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Turbulence Modeling

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ABS: A survey designed to provide an introduction to the subject of turbulence modeling, and to explain the need for such models is given. The subject is developed along chronological lines since this provides a logical development plan and also because it then moves from relatively simple phenomenological models through more complicated procedures and ultimately to the subject of large-eddy simulation.

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Turbulence Modeling

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ABSTRACT

The following is a survey article designed to provide an introduction to the subject of turbulence modeling, and to explain the need for such models.

The subject is developed along chronological lines since this provides a logical development plan and also because it then moves from relatively simple phenomenological models through more complicated procedures and ultimately to the subject of large-eddy simulation.

"...a numerical procedure without a turbulence model stands in the same relation to a complete calculation scheme as an ox does to a bull."

Peter Bradshaw

I. INTRODUCTION

For over a hundred years intelligent people have worked long hours and written thousands of technical papers on the last unsolved problem of classical physics, turbulent fluid flow. Indeed, there appears to be an emerging belief that the problem may not have a solution in the usual sense of the word. That is to say that turbulence seems to be simply the manifestation of ensemble or time averages of bounded instabilities resulting from multiple bifurcations in solutions of the Navier-Stokes equations. This is not exactly a helpful phenomenological explanation, but it may well be the best we will be able to give.

Because of the absence of a consistent and comprehensive theory of turbulent flows, their prediction cannot be made from first principles, but must be based on semiempirical models. Modeling the physics rather than solving the full, unsteady, three-dimensional Navier-Stokes equations is imposed by the fact that even the largest of today's computers are inadequate to predict any real flow.

In this chapter, rather than attempting to treat the multiple potential applications of turbulent-flow predictions, we will consider the background and development of our current predictive capability in a more or less general setting. Several other works presenting more specific information, or an alternative point of view, can be found in references [1]-[5].

Hinze [1] defines turbulence as follows: "Turbulent fluid motion is an irregular condition of flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be observed." A few minutes of observing smoke emerging from a tall stack will be sufficient to provide a feeling for the above definition.

There is a consensus among researchers of turbulence that the unsteady, three-dimensional Navier-Stokes equations solved on a fine enough mesh can describe turbulent flow from first principles. Eventually, given computers which are sufficiently large, sufficiently fast, and sufficiently cheap, we will be able to solve these equations directly, and the problem of turbulence modeling will simply vanish. Thus, the need for turbulence modeling is caused solely by our inability to solve the equations that describe the physical problem on a fine enough mesh to resolve all the relevant physical scales. The obvious question which arises is, why must anyone continue to work on it if, in time, the problem will go away? The answer is that the time scale for the development of the requisite computational power, on machines and algorithms, is very large. Lomax [6] has pointed out that existing computer memories will accommodate about a 64^3 mesh for compressible turbulent-flow calculations, whereas a computer memory on the order of 512^3 mesh (roughly a three-order-of-magnitude increase) would be required to compute a portion of an incompressible, homogeneous, turbulent flow. Furthermore, for any industrially significant flow, meaningful calculations of turbulence from first principles would probably require at least another two-order-of-magnitude increase of computer memory. Such a powerful computer would permit computation of eddies of a scale larger than that associated with the Kolmogorov equilibrium scale, coupled with a hypothesized universal model at smaller scales. Since the required machine time varies typically with the square of the number of mesh points, most likely 10 orders of magnitude increase in computer power is needed before we are in a position to handle real turbulent-flow problems from first principles on a routine basis. Without a breakthrough in either computer design or turbulence theory it would appear that the millennium is 50 to 100 years in the future. This bleak, long-term outlook provides the motivation to seek turbulence models for application to the Reynolds-averaged equations. These equations will be discussed in detail in subsequent sections.

The techniques of turbulent-flow prediction currently in use or under development follow traditional lines since much of the foundations of current theory were developed long before the digital computer came into significant use. To understand these traditional approaches, let us return to Hinze's definition of turbulence. The relevant elements, for our purposes, can be restated as: the flow is irregular, unsteady, three dimensional, and statistically distinct averages exist. It is this last property which makes the flow approachable at all.

In the following sections we will describe the equations of turbulent flow and show how the need for modeling arises, discuss some of the representative examples of today's turbulence models and some aspects of the computational schemes required, touch on some special topics (i.e., separation, unsteady flows, and three-dimensional flows), and, finally, briefly consider the topic of Large Eddy Simulation and its promise for the future.

II. EQUATIONS OF TURBULENT FLOW

In this section, and in those that follow, we will discuss the steady, incompressible Navier-Stokes equations. Consider, for example, the continuity and x-momentum equations

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho} \frac{dp}{dx} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \tag{2}$$

These equations contain most of the complexities associated with the turbulence itself, and the additional problems of compressibility, etc. do not serve to illuminate the discussion in any significant way.

Suppose that the various flow parameters have time histories like that shown in figure 1. The velocity u can be decomposed into two parts as

$$u(t) = \bar{u} + u'(t) \tag{3}$$

where $\bar{u}$ is a steady carrier and $u'(t)$ is a high-frequency oscillatory component with a zero mean. From this decomposition of the velocity we can define several time averages as

$$\bar{u} = \frac{1}{T} \int_t^{t+T} u(t)dt \tag{4}$$

and

$$\overline{u'} = \frac{1}{T} \int_t^{t+T} [u(t) - \bar{u}]dt = \frac{1}{T} \int_t^{t+T} u'(t)dt = 0 \tag{5}$$

Similarly,

$$\frac{1}{T}\int_t^{t+T} u(t)v(t)dt = \overline{uv} + \overline{u'v'} \tag{6}$$

and

$$\frac{1}{T}\int_t^{t+T} u(t)u(t)dt = \overline{u^2} + \overline{u'^2} \tag{7}$$

If we substitute these composite variables into the equations of motion, expand, and time-average, we obtain equations like the following

$$\begin{aligned}\bar{u}\frac{\partial\bar{u}}{\partial x} + \bar{v}\frac{\partial\bar{u}}{\partial y} + \bar{w}\frac{\partial\bar{u}}{\partial z} &= -\frac{1}{\rho}\frac{dp}{dx} + \nu\left(\frac{\partial^2\bar{u}}{\partial x^2} + \frac{\partial^2\bar{u}}{\partial y^2} + \frac{\partial^2\bar{u}}{\partial z^2}\right) \\ &\quad - \left(\frac{\partial\overline{u'^2}}{\partial x} + \frac{\partial\overline{u'v'}}{\partial y} + \frac{\partial\overline{u'w'}}{\partial z}\right)\end{aligned} \tag{8}$$

Note that we recover the original equation, in terms of time-averaged velocities, the barred quantities, but with the three additional terms, the so-called Reynolds stresses included in the rightmost parentheses. These additional unknowns result from time-averaging the nonlinear convective terms and have the appearance of stresses.

When we substitute this velocity decomposition in all three momentum equations, the stress tensor for these Reynolds-averaged equations becomes

$$\tau_{ij} = \begin{bmatrix} (\tau_{xx} - \overline{u'^2}) & (\tau_{xy} - \overline{u'v'}) & (\tau_{xz} - \overline{u'w'}) \\ (\tau_{yx} - \overline{v'u'}) & (\tau_{yy} - \overline{v'^2}) & (\tau_{yz} - \overline{v'w'}) \\ (\tau_{zx} - \overline{u'w'}) & (\tau_{zy} - \overline{w'v'}) & (\tau_{zz} - \overline{w'^2}) \end{bmatrix} \tag{9}$$

where the τ_{xx}, τ_{xy}, etc. are the usual viscous stresses. The nine new unknowns, the Reynolds stresses, have been introduced by the decomposition and time-averaging of the velocities. Because of the skew symmetry of the matrix only six of the nine new unknowns are independent. If we attempt to increase the number of equations, for example, by taking successive moments of the momentum equation, the number of unknowns continues to proliferate and this shortfall between equations and unknowns has been termed the "turbulence closure problem."

The complete Navier-Stokes equations in Reynolds-averaged, mass-averaged form, including the Reynolds stress and turbulent heat-transfer terms, are presented in orthogonal tensor form and cylindrical coordinates by Rubesin and Rose [7].

III. SIMILARITY LAWS

Before we turn our attention to the modeling of the Reynolds stresses, it is useful to consider some experimental results. One of the most important classes of flows, and hence one for which a great deal of experimental data exist, is the wall-bounded shear flow. If we restrict our attention to a two-dimensional steady flow, and without complications such as wall mass transfer or separation, virtually all the reliable experimental data can be correlated by the Law of the Wall and the Law of the Wake. These similarity laws can be written in combined form, following Coles [8], as

$$u^+ = \frac{\bar{u}}{u_\tau} = \frac{1}{\kappa} \ln y^+ + \frac{\Pi}{\kappa} 2 \sin^2 \left(\frac{\pi}{2} \frac{y}{\delta}\right) + C \tag{10}$$

where

$$\kappa = 0.41$$

$$C = 5.0$$

$$y^+ = \frac{y u_\tau}{\nu}$$

$u_\tau = \sqrt{\tau_w/\rho}$

δ = boundary-layer thickness

Π = wake strength

This expression provides a good fit to the experimental velocity data over a wide range of pressure gradients in terms of the three parameters, u_τ, Π, and δ. Figure 2 shows how well this correlation works: for a strong adverse pressure gradient, figure 2a; a zero pressure gradient, figure 2b; and a strong favorable pressure gradient, figure 2c. The data are taken from the 1968 Stanford Conference Proceedings [7]. The Law of the Wall, the linear portion of the correlation on the semilog plot, represents only about the inner 20% of the boundary layer in an adverse pressure gradient and about the inner 50% of the boundary layer in a favorable pressure gradient. The important point to note is that, despite the apparent chaos in the instantaneous values of the flow parameters, the time averages can be correlated by a judicious choice of variables; further, these averages are reproducible in different wind tunnels and with different instrumentation as long as the appropriate dimensionless parameters are held fixed. In addition, these correlations of experimental data provide insight into those parameters that govern the flow field. Such insight is critical to the construction of rational models of the turbulent mixing process. It should always be borne in mind that no improvement in turbulence modeling can take place without continued careful experimental studies being carried out, and without exhaustive analysis of the data resulting from these studies.

IV. TURBULENCE MODELING

Turbulence modeling concerns itself with the generation and testing of closure relations describing the Reynolds stresses. Since its inception, the goal of turbulence modeling has been the "universal model." We may define this as an equation, or system of equations, capable of accurately describing the Reynolds stresses of any turbulent flow without any previous experimental information. Such a model does not as yet exist and, in fact, may never exist. At present we can confidently predict only a relatively small class of turbulent flows or, more properly, a few small classes of flows. These predictable flows fall into the category of equilibrium flows, which in turn may be defined as flows in which the production and dissipation of turbulence energy, shear stresses, etc. are in balance. This balanced condition implies that there exists a one-to-one relationship between the Reynolds stresses and the mean flow. Physically, this means that the turbulent flow is independent of its history, or that it has existed for a significant time under the effects of mild pressure gradients and without recent or rapid changes in the boundary conditions, for example, blowing, bleed, slip, etc. Examples of such flows are boundary layers on mildly curved bodies at subsonic speeds, shock-free supersonic flows, far wakes, and mixing layers well downstream of their initiation.

Turbulence models can be divided into three major categories: models suitable for integral methods, eddy-viscosity models, and Reynolds-stress models. We will discuss each of these categories in the following paragraphs.

