eclipses themselves cannot explain all of the observed variations shown in Figure 1. The effect of eclipses in general is too small. For instance, the biggest eclipse on 28 March 1979 (the arrow in Figure 1 designates the date) did not have any obvious appreciable effect. Also, variations were observed during periods when there were no eclipses at all, such as between April and October of 1978. Most of the variations are believed to be due to terrestrial radiation (e.g., Anselmo *et al.*, 1983; Smith, 1983) and fluctuations in charge drag (Afonso *et al.*, 1984).

An eclipse can nevertheless have a sizeable effect on the along-track position. Since an average eclipse lasts only 18 minutes, on the time scale of a day or more the semimajor axis will appear to undergo a sudden change. In effect it is a step function, changing from one constant value to another (ignoring of course other influences on a). If the eclipse is not allowed for, then an along-track error $\Delta s = -3\Delta a\, nt/2$ will build up over time t. For the eclipse on 28 March 1979, this amounts to about 16 m over 15 days. This is big enough to make it worthwhile to include eclipses of the sun by the moon as seen by Lageos in programs such as GEODYN which integrate the orbit.

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Table 1
Data necessary to compute the change in semimajor axis.
Dashes (–) indicate quantities which vary from eclipse to eclipse.

Quantity	Symbol	Numerical Value
Lageos semimajor aixs	a	1.227×10^{7} m
Speed of light	c	2.9979×10^{8} m s^{-1}
Lageos radiation coefficient	C_R	1.13
Solar constant	F_S	1.36×10^{3} W m^{-2}
Gravitation constant times earth mass	GM_E	3.986×10^{14} m^{3} s^{-2}
Obliquity of ecliptic	I_S	23.4432 deg
Lageos inclination	I	109.9 deg
Lageos mass	M_L	411 kg
Lageos radius	R_L	0.3 m
Mean longitude of sun	U_S	–
Lageos mean longitude	U	–
U at eclipse beginning	U_1	–
U at eclipse end	U_2	–
Lageos node	Ω	–

Table 2
Semimajor axis change for eclipses occurring between 4 May 1976 and 31 December 1983.

Number	Date	Δa (mm)
1	21 November 1976	+ 0.0
2	18 April 1977	+ 1.5
3	12 October	- 0.4
4	12 October	- 0.9
5	9 March 1978	- 2.5
6	7 April	- 0.6
7	7 April	+ 1.0
8	2 October	+ 0.0
9	26 February 1979	- 1.4
10	26 February	- 0.2
11	28 March	+17.6
12	21 September	- 0.2
13	21 September	+ 0.3
14	16 February 1980	- 0.1
15	16 February	+ 0.1
16	10 August	- 4.6
17	5 February 1981	+ 2.2
18	1 July	- 6.4
19	31 July	+ 0.1
20	26 December	+ 1.0
21	26 December	+ 0.0
22	25 January 1982	+ 1.2
23	25 January	+ 0.6
24	21 June	+ [illegible]
25	20 July	- 1.[illegible]
26	15 December	+ 0.2
27	15 December	+11.2
28	14 January 1983	- 1.0
29	11 June	+ 1.3
30	4 December	- 0.1

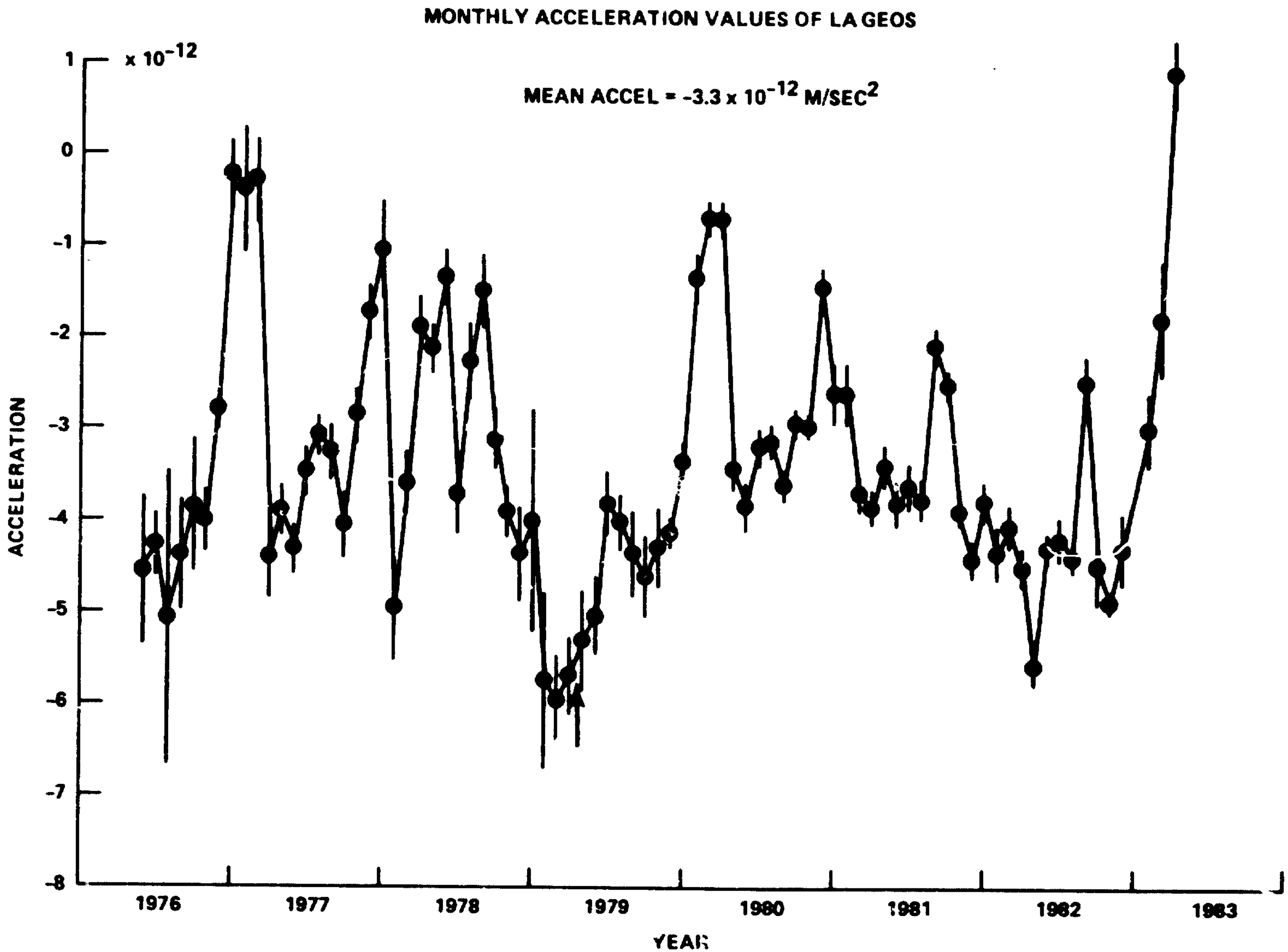

Figure 1. The unmodeled variations in along-track acceleration experienced by Lageos, for the period mid-1976 to mid-1983. The arrow marks the date of the eclipse with the largest effect on the semimajor axis: 28 March 1979.