A. Integral Method Models

Many of the practical calculation schemes, including integral methods, are based on a subset of the Navier-Stokes equations termed the boundary-layer equations. For flows in which v/u is everywhere a small quantity, this system of equations, although substantially simpler than the Navier-Stokes system, contains all the relevant physics. In subsequent sections of the text we will consider these equations almost exclusively, since they simplify the discussion without significant loss of generality. The fundamental simplifying assumption is that variations with respect to x are much smaller than those with respect to y. This reduces the number of equations by one, for the two-dimensional case, by deleting the y-momentum equation, thus making the pressure distribution a parameter of the problem rather than part of the solution and changing the remaining system of equations to parabolic in x rather than elliptic. This latter change permits substantial savings in computing costs since the resulting system can be solved by a marching procedure rather than by a sweeping method. The earliest methods developed for the computation of boundary-layer flows were integral methods, and such methods are still in use today. The principal use of these methods today is for design studies where many sets of calculations for the same kinds of flows are required. In this application where the calculation is used to interpolate between experimentally established bench-marks, integral methods are without peer. These methods are computationally very fast and permit the easy incorporation of empirical information via data correlations. In discussing these methods, we are called upon to discriminate between the requirements of engineering application and of science. In the first case, we are concerned with the effects of fluid flow on an object of arbitrary shape; in the second, we are concerned with the effects of an object of arbitrary shape on a fluid flow. The choice of viewpoint determines the type and detail of information sought.

In applied engineering one is concerned with such parameters as pressure distribution and skin friction, and any method which will provide this information accurately and cheaply for the particular flow configuration at hand, is satisfactory. Given an adequate data base and a limited range of performance requirements, integral methods are quite satisfactory.

One can obtain any of several integral methods by simply integrating the momentum and continuity equations with respect to y. Any text on classical boundary-layer theory will provide examples of how this can be carried out. Additional relations may be obtained by taking moments of this equation with respect to u and its powers before integration (ref. [9]).

The basis of most integral methods currently in use is Head's Entrainment Method [10]. In this method the momentum and continuity equations are integrated across the boundary layer to yield the Karman Integral Equation

$$\frac{d\theta}{dx} + (2 + H) \frac{\theta}{u_e} \frac{du_e}{dx} = \frac{C_f}{2} \tag{11}$$

where

$$\theta = \int_o^\delta \frac{u}{u_e}\left(1 - \frac{u}{u_e}\right) dy$$

$$\delta^* = \int_o^\delta \left(1 - \frac{u}{u_e}\right) dy$$

$$H = \frac{\delta^*}{\theta}$$

$$\frac{C_f}{2} = \frac{\tau_w}{\rho_e u_e^{\,2}}$$

This provides us with one ordinary differential equation in the three unknowns, skin friction coefficient C_f, momentum thickness θ, and shape factor H. A second equation is obtained by integrating the continuity equation across the boundary layer over the same limits, and combining the result with the definition of displacement thickness, δ^*; that is,

$$\frac{d}{dx}(\delta - \delta^*) = \bar{v}(\delta) - \frac{\delta - \delta^*}{u_e}\frac{du_e}{dx} \tag{12}$$

where $\bar{v}(\delta) = v_e/u_e$ is proportional to the rate at which free-stream fluid is entrained into the boundary layer. Using Head's notation, let $\bar{v}(\delta) = F$ and $(\delta - \delta^*) = \Delta$, we can define a new shape factor as

$$H_1 = \frac{\Delta}{\theta}$$

where, for power-law velocity distributions, $u/u_e = (y/\delta)^n$,

$$H_1 = \frac{2H}{H - 1}$$

Green [11] notes that experimental data for Head's $F(H_1)$ can be well correlated by the linear function of H

$$F = 0.025H - 0.022 \tag{13}$$

Differentiating equation (13) with respect to x and combining equations (11) and (12) yield

$$\theta \frac{dH}{dx} = H(H^2 - 1) \frac{\theta}{u_e} \frac{du_e}{dx} + \frac{H - 1}{2} [(H - 1)F - HC_f] \tag{14}$$

Equations (11) and (14), together with a skin-friction law, for example, the Ludwig-Tillman Law

$$C_f = 0.246 \exp(-1.561H)Re_\theta^{-0.268} \tag{15}$$

provide the necessary three equations. The system of ordinary differential equations is then solved by any standard integration routine, for example, Runge-Kutta, or Adams Moulton. For examples of these schemes any standard text in boundary-layer theory or the 1968 Stanford Conference Proceedings should be considered. The turbulence information is contained in equations (14) and (15) which reflect correlations of experimental data.

In reference [11], Green extended this method to compressible flows, and in reference [12], Green and his coworkers incorporated an ordinary differential equation, based on the turbulence energy equation, to describe the entrainment rate. This latter method, the Lag-Entrainment Method, is an effort to incorporate flow history into the calculation. It is a reasonably successful procedure for some types of flows and is in fairly common use today.

The advantages of methods of this type are computational simplicity and the ease with which the turbulence equations can be modified to incorporate new empirical information, that is, by simply introducing a different correlation of the skin-friction coefficient. The principal disadvantage is a lack of flexibility in dealing with flows even slightly different from those for which the correlations were developed. That is to say, when a new flow situation is encountered new data correlations obtained in like flows must be used.

Differential methods or, more properly, difference methods introduce substantially more flexibility in their description of the variation of the mean flow, but require more detailed information about the turbulence. The balance of the turbulence modeling techniques discussed here will be those appropriate to differential mean-flow descriptions.

B. Eddy-Viscosity Models

For boundary-layer flows it is generally agreed that the Reynolds normal stresses $\overline{u'^2}$ may be ignored in comparison with the Reynolds shear stresses $\overline{u'v'}$, except perhaps in the neighborhood of the separation point. This agreement is based on a combination of experimental data and general unwillingness to complicate the problem further by considering Reynolds normal stresses. If we argue that the turbulent shear stress should be defined in an equivalent manner to the laminar shear stress, we can propose a new turbulent viscosity. Applying this argument to the two-dimensional, incompressible boundary-layer equations, defined above, we can write after Boussinesq

$$\overline{-u'v'} = \varepsilon \frac{\partial \bar{u}}{\partial y} \tag{16}$$

where ε is the so-called eddy or turbulent viscosity. Note that this viscosity is a property of the fluid motion and not a physical property of the fluid itself, that is,

$$\varepsilon = \frac{\overline{-u'v'}}{\partial\bar{u}\partial y} \tag{17}$$

Substituting this into the momentum equation we obtain

$$\bar{u}\frac{\partial\bar{u}}{\partial x} + \bar{v}\frac{\partial\bar{u}}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial}{\partial y}\left(1 + \frac{\varepsilon}{\nu}\right)\frac{\partial\bar{u}}{\partial y} \tag{18}$$

From dimensional considerations and by analogy with kinetic theory, the eddy viscosity is proportional to the product of a length scale and a velocity scale. The various assumptions that go into defining these scales and the type and number of equations used for this purpose establish a classification scheme within the class of eddy-viscosity methods.

1. ZERO-EQUATION MODELS

By zero-equation models is meant that the model requires no partial differential equations to describe the eddy viscosity. The current most commonly used algebraic eddy viscosity model is that due to Cebeci and Smith [13], with subsequent improvements by Cebeci [14]. This is a composite model, in keeping with the experimental observations embodied in the similarity laws noted earlier, that is,

$$\varepsilon = \min\begin{cases}\varepsilon_i \\ \varepsilon_o\end{cases} \tag{19}$$

In the inner region, the length scale is taken as proportional to the distance from the wall, with a damping term near the wall due to van Driest [15], and the velocity scale is taken as this distance times the normal gradient of velocity, that is,

$$\varepsilon_i = \ell \cdot \ell \left|\frac{\partial\bar{u}}{\partial y}\right|$$

where

$$\ell = 0.4y\left(1 - e^{-y^+/A^+}\right)$$

is the length scale with near boundary damping, and where

$$A^+ = 26(1 - 11.8p^+)^{-1/2}$$

$$p^+ = \frac{-\nu}{\rho u_\tau^{\ 3}} \frac{dp}{dx}$$

The term p^+ was introduced to fit data better in pressure gradients. This results in an inner eddy viscosity of the form

$$\varepsilon_i = \left[0.4y\left(1 - e^{-y^+/A^+}\right)\right]^2 \left|\frac{\partial \bar{u}}{\partial y}\right| \frac{\partial \bar{u}}{\partial y} \tag{20}$$

In the outer region the length scale is taken as proportional to the boundary-layer thickness and the velocity scale is taken as the so-called velocity defect. These definitions can be combined as

$$\varepsilon_o = 0.0168 \int_o^\delta (u_e - u)dy \tag{21}$$

In figure 3 we show a comparison of the predicted and measured velocity distributions in physical variables, corresponding to the same cases shown in similarity variables in figure 2. It can be noted that presentation of data in these different forms emphasizes different regions of the boundary layer, and can provide additional insight into the physics of the flow. It can be seen that the calculations based on the zero-equation model provide a good representation of the zero and favorable pressure-gradient data while missing the adverse pressure-gradient results rather badly. This failure appears as a significant underprediction of the boundary-layer thickness coupled with an underprediction of the wall-shear stress. Failure in the accurate prediction of flows with strong adverse pressure gradients is a major shortcoming of these methods. As shown in figure 4 the skin friction as predicted with the Cebeci-Smith model is compared with the experimental data of Simpson, Chew, and Shivaprasad [16]. This flow is essentially a two-dimensional diffuser with a strong adverse pressure gradient leading to separation. The comparison is typical of algebraic eddy-viscosity models incorporated in a standard boundary-layer code in the neighborhood of separation. In these cases, the comparison is confused by the appearance of an additional complication, the separation-point singularity, which renders the mean-flow equations invalid at the separation point. This point will be discussed further in a subsequent section.

From the computational point of view, models of this type are the simplest models that satisfy the requirements of a robust difference solution to the boundary-layer equations. If such models are to be used within a Navier-Stokes formalism, some ad hoc assumption must be made regarding the location of the boundary-layer edge and the definition of the velocity there. These parameters lack a clear definition in any situation in which the normal derivative of velocity does not decrease monotonically as the outer flow is approached (cf. refs. [17] and [18]).

2. ONE-EQUATION MODELS

As noted above, the goal of turbulence modeling is to produce the universal model. To redress the shortcomings of zero-equation models it seems plausible to require that the scaling parameters of the turbulence be based on a property of the turbulence rather than on a property of the mean flow. In the mid 1960s Glushko [19] took a step in this direction by proposing that the velocity scale should be the square root of the kinetic energy of the turbulence, while retaining an algebraic length scale. The turbulence energy equation may be written (ref. [20]) as

$$\frac{\partial q^2/2}{\partial t} + \bar{u}_\ell \frac{\partial q^2/2}{\partial x_\ell} + \overline{u_i' u_\ell'} \frac{\partial \bar{u}}{\partial x_\ell} + \frac{\partial}{\partial x_\ell}\left(\overline{pu_\ell} + \frac{q^2 u_\ell'}{2}\right) = \nu u_i' \overline{\frac{\partial u_i}{\partial x_\ell}^2} \tag{22}$$

The various terms in this equation, from left to right, are the rate of increase of turbulence energy, the increase of this energy due to the convection by the mean flow, the production of turbulence energy, the transport of turbulence energy by turbulent and pressure diffusion, and, finally, a term that accounts for the dissipation of turbulence energy into heat. In attempting to solve this equation, the first three terms can be allowed to stand as they are while the latter two terms, comprising the turbulent and pressure diffusion and the dissipation, must be modeled.

Rubesin [21] gives a clear presentation of the original Glushko model and extends it to compressible flow. In addition, he introduces the elliptic terms required for consistency with a full Navier-Stokes solution. The Glushko model for steady mean flow, including the modeled terms, may be written as

$$\bar{u} \frac{\partial q^2/2}{\partial x} + \bar{v} \frac{\partial q^2/2}{\partial y} = \nu \frac{\partial}{\partial y} [1 + \bar{\varepsilon}(\lambda r)] \frac{\partial q^2/2}{\partial y} + \nu \bar{\varepsilon} S_{ij} \frac{\partial u_i}{\partial x_j}$$

$$- C\nu[1 + \bar{\varepsilon}(\lambda r)] \frac{q^2}{2\ell^2} \tag{23}$$

where

$$\frac{q^2}{2} = \frac{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}}{2} \qquad \text{the turbulence kinetic energy}$$

$$\bar{\varepsilon} = \frac{\varepsilon}{\nu} \qquad \text{the dimensionless turbulent viscosity}$$

$$S_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad \text{the mean strain rate}$$

and ℓ is the turbulence length scale. To complete the model the relations

$$\left.\begin{aligned}
r &= \frac{\sqrt{q^2/2}\,\ell}{\nu} \\
\bar{\varepsilon} &= H(r)\alpha r \\
H(r) &= \begin{cases} \dfrac{r}{r_o} & 0 \le \dfrac{r}{r_o} < 0.75 \\[2ex] \dfrac{r}{r_o} - \left(\dfrac{r}{r_o} - 0.75\right)^2 & 0.75 \le \dfrac{r}{r_o} < 1.25 \\[2ex] 1 & 1.25 \le \dfrac{r}{r_o} \end{cases} \\
\frac{\ell}{\delta} &= \begin{cases} \dfrac{y}{\delta} & 0 \le \dfrac{y}{\delta} < 0.23 \\[2ex] \dfrac{y/\delta + 0.37}{12.61} & 0.23 \le \dfrac{y}{\delta} < 0.57 \\[2ex] \dfrac{1.48 - y/\delta}{2.42} & 0.57 \le \dfrac{y}{\delta} < 1.48 \end{cases}
\end{aligned}\right\} \qquad (24)$$

where

$$\alpha = 0.2 \qquad C = 3.93$$

$$r_o = 110 \qquad \lambda = 0.4$$

were suggested by Glushko. He also proposed the boundary conditions

$$y = 0 \qquad q^2 = 0$$

$$y \to \infty \qquad q^2 = 0$$

This latter condition is required by the form of the equations at large y. The constants and functional relations were chosen to provide agreement with experimental results for flat-plate flows.

While this model is historically interesting, as the first real break from equilibrium concepts in a practical calculation scheme, it has a major conceptual shortcoming — that the length scale is still an algebraically defined quantity defined in terms of distance from the wall. For more geometrically complicated situations, for example, corner flows, backward-facing steps, etc., additional ad hoc relations for the length scale must be proposed.

From the computational point of view, this model, along with the other multi-equation models requires an implicit calculation scheme because of the large range of eigenvalues (i.e., stiffness) associated with these equations. Given a little care, however, serious computational difficulties can be avoided. One problem which cannot be avoided here is the need to specify an additional condition on the inflow boundary, that is, the initial turbulence energy profile. A procedure for obtaining such a profile (ref. [22]) is to start the calculation as a laminar flow with an initial energy distribution given by

$$\frac{q^2}{2} = \frac{q_o^{2*}}{2} \left(\frac{y}{y^*}\right)^2 \exp\left\{\frac{1}{2}\left[1 - \left(\frac{y}{y^*}\right)^2\right]\right\} \tag{25}$$

where $q_o^{2*}/2$ and y^*/δ are specified constants, and let the calculation simulate a transition process. Beckwith and Bushnell [22] conducted a parametric study on the effects of varying the initial energy and found that at too low a value the energy simply damps with increasing downstream distance. This procedure is satisfactory in the absence of any experimental data. Murphy and Rubesin [18] found that if experimental velocity profiles are to be matched in initiating a calculation, a slightly inconsistent energy profile can cause the solution to have an initial jump from the starting condition. This behavior is shown in figure 5 for the flat-plate data of Wieghardt (presented in ref. [5]).

3. TWO-EQUATION MODELS

A much more general eddy viscosity model can be proposed by the introduction of a differential equation to define the length scale as well as the velocity scale. This is particularly important when we consider flows such as those over a backward-facing step or over the trailing edge of an airfoil. In these cases the distance from the wall is not well defined and, if a zero- or one-equation model is being used, some ad hoc assumption must be made with regard to the length scale. Two-equation models provide more general models by permitting the same length-scale equation to be used, regardless of the flow configuration. The principal difficulty in proposing such a model is in arriving at the appropriate form for the new differential equation. Since mixing length is a wholly artificial concept it is necessary to identify the proposed length scale with some average-eddy size. Taking the position that all turbulence properties are probably transported in an analogous fashion, one can model the length-scale transport equation after the turbulence kinetic-energy equation. Some authors have chosen not to model the actual length scale, but rather a turbulence-field parameter which is related to it.

One of the earliest two-equation models and the one in most general use today is that due to Jones and Launder [23]. The second equation defines a new parameter, the dissipation, which is related to the length scale defined previously as $\ell = k^{3/2}/\varepsilon$. This model can be written as

$$\rho u_j \frac{\partial k}{\partial x_j} = \rho \tau_{ij} \frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_j}\left[\left(\mu + \frac{\mu_T}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] - \left[\rho\varepsilon + 2\mu\left(\frac{\partial k^{1/2}}{\partial x_{ij}}\right)^2\right]$$

$$\rho u_j \frac{\partial \varepsilon}{\partial x_j} = C_1 \frac{\varepsilon}{k} \tau_{ij} \frac{\partial u_i}{\partial x_j} + \frac{\partial}{\partial x_j}\left(\mu + \frac{\mu_T}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}\right) - C_2 \frac{\rho\varepsilon^2}{k} + \frac{2\mu\mu_T}{\rho}\left(\frac{\partial^2 u}{\partial x_2{}^2}\right)^2$$

where

$$\varepsilon = \mu_T = \frac{C_\mu \rho k^2}{\varepsilon} \qquad \sigma_k = 1$$

$$C_\mu = 0.09 \exp[-2.5/(1 + R_T/50)] \qquad \sigma_\varepsilon = 1.3$$

$$C_1 = 1.45 \qquad R_T = \frac{\rho k^2}{\mu\varepsilon}$$

$$C_2 = 2[1 - 0.3 \exp(-R_T{}^2)]$$

These constants and functional relations are obtained by matching experimental data in widely differing flow configurations. Note that we have retained the symbol k for turbulence energy in this case, since this model and its descendants are generally known as k-epsilon models.

As might be expected, this increase in generality is not obtained without some cost. In particular, the computational work required by these methods is substantial, and in both time-marching and steady-state calculations extreme care must be taken to prevent instabilities from propagating. These instabilities occur near surfaces where the turbulence-field variables are rapidly varying. The large variations in the scales of the problem occurring in this region is reflected in the computations as stiffness in the governing equations. In a recent paper, Viegas and Rubesin [24] overcame much of the numerical difficulty by defining wall functions to be satisfied by all the differential equations, for compressible flow, at small y. This procedure apparently makes a first-order improvement in terms of the ease with which these equations can be solved without degrading their accuracy. Some ad hoc assumptions must still be made, however, in flows where an unambiguous distance from the wall is not available, for example, the edge of a backward-facing step.

When the proper initial conditions are available for these higher-order methods their accuracy is, in general, comparable to that of the Cebeci-Smith model for flows for which the latter works well (fig. 5). In addition, they can be used in cases for which simpler models would require many changes. The cost of these methods is significant, however, and one should always use the simplest method consistent with the results sought. For a recent review of the performance of the higher-order methods Marvin's discussion in reference [25] is useful.

V. REYNOLDS STRESS MODELS

All of the preceding results effectively ignore the question of the basic validity of the eddy-viscosity concept. Experimental results obtained over the last 20 years or so have shown that while eddy viscosity is a convenient concept from the computational viewpoint, it has little physical basis. This should come as no surprise since the fundamental assumption of the arguments based on kinetic theory of the eddy-viscosity concept requires that, analogous to the mean free path, some measure of the typical eddy size be small compared with the dimensions of the container. Since the largest eddies have a dimension of the same order as the boundary-layer thickness, it is not surprising that, under some conditions, the theory fails to provide good predictions. What is surprising is that the concept works so well as long as we restrict our attention to flow in zero and favorable pressure gradients.

A logical next step in the hierarchy of turbulence models is the Reynolds stress model. We will consider three models in this category reflecting the development of these methods over the last 15 years.

The earliest Reynolds stress model used routinely in computation was that due to Bradshaw, Ferris, and Atwell [26]. In their method it is assumed that the Reynolds shear stress is proportional to the turbulence kinetic energy. A form of the turbulence-energy equation is then solved for the Reynolds shear stress directly. As with the Glushko model, the diffusion and dissipation terms must be modeled. This was a complete break from the eddy-viscosity concept, and the direct link between Reynolds stresses and the mean field embedded in all the models discussed previously.

The equations for the Bradshaw model may be written as

$$\frac{1}{2}\rho\left(\bar{u}\frac{\partial q^2}{\partial x} + \bar{v}\frac{\partial q^2}{\partial y}\right) = \tau_{xy}\frac{\partial \bar{u}}{\partial y} - \text{DIFF} - \text{DISS}$$

and

$$\frac{\tau}{\rho q^2} = a_1$$

where DIFF is the turbulent diffusion term, modeled as

$$\text{DIFF} = \left(\frac{\tau_{max}}{\rho}\right)^{1/2} \frac{\partial}{\partial y} G\left(\frac{\tau}{\rho}\right)$$

and DISS is the dissipation term modeled as

$$\text{DISS} = \left(\frac{\tau/\rho}{L}\right)^{3/2}$$

and the empirical functions L and G are shown in figure 6. Note that in an equilibrium condition, Production = Dissipation, the dissipation length scale L is exactly analogous to the mixing length, and this equation can be used to define it. While this model is a significant conceptual advance over the Glushko model it suffers a similar shortcoming in terms of generality in that the length scale is still defined algebraically in terms of the distance from the wall.

A substantially more complicated model is the algebraic Reynolds stress model proposed by Rodi et al. [27]. This model uses the basic two-equation model of Jones and Launder as a starting point, but rather than defining an eddy viscosity, Rodi et al. chose to define the individual Reynolds stresses in terms of algebraic relations between the turbulence energy and the dissipation, or length scale. In the limit of equilibrium flow this method reduces to the usual Jones-Launder model since the Reynolds shear-stress equation contains the same constitutive relation for stress as does the original model.

In reference [28], Launder, Reece, and Rodi propose a full Reynolds stress model providing differential equations for the transport of all six of the Reynolds stresses, which must be solved simultaneously with the dissipation equation. These last two methods are still in the relatively early research phase and have seen only very limited application as of this time. As a result their usefulness and generality remain an open question. For the equations describing these methods the reader should examine the cited references.

VI. SEPARATED FLOWS

Separated flows have, so far, been the most difficult aerodynamic flows to predict accurately because 1) reliable experimental data for separated flow are extremely difficult to acquire, and 2) classical boundary-layer calculations fail at the separation point as a result of singularity in the equations. The rationale for considering them here is that these flows provide a sufficiently hard test for turbulence models to permit discrimination between various models. As was pointed out by Goldstein [29], the singularity in the boundary-layer equations is associated with the specification of the streamwise pressure gradient at the separation point. If some parameter other than pressure, for example, displacement thickness, skin friction, or some bounding stream function, is specified the boundary-layer equations are well behaved despite the presence of the separation.

Solutions obtained in this manner, so-called inverse solutions to the boundary-layer equations, have been compared with solutions to the Navier-Stokes equations for the same distributions of skin friction and have been found to reproduce all the relevant physical parameters as long as v/u remains small, that is, as long as the flow is "slender." The use of inverse boundary-layer methods as test beds for turbulence models appears to be an economical approach to the evaluation of these models in the vicinity of separation. Unfortunately, however, if the pressure is not that which was observed experimentally, no conclusions with regard to the specific shortcomings of the turbulence model can be inferred. A subterfuge was proposed by Arielli and Murphy [30], which permits one to specify both the additional boundary condition required to avoid the separation point singularity and the streamwise pressure distribution. This permits the specification of streamwise pressure distribution while retaining the economy associated with boundary-layer calculations.

Several investigators, using inverse boundary-layer methods and the full Navier-Stokes equations, have considered the problem of turbulent separation and have found that, in general, current models tend to provide too much mixing, that is, too large a value of skin friction, as the separation point is approached and too little mixing or too slow a rate of recovery of skin friction, downstream of reattachment. In an important aerodynamic flow situation, the shock-wave boundary-layer interaction, the separation point itself is reasonably well predicted, but since this is an inertially dominated flow, this should not be taken as evidence in favor of the turbulence model. In fact, a reasonable prediction of the behavior of the mean flow in this case can be determined from inviscid considerations alone (cf. ref. [31]).

Calculations of the flow of reference [16] (the flow in a converging-diverging-converging channel, simulating a subsonic diffuser, but forcing reattachment in the final converging section) using inverse boundary-layer and full Navier-Stokes procedures, and specifying the channel configuration rather than the free-stream velocity, predict the separation point to be in about the correct location, but predict a velocity gradient in the free stream to be about twice that observed experimentally. These independent calculations (refs. [32] and [33]) are consistent with each other and with the observation that the zero-equation models overpredict turbulent mixing in the approach to separation. Simpson and Collins [34] suggest that the failure of the predictions is caused by inadequate accounting for the Reynolds normal stresses in the neighborhood of separation. Since it is extremely difficult to obtain a truly two-dimensional flow in the neighborhood of a separation point, and equally difficult to obtain reliable measurements in the near-wall region, evidence to support or reject this hypothesis is lacking.

Two useful references on the general topic of separated flows are the AGARD Conference of Separated Flow in 1976 (ref. [35]) and the Project Squid Summary Report on Colloquium on Flow Separation in 1979 (ref. [36]).

VII. UNSTEADY AND THREE-DIMENSIONAL FLOWS

In considering unsteady flows we must introduce a new concept, that of ensemble averaging. In this case we find that averaging over long times will introduce a confusion between actual turbulent motion and at least some of the high-frequency components of the mean motion. To avoid this confusion, we can consider the averages to take place over some large number of successive realizations of the same event. For example, if we consider a sinusoidally varying mean flow we can take a data reading every time we reach the same point in the cycle, and then take an average of these readings over many hundreds of cycles. This procedure is termed ensemble or phase averaging.

Only within the last 5 years or so has the computational capability become available to permit routine calculation of unsteady and three-dimensional viscous flows. As a result, turbulence modeling for these flows has been almost exclusively an adaptation of the two-dimensional steady models discussed above. As in the steady case, predictions of unsteady flows in favorable and mildly adverse pressure gradients can be made with reasonable accuracy as long as the dimensionless frequency is not too high. At higher frequencies the existing experimental data are inadequate to permit a firm judgment to be made. References [37] and [38] provide an introduction to the literature of unsteady turbulent flows.

For truly three-dimensional flow (e.g., ref. [39]) too few calculations have been made to permit any general conclusions to be drawn. An example of some of these calculations is the work of Kussoy, Viegas, and Horstmann [40].

VIII. LARGE EDDY SIMULATION

Within the last 5 years, investigators with access to very large computers have been obtaining solutions to the three-dimensional, unsteady Navier-Stokes equation on a relatively fine mesh. These solutions, for low Reynolds number turbulent flows, have sufficient spatial and temporal resolution to capture at least some of the turbulent eddy structure. Moin et al. [41], as reported by Rubesin [42], have obtained solutions of a channel flow at a Reynolds number, based on channel half width, of 13,800. These conditions correspond to the experiment of Hussain and Reynolds [43]. In figure 7 (taken from ref. [42]), we show two velocity profiles corresponding roughly to the maxima of the positive and negative excursions in the instantaneous velocity, together with the long time average of the resolved velocities. As can be seen, the appearance of these profiles gives a quite plausible representation of a turbulent flow. In figure 8 the same long time averaged profile plotted on the usual turbulent similarity coordinates are shown. Despite the relative crudeness of the calculation, and the various assumptions regarding initial conditions and subgrid modeling, these results provide substantial hope for the ability to compute turbulence in the long term, and in the nearer term such calculations may provide substantial insight into the modeling of parameters which presently cannot be measured.

IX. SOME CONCLUDING REMARKS

In the foregoing we have considered only relatively simple flows of the type used to evaluate turbulence models. In a situation in which the reader is called upon to make use of the information contained here, the flow situation will almost certainly be much more complicated. Since one cannot provide the appropriate information for every application, it seems more appropriate to provide the reader with an appreciation of where we are with respect to turbulence modeling, and of how we got there. With this information and an acquaintance with the material in the works referenced here, it is hoped that the reader will be in a better position to make rational judgments when confronted with more complex problems. Some additional references which may be found useful are references [44]-[46].

We have not considered any of the difficulties associated with such complicating factors as heat transfer, mass transfer, and/or chemical reactions since the problem of turbulence is "sufficient unto the day." In addition to the references cited here the author has frequently found the following journals to be helpful: the several Transactions of the ASME, the International Journal of Heat and Mass Transfer, and the various publications of the AIAA. Workers in specialized fields other than aerodynamics will undoubtedly require and be familiar with different sources.

LIST OF SYMBOLS

A^+	function used in defining near-wall damping in Cebeci-Smith eddy viscosity relation (ref. [13])
C_f	skin friction coefficient, $\tau_w/(1/2)\rho_e u_o{}^2$
F	function defined in equation (3)
G	diffusion function in Bradshaw et al. model (ref. [26])
H	boundary-layer form factor, δ^*/θ
H_1	Head's boundary layer form factor, Δ/θ (ref. [10])
k	turbulence kinetic energy in Jones-Launder model (ref. [23])
L	dissipation length scale, model of Bradshaw et al. (ref. [26])
ℓ	mixing length
p	pressure
p^+	dimensionless pressure gradient parameter in A^+
$q^2/2$	kinetic energy of turbulence
Re	Reynolds number
T	a time period, long relative to the turbulent motion period, but short compared with mean motion period
t	time variable
u,v,w	velocities
u_τ	shear velocity
x,y,z	spatial coordinates
y^+	a wall unit Reynolds number
δ	boundary-layer thickness
δ^*	displacement thickness = $\int_o^\delta 1 - (Au/\rho_e u_o)dy$
Δ	Head's parameter = $\delta - \delta^*$ (ref. [10])

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FIGURE CAPTIONS

FIG. 1. History of instantaneous velocity. a) Hot-wire signal. b) Reynolds decomposition.

FIG. 2. Comparison of experimentally determined velocity; profile with the Law of the Wall-Law of the Wake. a) Strong adverse pressure gradient (data of Ludwig-Tillman); ID 1200, x = 2.782 m. b) Zero pressure gradient (data of Wieghardt); ID 1400, x = 3.487 m. c) Strong favorable pressure gradient (data of Herring and Norbury); ID 2800, x = 2.0 ft.

FIG. 3. Comparison of experimental and predicted velocity profiles. a) Strong adverse pressure gradient. b) Zero pressure gradient. c) Strong favorable pressure gradient.

FIG. 4. Comparison of experimentally determined skin friction coefficient with the prediction of an algebraic eddy viscosity model.

FIG. 5. Comparison of experiment skin friction of a flat plate with predictions using three eddy viscosity closure models.

FIG. 6. Length scale and dissipation functions for the model of Bradshaw et al. (ref. [26]).

FIG. 7. Instantaneous velocity profiles and Reynolds averaged velocity profiles from large eddy simulation calculation.

FIG. 8. Reynolds averaged velocity profile of Fig. 7 compared with the experimental data of Hussain and Reynolds (ref. [43]).

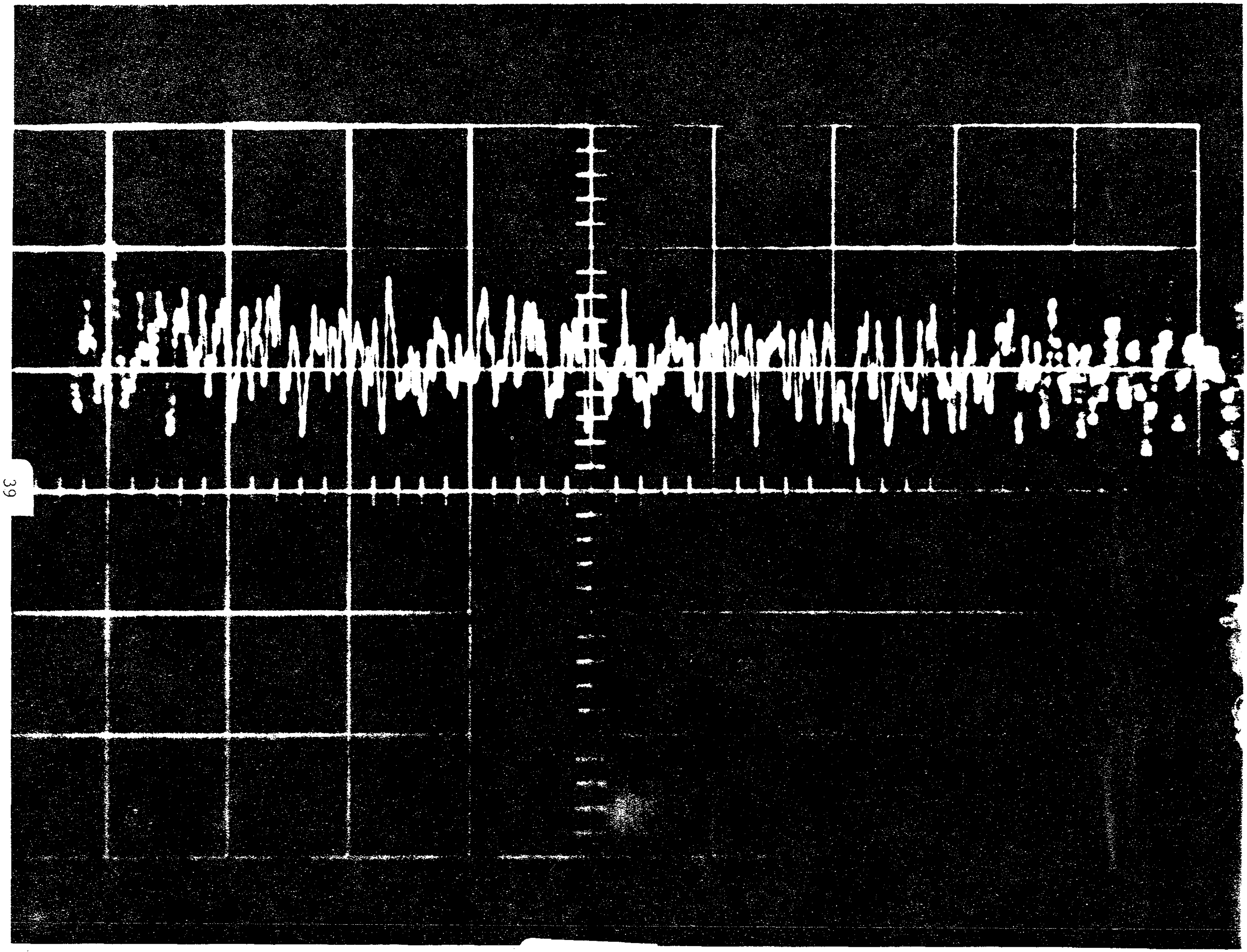


Fig. 1a

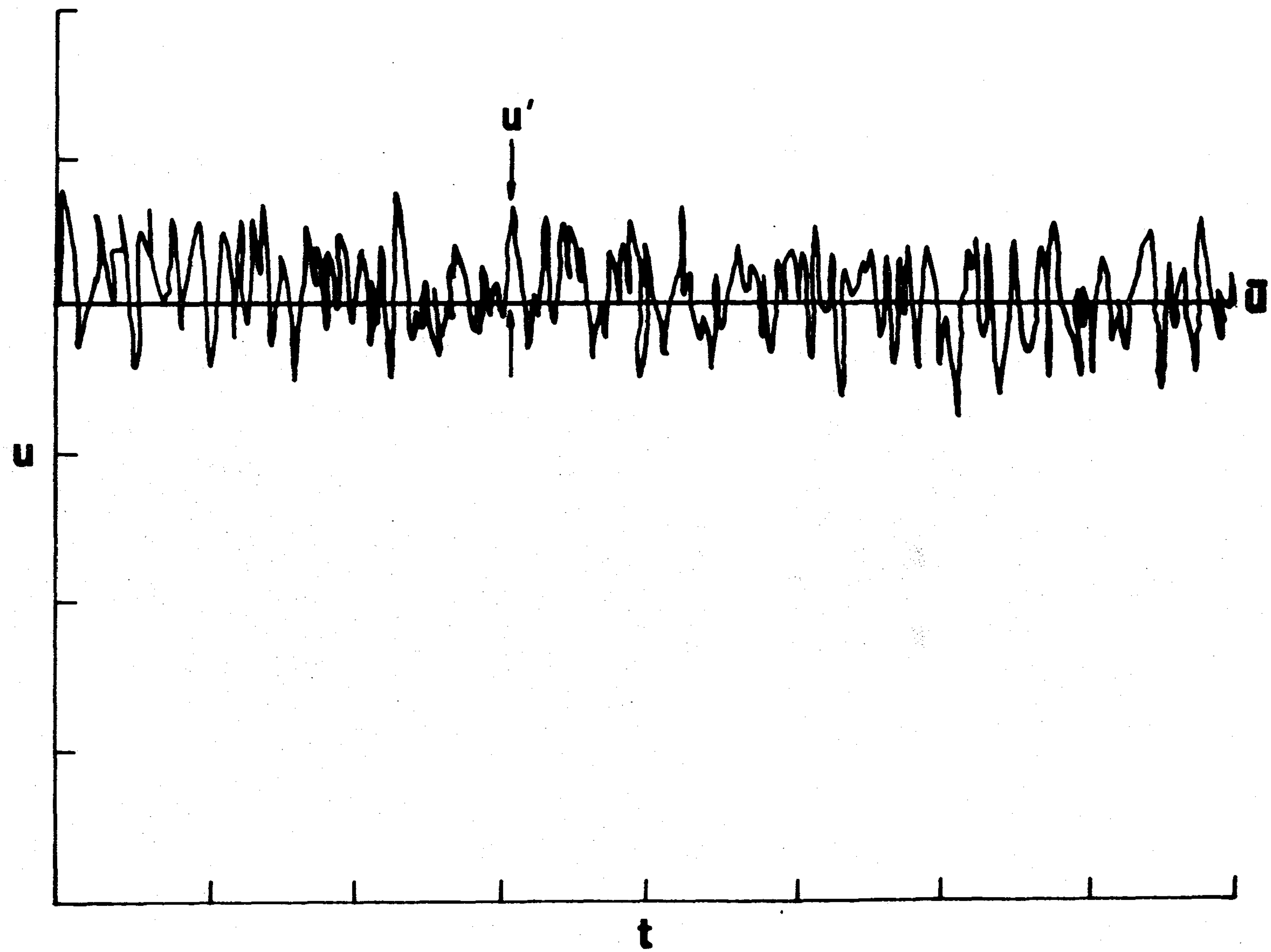


Fig. 1B

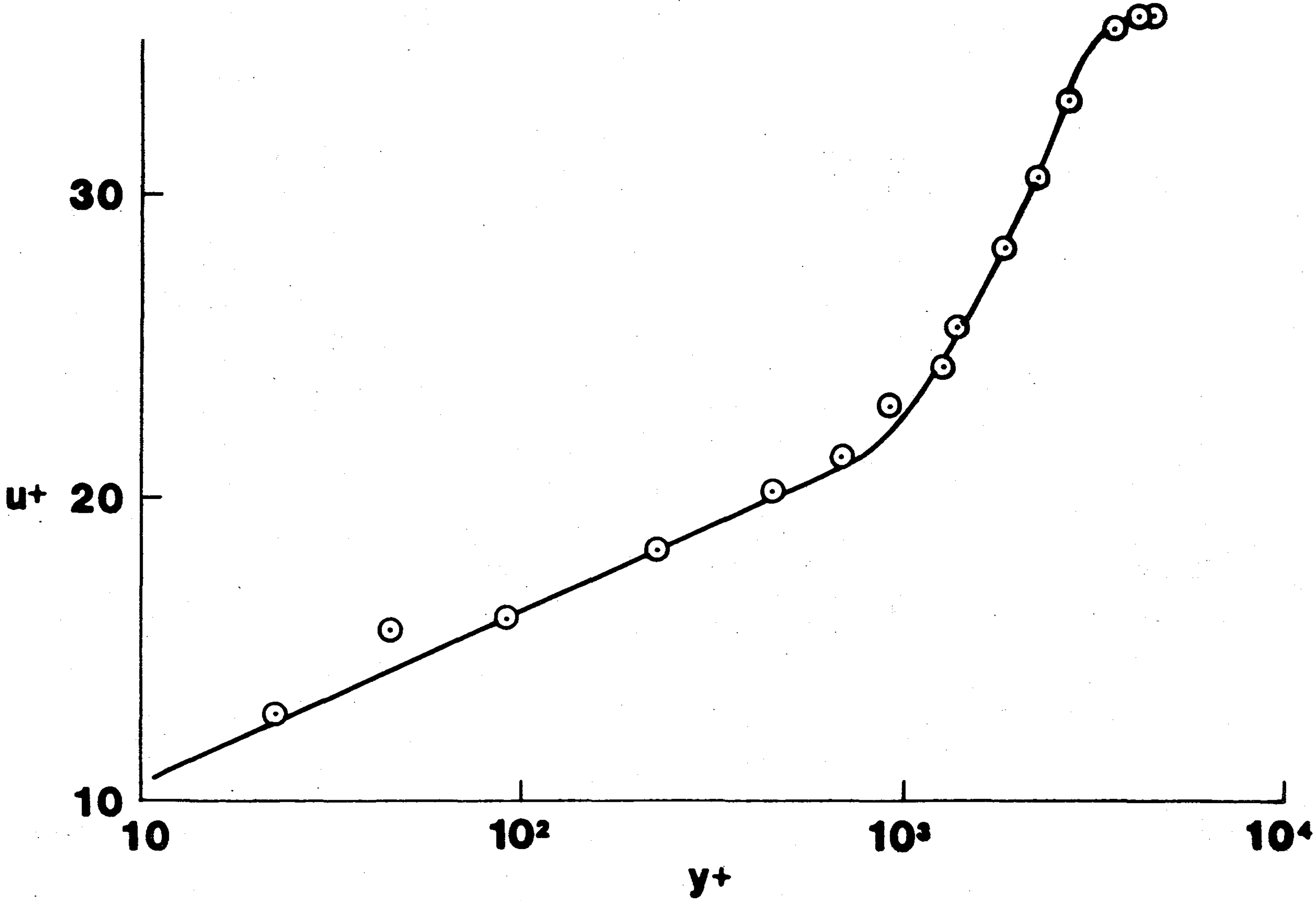


Fig. 2a

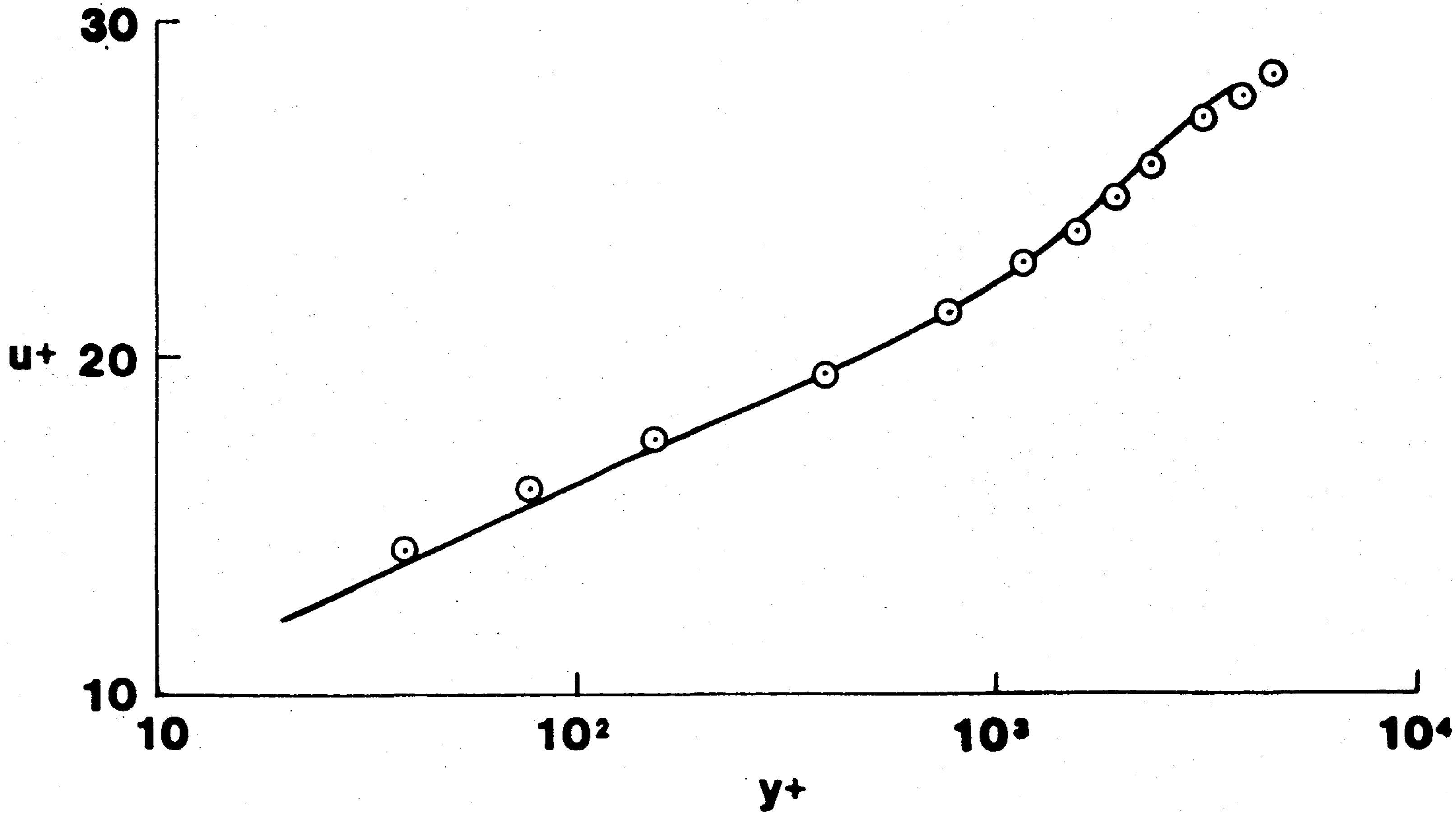


Fig. 2b

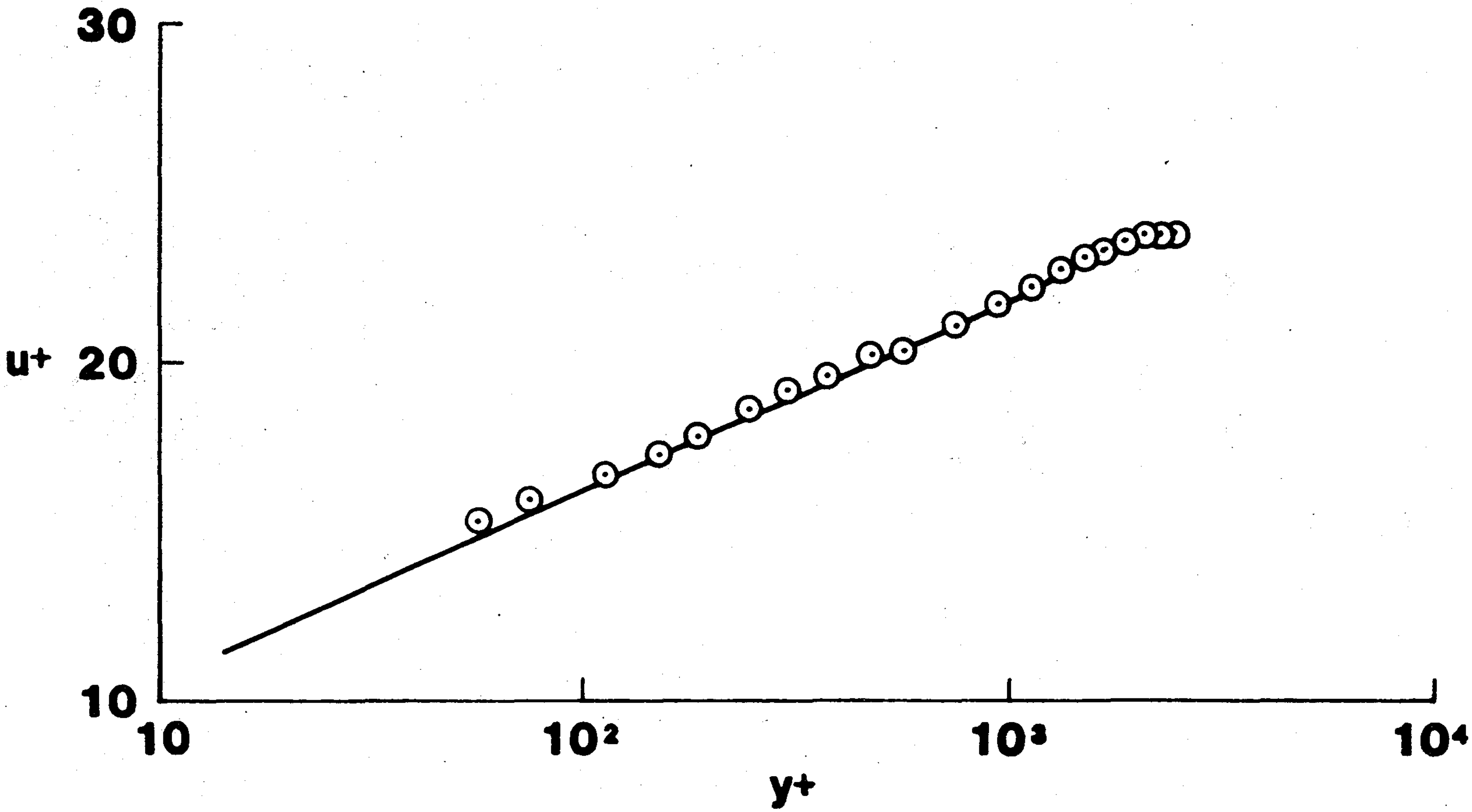
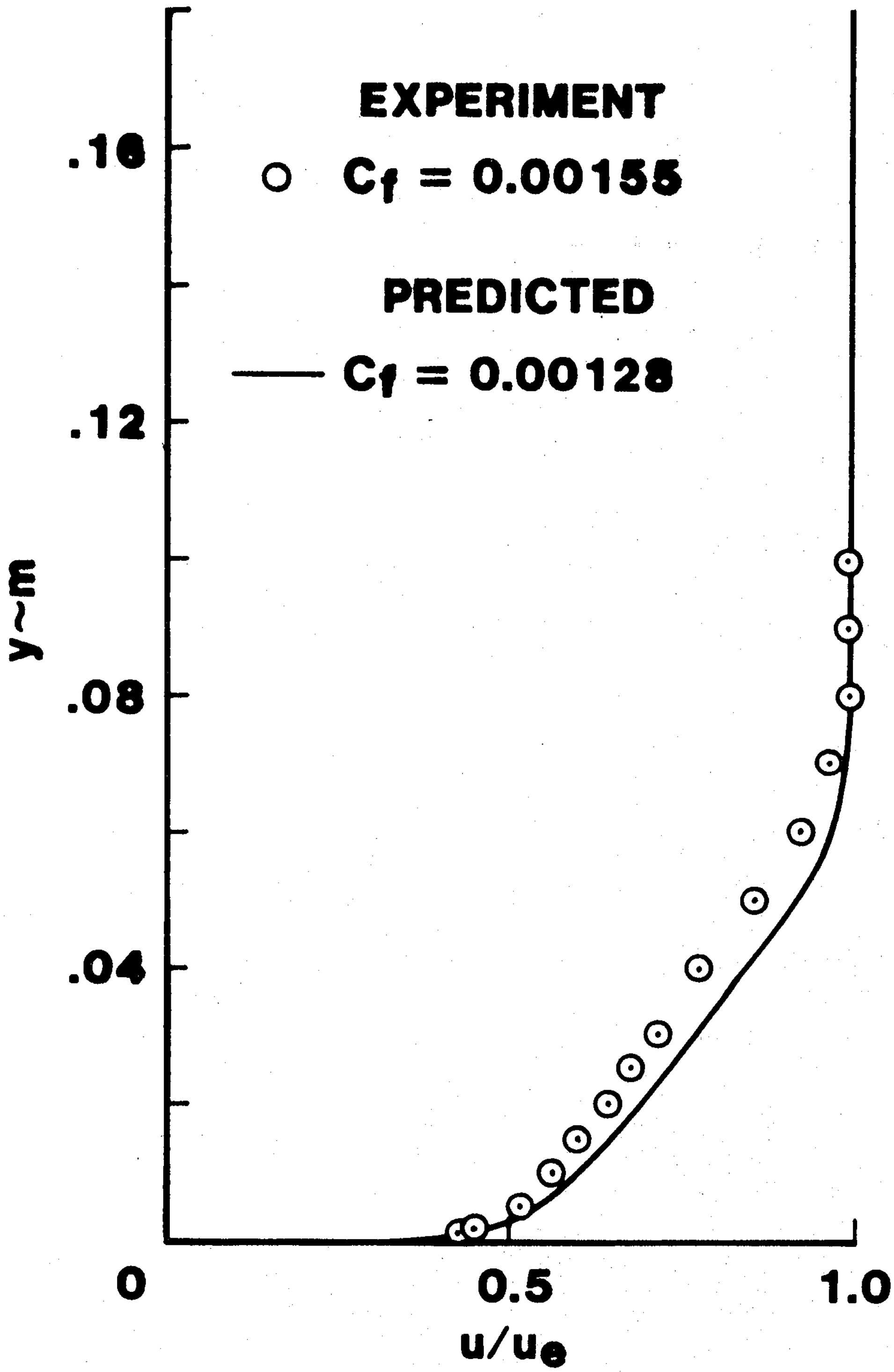


Fig. 2c


EXPERIMENT
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PREDICTED
—— C_f = 0.00128
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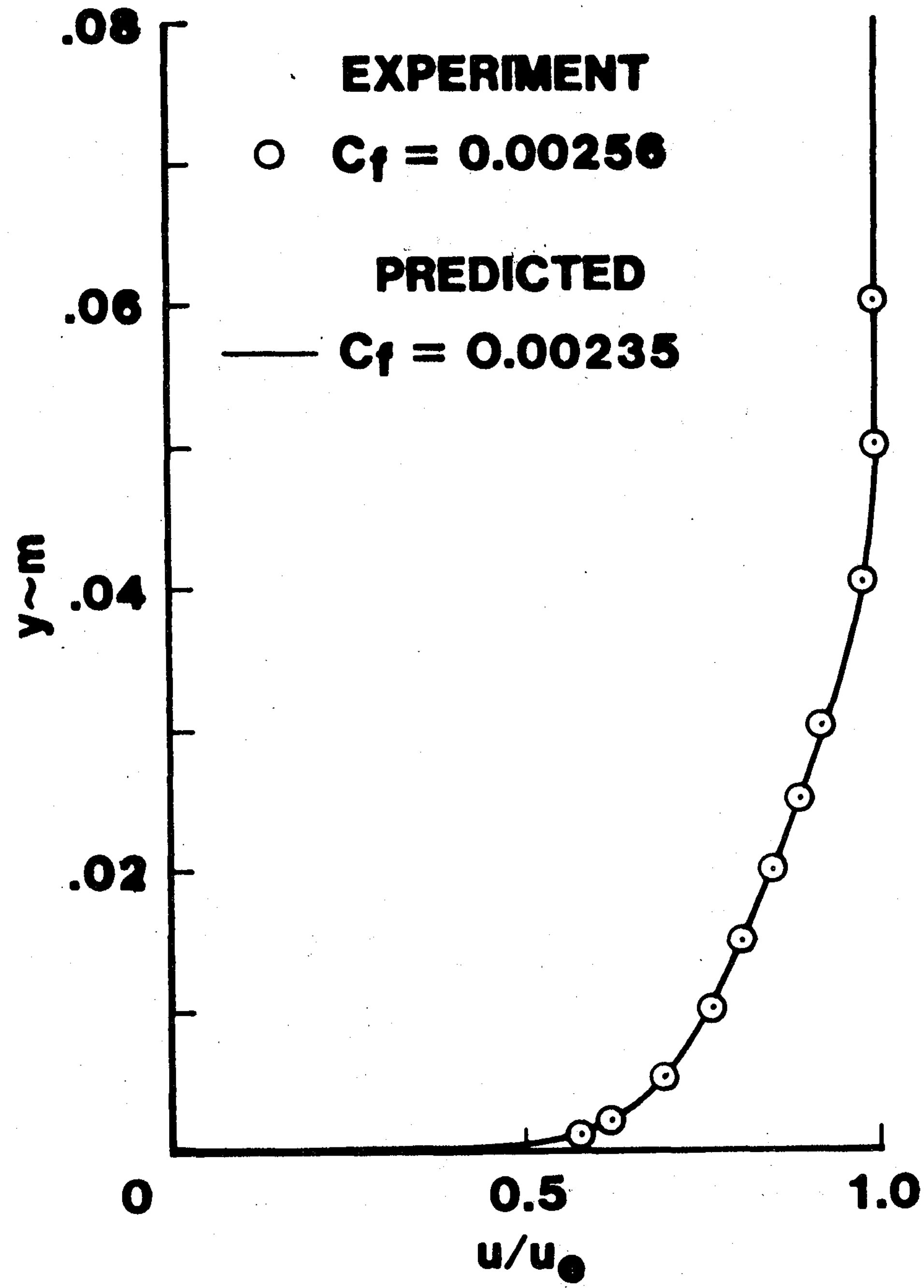
.12
.08
.04
y~m
0
0.5
1.0
u/u$_e$

Fig. 3a


.08
.06
.04
.02
y~m
EXPERIMENT
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PREDICTED
—— Cf = 0.00235
0
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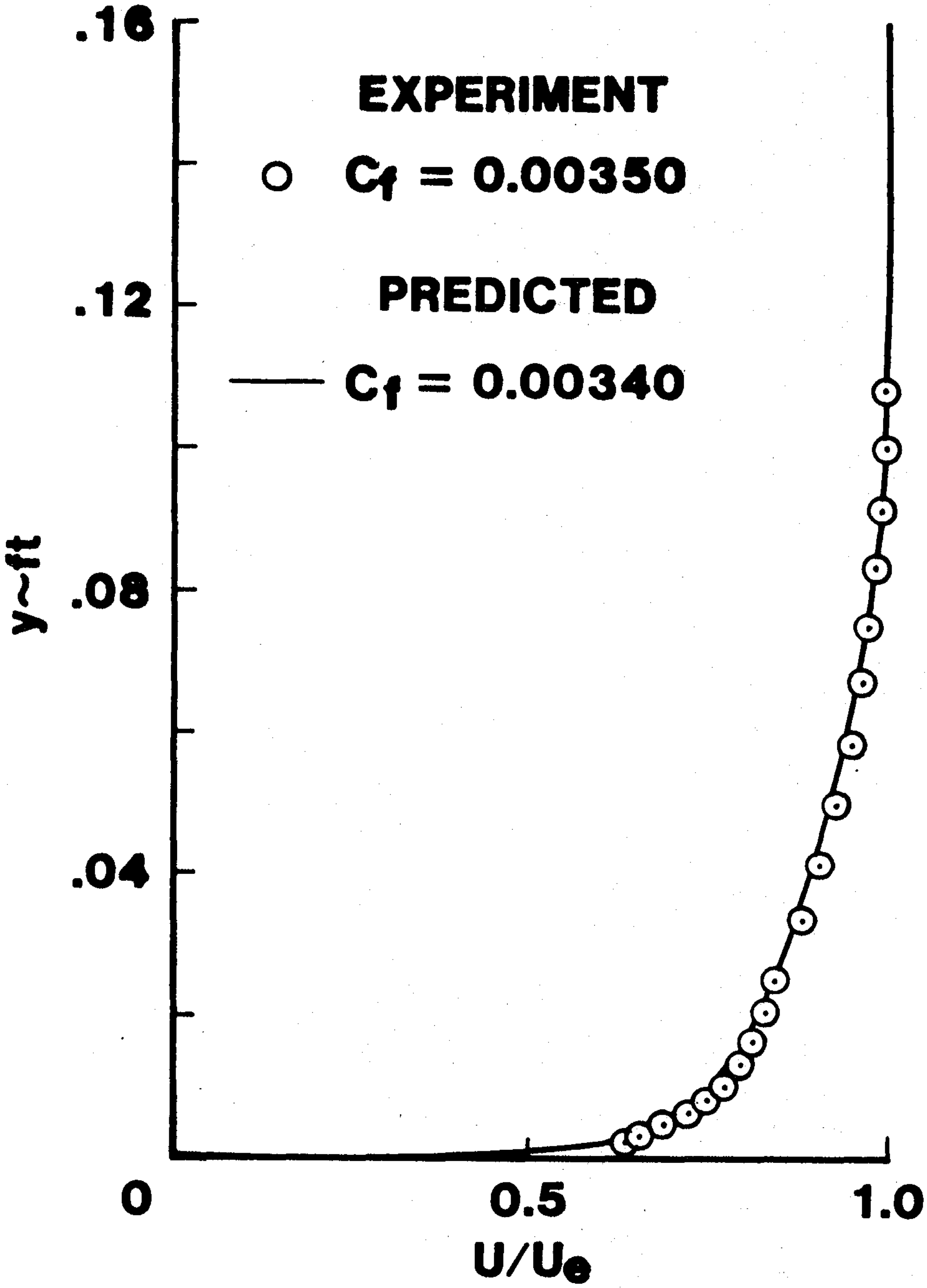
1.0
u/ue

Fig. 3b



.16
.12
.08
.04
y~ft
EXPERIMENT
○ Cf = 0.00350
PREDICTED
Cf = 0.00340
0
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1.0
U/Ue

Fig. 3c

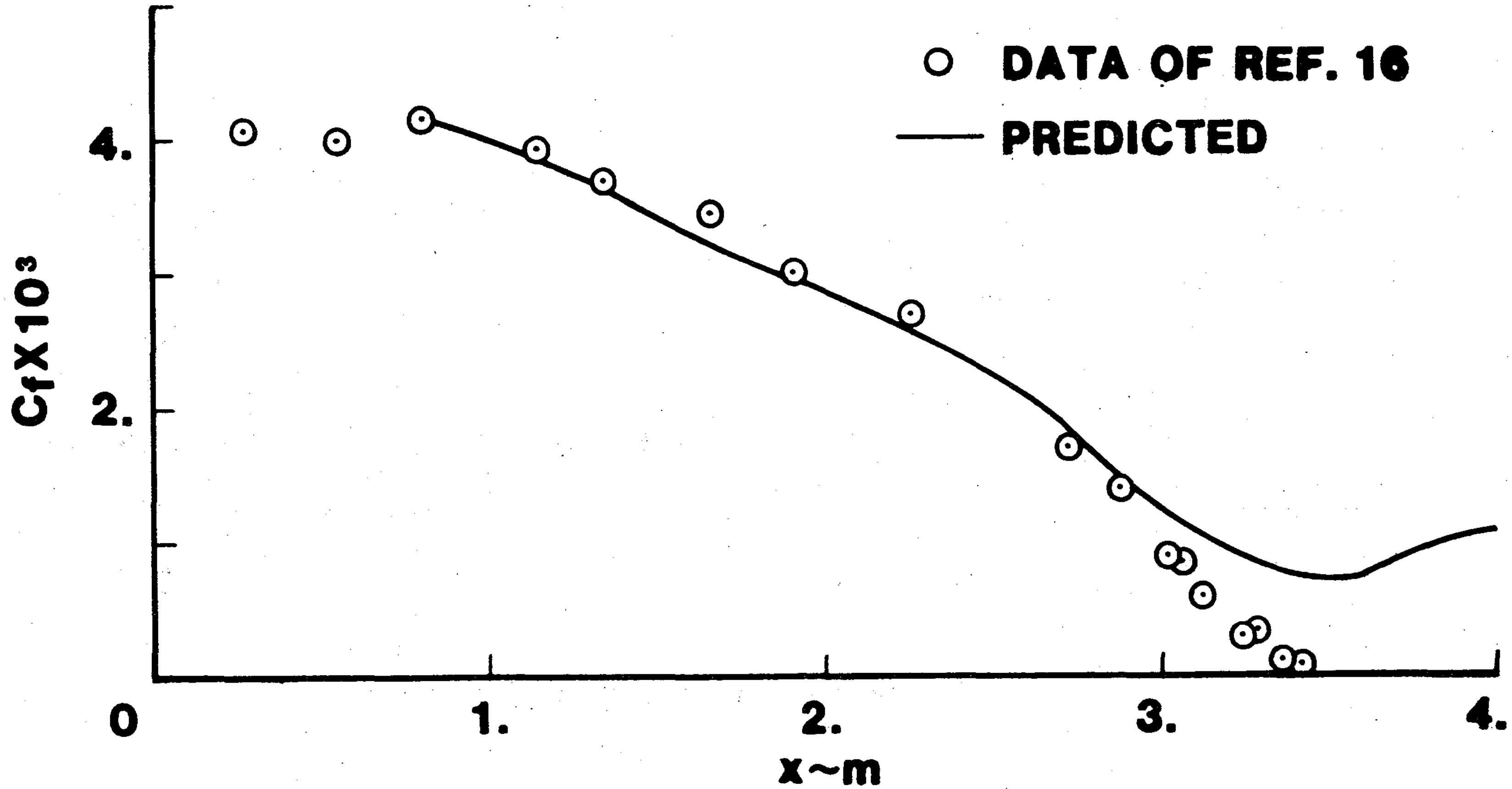
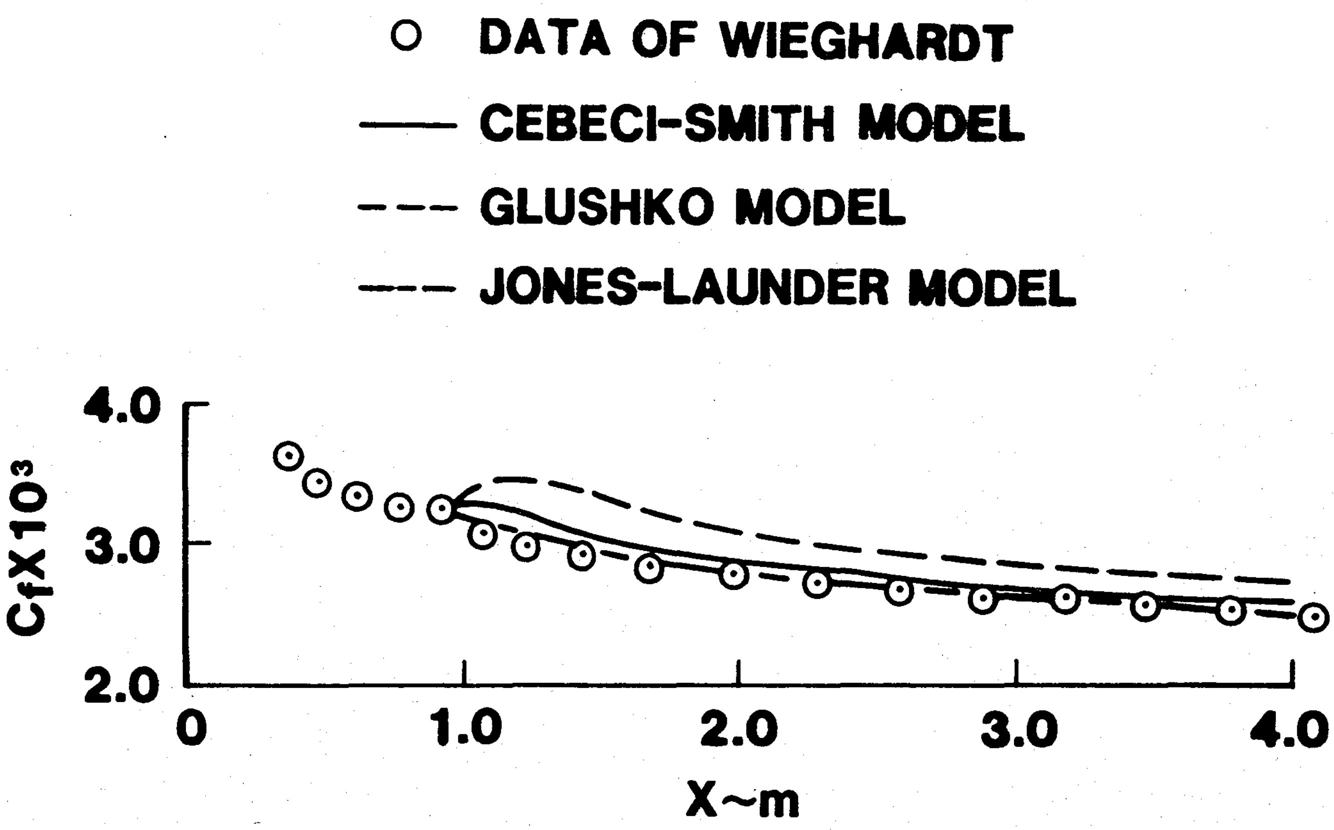


Fig. 4


DATA OF WIEGHARDT
CEBECI-SMITH MODEL
GLUSHKO MODEL
JONES-LAUNDER MODEL
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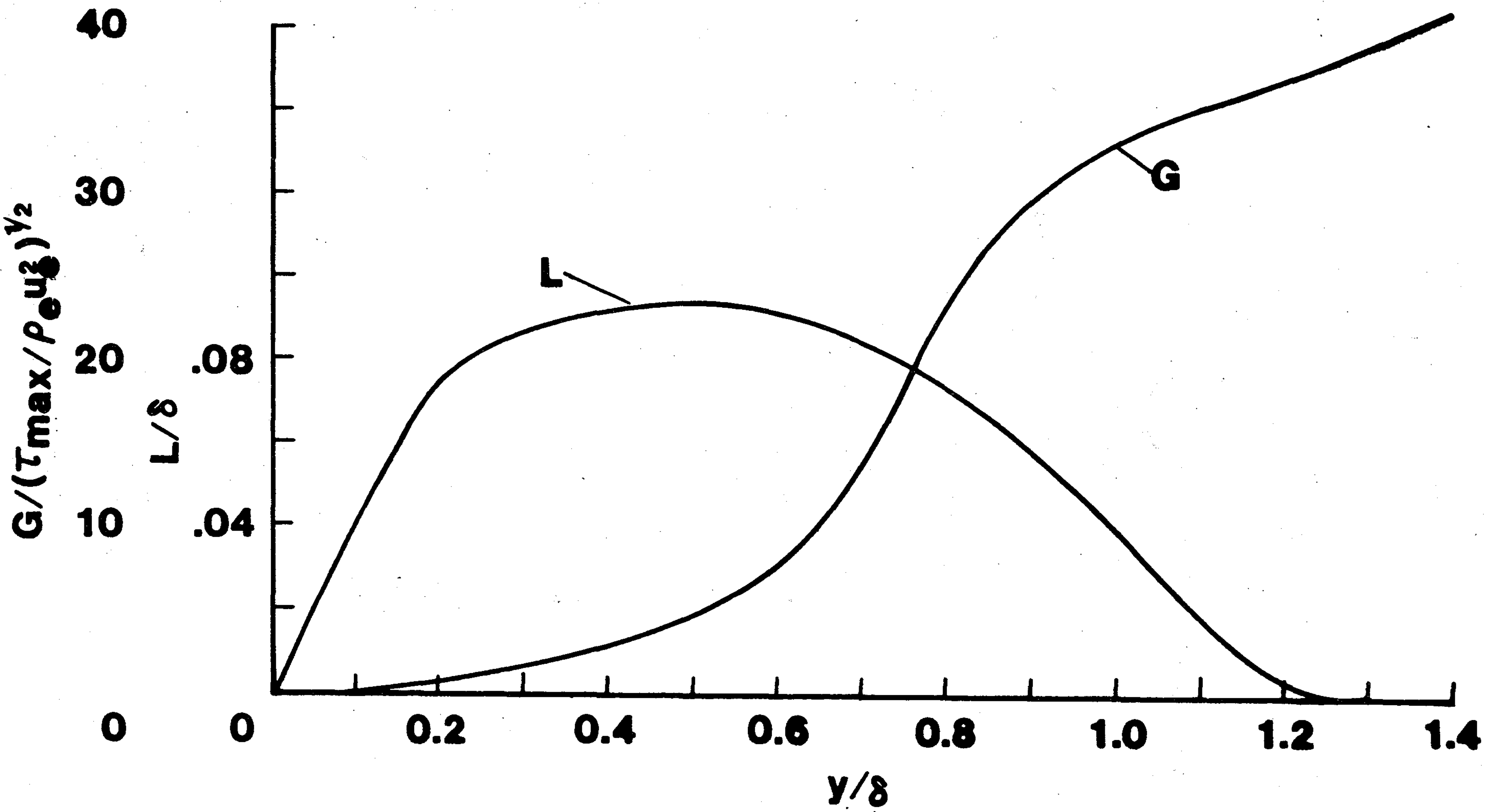
3.0
2.0
$C_f X 10^3$
0
1.0
2.0
3.0
4.0
X~m

Fig. 5



40
30
20
10
0
.08
.04
0
$G/(\tau_{max}/\rho_e u_e^2)^{1/2}$
L/δ
L
G
0.2
0.4
0.6
0.8
1.0
1.2
1.4
y/δ

Fig. 6

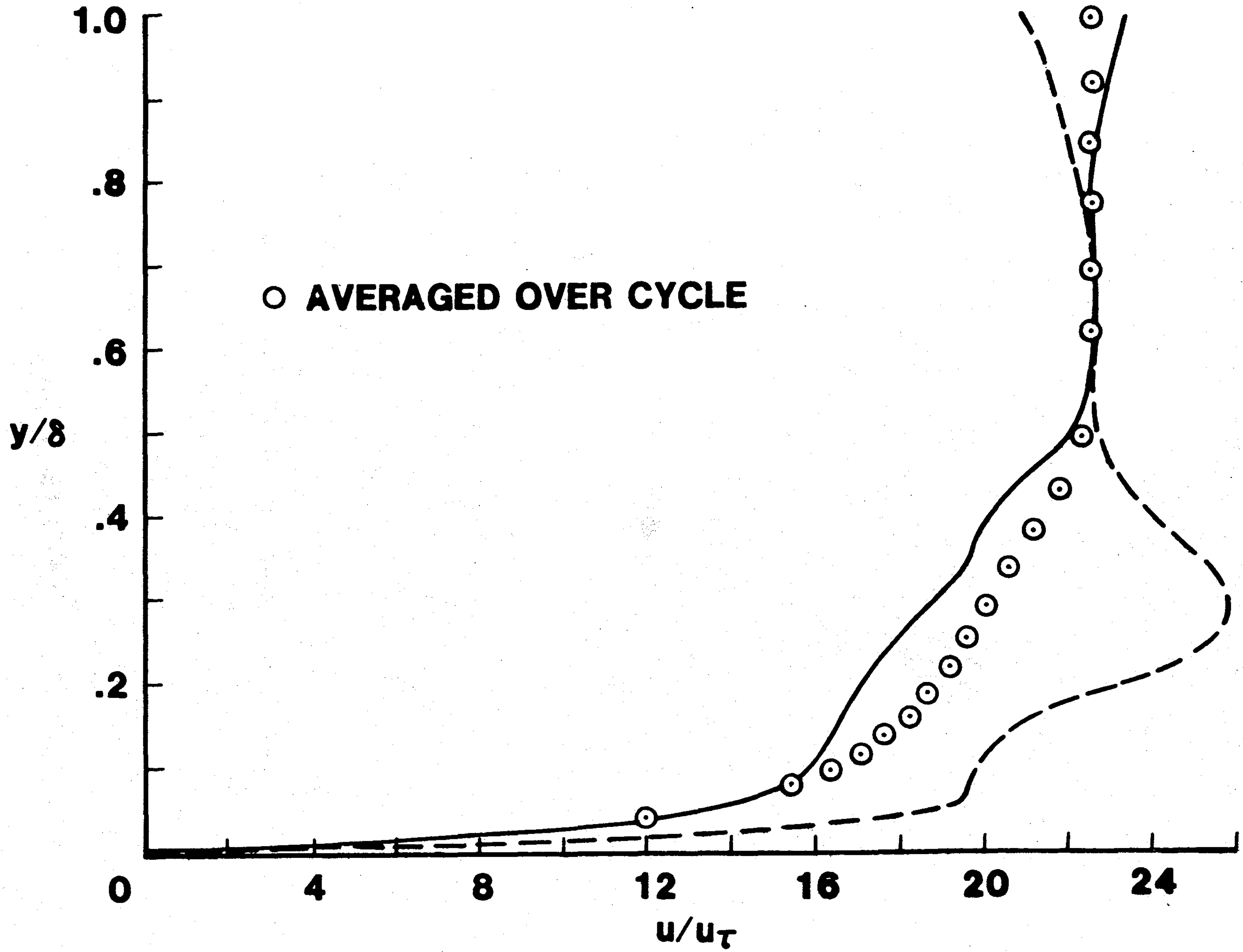


Fig. 7

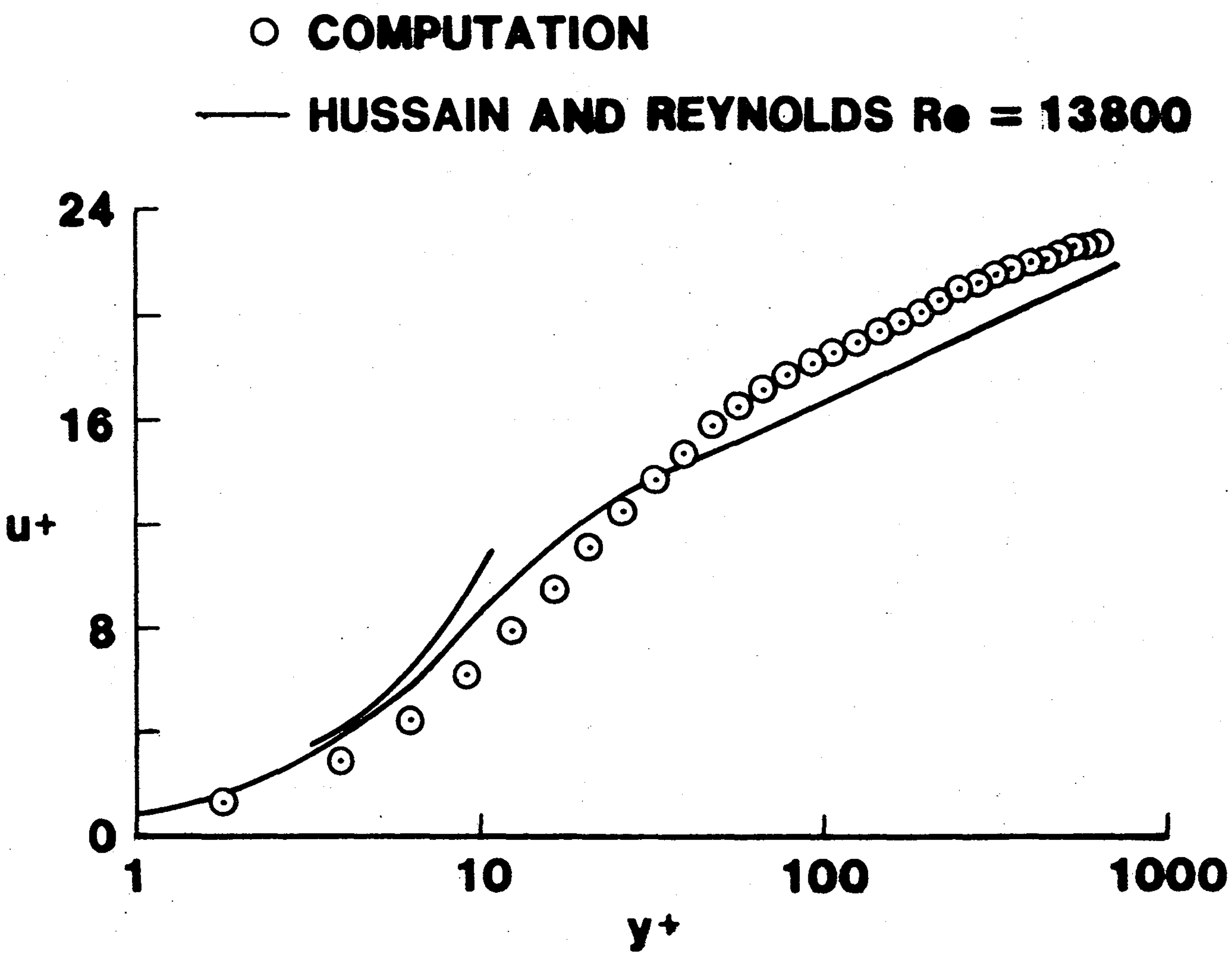

O COMPUTATION
—— HUSSAIN AND REYNOLDS Re = 13800
24
16
u+
8
0
1
10
100
1000
y+

Fig. 8

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15. Supplementary Notes

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16. Abstract

The following is a survey article designed to provide an introduction to the subject of turbulence modeling, and to explain the need for such models.

The subject is developed along chronological lines since this provides a logical development plan and also because it then moves from relatively simple phenomenological models through more complicated procedures and ultimately to the subject of large-eddy simulation.

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